

Dynamical symmetry enhancement near black hole horizons

Ulf Gran



Introduction

Background

The project is a part of the research program on the development of the theory of the structure of the universe.

Questions

What is the role of the structure of the universe in the development of the theory of the structure of the universe?

Conclusions of research & substantiation of the results of the research

The results of the research show that the structure of the universe is a part of the theory of the structure of the universe.

Method

Methodology

The methodology of the research is based on the theory of the structure of the universe.

Analysis

The analysis of the research shows that the structure of the universe is a part of the theory of the structure of the universe.

Results

The results of the research show that the structure of the universe is a part of the theory of the structure of the universe.

Conclusion

The conclusion of the research shows that the structure of the universe is a part of the theory of the structure of the universe.

Results

The results of the research show that the structure of the universe is a part of the theory of the structure of the universe.

Tools

The Role of the Principle

The role of the principle is to provide a basis for the development of the theory of the structure of the universe.

Gamma

The gamma is a part of the theory of the structure of the universe.

Index Theorems

The index theorems are a part of the theory of the structure of the universe.

Introduction

The importance of
(supersymmetric) black hole and
brane solutions

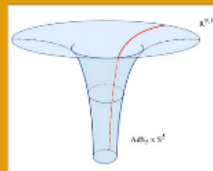
- Testing ground for quantum gravity, c.f. firewall paradox
- Testing conjectured dualities (e.g. AdS/CFT)
- Microscopic understanding of black hole entropy
- Applications in condensed matter physics (AdS/CMT)

Example of symmetry enhancement:
D3-branes in IIB supergravity

- Near the horizon the symmetry enhances to $SO(2,4)$, i.e. the conformal group in $D=4$
- In addition the number of preserved SUSY doubles (no SUSY broken)



Gauge/gravity duality



Questions

- General picture?
- What is required for (super)-symmetry enhancement near the black hole horizon?

Aim: General proof of (super)-symmetry enhancement for SUSY horizons

U. Gran, J. Gutowski, U. Kayani & G. Papadopoulos [arXiv:1411.5286 & 1409.6303]
U. Gran, J. Gutowski & G. Papadopoulos [arXiv:1306.5765]
J. Gutowski & G. Papadopoulos [arXiv:1303.0869]
J. Grover, J. Gutowski, G. Papadopoulos, & W. Sabra [arXiv:1303.0853]

The importance of (supersymmetric) black hole and brane solutions

- Testing ground for quantum gravity, c.f. firewall paradox
- Testing conjectured dualities (e.g. AdS/CFT)
- Microscopic understanding of black hole entropy
- Applications in condensed matter physics (AdS/CMT)

Introduction

The importance of (supersymmetric) black hole and brane solutions

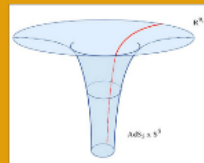
- Testing ground for quantum gravity, c.f. firewall paradox
- Testing conjectured dualities (e.g. AdS/CFT)
- Microscopic understanding of black hole entropy
- Applications in condensed matter physics (AdS/CMT)

Example of symmetry enhancement: D3-branes in IIB supergravity

- Near the horizon the symmetry enhances to $SO(2,4)$, i.e. the conformal group in $D=4$
- In addition the number of preserved SUSY doubles (no SUSY broken)



Gauge/gravity duality



Questions

- General picture?
- What is required for (super)-symmetry enhancement near the black hole horizon?

Aim: General proof of (super)-symmetry enhancement for SUSY horizons

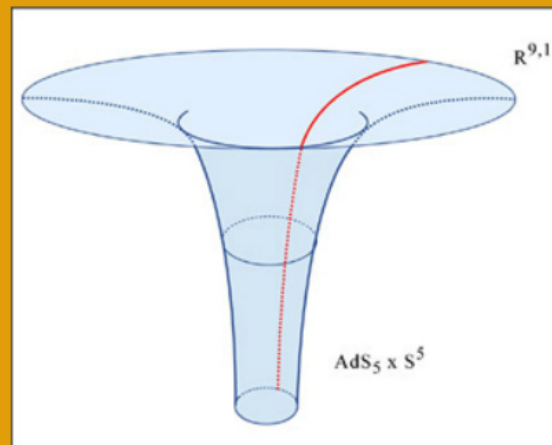
U. Gran, J. Gutowski, U. Kayani & G. Papadopoulos [arXiv:1411.5286 & 1409.6303]
 U. Gran, J. Gutowski & G. Papadopoulos [arXiv:1306.5765]
 J. Gutowski & G. Papadopoulos [arXiv:1303.0859]
 J. Grover, J. Gutowski, G. Papadopoulos, & W. Sabra [arXiv:1303.0853]

Example of symmetry enhancement: D3-branes in IIB supergravity

- Near the horizon the symmetry enhances to $SO(2,4)$, i.e. the conformal group in $D=4$
- In addition the number of preserved SUSY doubles (no SUSY broken)



Gauge/gravity duality



Introduction

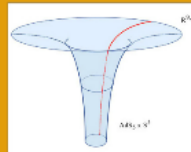
The importance of
(supersymmetric) black hole and
brane solutions

- Testing ground for quantum gravity, c.f. firewall paradox
- Testing conjectured dualities (e.g. AdS/CFT)
- Microscopic understanding of black hole entropy
- Applications in condensed matter physics (AdS/CMT)

Example of symmetry enhancement:
D3-branes in IIB supergravity

- Near the horizon the symmetry enhances to $SO(2,4)$, i.e. the conformal group in $D=4$
- In addition the number of preserved SUSY doubles (no SUSY broken)

→ Gauge/gravity duality



Questions

- General picture?
- What is required for (super)-symmetry enhancement near the black hole horizon?

Aim: General proof of (super)-symmetry enhancement for SUSY horizons

U. Gran, J. Gutowski, U. Kayani & G. Papadopoulos
[arXiv:1411.5286 & 1409.6303]
U. Gran, J. Gutowski & G. Papadopoulos [arXiv:1306.5765]
J. Gutowski & G. Papadopoulos [arXiv:1302.0869]
J. Grover, J. Gutowski, G. Papadopoulos, & W. Sabra
[arXiv:1303.0853]

Questions

- General picture?
- What is required for (super)-symmetry enhancement near the black hole horizon?

Aim: General proof of (super)-symmetry enhancement for SUSY horizons

U. Gran, J. Gutowski, U. Kayani & G. Papadopoulos
[arXiv:1411.5286 & 1409.6303]

U. Gran, J. Gutowski & G. Papadopoulos [arXiv:1306.5765]

J. Gutowski & G. Papadopoulos [arXiv:1303.0869]

J. Grover, J. Gutowski, G. Papadopoulos, & W. Sabra
[arXiv:1303.0853]

Introduction

The importance of the asymptotic expansion

- Asymptotic expansion is a powerful tool for approximating functions in the limit of a small parameter.
- It is used in many areas of physics and engineering, such as quantum mechanics, fluid dynamics, and celestial mechanics.
- The asymptotic expansion of a function $f(x)$ as $x \rightarrow \infty$ is a series of the form:

Example of asymptotic expansion

Here are two examples of asymptotic expansions:

- Example 1: The asymptotic expansion of the function $f(x) = \ln(x)$ as $x \rightarrow \infty$ is:

Questions

General question: What is the asymptotic expansion of the function $f(x)$ as $x \rightarrow \infty$?

More specific question: What is the asymptotic expansion of the function $f(x) = \ln(x)$ as $x \rightarrow \infty$?

Method

Problem statement

Find the asymptotic expansion of the function $f(x)$ as $x \rightarrow \infty$.

Assumptions

- The function $f(x)$ is analytic in the complex plane.
- The function $f(x)$ has a branch cut along the negative real axis.

Analysis

The asymptotic expansion of the function $f(x)$ as $x \rightarrow \infty$ is given by:

$$f(x) \sim \sum_{n=0}^{\infty} \frac{a_n}{x^n}$$

Results

The asymptotic expansion of the function $f(x)$ as $x \rightarrow \infty$ is:

$$f(x) \sim \ln(x) + \frac{1}{2x} + \frac{1}{12x^2} + \frac{1}{30x^3} + \dots$$

Results

There is a deep connection between the asymptotic expansion of a function and its analytic properties.

For example, the asymptotic expansion of the function $f(x) = \ln(x)$ as $x \rightarrow \infty$ is:

$$f(x) \sim \ln(x) + \frac{1}{2x} + \frac{1}{12x^2} + \frac{1}{30x^3} + \dots$$

This expansion is valid for all $x > 0$.

Tools

The Hopf Maximum Principle

The Hopf maximum principle is a powerful tool for proving the uniqueness of solutions to elliptic partial differential equations.

It states that if u is a function satisfying the elliptic equation $\Delta u = f$ in a domain Ω , and if u attains its maximum value at a point on the boundary of Ω , then u is constant in Ω .

GAMMA

The Gamma function is a generalization of the factorial function to non-integer values.

It is defined by the integral:

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

Index Theorems

The index of an elliptic operator is a topological invariant that is independent of the choice of the operator.

It is defined as the difference between the number of positive and negative eigenvalues of the operator.

Liouville's Theorem

Liouville's theorem states that any bounded entire function is constant.

It is a special case of the more general theorem of Picard, which states that any non-constant entire function takes on all complex values except for at most one.

Conclusion

The asymptotic expansion of a function is a powerful tool for approximating functions in the limit of a small parameter.

It is used in many areas of physics and engineering, such as quantum mechanics, fluid dynamics, and celestial mechanics.

Tools

The Hopf Maximum Principle

- The Hopf maximum principle states that if a function achieves its maximum in the interior of the domain, the function is a constant
- We assume *compact* horizons
- Example: Harmonic functions
- Strategy: Rewrite the equations in such a form that the maximum principle can be applied

GAMMA

- Mathematica package for performing Γ -matrix algebra
- Very lengthy calculations...
- Download from www.gran.name
- Compatible with Mathematica 10



Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds

$$\not{D}^2 \epsilon = \nabla^2 \epsilon + \frac{1}{4} R \epsilon$$

- Relates the Dirac and Laplace operator acting on a spinor, and the scalar curvature
- In $D=4$ important in Seiberg-Witten theory

Index Theorems

- Index theorems relate *analytic* quantities to *topological* invariants
- Since we have Dirac-like operators (modified by fluxes) we can use index theorems to count their *zero-modes*
- The zero-modes will be identified with the *Killing spinors* (c.f. Killing vectors)
- Through (modified) Lichnerowicz type of theorems we can combine analytical and topological constraints from spinor theory with constraints on elliptic PDEs for which the Hopf maximum principle applies

The Hopf Maximum Principle


- The Hopf maximum principle states that if a function achieves its maximum in the interior of the domain, the function is a constant
- We assume *compact* horizons
- Example: Harmonic functions
- Strategy: Rewrite the equations in such a form that the maximum principle can be applied

Tools

The Hopf Maximum Principle

- The Hopf maximum principle states that if a function achieves its maximum in the interior of the domain, the function is a constant
- We assume *compact* horizons
- Example: Harmonic functions
- Strategy: Rewrite the equations in such a form that the maximum principle can be applied

GAMMA

- Mathematica package for performing Γ -matrix algebra
- Very lengthy calculations... 
- Download from www.gran.name
- Compatible with Mathematica 10

Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds
- $$\mathcal{D}^2 \epsilon := \nabla^2 \epsilon + \frac{1}{4} R \epsilon$$
- Relates the Dirac and Laplace operator acting on a spinor, and the scalar curvature
 - In $D=4$ important in Seiberg-Witten theory

Index Theorems

- Index theorems relate *analytic* quantities to *topological* invariants
- Since we have Dirac-like operators (modified by fluxes) we can use index theorems to count their *zero-modes*
- The zero-modes will be identified with the *Killing spinors* (c.f. Killing vectors)
- Through (modified) Lichnerowicz type of theorems we can combine analytical and topological constraints from spinor theory with constraints on elliptic PDEs for which the Hopf maximum principle applies

Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds

$$\mathcal{D}^2\epsilon = \nabla^2\epsilon + \frac{1}{4}R\epsilon$$


- Relates the Dirac and Laplace operator acting on a spinor, and the scalar curvature
- In D=4 important in Seiberg-Witten theory

Tools

The Hopf Maximum Principle

- The Hopf maximum principle states that if a function achieves its maximum in the interior of the domain, the function is a constant
- We assume *compact* horizons
- Example: Harmonic functions
- Strategy: Rewrite the equations in such a form that the maximum principle can be applied

GAMMA

- Mathematica package for performing T-matrix algebra
- Very lengthy calculations... 
- Download from www.gran.name
- Compatible with Mathematica 10

Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds

$$\mathcal{D}^2 \epsilon = \nabla^2 \epsilon + \frac{1}{4} R \epsilon$$

- Relates the Dirac and Laplace operator acting on a spinor, and the scalar curvature
- In D=4 important in Seiberg-Witten theory

Index Theorems

- Index theorems relate *analytic* quantities to *topological* invariants
- Since we have Dirac-like operators (modified by fluxes) we can use index theorems to count their *zero-modes*
- The zero-modes will be identified with the *Killing spinors* (c.f. Killing vectors)
- Through (modified) Lichnerowicz type of theorems we can combine analytical and topological constraints from spinor theory with constraints on elliptic PDEs for which the Hopf maximum principle applies

Index Theorems


- Index theorems relate *analytic* quantities to *topological* invariants
- Since we have Dirac-like operators (modified by fluxes) we can use index theorems to count their *zero-modes*
- The zero-modes will be identified with the *Killing spinors* (c.f. Killing vectors)
- Through (modified) Lichnerowicz type of theorems we can combine analytical and topological constraints from spinor theory with constraints on elliptic PDEs for which the Hopf maximum principle applies

Tools

The Hopf Maximum Principle

- The Hopf maximum principle states that if a function achieves its maximum in the interior of the domain, the function is a constant
- We assume *compact* horizons
- Example: Harmonic functions
- Strategy: Rewrite the equations in such a form that the maximum principle can be applied

GAMMA

- Mathematica package for performing F-matrix algebra
- Very lengthy calculations... 
- Download from www.gran.name
- Compatible with Mathematica 10

Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds
- $$\mathcal{L}^2 \epsilon = \nabla^2 \epsilon + \frac{1}{4} R \epsilon$$
- Relates the Dirac and Laplace operator acting on a spinor, and the scalar curvature
 - In D=4 important in Seiberg-Witten theory

Index Theorems

- Index theorems relate *analytic* quantities to *topological* invariants
- Since we have Dirac-like operators (modified by fluxes) we can use index theorems to count their *zero-modes*
- The zero-modes will be identified with the *Killing spinors* (c.f. Killing vectors)
- Through (modified) Lichnerowicz type of theorems we can combine analytical and topological constraints from spinor theory with constraints on elliptic PDEs for which the Hopf maximum principle applies

GAMMA

- Mathematica package for performing Γ -matrix algebra
- Very lengthy calculations...
- Download from www.gran.name
- Compatible with Mathematica 10



Equations to analyse

- The Killing spinor equations (KSE) encode the requirement of preserving SUSY: linear
- Field equations encode the interplay between fluxes and geometry: non-linear
- Integrability conditions relate the KSEs and the field equations
- Usual approach: Remove the redundant field equations
- We will do the opposite! (Since some of the KSE are very complicated)
- Coordinate independent definition of an event horizon: *Killing horizon*
- Use an adapted metric [Wald]
- Integrate the KSE along the light-cone directions, leaving conditions on the horizon in terms of modified Dirac operators.

Method

Analysis II

- Use all available tools to show that if η_- is a Killing spinor, then so is

$$\eta_+ = \Gamma_+ \Theta_- \eta_-$$

$$\text{where } \Theta_- = \frac{1}{4} h_i \Gamma^i + \frac{1}{4} \Gamma_{11} L_i \Gamma^i - \frac{1}{16} e^\Phi \Gamma_{11} (-2S + \tilde{F}_{ij} \Gamma^{ij}) - \frac{1}{8 \cdot 4!} e^\Phi (-12 X_{ij} \Gamma^{ij} + \tilde{G}_{ijkl} \Gamma^{ijkl}) - \frac{1}{8} e^\Phi m$$

- The three Killing vectors that can be constructed form an $sl(2, \mathbb{R})$ algebra

Assumptions

- The horizons admit at least one *supersymmetry*
- The horizons are *compact*
- The near horizon geometries are *smooth*

Analysis I

- Identify the Killing spinors with the zero-modes of modified Dirac operators

$$\mathcal{D}^{(+)} : \nabla^+ \nabla^+ : \|\eta_+\|^2 = (2 \nabla^+ \nabla^+ + h^2) \nabla^+ : \|\eta_+\|^2 = 2 \|\nabla^{(+)} \eta_+\|^2 + (-4\kappa - 16\kappa^2) \|A^{(+)} \eta_+\|^2$$

$$\mathcal{D}^{(-)} : \nabla^+ (e^{-2\Phi} V_1) = -2e^{-2\Phi} \|\nabla^{(-)} \eta_-\|^2 + e^{-2\Phi} (4\kappa + 16\kappa^2) \|A^{(-)} \eta_-\|^2$$

$$\text{where } V = -d \|\eta_-\|^2 - \|\eta_-\|^2 h$$

- Now use the index theorem to arrive at $N_+ = N_-$ i.e. $N = N_+ + N_- = 2N_-$

Equations to analyse

- The Killing spinor equations (KSE) encode the requirement of preserving SUSY: linear
- Field equations encode the interplay between fluxes and geometry: non-linear
- Integrability conditions relate the KSEs and the field equations
- Usual approach: Remove the redundant field equations
- We will do the opposite! (Since some of the KSE are very complicated)
- Coordinate independent definition of an event horizon: *Killing horizon*
- Use an adapted metric [Wald]
- Integrate the KSE along the light-cone directions, leaving conditions on the horizon in terms of modified Dirac operators.

Method

Equations to analyse

- The Killing spinor equations (KSE) encode the requirement of preserving SUSY: linear
- Field equations encode the interplay between fluxes and geometry: non-linear
- Integrability conditions relate the KSEs and the field equations
- Usual approach: Remove the redundant field equations
- We will do the opposite! (Since some of the KSE are very complicated)
- Coordinate independent definition of an event horizon: *Killing horizon*
- Use an adapted metric [Wald]
- Integrate the KSE along the light-cone directions, leaving conditions on the horizon in terms of modified Dirac operators.

Assumptions

- The horizons admit at least one *supersymmetry*
- The horizons are *compact*
- The near horizon geometries are *smooth*

Analysis II

- Use all available tools to show that if η_- is a Killing spinor, then so is

$$\eta_+ = \Gamma_+ \Theta_- \eta_-$$

where $\Theta_- = \frac{1}{4} h_i \Gamma^i + \frac{1}{4} \Gamma_{11} L_i \Gamma^i - \frac{1}{16} e^\Phi \Gamma_{11} (-2S + \hat{F}_{ij} \Gamma^{ij})$
 $- \frac{1}{8 \cdot 4!} e^\Phi (-12 X_{ij} \Gamma^{ij} + \hat{G}_{ijkl} \Gamma^{ijkl}) - \frac{1}{8} e^\Phi m$

- The three Killing vectors that can be constructed form an $sl(2, \mathbb{R})$ algebra

Analysis I

- Identify the Killing spinors with the zero-modes of modified Dirac operators

$$\mathcal{D}^{(+)} : \nabla \eta_+ \parallel \eta_+ \parallel^2 = \frac{1}{2} \parallel \nabla^{(+)} \eta_+ \parallel^2 + (-4\kappa - 16\kappa^2) \parallel \mathcal{A}^{(+)} \eta_+ \parallel^2$$

$$\mathcal{D}^{(-)} : \nabla \eta_- \parallel \eta_- \parallel^2 = \frac{1}{2} \parallel \nabla^{(-)} \eta_- \parallel^2 + (-4\kappa + 16\kappa^2) \parallel \mathcal{A}^{(-)} \eta_- \parallel^2$$

where $\nabla^{(\pm)} = \nabla^{(\pm)} + \kappa \Gamma_{\pm} \mathcal{A}^{(\pm)}$
 where $V = -d \parallel \eta_{\pm} \parallel^2 = \parallel \eta_{\pm} \parallel^2 h$
 i.e. $N_+ = N_- + N_{\pm} = 2N_{\pm}$

- Now use the index theorem to arrive at $N_+ = N_-$

Assumptions

- The horizons admit at least one *supersymmetry*
- The horizons are *compact*
- The near horizon geometries are *smooth*

Method

Equations to analyse

- The Killing spinor equations (KSE) encode the requirement of preserving SUSY: linear
- Field equations encode the interplay between fluxes and geometry: non-linear
- Integrability conditions relate the KSEs and the field equations
- Usual approach: Remove the redundant field equations
- We will do the opposite! (Since some of the KSE are very complicated)
- Coordinate independent definition of an event horizon: *Killing horizon*
- Use an adapted metric [Wald]
- Integrate the KSE along the light-cone directions, leaving conditions on the horizon in terms of modified Dirac operators.

Assumptions

- The horizons admit at least one *supersymmetry*
- The horizons are *compact*
- The near horizon geometries are *smooth*

Analysis II

- Use all available tools to show that if η_- is a Killing spinor, then so is

$$\eta_+ = \Gamma_+ \Theta_- \eta_-$$

$$\text{where } \Theta_- = \frac{1}{4} h_i \Gamma^i + \frac{1}{4} F_{ij} L_i \Gamma^j - \frac{1}{16} e^\Phi \Gamma_{11} (-2S + \tilde{F}_{ij} \Gamma^{ij}) \\ - \frac{1}{8 \cdot 4!} e^\Phi (-12 X_{ij} \Gamma^{ij} + \tilde{G}_{ijkl} \Gamma^{ijkl}) - \frac{1}{8} e^\Phi m$$

- The three Killing vectors that can be constructed form an $\mathfrak{sl}(2, \mathbb{R})$ algebra

Analysis I

- Identify the Killing spinors with the zero-modes of modified Dirac operators

$$\mathcal{D}^{(+)} : \tilde{\nabla}^+ \tilde{\Psi} \cdot \|\eta_+\|^2 = (2\tilde{\nabla}^+ \Phi + \tilde{F} \cdot \tilde{\Gamma}) \tilde{\Psi} \cdot \|\eta_+\|^2 = 2 \|\tilde{\nabla}^{(+)} \eta_+\|^2 + (-4\kappa - 16\kappa^2) \|\mathcal{A}^{(+)} \eta_+\|^2$$

$$\mathcal{D}^{(-)} : \tilde{\nabla}^+ (e^{-2\Phi} V) = -2e^{-2\Phi} \|\tilde{\nabla}^{(-)} \eta_-\|^2 + e^{-2\Phi} (4\kappa + 16\kappa^2) \|\mathcal{A}^{(-)} \eta_-\|^2$$

$$\text{where } V = -d \|\eta_-\|^2 - \|\eta_-\|^2 h$$

$$\text{Now use the index theorem to arrive at } N_+ = N_-$$

$$\text{i.e. } N = N_+ + N_- = 2N_+$$

Analysis I

- Identify the Killing spinors with the zero-modes of modified Dirac operators

$\mathcal{D}^{(+)} :$

$$\begin{aligned} \tilde{\nabla}^i \tilde{\nabla}_i \parallel \eta_+ \parallel^2 - (2\tilde{\nabla}^i \Phi + h^i) \tilde{\nabla}_i \parallel \eta_+ \parallel^2 = \\ 2 \parallel \hat{\nabla}^{(+)} \eta_+ \parallel^2 + (-4\kappa - 16\kappa^2) \parallel \mathcal{A}^{(+)} \eta_+ \parallel^2 , \end{aligned}$$

$\mathcal{D}^{(-)} :$

where $\hat{\nabla}_i^{(\pm)} = \nabla_i^{(\pm)} + \kappa \Gamma_i \mathcal{A}^{(\pm)}$

$$\tilde{\nabla}^i (e^{-2\Phi} V_i) =$$

$$- 2e^{-2\Phi} \parallel \hat{\nabla}^{(-)} \eta_- \parallel^2 + e^{-2\Phi} (4\kappa + 16\kappa^2) \parallel \mathcal{A}^{(-)} \eta_- \parallel^2$$

where $V = -d \parallel \eta_- \parallel^2 - \parallel \eta_- \parallel^2 h$

- Now use the index theorem to arrive at $N_+ = N_-$

i.e. $N = N_+ + N_- = 2N_-$

Method

Equations to analyse

- The Killing spinor equations (KSE) encode the requirement of preserving SUSY: linear
- Field equations encode the interplay between fluxes and geometry: non-linear
- Integrability conditions relate the KSEs and the field equations
- Usual approach: Remove the redundant field equations
- We will do the opposite! (Since some of the KSE are very complicated)
- Coordinate independent definition of an event horizon: *Killing horizon*
- Use an adapted metric [Wald]
- Integrate the KSE along the light-cone directions, leaving conditions on the horizon in terms of modified Dirac operators.

Assumptions

- The horizons admit at least one *supersymmetry*
- The horizons are *compact*
- The near horizon geometries are *smooth*

Analysis II

- Use all available tools to show that if η_- is a Killing spinor, then so is

$$\eta_+ = \Gamma_+ \Theta_- \eta_-$$

where $\Theta_- = \frac{1}{4} h_i \Gamma^i + \frac{1}{4} \Gamma_{11} L_i \Gamma^i - \frac{1}{16} e^\Phi \Gamma_{11} (-2S + \hat{F}_{ij} \Gamma^{ij})$
 $- \frac{1}{8 \cdot 4!} e^\Phi (-12 X_{ij} \Gamma^{ij} + \hat{G}_{ijkl} \Gamma^{ijkl}) - \frac{1}{8} e^\Phi m$

- The three Killing vectors that can be constructed form an $sl(2, \mathbb{R})$ algebra

Analysis I

- Identify the Killing spinors with the zero-modes of modified Dirac operators

$$\mathcal{D}^{(+)} : \nabla \hat{\nabla} \eta_+ \parallel \eta_+ \parallel^2 = \frac{1}{2} \parallel \hat{\nabla}^{(+)} \eta_+ \parallel^2 + (-4\kappa - 16\kappa^2) \parallel \mathcal{A}^{(+)} \eta_+ \parallel^2$$

$$\mathcal{D}^{(-)} : \nabla \hat{\nabla} \eta_- \parallel \eta_- \parallel^2 = \frac{1}{2} \parallel \hat{\nabla}^{(-)} \eta_- \parallel^2 + (-4\kappa + 16\kappa^2) \parallel \mathcal{A}^{(-)} \eta_- \parallel^2$$

where $\hat{\nabla}_i^{(\pm)} = \nabla_i^{(\pm)} + \kappa \Gamma_i \mathcal{A}^{(\pm)}$
 where $V = -d \parallel \eta_- \parallel^2 = \parallel \eta_- \parallel^2 h$
 i.e. $N_+ = N_- + N_- = 2N_-$

- Now use the index theorem to arrive at $N_+ = N_-$

Analysis II

- Use all available tools to show that if η_- is a Killing spinor, then so is

$$\eta_+ = \Gamma_+ \Theta_- \eta_-$$

where $\Theta_- = \frac{1}{4}h_i\Gamma^i + \frac{1}{4}\Gamma_{11}L_i\Gamma^i - \frac{1}{16}e^\Phi\Gamma_{11}(-2S + \tilde{F}_{ij}\Gamma^{ij})$
 $-\frac{1}{8 \cdot 4!}e^\Phi(-12X_{ij}\Gamma^{ij} + \tilde{G}_{ijkl}\Gamma^{ijkl}) - \frac{1}{8}e^\Phi m$

- The three Killing vectors that can be constructed form an $\mathfrak{sl}(2, \mathbb{R})$ algebra

ols

MMA

package for
-mixed-integer
-by calculations...
A from water grain name
date with Mathematics 75

Lichnerowicz Theorems

- Fundamental equation in the analysis of spinors on pseudo-Riemannian manifolds
- $$\square \psi = \nabla^2 \psi + \frac{1}{2} R \psi$$
- Relates the Dirac and Laplace operator acting on a spinor ψ and the scalar curvature
- Is one important in Selberg-Witten theory

Index Theorems

- Index theorems relate analytic quantities to topological invariants
- Spinors are like Dirac operators (modified by magnetic field) can use index theorems to count zero eigenvalues
- The zero modes will be identified with the fermion ground states (Killing vectors)
- Through modified Lichnerowicz type of Killing spinors (i.e. Killing spinors) with
- Through modified Lichnerowicz type of Killing spinors (i.e. Killing spinors) with
- Through modified Lichnerowicz type of Killing spinors (i.e. Killing spinors) with

Method

Equations to analyze

- The Killing vector equations that describe the symmetries of the spacetime geometry
- The Einstein equations that describe the geometry of the spacetime
- The Dirac equations that describe the spinors
- The Bianchi identities that describe the conservation of energy and momentum

Assumptions

- The horizons are compact and smooth
- The horizons are connected
- The new horizon geometries are smooth

Analysis II

- Total of analysis from the fact that $\Delta \psi = 0$ and $\Delta \psi = 0$ where $\Delta = \nabla^2 + \frac{1}{2} R$ and $\nabla^2 = \nabla_a \nabla^a$ and R is the scalar curvature

Analysis I

- The Dirac equations that describe the spinors

Results

- There is always supersymmetry enhancement $N = 2N_+ + \text{Index}(D_\psi)$
- Killing horizons in 4d SUGRA always preserve an even number of supersymmetries
- Horizons with $N_+ = 0$ and non-trivial fluxes have an $sl(2, \mathbb{R})$ symmetry subalgebra
- Conjecture that this holds in general for all supergravity theories
- The $sl(2, \mathbb{R})$ symmetry provides evidence that all such horizons have an AdS/CFT dual
- Prediction from D=11 breaks an even number of supersymmetries

Outlook

- All information contained within
- Solving the Killing vector equations
- Solving the Dirac equations
- Solving the Bianchi identities
- Can we extend the $sl(2, \mathbb{R})$ symmetry to get information about the dual CFT?

Results

- There is always supersymmetry enhancement

$$N = 2N_- + \text{Index}(D_E)$$

- Killing horizons in IIA SUGRA always preserve an **even** number of supersymmetries.
- Horizons with $N_- \neq 0$ and non-trivial fluxes have an $\mathfrak{sl}(2, \mathbb{R})$ symmetry subalgebra
- Conjecture that this holds in general for all supergravity theories
- The $\mathfrak{sl}(2, \mathbb{R})$ symmetry provides evidence that all such horizons have an AdS/CFT dual
- Reduction from D=11 breaks an even number of supersymmetries

Analysis II

try to show that if $e_{\mu\nu}$ is a solution

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

$$e_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu} \epsilon^{\alpha\beta} \epsilon_{\alpha\beta}$$

Analysis I

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Identify the data relevant with the given

result of the given data

Results

- There is always supersymmetry enhancement
 $N = 2N_{\text{SUGRA}} + \text{Index}(D_E)$
- Killing horizons in IIA SUGRA always preserve and even number of supersymmetries.
- Horizons with $N_{\text{Killing}} \neq 0$ and non-trivial fluxes have an $sl(2, \mathbb{R})$ symmetry subalgebra
- Conjecture that this holds in general for all supergravity theories
- The $sl(2, \mathbb{R})$ symmetry provides evidence that all such horizons have an AdS/CFT dual
- Reduction from D=11 breaks an even number of supersymmetries

Outlook

- All information obtained without solving neither the (horizon) KSEs nor the field equations!
- Solving the Killing spinor equations is feasible
- Structure of generic black holes
- Can we exploit the $sl(2, \mathbb{R})$ symmetry to get information about the dual CFT?

Outlook

- All information obtained without solving neither the (horizon) KSEs nor the field equations!
- Solving the Killing spinor equations is feasible
- Structure of generic black holes
- Can we exploit the $sl(2, \mathbb{R})$ symmetry to get information about the dual CFT?

Dynamical symmetry enhancement near black hole horizons

Ulf Gran



Thank you!