Benasque, 2-5-16

Black holes & attractors in abelian gauged sugra

Plan: I) Motivation
II) $N=2$, $d=4$ gauged sugra coupled to vector- and hypermult.
III) 1st order flow eqns. from squaring of action
IV) A different squaring $\rightsquigarrow$ non-BPS flow eqns.
V) An explicit example: prepot. $F=-iL^0L^1$ + UHM

- based mainly on 1602.01334 w/ N. Petri & M. Rabbiosi
1503.09055 w/ S. Chimento & N. Petri

I) Motivation

AdS bl. holes interesting from 2 different points of view:

i) learn someth. on strongly coupled field theories by studying gravity solns. finite $T \rightsquigarrow$ black holes
e.g. AdS/cond-mat: hologr. superconductors (high-T_c), transition from Fermi-liquid to non-Fermi-liquid behaviour, ...
require bulk scalars charged under a $U(1)$
quark-gluon plasma
⋮

ii) learn someth. on black holes by doing exact computations in dual field theory $\rightsquigarrow$ microstate counting
e.g. Benini/Hristov/Zaffaroni 1511.04085: microscopic entropy calculation for the black holes in $N=2$, $d=4$ FI gauged sugra constructed in 0911.4926. These are dual to a topologically twisted ABJM theory, whose partition fct. can be computed exactly using supersymm. localization techniques.

II) $N=2$, $d=4$ gauged sugra coupled to vector- and hypermult.

bos. field content: graviton e^a_μ
n_V+1 vectors A^Λ_μ, $\Lambda = 0,\dots,n_V$ (graviphoton + n_V fields coming from vect. mult.)
n_V complex scalars z^i, $i=1,\dots,n_V$

$4n_H$ real hyperscalars q^u, $u = 1, \dots, 4n_H$

· z^i parametrize n_V-dim. special Kähler manif. w/ metr. $g_{i\bar{j}}(z,\bar{z})$, base of sympl. bundle w/ cov. hol. sections

$$V = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \quad D_{\bar{i}} V \equiv \partial_{\bar{i}} V - \frac{1}{2}(\partial_{\bar{i}} K) V = 0,$$

obeying the constr. $\langle V | \bar{V} \rangle \equiv \bar{L}^\Lambda M_\Lambda - L^\Lambda \bar{M}_\Lambda = -i$,
K = Kähler pot.

· q^u parametrize quatern. Kähler manif. w/ metr. $h_{uv}(q)$.

n_H-dim. Riem. manif. admitting locally defined triplet $\vec{K}_u{}^v$ of almost complex structures satisfying

$$h^{st} K^x{}_{us} K^y{}_{tw} = -\delta^{xy} h_{uw} + \varepsilon^{xyz} K^z{}_{uw}, \quad x, y = 1,2,3$$

Levi-Civita conn. preserves $\vec{K}$ up to rotation,

$$\nabla_w \vec{K}_u{}^v + \vec{\omega}_w \times \vec{K}_u{}^v = 0$$

connection of the $SU(2)$ bundle for which the quat. manif. is the base

· We shall only gauge abelian isometries of the quat. Kähler metric h_{uv}, generated by Killing vectors $k^u_\Lambda(q)$, $[k_\Lambda, k_\Sigma] = 0$. Requirement that quat. Kähler structure be preserved implies existence, for each Killing vector, of a triplet of Killing potentials (moment maps) P^x_Λ such that

$$D_u P^x_\Lambda \equiv \partial_u P^x_\Lambda + \varepsilon^{xyz} \omega^y{}_u P^z_\Lambda = -2\Omega^x{}_{uv} k^v_\Lambda$$

$SU(2)$ curv.

· bosonic Lagrangian:

$$e^{-1}\mathcal{L} = \frac{R}{2} - g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} - h_{uv} \hat{\partial}_\mu q^u \hat{\partial}^\mu q^v + \frac{1}{4} I_{\Lambda\Sigma} F^{\Lambda\mu\nu} F^\Sigma{}_{\mu\nu}$$

$$+ \frac{1}{4} R_{\Lambda\Sigma} F^{\Lambda\mu\nu} * F^\Sigma{}_{\mu\nu} - V(z, \bar{z}, q),$$

where $V = 4 h_{uv} k^u_\Lambda k^v_\Sigma L^\Lambda \bar{L}^\Sigma + (g^{i\bar{j}} D_i L^\Lambda D_{\bar{j}} \bar{L}^\Sigma - 3 L^\Lambda \bar{L}^\Sigma) \cdot P^x_\Lambda P^x_\Sigma$

scalar pot

$$\hat{\partial}_\mu q^u = \partial_\mu q^u + A^\Lambda_\mu k^u_\Lambda$$ cov. der. of hyperscal.

$$I_{\Lambda\Sigma} = \operatorname{Im} \mathcal{N}_{\Lambda\Sigma}, \quad R_{\Lambda\Sigma} = \operatorname{Re} \mathcal{N}_{\Lambda\Sigma}, \quad I^{\Lambda\Sigma} I_{\Sigma\Gamma} = \delta^\Lambda{}_\Gamma$$

period matrix $\mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + 2i \dfrac{N_{\Lambda\Gamma} L^\Gamma N_{\Sigma\Delta} L^\Delta}{L^\Omega N_{\Omega\Psi} L^\Psi}$,

$$F_{\Lambda\Sigma} = \partial_\Lambda \partial_\Sigma \underset{\uparrow}{F}, \quad N_{\Lambda\Sigma} = \operatorname{Im} F_{\Lambda\Sigma}$$

prepot., hom. fct. of degree 2 in L's

III) 1st order flow eqns. from squaring of action

· ansatz $ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + e^{2\psi(r)} d\Omega^2_\kappa)$,

$$d\Omega^2_\kappa = d\vartheta^2 + f^2_\kappa(\vartheta) d\varphi^2, \quad f_\kappa(\vartheta) = \frac{1}{\sqrt{\kappa}} \sin(\sqrt{\kappa}\,\vartheta), \quad \kappa = 0, \pm 1$$

$$z^i = z^i(r), \quad q^u = q^u(r), \quad A^\Lambda = A^\Lambda_t(r) dt - \kappa p^\Lambda f'_\kappa(\vartheta) d\varphi$$

magn. & el. charges: $p^\Lambda = \dfrac{1}{\text{vol}(\Sigma_\kappa)} \displaystyle\int_{\Sigma_\kappa(r)} F^\Lambda$, $\underset{\substack{\uparrow \\ r\text{-dep.}}}{e_\Lambda(r)} = \dfrac{1}{\text{vol}(\Sigma_\kappa)} \displaystyle\int_{\Sigma_\kappa(r)} G_\Lambda$,

2-surf. of const. r, t $\rightarrow \Sigma_\kappa(r)$

$$\text{vol}(\Sigma_\kappa) = \int f_\kappa(\vartheta) d\vartheta \wedge d\varphi, \quad G_\Lambda = -\frac{2}{e} * \frac{\delta \mathcal{L}}{\delta F^\Lambda}$$

Maxw. eqns. $\Rightarrow e'_\Lambda(r) = -2 e^{2\psi - 4U} \underbrace{h_{uv} k^u_\Lambda k^v_\Sigma}_{=: \mathcal{H}_{\Lambda\Sigma}} A^{\Sigma\,(*)}_t, \quad p^\Lambda k^u_\Lambda = 0, \quad k_{\Lambda u} q'^u = 0$

$\rightarrow$ eom can be obtained from eff. action

$$S = \int dr L = \int dr \Big[e^{2\psi} (U'^2 - \psi'^2 + h_{uv} q'^u q'^v + g_{i\bar{j}} z'^i \bar{z}'^{\bar{j}}) + e_\Lambda A'^\Lambda_t + e^{2(U-\psi)} V_{BH}$$
$$- e^{2\psi - 4U} h_{uv} k^u_\Lambda k^v_\Sigma A^\Lambda_t A^\Sigma_t - \kappa + e^{2(\psi - U)} V \Big]$$

$$V_{BH} = -\tfrac{1}{2} Q^T \mathcal{M} Q \qquad \text{black hole pot.}$$

$$Q = \begin{pmatrix} p^\Lambda \\ e_\Lambda \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} I + R I^{-1} R & -R I^{-1} \\ -I^{-1} R & I^{-1} \end{pmatrix}$$

- action is <u>1st order</u> in A'^{Λ}_t (or e'_Λ). Want <u>2nd order</u>, since we want to square the action
  $\rightsquigarrow$ use constraint (*) to express A^Σ_t in terms of $e'_\Lambda(r)$.

Note: $\mathcal{H}_{\Lambda\Sigma}$ is in general not invertible. Let us assume that it is, w/ inverse $\mathcal{H}^{\Lambda\Sigma}$ (the general non-inv. case is treated in 1602.01334).

$\rightsquigarrow$ 2nd order action
$$S = \int dr \Big[e^{2\psi}\big(U'^2 - \psi'^2 + h_{uv} q'^u q'^v + g_{i\bar{\jmath}} z'^i \bar{z}'^{\bar{\jmath}} + \tfrac{1}{4} e^{4(U-\psi)} \mathcal{H}^{\Lambda\Sigma} e'_\Lambda e'_\Sigma\big) + e^{2(U-\psi)} V_{BH} - \mathcal{H} + e^{2(\psi-U)} V \Big] \qquad (\Delta)$$

- need also impose Ham. constr. $H=0$ ← Legendre-transf. of L in (Δ)

- Inspired by FI gauged case (Dall'Agata / Gnecchi 1012.3756), define
$$W = e^U \left| \mathcal{Z} + i\mathcal{H} e^{2(\psi-U)} \mathcal{L} \right|,$$

where $\mathcal{Z} = \langle Q, \mathcal{V} \rangle$ (central charge), $\mathcal{L} = \mathcal{Q}^x \mathcal{W}^x$,

$$\mathcal{Q}^x = \langle \underset{\begin{pmatrix} 0 \\ P^x_\Lambda \end{pmatrix}}{\underset{\uparrow}{P^x}}, Q \rangle = p^\Lambda P^x_\Lambda, \quad \mathcal{W}^x = \langle P^x, \mathcal{V} \rangle = L^\Lambda P^x_\Lambda$$

Then, if $\mathcal{Q}^x \mathcal{Q}^x = 1$ (note that $\mathcal{Q}^x\mathcal{Q}^x$ is a const. of motion; this cond. generalizes the Dirac-type charge quant. cond. of the FI case), one can show that (2) W solves the HJ equ. associated to (Δ) w/ zero energy, namely

$$e^{-2\psi}\Big((\partial_U W)^2 - (\partial_\psi W)^2 + 4 g^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} W + h^{uv} \partial_u W \partial_v W + 4 e^{4(\psi-U)} \mathcal{H}_{\Lambda\Sigma} \partial_{e_\Lambda} W \partial_{e_\Sigma} W \Big) - e^{2(\psi-U)} V - e^{2(U-\psi)} V_{BH} + \mathcal{H} = 0.$$

Using this, one can (up to a total der.) rewrite the action (Δ) as a "sum" of squares,

$$S = \int dr \Big[e^{2\psi}\left(U' + e^{-2\psi}\partial_U W\right)^2 - e^{2\psi}\left(\psi' - e^{-2\psi}\partial_\psi W\right)^2 +$$

$$e^{2\psi} g_{i\bar{j}} \left(z'^i + 2e^{-2\psi} g^{i\bar{k}}\partial_{\bar{k}} W\right)\left(\bar{z}'^{\bar{j}} + 2e^{-2\psi} g^{\bar{j}l}\partial_l W\right) +$$

$$e^{2\psi} h_{uv}\left(q'^u + e^{-2\psi} h^{us}\partial_s W\right)\left(q'^v + e^{-2\psi} h^{vt}\partial_t W\right) +$$

$$\frac{1}{4} e^{4U-2\psi} \mathcal{H}^{\Lambda\Gamma}\left(e'_\Lambda + 4e^{2\psi-4U}\mathcal{H}_{\Lambda\Sigma}\partial_{e_\Sigma} W\right)\left(e'_\Gamma + 4e^{2\psi-4U} \mathcal{H}_{\Gamma\Omega}\partial_{e_\Omega} W\right)\Big]$$

$\rightsquigarrow$ stationary if the terms in the brackets vanish $\rightsquigarrow$ 1st order flow eqns.

The latter can also be obtained from the results of Meessen / Ortín 1204.0493, where all (timelike) susy solns. of this theory were classified, or from Halmagyi / Petrini / Zaffaroni 1305.0730, where the KSE were solved for a static spher. symm. ansatz.
However: Our formalism allows to go beyond BPS case:

IV) A different squaring $\rightsquigarrow$ non-BPS flow eqns.

· squaring of action not unique

$\rightsquigarrow$ alternative set of 1st order eqns. driven by HJ fct.

$$W = e^U \left| \langle\tilde{Q}, \mathcal{V}\rangle + i\kappa e^{2(\psi-U)}\langle W^x\tilde{Q}^x, \mathcal{V}\rangle \right| \quad ,$$

where $\tilde{Q} = S Q$, $S \in Sp(2n_V+2, \mathbb{R})$ "field rotation matrix"
S must satisfy the compatibility conditions

$$S\mathcal{H}S^T = \mathcal{H} \quad , \quad S^T \mathcal{M} S = \mathcal{M} \qquad (\square) \quad ,$$

and the rotated charges must satisfy $\tilde{Q}^x\tilde{Q}^x = 1$.
Comments: i) technique of "rotating charges" was first introduced in Ceresole / Dall'Agata 0702088 and Cardoso / Ceresole / Dall'Agata / Oberreuter / Perz 0706.3373 and generalizes sign-flipping procedure of Ortín 9612142. It was applied to $U(1)$ FI-gauged sugra in

DK / Vaughan 1211.1618 and Gnecchi / Toldo 1211.1966

ii) In general it is not guaranteed that a nontriv. sol. to ($\square$) exists.

$\underline{\text{V}}$) An explicit example: $F = -iL^0L^1 + \text{UHM}$

one scalar z

$$h_{uv}\,dq^u dq^v = d\phi^2 + \frac{1}{4}e^{4\phi}\left(da - \frac{1}{2}\xi_0 d\xi^0 + \frac{1}{2}\xi^0 d\xi_0\right)^2 + \frac{1}{4}e^{2\phi}\left(d\xi^{0\,2} + d\xi_0^2\right)$$

· gauging: $k^u_\Lambda \partial_u = k_\Lambda \partial_a + \delta^0{}_\Lambda c\left(\xi_0 \partial_{\xi^0} - \xi^0 \partial_{\xi_0}\right)$, k_Λ, c const. $\Lambda = 0, 1$

axionic shift symm. rot. in $\xi^0 - \xi_0$ plane

(can be obtained by compactifying 11d sugra on Sasaki-Einstein manif. Q^{111} (leads to $F = 2i(L^0L^1L^2L^3)^{\frac{1}{2}}$) and then truncating)

· truncate z real, a const., $\xi^0 = \xi_0 = 0$. Then the flow eqns. admit the sol.

$$ds^2 = \frac{16r^2}{k_1 c}\left[-\left(1 + \frac{k_0}{cr^2}\right)^2 r^2 dt^2 + \left(1 + \frac{k_0}{cr^2}\right)^{-2}\frac{dr^2}{r^2} + \frac{1}{2}\left(d\vartheta^2 + \sinh^2\vartheta\, d\varphi^2\right)\right]$$

$\mathcal{H} = 1$

$$A^0 = -\frac{\cosh\vartheta}{c}d\varphi \,, \quad A^1 = \frac{k_0}{k_1}\frac{\cosh\vartheta}{c}d\varphi$$

$$\phi = -\log r \,, \quad z = \frac{c}{k_1}r^2$$

· no free par., all the const. completely fixed by gauging
· correct signature & $z > 0$ (needed for real Kähler pot.) $\Rightarrow k_1 c > 0$
· horizon at $r = \sqrt{-k_0/c}$ $(k_0/c < 0)$, curv. sing. at $r = 0$
· sol. interpolates between $AdS_2 \times H^2$ at hor. and conformal to $AdS_2 \times H^2$ (hyperscaling violating) for $r \to \infty$.

- Final remarks:
  · Can extend formalism to <u>symplectically covariant</u> form ($\to$ also magn. gaugings)
  · assuming a geom. of the form $AdS_2 \times \Sigma_\kappa$, one obtains from the 1st order flow eqns. the <u>attractor eqns. for gauged sugra w/ hypers.</u>