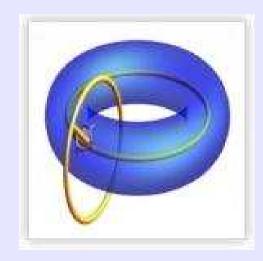
quasi-particle pictures from continuous unitary transformations

Kai Phillip Schmidt





24.02.2016



Entanglement in Strongly Correlated Systems

The study of entanglement in strongly correlated systems has lived a series of important advances in recent years, in turn underpinning a better understanding of the quantum properties of matter. Concerning numerics, examples of these are e.g. new numerical methods based on tensor networks, as well as advances in quantum Monte Carlo, exact diagonalizations, and **continuous unitary transformations**....

overview

- continuous unitary transformations (CUTs) spirit quasi-particle pictures
- perturbative CUTs (pCUTs)
 transverse-field Ising model on the three-dimensional
 Swedenborgite lattice
- graph-based CUTs (gCUTs)
 generalized notion of linked-cluster theorem
 quasi-particle decay
- momentum-space CUTs
 magnon and Higgs particles in an ordered quantum magnet

numerical tools for quantum-lattice models

- exact diagonalization (ED)
- quantum Monte Carlo (QMC)
- tensor-network approaches DMRG, iPEPS, ...
- ...

numerical tools for quantum-lattice models

- exact diagonalization (ED)
- quantum Monte Carlo (QMC)
- tensor-network approaches DMRG, iPEPS, ...
- ...
- continuous unitary transformations (CUTs) linked-cluster expansions (LCEs) non-perturbative LCEs

continuous unitary transformations (CUTs)

CUT

$$\mathcal{H}(\ell) = U(\ell)\mathcal{H}U^{\dagger}(\ell)$$

flow equation

$$\frac{d\mathcal{H}(\ell)}{d\ell} = [\eta(\ell), \mathcal{H}(\ell)]$$

effective model

$$\mathcal{H}(\ell=\infty)=\mathcal{H}_{\mathrm{eff}}$$

'94 Wegner '93/94 Glazek/Wilson

infinite hierarchy of coupled equations truncation schemes necessary!

see also works by Wegner, Stein, Toda, Mielke, Kehrein, ...

effective models and quasi-particles

photobucket.com

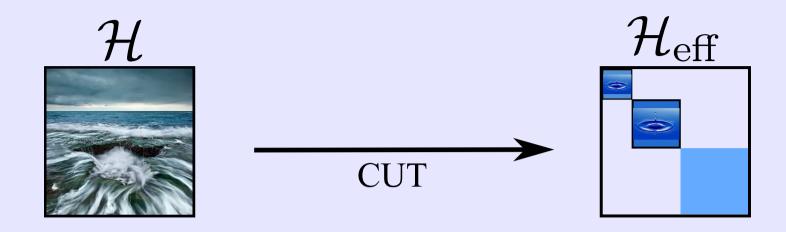
correlated system





low energy

quasi-particle conserving CUT



- lacktriangle counting operator $\mathcal Q$ with $\mathcal Q|i
 angle=q_i|i
 angle$
- quasi-particle generator

$$\eta_{i,j}^{\mathcal{Q}} = \operatorname{sgn}(q_i - q_j)\mathcal{H}_{i,j}$$

quasi-particle conserving effective Hamiltonian with $[\mathcal{Q}, \mathcal{H}_{\text{eff}}] = 0$ '97 Stein '98 Mielke '00 Knetter/Uhrig

truncation schemes

perturbative CUT

high-order series expansions thermodynamic limit

Knetter/Uhrig, EPJB (2000) Knetter/KPS/Uhrig, J. Phys. A (2003) Coester/KPS, PRE (2015)

graph-based CUT

combine graph theory with CUTs thermodynamic limit

Yang/KPS, EPL (2011)

Coester/Clever/Herbst/Capponi/KPS, EPL (2015)

Coester/KPS (2016)



self-similar CUT

operator flow thermodynamic limit Heidbrink/Uhrig, PRL (2002) Fischer/Duffe/Uhrig, NJP (2010) Krull/Drescher/Uhrig, PRB (2013) Powalski/Uhrig/KPS, PRL (2015)

see also posters by David Schneider and Serkan Sahin!

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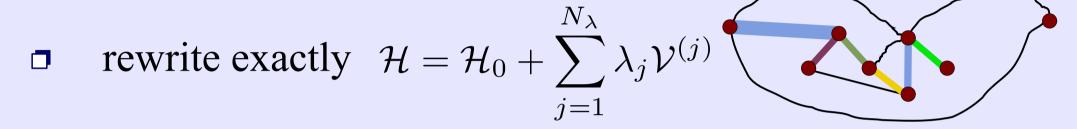
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set up

consider lattice of supersites (spin, dimer, ...)



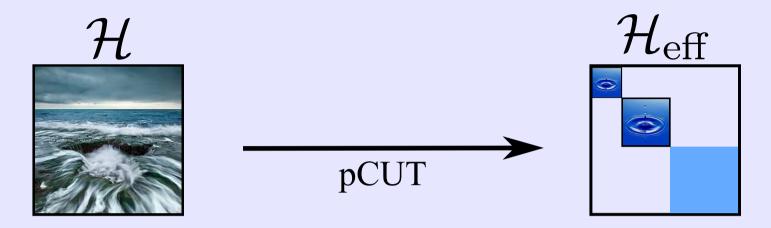
unperturbed part diagonal in supersites

$$\mathcal{H}_0 = E_0 + \mathcal{Q} \quad \text{mit} \quad \mathcal{Q} \equiv \sum_i \hat{n}_i \equiv \sum_{i,\alpha} \hat{f}_{i,\alpha}^{\dagger} \hat{f}_{i,\alpha}$$

interaction $\mathcal{V}^{(j)}$ couples supersites (colors)

$$\mathcal{H} = \mathcal{H}_0 + \sum_{n=-N}^{N} T_n \quad \text{mit} \quad [\mathcal{Q}, T_n] = nT_n$$

perturbative CUT



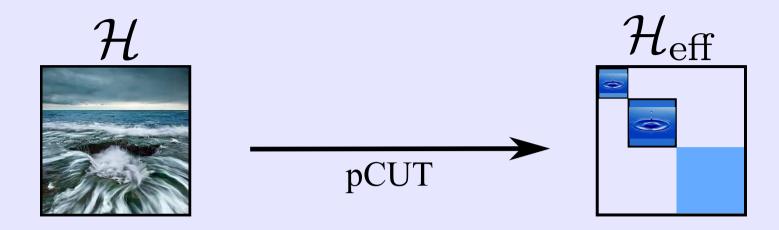
 \blacksquare if spectrum of \mathcal{H}_0 is equidistant in

$$\mathcal{H} = \mathcal{H}_0 + \sum_{n=-N}^{N} T_n \quad \text{with} \quad [\mathcal{Q}, T_n] = nT_n$$

then perturbative CUT yields model-independently

$$\mathcal{H}_{\text{eff}}(\{\lambda_j\}) = \mathcal{H}_0 + \sum_{\sum_j k_j = k}^{\infty} \lambda_1^{k_1} \dots \lambda_{N_{\lambda}}^{k_{N_{\lambda}}} \sum_{|\underline{m}| = k \sum_i m_i = 0} C(\underline{m}) T(\underline{m})$$

perturbative CUT



model-independent effective Hamiltonian

$$\mathcal{H}_{\text{eff}}(\{\lambda_j\}) = \mathcal{H}_0 + T_0 + T_1 T_{-1} - T_{-1} T_1 + \frac{1}{2} T_2 T_{-2} - \frac{1}{2} T_{-2} T_2 \dots$$

 \blacksquare \mathcal{H}_{eff} not normal-ordered (model-dependent part!)

pCUT as a linked-cluster expansion

pCUT as a linked-cluster expansion

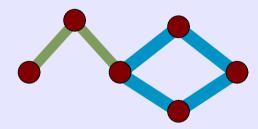


divide in finite clusters and embed in thermodynamic limit

quantum many-body problem

$$\mathcal{H} = \mathcal{H}_0 + \lambda V$$

lattice with sites and links



focus on zero temperature



Goal

calculate high-order series expansion (order 10-20) in λ

$$\mathcal{H} = \mathcal{H}_0 + \lambda V$$

- use extrapolation schemes to
 - access quantum critical regime
 - determine properties of elementary excitations



Extensive quantities

• e.g. ground-state energy per site (also entanglement entropies)

$$\epsilon_0 \equiv \frac{E_0}{N} = \sum_{\mathcal{G}_{\nu}} c(\mathcal{G}_{\nu}) \epsilon(\mathcal{G}_{\nu})$$

- sum over linked clusters \mathcal{G}_{ν}
- reduced energies via subcluster subtractions

$$\epsilon(\mathcal{G}_{\nu}) = E(\mathcal{G}_{\nu}) - \sum_{\mathcal{G}_{\nu'} \in \mathcal{G}_{\nu}} c(\mathcal{G}_{\nu'}/\mathcal{G}_{\nu}) \epsilon(\mathcal{G}_{\nu'})$$

Excitations

quasi-particles, e.g. magnons, triplons, anyons, ...

lacksquare momentum $ec{k}$, dispersion $\omega(ec{k})$

• one quasi-particle dispersion '96 Gelfand

• two quasi-particle interaction '00 Trebst et al.

'01 Zheng et al.

'01 Knetter/KPS/Uhrig

• spectral densities '01 Knetter et al...

'01 KPS/Knetter/Uhrig

topological order / anyons '08- Dusuel/Vidal/KPS

Properties

- thermodynamic limit
- scales well with increasing dimensions
- perturbative, but extrapolations



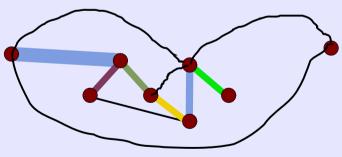
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Challenges

- systems with various couplings
- long-range interactions
- disorder
- non-perturbative extensions



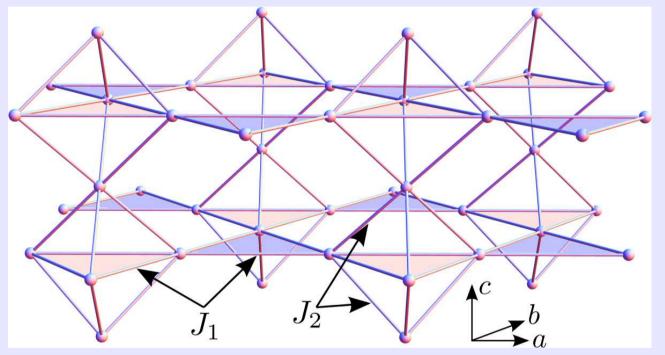


→ white graphs

Coester/KPS, PRE (2015)

Sikkenk/Coester/Fritz/KPS (2016)

sweden borgites



CaBaCo₂Fe₂O₇
YBaCo₄O₇
(not Ising spins)

- transverse field Ising model
- limits

h = 0 pure Ising (classical)

 $J_1 = 0$ TFIM on quasi 1d chains

 $J_2 = 0$ TFIM kagome lattice

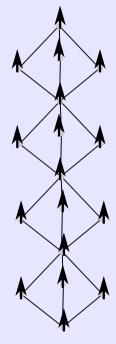
Buhrandt/Fritz, PRB (2014)

Powalski/Coester/Moessner/KPS, PRB (2013)

decoupled chains on the Swedenborgite lattice ($J_1 = 0$)

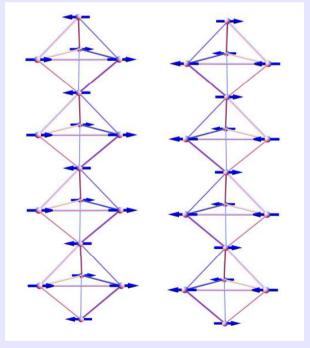
- unfrustrated TFIM on quasi-1d chain
- quantum phase transition in 2d Ising universality class

polarized state



$$J_2 = 0$$

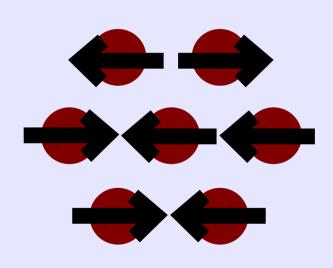
Ising macro-spin



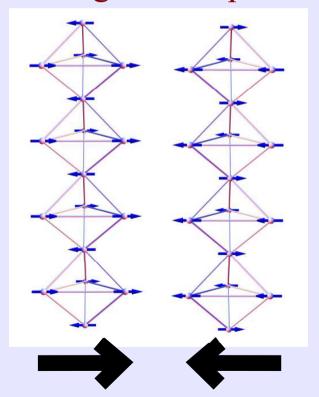
$$h = 0$$

decoupled chains on the Swedenborgite lattice ($J_1 = 0$)

- \bullet h = 0: atomic limit for Ising macro-spins
- Ising macro-spins build effective triangular lattice

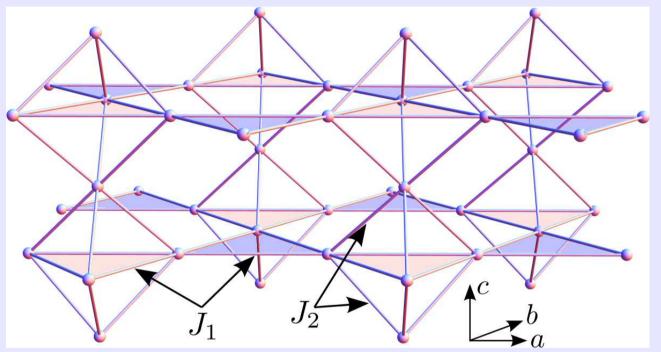


Ising macro-spin



Sikkenk/Coester/Fritz/KPS (2016)

sweden borgites



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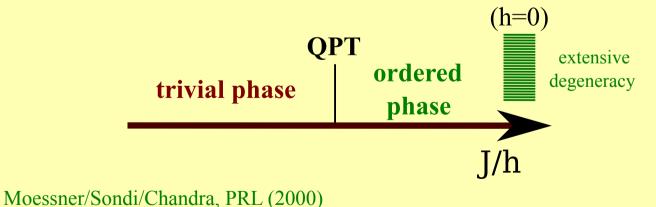
Buhrandt/Fritz, PRB (2014)

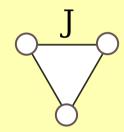
Powalski/Coester/Moessner/KPS, PRB (2013)

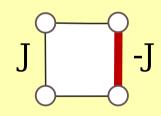
fully frustrated Ising models

$$H = \sum_{\langle i,j \rangle} J_{ij} \, \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

- h=0: extensively many ground states
- J=0: fully polarized phase (trivial)
- generically "order by disorder"





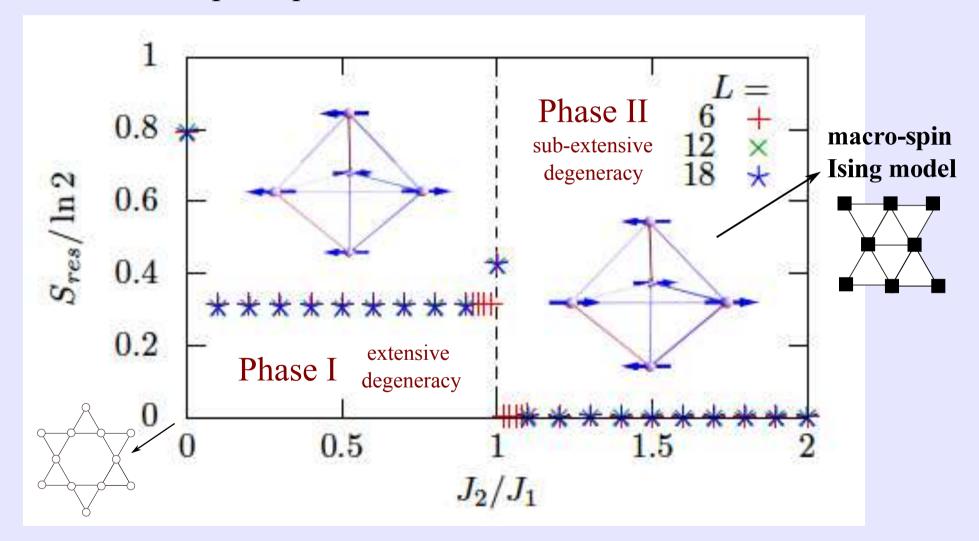


Moessner/Sondi, PRB (2001)

Ising model on the Swedenborgite lattice (h = 0)

Buhrandt/Fritz, PRB (2014)

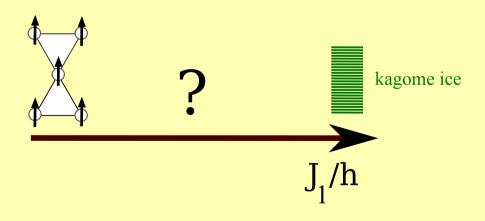
two classical spin liquids I and II

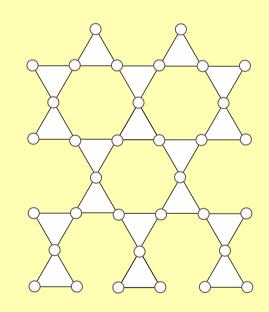


kagome ice in a transverse field ($J_2 = 0$)

$$H = J_1 \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

- $J_1=0$: fully polarized phase





kagome ice in a transverse field ($J_2 = 0$)

$$H = J_1 \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

- $J_1=0$: fully polarized phase

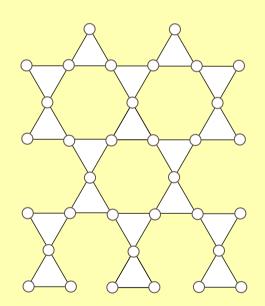
polarized phase

polarized phase

J₁/h

Moessner/Sondi/Chandra, PRL (2000)

Moessner/Sondi, PRB (2001)



disorder by disorder $(J_2 = 0)$

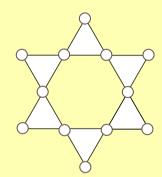
$$H = J_1 \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x$$

polarized phase

J₁/h

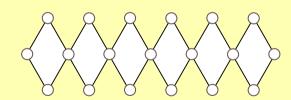
disorder by disorder scenario for kagome TFIM

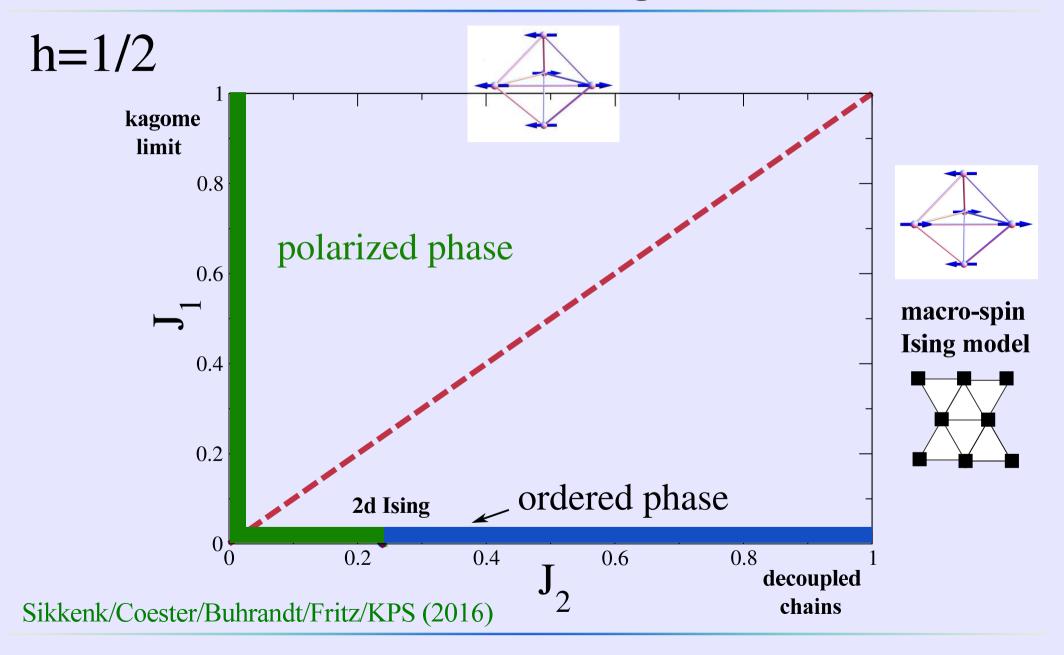
Powalski/Coester/Moessner/KPS, PRB (2013)



 disorder by disorder for fully-frustrated diamond chain can be deduced analytically

Coester/Malitz/Fey/KPS, PRB (2013)





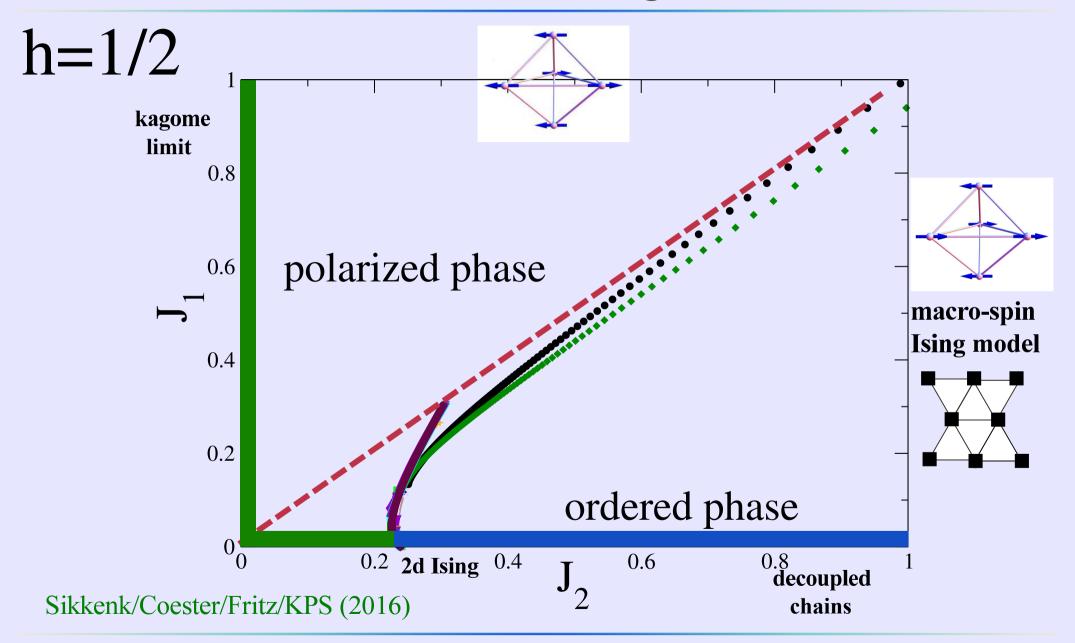
LCEs on the Swedenborgite lattice

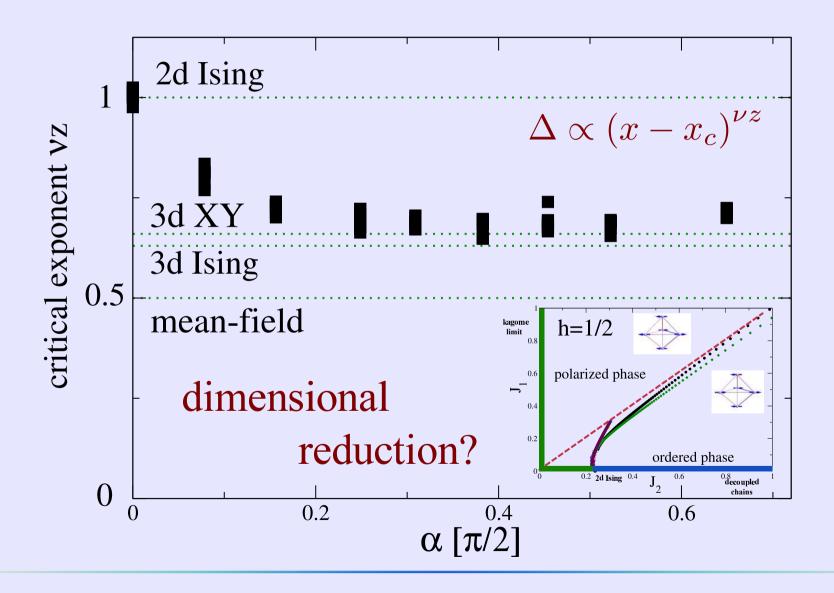
$$\mathcal{H} = h \sum_{i} \sigma_{i}^{x} + J_{1} \sum_{i} \sigma_{i}^{z} \sigma_{j}^{z} + J_{2} \sum_{i} \sigma_{i}^{z} \sigma_{j}^{z}$$

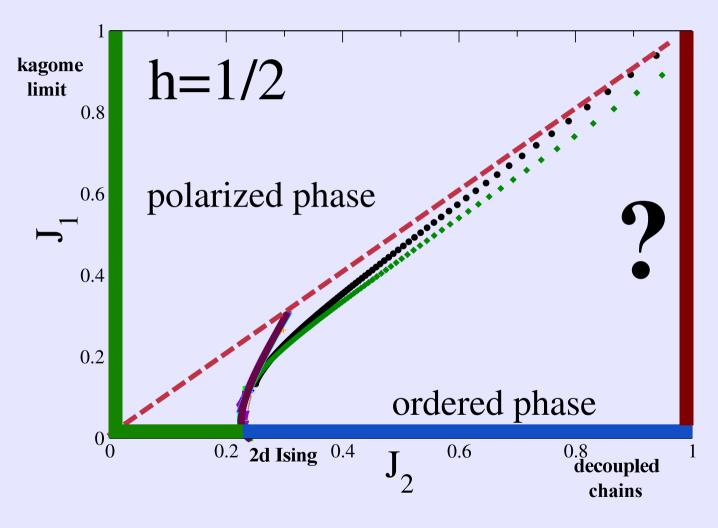
$$\langle i, j \rangle \in \text{same layer} \quad \langle i, j \rangle \in \text{different layer}$$

- lacktriangledown high-field expansion in J_1 and J_2 up to order 11
- □ linktypes = "color"
- white-graph expansion
 ignore color for generating topologically distinct graphs, but do an optimal bookkeeping!

Coester/KPS, PRE (2015)

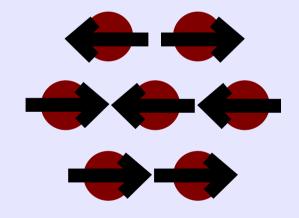






degenerate macro spins

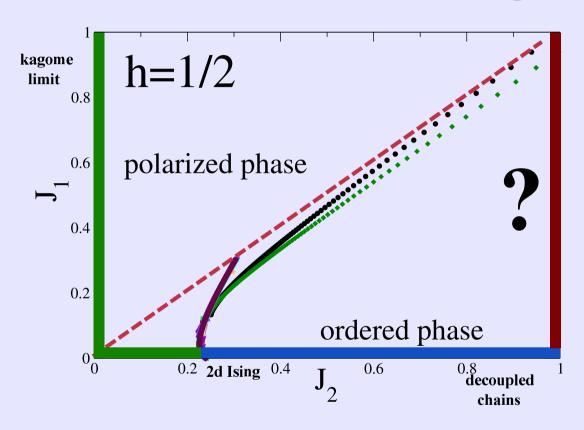
$$J_2 > J_1 \gg h$$



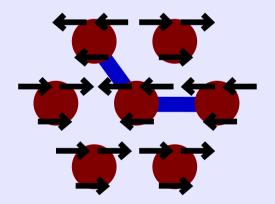
ground states of triangular Ising model

$$J_2 > J_1 \gg h$$

degenerate macro spins

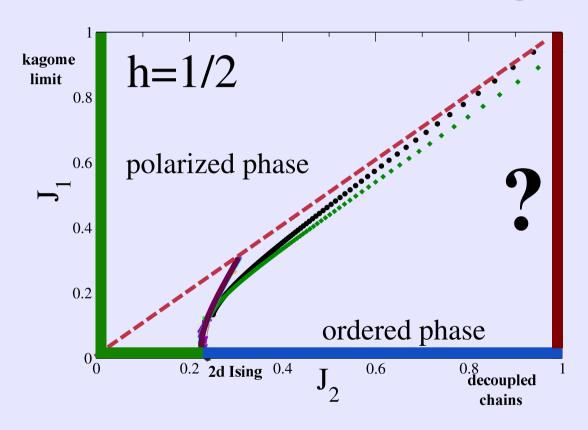


 effective diagonal 2d classical dimer model in frustrated links

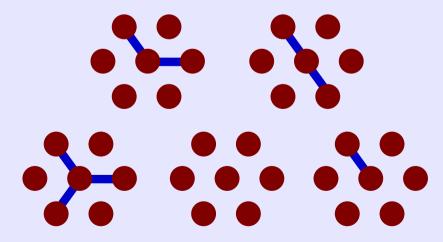


$$J_2 > J_1 \gg h$$

degenerate macro spins



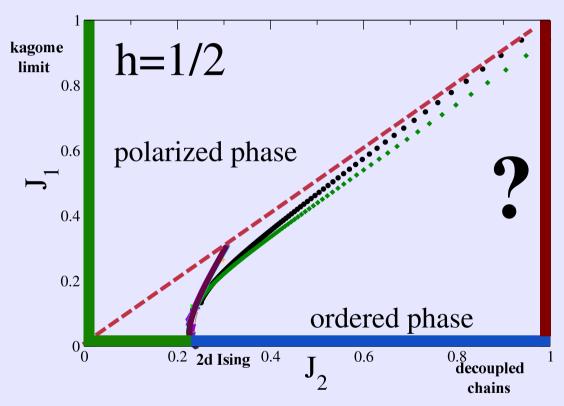
- effective diagonal 2d classical dimer model in frustrated links
- all configurations stay degenerate in $\mathcal{O}(h^2)$



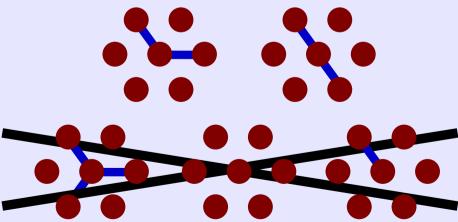
TFIM on the Swedenborgite lattice

$$J_2 > J_1 \gg h$$

degenerate macro spins



- effective diagonal 2d classical dimer model in frustrated links
- configurations with 2 frustrated links in $\mathcal{O}(h^4)$

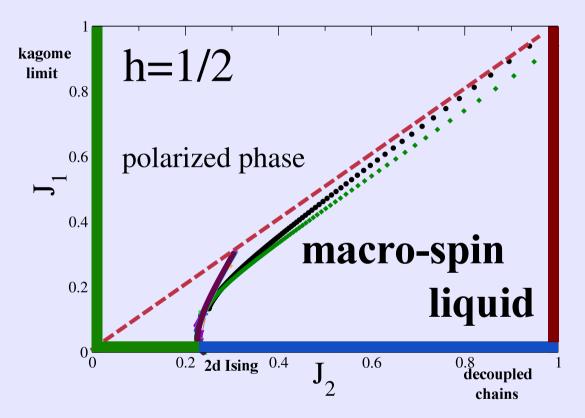


Sikkenk/Coester/Fritz/KPS (2016)

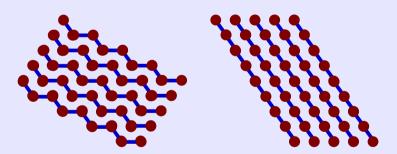
TFIM on the Swedenborgite lattice

$$J_2 > J_1 \gg h$$

degenerate macro spins



- effective diagonal 2d classical dimer model in frustrated links
- emergent stripe structures sub-extensively degenerate



exact degeneracy on finite clusters and at least up $\mathcal{O}(h^{10})$

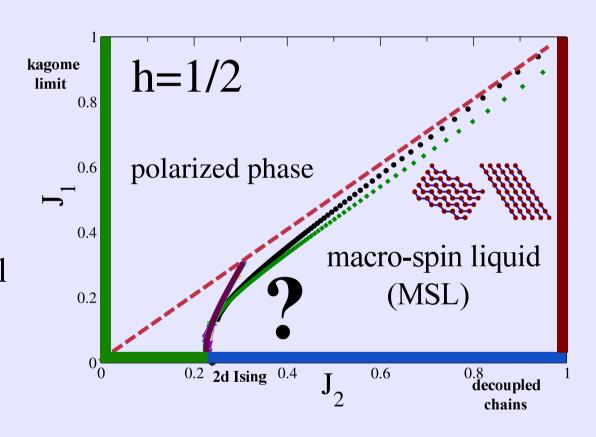
Sikkenk/Coester/Fritz/KPS (2016)

TFIM on the Swedenborgite lattice

Sikkenk/Coester/Fritz/KPS (2016)

- white graphs efficient
- disorder by disorder
- dimensional reduction (2+1)d from high-field expansion
- effective diagonal
 2d classical dimer model
 in terms of macro-spins

dimensional reduction (3+1)d to 2d



perturbative CUTs and LCEs

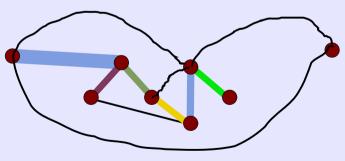
Properties

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- scales well with increasing dimensions
- perturbative, but extrapolations

Challenges

- systems with various couplings
- long-range interactions
- disorder
- non-perturbative extensions





→ white graphs
Coester/KPS, PRE (2015)

truncation schemes

perturbative CUT

high-order series expansions thermodynamic limit

Knetter/Uhrig, EPJB (2000) Knetter/KPS/Uhrig, J. Phys. A (2003) Coester/KPS, PRE (2015)

graph-based CUT

combine graph theory with CUTs thermodynamic limit

Yang/KPS, EPL (2011)

Coester/Clever/Herbst/Capponi/KPS, EPL (2015)

Coester/KPS (2016)

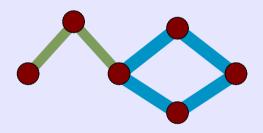


self-similar CUT

operator flow thermodynamic limit Heidbrink/Uhrig, PRL (2002) Fischer/Duffe/Uhrig, NJP (2010) Krull/Drescher/Uhrig, PRB (2013) Powalski/Uhrig/KPS, PRL (2015)

non-perturbative linked-cluster expansions (NLCEs)

- do graph expansion as before
- calculate non-perturbatively(e.g. ED) on graphs!





divide in finite clusters and embed in thermodynamic limit

non-perturbative linked-cluster expansions (NLCEs)

see also poster by Dominik Ixert!

History and developments

- ground-state energy for lattice gauge theories (ED) '84 Irving/Hamer
- thermodynamic quantities for quantumspin models (ED)'06 Rigol/Bryant/Singh
- excitations and effective models
 (gCUT)
 '11 Yang/KPS
 '15 Coester/Clever/Herbst/Capponi/KPS
- entanglement entropies
 (ED,DMRG)
 '14 Stoudenmire/Gustainis/Johal/Wessel/Melko
- quantum quenches and many-body localization '14 Rigol in the thermodynamic limit (ED)
 '15 Tang/Iyer/Rigol

graph-based CUTs

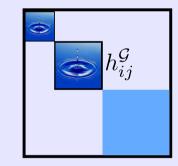
Idea

- generate topologically distinct graphs
- solve flow-equation on each graph

$$\partial_l \mathcal{H}^{\mathcal{G}_{
u}} = [\eta^{\mathcal{G}_{
u}}, \mathcal{H}^{\mathcal{G}_{
u}}]$$







block-diagonal in particle-number operator

Yang/KPS EPL (2011)

graph-based CUTs

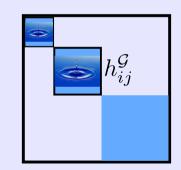
Idea

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$$\partial_l \mathcal{H}^{\mathcal{G}_{
u}} = [\eta^{\mathcal{G}_{
u}}, \mathcal{H}^{\mathcal{G}_{
u}}]$$



block-diagonalization unitary transformation



- subgraph subtractions for matrix elements (gse, hopping elements, ...)
- embedding in the thermodynamic limit

Yang/KPS EPL (2011)

graph-based CUTs

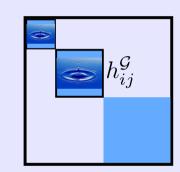
Idea

- generate topologically distinct graphs
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$$\partial_l \mathcal{H}^{\mathcal{G}_
u} = [\eta^{\mathcal{G}_
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u}]$$



block-diagonalization
unitary transformation

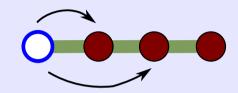


- on graph numerically exact solution of CUT (finite matrices ...)
- for ground-state energy: just identical to NLCE with ED
- convergence expected for finite correlation length

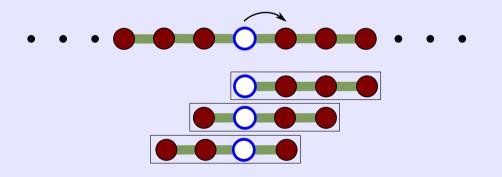
Yang/KPS EPL (2011)

one-particle dispersion

lacktriangle hopping matrix $h_{ij}^{\mathcal{G}}$ on each cluster



embedding restores translation symmetry

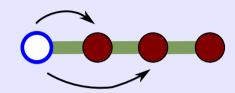




lacksquare Fourier transformation yields $\omega(ec{k})$

one-particle dispersion

lacktriangle hopping matrix $h_{ij}^{\mathcal{G}}$ on each cluster



embedding restores translation symmetry

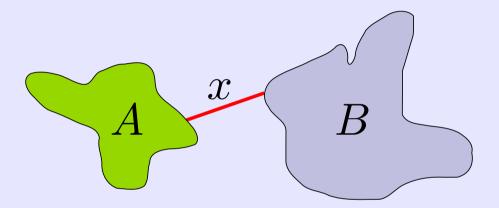




really true non-perturbatively?

$$\mathcal{H} = \mathcal{H}_0 + \lambda V$$

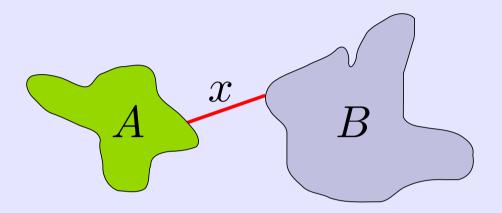
consider two clusters



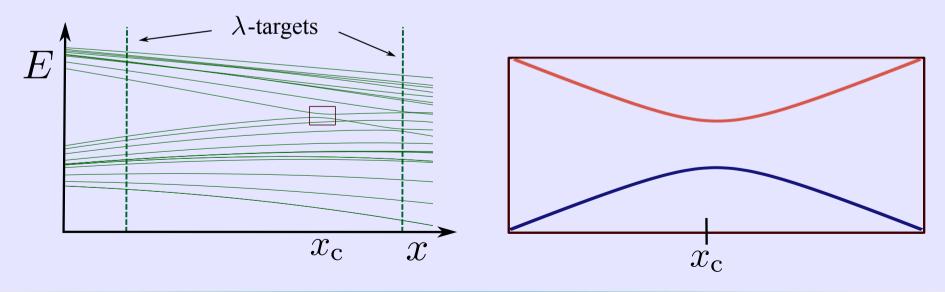
Coester/Clever/Herbst/Capponi/KPS EPL (2015)

Coester/Clever/Herbst/Capponi/KPS EPL (2015)

consider two clusters

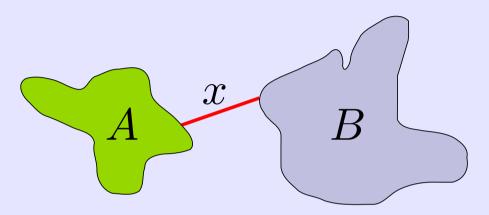


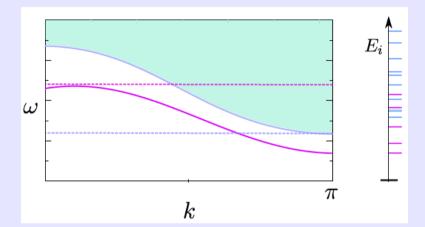
anti-level crossings



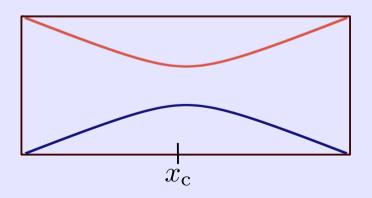
Coester/Clever/Herbst/Capponi/KPS (2015)

consider two clusters





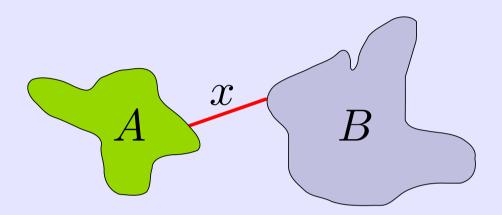
artificial anti-level crossings (assume no true decay, "pseudo-decay")



- drastic change of hopping elements
- non-perturbative effect
- relation to intruder states in quantum chemistry Malrieu et al. (1985)
- breakdown of conventional NLCE

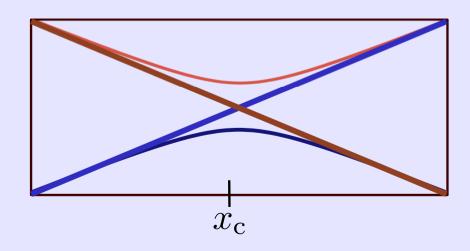
Coester/Clever/Herbst/Capponi/KPS (2015)

consider two clusters



need level crossing!

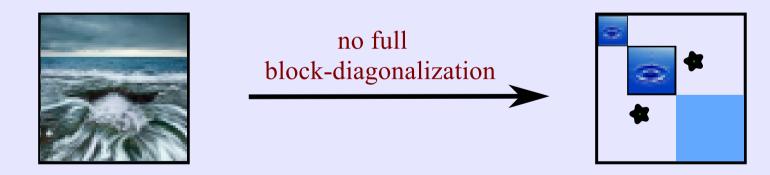
generalized notion of cluster additivity needed!



Coester/Clever/Herbst/Capponi/KPS (2015)

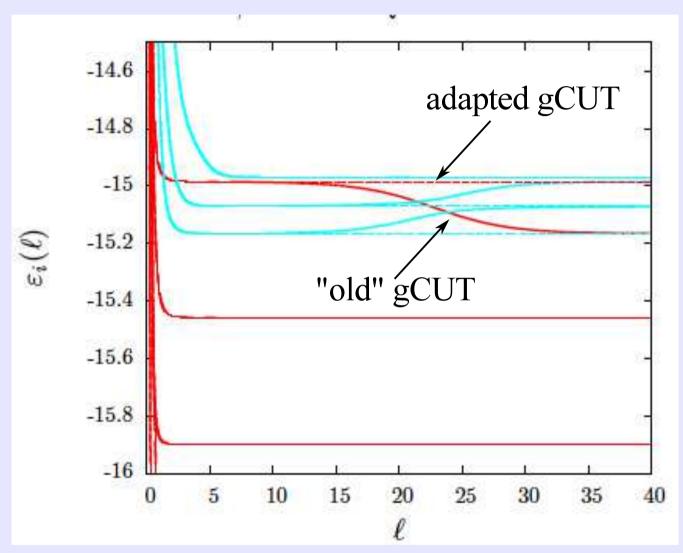
generalized cluster additivity

- reduced graph symmetry yields artificial entanglement
- one has to disentangle such levels
- adapted graph-based CUTs



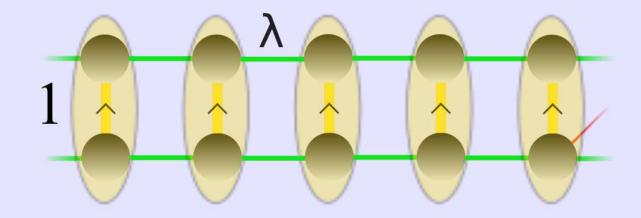
beyond the paradigm of using exact eigenvectors!

Coester/Clever/Herbst/Capponi/KPS (2015)



flow axis gives freedom which one can use!

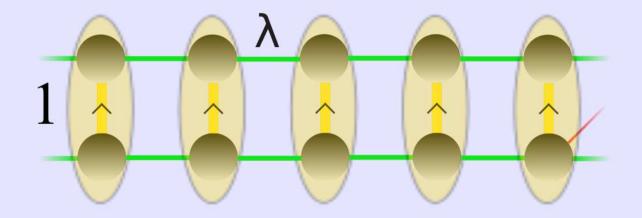
two-leg Heisenberg ladder



- $\lambda = 0$: product state of singlets, excitations local triplets
- \Box finite λ : triplon quasi-particles, finite dispersion

'03 KPS/Uhrig

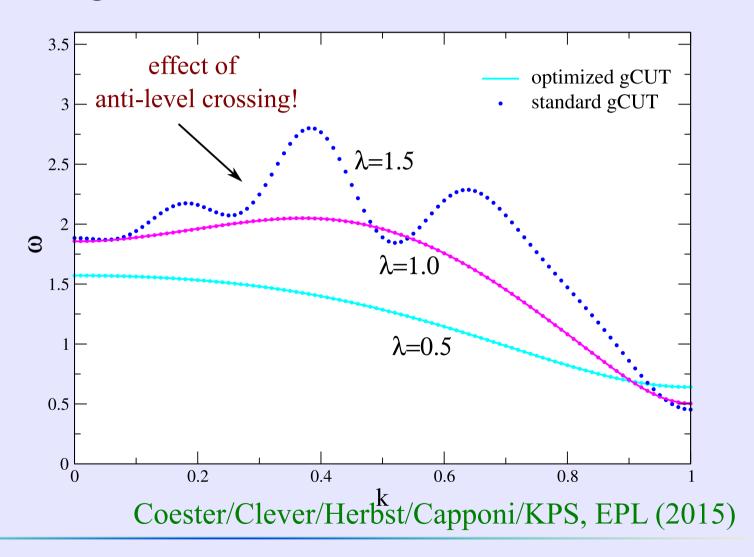
two-leg Heisenberg ladder



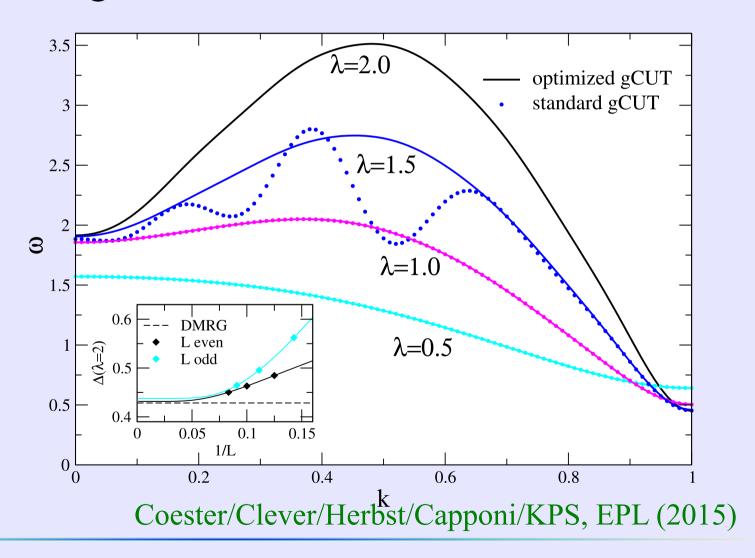
- apply standard and optimized gCUTs
- consider graphs up to 12 dimers

Coester/Clever/Herbst/Capponi/KPS, EPL (2015)

two-leg Heisenberg ladder



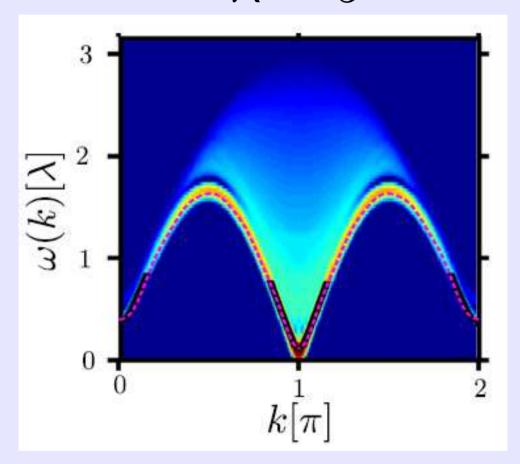
two-leg Heisenberg ladder



two-leg Heisenberg ladder

$$\lambda = 5$$

comparison gCUT/DMRG



DMRG data from Schmidinger et al., PRB (2013)

Coester/KPS (2016)

summary - graph-based CUTs



non-perturbative LCEs

- for excited states, fundamental challenge due to reduced symmetry of graphs (intruder states)
- generalized notion of cluster additivity
- challenge can be solved with graph-based
 CUTs (also possible for true quasi-particle decay!)

truncation schemes

perturbative CUT

high-order series expansions thermodynamic limit

Knetter/Uhrig, EPJB (2000) Knetter/KPS/Uhrig, J. Phys. A (2003) Coester/KPS, PRE (2015)

graph-based CUT

combine graph theory with CUTs thermodynamic limit

Yang/KPS, EPL (2011)

Coester/Clever/Herbst/Capponi/KPS, EPL (2015)

Coester/KPS (2016)



self-similar CUT

operator flow thermodynamic limit Heidbrink/Uhrig, PRL (2002) Fischer/Duffe/Uhrig, NJP (2010) Krull/Drescher/Uhrig, PRB (2013) Powalski/Uhrig/KPS, PRL (2015)

set up

- pCUT/gCUT good for gapped quantum phases
 - gapless phases ? (here: ordered magnets)
- momentum space

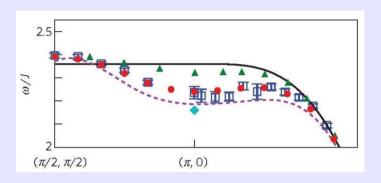
$$\mathcal{H} = E_0 + \sum_k \omega_k b_k^{\dagger} b_k + \sum_k \Gamma_k \left(b_k^{\dagger} b_{-k}^{\dagger} + \text{h.c.} \right) + \dots$$

- truncation scheme?
 - \longrightarrow idea: scaling dimension d of operators $k_i \to \lambda k_i$

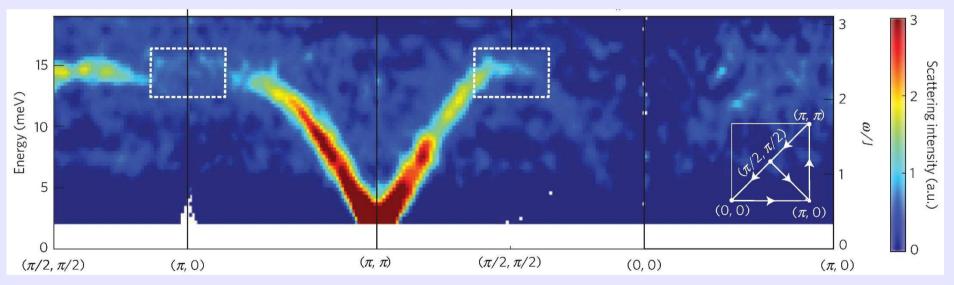
$$d=1$$
 self-consistent mean-field theory $d=2$ beyond

square lattice spin-1/2 Heisenberg model

Fractional spinons in the square lattice Heisenberg?



spin wave theory magnon perturbation up to 1/S³ spinon calculation experiment series expansion around Ising limit quantum Monte Carlo

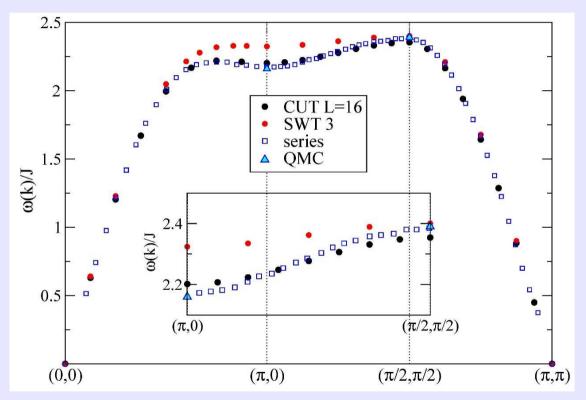


 $Cu(DCOO)_24D_2O$

Dalla Piazza et al., Nat. Phys. (2015)

momentum space CUTs

quantitative magnon description



roton minimum from magnon-Higgs scattering

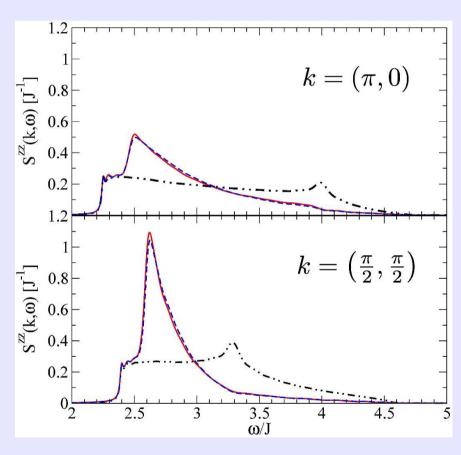
$$\mathcal{H} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

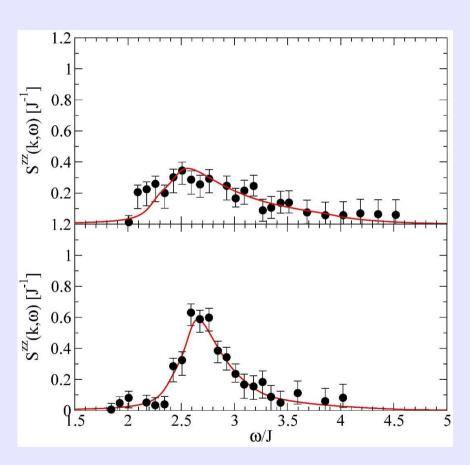
- **□** square lattice, spin 1/2
- Dyson-Maleev representation
- **CUT** in momentum space
- truncation based on scaling dimensions

Powalski/Uhrig/KPS, PRL (2015)

momentum space CUTs

longitudinal dynamic structure factor





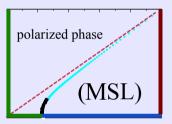
"Higgs" resonance

Powalski/Uhrig/KPS, PRL (2015)

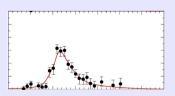
summary

- continuous unitary transformations (CUTs) spirit quasi-particle pictures
- perturbative CUTs (pCUTs)
 transverse-field Ising model on the three-dimensional
 Swedenborgite lattice
- graph-based CUTs (gCUTs)
 generalized notion of linked-cluster theorem quasi-particle decay
- momentum-space CUTs
 magnon and Higgs particles in an ordered quantum magnet









collaborators

• pCUTs - Swedenborgites

Kris Coester (Dortmund) Tycho Sikkenk (Utrecht) Lars Fritz (Utrecht)





Kris

Lars

• gCUTs

Kris Coester (Dortmund)
Frederik Herbst (Dortmund)
Sebastian Clever (Dortmund)
Sylvain Capponi (Toulouse)



Kris



Frederik



Sebastian



Sylvain

momentum-space CUTs

Michael Powalski (Dortmund) Götz Uhrig (Dortmund)



Michael



Götz

