Invariant solutions to the Strominger system on solvmanifolds

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(Joint work with Luis Ugarte and Raquel Villacampa (Univ. Zaragoza))

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Superstring solutions, supersymmetry and geometry, 03/05/2016

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- The model is based on a Hermitian manifold X generalizing CY-manifolds.
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For us, $X = (G/\Gamma, J, F)$ is a Hermitian homogeneus space:

- G is 6-dimensional Lie group, $\mathfrak{g}$ its Lie algebra.
- Γ lattice subgroup of $G \Rightarrow G/\Gamma$ is compact.
- J is G-invariant complex structure.
- F is the fundamental form of a G-invariant Hermitian metric:

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The G-invariance of the solution implies:

- solutions with constant dilaton.
- analysis at the level of $\mathfrak{g}$.

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d) The Green-Schwarz anomally cancellation condition:

$$dT = 2\pi^2\alpha' \left(p_1(\nabla) - p_1(A)\right) = \frac{\alpha'}{4}\left(\mathrm{tr}\Omega \wedge \Omega - \mathrm{tr}\Omega^A \wedge \Omega^A\right)$$

where:

- $\alpha' \in \mathbb{R}\backslash\{0\}$ (better in physics $\alpha' > 0$).
- T is the torsion 3-form.
- Ω is the curvature form for ∇.
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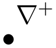
Theorem [FIUV09,Iva10]

A solution to Strominger satisfies heterotic eq. motion $\Leftrightarrow$ ∇ is SU(3)-instanton.

Some particular metric connections ∇:

- Bismut $\nabla^+ = \nabla^{\mathrm{LC}} + \frac{1}{2}T$ ([Car03],[DFG08]): Hermitian ($\nabla^+ J = 0$), torsion $T(\cdot,\cdot,\cdot) = JdF(\cdot,\cdot,\cdot)$.



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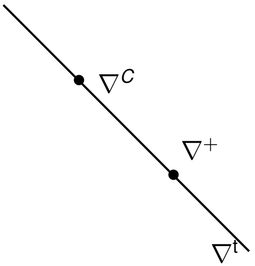
- [FY15] Hermitian connections ∇^{t} ([Gau77]):

$$g(\nabla^{\mathrm{t}}_X Y, Z) = g(\nabla^{LC}_X Y, Z) + \frac{1-\mathrm{t}}{4} T(X,Y,Z) + \frac{1+\mathrm{t}}{4} C(X,Y,Z), \quad \mathrm{t} \in \mathbb{R}.$$

$\nabla^+ = \nabla^{\mathrm{t}=-1}$, $\nabla^C = \nabla^{\mathrm{t}=1}$.



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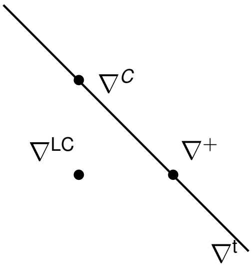

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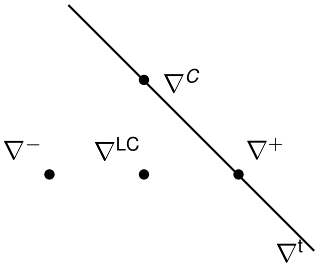
- $\nabla^{-} = \nabla^{\mathrm{LC}} - \frac{1}{2}T$ ([Hul86],[BR89]): torsion= $-T$.



Proposition [–UV16]

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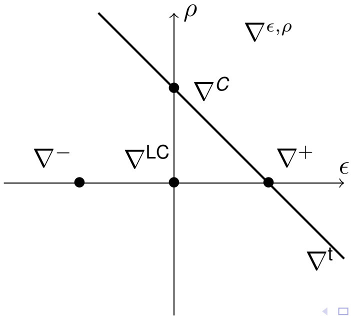

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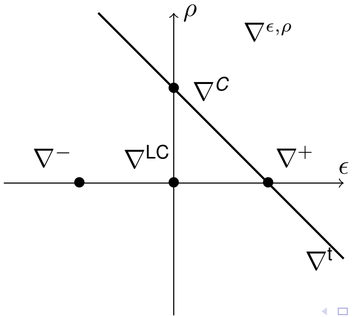

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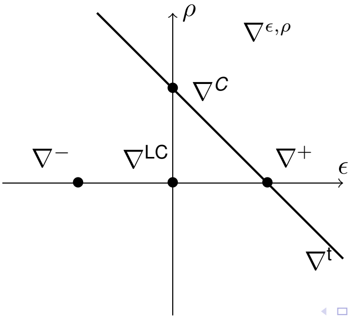
- ∇^{t} corresponds to $\nabla^{\varepsilon,\frac{1}{2}-\varepsilon}$ (i.e. $\varepsilon + \rho - \frac{1}{2} = 0$), and

$$\nabla^{c} = \nabla^{0,\frac{1}{2}}, \quad \nabla^{\pm} = \nabla^{\pm\frac{1}{2},0}, \quad \nabla^{LC} = \nabla^{0,0}.$$



Complex nilmanifolds $(G/\Gamma, J)$ are a rich source of examples as they are not-Kähler (except the torus) and have holomorphically trivial canonical bundle. We have a complete map of the $\mathfrak{g}'s$ [Sal01] and the cmpx. str. up to isomorphism. [ABD11,UV14,C–UV14]

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[Uga07,FIUV09,UV14,UV15]

If a 6-dim. nilmanifold admits Hermitian balanced metrics then $\mathfrak{g} \cong \mathfrak{h}_2, \ldots, \mathfrak{h}_6, \mathfrak{h}_{19}^-$.

- $\mathfrak{h}_2, \ldots, \mathfrak{h}_5$ provide solutions to the Strominger system with respect to ∇^+ and ∇^{LC}. $\mathfrak{h}_3$ provides also sols. to the motion eqs. with respect to ∇^+ .
- $\mathfrak{h}_6$ provides solutions to the Strominger system with respect to ∇^+.
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Solutions with $\alpha' > 0$ and non-flat instanton for all the cases.

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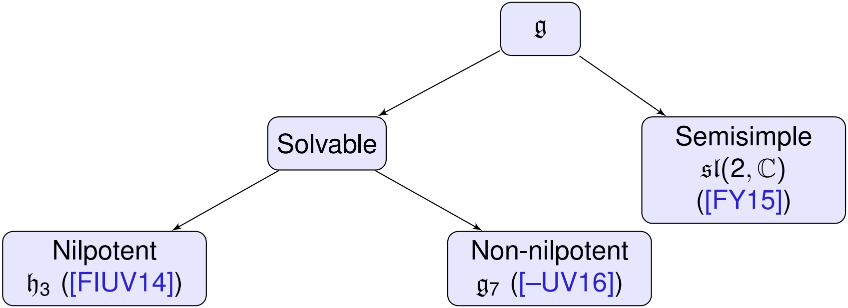
[FY15]

There are solutions to the Strominger system on $\mathfrak{sl}(2, \mathbb{C})$:

- with $\alpha' > 0$ and flat instanton for any ∇^t with $t < 0$ (∇^+ included).
- with $\alpha' > 0$ and non-flat instanton for any ∇^t with $t < -1$ (∇^+ not included).

- We are mainly interested in considering Strominger + motion equations.
- We revisit the $\mathfrak{h}_3$ and $\mathfrak{sl}(2,\mathbb{C})$.
- We provide the new solvable non-nilpotent example $\mathfrak{g}_7$.



The nilpotent Lie algebra $\mathfrak{h}_3 = \mathfrak{h}(2,1) \oplus \mathbb{R}$ is given by:

$$d\beta^j = 0,\ j = 1, \dots, 5, \quad d\beta^6 = \beta^{12} + \beta^{34}.$$

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The Lie group $H_3 = H(2,1) \times \mathbb{R}$, $H(2,1)$ is the 5-dim. Heisenberg Lie group:

$$H(2,1) = \left\{ \begin{pmatrix} 1 & a_1 & a_2 & c \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & 1 & b_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mid a_1, a_2, b_1, b_2, c \in \mathbb{R} \right\}.$$

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Malcev's Theorem $\Rightarrow$ H_3 admits lattices ([Mal62]).

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Proposition [ABD11]

There are 2 non-isomorphic cmpx. str. $J^{\pm}$ on $\mathfrak{h}_3$. They are given in terms of a $(1,0)$-basis $\{\omega^1, \omega^2, \omega^3\}$:

$$J^{\pm}: \quad d\omega^1 = d\omega^2 = 0, \quad d\omega^3 = \omega^{1\bar{1}} \pm \omega^{2\bar{2}}.$$

Proposition [Uga07],[UV15]

- Only J^- admits balanced metrics.
- The non-isomorphic balanced metrics on J^- are given by:

$$2F_t = i\left(\omega^{1\bar{1}} + \omega^{2\bar{2}} + t^2\omega^{3\bar{3}}\right), \quad 0 \neq t \in \mathbb{R}.$$

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Proposition [UV15]

For any SU(3)-str. the connection A_λ:

$$(\sigma^{A_\lambda})^1_2 = -(\sigma^{A_\lambda})^2_1 = -(\sigma^{A_\lambda})^3_4 = (\sigma^{A_\lambda})^4_3 = \lambda(e^5 + e^6),$$

is SU(3)-instanton for any $\lambda \in \mathbb{R}$. Furthermore,

$$p_1(A_\lambda) = -\frac{2\,t^2\lambda^2}{\pi^2}\,e^{1234}.$$

Proposition [–UV16]

Given the SU(3)-str. $(J^-, F_t, \Psi_t = t\omega^{123})$, $\nabla^{\epsilon,\rho}$ is SU(3)-instanton $\Leftrightarrow$ $(\epsilon, \rho) = (\frac{1}{2}, 0)$, that is, ∇^+.

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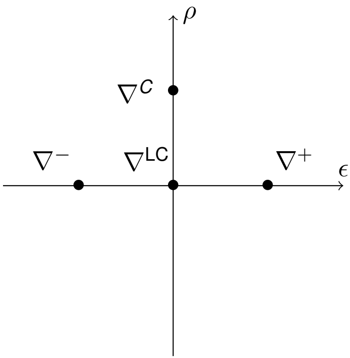

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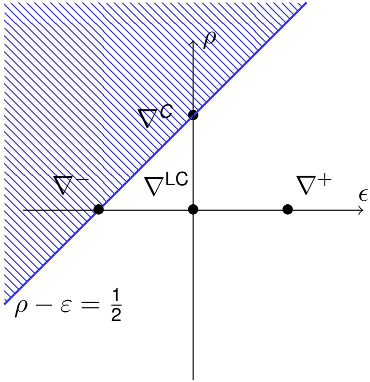

(i) If $\rho \geq \varepsilon + \frac{1}{2}$, then $\alpha' < 0$.

Particular cases: ∇^- and the Hermitian connection ∇^t for any $t \geq 1$ (Chern included).

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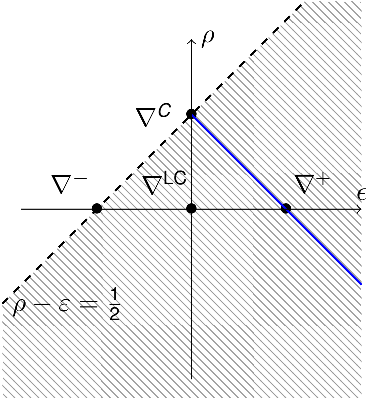

(ii) If $\rho < \varepsilon + \frac{1}{2}$, then $\alpha' > 0$.

Particular cases: ∇^{LC} and the Hermitian connection ∇^t for any $t < 1$ (Bismut included).

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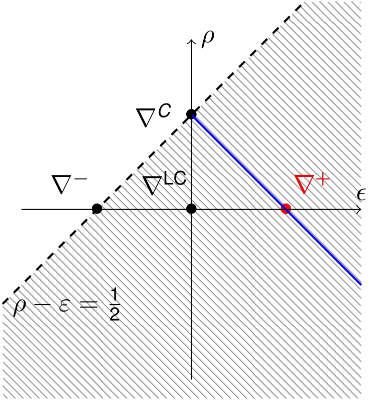

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The analysis of the cancellation of anomalies depends on the function:

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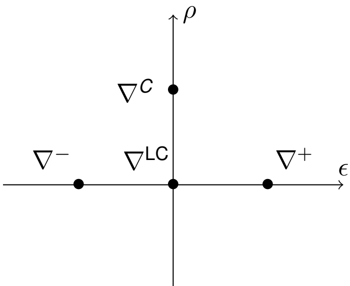

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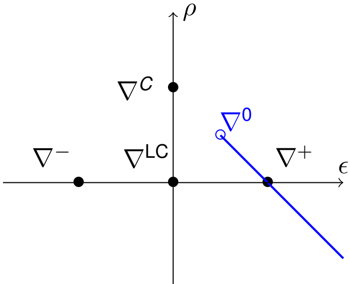

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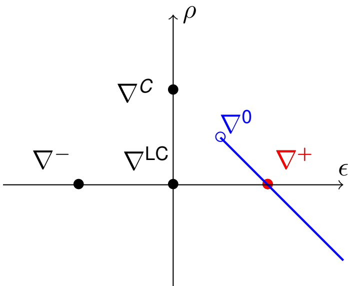

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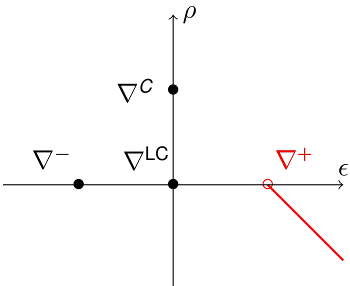

(ii) If $\beta(\epsilon,\rho) \neq 8$ there are solutions to Strominger with non-flat instanton ($A = \nabla^+$) and $\mathrm{sign}(\alpha') = \mathrm{sign}(\beta(\epsilon,\rho) - 8)$.

In particular for any Hermitian connection ∇^t with $t < -1$ (∇^+ not included).

$\mathfrak{g}_7$ is the solvable Lie algebra given by:

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In terms of the (1,0)-basis:

$$\omega_\delta^1 = \beta^4 + i\,\beta^2, \quad \omega_\delta^2 = \beta^3 + i\,\beta^5, \quad \omega_\delta^3 = \tfrac{1}{2}\beta^6 + 2i\,\delta\beta^1.$$

J_δ is expressed by:

$$J_\delta : \quad \begin{array}{ll} d\omega_\delta^1 = i\,\omega_\delta^1 \wedge (\omega_\delta^3 + \omega_\delta^{\bar{3}}), & d\omega_\delta^2 = -i\,\omega_\delta^2 \wedge (\omega_\delta^3 + \omega_\delta^{\bar{3}}), \\ d\omega_\delta^3 = \delta\,(\omega_\delta^{1\bar{1}} - \omega_\delta^{2\bar{2}}), & \end{array}$$

Proposition [F–U15]

Any balanced metric on $(\mathfrak{g}_7, J_\delta)$ is given by

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The real basis $\{e^1, \ldots, e^6\}$:

$$e^1 + i\,e^2 = \frac{\sqrt{r^4 - |u|^2}}{r}\,\omega^1_\delta, \qquad e^3 + i\,e^4 = \frac{u}{r}\,\omega^1_\delta + ir\,\omega^2_\delta, \qquad e^5 + i\,e^6 = t\,\omega^3_\delta.$$

is adapted to $(J_\delta,\ F^{\delta}_{r,t,u})$. We consider the SU(3)-structures on $\mathfrak{g}_7$:

$$\left(J_\delta,\ F^{\delta}_{r,t,u},\ \Psi^{\delta}_{r,t,u} = (e^1 + i\,e^2) \wedge (e^3 + i\,e^4) \wedge (e^5 + i\,e^6)\right).$$

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For each $\lambda, \mu \in \mathbb{R}$ the connection $A_{\lambda,\mu}$ given by:

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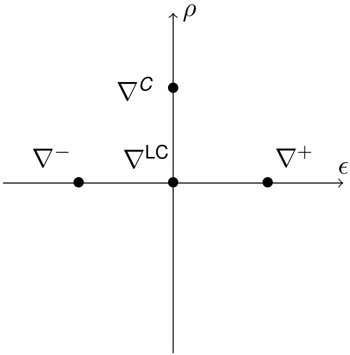
From now on, we divide the study according to the vanishing of the parameter u.

We start by looking for solutions to the motion equations for SU(3)-structures with $u = 0$.

Theorem [–UV16]

Let M be a $\mathfrak{g}_7$-solvmanifold endowed with the SU(3)-str. $(J_\delta,\ F^\delta_{r,t,u=0},\ \Psi^\delta_{r,t,u=0})$, the Strominger system has invariant solutions for any connection $\nabla^{\varepsilon,\rho}$ and with non-flat instanton.



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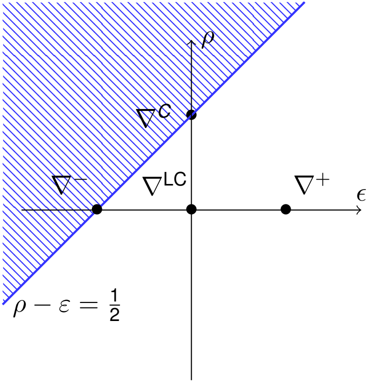

(i) If $\rho \geq \varepsilon + \frac{1}{2}$, then $\alpha' < 0$.

Particular cases: ∇^- and the Hermitian connection ∇^t for any $t \geq 1$ (Chern included).

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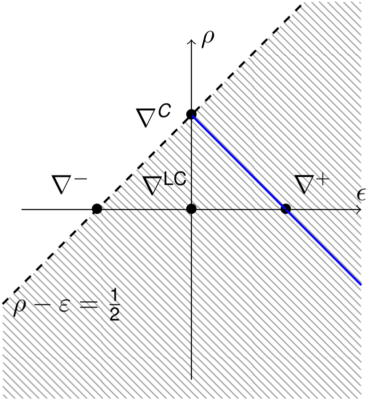

(ii) If $\rho < \varepsilon + \frac{1}{2}$, then $\alpha' > 0$.

Particular cases: ∇^{LC} and the Hermitian connection ∇^t for any $t < 1$ (Bismut included).

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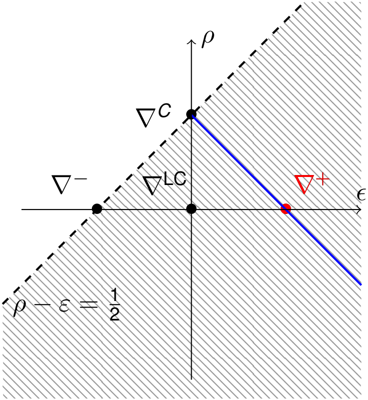


(iii) for $\nabla = \nabla^+$, the solutions satisfy the heterotic equations of motion with $\alpha' > 0$ and non-flat instanton.

After studying the cancellation of anomalies for the SU(3)-str. with $u \neq 0$:

$$dT = \frac{\alpha'}{4}\left(p_1(\nabla^{\epsilon,\rho}) - p_1(A_{\lambda,\mu})\right)$$

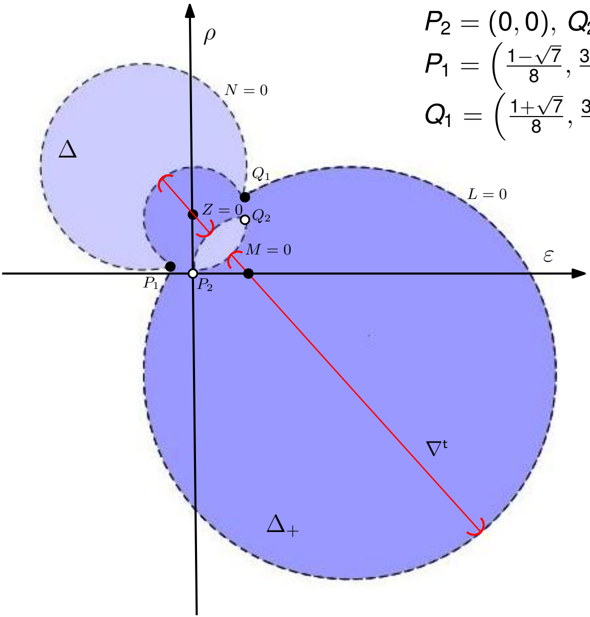
the results for the solutions to Strominger system are expressed in terms of the regions:

$$\begin{aligned}\Delta &= L^- \cup N^- - \{P_2, Q_2\},\\ \Delta_+ &= L^- \cup M^- - (\overline{M^-} \cap \overline{Z^-}),\end{aligned}$$

where $L^- := \{(\epsilon,\rho) \mid L(\epsilon,\rho) < 0\}$, for the functions

$$\begin{aligned}L(\epsilon,\rho) &= (\varepsilon - \tfrac{3}{2})^2 + (\rho+1)^2 - 4,\\ N(\epsilon,\rho) &= (\varepsilon + \tfrac{1}{2})^2 + (\rho-1)^2 - 1,\\ M(\epsilon,\rho) &= \varepsilon^2 + (\rho - \tfrac{1}{2})^2 - \tfrac{1}{4},\\ Z(\epsilon,\rho) &= (1+\epsilon-\rho)\left[4\epsilon^2 + (1-2\rho)^2 - 4\right] + 3.\end{aligned}$$

$L = 0$ and $N = 0$ intersect at the points $P_1 = \left(\frac{1-\sqrt{7}}{8}, \frac{3-\sqrt{7}}{8}\right)$ and $Q_1 = \left(\frac{1+\sqrt{7}}{8}, \frac{3+\sqrt{7}}{8}\right)$.



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(ii) For any $(\epsilon, \rho) \in \Delta$, there exist solutions to the Strominger system with non-flat instanton. Moreover, $\alpha' > 0$ if and only if $(\epsilon, \rho) \in \Delta_+$.

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Corollary [–UV16]

A solvmanifold with underlying Lie algebra $\mathfrak{g}_7$ provides invariant solutions to the Strominger system with $\alpha' > 0$ and non-flat instanton with respect to a Hermitian connection ∇^{t} for

$$\mathrm{t} \in (-5 - 4\sqrt{2}, 1 - \sqrt{2}) \cup (\tfrac{5-\sqrt{17}}{2}, 1 + \sqrt{2}).$$

In particular, there are solutions for the Bismut connection ($\mathrm{t} = -1$) and for the Chern connection ($\mathrm{t} = 1$).

$\mathfrak{g}_7$ has also a behaviour observed for the nilpotent Lie algebra $\mathfrak{h}_{19}^{-}$ in [UV14]:

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Let us consider a solvmanifold with underlying Lie algebra $\mathfrak{g}_7$. There is an $\mathrm{SU}(3)$-structure and a non-flat instanton solving at the same time the Strominger system for ∇^+ and ∇^C,both with positive α''s.

That is:

$$(G_7/\Gamma, J_\delta, F^\delta_{r,t,u\neq 0}, \Psi^\delta_{r,t,u\neq 0}, \nabla, A_{\lambda,\mu} \neq 0) : \left\{ \begin{array}{l} \nabla = \nabla^+, \, (\alpha' > 0) \\ \\ \nabla = \nabla^C, \, (\tilde{\alpha}' > 0) \end{array} \right.$$

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Concluding $\mathfrak{g}_7$...

$\mathfrak{g}_7$ has a very rich space of solutions: $\nabla = \nabla^{\mathrm{LC}}, \nabla^+, \nabla^C$, heterotic motion equations. Comparing with the NLA's:

- $\mathfrak{h}_2, \ldots, \mathfrak{h}_6$ have solutions to Strominger for $\nabla^{\mathrm{LC}}, \nabla^+$, but not for ∇^C.
- $\mathfrak{h}_{19}^-$ has solutions to Strominger for ∇^+, ∇^C but not to the heterotic motion equations.