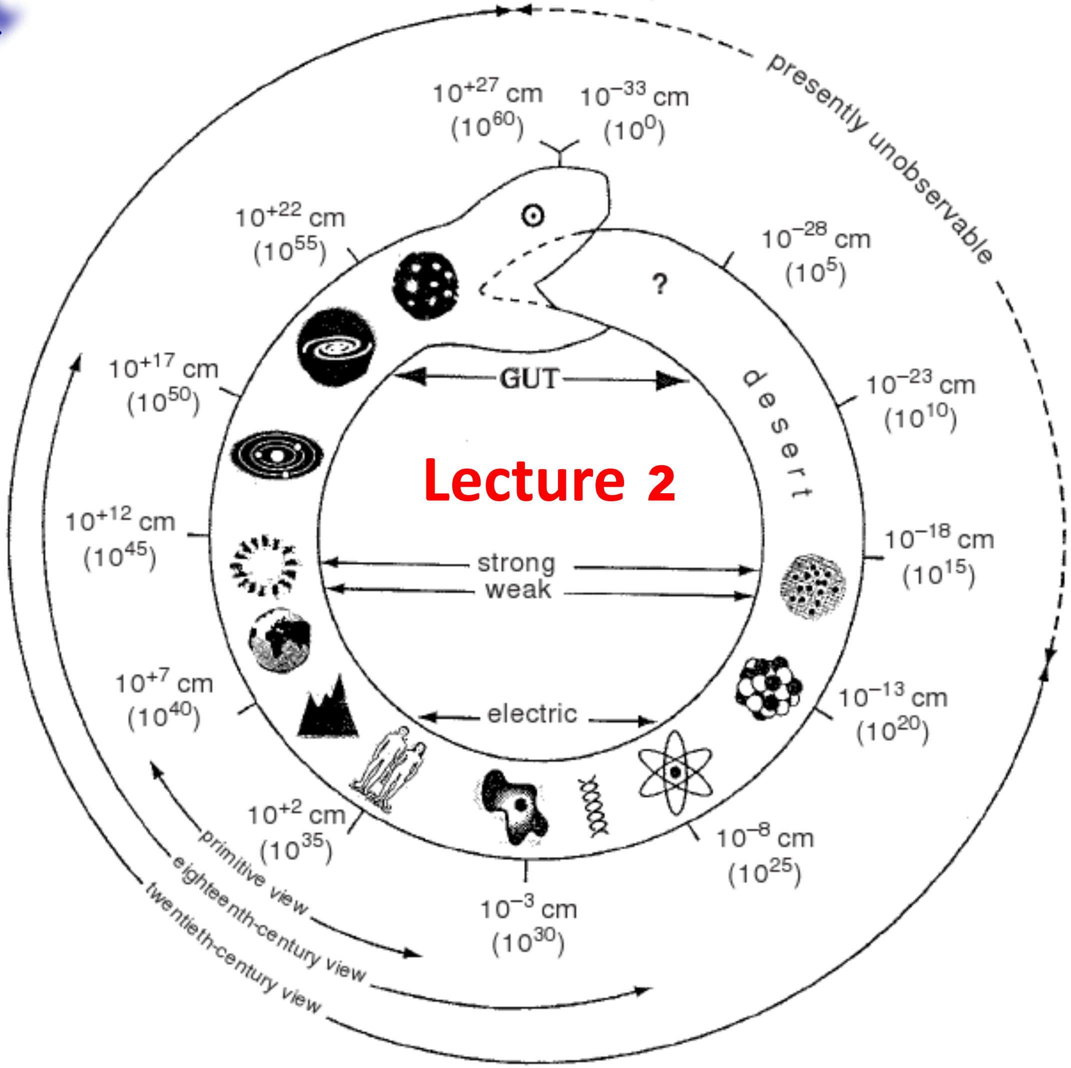
The late universe

Subir Sarkar



Lecture 2

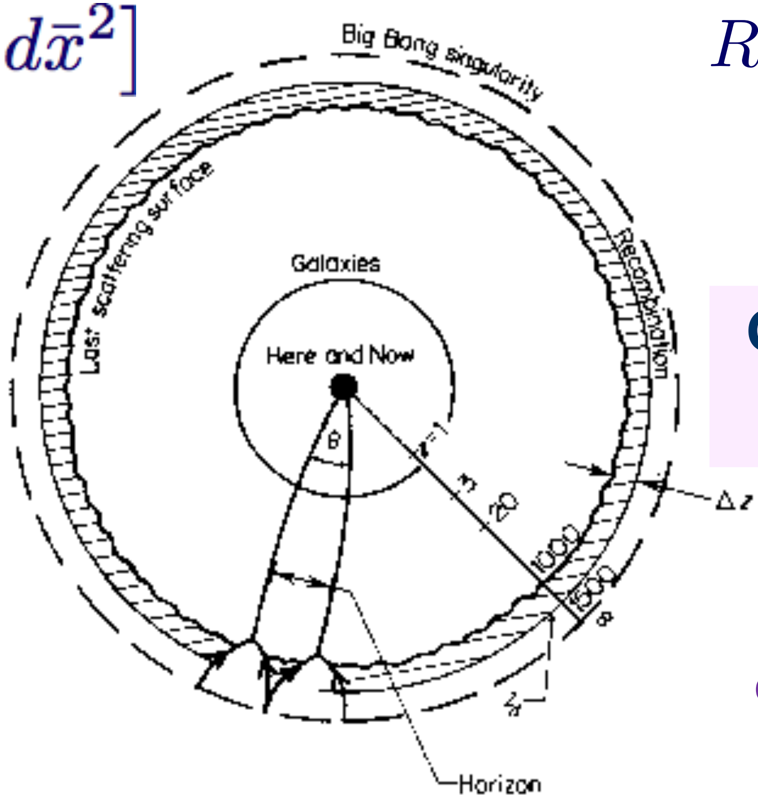
Taller de Altas Energías, 4-17 September 2016, Benasque

The **standard cosmological model** is based on several key assumptions:
maximally symmetric space-time + general relativity + *ideal fluids*

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\bar{x}^2\right]$$

$$a^2(\eta)d\eta^2 \equiv dt^2$$

Space-time metric
Robertson-Walker



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G_{\rm N} T_{\mu\nu}$$

Geometrodynamics
Einstein

$$T_{\mu\nu} = -\langle\rho\rangle_{\rm fields}\; g_{\mu\nu}$$

$$\Lambda = \lambda + 8\pi G_{\rm N}\langle\rho\rangle_{\rm fields}$$

Classical *and* quantum contributions

Friedmann Lemaitre

$$\Rightarrow H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{\rm N}\rho_{\rm m}}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Hubble equation

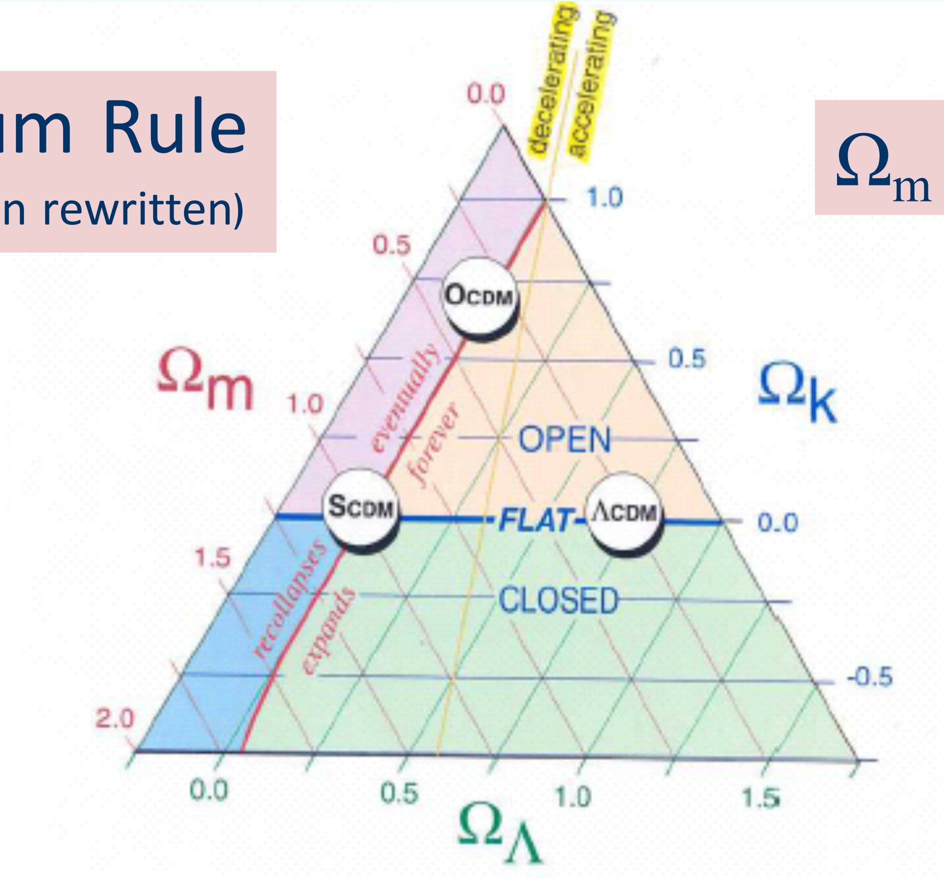
$$\equiv {H_0}^2\left[\Omega_{\rm m}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

$$\text{where} : z \equiv \frac{a_0}{a} - 1, \Omega_{\rm m} \equiv \frac{\rho_{\rm m}}{3H_0^2/8\pi G_{\rm N}}, \Omega_k \equiv \frac{k}{a_0^2 H_0^2}, \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$$

So it is *natural* for data interpreted in this idealised model to imply that $\Omega_\Lambda\,(\equiv 1 - \Omega_m - \Omega_k)$ is of $O(1)$, i.e. $\Lambda \sim H_0^2$, given the uncertainty in measuring Ω_m and the possibility of other components (Ω_x) e.g. the 'back reaction' of inhomogeneities which are *unaccounted* for in the standard Hubble equation

Cosmic Sum Rule

(Hubble equation rewritten)



$$\Omega_m + \Omega_k + \Omega_\Lambda \equiv 1$$

Bahcall, Ostriker, Perlmutter & Steinhardt (1999)

Nevertheless this has been interpreted as evidence for vacuum energy!

$$\Rightarrow \rho_\Lambda = 8\pi G\Lambda \sim H_0^2 M_p^2 \sim (10^{-12}\ \mathrm{GeV})^4$$

(NB: The *real* energy scale of the problem is: $H_0 \sim 10^{-42}$ GeV)

The **Standard $SU(3)_c$ x $SU(2)_L$ x $U(1)_Y$ Model** (viewed as an effective field theory up to some high energy cut-off scale M) describes *all* of microphysics

$$+M^4 + M^2\Phi^2$$

$m_H^2 \simeq \frac{h_t^2}{16\pi^2}\int_0^{M^2} \mathrm{d}k^2 = \frac{h_t^2}{16\pi^2}M^2$

hierarchy problem

vacuum energy problem

super-renormalisable

$-\mu^2\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2, m_H^2 = \lambda v^2/2$ → Higgs

$$\mathcal{L}_{\rm eff} = F^2 + \bar{\Psi}\,\not{D}\Psi + \bar{\Psi}\Psi\Phi + (D\Phi)^2 + V(\Phi)$$

renormalisable

$$+\frac{\bar{\Psi}\Psi\Phi\Phi}{M} + \frac{\bar{\Psi}\Psi\bar{\Psi}\Psi}{M^2} + \cdots$$

non-renormalisable

neutrino mass

proton decay, FCNC …

New physics beyond the SM ⇒ non-renormalisable operators suppressed by M^n which decouple as $M \to M_P$ … so a small Majorana ν mass, metastable proton etc is *natural*

But as M is raised, the effects of the super-renormalisable operators are *exacerbated*

(One solution for Higgs mass divergence → 'softly broken' *supersymmetry* at O(TeV)

… or the Higgs could be *composite* – a pseudo Nambu-Goldstone boson)

1st SR term **couples to gravity** so the *natural* expectation is $\rho_\Lambda \sim (1\ \mathrm{TeV})^4 >> (1\ \mathrm{meV})^4$

… *i.e.* the universe should have been inflating since (or collapsed at): $t \sim 10^{-12}$ s!

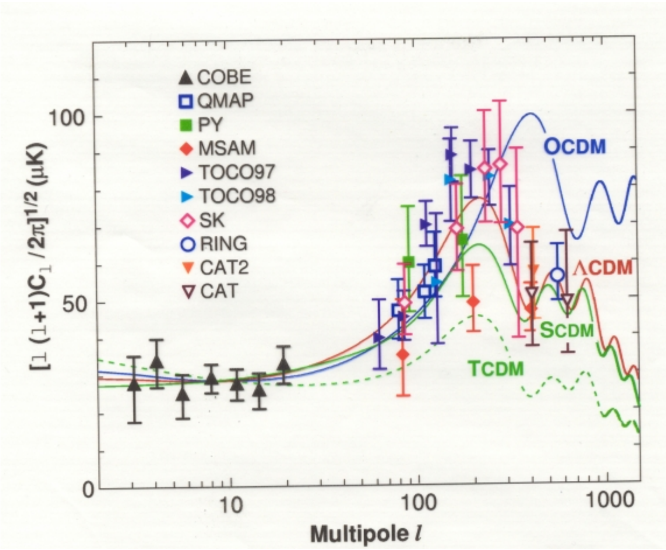
There must be some reason why this did *not* happen ($\Lambda \to 0$?)

"Also, as is obvious from experience, the [zero-point energy] does not produce any gravitational field" - Wolfgang Pauli

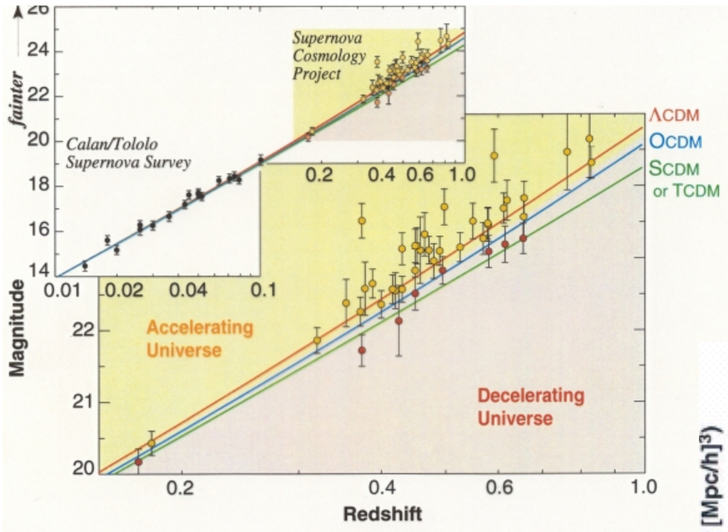
Die allgemeinen Prinzipien der Wellenmechanik, Handbuch der Physik, Vol. XXIV, 1933

However complementary observations indicated that: $\Omega_\Lambda \sim 0.7$, $\Omega_m \sim 0.3$

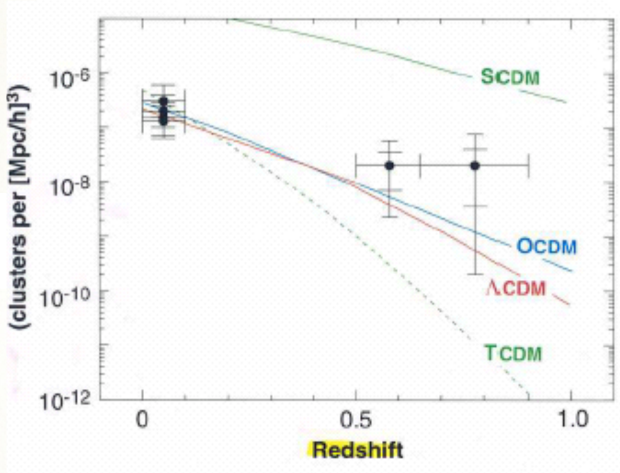
(*assuming* the 'Cosmic Sum Rule': $\Omega_m + \Omega_k + \Omega_\Lambda \equiv 1$)



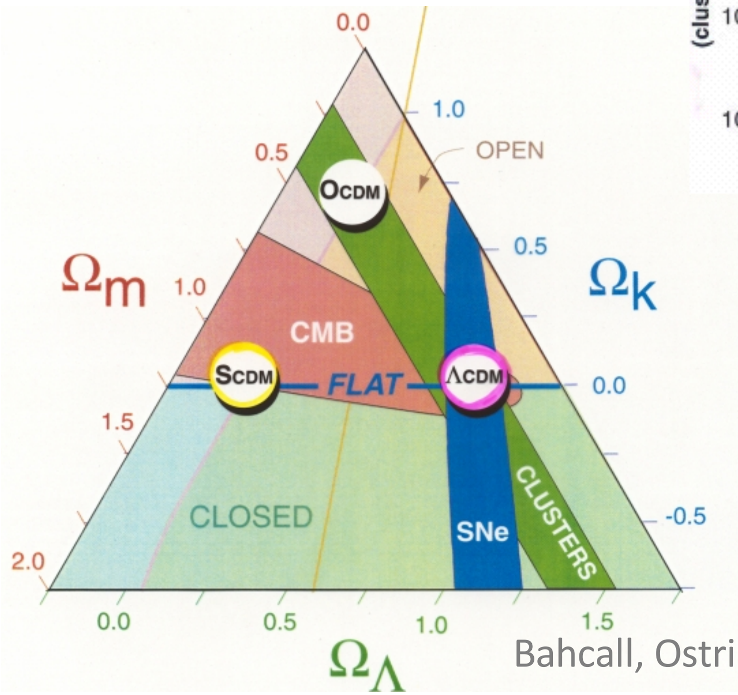
$\Omega_m + \Omega_\Lambda \approx 1.0 \pm 0.03$



$0.8\Omega_m - 0.6\Omega_\Lambda \approx -0.2 \pm 0.1$

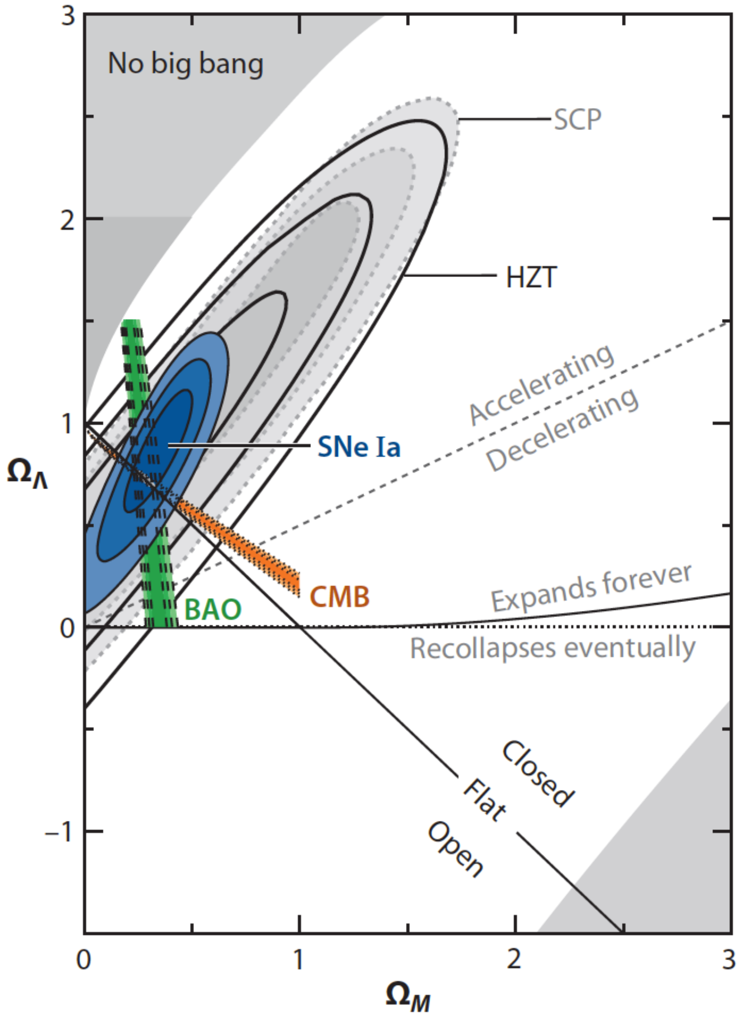


$\Omega_m \approx 0.3$



Bahcall, Ostriker, Perlmutter, Steinhardt (1999)

CMB data indicate $\Omega_k \approx 0$ so the FLRW model is simplified further, leaving only two free parameters (Ω_Λ and Ω_m) to be fitted to data



Goobar & Leibundgut, ARNPS 61:251,2011

But if we *underestimate* Ω_m, or if there is a Ω_x (e.g. "back reaction") which the Cosmic Sum Rule does *not* include, then we will *necessarily* infer $\Omega_\Lambda \neq 0$

(and the plot above will be misleading since flatness now $\Rightarrow \Omega_\Lambda + \Omega_m + \Omega_x = 1$)

Could ‘dark energy’ be an artifact of approximating the universe as homogeneous?

Quantities averaged over a domain $\mathcal{D}$ obey modified Friedmann equations Buchert 1999:

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -4\pi G\langle\rho\rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} \ ,$$
$$3\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = 8\pi G\langle\rho\rangle_{\mathcal{D}} - \frac{1}{2}\langle{}^{(3)}R\rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q}_{\mathcal{D}} \ ,$$

where $\mathcal{Q}_{\mathcal{D}}$ is the backreaction term,

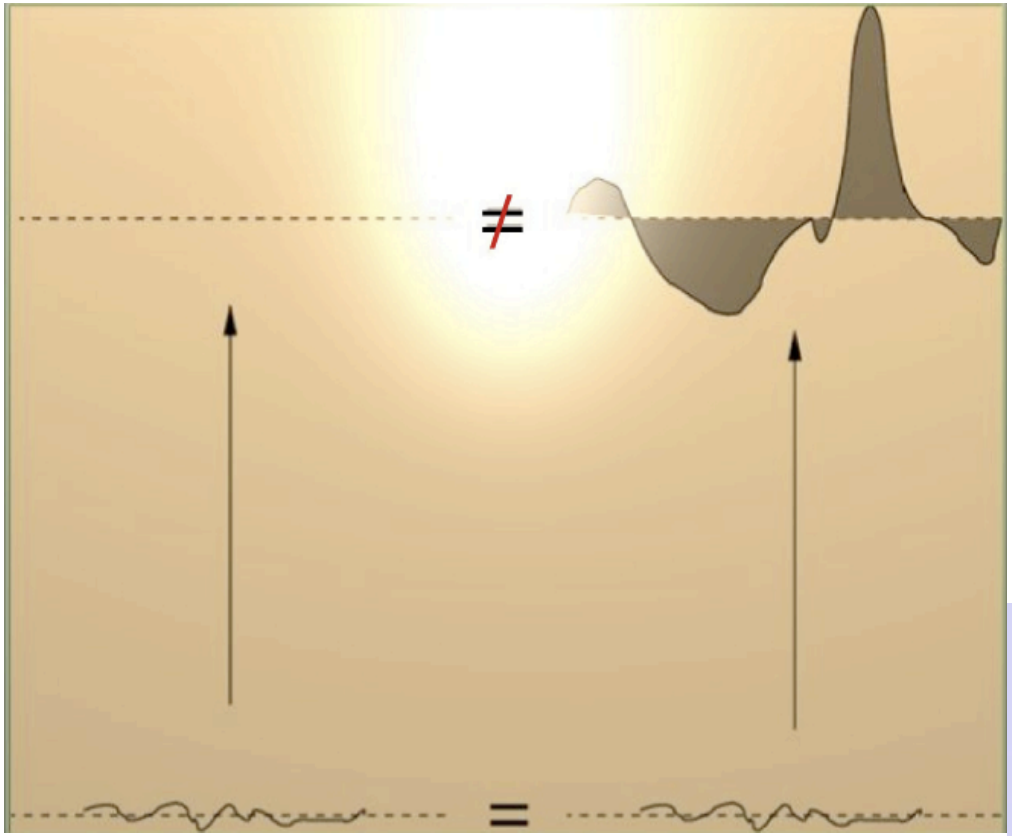
$$\mathcal{Q}_{\mathcal{D}} = \frac{2}{3}(\langle\theta^2\rangle_{\mathcal{D}} - \langle\theta\rangle^2_{\mathcal{D}}) - \langle\sigma^{\mu\nu}\sigma_{\mu\nu}\rangle_{\mathcal{D}} \ .$$

Variance of the expansion rate. Average shear.

If $\mathcal{Q}_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$ then $a_{\mathcal{D}}$ accelerates.

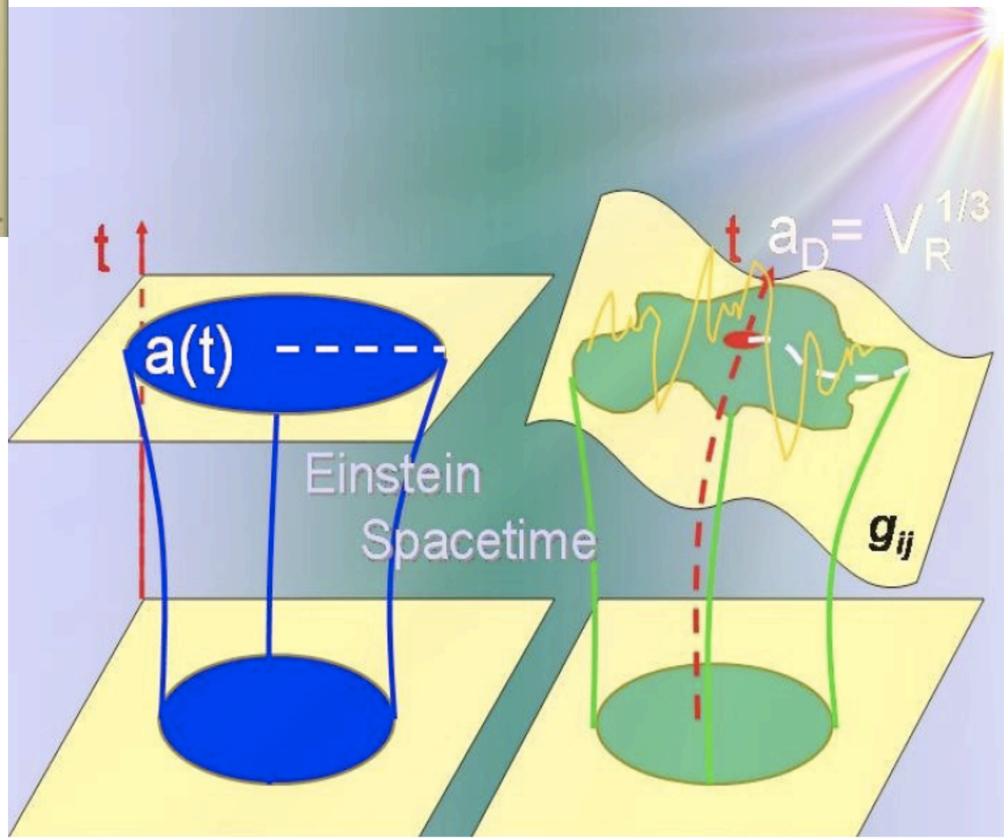
Can mimic a cosmological constant if $\mathcal{Q}_{\mathcal{D}} = -\frac{1}{3}\langle{}^{(3)}R\rangle_{\mathcal{D}} = \Lambda_{\rm eff}$.

Whether the backreaction can be sufficiently large is an ***open question***



'Back reaction' is hard to compute because spatial averaging and time evolution (along our past light cone) do *not* commute in general relativity

Due to structure formation, the homogeneous solution of Einstein's eqs. is distorted - its average must be taken over the *actual* geometry … the result is *different* from the standard FRW model



Courtesy: Thomas Buchert

Does it make sense to interpret Λ as vacuum energy?

"The interpretation, we feel, should be left to you and the very few others who are competent to discuss the matter with authority"

Edwin Hubble in letter to Wilhelm De Sitter (1931)

(concerning interpretation of cosmological redshifts … after he had mistakenly fitted the redshift-distance data to a *quadratic* relationship: $z \propto r^2$ – 'the De Sitter effect')

For a clock in De Sitter space, $\mathrm{d}s^2 = \left(1 - \frac{r^2}{\mathcal{R}^2}\right)\mathrm{d}t^2 - \mathrm{d}r^2/\left(1 - \frac{r^2}{\mathcal{R}^2}\right) - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$, at rest ($\mathrm{d}r = \mathrm{d}\theta = \mathrm{d}\varphi = 0$), the time-like interval, $\mathrm{d}s^2 = \mathrm{d}t^2\,(1 - r^2/R^2)$, depends on radial distance, becoming smaller as r increases $\Rightarrow$ redshift of light from distant sources with: $\frac{\mathrm{d}t}{\mathrm{d}t_0} = \sqrt{1 - \frac{r^2}{\mathcal{R}^2}} = \frac{\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} \Rightarrow z \simeq \frac{1}{2}\frac{r^2}{\mathcal{R}^2}, \text{ for } r \ll \mathcal{R}$

(NB: This is misleading because there are in fact no inertial observers in De Sitter space!)

"Raffiniert ist der Herrgott, aber boshaft ist er nicht!"

(*Subtle is the Lord, but malicious He is not!*)

Albert Einstein (1921)

Interpreting Λ as vacuum energy raises the 'coincidence problem':

why is $\Omega_\Lambda \approx \Omega_m$ *today*?

An evolving ultralight scalar field ('**quintessence**') can display 'tracking' behaviour: this requires $\mathbf{V(\varphi)^{1/4} \sim 10^{-12}}$ **GeV,** but $\sqrt{\mathbf{d^2V/d\varphi^2}} \sim \boldsymbol{H_0} \mathbf{\sim 10^{-42}}$ **GeV** to ensure slow-roll *...*
i.e. just as much fine-tuning as a bare cosmological constant

A similar comment applies to models (e.g. '**DGP brane-world'**) wherein gravity is modified on the scale of the present Hubble radius so as to mimic vacuum energy …
this scale is unnatural in a fundamental theory and is simply put in by hand

(Similar fine-tuning in *every* other attempt: **massive gravity**, **chameleon fields** …)

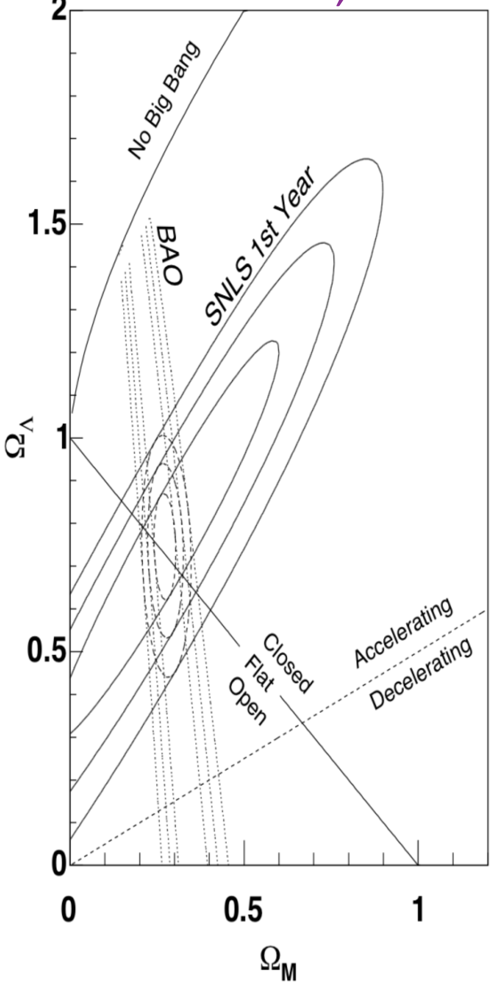
The only *natural* option is if $\Lambda \sim H^2$ *always*, but this is just a renormalisation of G_N!
Recall: $H^2 = 8\pi G_N/3 + \Lambda/3$ … this is *ruled out* by e.g. Big Bang nucleosynthesis (which requires $G_N^{\text{cosmic}} \sim G_N^{\text{laboratory}}$) and in any case does *not* yield accelerated expansion

There can be no *physical* explanation for the coincidence problem

Do we infer $\Lambda \sim H_0^2$ because that is just the **observational sensitivity**?
(if $\Lambda << H_0^2$ we would not measure it, if $\Lambda >> H_0^2$ we would not be here!)

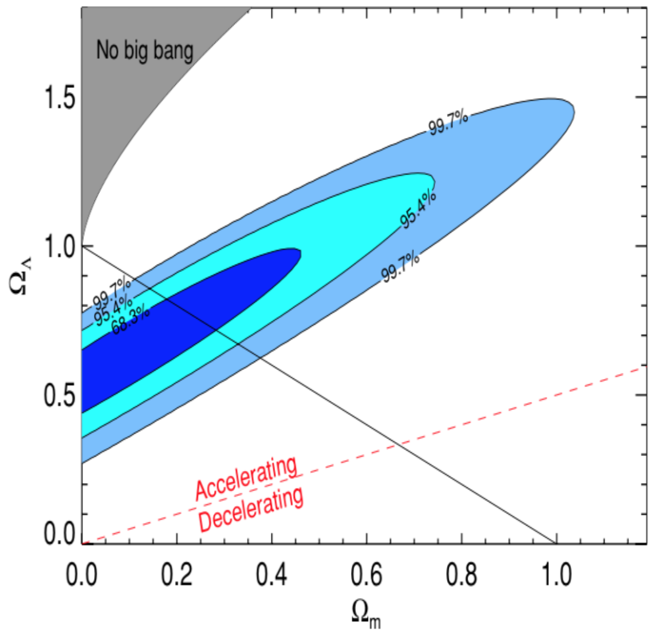
How strong is the evidence for cosmic acceleration?

Astier *et al*, 2006



"SN data alone require* cosmic acceleration at >99.999% confidence, including systematic effects" (Conley *et al*, 2011)

*from the magnitude-redshift plot



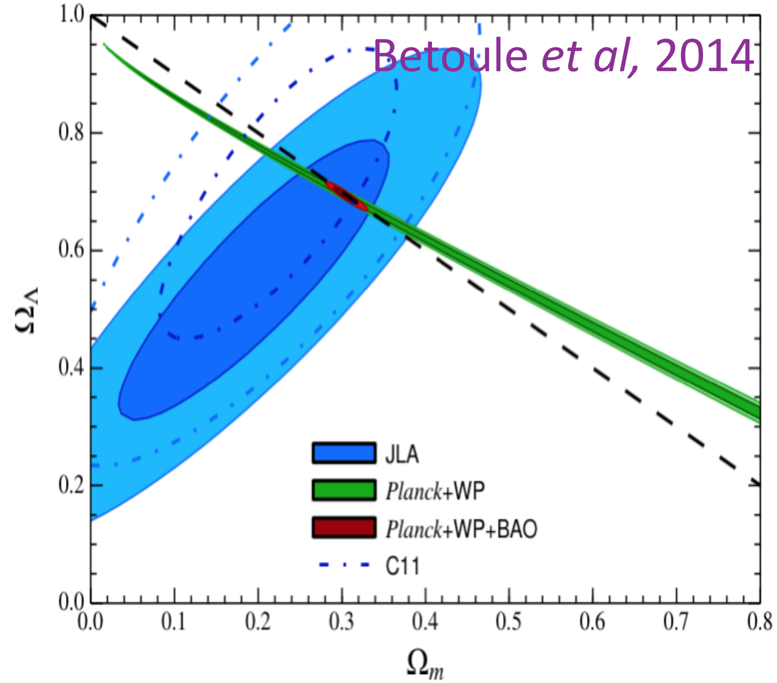
$$\mu \equiv 25 + 5\log_{10}(d_L/\mathrm{Mpc}), \quad \text{where:}$$

$$d_L = (1+z)\frac{d_H}{\sqrt{\Omega_k}}\,\mathrm{sinn}\left(\sqrt{\Omega_k}\int_0^z \frac{H_0\,dz'}{H(z')}\right),$$

$$d_H = c/H_0, \quad H_0 \equiv 100h\ \mathrm{km\,s^{-1}Mpc^{-1}},$$

$$H = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda},$$

$\mathrm{sinn} \to \sinh$ for $\Omega_k > 0$ and $\mathrm{sinn} \to \sin$ for $\Omega_k < 0$

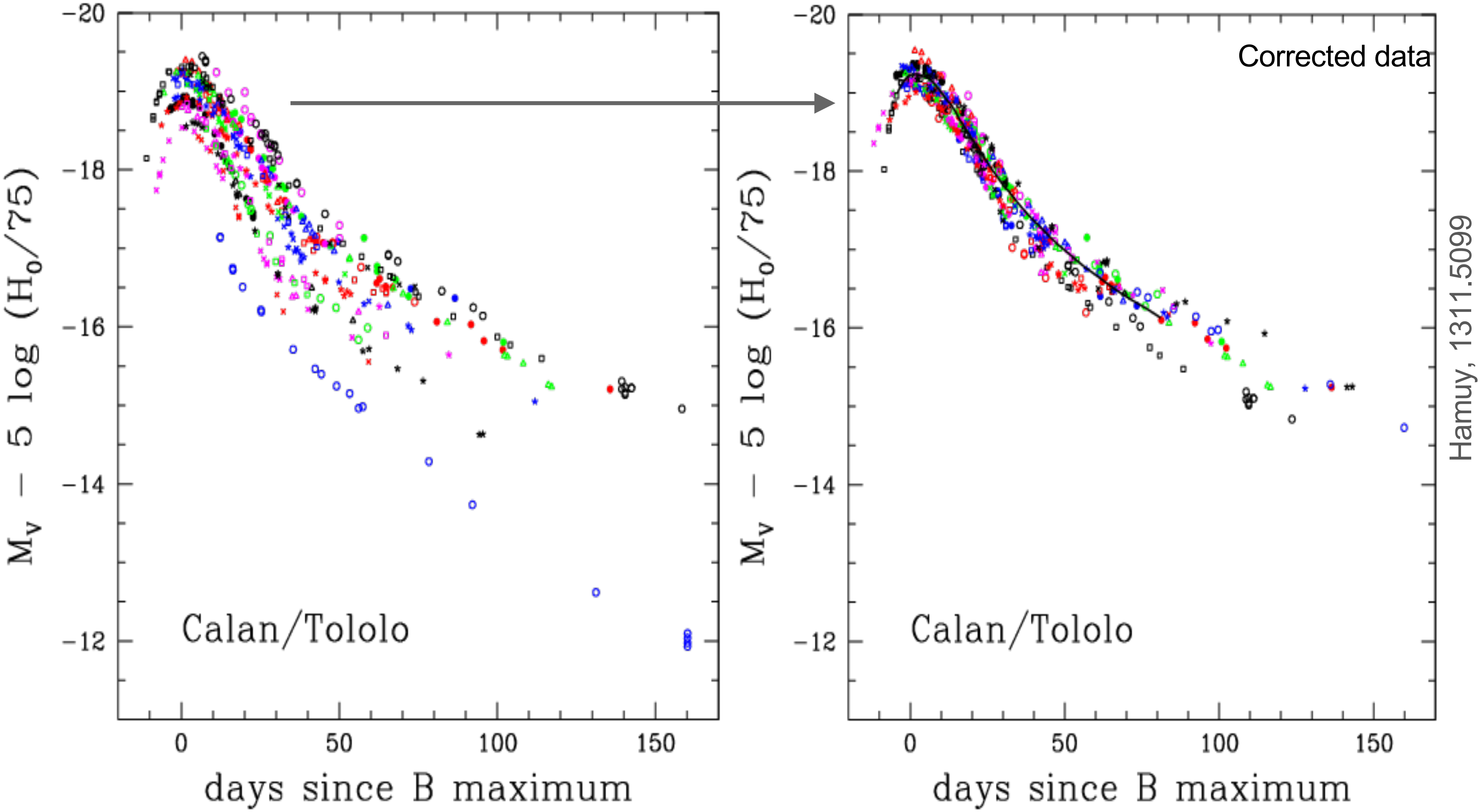


$$\boxed{\mu_B = m_B^* - M + \alpha X_1 - \beta C}$$

$$\chi^2 = \sum_{objects} \frac{(\mu_B - 5\log_{10}(d_L(\theta, z)/10pc))^2}{\sigma^2(\mu_B) + \sigma_{int}^2}$$

But they *assume* ΛCDM and adjust σ_{int} to get chi-squared of 1 per d.o.f. for the fit!

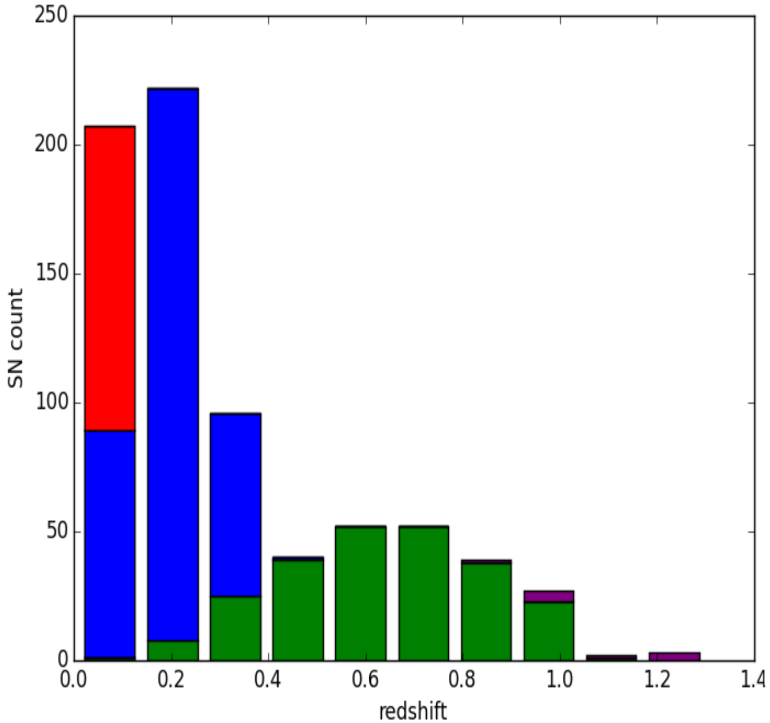
Type Ia supernovae as 'standardisable candles'

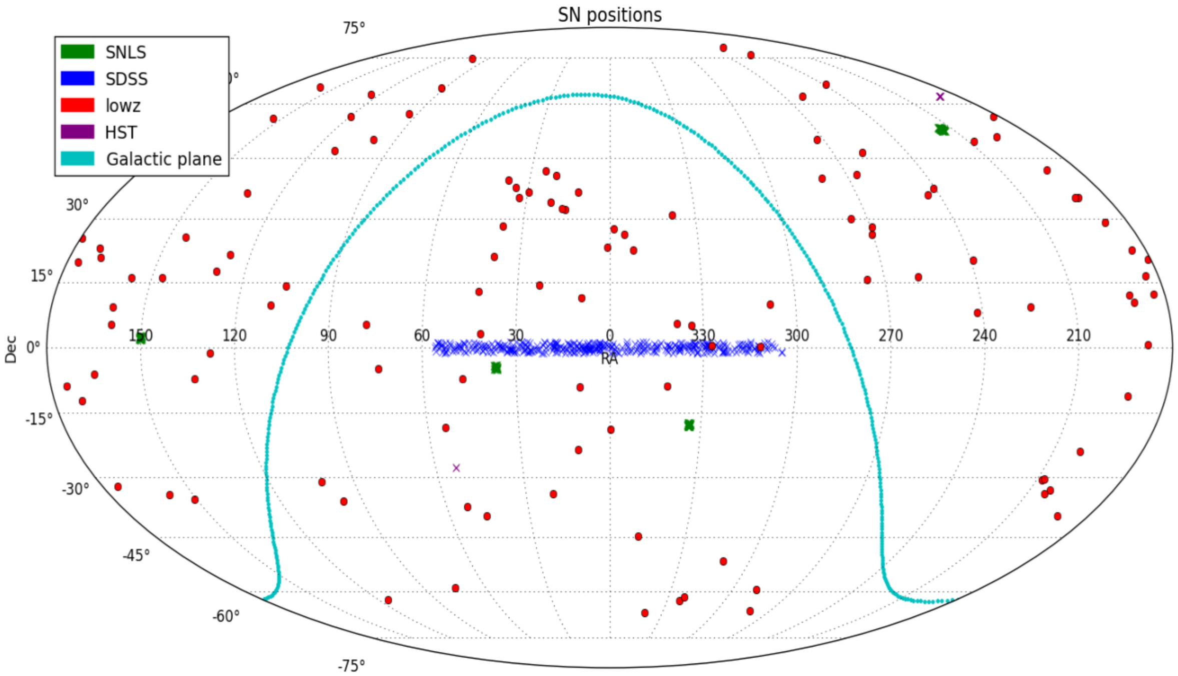


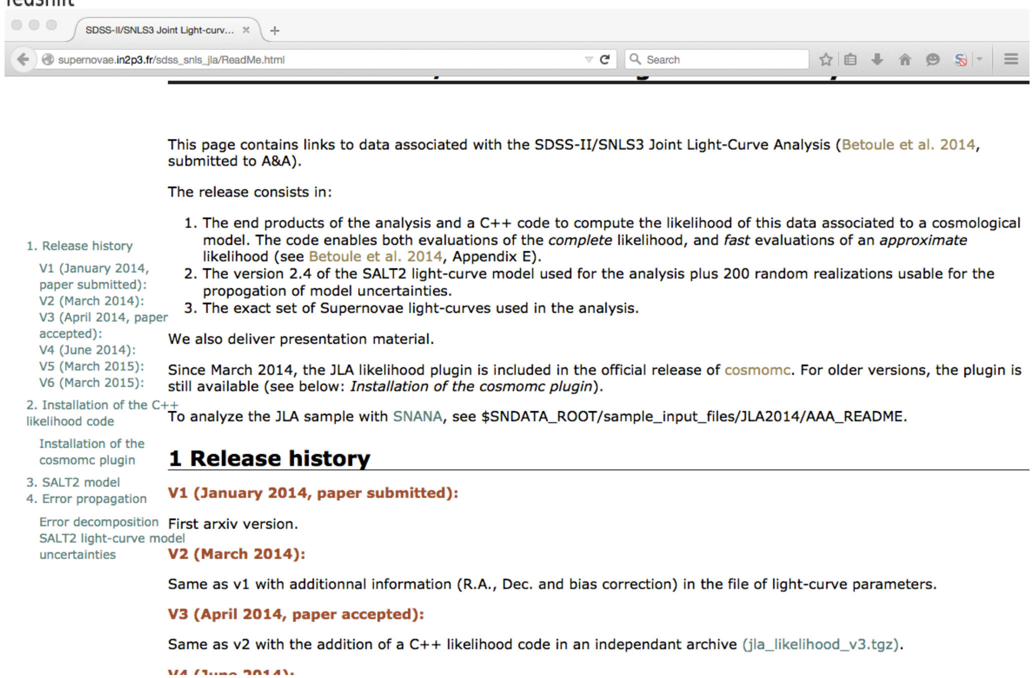
$$\mu_B = m_B^* - M + \alpha X_1 - \beta \mathcal{C}$$

Use a standard template (e.g. SALT 2) to make 'stretch' and 'colour' corrections ...

Joint Lightcurve Analysis data (740 SNe)






SDSS-II/SNLS3 Joint Light-curv...

supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html

This page contains links to data associated with the SDSS-II/SNLS3 Joint Light-Curve Analysis (Betoule et al. 2014, submitted to A&A).

The release consists in:

1. The end products of the analysis and a C++ code to compute the likelihood of this data associated to a cosmological model. The code enables both evaluations of the *complete* likelihood, and *fast* evaluations of an *approximate* likelihood (see Betoule et al. 2014, Appendix E).
2. The version 2.4 of the SALT2 light-curve model used for the analysis plus 200 random realizations usable for the propogation of model uncertainties.
3. The exact set of Supernovae light-curves used in the analysis.

We also deliver presentation material.

Since March 2014, the JLA likelihood plugin is included in the official release of cosmomc. For older versions, the plugin is still available (see below: *Installation of the cosmomc plugin*).

To analyze the JLA sample with SNANA, see $SNDATA_ROOT/sample_input_files/JLA2014/AAA_README.

1 Release history

V1 (January 2014, paper submitted):

First arxiv version.

V2 (March 2014):

Same as v1 with additionnal information (R.A., Dec. and bias correction) in the file of light-curve parameters.

V3 (April 2014, paper accepted):

Same as v2 with the addition of a C++ likelihood code in an independant archive (jla_likelihood_v3.tgz).

Data publicly available

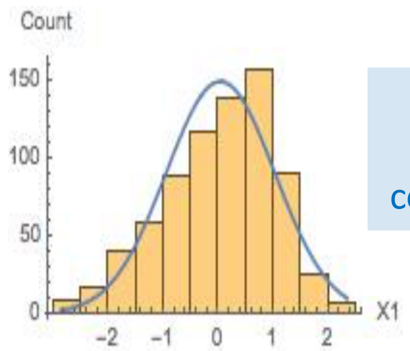
Betoule *et al*, 1401.4064

Construct a Maximum Likelihood Estimator

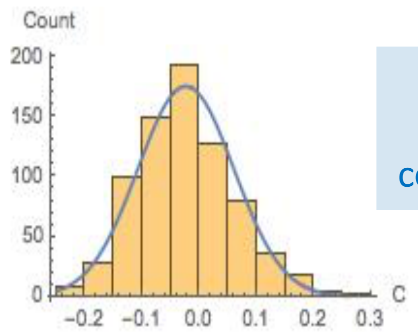
$$\mathcal{L} = \text{probability density}(\text{data}|\text{model})$$

$$\mathcal{L} = p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|\theta]$$
$$= \int p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta_{\text{cosmo}}]$$
$$\times p[(M, x_1, c)|\theta_{\text{SN}}]dM dx_1 dc$$

Well-approximated as Gaussian



JLA data 'Stretch' corrections



JLA data 'Colour' corrections

$$p[(M, x_1, c)|\theta] = p(M|\theta)p(x_1|\theta)p(c|\theta),$$

$$p(M|\theta) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left(-\left[\frac{M - M_0}{\sigma_{M0}}\right]^2 / 2\right)$$

$$p(x_1|\theta) = \frac{1}{\sqrt{2\pi\sigma_{x0}^2}} \exp\left(-\left[\frac{x_1 - x_{10}}{\sigma_{x0}}\right]^2 / 2\right)$$

$$p(c|\theta) = \frac{1}{\sqrt{2\pi\sigma_{c0}^2}} \exp\left(-\left[\frac{c - c_0}{\sigma_{c0}}\right]^2 / 2\right)$$

Nielsen *et al*, arXiv: 1506.01354

Likelihood

$$p(Y|\theta) = \frac{1}{\sqrt{|2\pi\Sigma_l|}} \exp\left[-\frac{1}{2}(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^{\mathrm{T}}\right]$$

$$p(\hat{X}|X,\theta) = \frac{1}{\sqrt{|2\pi\Sigma_d|}} \exp\left[-\frac{1}{2}(\hat{X} - X)\Sigma_d^{-1}(\hat{X} - X)^{\mathrm{T}}\right]$$

$$\mathcal{L} = \frac{1}{\sqrt{|2\pi(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)|}} \times \exp\left(-\frac{1}{2}(\hat{Z} - Y_0 A)(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^{\mathrm{T}}\right)$$

intrinsic distributions

cosmology

SALT2

Confidence regions

Nielsen *et al*, arXiv: 1506.01354

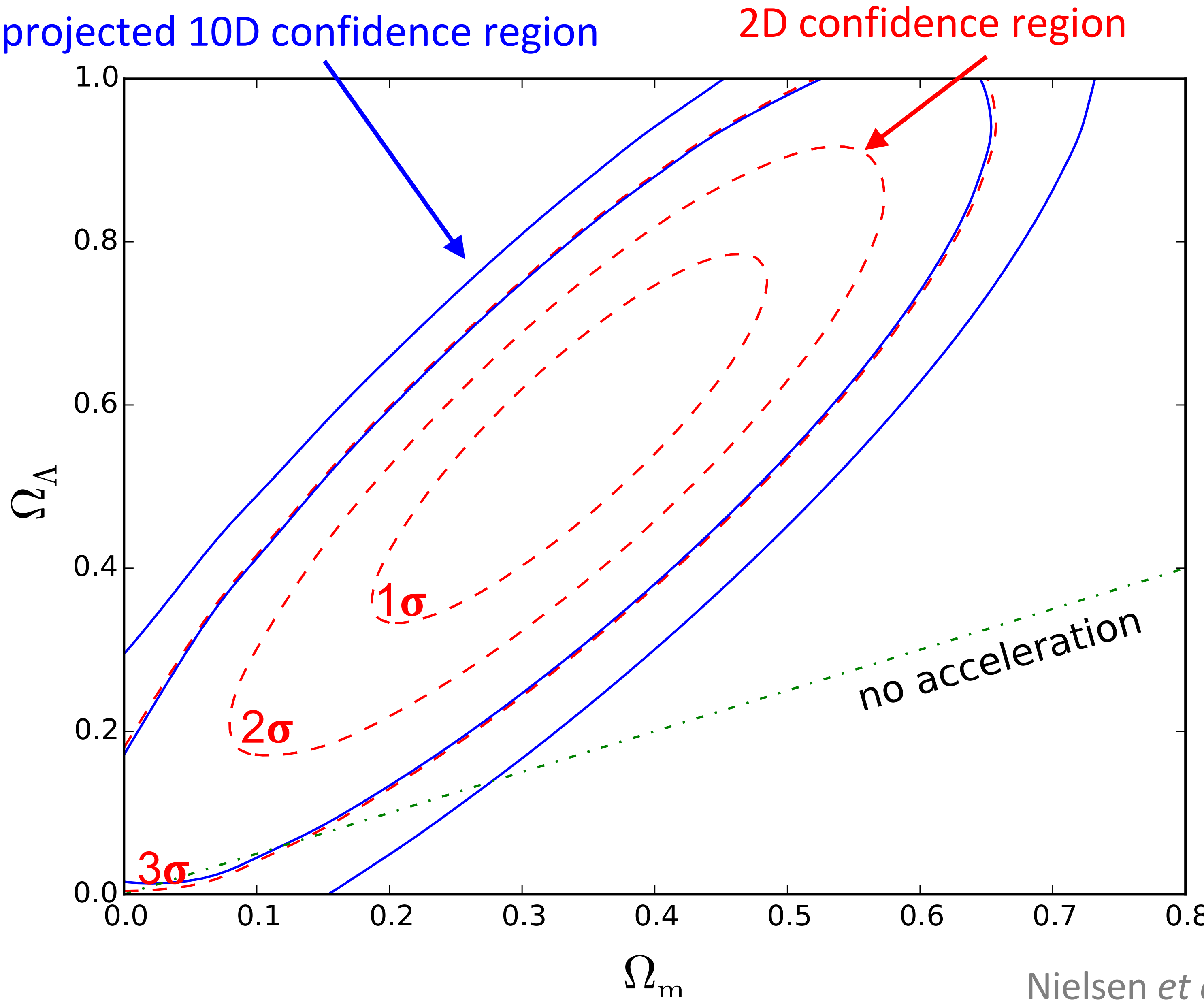
$$p_{\mathrm{cov}} = \int_0^{-2\log\mathcal{L}/\mathcal{L}_{\mathrm{max}}} \chi^2(x;\nu)dx$$

$$\mathcal{L}_p(\theta) = \max_{\phi} \mathcal{L}(\theta,\phi)$$

1,2,3-sigma

solve for Likelihood value

Data consistent with *uniform expansion* @3σ!



profile likelihood

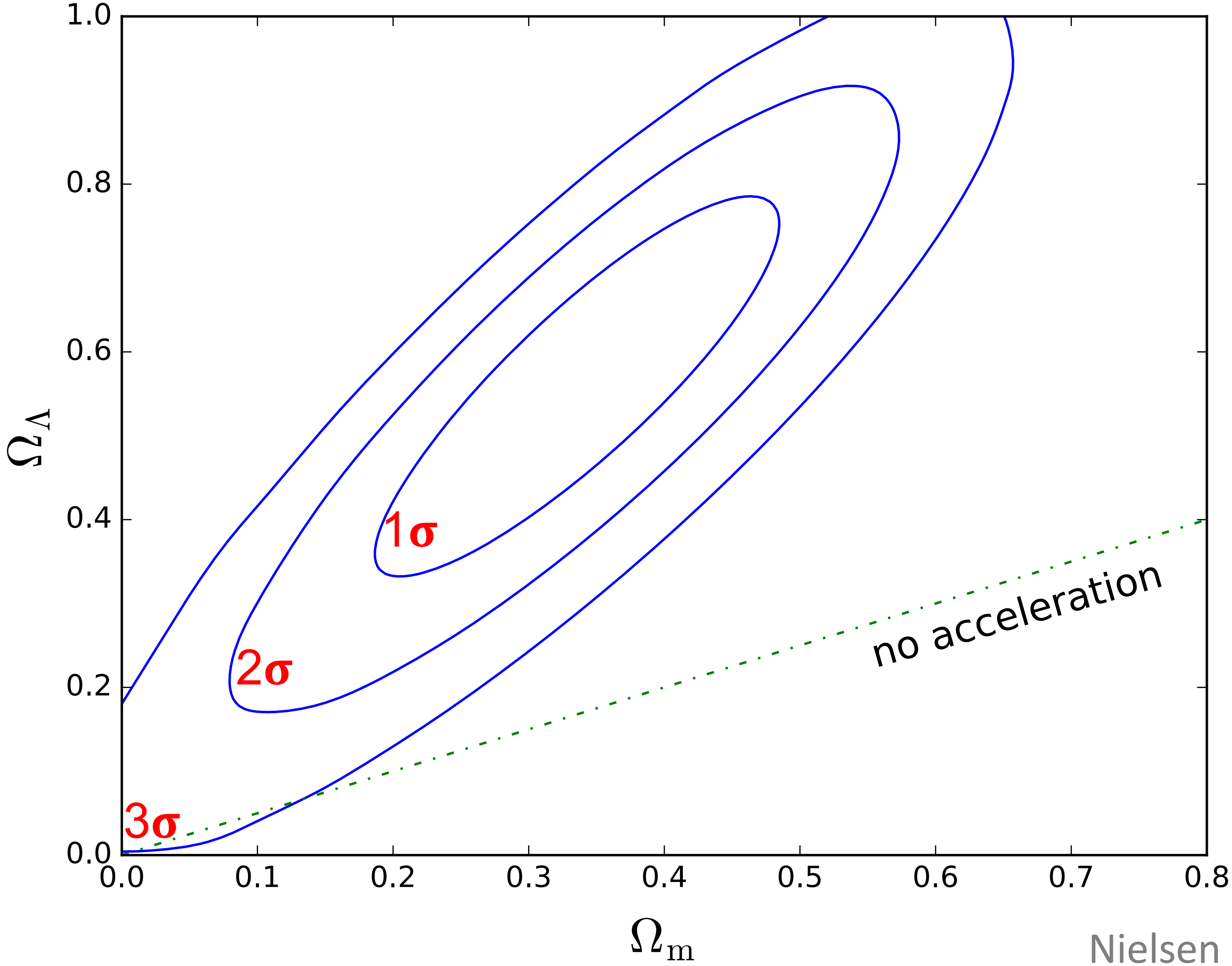
MLE, *best fit*

Ω_M	0.341
Ω_Λ	0.569
α	0.134
x_0	0.038
σ^2_{x0}	0.931
β	3.058
c_0	-0.016
σ^2_{c0}	0.071
M_0	-19.05
σ^2_{M0}	0.108

Nielsen *et al*, arXiv: 1506.01354

Data consistent with *uniform expansion* @3σ!

Opens up interesting possibilities e.g. viscosity of cosmic fluid (associated with structure formation)



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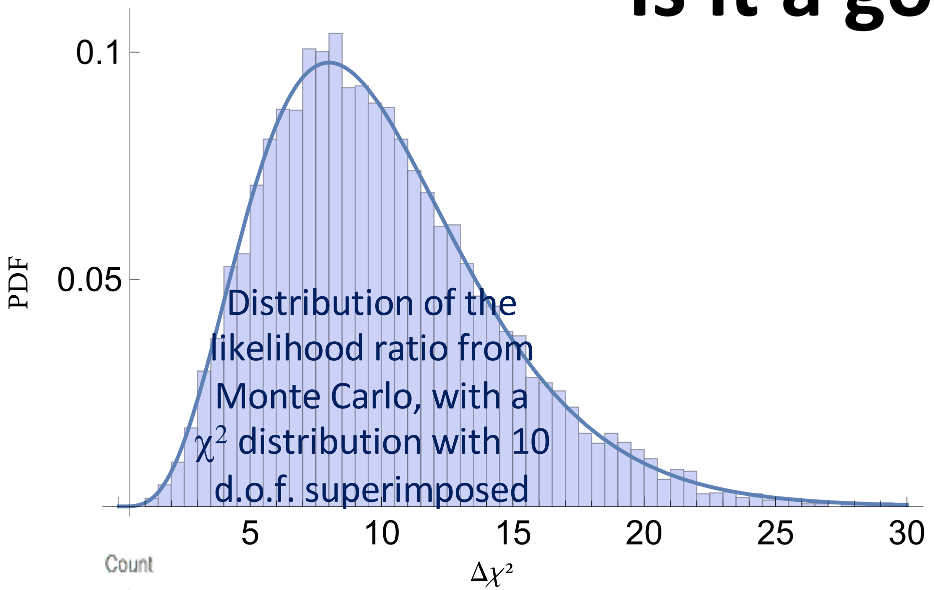
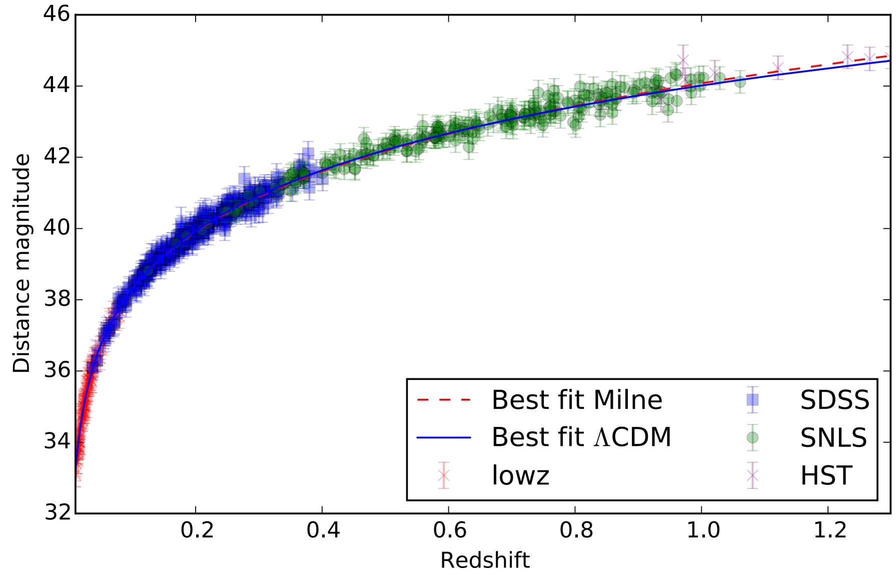
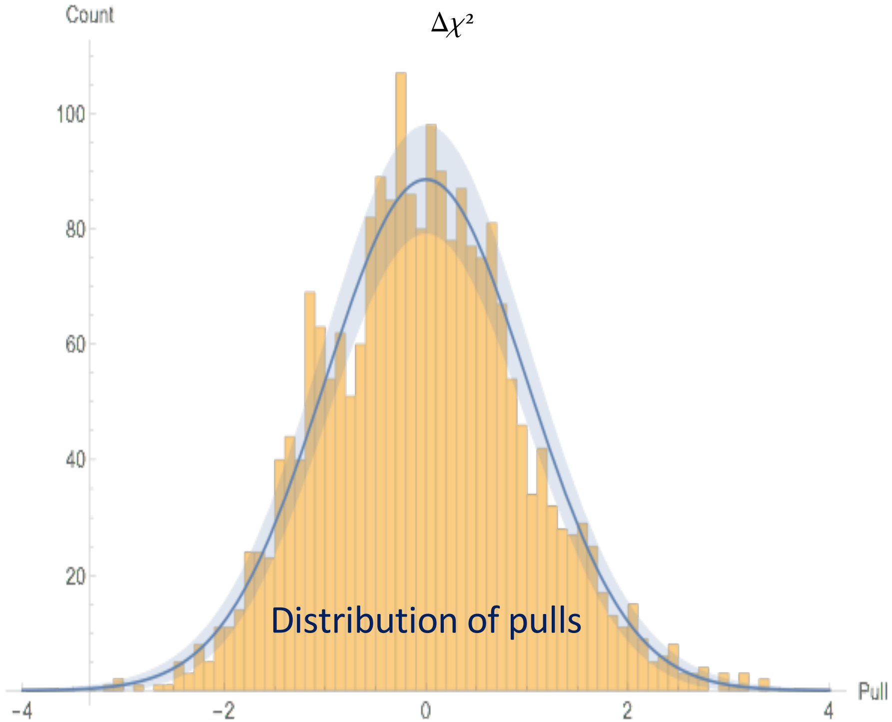
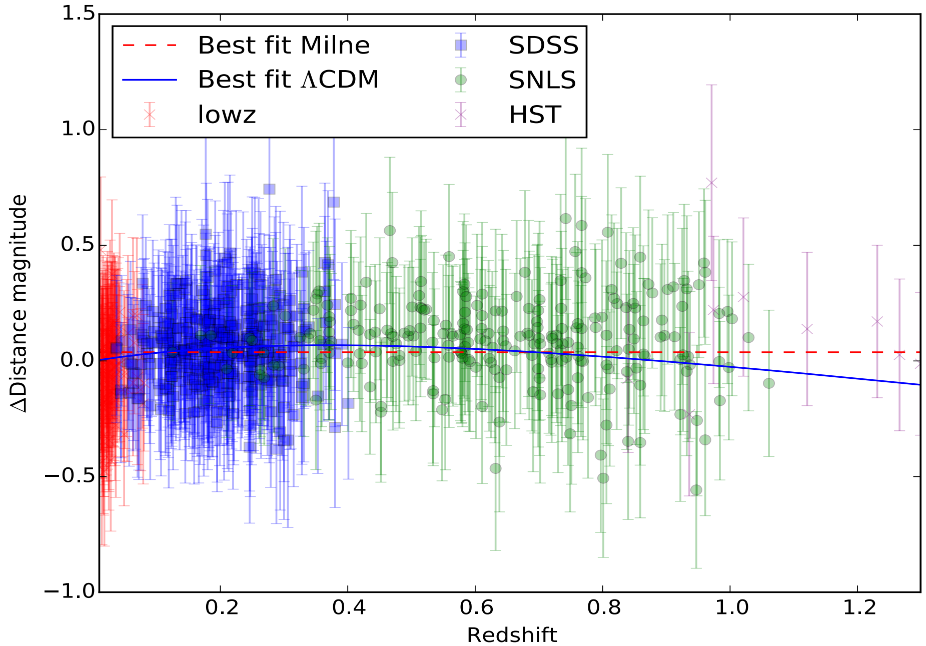
Nielsen *et al*, arXiv: 1506.01354

Is it a good fit ?





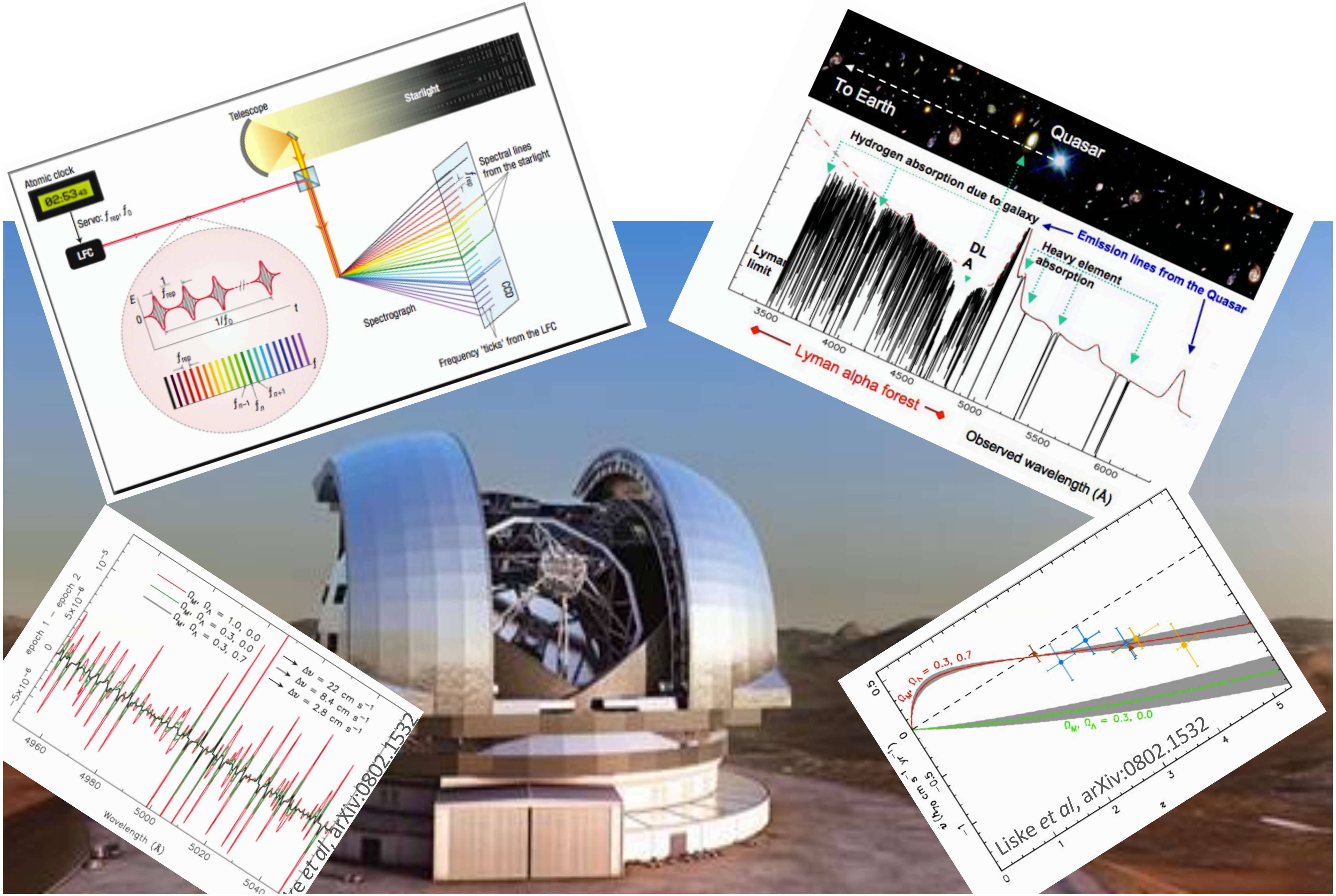




$$\text{pull} = (\Sigma_d + A^{\mathrm{T}}\Sigma_l A)^{-1/2}(\hat{Z} - Y_0 A)$$

Nielsen *et al*, arXiv: 1506.01354

A direct test of cosmic acceleration (using a 'Laser Comb' on the European Extremely Large Telescope) to measure the redshift drift of the Lyman-a forest over 15 years



But is not dark energy (cosmic acceleration) independently established from CMB and large-scale structure observations? Answer: No!

The formation of large-scale structure is akin to a scattering experiment

The **Beam:** **inflationary density perturbations**

No 'standard model' – *assumed* to be adiabatic and close to scale-invariant

The **Target:** **dark matter (+ baryonic matter)**

Identity unknown - usually taken to be cold and collisionless

The **Detector:** **the universe**

Modelled by a 'simple' FRW cosmology with parameters h, $\Omega_{\rm CDM}$, $\Omega_{\rm B}$, Ω_Λ , Ω_k

The **Signal:** **CMB anisotropy, galaxy clustering, weak lensing …**

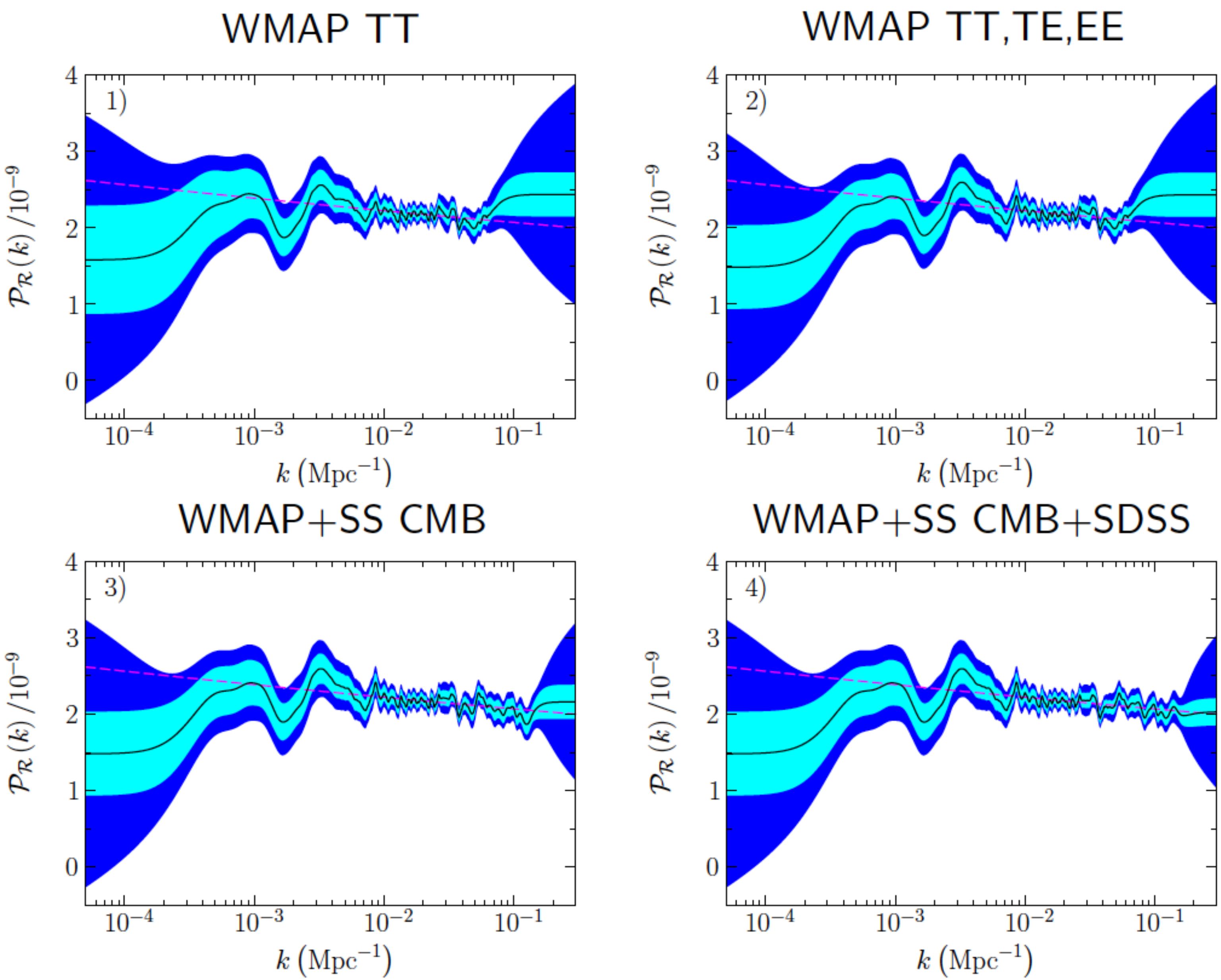
measured over scales ranging from ~1 – 10000 Mpc ($\Rightarrow$ ~8 e-folds of inflation)

But we *cannot* uniquely determine the properties of the **detector** with an unknown **beam** *and* **target**!

… hence need to adopt 'priors' on $\boldsymbol{h}$, $\boldsymbol{\Omega_{\rm CDM}}$ …, and *assume* a primordial power-law spectrum, in order to break inevitable **parameter degeneracies**

Hence evidence for Λ is *indirect* (can match same data without it e.g. arXiv:0706.2443)

The ‘inverse problem’ of inferring the primordial spectrum of perturbations generated by inflation is necessarily “ill-conditioned” … ‘Tikhonov regularisation’ can be used to do this in a non-parametric manner (Hunt & Sarkar, JCAP **01**:025,2014)



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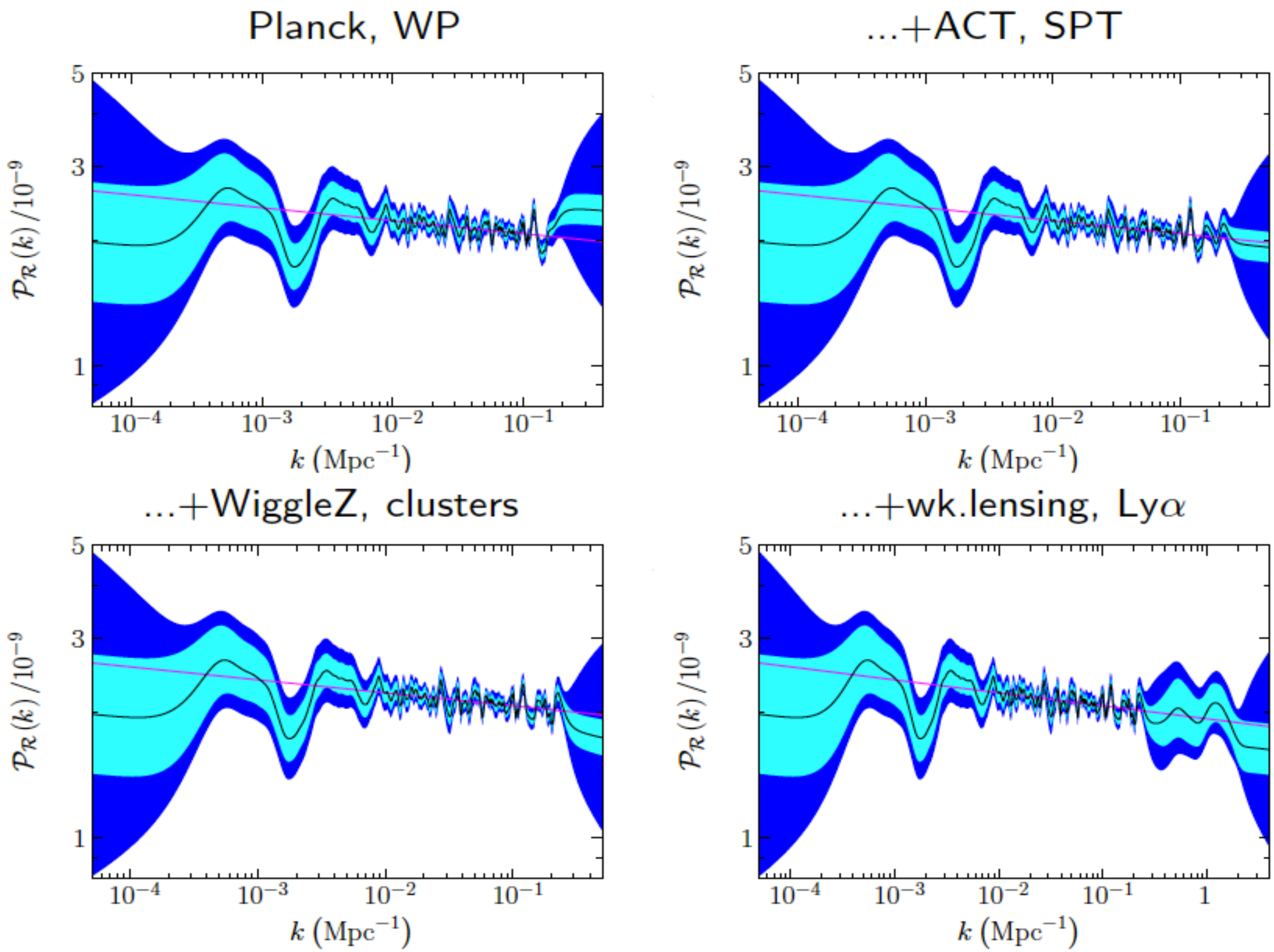
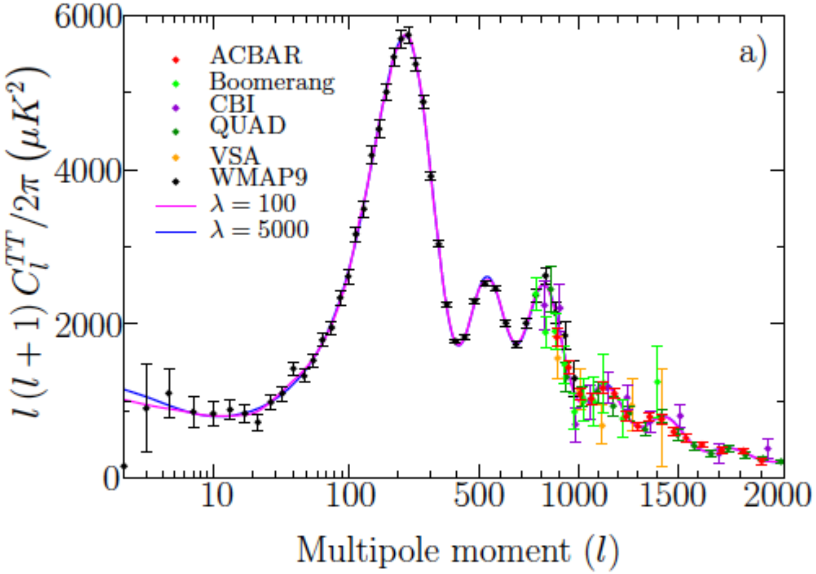
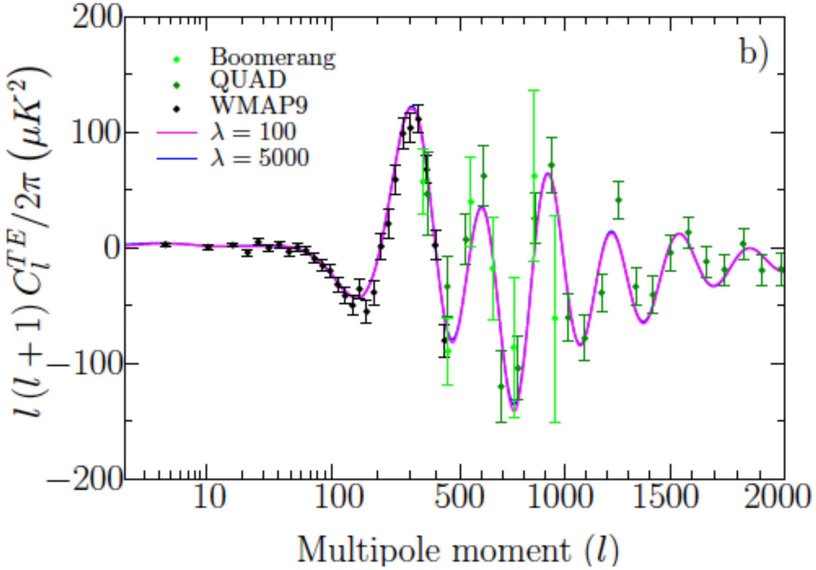


The fit to all the data is just as good as the usually (assumed) power-law spectrum …
but the inferred cosmological parameters can be *very* different

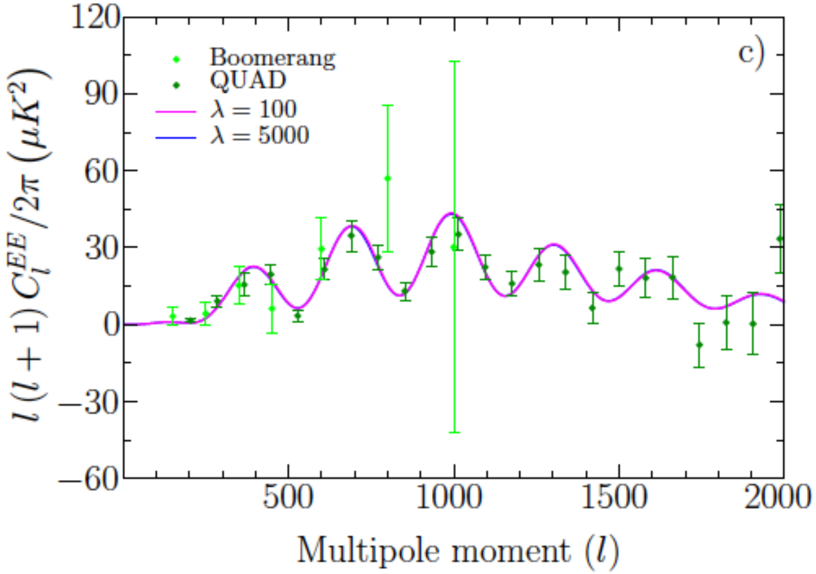
The fit to CMB TT data



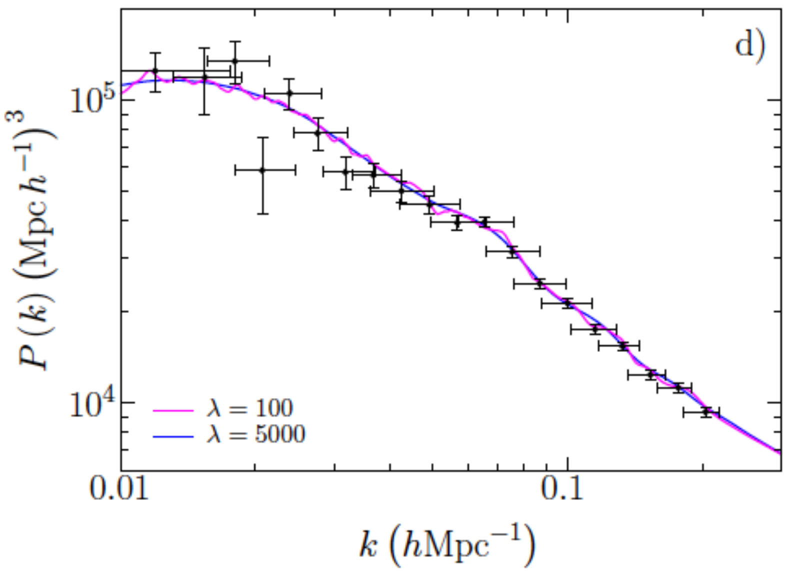
The fit to CMB TE data



The fit to CMB EE data



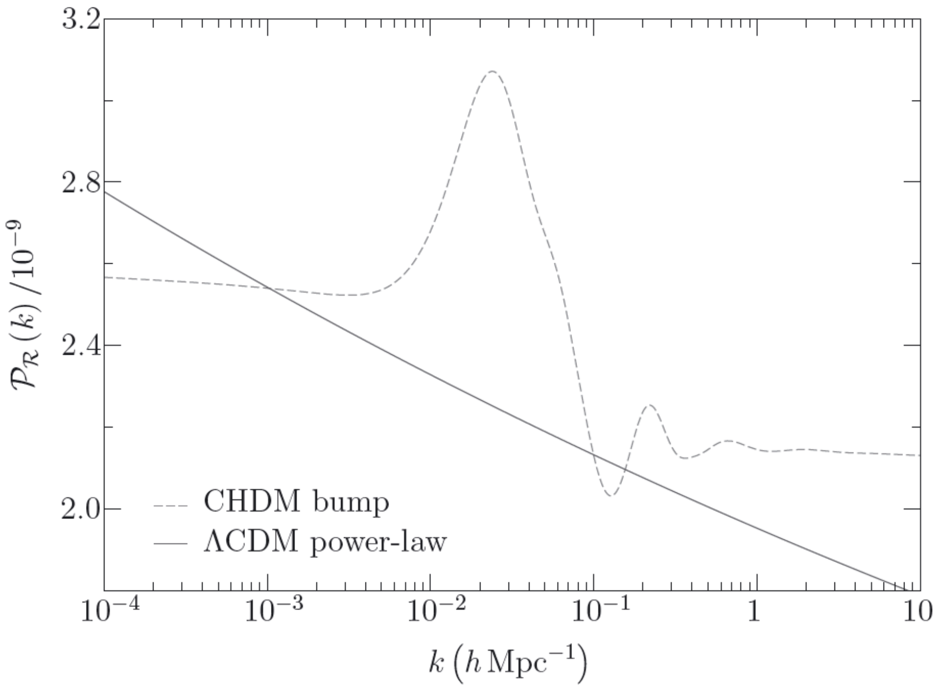
The fit to SDSS LRG data



Hunt & Sarkar, JCAP **01**:025,2014

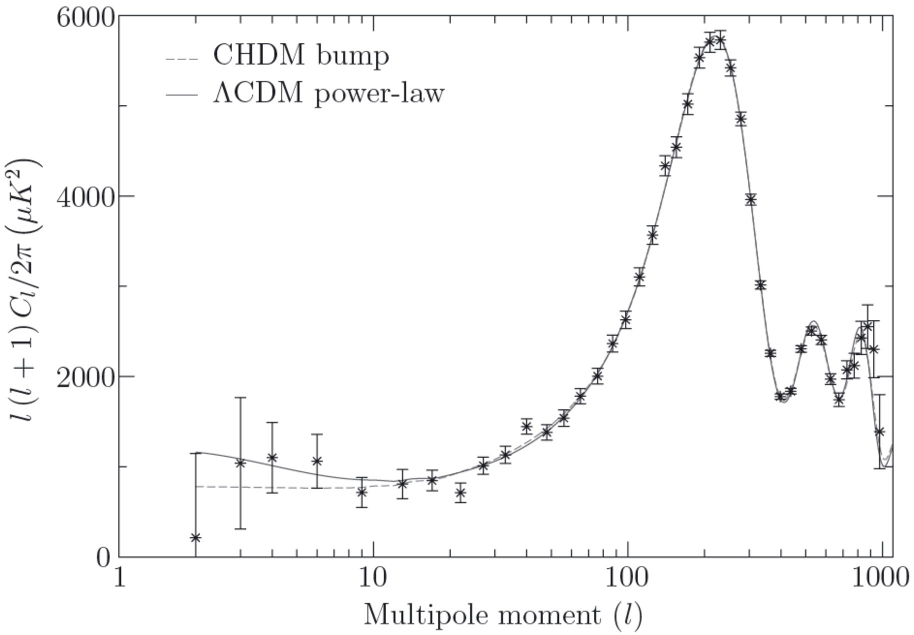
E.g. if there is a 'bump' in the spectrum (around the first acoustic peak), the CMB data can be fitted *without dark energy* ($\Omega_m = 1, \Omega_\Lambda = 0$) if $h \sim 0.45$

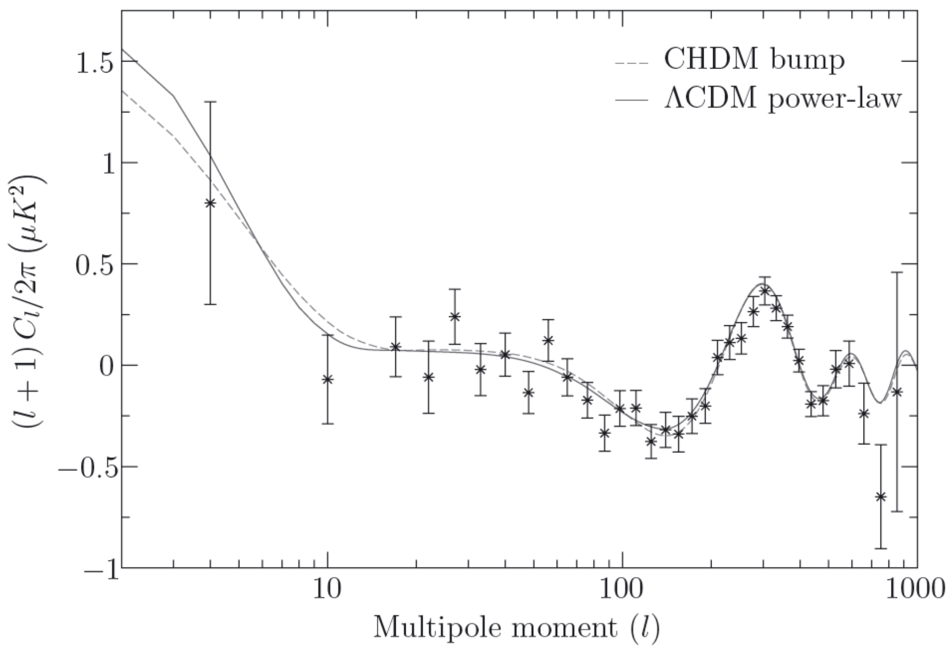
(Hunt & Sarkar arXiv:0706.2443, 0807.4508)



While significantly below the local value of $h \sim 0.7$ this is consistent with its 'global' value in the effective EdeS model fitted to an inhomogeneous, relativistic cosmology

(Roukema *et al*, arXiv:1608.06004)

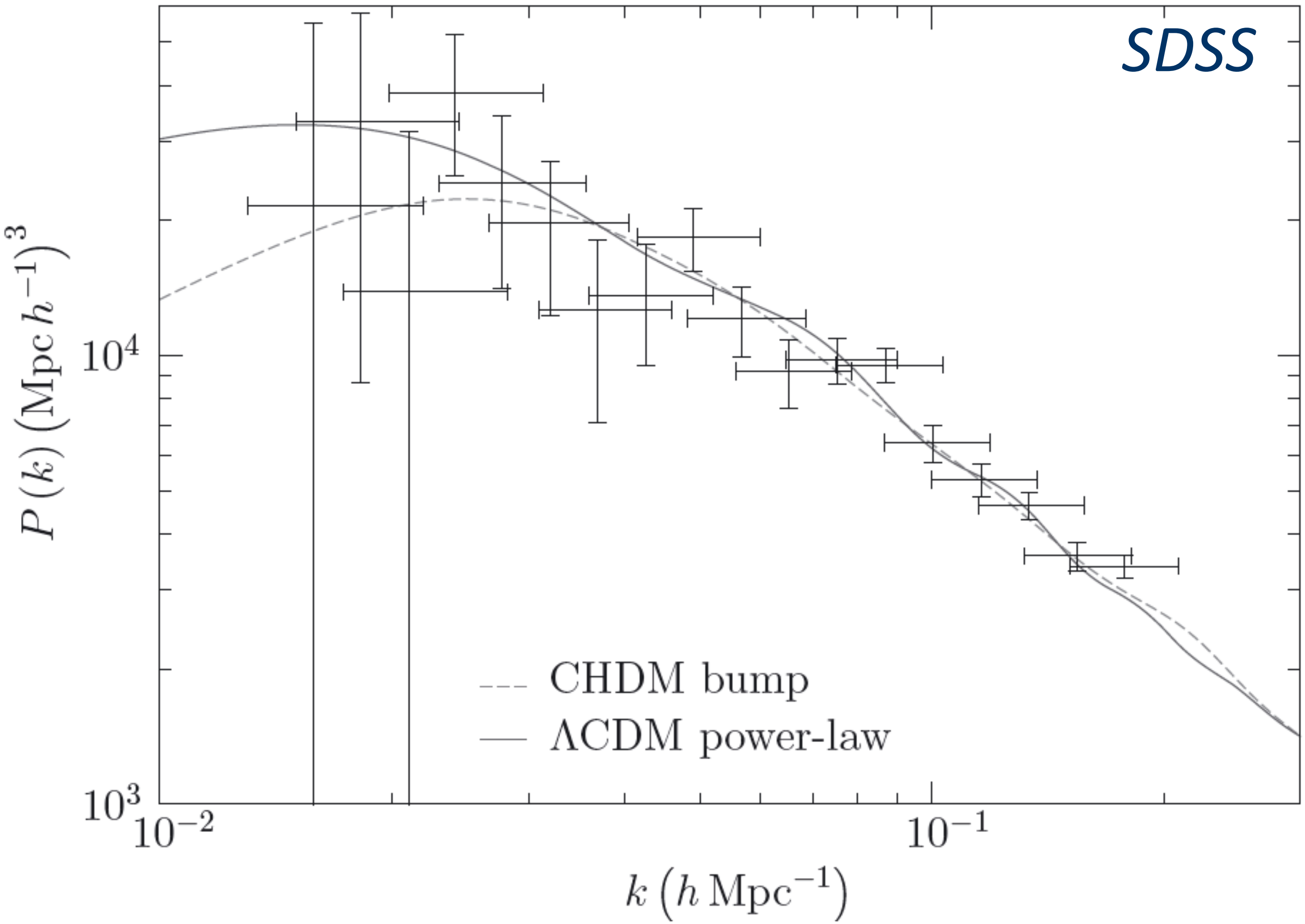




The small-scale power would be excessive unless damped by free-streaming

But adding 3 νs of mass ~0.5 eV ($\Rightarrow \Omega_\nu \approx 0.1$) gives *good* match to large-scale structure

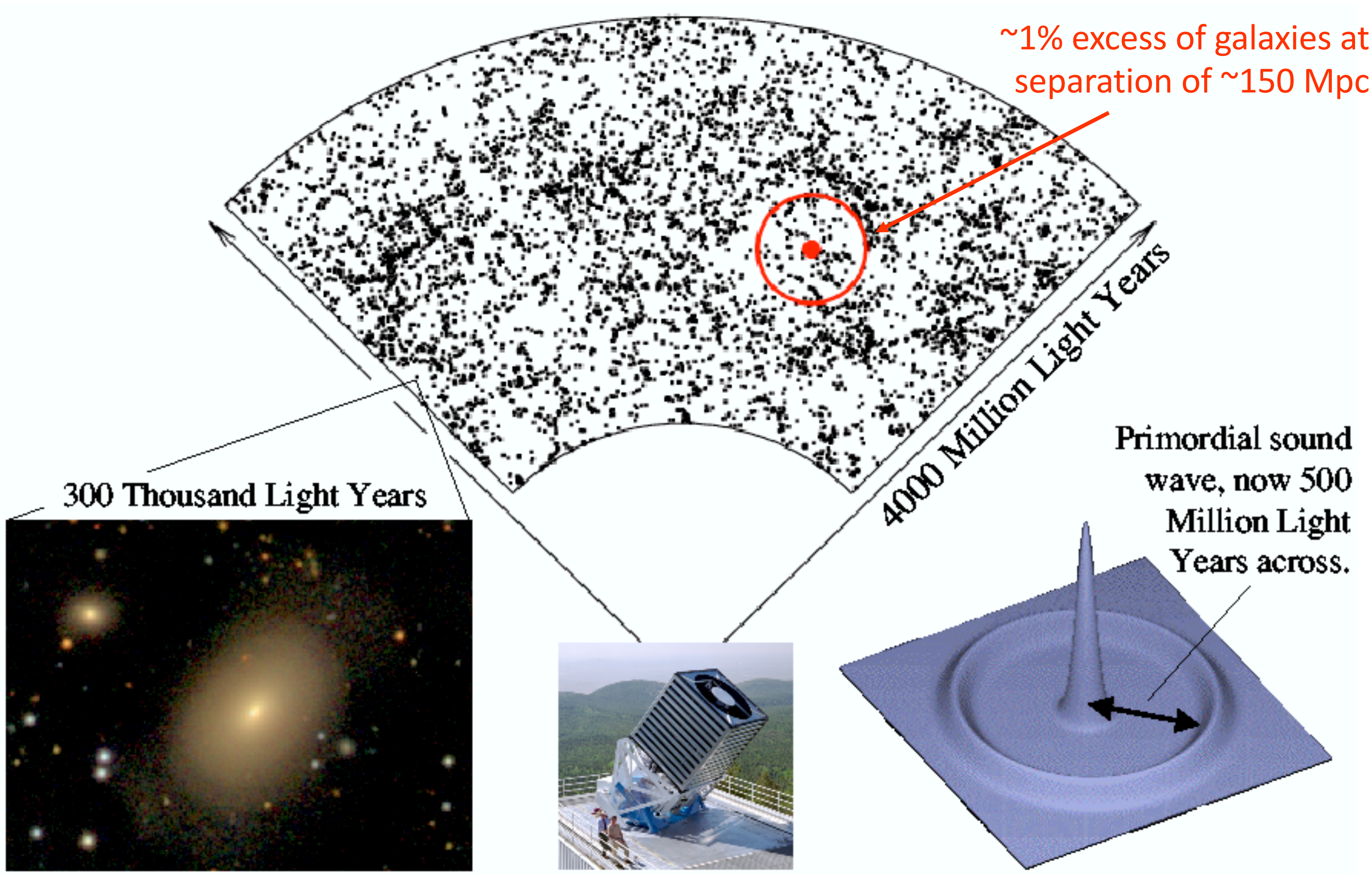
(note that $\Sigma\, m_\nu \approx 1.5$ eV … well above 'CMB bound' – but detectable by KATRIN!)



Hunt & Sarkar arXiv:0706.2443, 0807.4508

Fit gives $\Omega_b h^2 \approx 0.021 \rightarrow$ BBN √ $\Rightarrow$ baryon fraction in clusters predicted to be ~11% √

New Test: Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies



But is the galaxy distribution homogeneous (to better than 1%) on these scales?

Summary

- The 'standard model' of cosmology was established long *before* there was any observational data … and its empirical foundations (homogeneity, ideal fluids) have never been rigorously tested. Now that we have data this should be a priority!

- It is *not* simply a choice between a cosmological constant ('dark energy') and 'modified gravity' – there are other possibilities which should be explored (exact solutions of Einstein's equations are hard to find unless a great deal of symmetry is assumed … so alternative models are not as easy to formulate and confront with observations - but that does *not* make them less plausible as a description of Nature)

- The fact that the standard model implies an *unnatural* value for the cosmological constant, $\Lambda \sim H_0^2$, ought to motivate further work on developing and testing alternative models … rather than pursuing "precision cosmology" of what may well turn out to be an illusion