Spontaneous Symmetry Breaking | gauge symmetry

- Consider a U(1) gauge invariant Lagrangian for a complex scalar field $\phi(x)$:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2\,, \quad D_\mu = \partial_\mu + \mathrm{i}eqA_\mu$$

inv. under $\phi(x) \mapsto \phi'(x) = \mathrm{e}^{-\mathrm{i}q\theta(x)}\phi(x)\,, \quad A_\mu(x) \mapsto A'_\mu(x) = A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x)$

If $\lambda > 0$, $\mu^2 < 0$, the $\mathcal{L}$ in terms of quantum fields η and χ with null VEVs:

$$\phi(x) \equiv \frac{1}{\sqrt{2}}[v + \eta(x) + \mathrm{i}\chi(x)]\,, \quad \mu^2 = -\lambda v^2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \frac{1}{2}(\partial_\mu\chi)(\partial^\mu\chi)$$
$$- \lambda v^2\eta^2 - \lambda v\eta(\eta^2+\chi^2) - \frac{\lambda}{4}(\eta^2+\chi^2)^2 + \frac{1}{4}\lambda v^4$$
$$\boxed{+ eqvA_\mu\partial^\mu\chi} + eqA_\mu(\eta\partial^\mu\chi - \chi\partial^\mu\eta)$$
$$\boxed{+ \frac{1}{2}(eqv)^2A_\mu A^\mu} + \frac{1}{2}(eq)^2A_\mu A^\mu(\eta^2 + 2v\eta + \chi^2)$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\,v$

$m_\chi = 0$

(ii) $M_A = |eqv|$ (!)

(iii) Term $A_\mu\partial^\mu\chi$ (?)

(iv) Add $\mathcal{L}_{\mathrm{GF}}$

Spontaneous Symmetry Breaking | gauge symmetry

- Removing the cross term and the (new) gauge fixing Lagrangian:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial_\mu A^\mu - \xi M_A \chi)^2$$

$$\Rightarrow \quad \mathcal{L} + \mathcal{L}_{\rm GF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2 A_\mu A^\mu - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \overbrace{M_A \partial_\mu(A^\mu \chi)}^{\substack{M_A[A_\mu \partial^\mu \chi + \partial_\mu A^\mu \chi] \\ \text{total deriv.}}}$$

$$+ \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) - \frac{1}{2}\xi M_A^2 \chi^2 + \ldots$$

and the propagators of A_μ and χ are:

$$\widetilde{D}_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_A^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M_A^2}\right]$$

$$\widetilde{D}(k) = \frac{\mathrm{i}}{k^2 - \xi M_A^2}$$

$\Rightarrow$ χ has a gauge-dependent mass: actually it is not a physical field!

Spontaneous Symmetry Breaking | gauge symmetry

- A more transparent parameterization of the quantum field ϕ is

$$\phi(x) \equiv e^{iq\zeta(x)/v} \frac{1}{\sqrt{2}}[v + \eta(x)], \quad \langle 0|\,\eta\,|0\rangle = \langle 0|\,\zeta\,|0\rangle = 0$$

$$\phi(x) \mapsto e^{-iq\zeta(x)/v}\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x)] \quad \Rightarrow \quad \zeta \text{ gauged away!}$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta)$$
$$- \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{\lambda}{4}\eta^4 + \frac{1}{4}\lambda v^4$$
$$+ \frac{1}{2}(eqv)^2 A_\mu A^\mu + \frac{1}{2}(eq)^2 A_\mu A^\mu (2v\eta + \eta^2)$$

Comments:

(i) $m_\eta = \sqrt{2\lambda}\, v$

(ii) $M_A = |eqv|$

(iii) No need for $\mathcal{L}_{\rm GF}$

$\Rightarrow$ This is the unitary gauge ($\xi \to \infty$): just physical fields

$$\widetilde{D}_{\mu\nu}(k) \to \frac{i}{k^2 - M_A^2 + i\epsilon}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right] \quad \text{and} \quad \widetilde{D}(k) \to 0$$

Spontaneous Symmetry Breaking | gauge symmetry

⇒ Brout-Englert-Higgs mechanism: [Anderson '62]

[Higgs '64; Englert, Brout '64; Guralnik, Hagen, Kibble '64]

The gauge bosons associated with the spontaneously broken generators become massive, the corresponding would-be Goldstone bosons are unphysical and can be absorbed, the remaining massive scalars (Higgs bosons) are physical (the smoking gun!)

- The would-be Goldstone bosons are 'eaten up' by the gauge bosons ('get fat') and disappear (gauge away) in the unitary gauge ($\xi \to \infty$)
  ⇒ Degrees of freedom are preserved

  Before SSB: 2 (massless gauge boson) + 1 (Goldstone boson)

  After SSB: 3 (massive gauge boson) + 0 (absorbed would-be Goldstone)

- For loops calculations, 't Hooft-Feynman gauge ($\xi = 1$) is more convenient:
  ⇒ Gauge boson propagators are simpler, but
  ⇒ Goldstone bosons must be included in internal lines

Spontaneous Symmetry Breaking | gauge symmetry

- Comments:

  – After SSB the FP ghost fields (unphysical) acquire a gauge-dependent mass, due to interactions with the scalar field(s):

$$\widetilde{D}_{ab}(k) = \frac{\mathrm{i}\delta_{ab}}{k^2 - \xi_a M_{W^a}^2 + \mathrm{i}\epsilon}$$

  – Gauge theories with SSB are renormalizable ['t Hooft, Veltman '72]

    UV divergences appearing at loop level can be removed by renormalization of parameters and fields of the classical Lagrangian ⇒ predictive!

2. The Standard Model

Gauge group and particle representations

[Glashow '61; Weinberg '67; Salam '68]
[D. Gross, F. Wilczek; D. Politzer '73]

- The Standard Model is a gauge theory based on the local symmetry group:

$$\underbrace{SU(3)_c}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electroweak}} \to SU(3)_c \otimes \underbrace{U(1)_Q}_{\text{em}}$$

  with the electroweak symmetry spontaneously broken to the electromagnetic $U(1)_Q$ symmetry by the Brout-Englert-Higgs mechanism

- The particle (field) content: (ingredients: 12 *flavors* + 12 gauge bosons + H)

Fermions			I	II	III	Q
spin $\frac{1}{2}$	Quarks	f	uuu	ccc	ttt	$\frac{2}{3}$
		f'	ddd	sss	bbb	$-\frac{1}{3}$
	Leptons	f	ν_e	ν_μ	ν_τ	0
		f'	e	μ	τ	-1

$$Q_f = Q_{f'} + 1$$

Bosons		
spin 1	8 gluons	strong interaction
	$W^\pm$, Z	weak interaction
	γ	em interaction
spin 0	Higgs	origin of mass

Gauge group and particle representations

- The fields lay in the following representations (color, weak isospin, hypercharge):

Multiplets	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$	I	II	III	$Q = T_3 + Y$
Quarks	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ $-\frac{1}{3} = -\frac{1}{2} + \frac{1}{6}$
	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	u_R	c_R	t_R	$\frac{2}{3} = 0 + \frac{2}{3}$
	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	d_R	s_R	b_R	$-\frac{1}{3} = 0 - \frac{1}{3}$
Leptons	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$	$0 = \frac{1}{2} - \frac{1}{2}$ $-1 = -\frac{1}{2} - \frac{1}{2}$
	$(\mathbf{1}, \mathbf{1}, -1)$	e_R	μ_R	τ_R	$-1 = 0 - 1$
	$(\mathbf{1}, \mathbf{1}, 0)$	ν_{e_R}	ν_{μ_R}	ν_{τ_R}	$0 = 0 + 0$
Higgs	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	(3 families of quarks & leptons)			

$\Rightarrow$ From now on just the electroweak part (EWSM): $SU(2)_L \otimes U(1)_Y$

The EWSM with one family (of quarks or leptons)

- Consider two massless fermion fields $f(x)$ and $f'(x)$ with electric charges $Q_f = Q_{f'} + 1$ in three irreps of SU(2)$_L\otimes$U(1)$_Y$:

$$\mathcal{L}_F^0 = \mathrm{i}\overline{f}\partial\!\!\!/ f + \mathrm{i}\overline{f}'\partial\!\!\!/ f' \qquad f_{R,L} = \frac{1}{2}(1\pm\gamma_5)f\ , \quad f'_{R,L} = \frac{1}{2}(1\pm\gamma_5)f'$$

$$= \mathrm{i}\overline{\Psi}_1\partial\!\!\!/\Psi_1 + \mathrm{i}\overline{\psi}_2\partial\!\!\!/\psi_2 + \mathrm{i}\overline{\psi}_3\partial\!\!\!/\psi_3 \quad ; \quad \Psi_1 = \underbrace{\begin{pmatrix} f_L \\ f'_L \end{pmatrix}}_{(\mathbf{2},y_1)}\ , \quad \psi_2 = \underbrace{f_R}_{(\mathbf{1},y_2)}\ , \quad \psi_3 = \underbrace{f'_R}_{(\mathbf{1},y_3)}$$

- To get a Langrangian invariant under gauge transformations:

$$\Psi_1(x) \mapsto U_L(x)\mathrm{e}^{-\mathrm{i}y_1\beta(x)}\Psi_1(x), \quad U_L(x) = \mathrm{e}^{-\mathrm{i}T_i\alpha^i(x)}, \quad T_i = \frac{\sigma_i}{2} \quad \text{(weak isospin gen.)}$$

$$\psi_2(x) \mapsto \mathrm{e}^{-\mathrm{i}y_2\beta(x)}\psi_2(x)$$

$$\psi_3(x) \mapsto \mathrm{e}^{-\mathrm{i}y_3\beta(x)}\psi_3(x)$$

The EWSM with one family **gauge invariance**

⇒ Introduce gauge fields $W^i_\mu(x)$ $(i=1,2,3)$ and $B_\mu(x)$ through covariant derivatives:

$$\left.\begin{aligned} D_\mu\Psi_1 &= (\partial_\mu - \mathrm{i}g\widetilde{W}_\mu + \mathrm{i}g' y_1 B_\mu)\Psi_1 \,, \quad \widetilde{W}_\mu \equiv \frac{\sigma_i}{2}W^i_\mu \\ D_\mu\psi_2 &= (\partial_\mu + \mathrm{i}g' y_2 B_\mu)\psi_2 \\ D_\mu\psi_3 &= (\partial_\mu + \mathrm{i}g' y_3 B_\mu)\psi_3 \end{aligned}\right\} \Rightarrow \boxed{\mathcal{L}_F}$$

where two couplings g and g' have been introduced and

$$\widetilde{W}_\mu(x) \mapsto U_L(x)\widetilde{W}_\mu(x)U_L^\dagger(x) - \frac{\mathrm{i}}{g}(\partial_\mu U_L(x))U_L^\dagger(x)$$

$$B_\mu(x) \mapsto B_\mu(x) + \frac{1}{g'}\partial_\mu\beta(x)$$

⇒ Add Yang-Mills: gauge invariant kinetic terms for the gauge fields

$$\boxed{\mathcal{L}_{\mathrm{YM}}} = -\frac{1}{4}W^i_{\mu\nu}W^{i,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \,, \quad W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon_{ijk}W^j_\mu W^k_\nu$$

(include self-interactions of the SU(2) gauge fields) and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

The EWSM with one family | mass terms forbidden

⇒ Note that mass terms are not invariant under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, since LH and RH components do not transform the same:

$$m\overline{f}f = m(\overline{f_L}f_R + \overline{f_R}f_L)$$

⇒ Mass terms for the gauge bosons are not allowed either

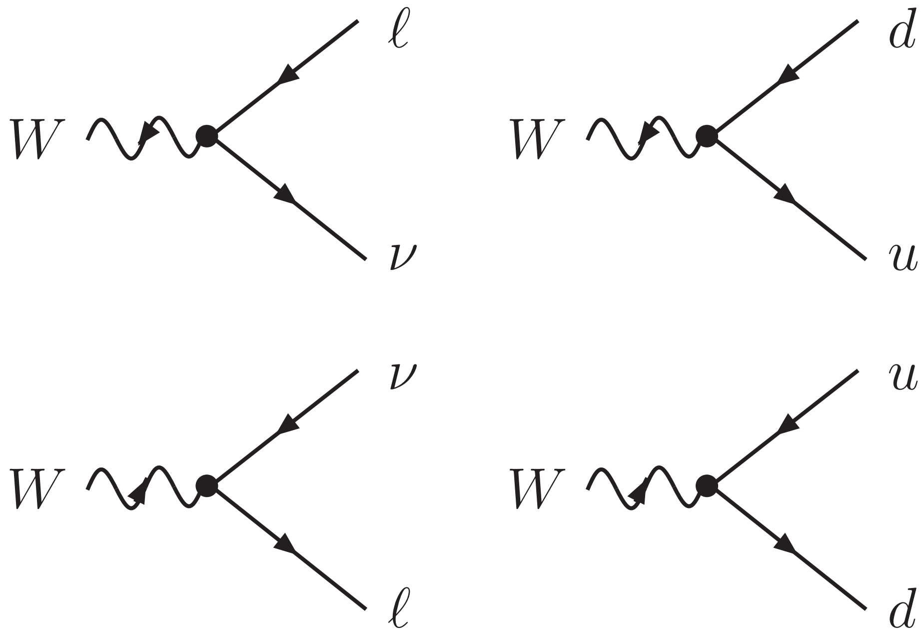
⇒ Next the different types of interactions are analyzed

The EWSM with one family | charged current interactions

- $$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu\widetilde{W}_\mu\Psi_1\,, \quad \widetilde{W}_\mu = \frac{1}{2}\begin{pmatrix} W^3_\mu & \sqrt{2}W^\dagger_\mu \\ \sqrt{2}W_\mu & -W^3_\mu \end{pmatrix}$$

$\Rightarrow$ charged current interactions of LH fermions with complex vector boson field W_μ:

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}\overline{f}\gamma^\mu(1-\gamma_5)f'W^\dagger_\mu + \text{h.c.}\,, \quad W_\mu \equiv \frac{1}{\sqrt{2}}(W^1_\mu + \mathrm{i}W^2_\mu)$$



The EWSM with one family | neutral current interactions

- The diagonal part of

$$\mathcal{L}_F \supset g\overline{\Psi}_1\gamma^\mu \widetilde{W}_\mu \Psi_1 - g' B_\mu (y_1 \overline{\Psi}_1 \gamma^\mu \Psi_1 + y_2 \overline{\psi}_2 \gamma^\mu \psi_2 + y_3 \overline{\psi}_3 \gamma^\mu \psi_3)$$

$\Rightarrow$ neutral current interactions with neutral vector boson fields W^3_μ and B_μ

We would like to identify B_μ with the photon field A_μ but that requires:

$$y_1 = y_2 = y_3 \quad \text{and} \quad g' y_j = eQ_j \quad \Rightarrow \quad \text{impossible!}$$

$\Rightarrow$ Since they are both neutral, try a combination:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \qquad s_W \equiv \sin\theta_W\,, \quad c_W \equiv \cos\theta_W$$

$\theta_W =$ weak mixing angle

$$\mathcal{L}_{\rm NC} = \sum_{j=1}^{3} \overline{\psi}_j \gamma^\mu \left\{ -\left[gT_3 s_W + g' y_j c_W \right] A_\mu + \left[gT_3 c_W - g' y_j s_W \right] Z_\mu \right\} \psi_j$$

with $T_3 = \dfrac{\sigma_3}{2}$ (0) the third weak isospin component of the doublet (singlet)

The EWSM with one family | neutral current interactions

- To make A_μ the photon field:

$$(1)\ \boxed{e = g s_W = g' c_W} \qquad (2)\ \boxed{Q = T_3 + Y}$$

where the electric charge operator is: $Q_1 = \begin{pmatrix} Q_f & 0 \\ 0 & Q_{f'} \end{pmatrix}, \quad Q_2 = Q_f\,, \quad Q_3 = Q_{f'}$

⇒ (1) Electroweak unification: g of SU(2) and g' of U(1) are related

⇒ (2) The hyperchages are fixed in terms of electric charges and weak isospin:

$$y_1 = Q_f - \frac{1}{2} = Q_{f'} + \frac{1}{2}\,, \qquad y_2 = Q_f\,, \qquad y_3 = Q_{f'}$$

$$\mathcal{L}_{\text{QED}} = -e\, Q_f \bar{f} \gamma^\mu f\, A_\mu \quad + (f \to f')$$

⇒ RH neutrinos are sterile: $y_2 = Q_f = 0$

The EWSM with one family | neutral current interactions

- The Z_μ is the neutral weak boson field:

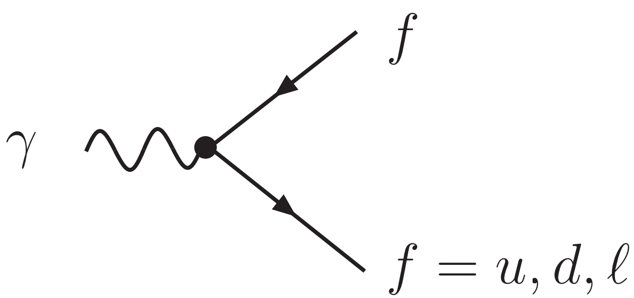
$$\mathcal{L}_{\rm NC}^{Z} = e\,\overline{f}\gamma^\mu(v_f - a_f\gamma_5)f\;Z_\mu \quad + (f \to f')$$

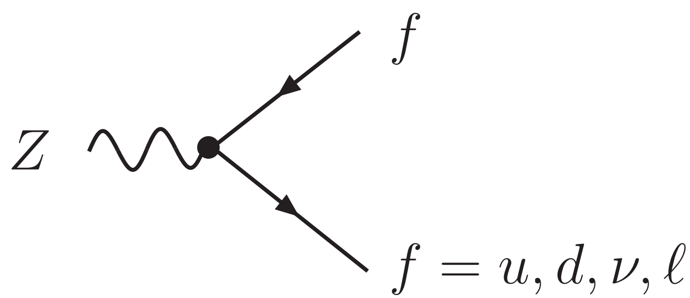
with

$$v_f = \frac{T_3^{f_L} - 2Q_f s_W^2}{2s_W c_W}\,, \quad a_f = \frac{T_3^{f_L}}{2s_W c_W}$$

- The complete neutral current Lagrangian reads:

$$\mathcal{L}_{\rm NC} = \mathcal{L}_{\rm QED} + \mathcal{L}_{\rm NC}^{Z}$$





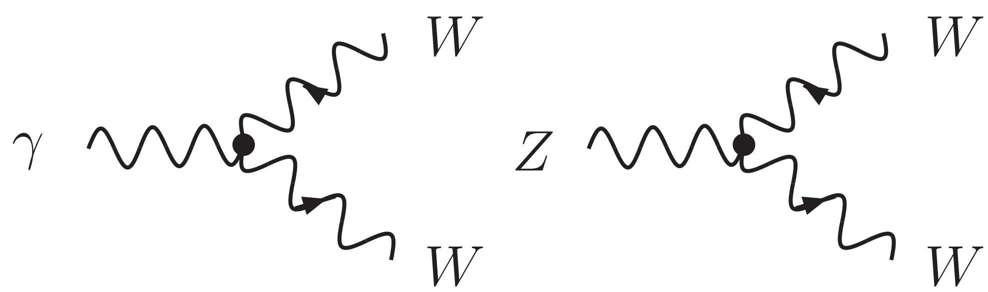
The EWSM with one family | gauge boson self-interactions

- Cubic:

$$\mathcal{L}_{\rm YM} \supset \mathcal{L}_3 = -\frac{{\rm i}ec_W}{s_W}\left\{W^{\mu\nu}W^\dagger_\mu Z_\nu - W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu W_\nu Z^{\mu\nu}\right\}$$
$$+\,{\rm i}e\left\{W^{\mu\nu}W^\dagger_\mu A_\nu - W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W_\nu F^{\mu\nu}\right\}$$

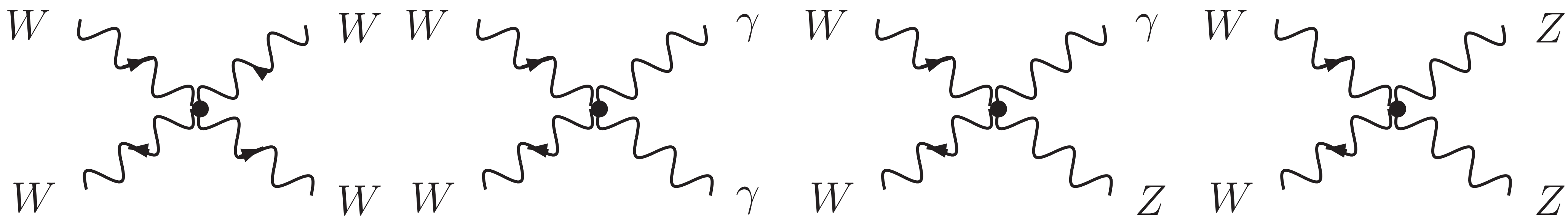
with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \qquad W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



- Quartic:

$$
\begin{aligned}
\mathcal{L}_{\rm YM} \supset \mathcal{L}_4 = &-\frac{e^2}{2s_W^2}\left\{\left(W_\mu^\dagger W^\mu\right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu\right\} \\
&-\frac{e^2 c_W^2}{s_W^2}\left\{W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu\right\} \\
&+\frac{e^2 c_W}{s_W}\left\{2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu\right\} \\
&- e^2\left\{W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu\right\}
\end{aligned}
$$



Note: even number of W and no vertex with just γ or Z

Electroweak symmetry breaking | setup

- Out of the 4 gauge bosons of $SU(2)_L \otimes U(1)_Y$ with generators $T_1,\ T_2,\ T_3,\ Y$ we need all to be broken except the combination $Q = T_3 + Y$ so that A_μ remains massless and the other three gauge bosons get massive after SSB

  $\Rightarrow$ Introduce a complex SU(2) Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \quad \langle 0 | \, \Phi \, | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

with gauge invariant Lagrangian ($\mu^2 = -\lambda v^2$):

$$\boxed{\mathcal{L}_\Phi} = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 , \qquad D_\mu \Phi = (\partial_\mu - \mathrm{i} g \widetilde{W}_\mu + \mathrm{i} g' y_\Phi B_\mu) \Phi$$

$$\text{take } y_\Phi = \frac{1}{2} \quad \Rightarrow \quad (T_3 + Y) \, |0\rangle = Q \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

$$\{T_1, T_2, T_3 - Y\} \, |0\rangle \neq 0$$

Electroweak symmetry breaking | gauge boson masses

- Quantum fields in the unitary gauge:

$$\Phi(x) \equiv \exp\left\{ \mathrm{i}\frac{\sigma_i}{2v}\theta^i(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\Phi(x) \mapsto \exp\left\{ -\mathrm{i}\frac{\sigma_i}{2v}\theta^i(x) \right\} \Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \Rightarrow$$

1 physical Higgs field $H(x)$

3 would-be Goldstones $\theta^i(x)$ gauged away

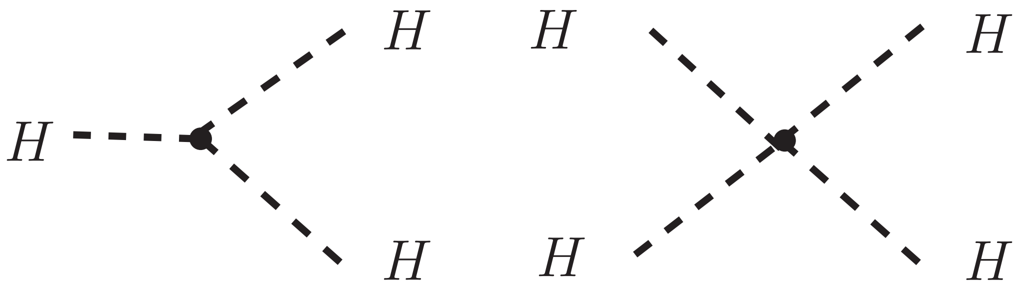
– The 3 dof apparently lost become the longitudinal polarizations of $W^\pm$ and Z that get massive after SSB:

$$\mathcal{L}_\Phi \supset \mathcal{L}_M = \underbrace{\frac{g^2 v^2}{4}}_{M_W^2} W_\mu^\dagger W^\mu + \underbrace{\frac{g^2 v^2}{8 c_W^2}}_{\frac{1}{2}M_Z^2} Z_\mu Z^\mu \quad \Rightarrow \quad M_W = M_Z c_W = \frac{1}{2} g v$$

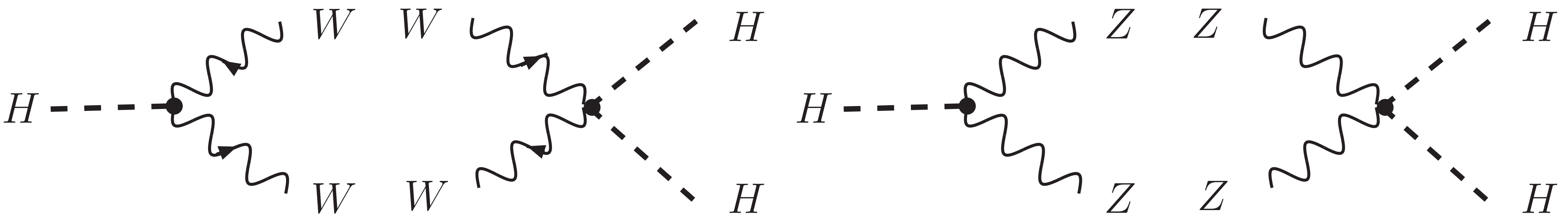
Electroweak symmetry breaking | Higgs sector

⇒ In the unitary gauge (just physical fields): $\mathcal{L}_\Phi = \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2}M_H^2 H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}H^4 \,, \quad M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$$



$$\mathcal{L}_M + \mathcal{L}_{HV^2} = M_W^2 W_\mu^\dagger W^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$



Electroweak symmetry breaking | Higgs sector

- Quantum fields in the R_ξ gauges:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}[v + H(x) + \mathrm{i}\chi(x)] \end{pmatrix} , \quad \phi^-(x) = [\phi^+(x)]^*$$

$$\begin{aligned} \mathcal{L}_\Phi = {} & \mathcal{L}_H + \mathcal{L}_M + \mathcal{L}_{HV^2} + \frac{1}{4}\lambda v^4 \\ & + (\partial_\mu \phi^+)(\partial^\mu \phi^-) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \\ & + \mathrm{i}M_W \, (W_\mu \partial^\mu \phi^+ - W_\mu^\dagger \partial^\mu \phi^-) + \, M_Z \, Z_\mu \partial^\mu \chi \\ & + \text{trilinear interactions [SSS, SSV, SVV]} \\ & + \text{quadrilinear interactions [SSSS, SSVV]} \end{aligned}$$

Electroweak symmetry breaking | gauge fixing

- To remove the cross terms $W_\mu \partial^\mu \phi^+$, $W_\mu^\dagger \partial^\mu \phi^-$, $Z_\mu \partial^\mu \chi$ and define propagators add:

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi_\gamma}(\partial_\mu A^\mu)^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z M_Z \chi)^2 - \frac{1}{\xi_W}|\partial_\mu W^\mu + \mathrm{i}\xi_W M_W \phi^-|^2$$

$\Rightarrow$ Massive propagators for gauge and (unphysical) would-be Goldstone fields:

$$\widetilde{D}^\gamma_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_\gamma)\frac{k_\mu k_\nu}{k^2}\right]$$

$$\widetilde{D}^Z_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_Z^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_Z)\frac{k_\mu k_\nu}{k^2 - \xi_Z M_Z^2}\right] \quad ; \quad \widetilde{D}^\chi(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}$$

$$\widetilde{D}^W_{\mu\nu}(k) = \frac{\mathrm{i}}{k^2 - M_W^2 + \mathrm{i}\epsilon}\left[-g_{\mu\nu} + (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right] \quad ; \quad \widetilde{D}^\phi(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_\gamma = \xi_Z = \xi_W = 1$)

Electroweak symmetry breaking | Faddeev-Popov ghosts

- The SM is a non-Abelian theory $\Rightarrow$ add Faddeev-Popov ghosts $c_i(x)$ $(i = 1, 2, 3)$

$$c_1 \equiv \frac{1}{\sqrt{2}}(u_+ + u_-)\ , \quad c_2 \equiv \frac{\mathrm{i}}{\sqrt{2}}(u_+ - u_-)\ , \quad c_3 \equiv c_W\, u_Z - s_W\, u_\gamma$$

$$\mathcal{L}_{\text{FP}} = \underbrace{(\partial^\mu \bar{c}_i)(\partial_\mu c_i - g\epsilon_{ijk} c_j W_\mu^k)}_{\text{U kinetic + [UUV]}} + \underbrace{\text{interactions with } \Phi}_{\text{U masses + [SUU]}}$$

$\Rightarrow$ Massive propagators for (unphysical) FP ghost fields:

$$\widetilde{D}^{u_\gamma}(k) = \frac{\mathrm{i}}{k^2 + \mathrm{i}\epsilon}\ , \quad \widetilde{D}^{u_Z}(k) = \frac{\mathrm{i}}{k^2 - \xi_Z M_Z^2 + \mathrm{i}\epsilon}\ , \quad \widetilde{D}^{u_\pm}(k) = \frac{\mathrm{i}}{k^2 - \xi_W M_W^2 + \mathrm{i}\epsilon}$$

('t Hooft-Feynman gauge: $\xi_Z = \xi_W = 1$)

Electroweak symmetry breaking | Faddeev-Popov ghosts

$$
\begin{aligned}
\mathcal{L}_{\mathrm{FP}} = \quad & (\partial_\mu \bar{u}_\gamma)(\partial^\mu u_\gamma) + (\partial_\mu \bar{u}_Z)(\partial^\mu u_Z) + (\partial_\mu \bar{u}_+)(\partial^\mu u_+) + (\partial_\mu \bar{u}_-)(\partial^\mu u_-) \\
[\mathrm{UUV}] \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & + \mathrm{i}e[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]A_\mu - \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_+)u_+ - (\partial^\mu \bar{u}_-)u_-]Z_\mu \\
& - \mathrm{i}e[(\partial^\mu \bar{u}_+)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_-]W_\mu^\dagger + \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_+)u_Z - (\partial^\mu \bar{u}_Z)u_-]W_\mu^\dagger \\
& + \mathrm{i}e[(\partial^\mu \bar{u}_-)u_\gamma - (\partial^\mu \bar{u}_\gamma)u_+]W_\mu - \frac{\mathrm{i}e c_W}{s_W}[(\partial^\mu \bar{u}_-)u_Z - (\partial^\mu \bar{u}_Z)u_+]W_\mu \\
& - \xi_Z M_Z^2\, \bar{u}_Z u_Z - \xi_W M_W^2\, \bar{u}_+ u_+ - \xi_W M_W^2\, \bar{u}_- u_- \\
[\mathrm{SUU}] \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. & - e\xi_Z M_Z\, \bar{u}_Z \left[\frac{1}{2 s_W c_W} H u_Z - \frac{1}{2 s_W} \left(\phi^+ u_- + \phi^- u_+ \right) \right] \\
& - e\xi_W M_W\, \bar{u}_+ \left[\frac{1}{2 s_W} (H + \mathrm{i}\chi) u_+ - \phi^+ \left(u_\gamma - \frac{c_W^2 - s_W^2}{2 s_W c_W} u_Z \right) \right] \\
& - e\xi_W M_W\, \bar{u}_- \left[\frac{1}{2 s_W} (H - \mathrm{i}\chi) u_- - \phi^- \left(u_\gamma - \frac{c_W^2 - s_W^2}{2 s_W c_W} u_Z \right) \right]
\end{aligned}
$$