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Chiral spin liquids

Bela Bauer

Based on work with:

- Lukasz Cinco & Guifre Vidal (Perimeter Institute)
- Andreas Ludwig & Brendan Keller (UCSB)
- Simon Trebst (U Cologne)
- Michele Dolfi (ETH Zurich)

- *Nature Communications 5, 5137 (2014)*
- *arXiv:1506.03351*

Chiral Spin liquids

Everything has been said!

Chiral spin liquids: numerical detection of universal behavior using entanglement

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- *Nature Communications 5, 5137 (2014)*
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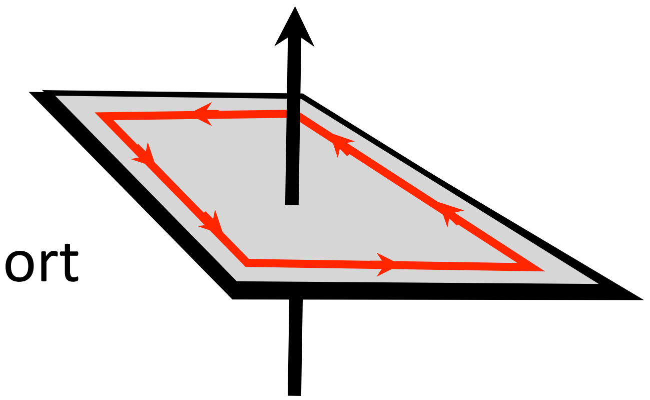
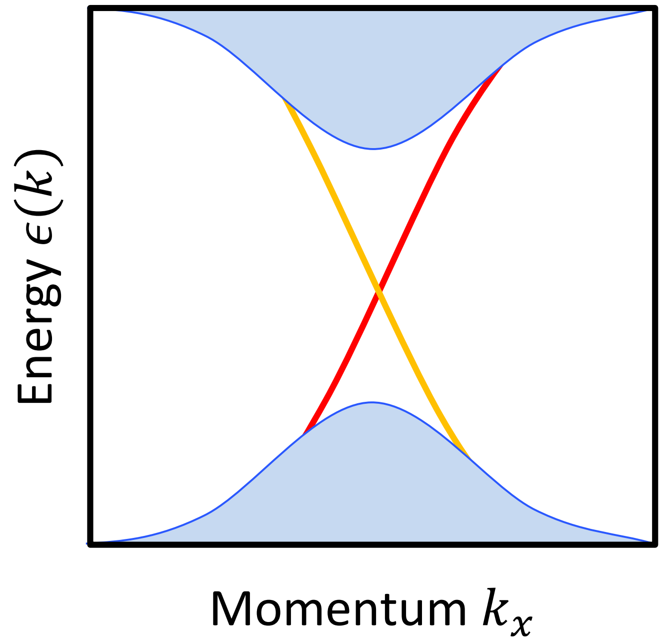
What is left to do?

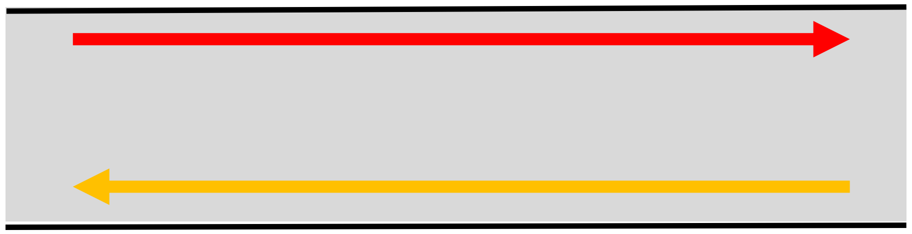
1. Reminder: what is a CSL?
2. An example: universal behavior of the $\nu = 1/2$ Laughlin state
3. Numerical detection using iDMRG
4. The semion, and the CSL as an SET
5. Possible phase diagrams on the Kagome lattice, and other speculations

Chiral edges

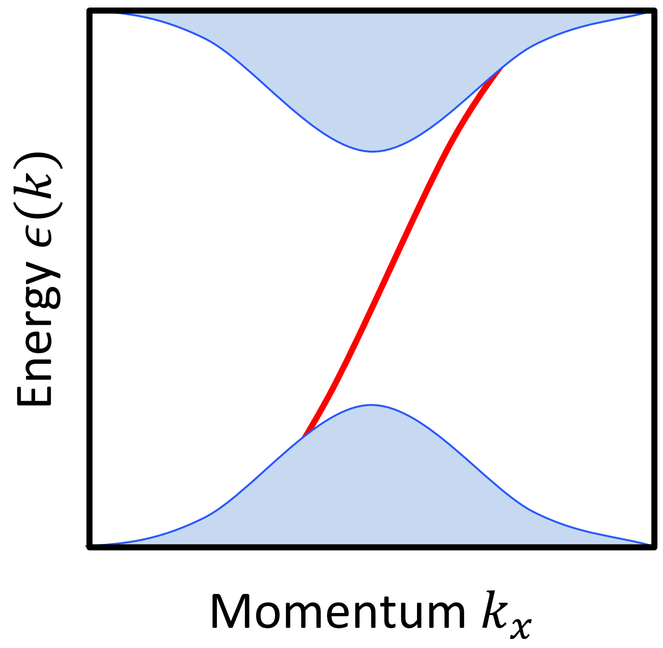
QHE: *2d* electron gas in perpendicular magnetic field
→ gapped phase with “chiral” edge: unidirectional transport

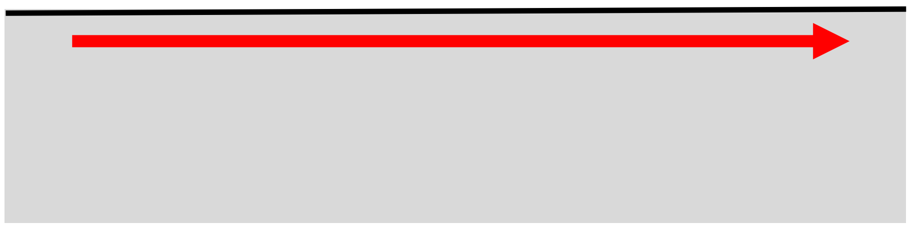






Bulk-edge correspondence

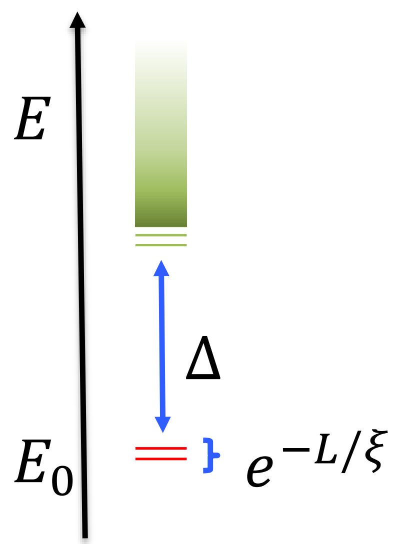




- Topological invariants associated with the edge:
  - Chiral central charge $\bar{c}$ "=" (# of right-moving modes) – (# of left-moving modes)
- Cannot be changed without a **bulk** phase transition!
- Can be detected experimentally: charge & thermal transport

Chiral topological phases

- Chiral: ✓
- Topological phase:



- Local *2d* Hamiltonian which, when placed on a torus, has $N_a(M) > 1$ near-degenerate low-energy states and a finite gap above these states.
- Exponentially small splitting:

$$H|n\rangle = (E_0 + O(e^{-L/\xi}))|n\rangle$$

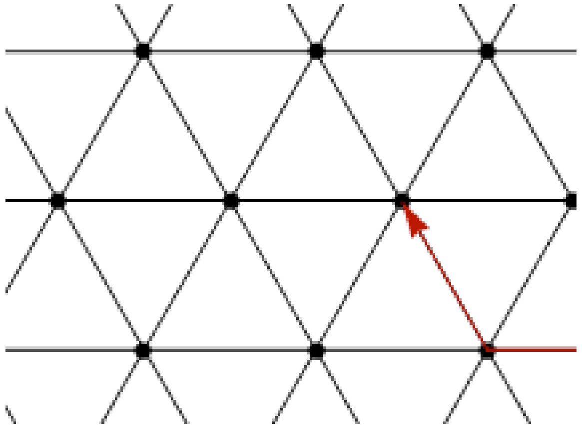
- Robust:

$$\langle n|O_{loc}|m\rangle = o\delta_{mn} + O(e^{-L/\xi})$$

Kalmeyer-Laughlin proposal

Can chiral topological phases be realized with spins?

- Kalmeyer & Laughlin, 1987
  - *Paradigm:* frustration & quantum fluctuations prevent ordering



$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

  - *Conjecture:* Ground state of triangular lattice Heisenberg antiferromagnet has universal properties of ν=1/2 Laughlin state

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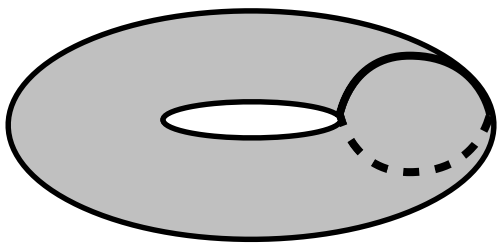
The $\nu = 1/2$ Laughlin state

Universal properties

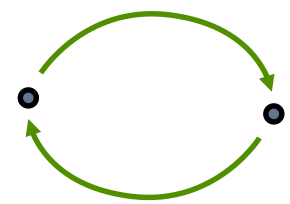
Degeneracy



N_a locally indistinguishable ground states with exponential splitting

Wen, 1990

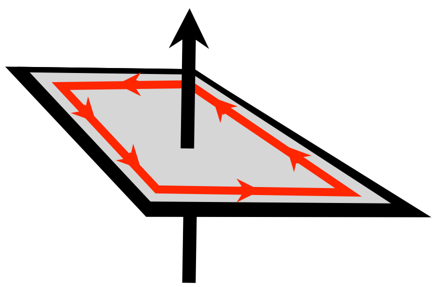
Excitations



Bosons, fermions: ±1
Abelian anyons: $e^{i\phi}$
Non-Abelian anyons: U

Wilczek, 1983

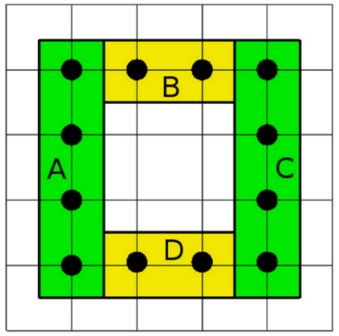
Edge states



Bulk is insulating. Edge is conducting in one direction.

Wen, 1991

Long-range entanglement



Subleading entropy contribution:
$S = \alpha L - \gamma$

Levin-Wen/Kitaev-Preskill 2005

$\nu = 1/2$ Laughlin state

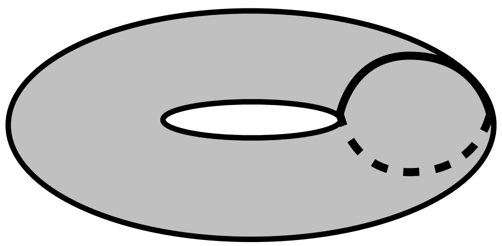
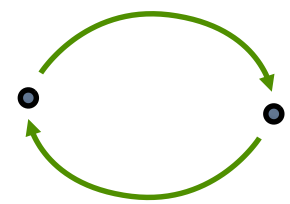
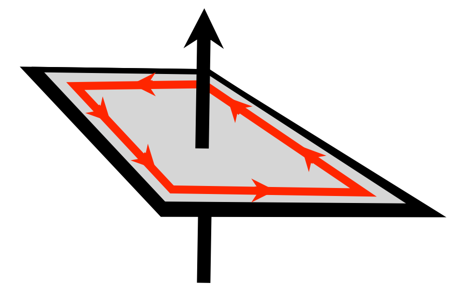
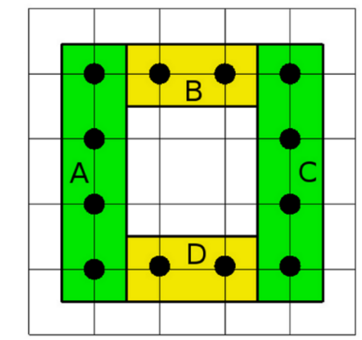
$N_a = 2$

Semion:
$\phi = \pi/2$
$s \otimes s = 1$

Chiral $SU(2)_1$ WZW CFT
$(\bar{c} = 1)$

$\gamma = \log\sqrt{2}$

Universal properties

Degeneracy	Excitations	Edge states	Long-range entanglement
			

$\nu = 1/2$ Laughlin state

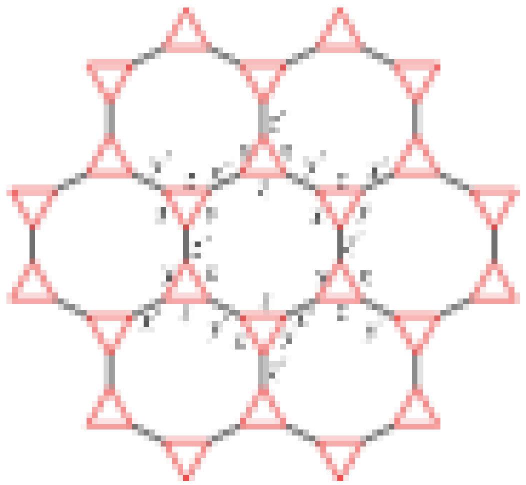
Degeneracy	Excitations	Edge states	Long-range entanglement
$N_a = 2$	Semion	Chiral $SU(2)_1$ WZW CFT	$\gamma = \log\sqrt{2}$

$\mathbb{Z}_2$ spin liquid (toric code)

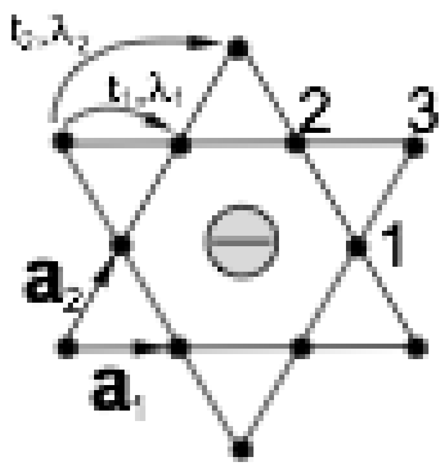
Degeneracy	Excitations	Edge states	Long-range entanglement
$N_a = 4$	3 Abelian anyons $a = \{1, e, m, f\}$	Gapped edge	$\gamma = \log\sqrt{4}$

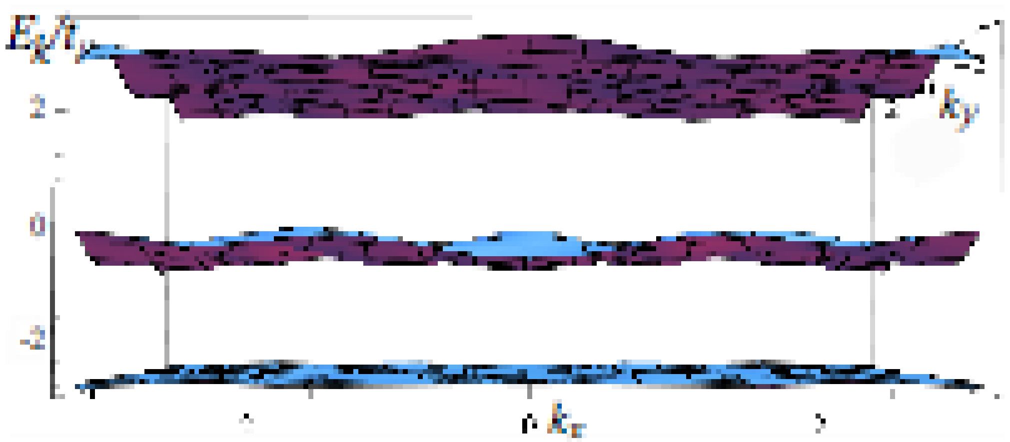
Realizations of a CSL

- Exact parent Hamiltonians:
  - Schroeter, Thomale, Kapit, Greiter (2007-2012)
  - Yao & Kivelson (2007)



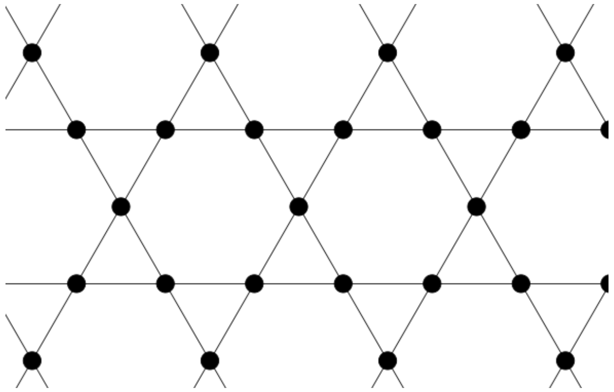
- Bosonic topological flat band models
  - Tang *et al* (2011), Sun *et al* (2011), Neupert *et al* (2011)





Simple spin models

- Our proposal (*BB et al, 2014; also BB et al 2013; Nielsen et al 2013*):



Wen, Zee, Wilczek 1989

Scalar spin chirality
$\chi_{ijk} = \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$

$$H = \sum_{\triangle} \chi_{ijk}$$

- Other models:
  - Deformations of the above (see later; *He, Bhattacharjee, Pollmann & Moessner 2015; Kumar, Sun, Fradkin 2015*)
  - Kagome $J_1 - J_2 - J_3$ model (*Gong, Zhu & Sheng 2014*): **spontaneous** breaking of time-reversal symmetry
  - $SU(N)$ models, …

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Numerical detection of universal physics

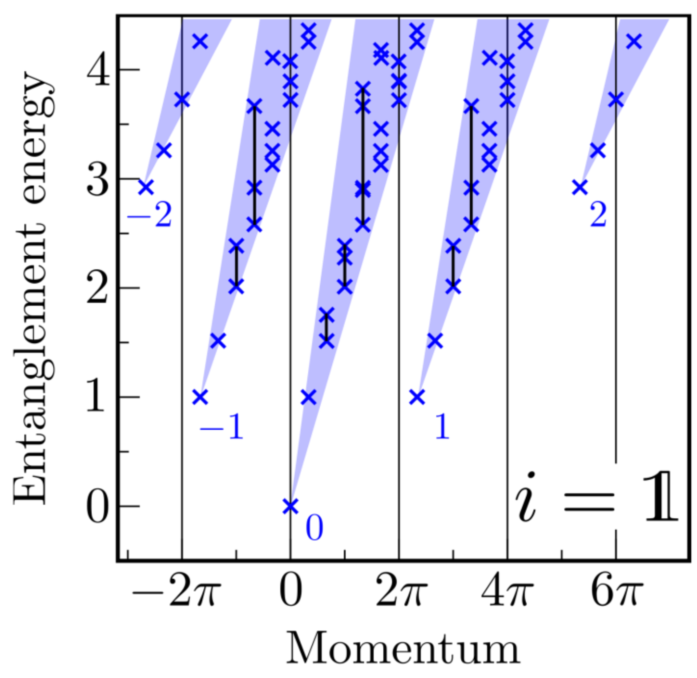
Based on L. Cincio and G. Vidal,

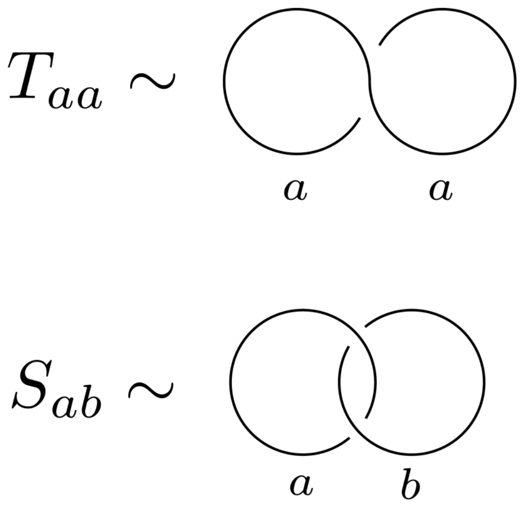
Phys. Rev. Lett. 110, 067208 (2013)

How do we know?

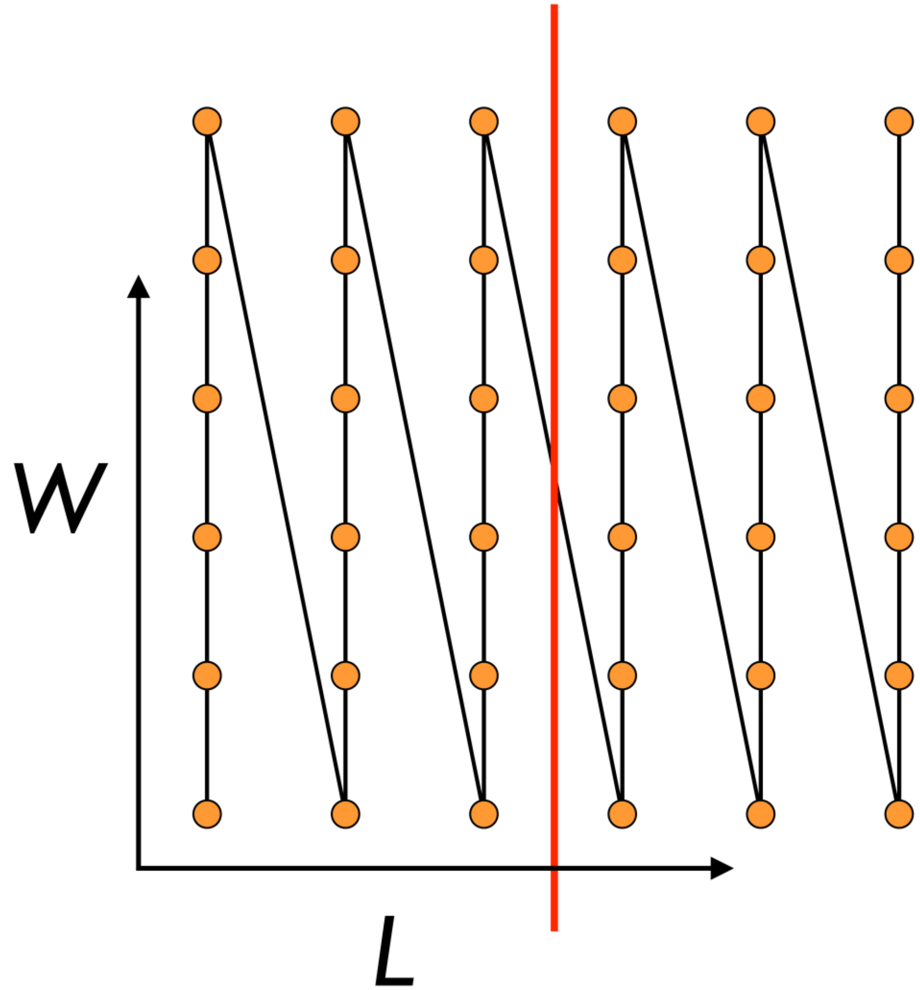
- Many methods give convincing results, in particular iDMRG

- Most powerful for 2d topological phases:





DMRG in *2d*: entanglement



- Bond dimension of the MPS:

$$M \sim \exp S$$

- Scaling of entanglement (area law):

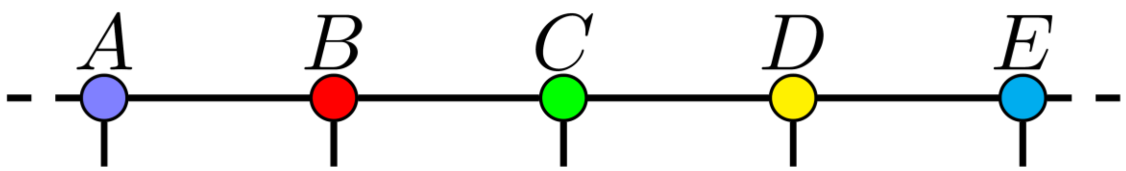
$$S \sim W$$

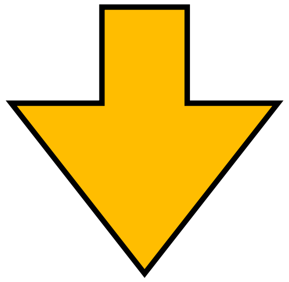
$$\cancel{S \sim L}$$

- *There is an easy (L) and a hard (W) direction!*

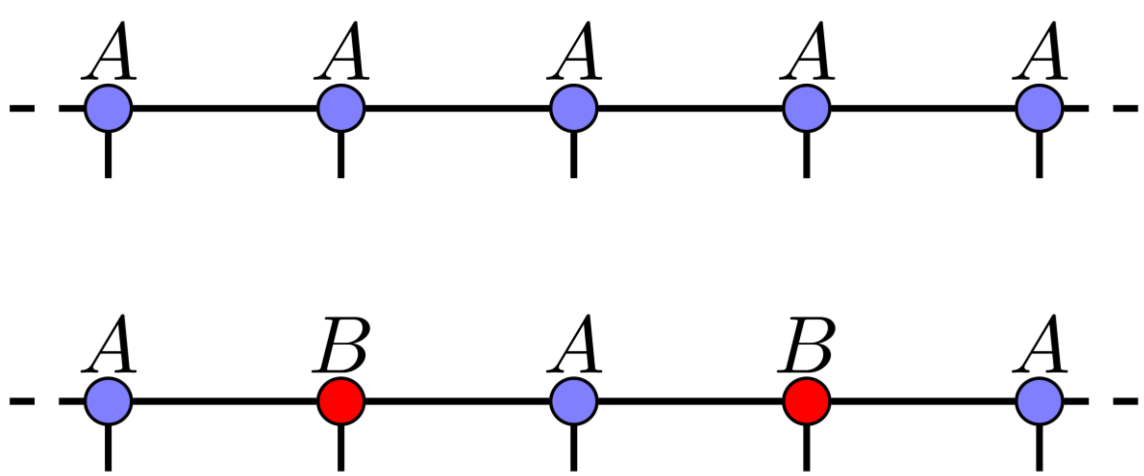
Use long, narrow systems!

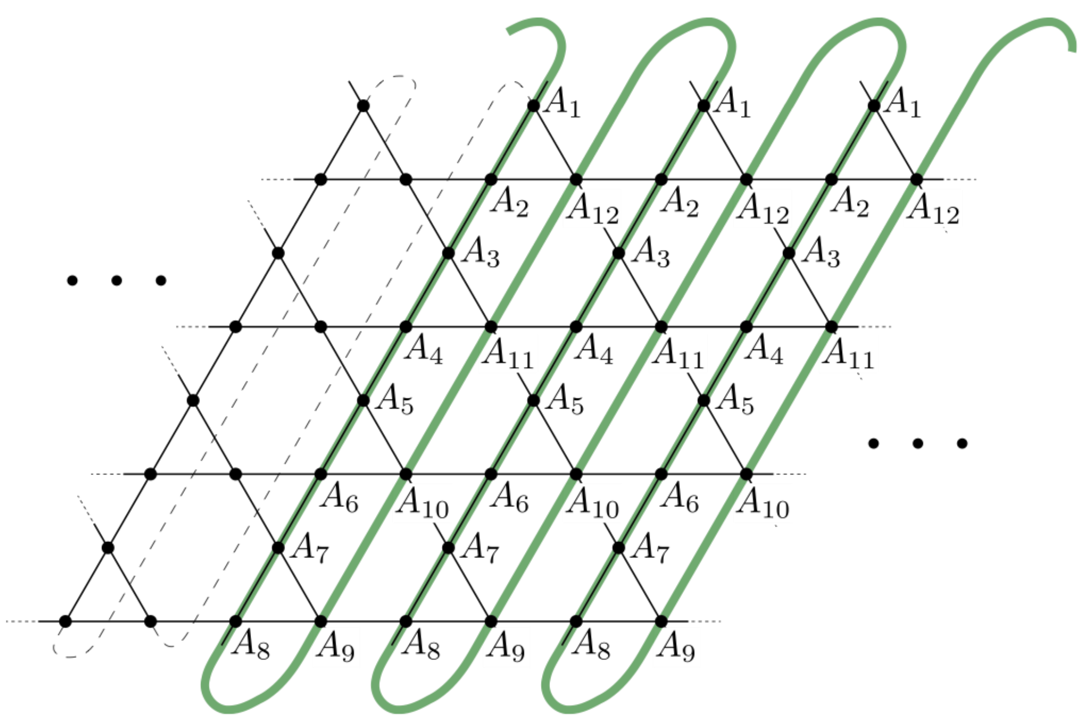
Infinite DMRG





Assume translational invariance





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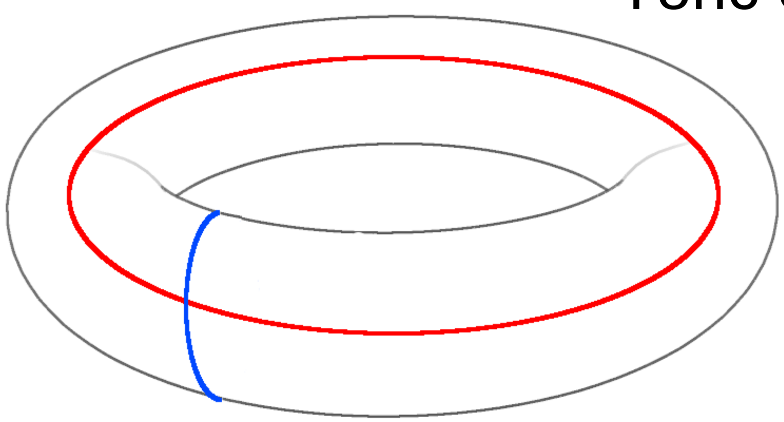
Minimally entangled states

Topological degeneracy – ground state manifold: $H = \text{span}\{|\psi_1\rangle, |\psi_2\rangle, \dots\}$

Toric code: $H = \text{span}\{|\psi_{ee}\rangle, |\psi_{eo}\rangle, |\psi_{oe}\rangle, |\psi_{oo}\rangle\}$



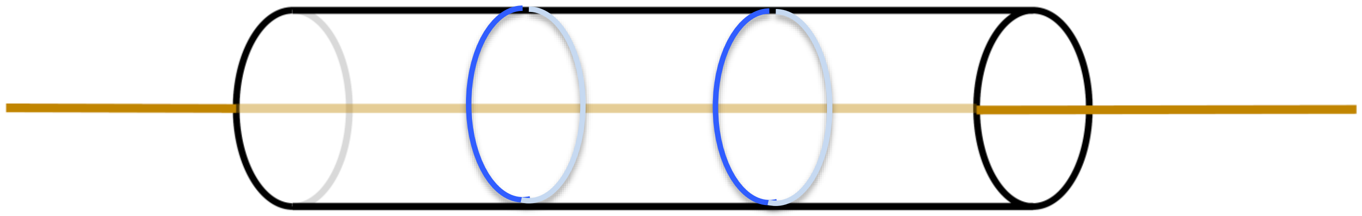
even/odd # of loops

Are all basis choices for g.s. manifold equally useful?

Minimally Entangled States

- Minimally entangled states = states with well-defined flux / eigenstates of the Wilson operator

$$W_i|\psi\rangle = W|\psi\rangle$$



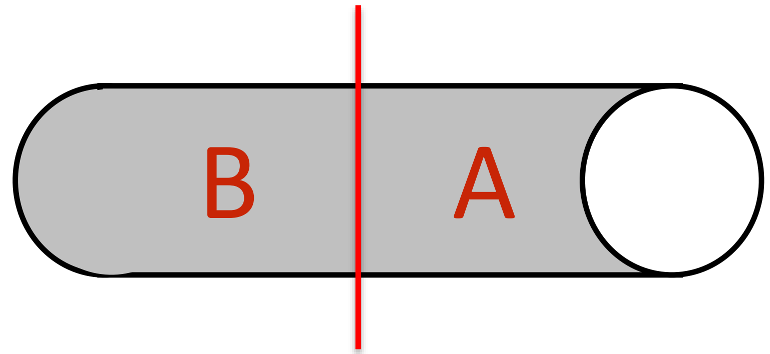
$$\langle W_i W_j \rangle = W^2$$

- MES in quasi-1d are long-range ordered states of the quasi-local Wilson operator!
- Non-MES are like CAT states:

$$(|ee\rangle + |eo\rangle) \sim (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

iMPS naturally collapses CAT states.

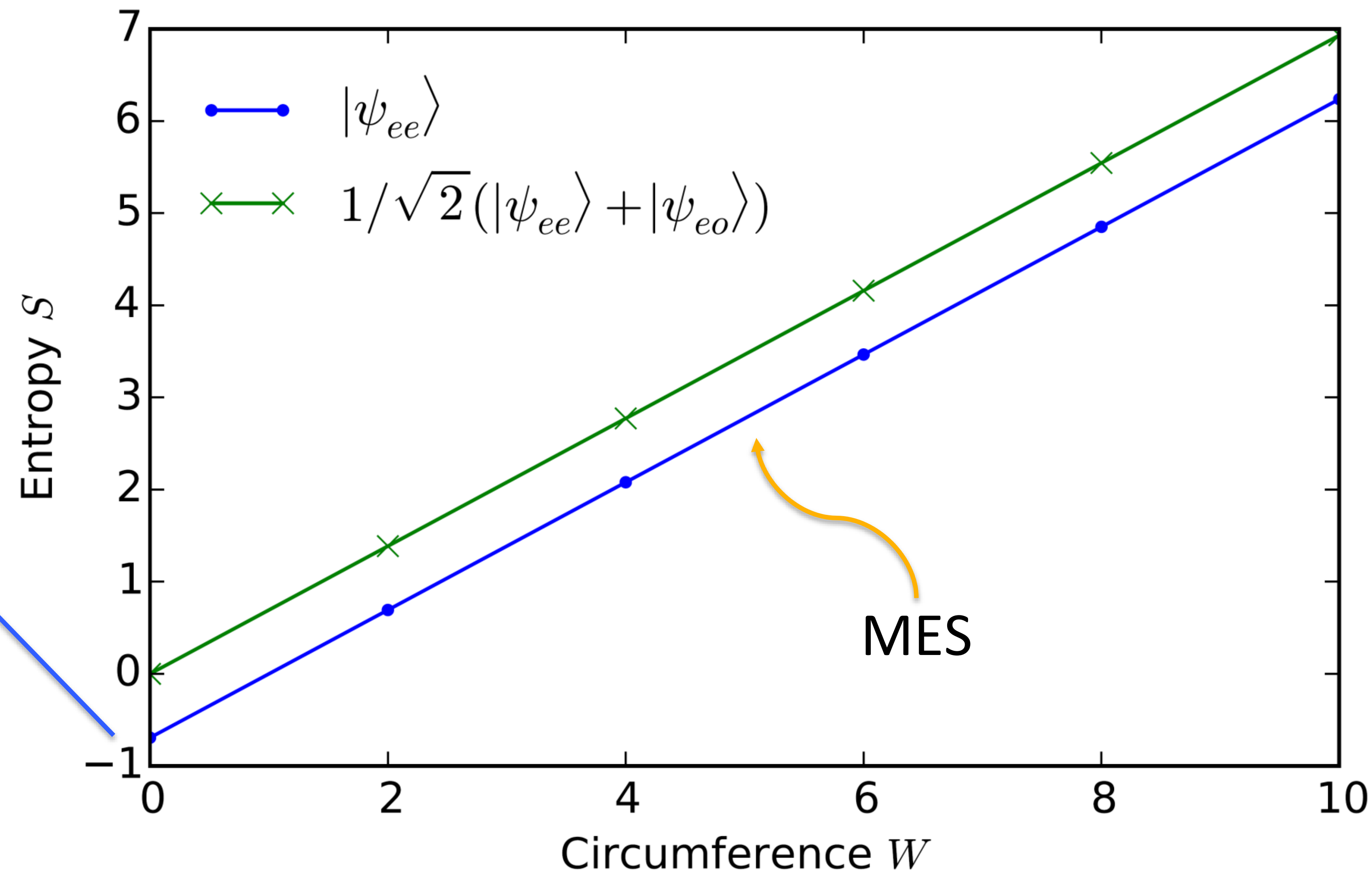
Topological Entanglement Entropy



$$S = \alpha W - \gamma$$

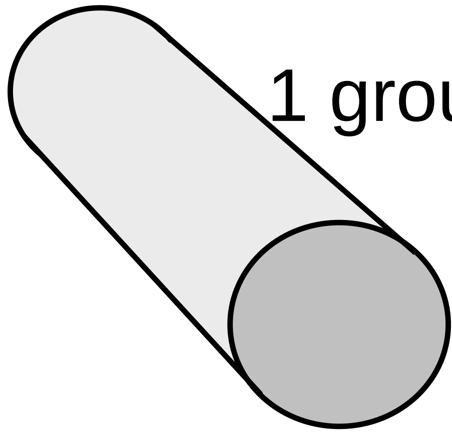
$$\gamma = \log\left(\frac{\mathcal{D}}{d_i}\right)$$

Non-Abelian phase: not all MES have the same entropy!

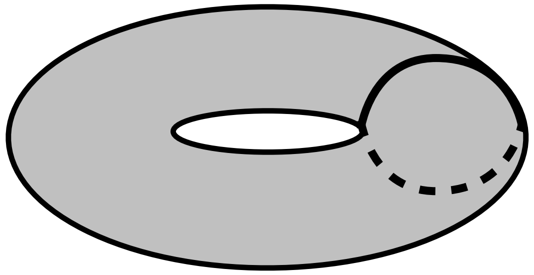


Identification of the CSL

Degeneracy

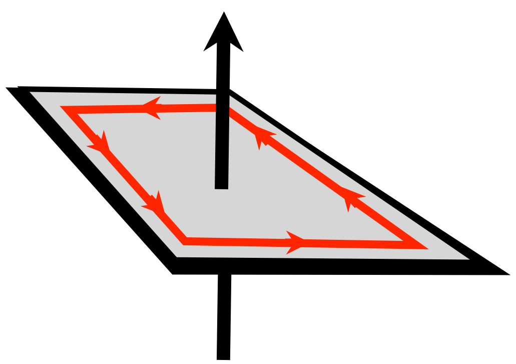


1 ground state



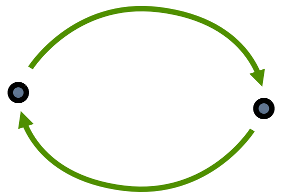
2 ground states

Edge states



Bulk is insulating.
Edge is conducting in one direction.

Excitations



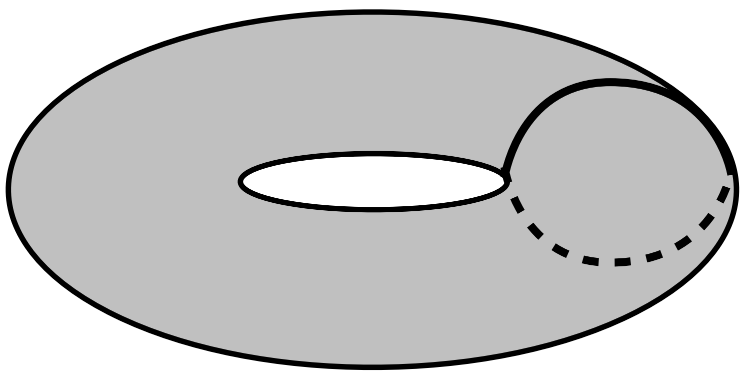
Fractional statistics

Bosons, fermions: ±1

Anyon: $e^{i\varphi}$

Spectrum

1 state



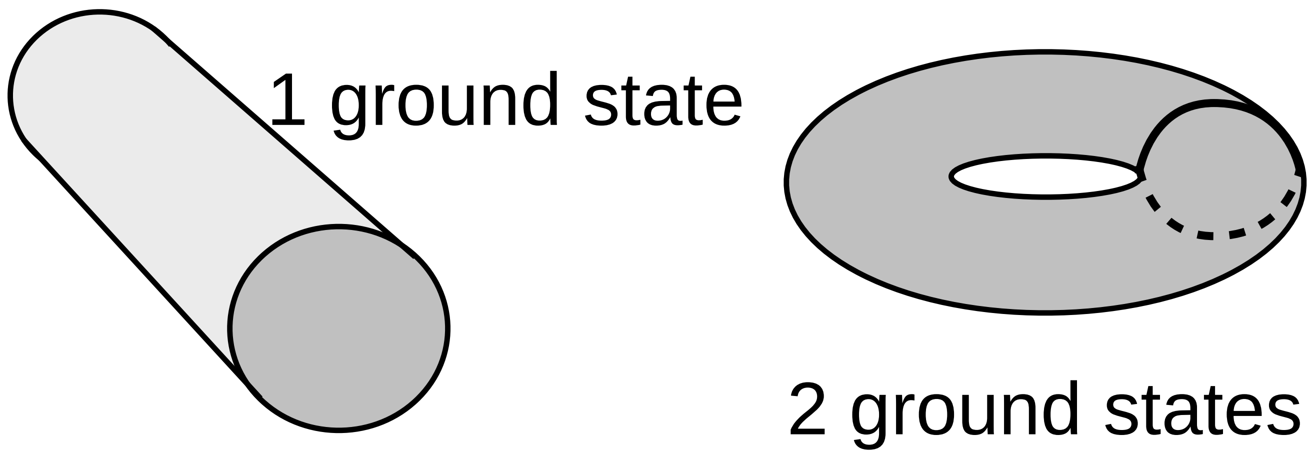
2 states

$\Delta \approx 0.05J$

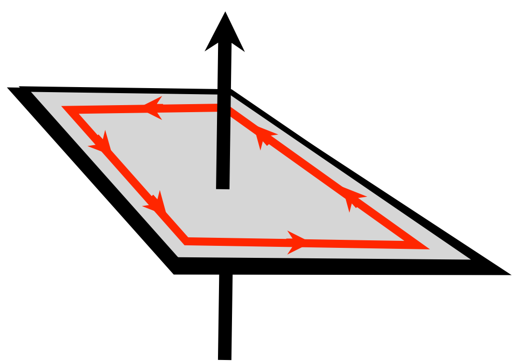
2 states

Identification of the CSL

Degeneracy



Edge states



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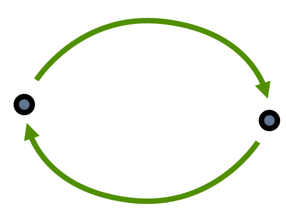
Excitations



Fractional statistics

Bosons, fermions: ± 1
Anyon: $e^{i\varphi}$

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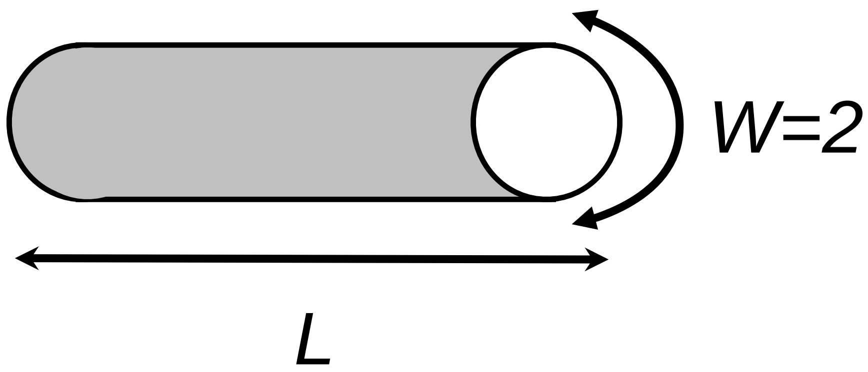
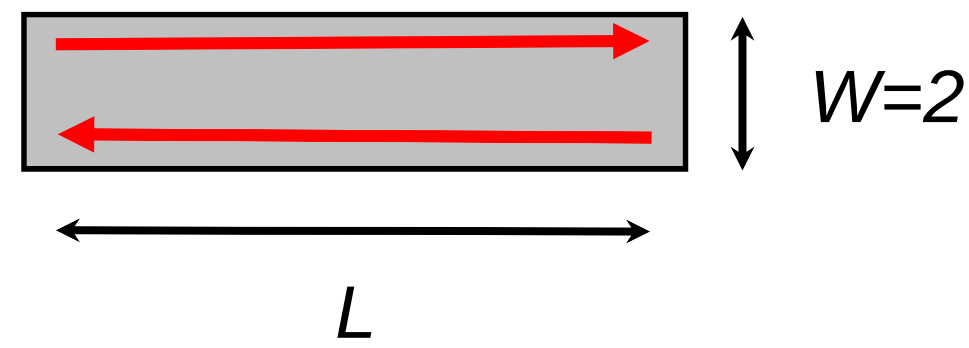
Edge state

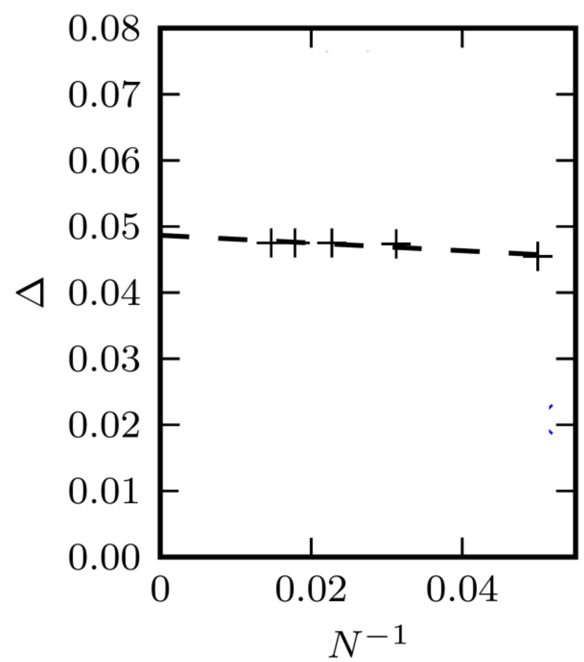


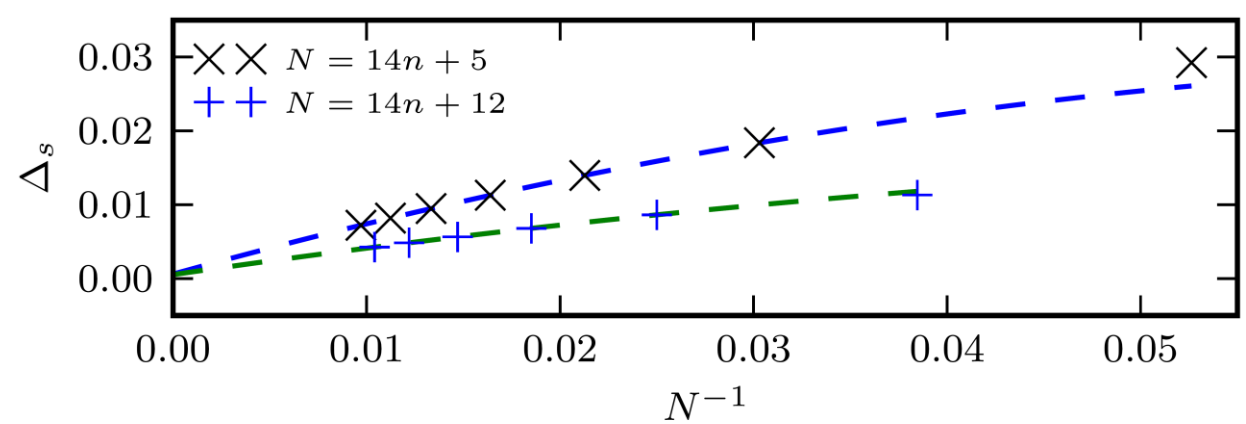


Cylinder

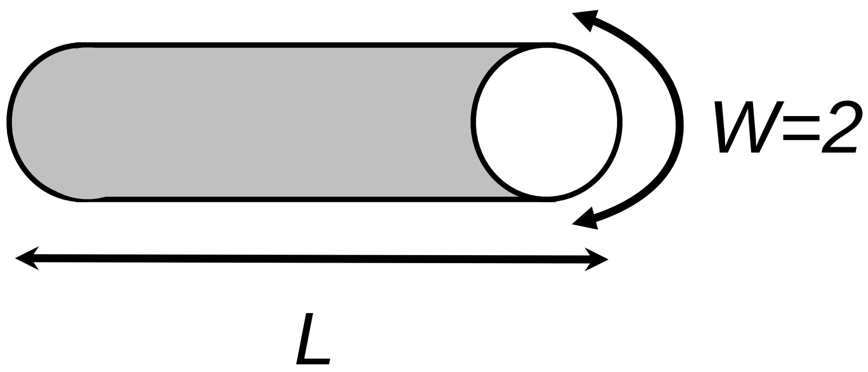
Strip

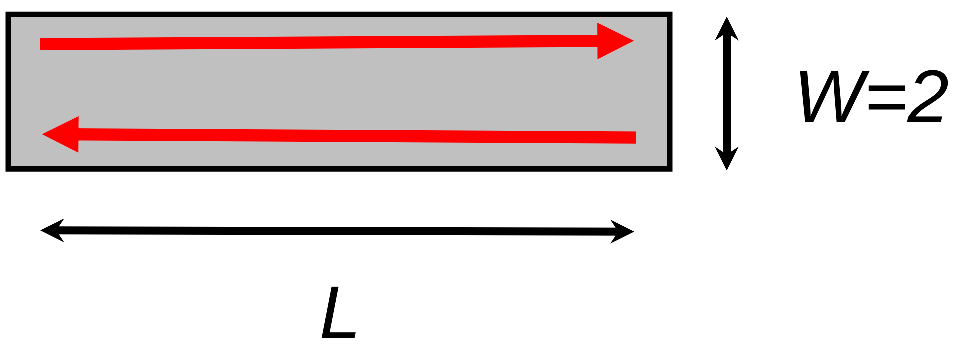
- Spin gap for a fully open system:
  $\Delta(L) \sim L^{-1}$





Edge state

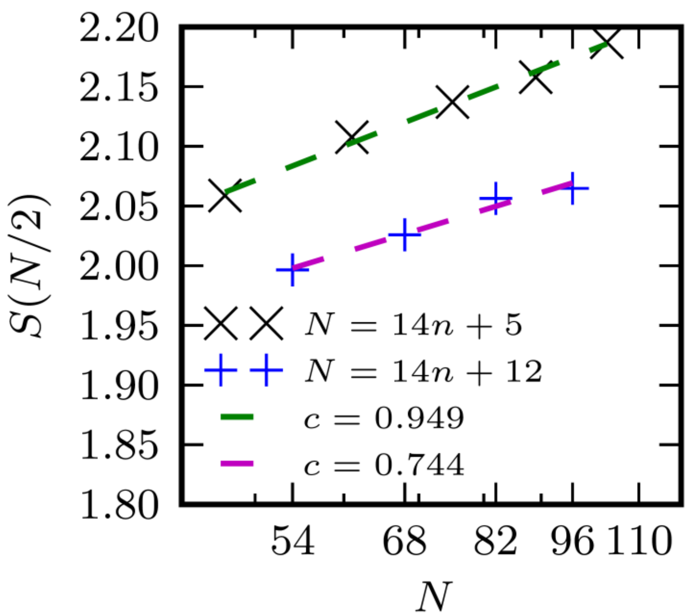


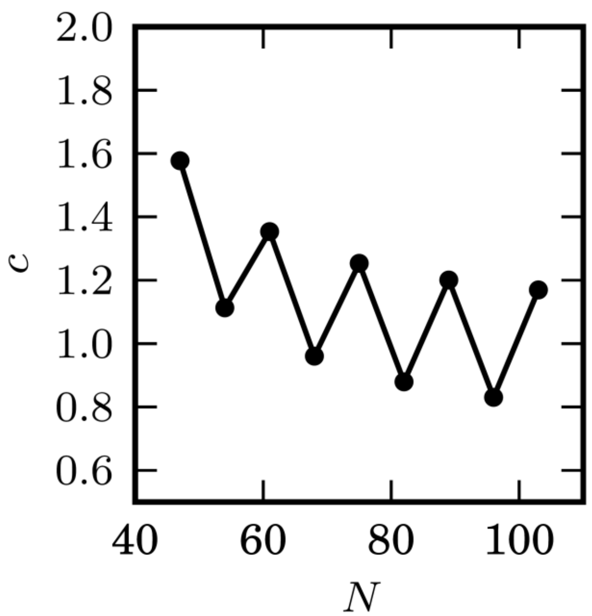


- Central charge from entanglement entropy

$$S(L/2) = S_0 + \frac{c}{6} \log L \qquad S(l) = S_0 + \frac{c}{6} \log\left(\frac{2L}{\pi} \sin\frac{\pi l}{L}\right)$$

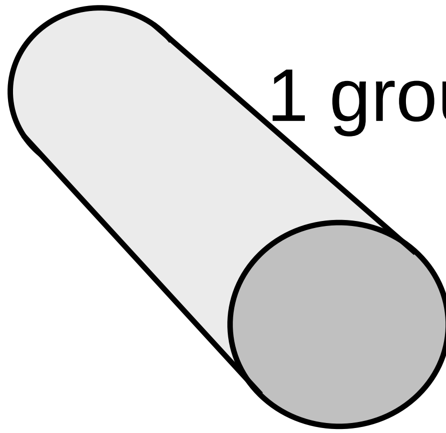
- Fit consistent with central charge *c=1*



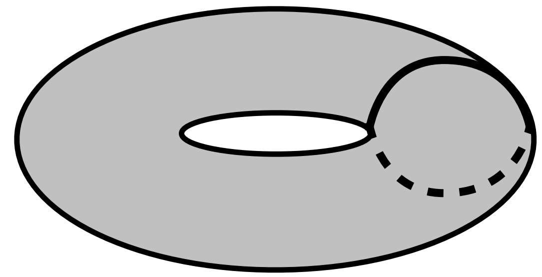


Identification of the CSL

Degeneracy

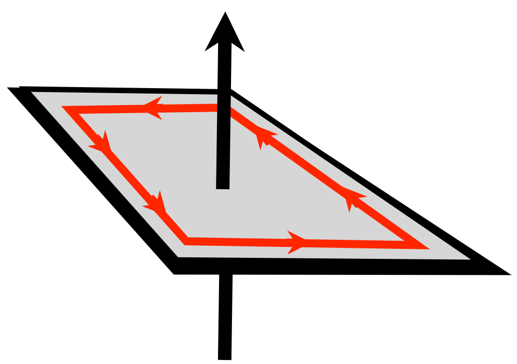


1 ground state



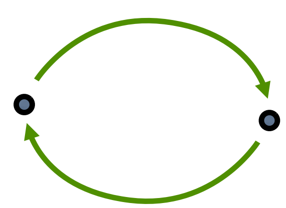
2 ground states

Edge states



Bulk is insulating.
Edge is conducting in one direction.

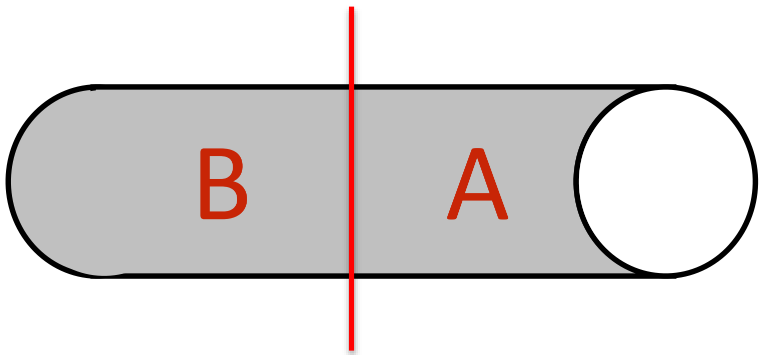
Excitations



Fractional statistics

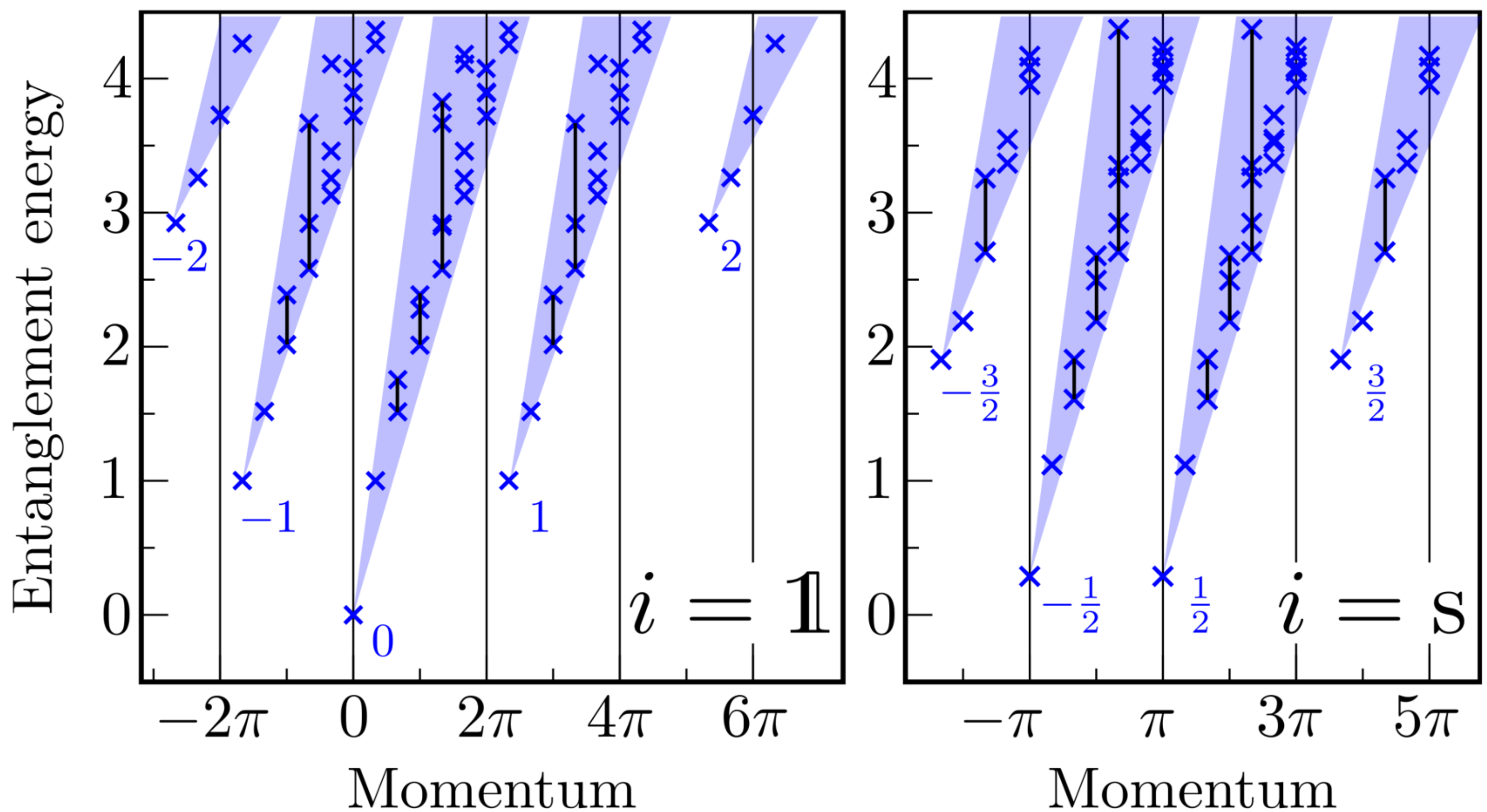
Bosons, fermions: ±1
Anyon: $e^{i\varphi}$

Entanglement spectrum



$$\rho_A = \sum_\alpha e^{-E_\alpha} |\alpha\rangle\langle\alpha|$$

Li & Haldane, PRL 2008: Entanglement manifestation of bulk-edge correspondence



Spectrum of SU(2)₁ WZW model

Entanglement spectrum

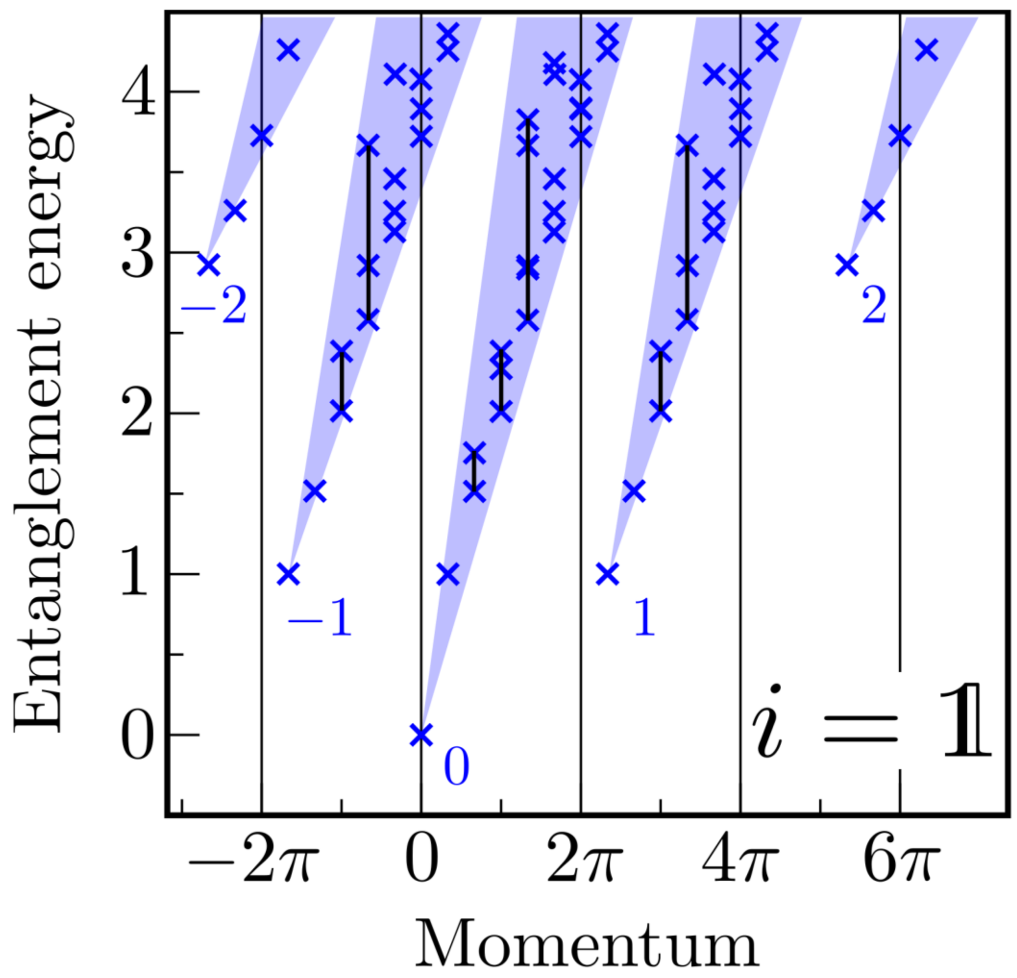


Table 15.1. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[1,0]}$.

L_0	m −2	−1	0	1	2	$su(2)$ decomposition
0			1			(0)
1		1	1	1		(2)
2		1	2	1		(2)+(0)
3		2	3	2		2(2)+(0)
4	1	3	5	3	1	(4)+2(2)+2(0)
5	1	5	7	5	1	(4)+4(2)+2(0)
6	2	7	11	7	2	2(4)+5(2)+4(0)

Di Francesco et al

Entanglement spectrum

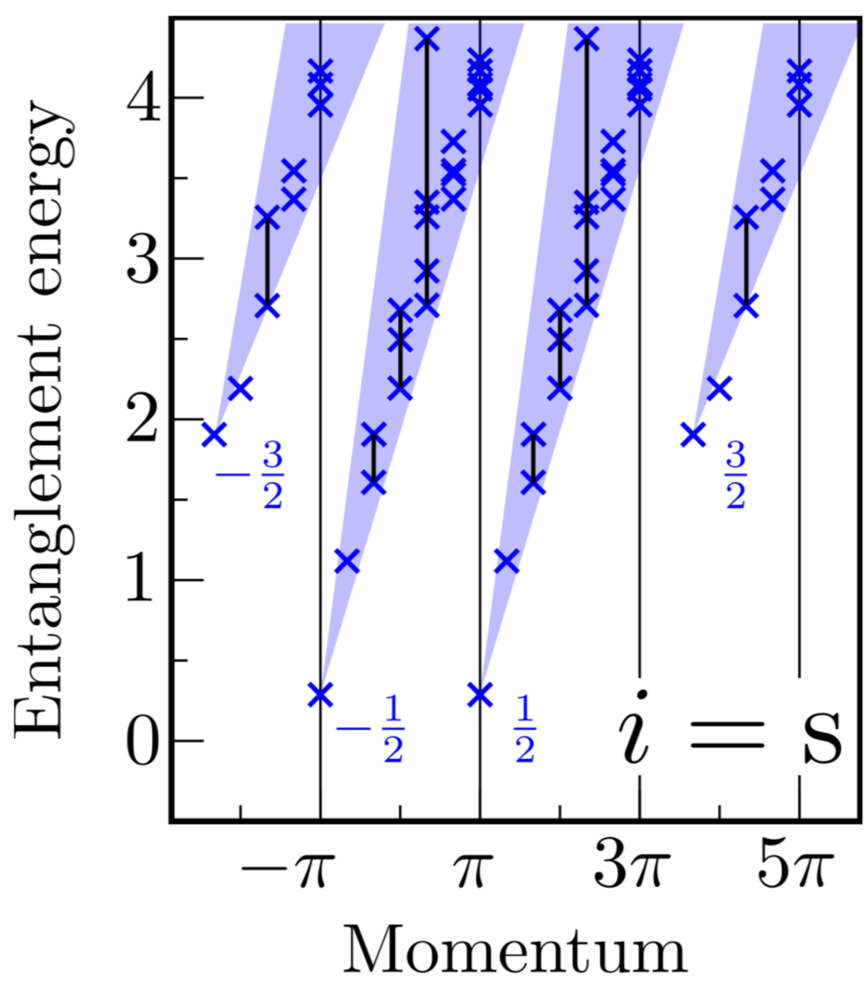


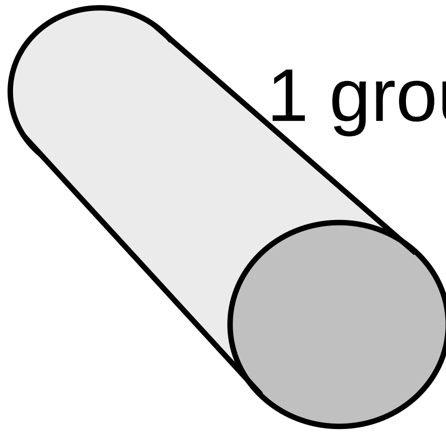
Table 15.2. States in the lowest grades of the $\widehat{su}(2)_1$ module $L_{[0,1]}$.

L_0	m: −2	−1	0	1	2	3	$su(2)$ decomposition
$\frac{1}{4}$			1	1			(1)
$\frac{5}{4}$			1	1			(1)
$\frac{9}{4}$		1	2	2	1		(3)+(1)
$\frac{13}{4}$		1	3	3	1		(3)+2(1)
$\frac{17}{4}$		2	5	5	2		2(3)+3(1)
$\frac{21}{4}$		3	7	7	3		3(3)+4(1)
$\frac{25}{4}$	1	5	11	11	5	1	(5)+4(3)+6(1)

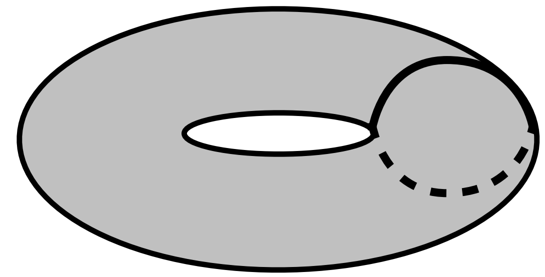
Di Francesco et al

Identification of the CSL

Degeneracy

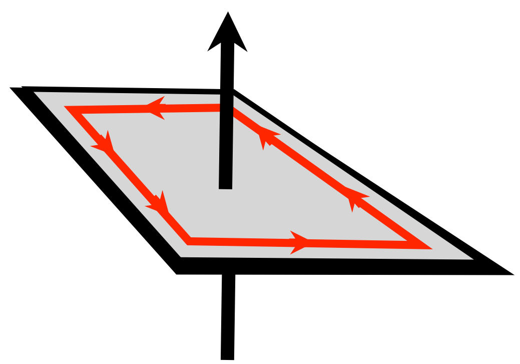


1 ground state



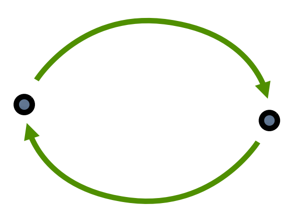
2 ground states

Edge states



Bulk is insulating.
Edge is conducting in one direction.

Excitations

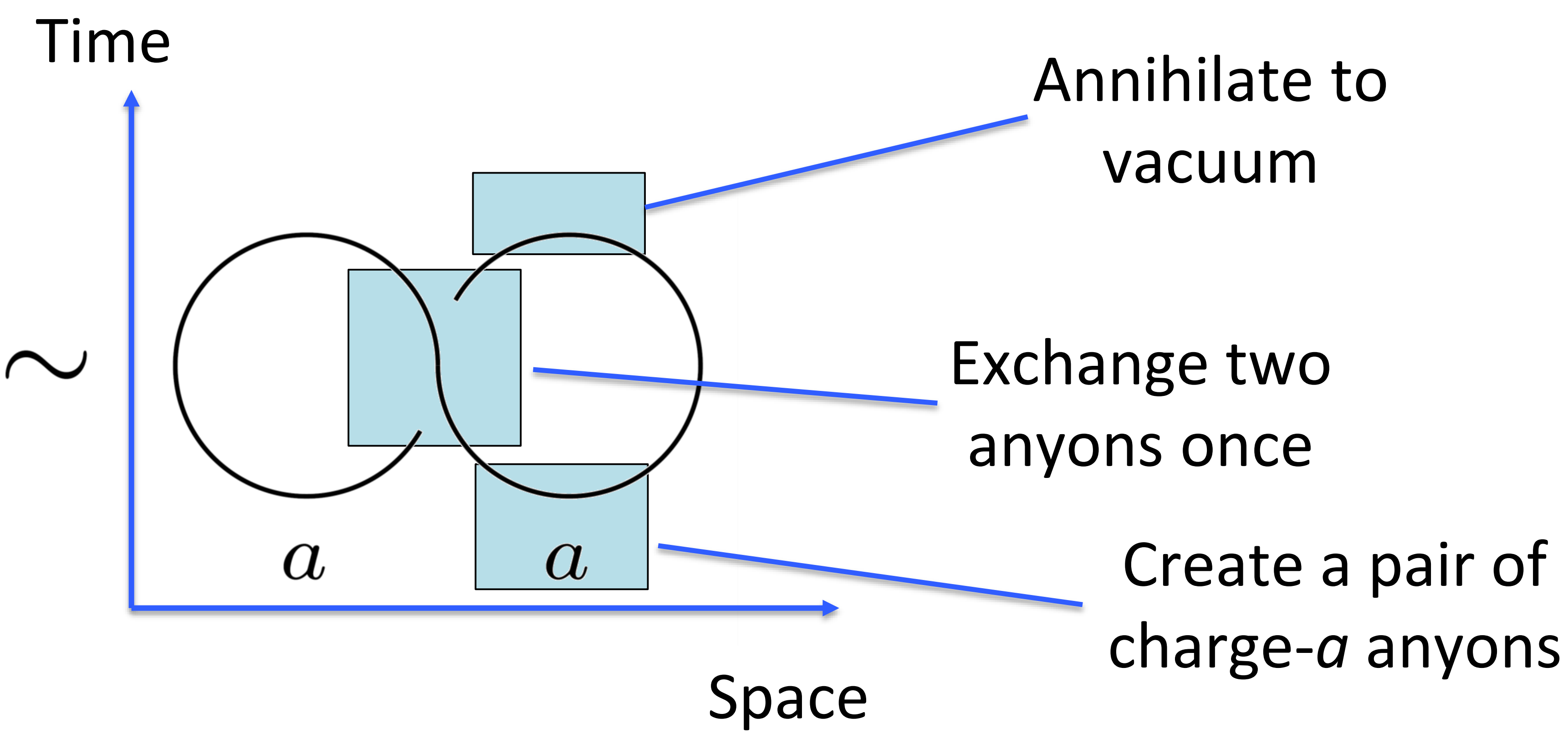


Fractional statistics

Bosons, fermions: ±1
Anyon: $e^{i\varphi}$

Modular matrices

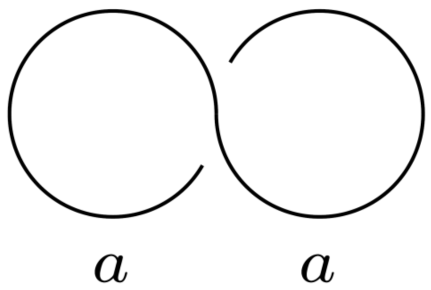
$$T_{aa} \sim$$



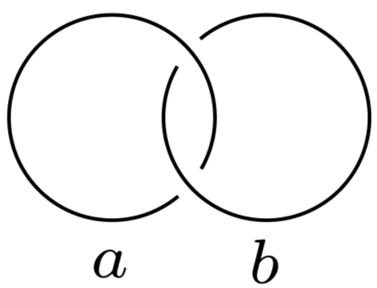
$$\begin{aligned} \text{Bosons}: \quad & T_{bb} = 1 \\ \text{Fermions}: \quad & T_{ff} = -1 \\ \text{Semions}: \quad & T_{ss} = i \end{aligned}$$

Modular matrices

$T_{aa} \sim$



$S_{ab} \sim$



- Conjecture: T & S matrices uniquely identify TQFT
- Recent proof for up to 4 particle types: *Rowell et al 2009, Bruillard et al 2013*

Chiral spin liquid:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Fibonacci anyons:

$$\phi = \frac{1+\sqrt{5}}{2}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{4\pi i}{5}} \end{pmatrix} \qquad S = \frac{1}{\sqrt{2+\phi}} \begin{pmatrix} 1 & \phi \\ \phi & -1 \end{pmatrix}$$

$\mathbb{Z}_2$ spin liquid (Toric code):

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

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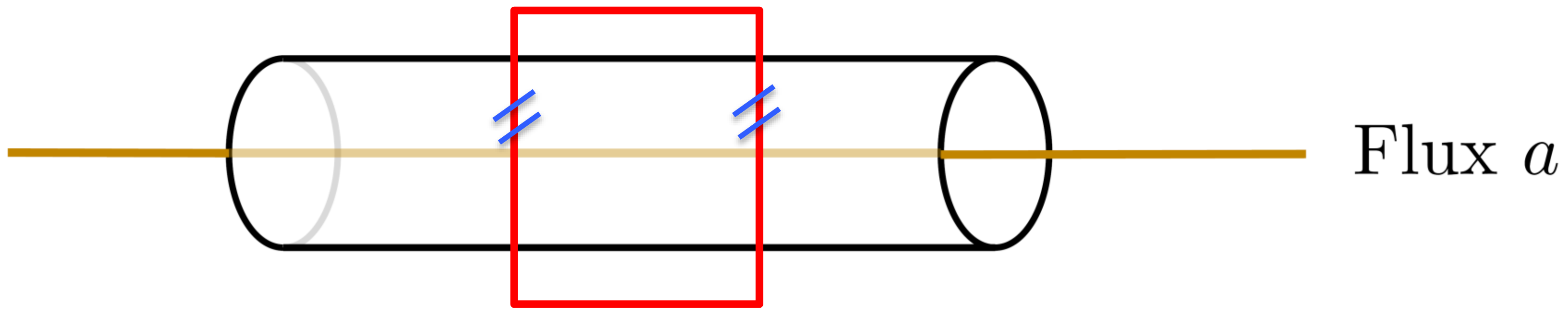
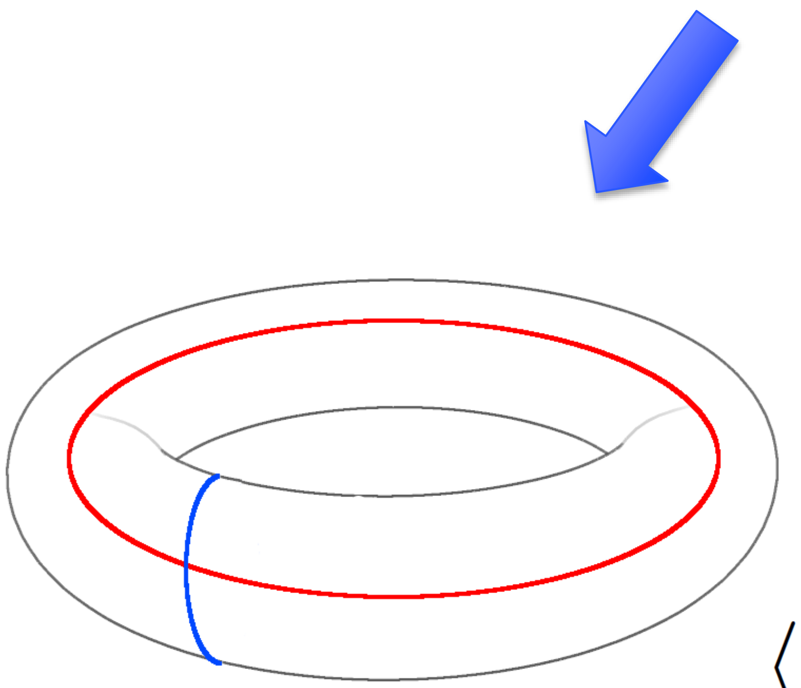
Modular matrices

MES basis on infinite (translationally invariant) cylinder



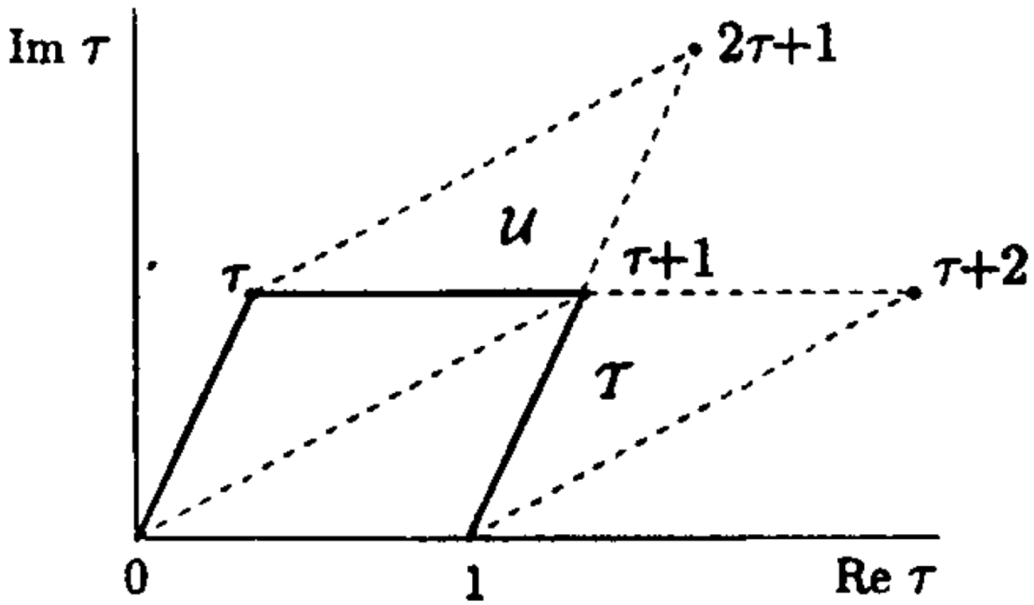


Given complete set of MES on a torus with $\pi/3$ rotation symmetry:

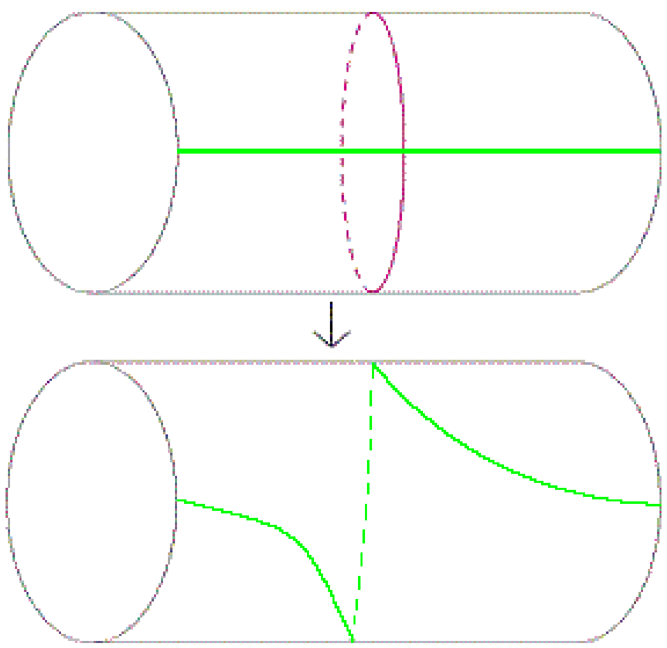
$$\langle \Psi_a^{\mathrm{tor}} | R_{\pi/3} | \Psi_b^{\mathrm{tor}} \rangle = (D T S^{-1} D^\dagger)_{ab}$$

Modular matrices & CFT

- Conjecture: Everything important is somewhere in The Big Yellow Book (in this case, p. 338, Sec. 10.1.2).

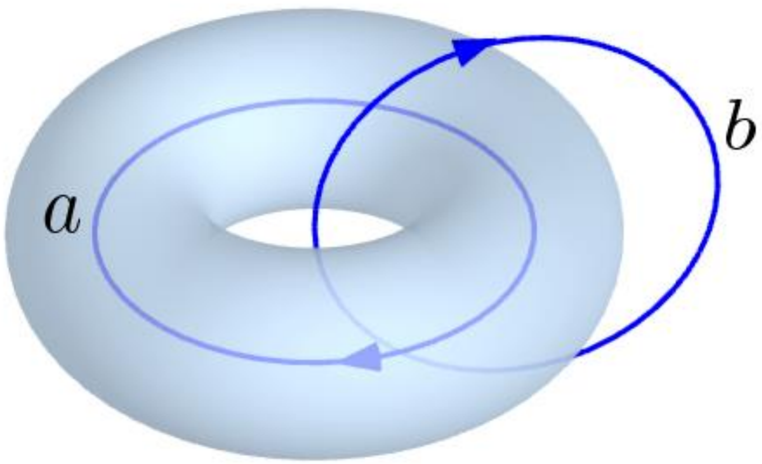


Dehn twist $\mathcal{T}$



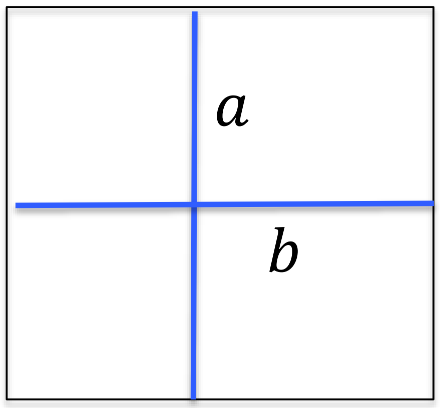
Modular matrices

$$S_{ab} = (90^\circ \text{ rotation of MES})$$
$$= \langle a | R_{\pi/2} | b \rangle$$



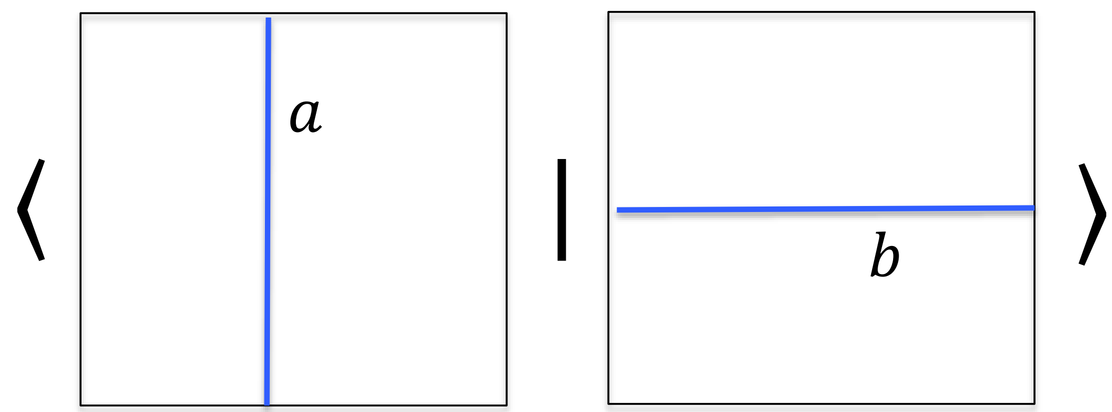
Two sets of MES:

1. *a* well-defined, *b* undefined
2. *b* well-defined, *a* undefined



Can be obtained by rotating states or minimizing entropy along two cuts (*Zhang, Grover, Turner, Oshikawa, Vishwanath 2012*)

Modular matrices



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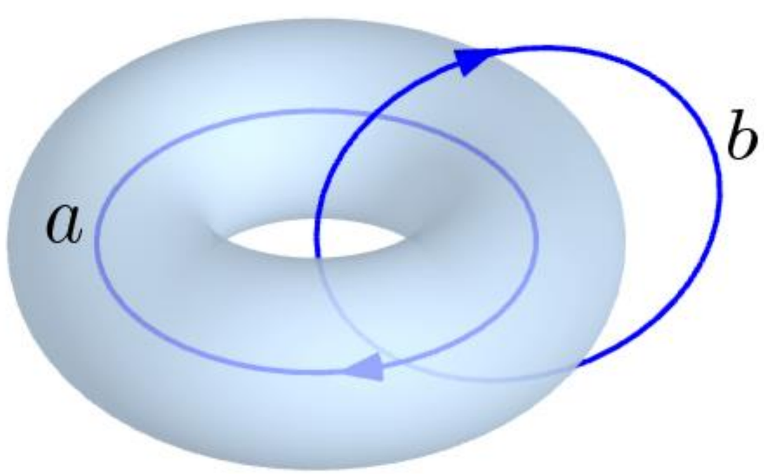

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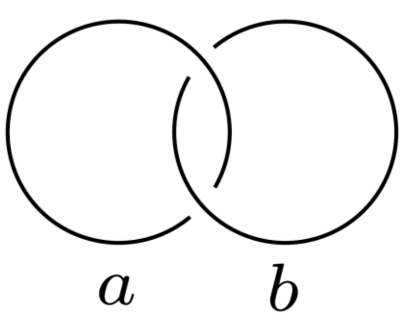
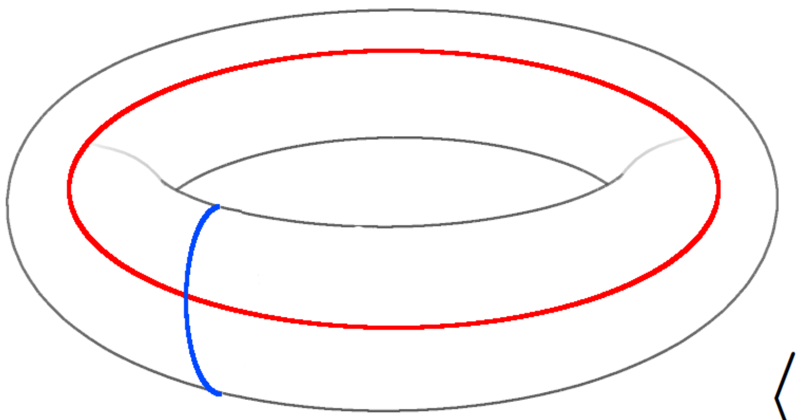


$\sim S_{ab}$

Modular matrices



Given complete set of MES on a torus with $\pi/3$ rotation symmetry:

$$\langle \Psi_a^{\text{tor}} | R_{\pi/3} | \Psi_b^{\text{tor}} \rangle = (DTS^{-1}D^\dagger)_{ab}$$

Chiral spin liquid:

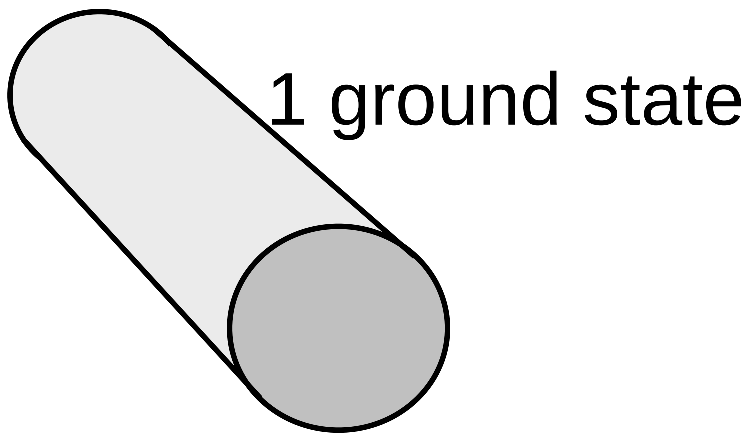
$$T = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

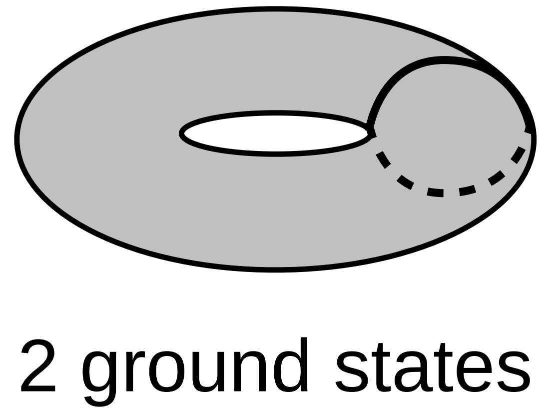
$$T = e^{-i\frac{2\pi}{24}0.988} \begin{bmatrix} 1 & 0 \\ 0 & i \cdot e^{-i0.0021\pi} \end{bmatrix},$$

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 0.996 & 0.995 \\ 0.996 & -0.994e^{-i0.0019\pi} \end{bmatrix}.$$

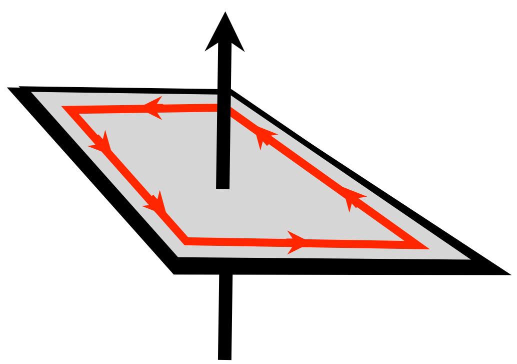
Identification of the CSL

Degeneracy



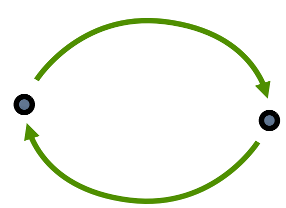


Edge states



Bulk is insulating.
Edge is conducting in one direction.

Excitations



Fractional statistics

Bosons, fermions: ±1

Anyon: $e^{i\varphi}$

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The semion

Symmetry fractionalization

- Anyons can carry fractional quantum numbers!
- Symmetry quantum numbers must be consistent with fusion rules

(Abelian case) $a \otimes b = c$

- Toric code: $a \in \{1, e, m, f\}$

$$e \otimes e = m \otimes m = f \otimes f = 1$$

$$e \otimes m = f$$

- SU(2) fractionalization?

1	e	m	f
0	1/2	1/2	1/2

$$1/2 \otimes 1/2 \neq 1/2$$

1	e	m	f
0	1/2	0	1/2

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Symmetry of the semion

- Abelian particle (quantum dimension $d_s = 1$) and exchange phase $e^{i\pi/2} = i$
- Fusion: $s \otimes s = 1$ (hence called $\mathbb{Z}_2$ anyons)

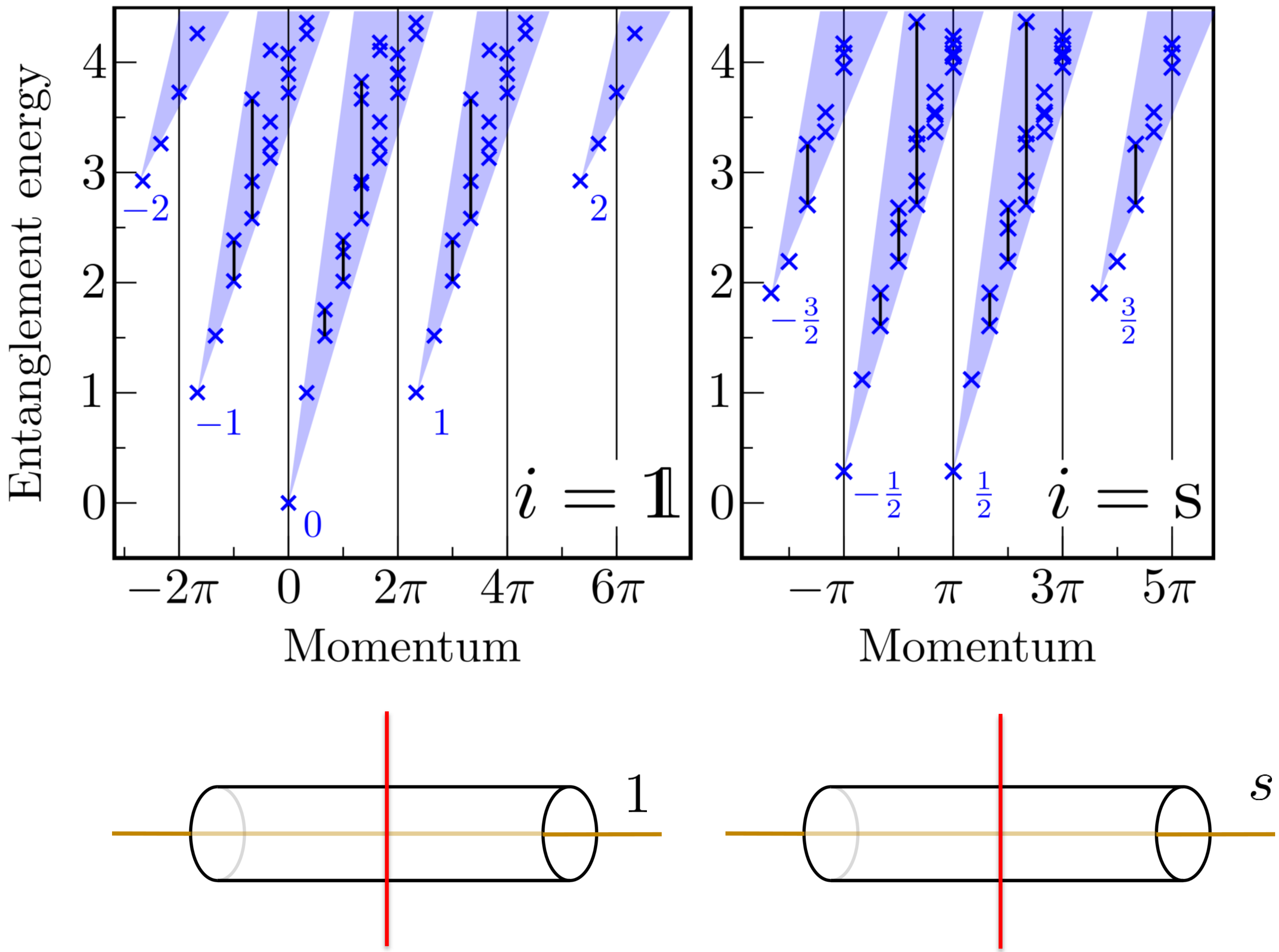
- Local (topologically trivial) excitations have integer spin
  - Example: spin flip $S = 1$
- Two consistent fractionalizations of SU(2):

1	s
0	0

1	s
0	1/2

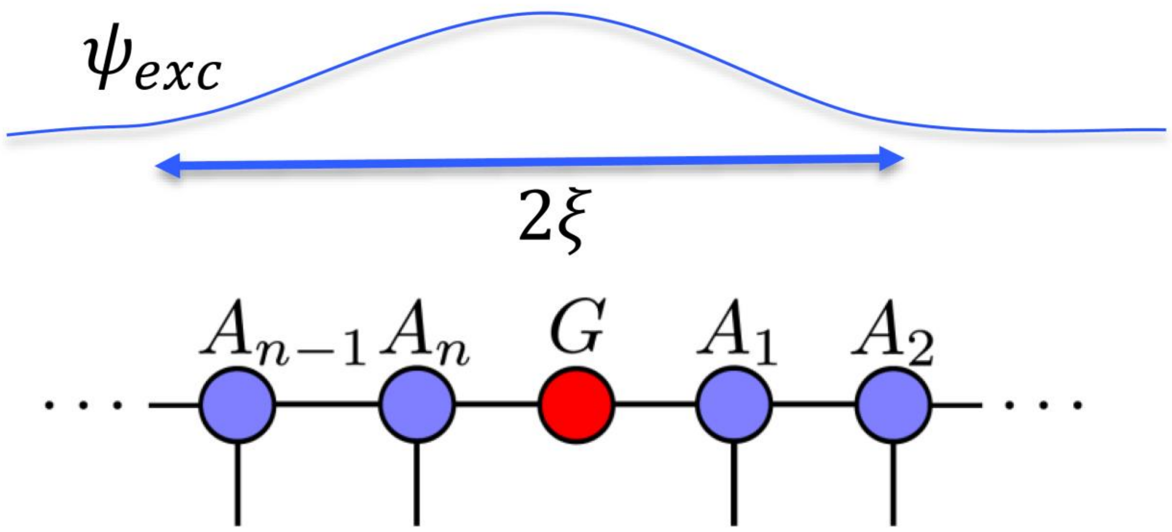
$$\frac{1}{2} \otimes \frac{1}{2} = 0 \mod 1$$

Entanglement spectrum



Ansatz states for excitations

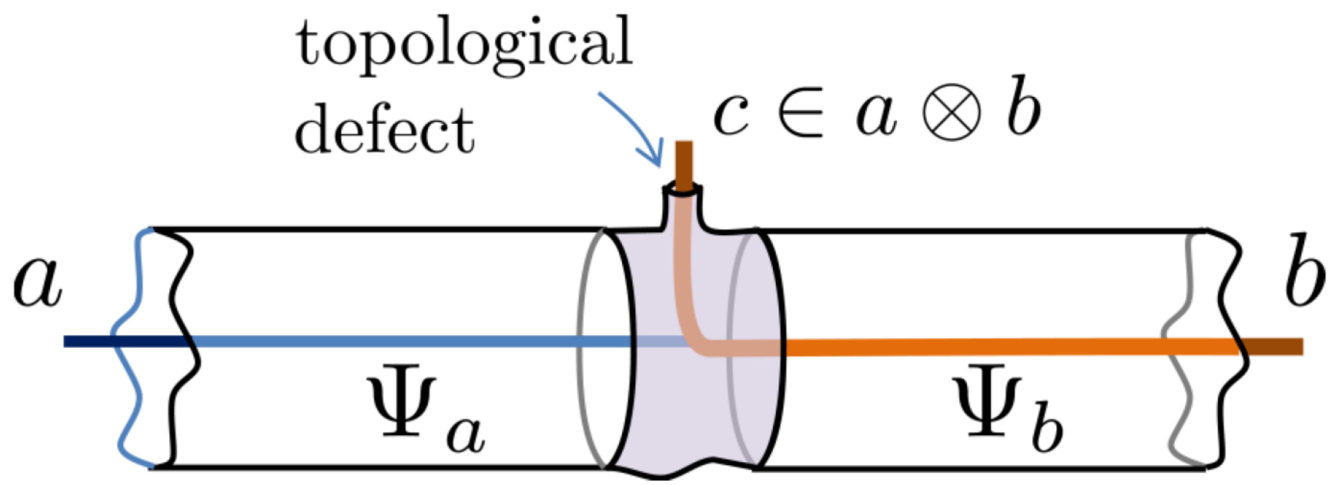
Local excitation



Local excitation energy Δ^{loc}

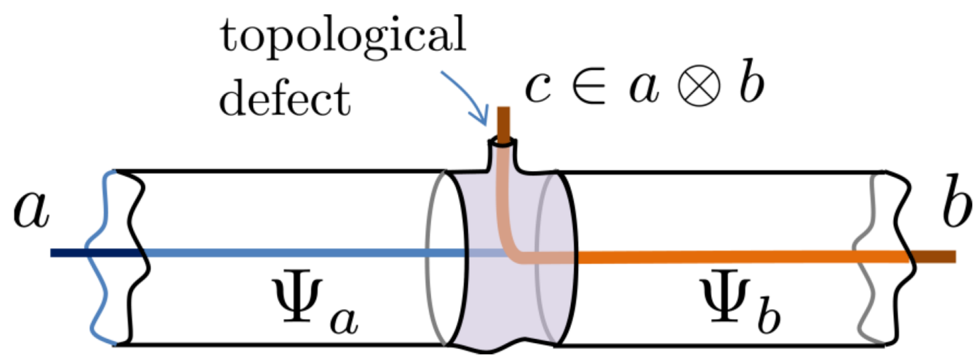
Topological excitation

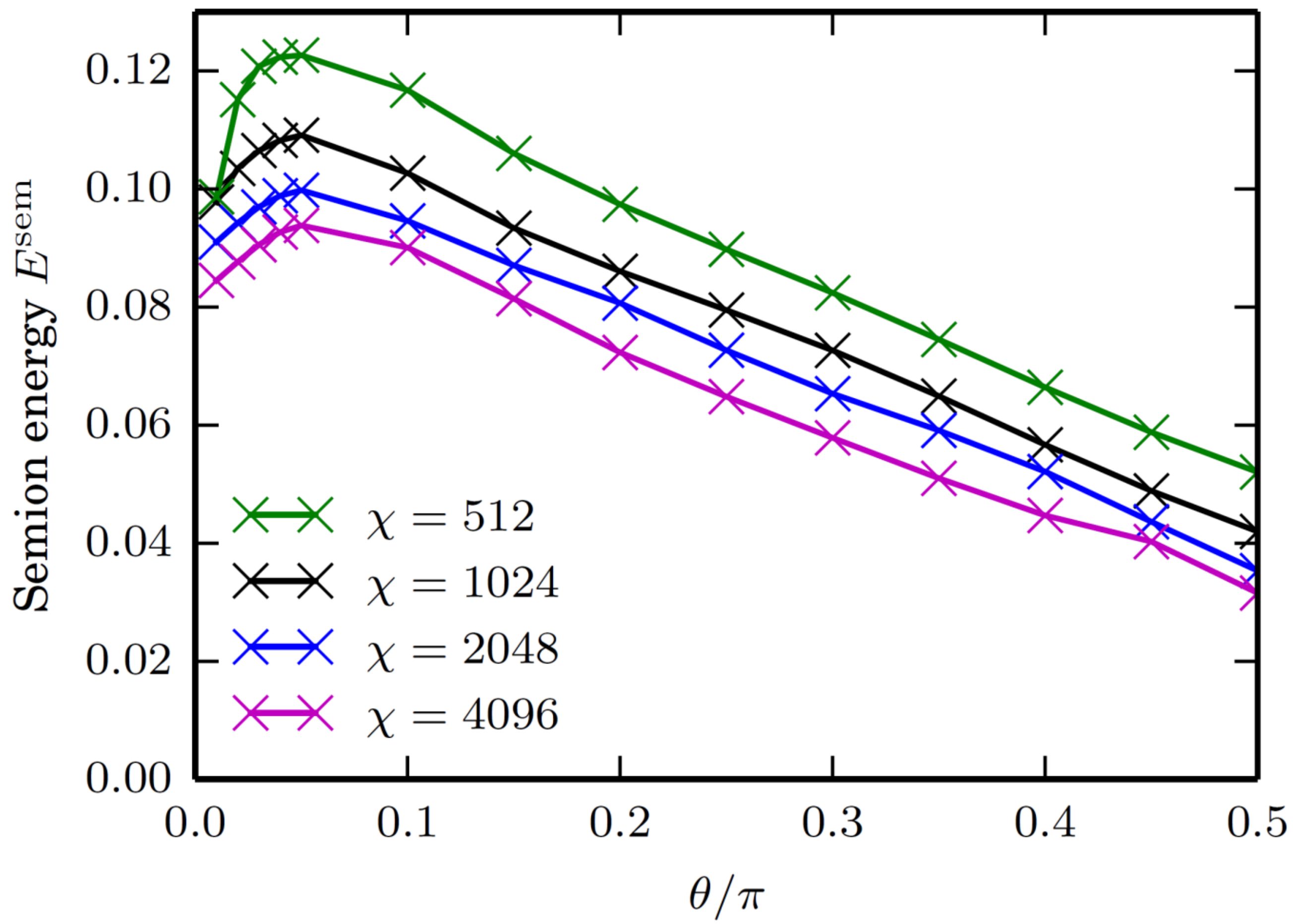
Cincio & Vidal 2012;
Zaletel et al 2012; Cincio, Vidal & BB 2015



Topological excitation energy E^a

Semion properties

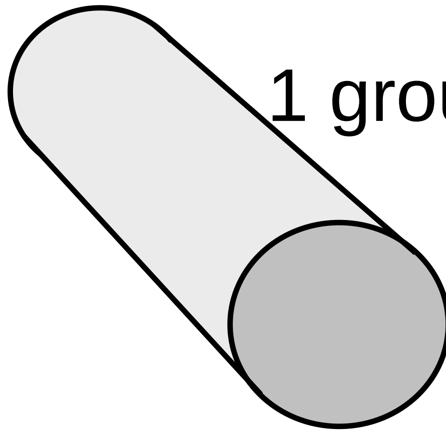




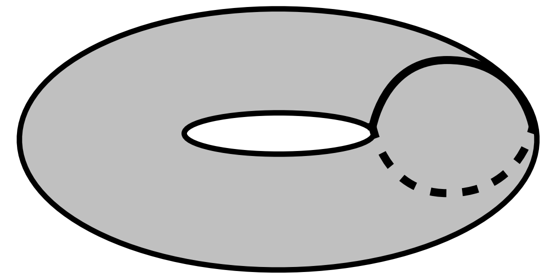
- Semion carries $S = 1/2$: fractionalized excitation!
  - CSL is a "symmetry-enriched topological phase"

Identification of the CSL

Degeneracy

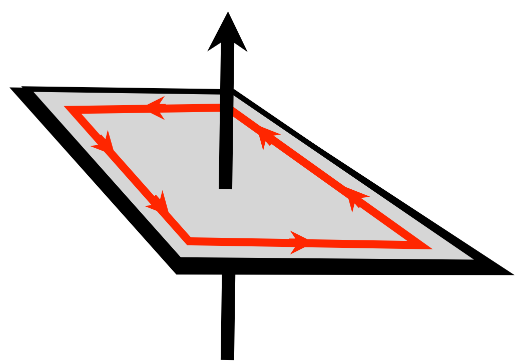


1 ground state



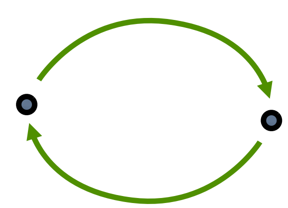
2 ground states

Edge states



Bulk is insulating.
Edge is conducting in one direction.

Excitations



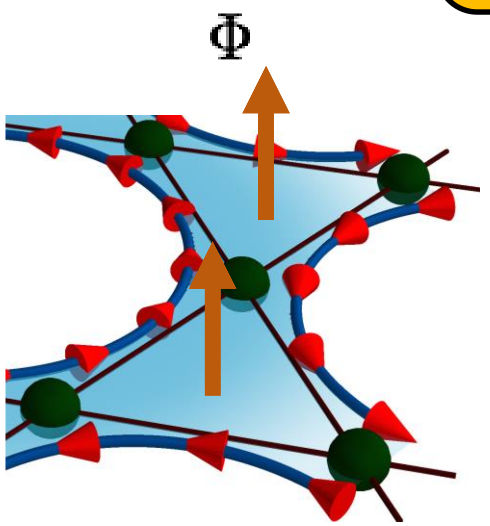
Fractional statistics

Bosons, fermions: ±1
Anyon: $e^{i\varphi}$

Phase diagram of the $J - J_\chi$ model

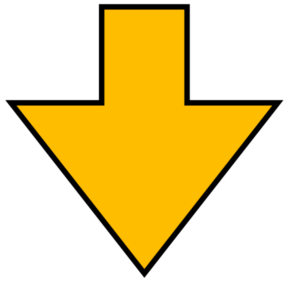
Mott insulator

$$H = -\sum_{\langle i,j\rangle,\sigma} (t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + t^*_{ij} c^\dagger_{j\sigma} c_{i\sigma}) + \frac{h_z}{2}\sum_i (n_{i\uparrow} - n_{i\downarrow}) + U\sum_i n_{i\uparrow} n_{i\downarrow}$$



$$t_{ij} t_{jk} t_{ki} = t^3 \exp(i\Phi)$$

Flux Φ through each triangle



- Half filling
- Large U

$$H = J_{\mathrm{HB}} \sum_{\langle i,j\rangle} \vec{S}_i \cdot \vec{S}_j + h_z \sum_i S^z_i + J_\chi \sum_{i,j,k\in\triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k) + \dots$$

$$J\cos\theta = J_{\mathrm{HB}} \sim t^2/U$$
$$J\sin\theta = J_\chi \sim \Phi t^3/U^2$$

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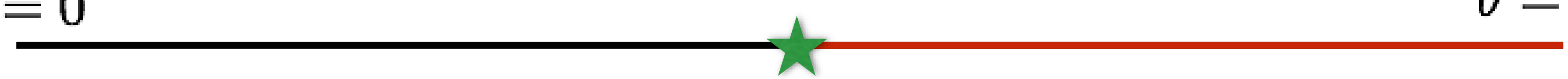
Phase diagram

$$H = \cos\theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \sin\theta \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

$\theta = 0$ $\theta = \pi/2$



TR-symmetric Heisenberg model:

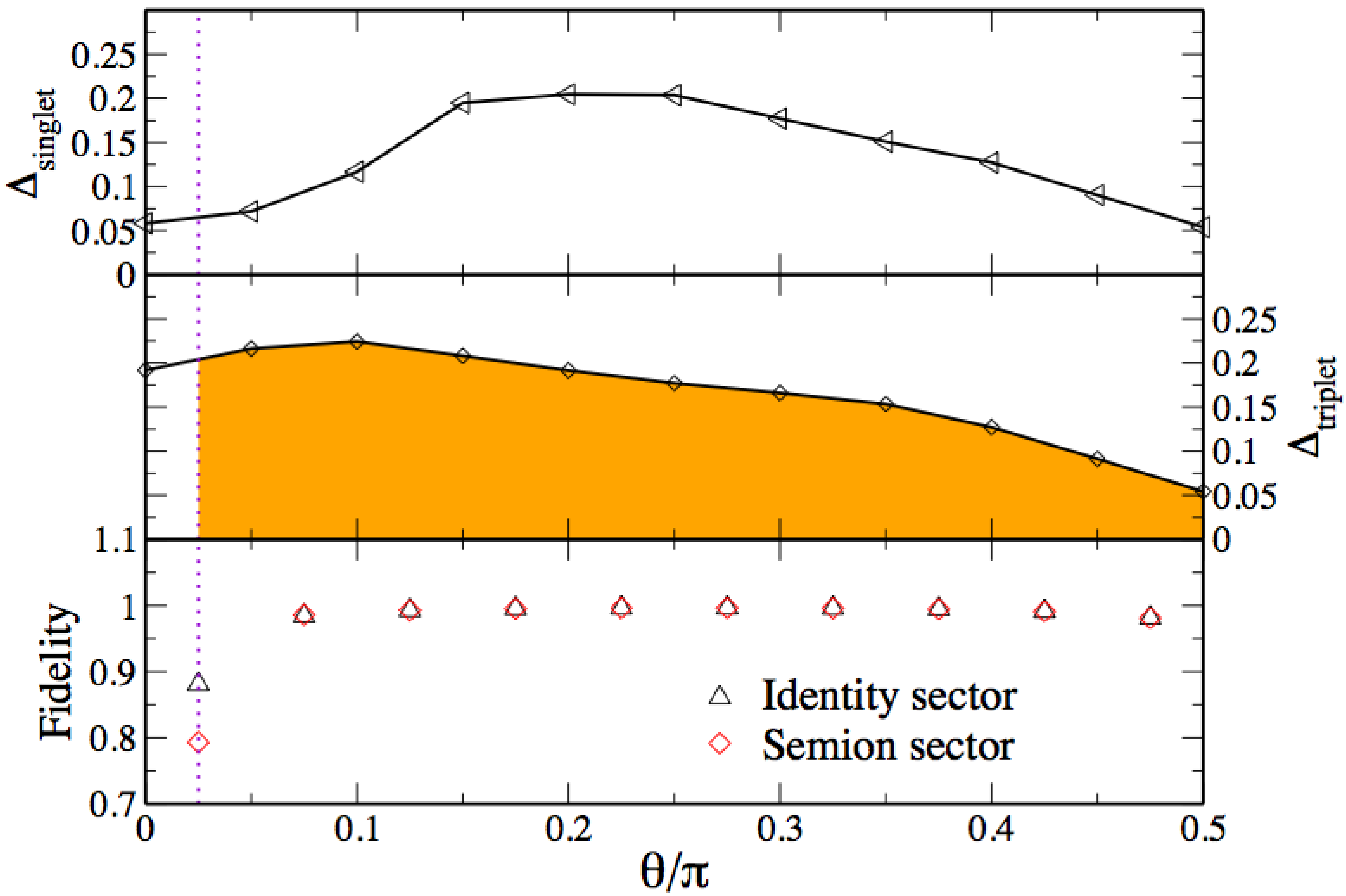
- Several promising SL candidates
- No indication of spontaneous TR breaking

Phase transition

TR-broken Chiral Spin Liquid

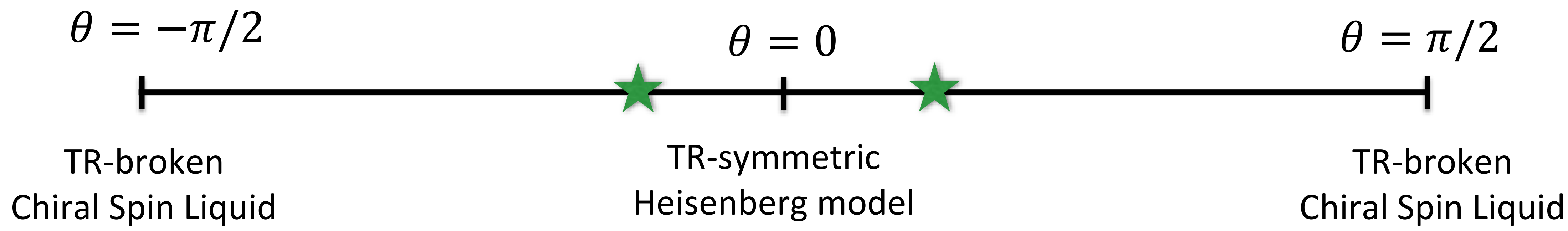
Phase diagram

$$H = \cos\theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \sin\theta \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



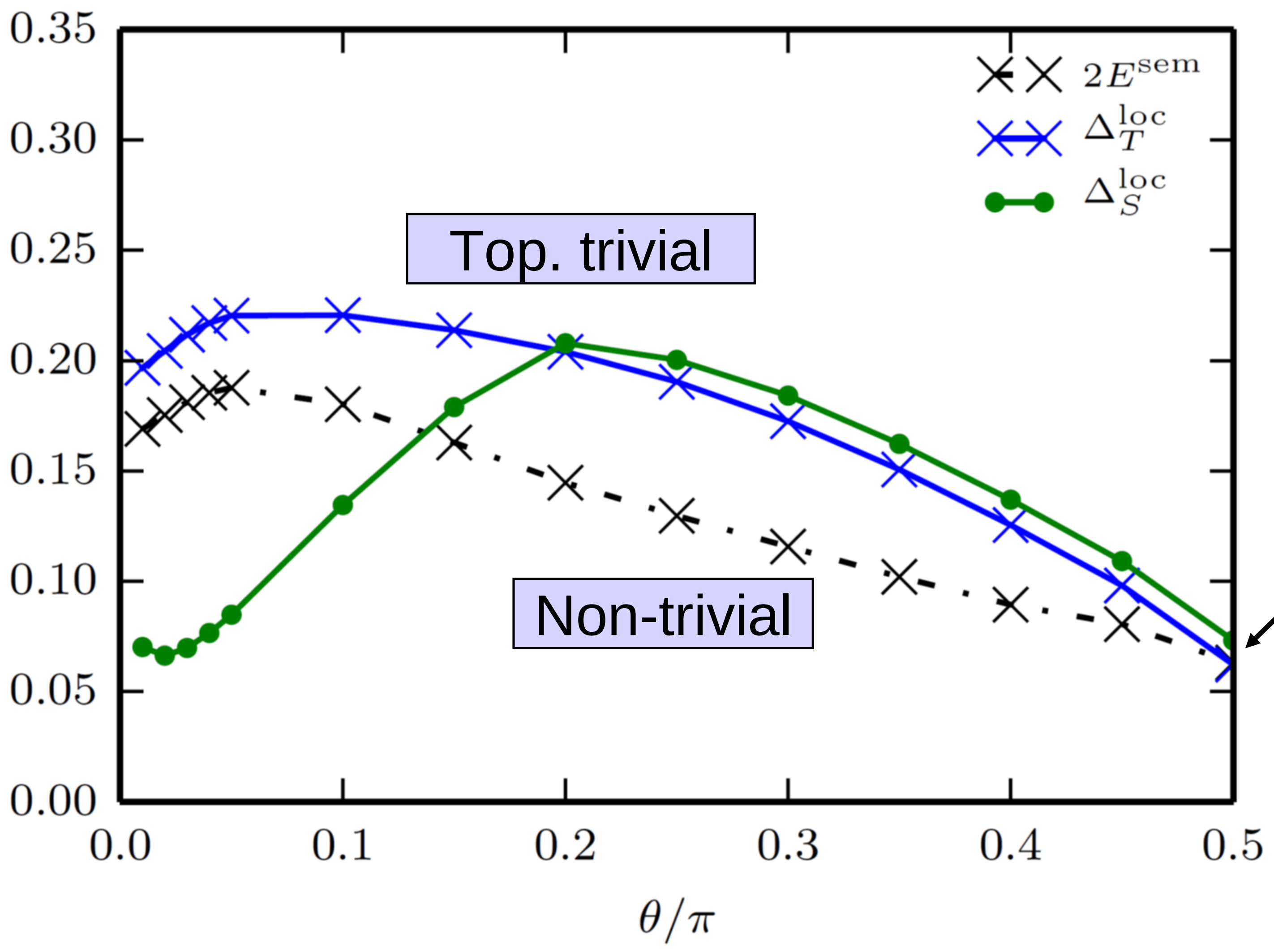
Phase diagram

$$H = \cos\theta \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \sin\theta \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



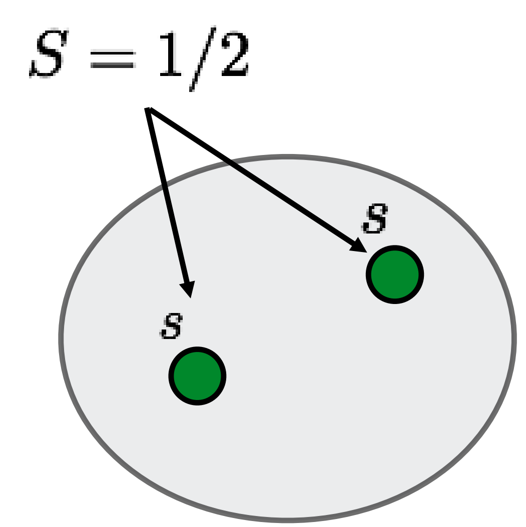
- Transition from CSL to $\mathbb{Z}_2$ spin liquid seems continuous and near the Heisenberg point
  - No field theoretic understanding of transition from CSL to toric code, only to double semion (*Barkeshli 2012*)
  - Double-semion transition seems unlikely due to recent no-go theorem in the absence of enlarged unit cell (*Zaletel & Vishwanath 2015*)
  - TR-symmetric point as first-order transition between two CSLs? Unlikely.
- Weakly first order or tiny intermediate phase can never be ruled out.

Excitation energies



At the chiral point:

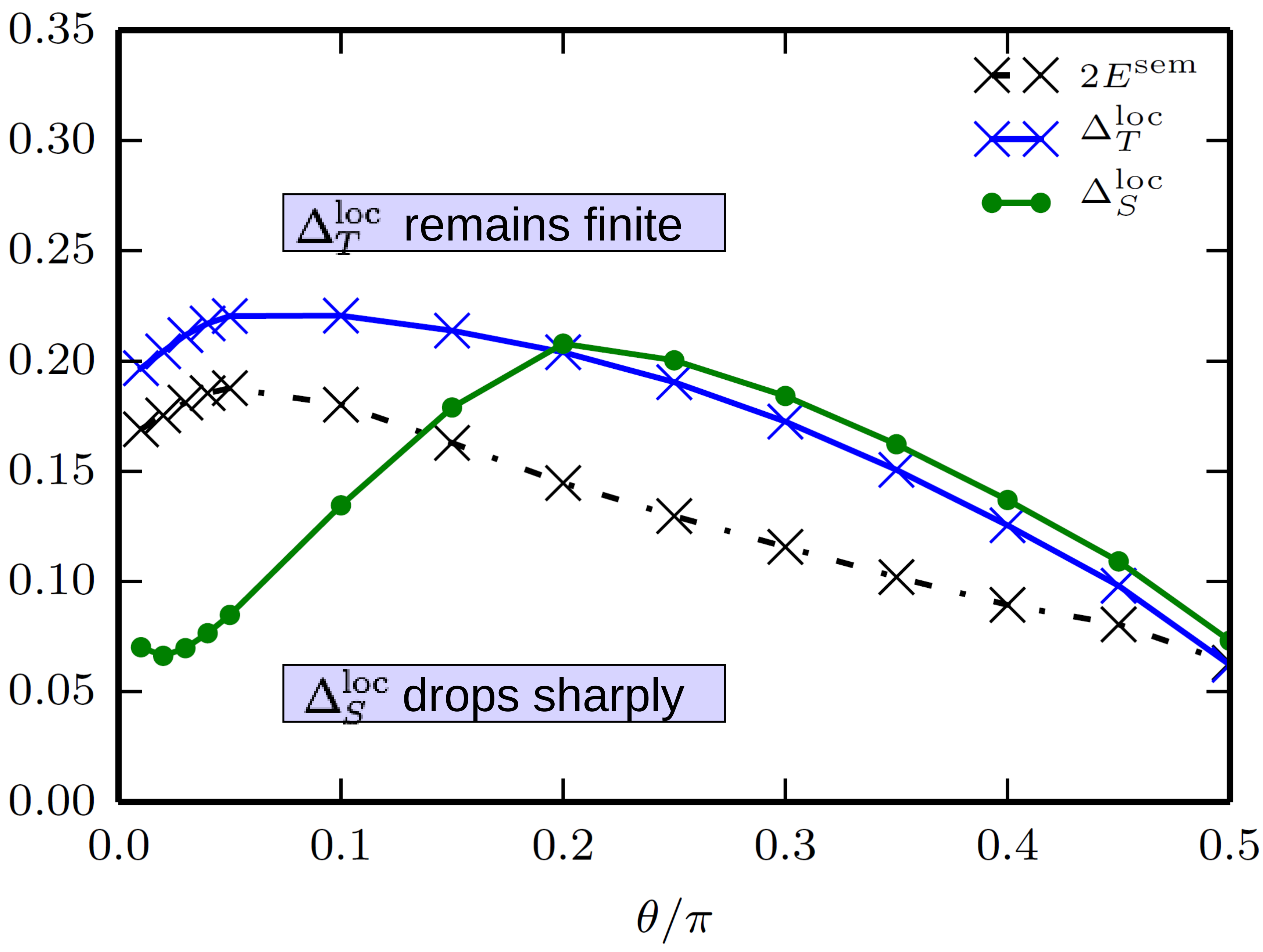
$$2E^{\mathrm{sem}} = \Delta_S^{\mathrm{loc}} = \Delta_T^{\mathrm{loc}}$$



Enclosed spin

$$S_{\mathrm{tot}} = 1/2 \oplus 1/2 = 0 \otimes 1$$

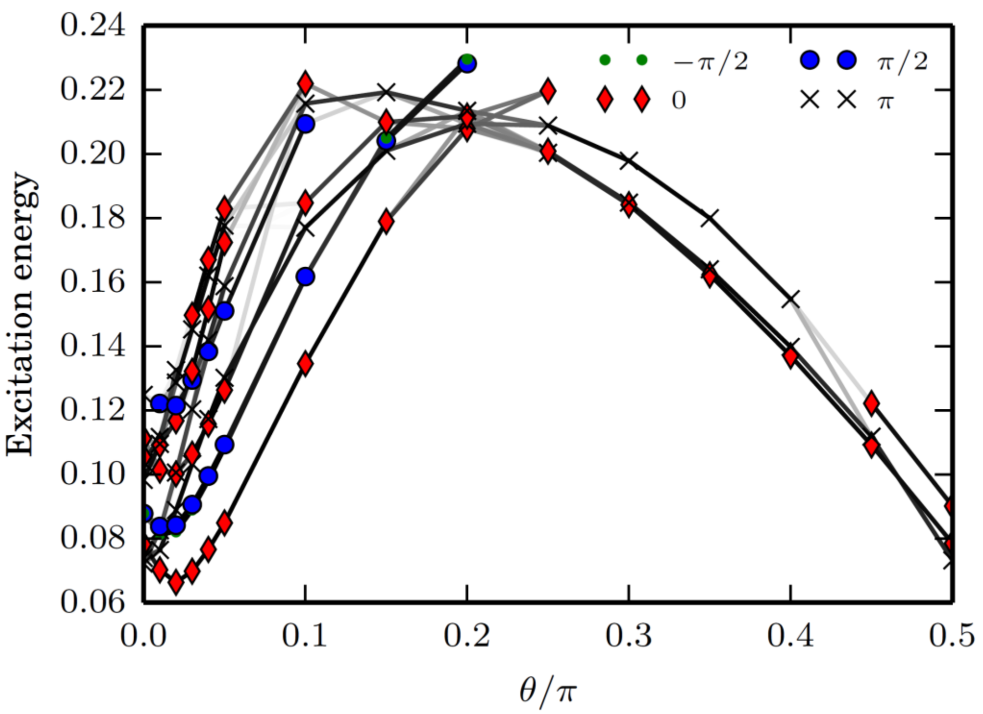
Excitation energies



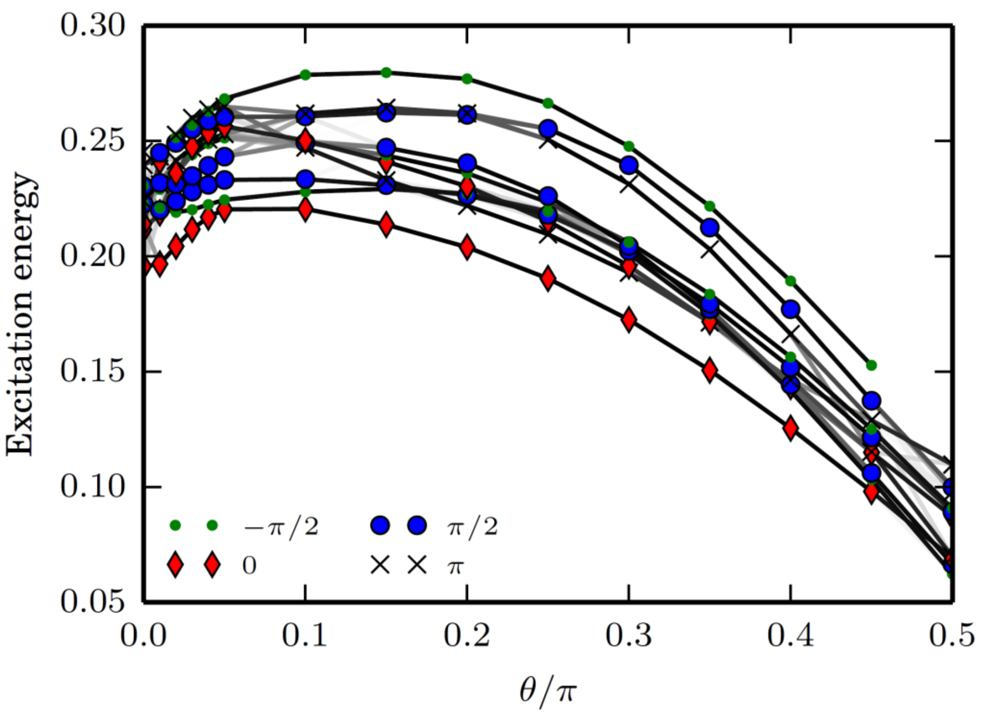
Anyons do not interact

Excitations

Singlets



Triplets



- Calculate several excited states

$$\{|S_\alpha(\theta)\rangle\}, \{|T_\alpha(\theta)\rangle\}$$

- Calculate overlaps

$$|\langle S_\alpha(\theta)|S_{\alpha'}(\theta')\rangle|^2$$

$$|\langle T_\alpha(\theta)|T_{\alpha'}(\theta')\rangle|^2$$

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Open questions and future directions

Experimental relevance

TR-invariant spin liquids:
- Realistic theoretical models contested
- Good material candidates (Herbertsmithite, Kapellasite, …)

TR-invariant spin liquids:
- Firmly established models
- Material candidates?

Dichotomy?

- Our model: Kagome Hubbard + orbital magnetic field
  - Can orbital field be strong enough? Not for Herbertsmithite (J too large, lattice spacing too small).
  - Other materials?
- Materials with spontaneous TR-symmetry breaking?
- Cold atoms (see *Nielsen, Sierra & Cirac '14*)
- Detection: thermal Hall realistic? Other probes?

Conclusions & Outlook

- Proposed a physically motivated model for a Mott insulator where time-reversal symmetry is broken by a three-spin chiral interaction
- Not today: Provided an intuitive network-model perspective on the emergent phase
- Unambiguous identification of the chiral spin liquid phase by its universal properties
  - Ground state degeneracy
  - Chiral edge state
  - Topological excitations
- Open questions remain regarding the phase diagram and transition into TR-symmetric phases