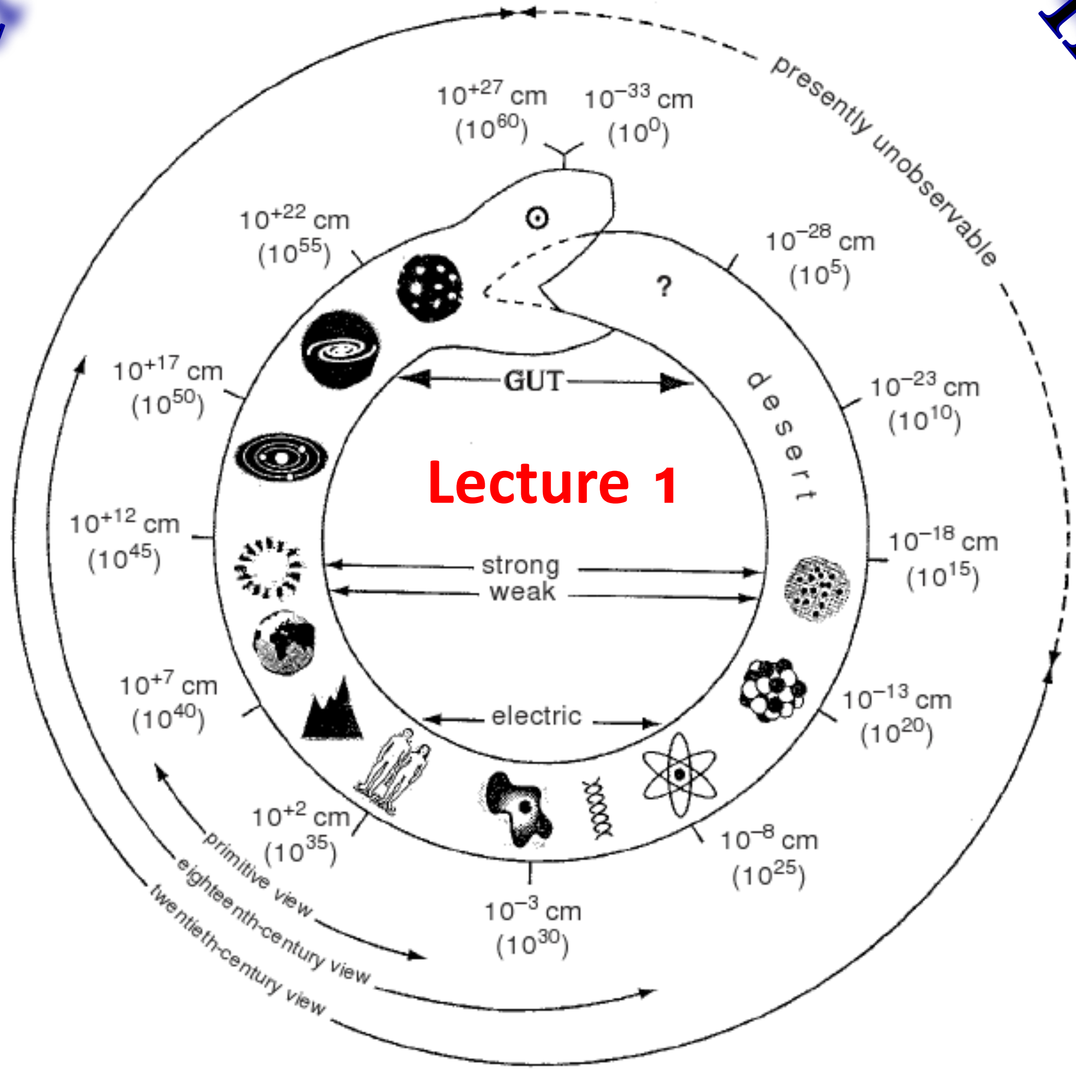
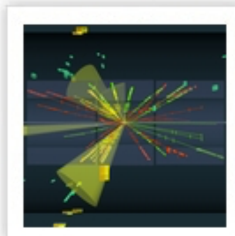
Introduction to cosmology

Subir Sarkar

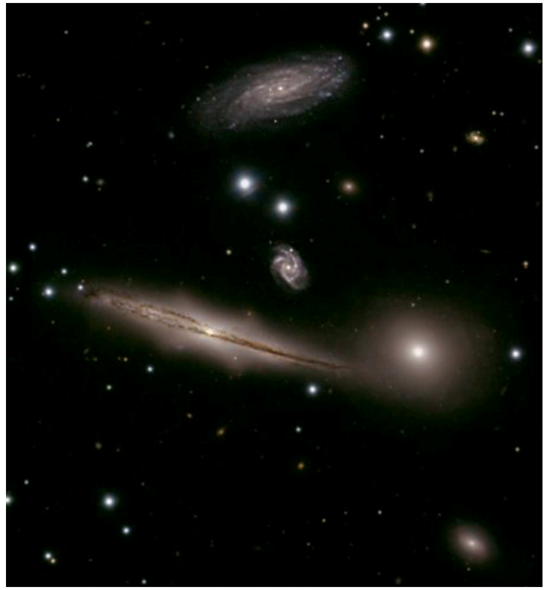


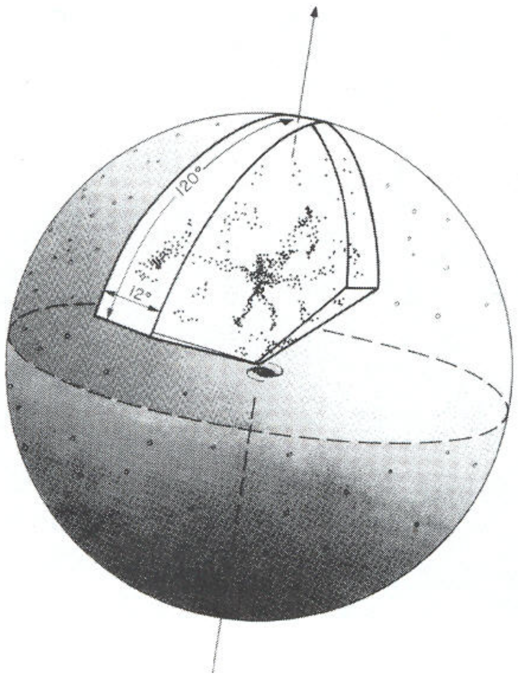
Lecture 1

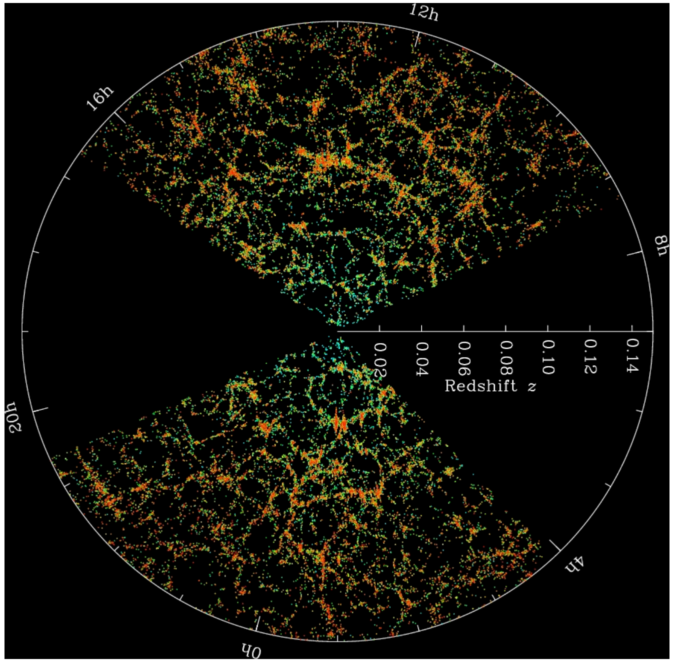


Taller de Altas Energías, 4-17 September 2016, Benasque

The universe appears complex and structured on many scales ...

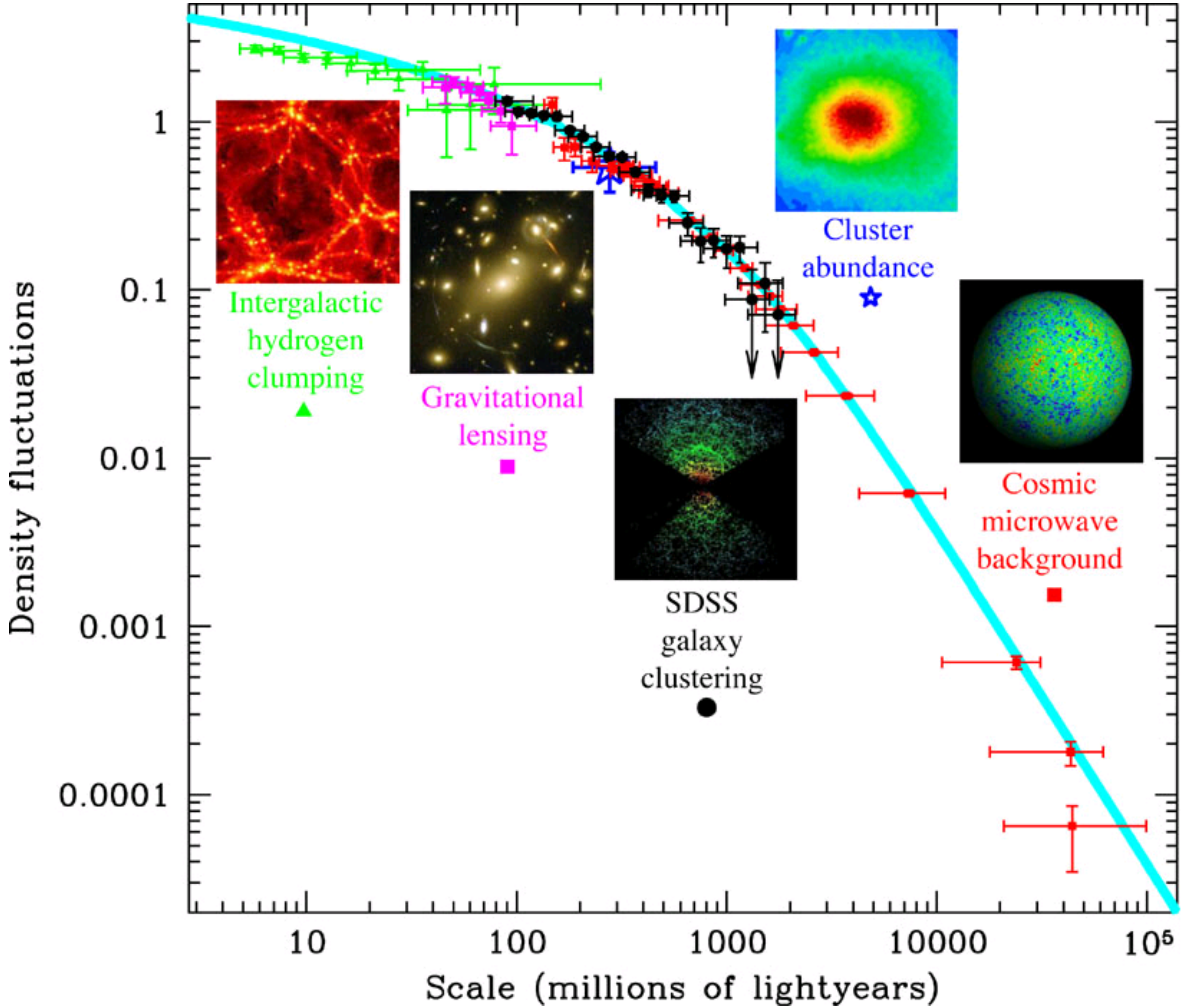






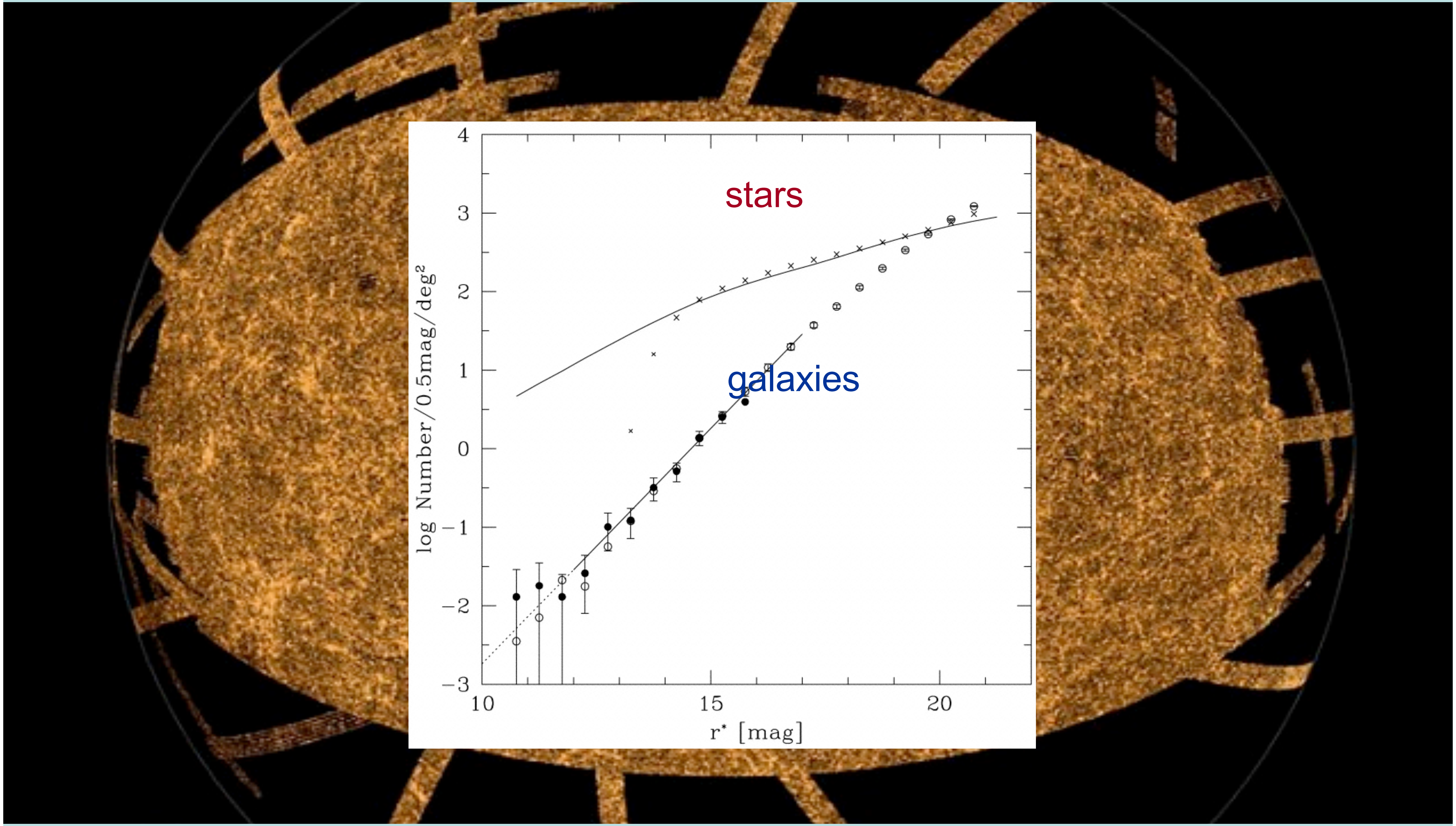
How can we describe it by a simple mathematical model?

Although the universe is lumpy, it seems to become smoother and smoother when averaged over larger and larger scales …



Courtesey: Max Tegmark

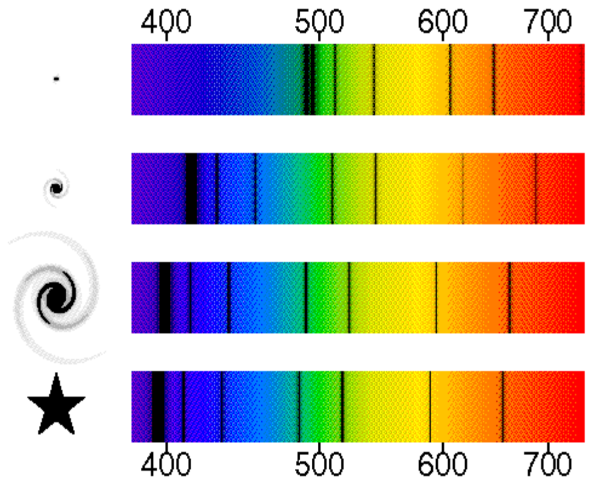
Hubble (1926) showed that the distribution of *faint* galaxies is **homogeneous**, i.e. $N\,(>S) \propto S^{-3/2} \Rightarrow N\,(<m) \propto 10^{0.6m}$, where $m \equiv -2.5 \log (S/S_0)$

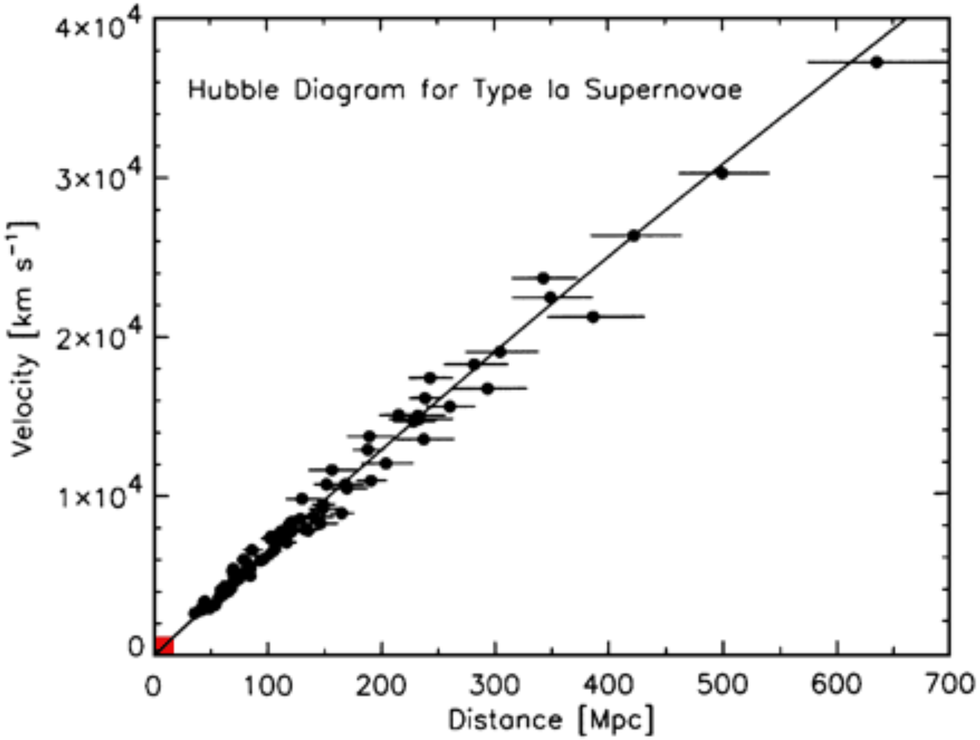


Here is the test done on galaxies in the **Sloan Digital Sky Survey**

NB: For stars, $N\,(<m) \propto 10^{0.4m}$, reflecting their 2D distribution

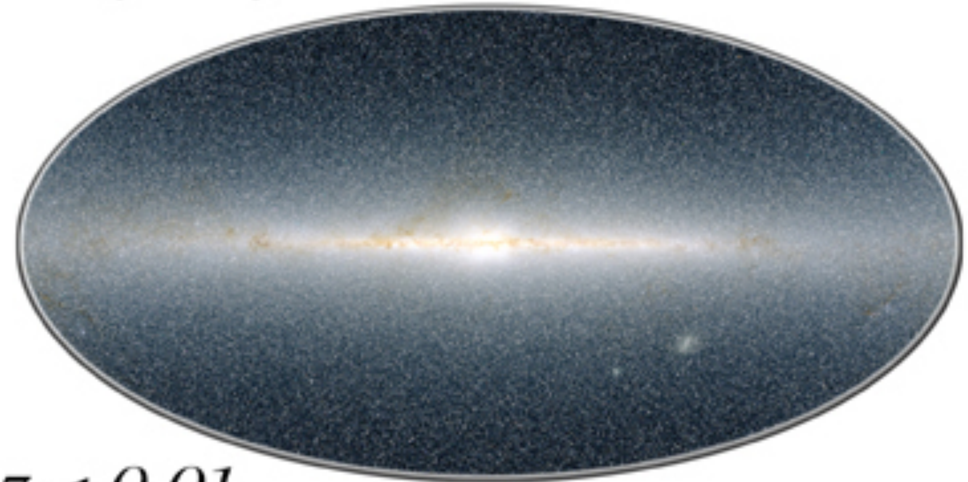
Red Shift: $z = \frac{\lambda_{\text{emitted}} - \lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \simeq v/c$, for $z \ll 1$



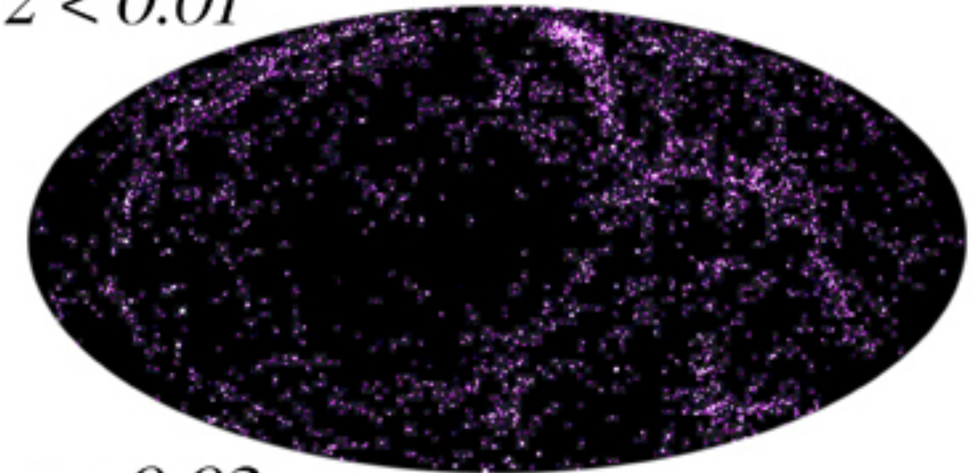


Velocity (redshift) is *proportional* to distance, so $z = 0.1 \Rightarrow d \sim 500$ Mpc

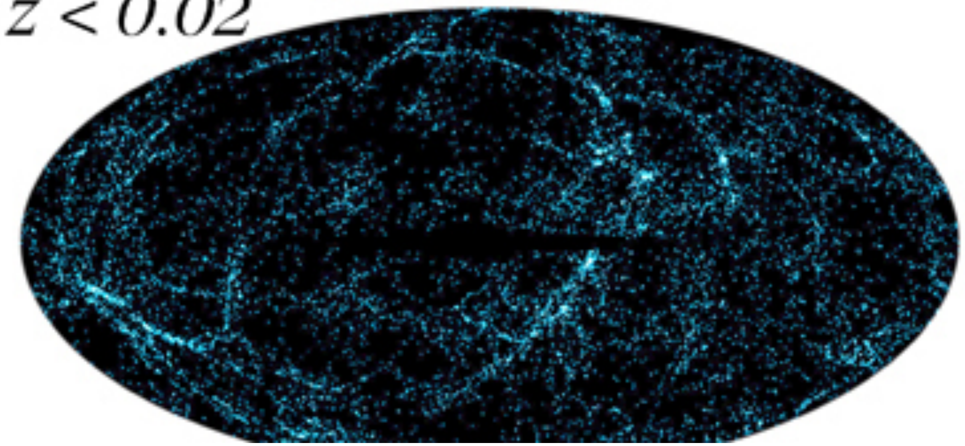
Milky Way

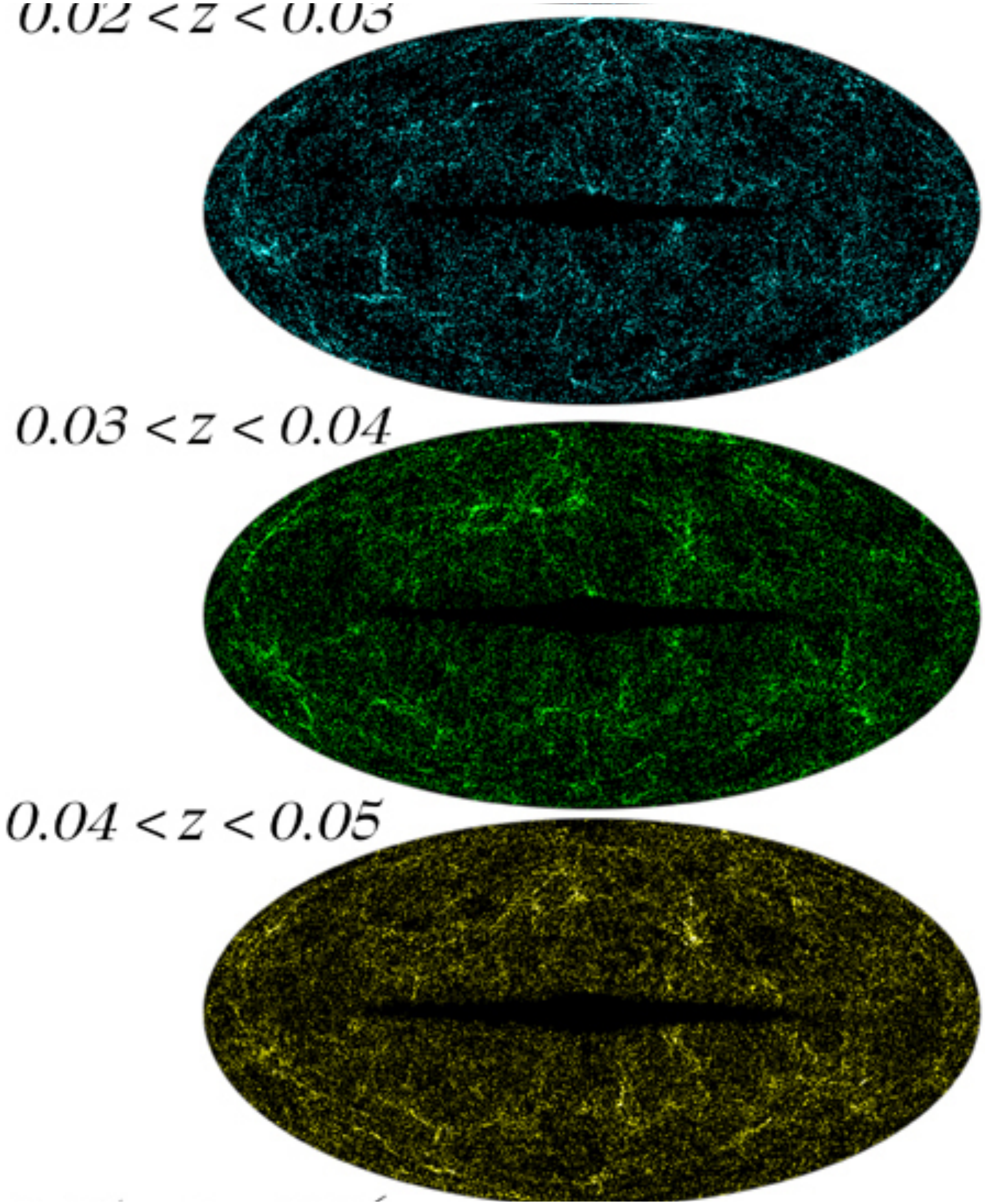


$z < 0.01$



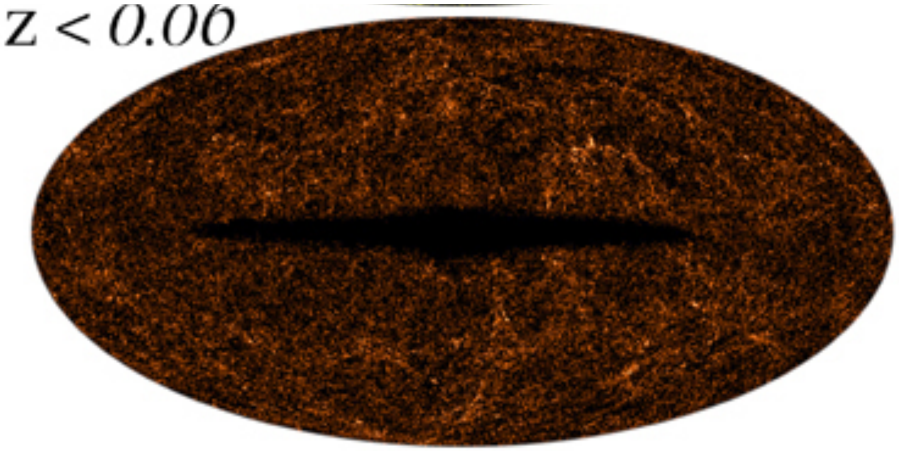
$0.01 < z < 0.02$



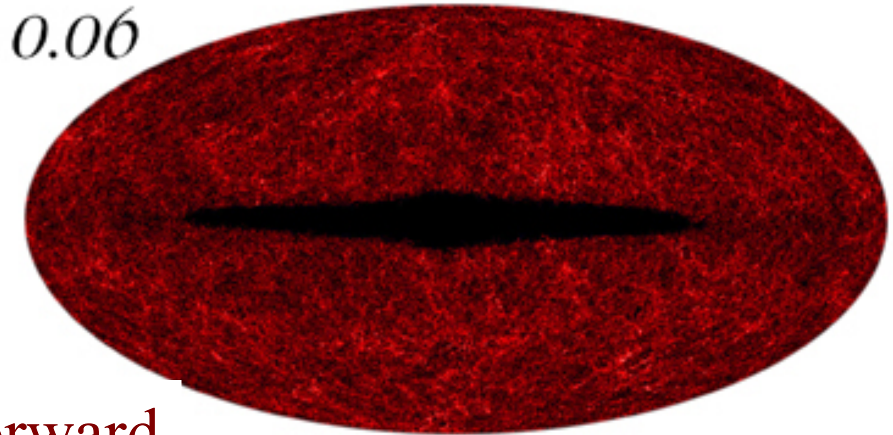


0.02 < z < 0.03
0.03 < z < 0.04
0.04 < z < 0.05

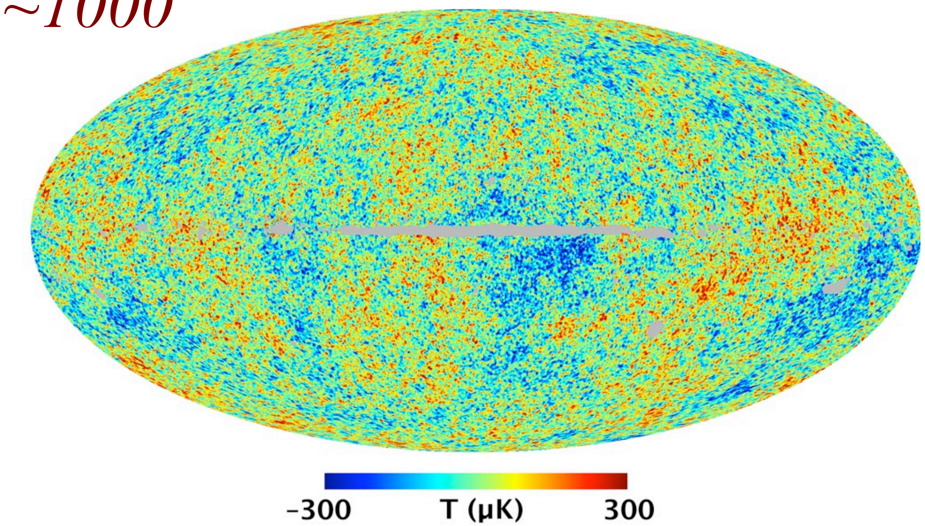
$0.05 < z < 0.06$



$z > 0.06$

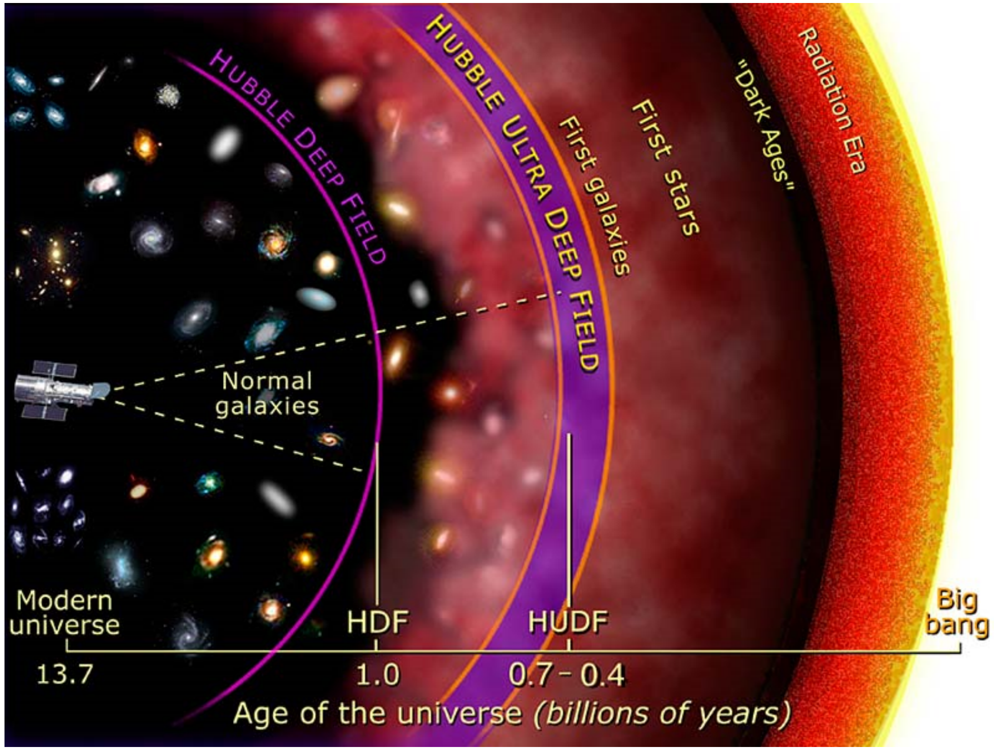


… fast forward to: $z \sim 1000$



The **cosmic microwave background** (blackbody spectrum with T=2.7255 ± 0.0006 K) is isotropic to 1 part in ~10^5 (*after* a x100 larger dipole anisotropy is removed)

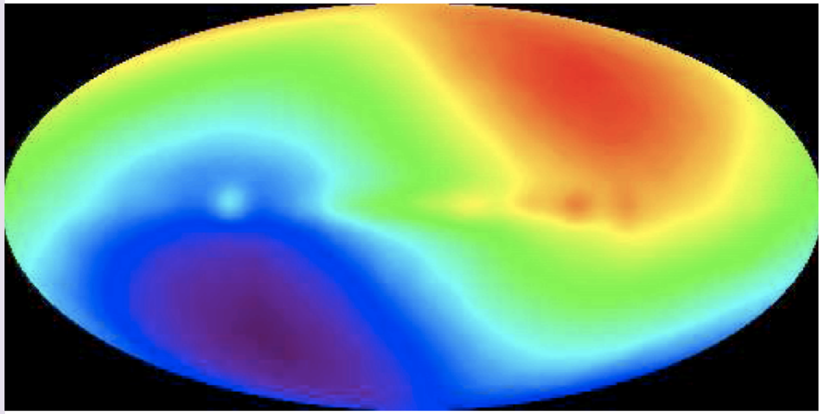
When we look out in distance, we look *back in time* so what we see makes sense if the universe was denser (hence hotter) when it was younger … back to the Big Bang:



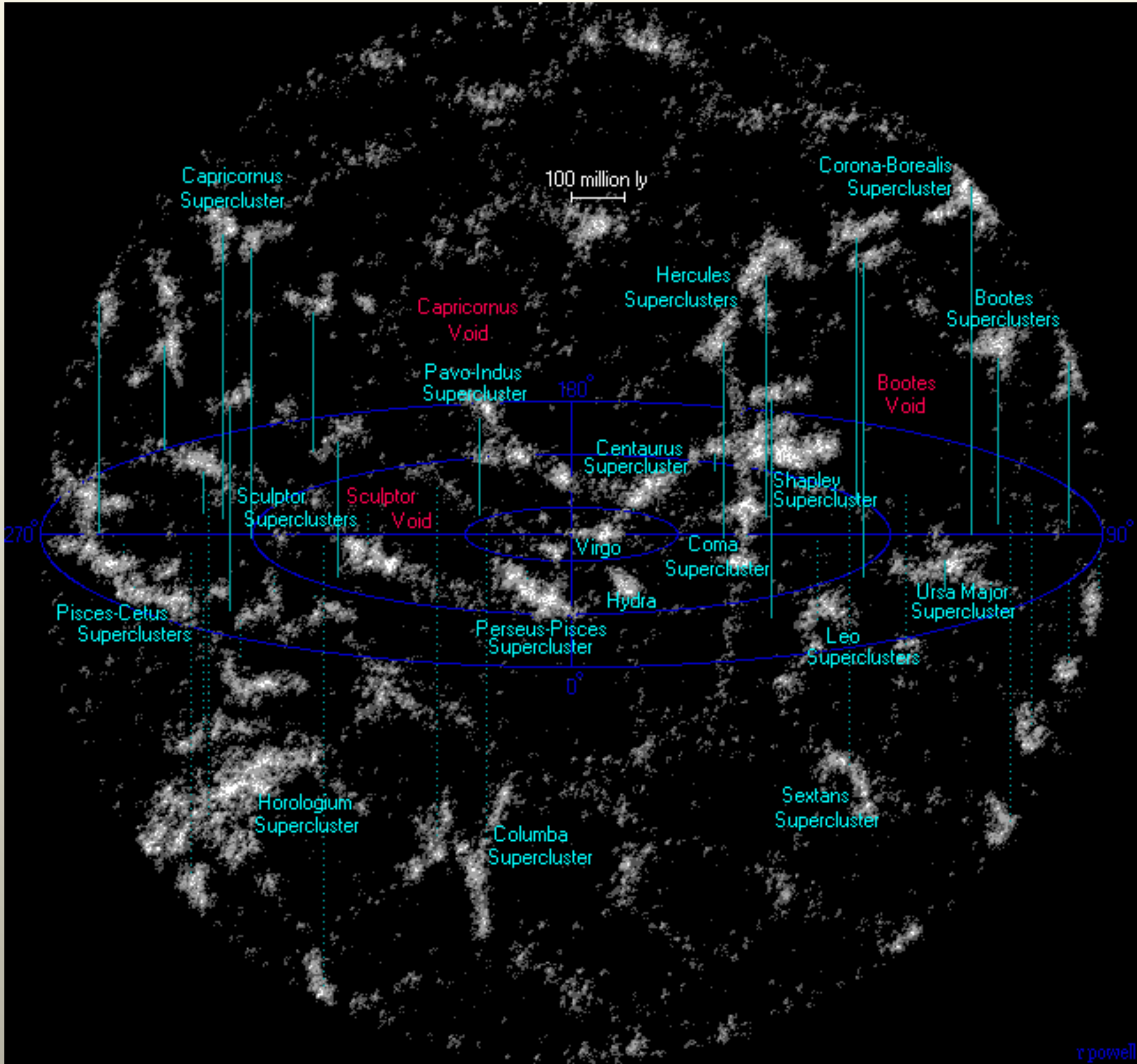
<u>Open Question</u>: The CMB exhibits a dipole anisotropy with amplitude 3.355±0.008 mK, ascribed to our local 'peculiar' motion with $v = 369.0 \pm 0.9$ km/s towards the Shapley supercluster. It is *after* we boost to this frame that we see the CMB as isotropic. However using SNe Ia to trace the Hubble flow, convergence to the CMB frame has *not* occured even as far out as $z \sim 0.07$ (~300 Mpc)!

Colin *et al*, MNRAS **414**:264,2011, Feindt *et al*, A&A **560**:A90,2013

Eppur si muove!

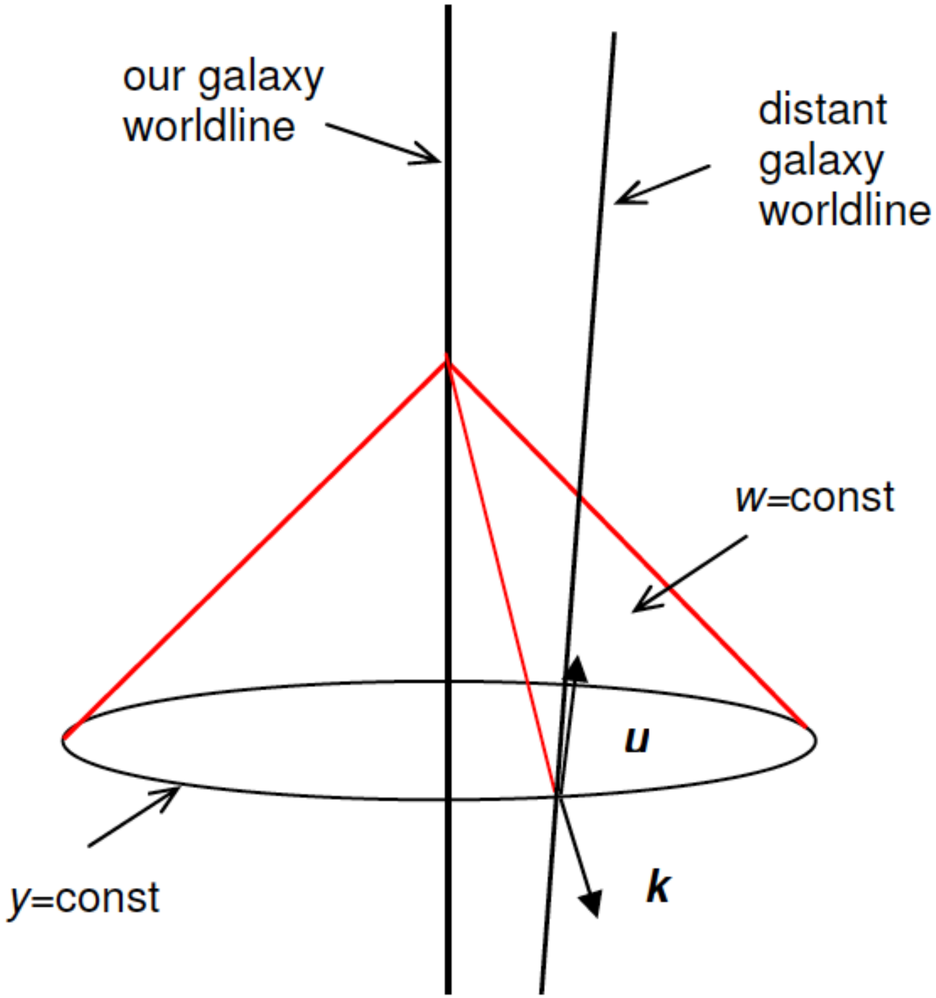


This is what our universe *actually* looks like out to a few hundred Mpc



It is *not* clear where the transition to homogeneity occurs (often quoted as ~100 Mpc)

Crucially all we can ever learn about the universe is contained within our past light cone



We *cannot* move over cosmological distances and check that the universe looks the same from other view points … so we must *assume* the validity of the 'Cosmological Principle' (Milne 1935) viz. our position is *typical*

Special relativity

$$ds^2 = \sum g_{ij} dx^i dx^j \ldots \text{ interval between events } x^i \text{ and } x^j (i, j = 0, 1, 2, 3)$$

$$g_{ij}(x) \equiv g_{ji}(x) \rightarrow 10 \text{ independent functions}$$

Minkowski metric

$$\eta_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \frac{\delta g_{ij}}{\delta x^k} = 0 \quad \Rightarrow ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

… invariant under **Lorentz velocity transformations**, *i.e. equivalent* to **local inertial coordinates** of **Newtonian mechanics**

General relativity

Now g_{ij} is related to the **distribution of matter** … but $g_{ij} = \eta_{ij}$ is a solution in the *absence* of matter – contrary to **Mach's principle***!

* inertial frames are determined relative to the motion of the matter ("distant stars") in the universe

Einstein (1919) saw two possible ways out:

* add suitable boundary conditions to eliminate anti-Machian solution, *viz.* let g_{ij} take some pathlogical form (rather than becoming η_{ij}) when far away from all matter ✗
… however de Sitter pointed out pheonomenological problems with this idea!

- Postulate that the matter distribution is **homogeneous** (in the average) and that matter causes space to curve so as to close in on itself (3D analogue of a 2D balloon)

→ Spatial volume finite but *no boundaries* and a non-singular metric everywhere ✔

Einstein's world model

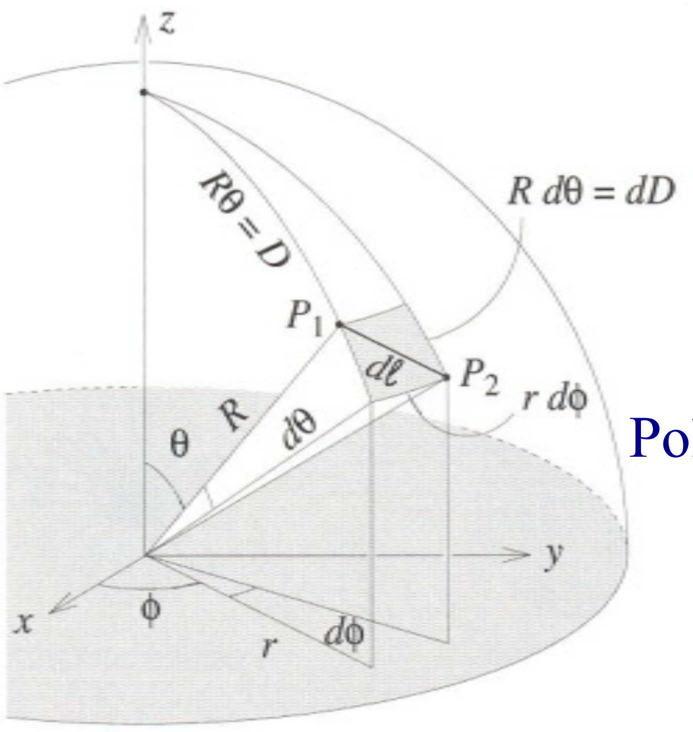
Homogeneity $\Rightarrow \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}m} \propto 10^{0.6m}$ … as observed later (Hubble 1926)

… incorporating Milne's 'Cosmological Principle'

$\mathrm{d}s^2 = \mathrm{d}t^2 + g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ … synchronous gauge (dense set of comoving observers)

This is the 'standard model' we are still using *today* to interpret all observations

Picture the spatial part as $\mathbf{S}^3$ (3D analogue of balloon, embedded in flat 4D space)



Set of points defining $\mathbf{S}^3$: $R^2 = x^2 + y^2 + z^2 + w^2$

where: $r^2 = x^2 + y^2 + z^2$

Line element: $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \mathrm{d}w^2$

i.e. $\mathrm{d}l^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$

Polar coordinates ($z=r\cos\theta$, $x=r\sin\theta\cos\varphi$, $y=r\sin\theta\sin\varphi$):

$$\mathrm{d}l^2 = \mathrm{d}r^2 + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) + r^2\mathrm{d}r^2/(R^2\text{-}r^2)$$
$$= \mathrm{d}r^2/(1 - r^2/R^2) + r^2\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

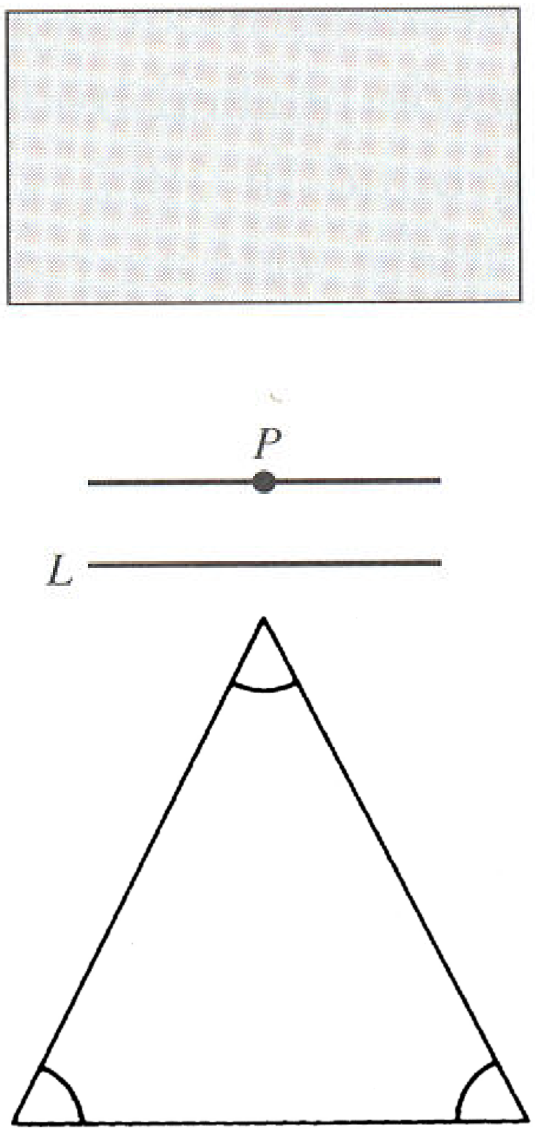
or, $\mathrm{d}s^2 = \mathrm{d}t^2 - R^2\,[\mathrm{d}\chi^2 + \sin^2\chi\,(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)]$,

where, $r = R\sin\chi$, $\chi \Rightarrow$ polar angle of hypersphere

Note interesting visual effects in curved space (when $r \sim R$), e.g. the angular size $\delta = D/R\sin\chi$ reaches minimum at $\chi = \pi/2$ and diverges to fill the entire sky when $\chi = \pi$ (this point is the just the 'Big Bang' – the antipodal point of the hypersphere)

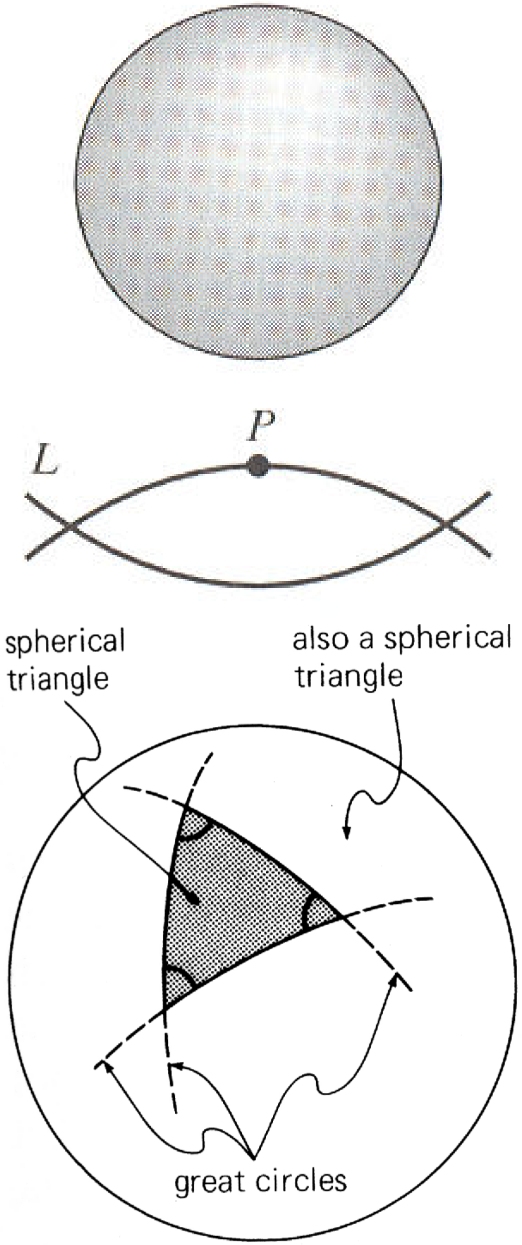
Also the parallax, $\varepsilon = A\cot\varphi/R$, *vanishes* at $\chi = \pi/2$

The 3 possible geometries of maximally-symmetric space

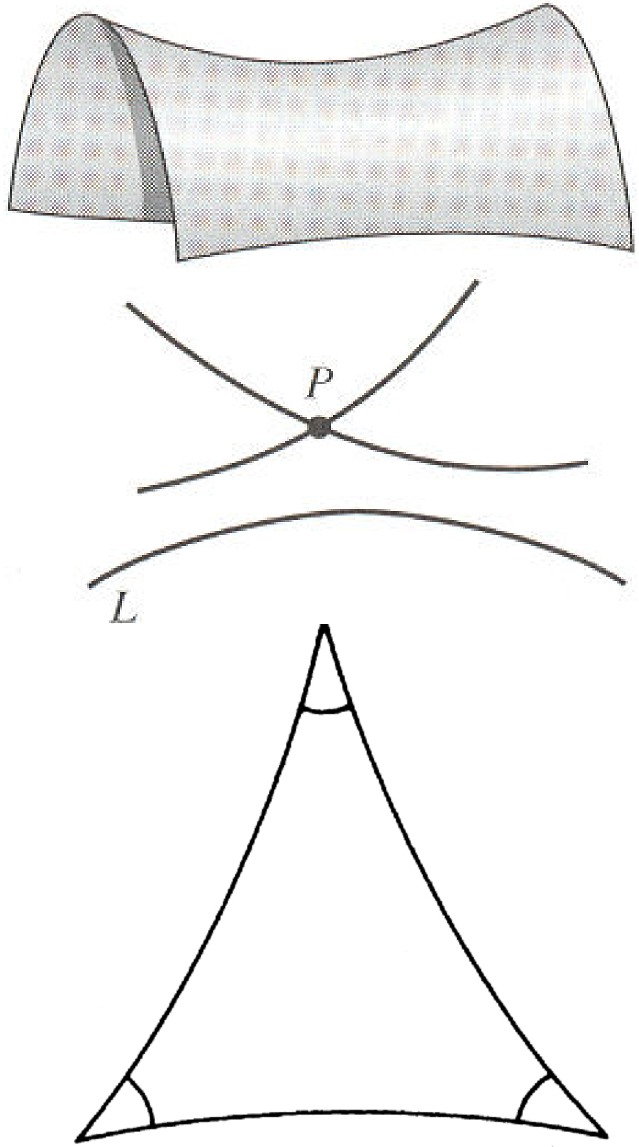


flat space

= 180°



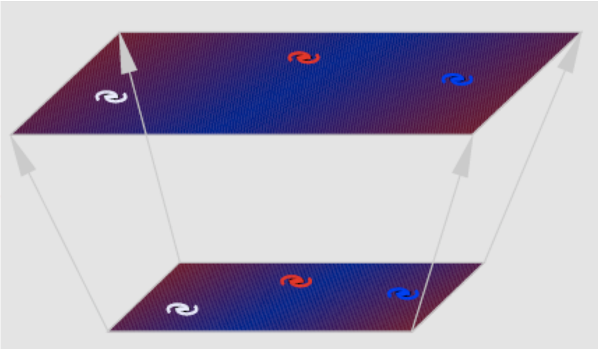
> 180°



hyperbolic space

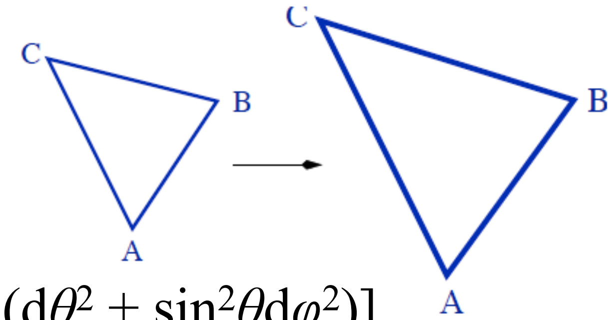
< 180°

The expanding universe (Friedmann 1922, Lemaitre 1931)



Generalise line element:

$R\ (t)\ = R_0\ a(t)$



$ds^2 = dt^2 - a^2(t)\, R_0^2\, [d\chi^2 + \sin^2\chi\ (d\theta^2 + \sin^2\theta d\varphi^2)]$

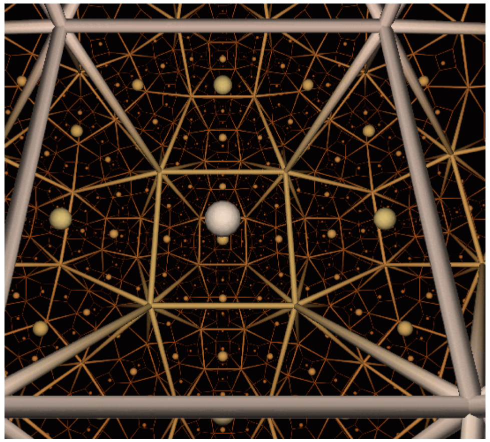
… a spatially **closed** expanding universe

To describe a spatially **open** expanding universe, change: $\chi \to i\chi$, $R_0 \to iR_0$, so

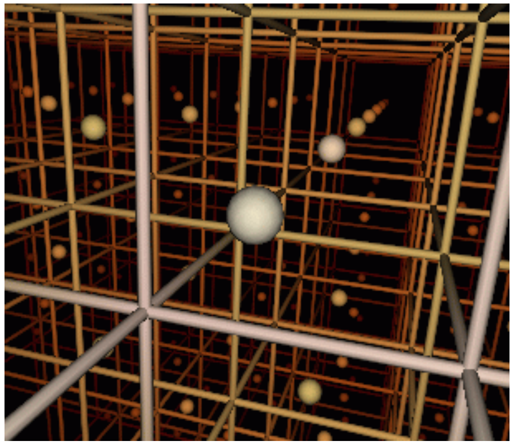
$$ds^2 = dt^2 - a^2(t)\, R_0^2\, [d\chi^2 + \sinh^2\chi\ (d\theta^2 + \sin^2\theta d\varphi^2)]$$

This is the Robertson-Walker line element (*maximally*-symmetric space-time):

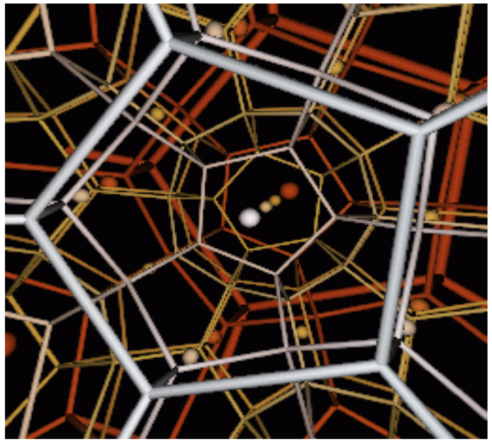
$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$



$k = -1$

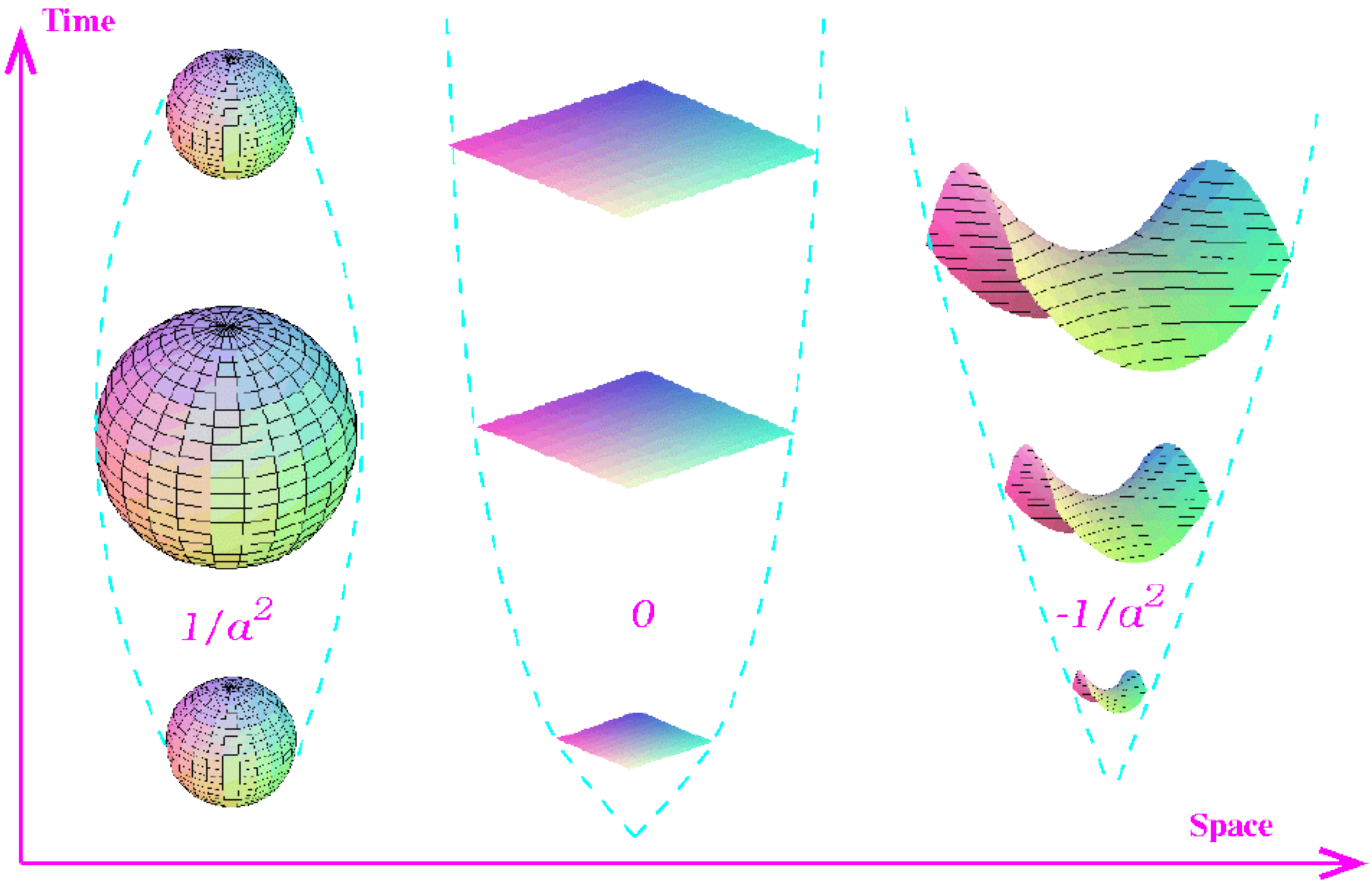


$k = 0$



$k = +1$

Homogeneous and isotropic world models

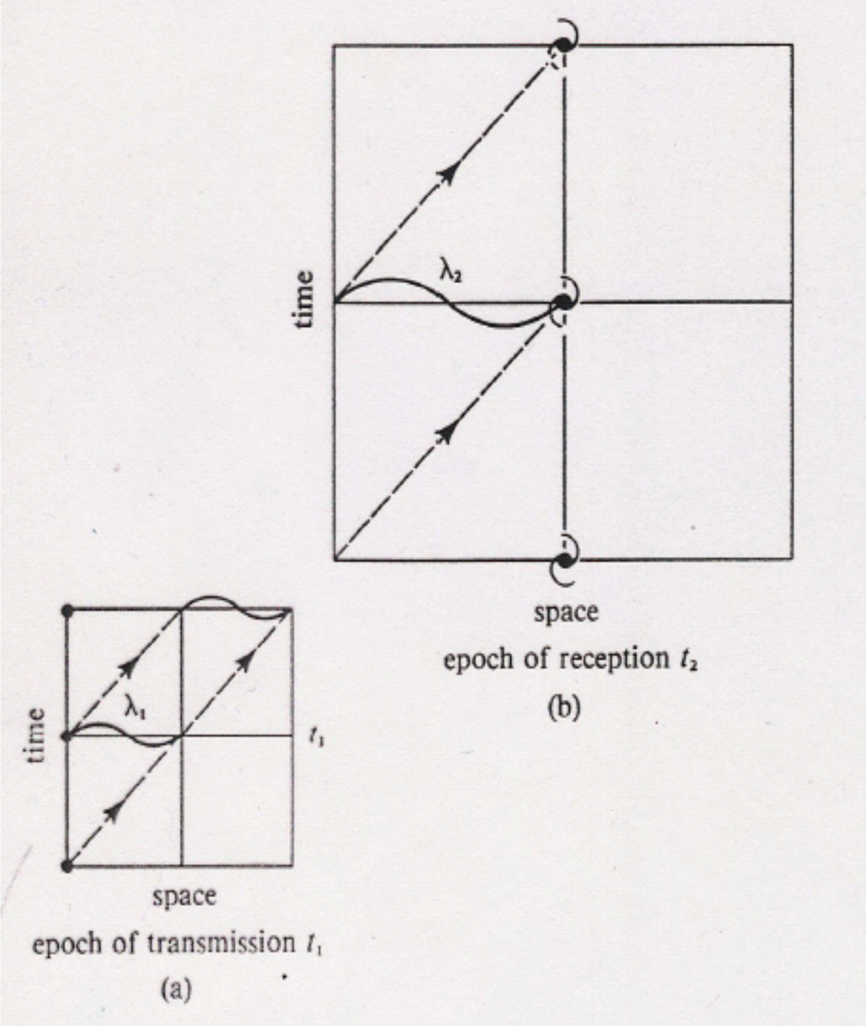


The redshift happens because, for null geodesics:

$$\int_t^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_0^r \frac{\mathrm{d}r}{\sqrt{1-kr^2}} = \text{const}$$

… for a galaxy (in co-moving coordinates), so crests of adjacent waves, separated by Δt at emission, will be received with separation, Δt_0:

$$\frac{\Delta t_0}{\Delta t} = 1 + \frac{\Delta\lambda}{\lambda_0} \equiv 1 + z = \frac{a(t_0)}{a(t)}$$

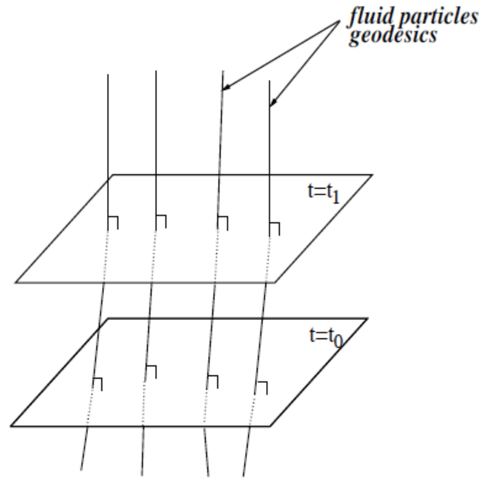


This is the cosmological time dilation or redshift: $z = \infty$ is the 'Big Bang' at $t = 0$ (the antipodal point of the hypersphere $\Rightarrow$ the furthest we can look back in principle)

Everything is not expanding (how would we know?) … certainly not bound structures like atoms or planets or galaxies – it is only the large-scale *smoothed* space-time metric which is stretching with cosmic time (and there is no restriction on the rate!)

The 'expansion' is in a sense *illusory* … because we can always transform to a "comoving" coordinate system where *galaxies are at rest* wrt each other

Ideal fluid: $T_{ij} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$



Poisson's equation: $\nabla . g = -4\pi G_{\mathrm{N}}(\rho + 3p)$

Birkhoff's theorem: If $T_{ij} = 0$ in some region within a spherically symmetric distribution of matter, then the solution in the hole $\Rightarrow$ flat space-time

Einstein's field equations

$$R_{ij} + \tfrac{1}{2} g_{ij} R_{\mathrm{c}} = 8\pi G_{\mathrm{N}} T_{ij}, \text{ where } R_{ij} \equiv g^{\lambda k} R_{\mu\nu\lambda k} \text{ and } R_{\mathrm{c}} \equiv g^{\mu\nu} R_{\mu\nu}$$

For the RW metric, the 00 and 11 components simplify to the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\mathrm{N}}}{3}\rho - \frac{k}{a^2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G_{\mathrm{N}}}{3}(\rho + 3p)$$

‘Newtonian’ cosmology

Consider sphere of radius *l* embedded in homogeneous background (McCrea & Milne 1934):

$$\ddot{\ell} = -G_{\rm N}M/r^2 = -\tfrac{4\pi}{3}G_{\rm N}(\rho+3p)\ell; \;\; \text{also } {\rm d}U \equiv \rho{\rm d}V + V{\rm d}\rho = -p{\rm d}V$$

$$\Rightarrow \dot{\rho} = -(\rho+p)\frac{\dot{V}}{V} = -3(\rho+p)\frac{\dot{\ell}}{\ell} \;\; \ldots \text{ energy eq. for ideal fluid}$$

$$\text{So, } \ddot{\ell} = \tfrac{8\pi}{3}G_{\rm N}\rho\ell + \tfrac{4\pi}{3}G_{\rm N}\dot{\rho}\frac{\ell^2}{\dot{\ell}} \Rightarrow \dot{\ell}^2 = \tfrac{8\pi}{3}G_{\rm N}\rho\ell^2 + K$$

To obtain a *static* solution (Einstein’s “greatest blunder”) we have to set:

$$\rho + 3p = 0 \text{ i.e. } p = -\tfrac{\rho}{3} \;(!) \Rightarrow \text{ universe of radius: } \mathcal{R}^2 = -\frac{\ell^2}{k} = [\tfrac{8\pi}{3}G_{\rm N}\rho]^{-1}$$

The static solution is in fact *unstable* (metric perturbations grow exponentially fast) but we do *not* have the freedom, as Einstein said, to “do away with the cosmological constant” … it is a necessary consequence of **general coordinate invariance** which allows an *arbitrary* constant multiplying the metric tensor to be added to the l.h.s.

So must modify the field equations to: $R_{ij} + \frac{1}{2}g_{ij}R_{\rm c} - \Lambda g_{ij} = 8\pi G_{\rm N}T_{ij}$

The Λ term can be *interpreted* (when moved to r.h.s.) as a fluid with: $\rho_\Lambda = -p_\Lambda = \Lambda/8\pi G_{\rm N}$

FLRW Dynamics

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_\mathrm{N}}{3}(\rho + 3p) \pm \frac{1}{a^2\mathcal{R}^2} \rightarrow -\frac{4\pi G_\mathrm{N}}{3}(\rho_\mathrm{b} + 3p_\mathrm{b}) \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}$$

b $\Rightarrow$ 'background' (i.e. "ordinary" matter/radiation)

Conservation of energy-momentum: $$\dot{\rho_\mathrm{b}} = -3(\rho_\mathrm{b} + p_\mathrm{b})\frac{\dot{a}}{a}$$

$$\Rightarrow \left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G_\mathrm{N}}{3}\rho_\mathrm{b} \pm \frac{1}{a^2\mathcal{R}^2} + \frac{\Lambda}{3}, \quad \text{where } + \text{ is open/- is closed universe}$$

Two interesting solutions describing an expanding universe:

Einstein-de Sitter: ... excellent description of early universe e.g. decoupling of CMB

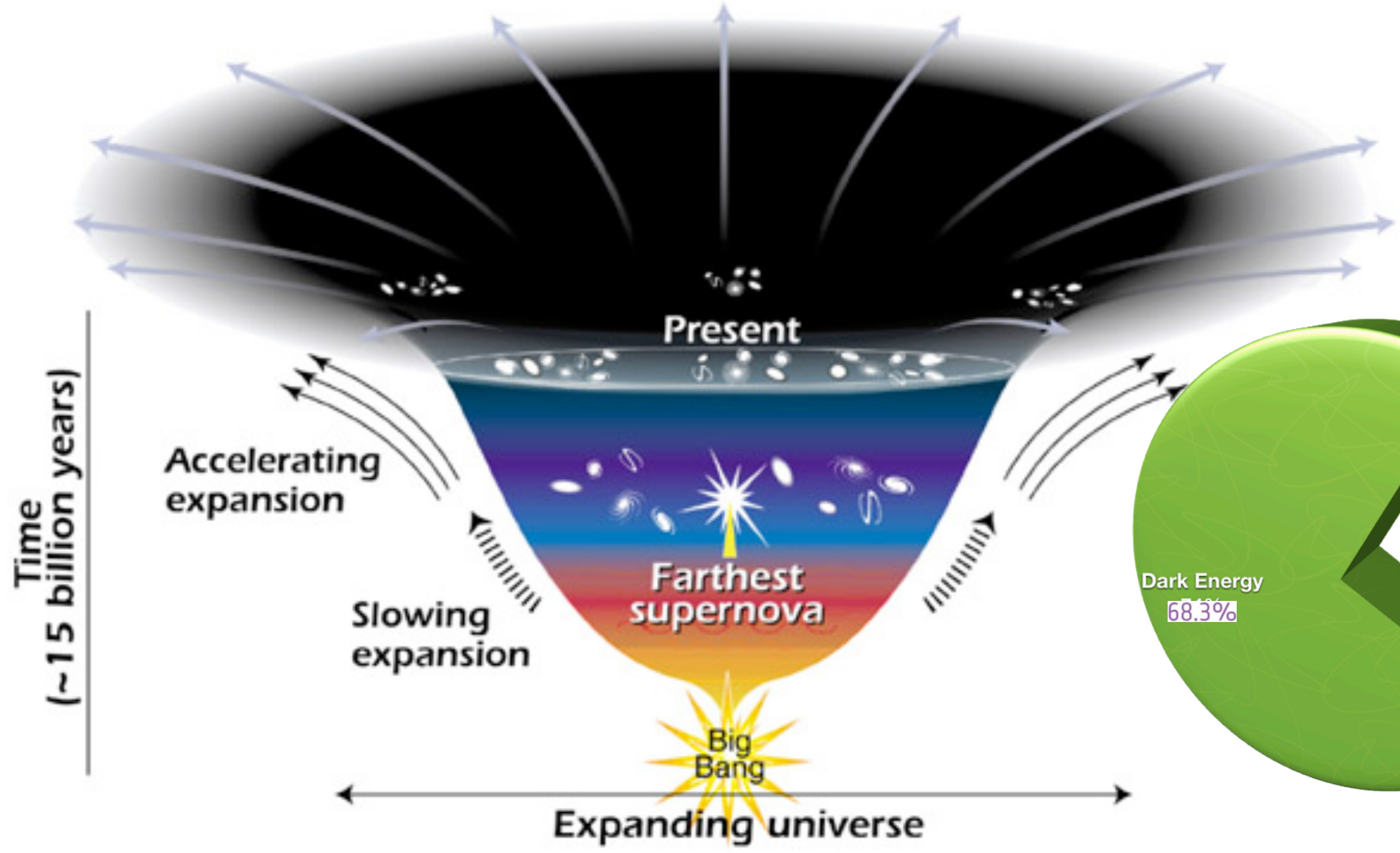
$$p_\mathrm{b} \ll \rho_\mathrm{b}, \Lambda = \frac{1}{a^2\mathcal{R}^2} = 0 \Rightarrow a(t) \propto t^{2/3}, t = \frac{2}{3H} = \frac{1}{\sqrt{6\pi G_\mathrm{N}\rho}}$$

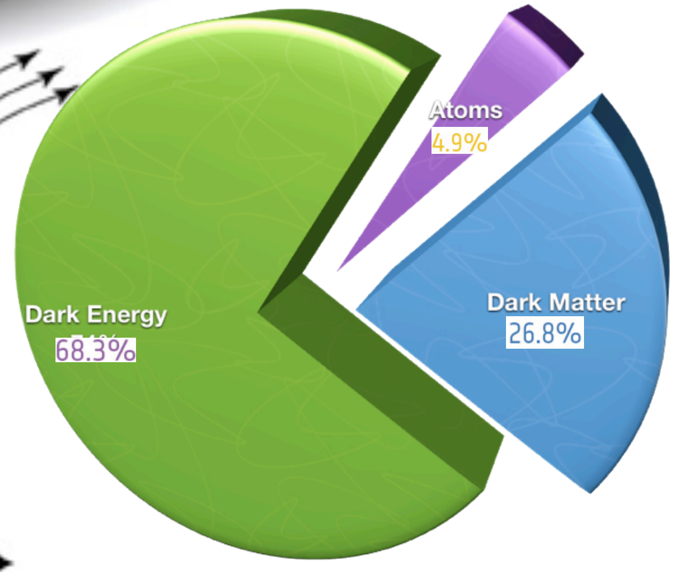
de Sitter: ... used to describe early era of accelerated expansion: 'inflation'

$$\rho_\mathrm{b} = p_\mathrm{b} = 0 \Rightarrow a(t) = \exp\left(H_\Lambda t\right), \text{ where } H_\Lambda = \sqrt{\frac{\Lambda}{3}}$$

The general solution is for an universe expanding under the influence of both matter (including radiation) and a cosmological constant ("dark energy")

This is today our 'standard model' of the universe …
dominated by Λ ("dark energy") and undergoing accelerated expansion





But because it is 'simple' and fits "all the observational data" does *not* mean it is right ... when we lack a *physical* understanding of Λ