1) **Olber's Paradox:** Calculate the likelihood that in a random direction in an isotropic, infinite, static universe uniformly filled with stars which have radius R and spatial density n, our view will be blocked by a star at distance r. (Assume that a typical star is like our Sun with radius $R_\odot \sim 7 \times 10^{10}$ cm and mass $M_\odot \sim 2 \times 10^{33}$ gm, and that the average density if luminous matter in the universe is $\sim 1\%$ of the critical density i.e. $\rho_{\rm lum} \sim 2 \times 10^{-31}$ gm cm^{-3}.)

Given that the sky is dark at night, what bound can be inferred on the age of such an universe (and/or of the stars contained in it)? Discuss the resolution of Olber's Paradox in the universe we inhabit.

For comparison, calculate the brightness of the night sky for a matter-dominated Friedmann cosmology (with $k = 0$) expanding presently at a rate H_0.

In reality stars are not uniformly distributed in space but are clustered in galaxies (of typical radius $\sim 10h^{-1}$ kpc and present number density $\sim 0.02h^3$ Mpc^{-3}, where $h \equiv H_0/100$ km/s/Mpc is the Hubble parameter. Calculate the optical depth due to galaxies up to a redshift z, assuming their comoving number is conserved. At what redshift would it become unity?

2) **Eppur si muove:** The Cosmic Microwave Background (CMB) is observed to have a black-body spectrum, i.e. the number of photons per unit volume with (angular) frequency ω in the range $\mathrm{d}\omega$ and moving in any direction (the solid angle $\mathrm{d}\Omega$ integrated to 4π) is

$$n(\omega)\mathrm{d}\omega = \frac{1}{\pi^2 c^3} \frac{\omega^2 \mathrm{d}\omega}{\exp\left(\hbar\omega/kT_0\right) - 1} \tag{1}$$

where the present temperature is measured to be $T_0 = 2.7255 \pm 0.0006$ K.

In the standard cosmological model this radiation should be isotropic over the sky in the 'cosmic rest frame' (CRF). Show that an inertial observer moving at velocity v $(= \beta c)$ with respect to this frame, the radiation will still have the blackbody form but with a temperature

$$T(\theta) = T_0 \frac{\sqrt{1-\beta^2}}{1 - \beta\cos\theta}, \tag{2}$$

where θ is the angle between the direction of motion and the line-of-sight.

Expand this to show that the leading term in v/c is a dipole anisotropy. If the maximum change in the temperature we observe is $\Delta T = 3.352$ mK then how does our velocity with respect to the CRF compare with the expected 'peculiar' (non-Hubble) velocity due to the growth of inhomogeneities?

Elaborate on whether the existence of the CRF conflicts with the principle of relativity and generally on the possible cosmological implications of our motion with respect to this frame.