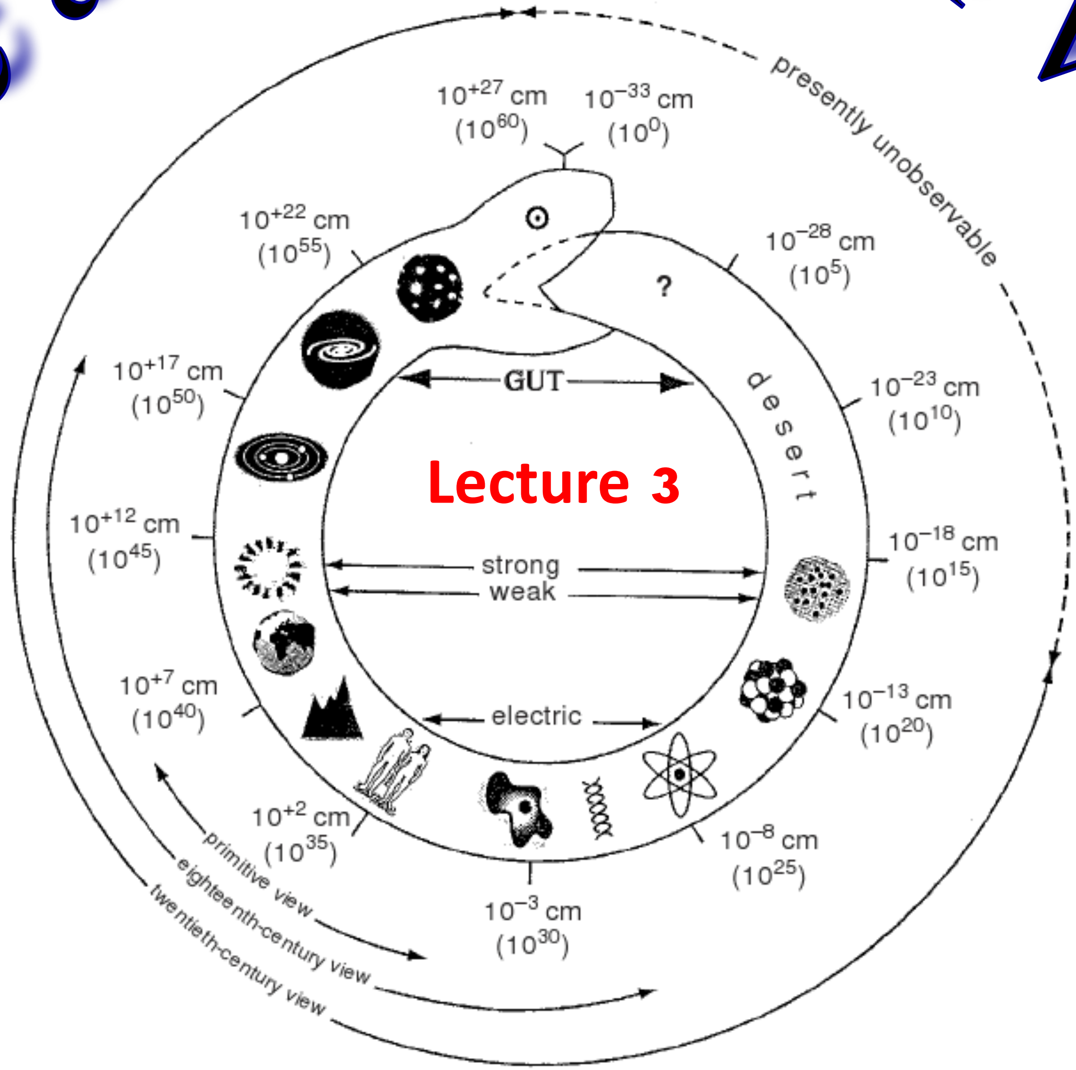
The early universe

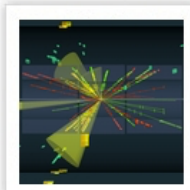
Subir Sarkar

Oxford Physics

The Niels Bohr International Academy

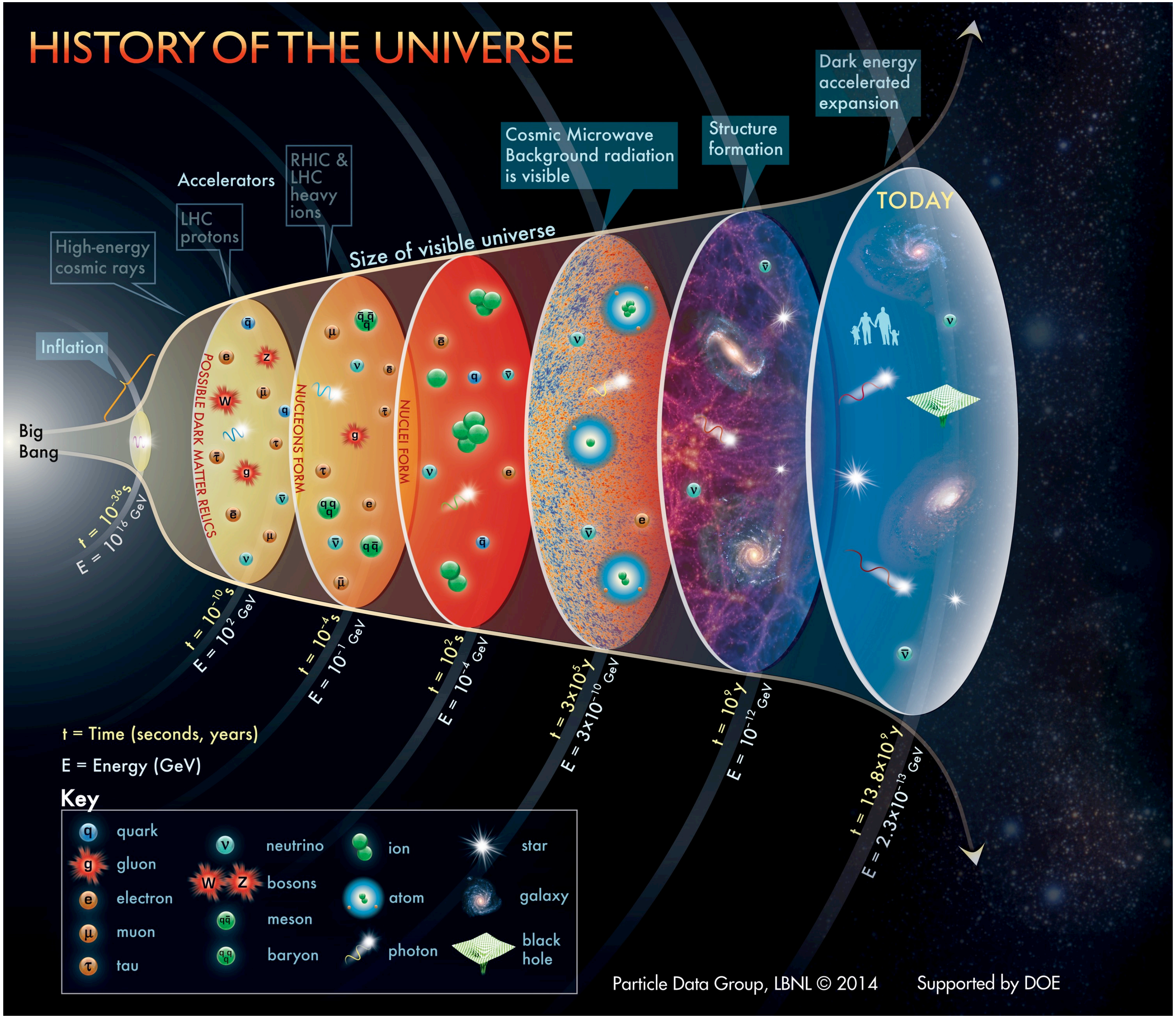


CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

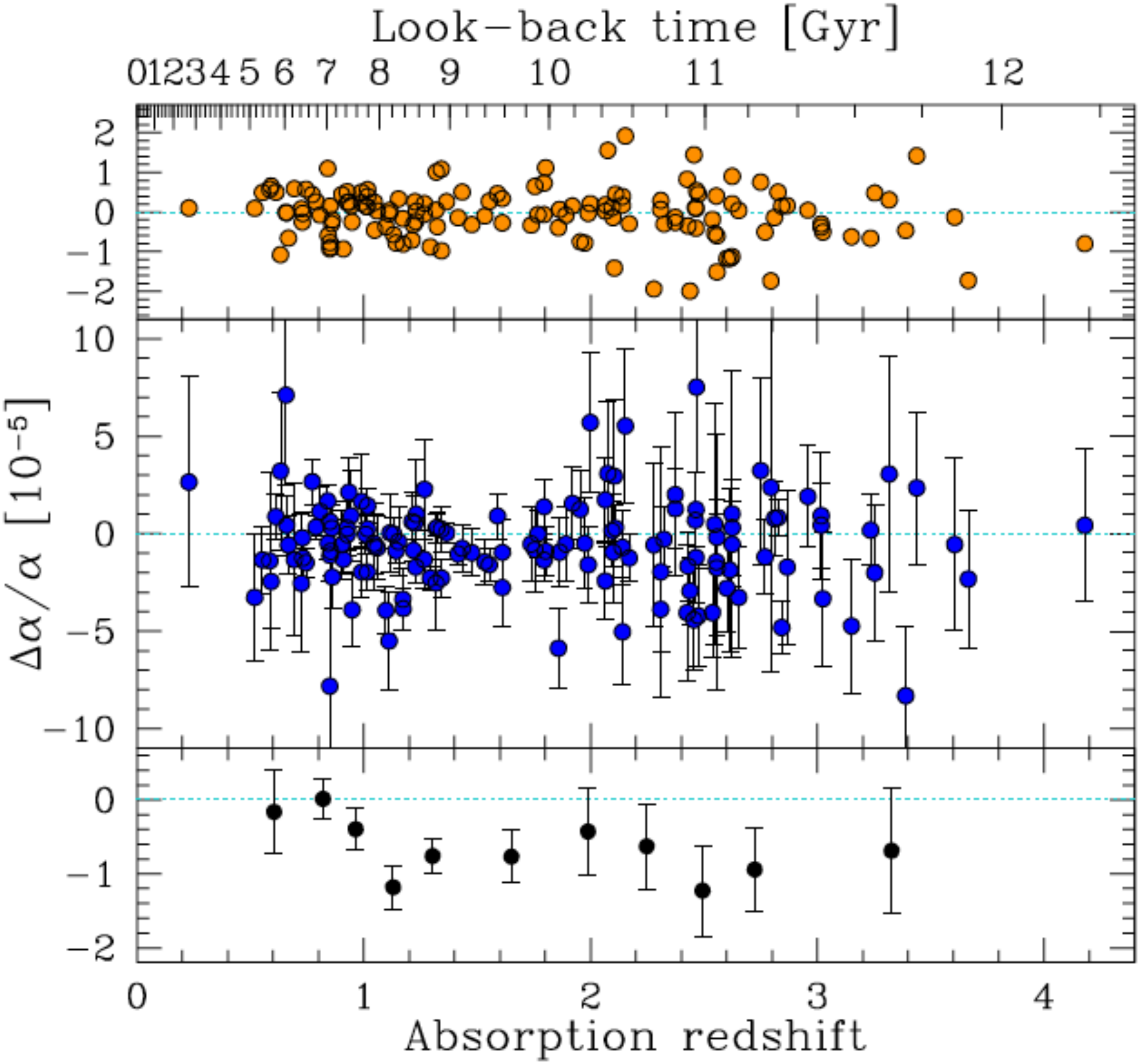


Taller de Altas Energías, 4-17 September 2016, Benasque

How are pretty pictures such as this one actually constructed?



We can check *experimentally* that physical 'constants' such as α have been sensibly constant for the past ~12 billion years …



Webb *et al*, PRL 82:884,1999, arXiv:0911.4512

So we are entitled to extrapolate known physical laws back in time with confidence

Knowing the equation of state, we can solve the Friedman equation ...

For matter: $\frac{d}{dt}(\rho a^3) = 0 \;\Rightarrow\; \rho = \rho_0/a^3 = \rho_0(1+z)^3$

Hence $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_0}{3a^3} \;\Rightarrow\; a(t) = \left(\frac{t}{t_0}\right)^{2/3}$

For radiation: $\frac{d}{dt}(\rho a^4) = 0 \Rightarrow \rho = \rho_0/a^4 = \rho_0(1+z)^4$

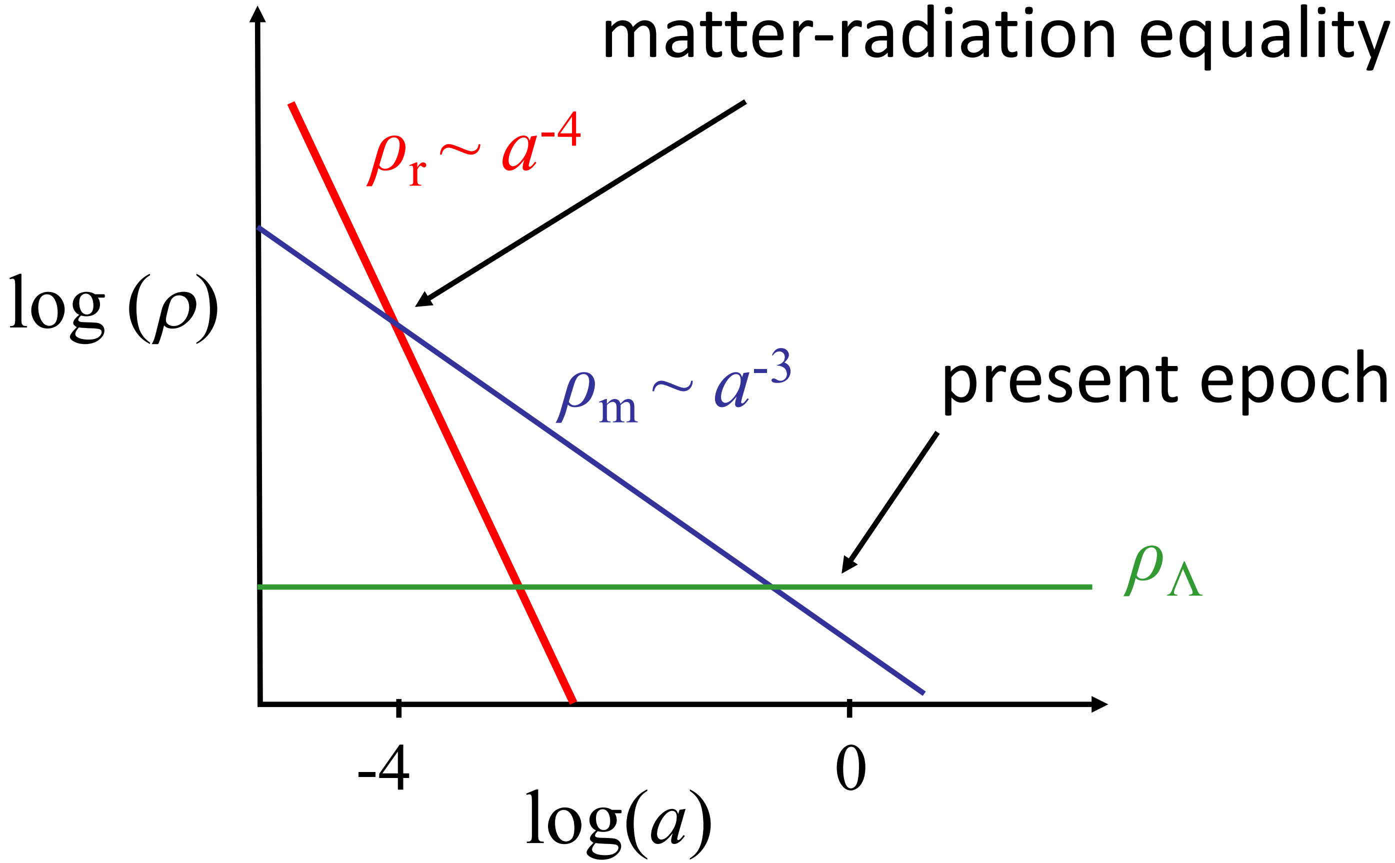
So radiation will dominate over other components as we go to early times

$a(t) = \left(\frac{t}{t_0}\right)^{1/2} \Rightarrow \rho_r \propto t^{-2}$ **Radiation-dominated era**

But at $a_{eq} = \rho_{r,0}/\rho_{m,0}$ natter density will come to dominate

Note that $\rho_m \propto t^{-2}$ uring the **Matter-dominated era as well**

Evolution of different energy components

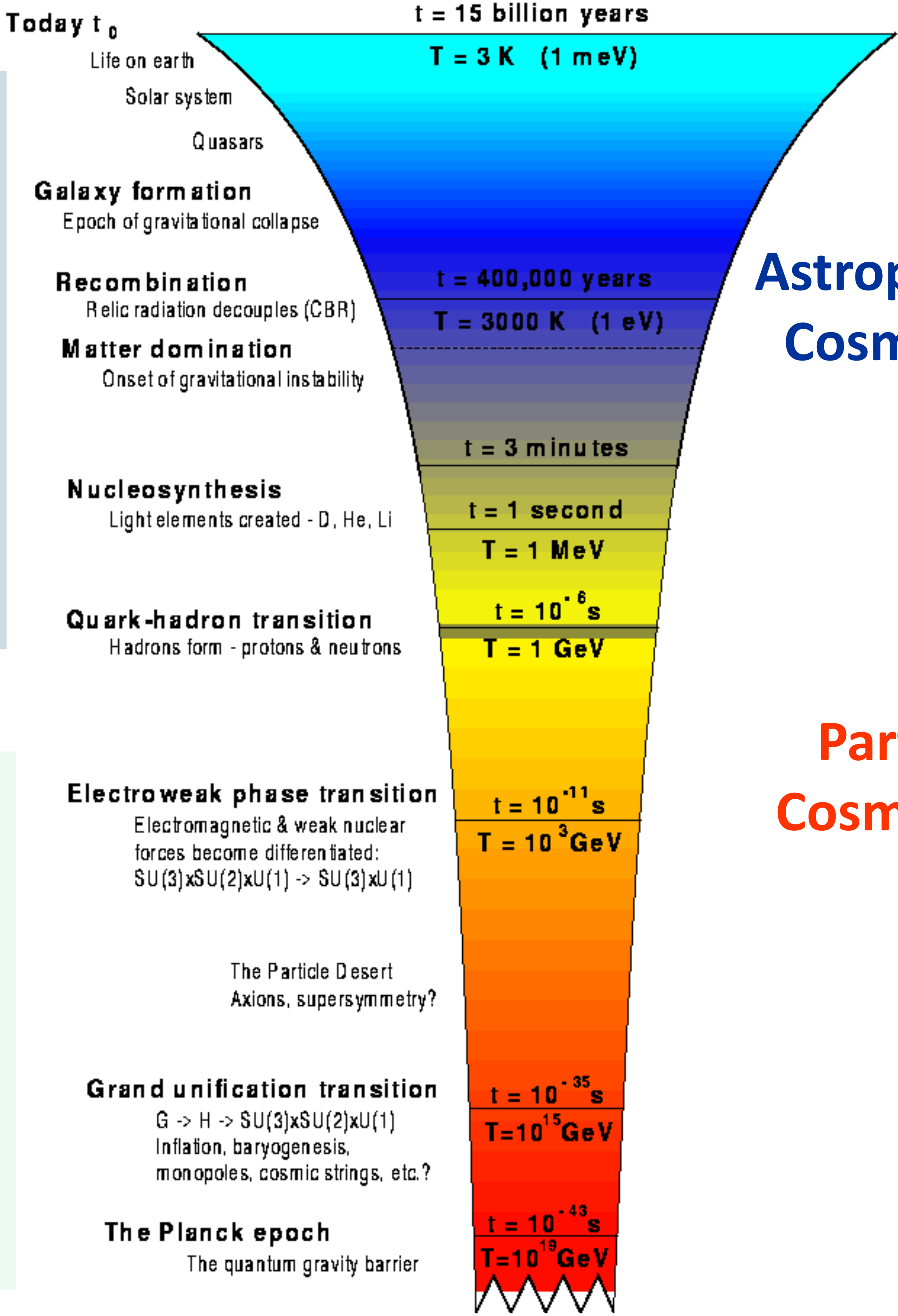


The early universe was therefore radiation-dominated

Very recently (at $z \sim 1$), the expansion has supposedly become dominated by a 'cosmological constant': $\Lambda \sim 2\, H_0^2 \Rightarrow \rho_\Lambda \sim 2\, H_0^2\, \mathrm{M_P}^2$

(This creates a severe **'why now?'** problem as $\rho_\Lambda << \rho_{m,\,r}$ at earlier epochs)

On the basis of known physics, the evolution of the universe can be extrapolated into our past, quite reliably up to the nucleosyntheis era and (with some caveats) back through the QCD phase transition up to the electroweak unification scale



Astrophysical Cosmology

Particle Cosmology

New physics is required to account for the observed asymmetry between matter and antimatter, to explain dark matter, and also generate the density fluctuations which seeded the formation of structure

Does the universe have any net quantum numbers?

The chemical potential is additively conserved in all reactions

hence zero for photons and Z^0 bosons which can be emitted or absorbed in any number (at high enough temperatures) – and consequently *equal and opposite* for a particle and its antiparticle, which can annihilate into such gauge bosons

A finite chemical potential corresponds to a *particle-antiparticle asymmetry*, i.e. a non-zero value for any associated conserved quantum number

The net electric charge of the universe is consistent with being zero

e.g. $q_{\mathrm{e-p}} < 10^{-26} e$ from the isotropy of the CMB (Caprini & Ferreira, JCAP **02**:006,2005)

The net baryon number is very small relative to the number of photons:

$$\boxed{\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim \frac{n_B}{n_\gamma} \simeq 5 \times 10^{-10}}$$

… and presumably so is any net lepton number

There can be a large lepton asymmetry in *neutrinos* (if $B - L$ is non-zero) but this is constrained to be small due to ν oscillations (Dolgov *et al*, Nucl.Phys. **B632**:363,2002)

(NB: The dark matter may be a particle with a relic asymmetry similar to that of baryons)

Thermodynamics of ultra-relativistic plasma in equilibrium:

$$\boxed{f_i^{\rm eq}(q,T) = \left[\exp\left(\frac{E_i-\mu_i}{T}\right)\mp 1\right]^{-1}}$$

For negligible chemical potential, this integrates to:

Number density: $$n_i^{\rm eq}(T) = g_i \int f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3q}{(2\pi)^3} = \frac{g_i}{2\pi^2}T^3 I_i^{11}(\mp)\ ,$$

Energy density: $$\rho_i^{\rm eq}(T) = g_i \int E_i(q)\, f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3q}{(2\pi)^3} = \frac{g_i}{2\pi^2}T^4 I_i^{21}(\mp)\ ,$$

Pressure density: $$p_i^{\rm eq}(T) = g_i \int \frac{q^2}{3E_i(q)}\, f_i^{\rm eq}(q,T)\,\frac{{\rm d}^3q}{(2\pi)^3} = \frac{g_i}{6\pi^2}\,T^4 I_i^{03}(\mp)$$

where: $$I_i^{mn}(\mp) \equiv \int_{x_i}^{\infty} y^m (y^2-x_i^2)^{n/2}\,({\rm e}^y \mp 1)^{-1} dy\ , \qquad x_i \equiv \frac{m_i}{T}$$

bosons :	$I_{\rm R}^{11}(-) = 2\zeta(3)$,	$I_{\rm R}^{21}(-) = I_{\rm R}^{03}(-) = \frac{\pi^4}{15}$
fermions :	$I_{\rm R}^{11}(+) = \frac{3\zeta(3)}{2}$,	$I_{\rm R}^{21}(+) = I_{\rm R}^{03}(+) = \frac{7\pi^4}{120}$

Non-relativistic particles (x >>1) have the Boltzmann distribution:

$$n_{\rm NR}^{\rm eq}(T) = \frac{\rho_{\rm NR}^{\rm eq}(T)}{m} = \frac{g}{(2\pi)^{3/2}}T^3 x^{3/2}{\rm e}^{-x}, \qquad p_{\rm NR} \simeq 0$$

The particle i will stay in *kinetic* equilibrium with the plasma (i.e. $T_i = T$) as long as the **scattering rate** $\Gamma_s = n\langle\sigma v\rangle$ **exceeds the Hubble rate** $H = (8\pi G\rho/3)^{1/2} \sim 1.66\sqrt{g}\, T^2/M_P$

It will decouple at $T_i = T_D$ when $\Gamma_s(T_D) = H(T_D)$

If it is *relativistic* at this time (i.e. $m_i \ll T_d$) then it would also have been in *chemical* equilibrium $(\mu_i + \mu_{\bar{i}} = \mu_{l^+} + \mu_{l^-} = \mu_\gamma = 0)$ and its abundance will just be:

$$\boxed{n_i^{\text{eq}}(T_D) = \frac{g_i}{2} n_\gamma(T_D) f_{B,\,F} \quad (f_B = 1, f_F = 3/4)}$$

Subsequently, the decoupled i particles will expand freely without interactions so that their **number in a comoving volume is conserved** and their pressure and energy density are functions of the scale-factor a alone. Although non-interacting, their phase space distribution will retain the equilibrium form, with T substituted by T_i, as long as the particles remain *relativistic*, which ensures that both E_i and T_i will scale as a^{-1}

Subsequently T_i will continue to track the photon temperature T but as the universe cools below various mass thresholds, the corresponding particles will become non-relativistic and annihilate – this will heat the photons (and any other interacting particles), but *not* the decoupled i particles, so that T_i will now *drop below* T and therefore n_i/n_γ will *decrease* below its value at decoupling

To calculate this write: $p = p_{\mathrm{I}}(T) + p_{\mathrm{D}}(a)$, $\rho = \rho_{\mathrm{I}}(T) + \rho_{\mathrm{D}}(a)$

(Alpher, Follin & Herman, Phys.Rev.**92**:1347,1953)

The energy conservation equation: $$a^3 \frac{\mathrm{d}p}{\mathrm{d}T} = \frac{\mathrm{d}}{\mathrm{d}T}\left[a^3(\rho + p)\right]$$

then reduces to: $$\frac{\mathrm{d}\ln a}{\mathrm{d}\ln T} = -\frac{1}{3}\frac{(\mathrm{d}\rho_{\mathrm{I}}/\mathrm{d}\ln T)}{(\rho_{\mathrm{I}} + p_{\mathrm{I}})}$$ (using $n_{\mathrm{D}}a^3 = \mathrm{const}$)

Combining with the 2nd law of thermodynamics, this yields:

$$\frac{\mathrm{d}\ln a}{\mathrm{d}\ln T} = -1 - \frac{1}{3}\frac{\mathrm{d}\ln\left(\frac{\rho_{\mathrm{I}}+p_{\mathrm{I}}}{T^4}\right)}{\mathrm{d}\ln T}$$

which integrates to: $$\ln a = -\ln T - \frac{1}{3}\ln\left(\frac{\rho_{\mathrm{I}} + p_{\mathrm{I}}}{T^4}\right) + \mathrm{constant}$$

Hence if $(\rho_{\mathrm{I}} + p_{\mathrm{I}})/T^4$ is constant (as for a gas of blackbody photons), this yields the *adiabatic invariant*: $\boldsymbol{aT} = \mathbf{constant}$

Epochs where the number of interacting species is *different* can now be related through the conservation of specific **entropy** in a comoving volume, i.e. $\mathrm{d}(s_{\mathrm{I}}\, a^3)/\mathrm{d}T = 0$, where:

$$s_{\mathrm{I}} \equiv \frac{\rho_{\mathrm{I}} + p_{\mathrm{I}}}{T} = \sum_{\mathrm{int}} s_i \ , \quad s_i(T) = g_i \int \frac{3m_i^2 + 4q^2}{3E_i(q)\, T}\, f_i^{\mathrm{eq}}(q, T)\, \frac{\mathrm{d}^3 q}{(2\pi)^3}$$

Here s_i can be parameterised in terms of the value for photons:

$$s_i(T) \equiv \left(\frac{g_{s_i}}{2}\right)\left(\frac{4}{3}\frac{\rho_\gamma}{T}\right) \ , \qquad g_{s_i} = \frac{45}{4\pi^4} g_i \left[I_i^{21}(\mp) + \frac{1}{3} I_i^{03}(\mp)\right]$$

So the number of *interacting* degrees of freedom is:

$$g_{s_{\mathrm{I}}} \equiv \frac{45}{2\pi^2}\frac{s_{\mathrm{I}}}{T^3} = \sum_{\mathrm{int}} g_{s_i}$$

… analogous to the *total number* of degrees of freedom:

$$\rho_i^{\mathrm{eq}}(T) \equiv \left(\frac{g_{\rho_i}}{2}\right)\rho_\gamma \ , \qquad g_{\rho_i} = \frac{15}{\pi^4} g_i\, I_i^{21}(\mp) \ = \sum_{\mathrm{B}} g_i + \frac{7}{8}\sum_{\mathrm{F}} g_i$$

We can now calculate how the temperature of a particle i which decoupled at T_D relates to the photon temperature T at a later epoch

For $T < T_D$, the entropy in the decoupled i particles and the entropy in the still interacting j particles are *separately conserved*:

$$S - S_I = s_i\, a^3 \quad = \frac{2\pi^2}{45} g_{s_i}(T)\,(a\,T)_i^3,$$
$$S_I = \sum_{j \neq i} s_j(T)\, a^3 \quad = \frac{2\pi^2}{45} g_{s_I}(T)\,(a\,T)^3$$

Since $T_i = T$ at decoupling, this yields for the subsequent ratio of temperatures (Srednicki *et al,* Nucl.Phys.B**310**:693,1988, Gondolo & Gelmini, *ibid* B**360**:145,1991):

$$\frac{T_i}{T} = \left[\frac{g_{s_i}(T_D)}{g_{s_i}(T)} \frac{g_{s_I}(T)}{g_{s_I}(T_D)}\right]^{1/3}$$

Following decoupling, the degrees of freedom specifying the conserved total entropy is:

$$g_s(T) \equiv \frac{45}{2\pi^2} \frac{S}{T^3 a^3} = g_{s_I}(T)\left[1 + \frac{g_{s_i}(T_D)}{g_{s_I}(T_D)}\right]$$

We now have an useful fiducial in the total entropy density, which *always* scales as a^{-3}:

$$\boxed{s(T) \equiv \frac{2\pi^2}{45} g_s(T) T^3}$$

Therefore the ratio of the decoupled particle density to the blackbody photon density is subsequently related to its value at decoupling as:

$$\frac{(n_i/n_\gamma)_T}{(n_i^{\mathrm{eq}}/n_\gamma)_{T_\mathrm{D}}} = \frac{g_s(T)}{g_s(T_\mathrm{D})} = \frac{N_\gamma(T_\mathrm{D})}{N_\gamma(T)}$$

where $N_\gamma = a^3 n_\gamma$ is the total number of blackbody photons in a comoving volume

The total energy density may similarly be parameterised as:

$$\rho(T) = \sum \rho_i^{\mathrm{eq}} \equiv \left(\frac{g_\rho}{2}\right) \rho_\gamma = \frac{\pi^2}{30} g_\rho T^4 \,, \qquad g_\rho \simeq \sum\nolimits_\mathrm{B} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum\nolimits_\mathrm{F} g_i \left(\frac{T_i}{T}\right)^4$$

So the relationship between a and T writes: $\dfrac{\mathrm{d}a}{a} = -\dfrac{\mathrm{d}T}{T} - \dfrac{1}{3}\dfrac{\mathrm{d}g_{s_\mathrm{I}}}{g_{s_\mathrm{I}}}$

During the radiation-dominated era, the expansion rate is:

$$H \equiv \frac{\dot{a}}{a} \simeq \sqrt{\frac{8\pi G_{\mathrm{N}}\rho}{3}}$$

Integrating this yields the time-temperature relationship:

$$t = -\int \left(\frac{45\, M_{\mathrm{P}}^2}{4\pi^3}\right)^{1/2} g_\rho^{-1/2} \left(1 + \frac{1}{3}\frac{\mathrm{d}\ln g_{s_{\mathrm{I}}}}{\mathrm{d}\ln T}\right)\frac{\mathrm{d}T}{T^3}$$

During the periods when $\mathrm{d}g_{s\mathrm{I}}/\mathrm{d}T \simeq 0$, i.e. away from mass thresholds and phase transitions, this yields the useful commonly used approximation:

$$(t/\mathrm{s}) = 2.42\, g_\rho^{-1/2}\, (T/\mathrm{MeV})^{-2}$$

So we can work out when events of physical significance occurred (according to the Standard $SU(3)_c \mathrm{x} SU(2)_L \mathrm{x} U(1)_Y$ Model … and beyond)

The above discussion is usually illustrated by the example of the decoupling of massless neutrinos in the Standard Model

The thermally-averaged #-section is: $\langle\sigma v\rangle \sim G_F^{\,2}\, E^2 \sim G_F^{\,2}\, T^2$ ($m_\nu \ll T$)
so the interaction rate is: $\Gamma = n\langle\sigma v\rangle \sim G_F^{\,2} T^5$ (since $n \approx T^3$)

This equals the expansion rate $H \sim T^2/M_P$ at the decoupling temperature

$$T_D(\nu) \sim (G_F^2 M_P)^{-1/3} \sim 1\ \mathrm{MeV}$$

At this time $n_\nu^{eq} = (3/4)n_\gamma$ since $T_\nu = T$ and $g_\nu = 2$. Subsequently as T drops below m_e, the electrons and positrons annihilate (almost) totally, heating the photons but *not* the decoupled neutrinos. While g_ν does not change, the number of other interacting degrees of freedom decreases from 11/2 (γ, $e^\pm$) to 2 (γ only), hence the comoving number of blackbody photons *increases* by the

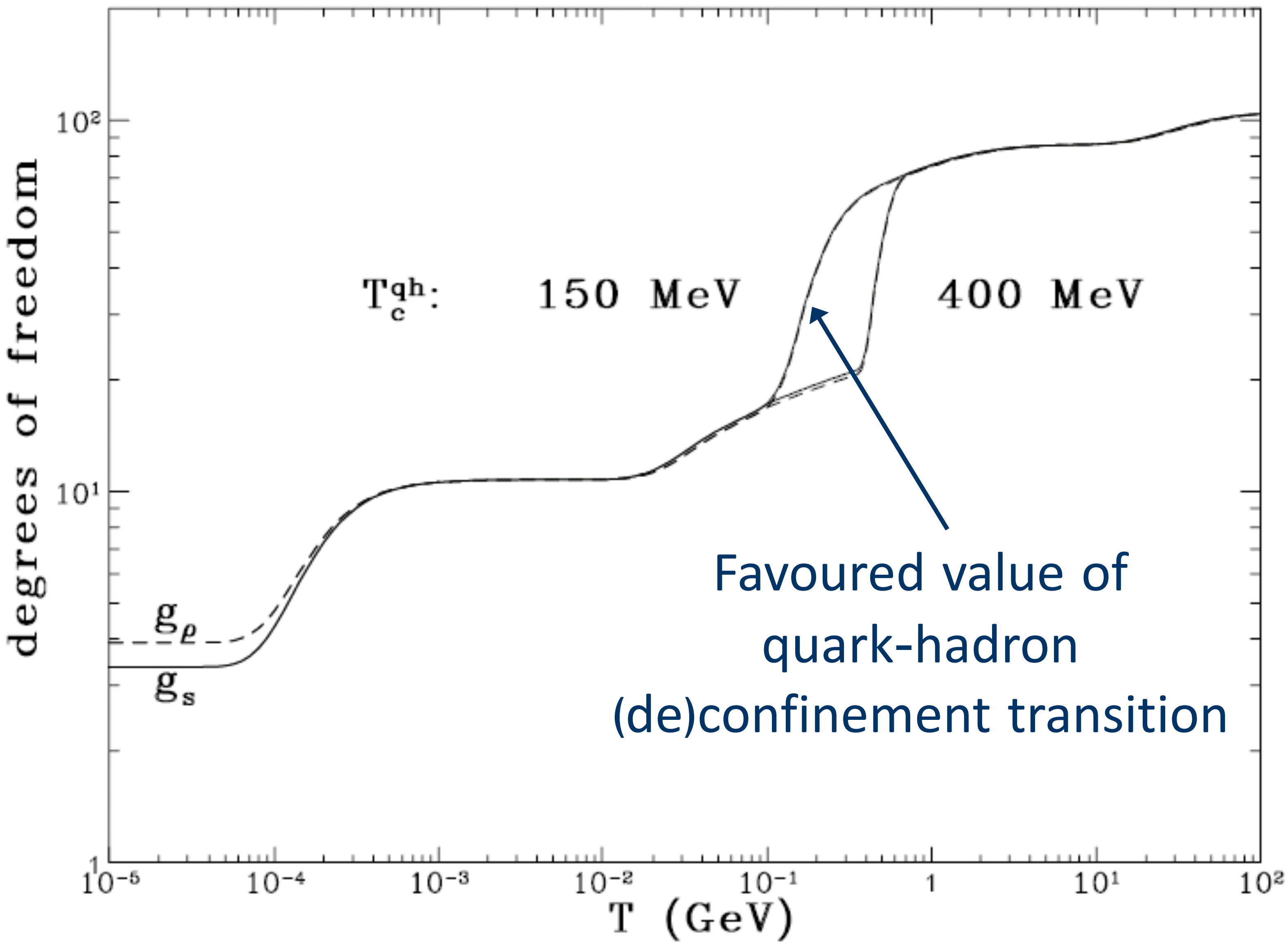
$$\frac{N_\gamma\,(T \ll m_\mathrm{e})}{N_\gamma\,(T = T_\mathrm{D}(\nu))} = \left[\frac{(aT)_{T \ll m_\mathrm{e}}}{(aT)_{T = T_\mathrm{D}(\nu)}}\right]^3 = \frac{11}{4} \quad \text{so} \quad \left(\frac{n_\nu}{n_\gamma}\right)_{T \ll m_\mathrm{e}} = \frac{4}{11}\left(\frac{n_\nu^{\mathrm{eq}}}{n_\gamma}\right)_{T = T_\mathrm{D}(\nu)} = \frac{3}{11}$$

Hence the degrees of freedom characterising the entropy and energy densities today are:

$$g_s\,(T \ll m_\mathrm{e}) \quad = g_\gamma + \tfrac{7}{8}\, N_\nu\, g_\nu \left(\frac{T_\nu}{T}\right)^3 = \frac{43}{11}\,,$$

$$g_\rho\,(T \ll m_\mathrm{e}) \quad = g_\gamma + \tfrac{7}{8}\, N_\nu\, g_\nu \left(\frac{T_\nu}{T}\right)^4 = 3.36$$

To construct our thermal history we must then count all boson and fermion species contributing to the number of relativistic degrees of freedom … and take into account (our uncertain knowledge of) possible phase transitions



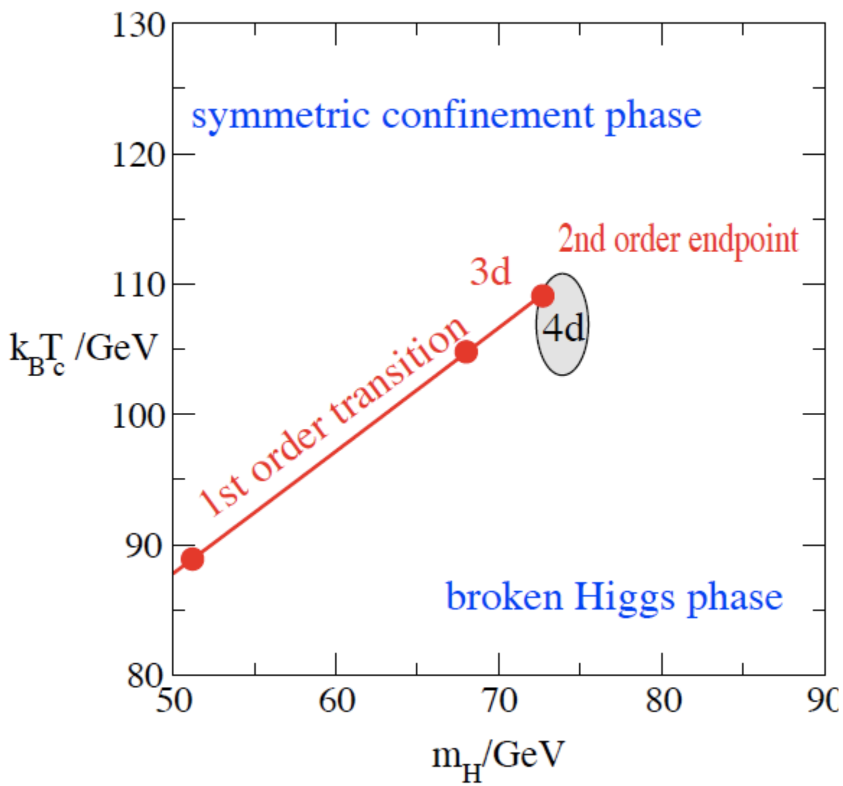
The Standard Model of the Early Universe

$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top-annihilation	96.25	
$T < 80$ GeV	$W^{\pm}$, Z^0, H^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\rightarrow \pi^{\pm,0}$, $37 \rightarrow 3$)
$T < 100$ MeV	$\pi^{\pm}$, π^0, μ^-	10.75	$e^{\pm}$, ν, $\bar{\nu}$, γ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

History of $g(T)$

The phase diagram of the Standard Model (based on a dimensionally reduced $SU(2)_L$ theory with quarks and leptons, with the Abelian hypercharge symmetry $U(1)_Y$ neglected). The 1st-order transition line ends at the 2nd-order endpoint: $m_H \simeq 72 \pm 2$ GeV/c^2, $k_B T_E \simeq 110$ GeV; for higher Higgs mass it is a 'crossover'

Rummukainen *et al*, Nucl.Phys.B**532**:283,1998



What is the highest temperature the Universe could have reached?

On dimensional grounds, the 2 ➛2 scattering/annihilation cross-section (at temperatures higher than the masses of particles) must go as $\sim\alpha^2/T^2$, i.e. the rate will go as: $\Gamma \sim n\langle\sigma v\rangle \sim \alpha^2 T$

Comparing this to the Hubble expansion rate, $H \sim (g_* T^4/10\, M_P^4)^{1/2}$, we see that the thermalisation temperature *cannot* exceed:

$T_{\rm therm} \sim \alpha^2 M_P/3\sqrt{g_*} \sim 10^{-4} M_P$ (taking: $\alpha = 1/24$, $g_* \sim 200$)

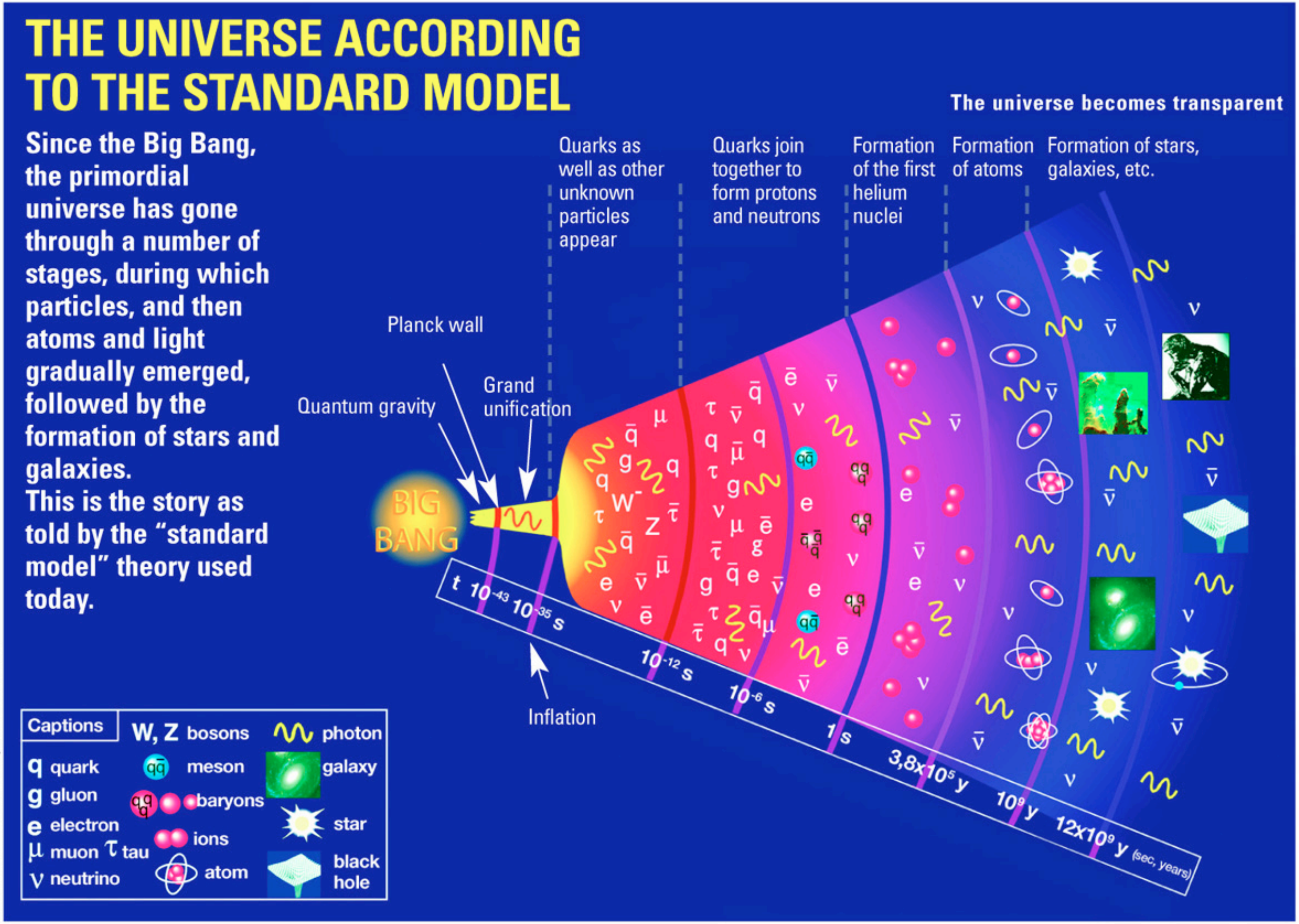
So the universe could never have been as hot as even the GUT scale!

A careful calculation (incl. the temperature dependence of $\alpha_{\rm QCD}$) gives:

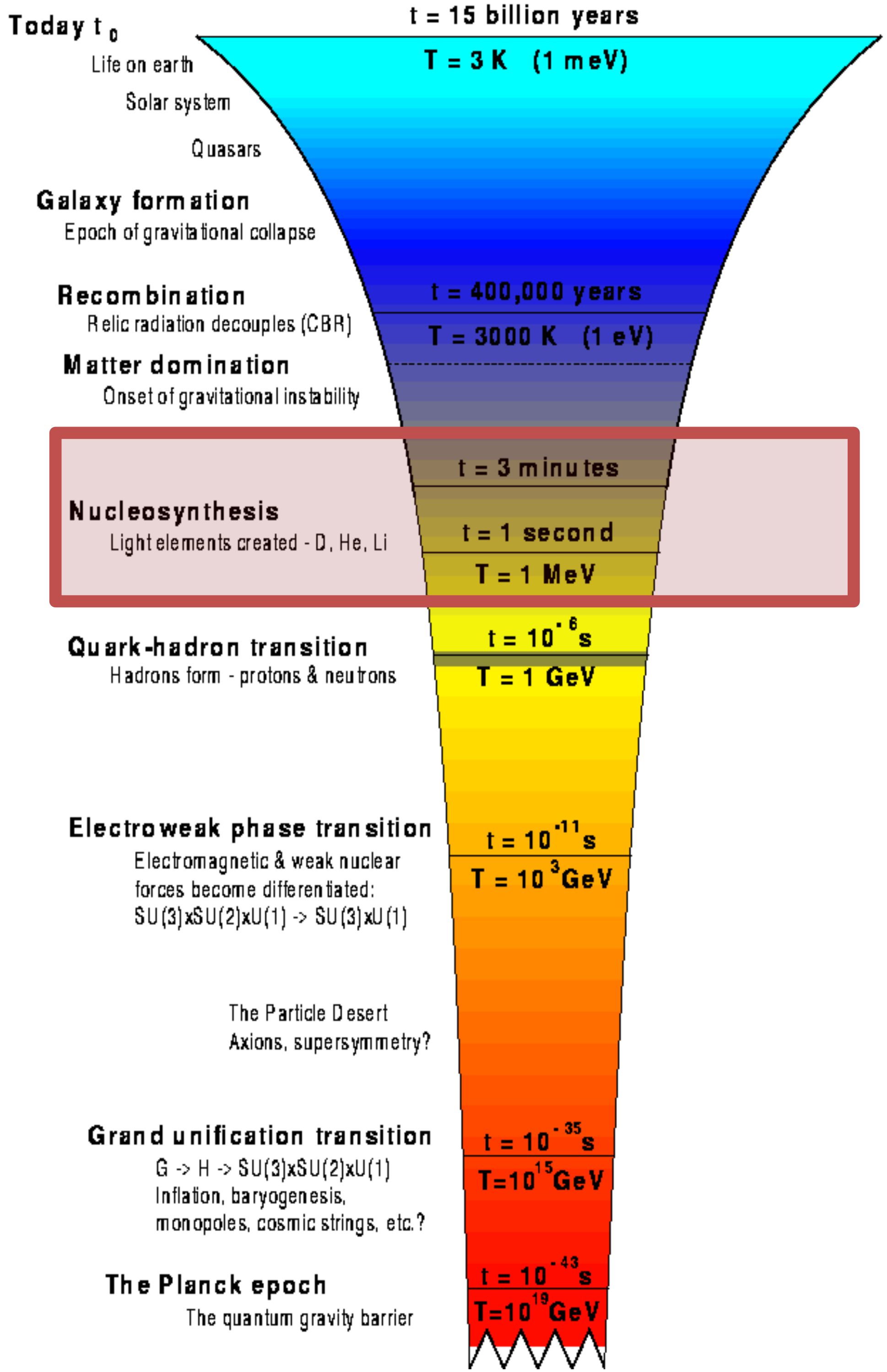
$T_{\rm therm} \sim 3\text{x}10^{14}$ GeV (Enqvist & Sirkaa, Phys. Lett. B**314**:298,1993)

Ought to revisit earlier discussions of GUT-scale baryogenesis, monopole problem …

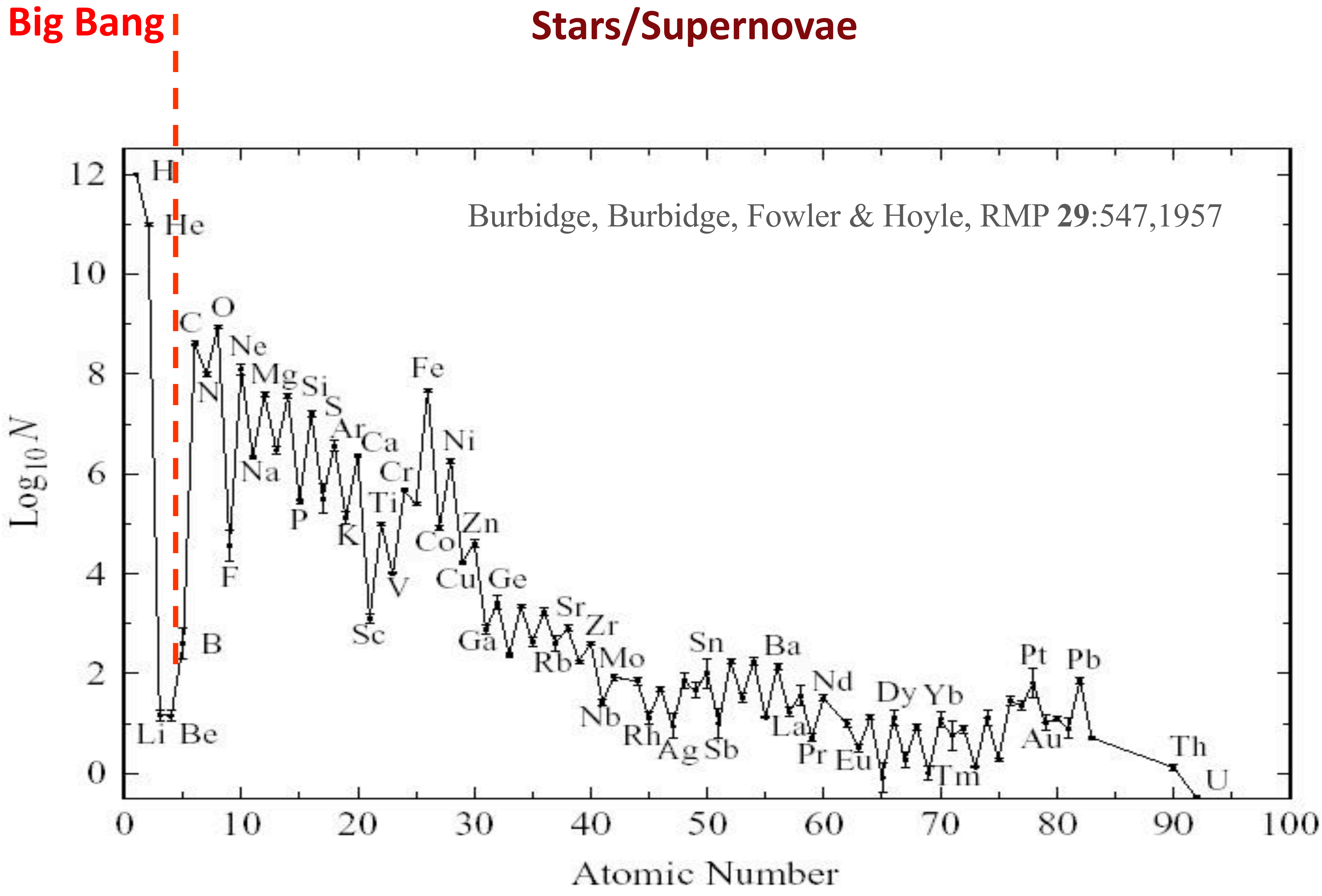
… so now you know how pretty pictures such as this one are actually constructed



Big Bang Nucleosynthesis



Where did all the elements come from?



PHYSICAL REVIEW VOLUME 92, NUMBER 6 DECEMBER 15, 1953

Physical Conditions in the Initial Stages of the Expanding Universe*,†

RALPH A. ALPHER, JAMES W. FOLLIN, JR., AND ROBERT C. HERMAN
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland

(Received September 10, 1953)

The detailed nature of the general nonstatic homogeneous isotropic cosmological model as derived from general relativity is discussed for early epochs in the case of a medium consisting of elementary particles and radiation which can undergo interconversion. The question of the validity of the description afforded by this model for the very early super-hot state is discussed. The present model with matter-radiation interconversion exhibits behavior different from non-interconverting models, principally because of the successive freezing-in or annihilation of various constituent particles as the temperature in the expanding universe decreased with time. The numerical results are unique in that they involve no disposable parameters which would affect the time dependence of pressure, temperature, and density.

The study of the elementary particle reactions leads to the time dependence of the proton-neutron concentration ratio, a quantity required in problems of nucleogenesis. This ratio is found to lie in the range ~4.5:1–~6.0:1 at the onset of nucleogenesis. These results differ from those of Hayashi mainly as a consequence of the use of a cosmological model with matter-radiation interconversion and of relativistic quantum statistics, as well as a different value of the neutron half-life.

The modern theory of primordial nucleosynthesis is based essentially on this paper … which followed the crucial observation by Hayashi (Prog.Theoret.Phys.**5:**224,1950) that **neutrons and protons were in chemical equilibrium in the hot early universe**

Alpher was awarded the US National Medal of Science in 2005:

"For his unprecedented work in the areas of nucleosynthesis, for the prediction that universe expansion leaves behind background radiation, and for providing the model for the Big Bang theory."

Weak interactions and nuclear reactions in expanding, cooling universe
(Hayashi 1950, Alpher, Follin & Herman 1953, Peebles 1966, Wagoner, Fowler & Hoyle 1967)

Dramatis personae:

Radiation (dominates) $\gamma, e^{\pm}, 3\nu\bar{\nu}$

Matter n, p

baryon-to-photon ratio (only free parameter) $n_B/n_\gamma \equiv \eta \simeq 2.74 \times 10^{-8}\Omega_B h^2$

Initial conditions: *T* >> 1 MeV, *t* << 1 s

n-*p* weak equilibrium: $n + \nu_e \leftrightarrow p + e^-$

neutron-to-proton ratio: $p + \nu_e \leftrightarrow n + e^+$

$$\tau_{\text{weak}}(n \leftrightarrow p) \geq t_{\text{universe}} \Rightarrow T_{\text{freeze-out}} \sim \left(G_N / G_F^2\right)^{1/3}$$

Weak freeze-out: $T_f \sim 1$ MeV, $t_f \sim 1$ s

which fixes: $n/p = e^{-(m_n - m_p)/T_f} \approx 1/6$

Deuterium bottleneck: $T \sim 1 \to 0.07$ MeV

D created by $np \to D\gamma$

but destroyed by high-E photon tail: $D\gamma \to np$

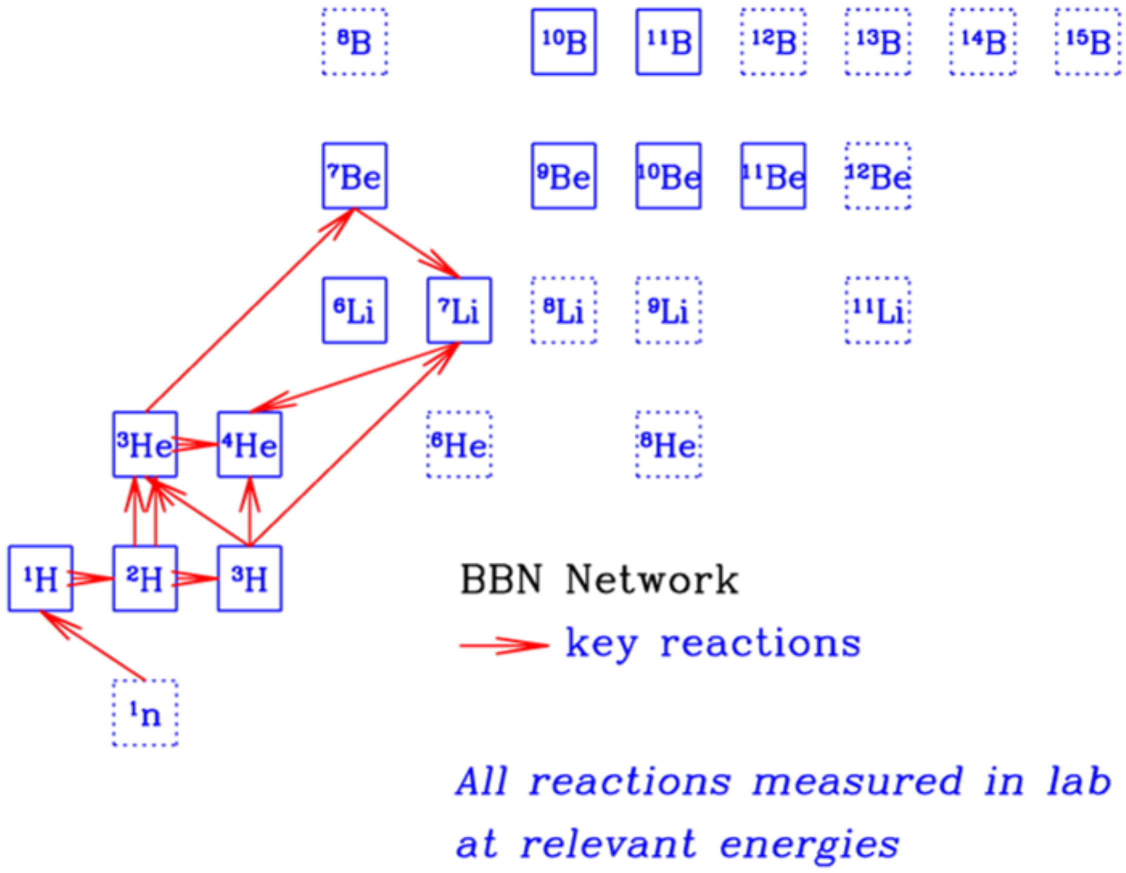
so nucleosynthesis halted until: $T_{\text{nuc}} \sim \Delta_D/-\ln(\eta)$

Element synthesis: $T_{\text{nuc}} \sim 0.07$ MeV, $t_{\text{nuc}} \sim 3$ min

(meanwhile $n/p \to 1/7$ through neutron β-decay)

nearly all $n \to {}^4$He ($Y_P \sim 25\%$ by mass) + left-over traces of D, ^{3}He, ^{7}Li (with ^{6}Li/^{7}Li ~ 10^{-5})

No heavier nuclei formed in standard, homogeneous hot Big Bang … must wait for stars to form after a ~billion years and synthesise all the other nuclei in the universe (s-process, r-process, …)



- Computer code by Wagoner (1969, 1973) .. updated by Kawano (1992)
- Coulomb & radiative corrections, ν heating et cetera (Dicus *et al* 1982)
- Nucleon recoil corrections (Seckel 1993)
- Covariance matrix of correlated uncertainties (Fiorentini *et al* 1998)
- Updated nuclear cross-sections (NACRE 2003)

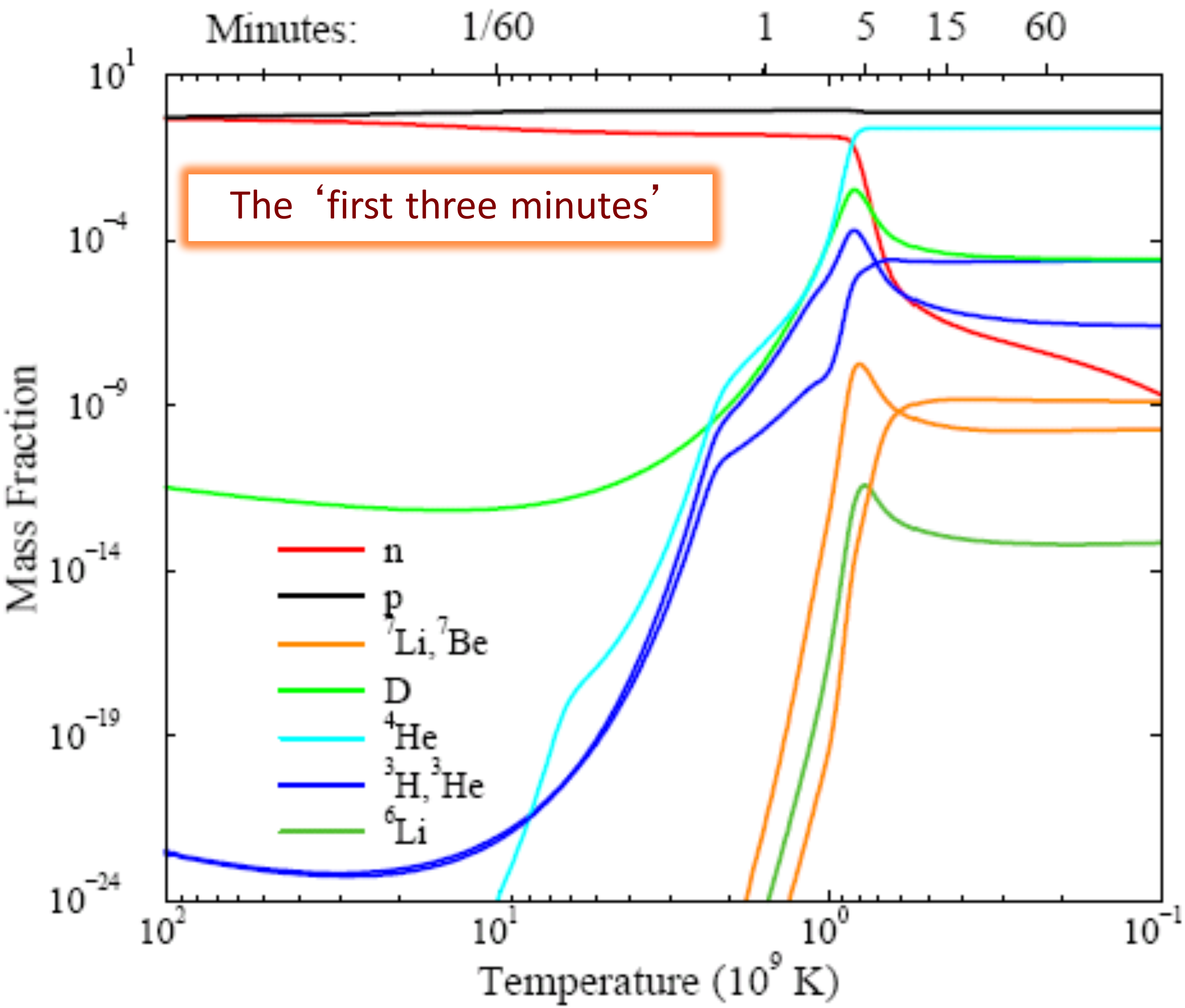
- **Time < 15 s, Temperature > 3 x 10^9 K**
  - universe is soup of protons, electrons and other particles … so hot that nuclei are blasted apart by high energy photons as soon as they form

- **Time = 15 s, Temperature = 3 x 10^9 K**
  - Still too hot for Deuterium to survive
  - Cool enough for Helium to survive, but too few building blocks

- **Time = 3 min, Temperature = 10^9 K**
  - Deuterium survives and is quickly fused into He
  - no stable nuclei with 5 or 8 nucleons, and this restricts formation of elements heavier than Helium
  - trace amounts of Lithium are formed

- **Time = 35 min, Temperature = 3 x 10^7 K**
  - nucleosynthesis essentially complete
  - Still hot enough to fuse He, but density too low for appreciable fusion

Model makes predictions about the relative abundances of the light elements ^{2}H, ^{3}He, ^{4}He and ^{7}Li, as a function of the nucleon density



Minutes:
1/60
1
5
15
60
The 'first three minutes'
Mass Fraction
10^{1}
10^{-4}
10^{-9}
10^{-14}
10^{-19}
10^{-24}
n
p
^{7}Li, ^{7}Be
D
^{4}He
^{3}H, ^{3}He
^{6}Li
10^{2}
10^{1}
10^{0}
10^{-1}
Temperature (10^{9} K)

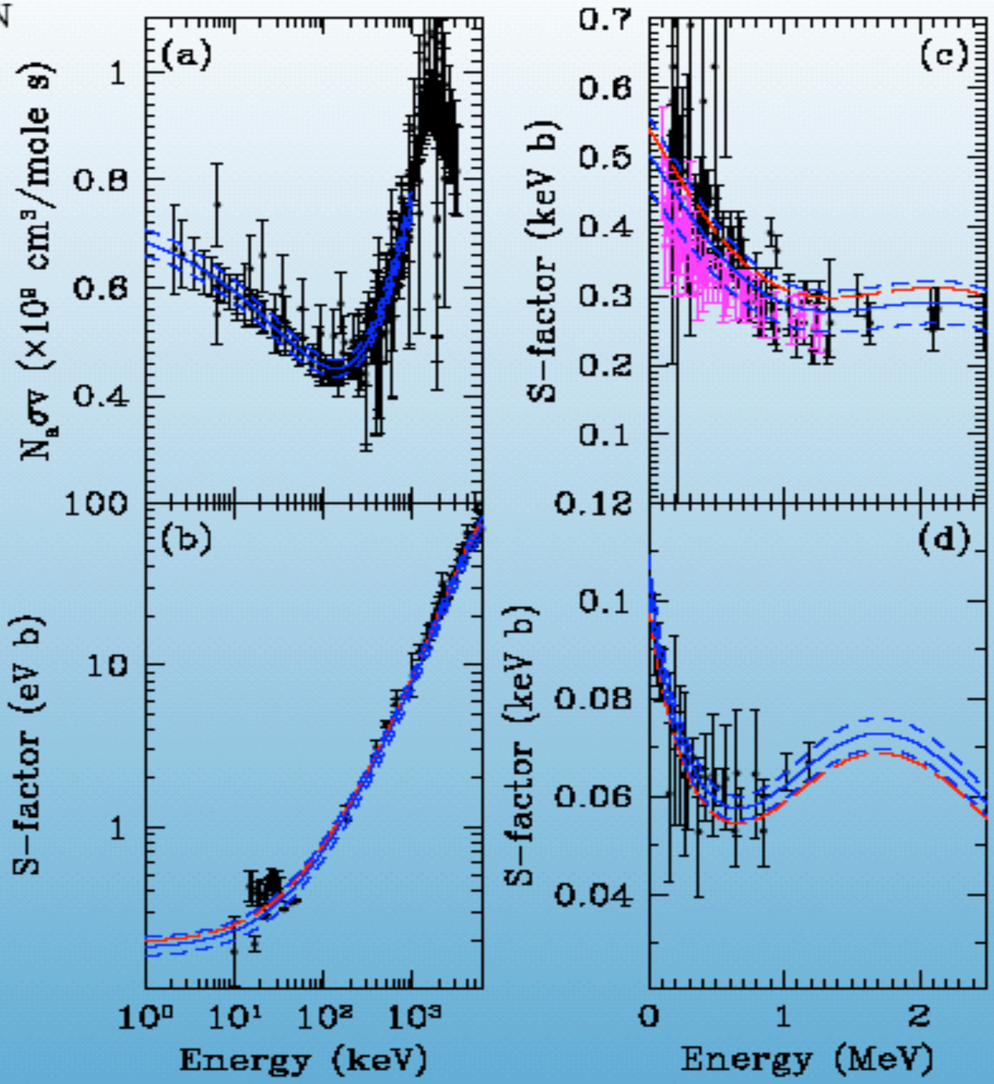
The neutron lifetime normalises the "weak" interaction rate: $\tau_n = 880.0 \pm 0.9$ s

(… has recently dropped in value by 6σ because of *one* new measurement!)

Table 1: Key Nuclear Reactions for BBN

Source	Reactions	
NACRE	$d(p,\gamma)^3$He	(b)
	$d(d,n)^3$He	
	$d(d,p)t$	
	$t(d,n)^4$He	
	$t(\alpha,\gamma)^7$Li	(d)
	^{3}He$(\alpha,\gamma)^7$Be	(c)
	^{7}Li$(p,\alpha)^4$He	
SKM	$p(n,\gamma)d$	
	^{3}He$(d,p)^4$He	
	^{7}Be$(n,p)^7$Li	
This work	^{3}He$(n,p)t$	(a)
PDG	τ_n	

NACRE
Cyburt, Fields, KAO
Nollett & Burles
Coc et al.



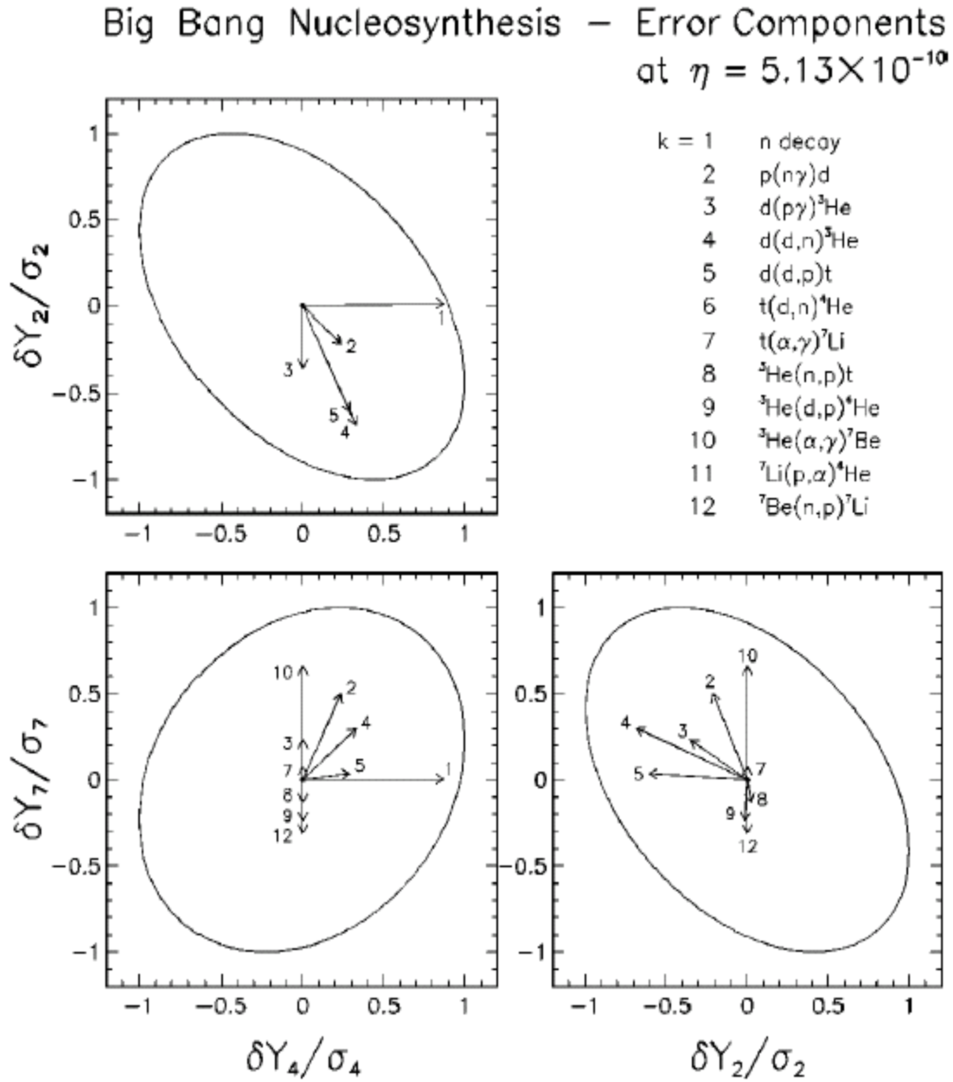
Uncertainties in synthesized abundances are *correlated* … estimate using Monte Carlo methods

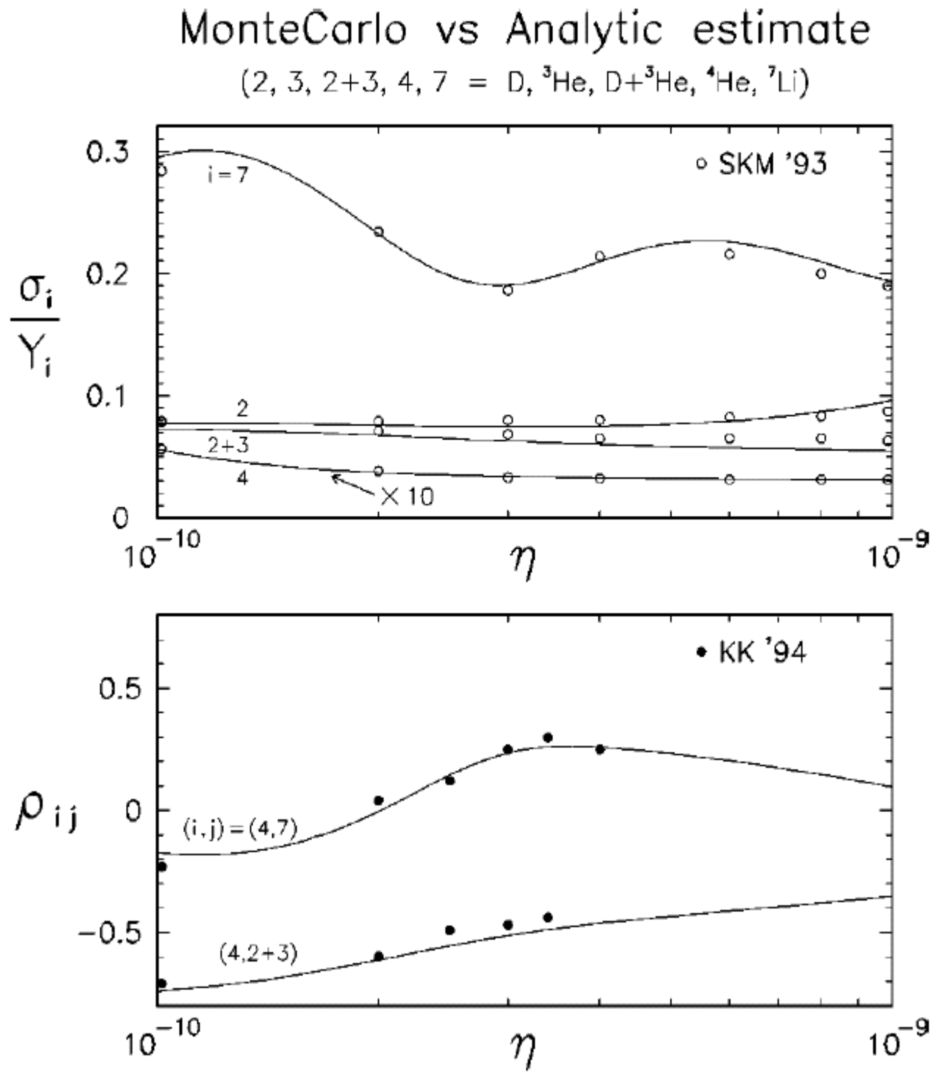
(Smith, Kawano, Malaney 1993; Krauss, Kernan 1994; Cyburt, Fields, Olive 2004)

Linear propagation of errors → **covariance matrix** (agrees with Monte Carlo results)

$$Y_i = Y_i(\eta) \pm \sigma_i(\eta) \;\Rightarrow\; \delta Y_i(\eta) = Y_i(\eta) \sum_k \lambda_{ik}(\eta) \frac{\delta R_k}{R_k}\,, \quad \lambda_{ik}(\eta) = \frac{\partial \ln Y_i(\eta)}{\partial \ln R_k(\eta)}$$

$$\sigma^2_{ij}(\eta) = Y_i(\eta) Y_j(\eta) \sum_k \lambda_{ik}(\eta) \lambda_{jk}(\eta) \left(\frac{\Delta R_k}{R_k}\right)^2 \;\Rightarrow\; \sigma_i(\eta) = \sqrt{\sigma^2_{ii}(\eta)}\,, \quad \rho_{ij}(\eta) = \frac{\sigma^2_{ij}(\eta)}{\sigma_i(\eta)\sigma_j(\eta)}$$

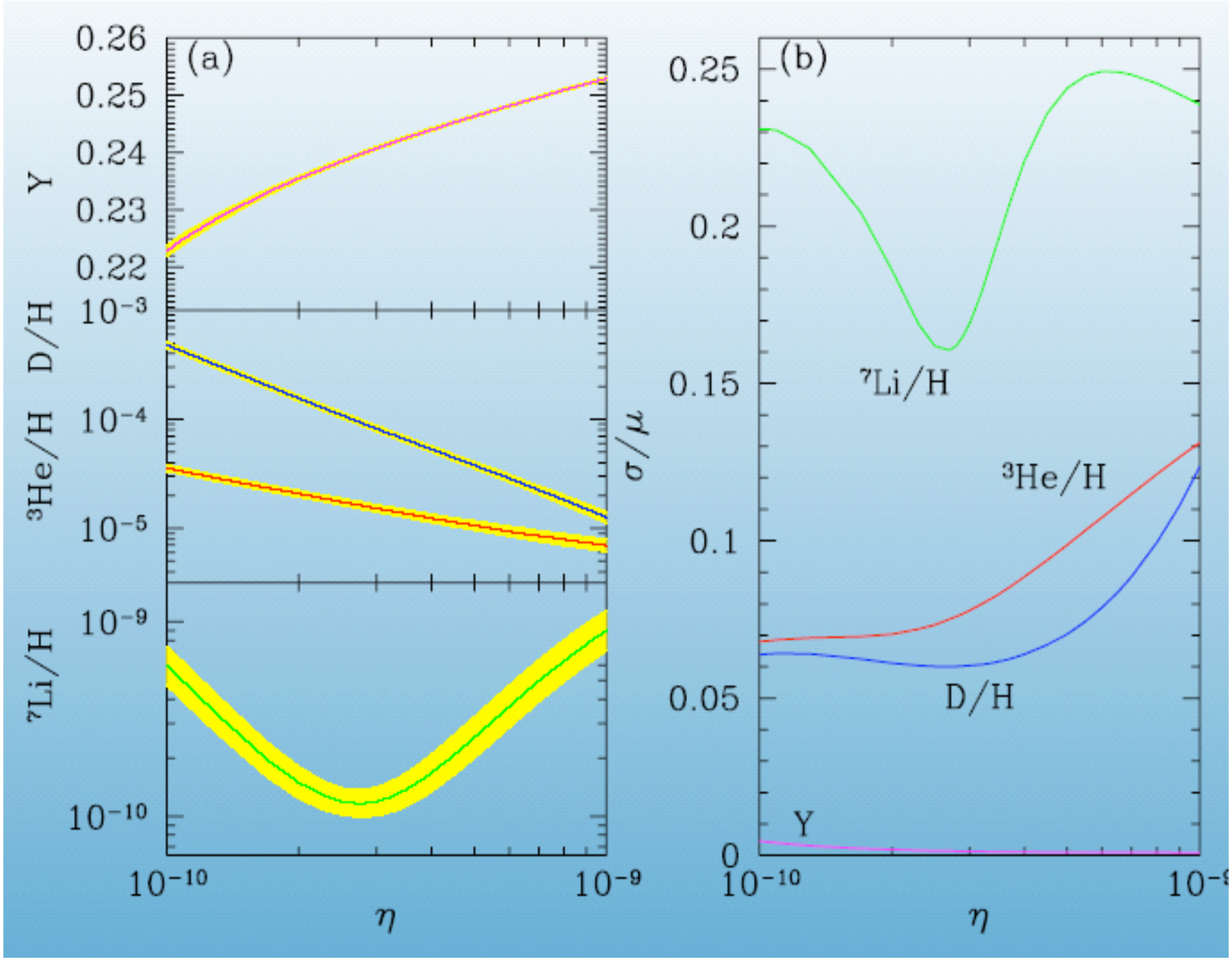




Fiorentini, Lisi, Sarkar, Villante, Phys.Rev.D58:063506,1998

BBN Predictions

line widths ⇒ theoretical uncertainties (neutron lifetime, nuclear cross sections)



Nucleosynthesis *without* a computer

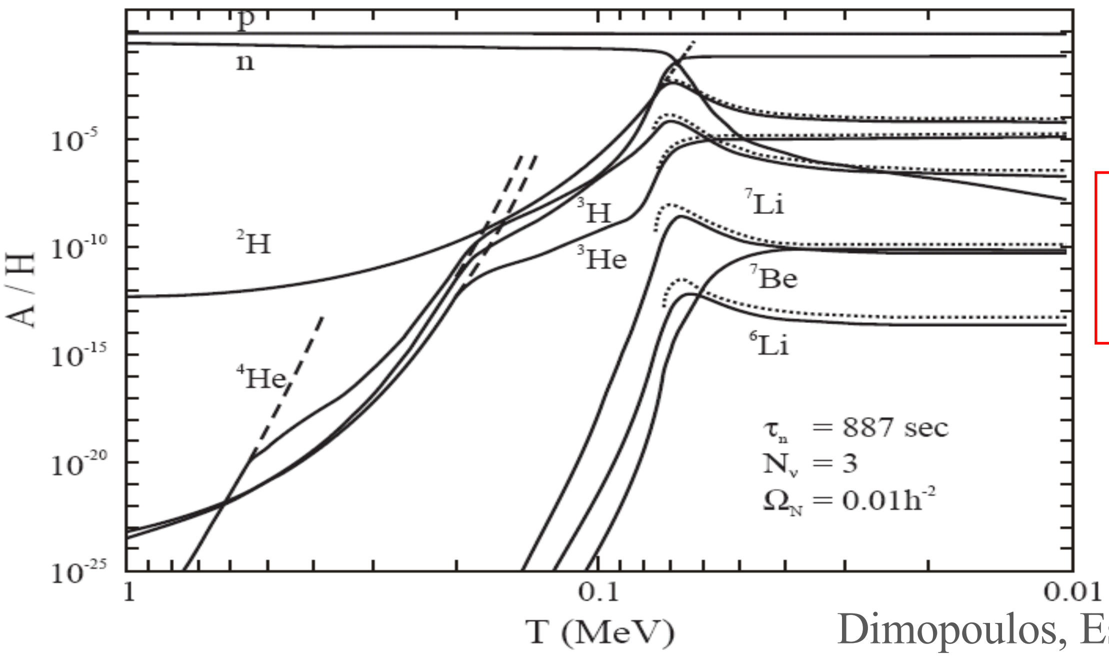
$$\frac{\mathrm{d}X}{\mathrm{d}t} = \underbrace{J(t)}_{\text{source}} - \underbrace{\Gamma(t)X}_{\text{sink}} \quad \Longrightarrow \quad X^{\mathrm{eq}} = \frac{J(t)}{\Gamma(t)}$$

… but general solution is:

$$X(t) = \exp\left(-\int_{t_\mathrm{i}}^{t} \mathrm{d}t'\ \Gamma(t')\right)\left[X(t_\mathrm{i}) + \int_{t_\mathrm{i}}^{t} \mathrm{d}t'\ J(t')\ \exp\left(-\int_{t_\mathrm{i}}^{t} \mathrm{d}t''\ \Gamma(t'')\right)\right]$$

If $\left|\frac{\dot{J}}{J} - \frac{\dot{\Gamma}}{\Gamma}\right| \ll \Gamma$ … then abundances approach equilibrium values

Freeze-out occurs when: $\Gamma \simeq H \quad \Longrightarrow \quad X(t \to \infty) \simeq X^{\mathrm{eq}}(t_{\mathrm{fr}}) = \frac{J(t_{\mathrm{fr}})}{\Gamma(t_{\mathrm{fr}})}$



Dimopoulos, Esmailzadeh, Hall, Starkman, ApJ **378**:504,1991

……….analytic solution

Examine reaction network to identify the largest 'source' and 'sink' terms

obtain D, ^{3}He and ^{7}Li to within a factor of 2 of exact numerical solution, and ^{4}He to within a few %

… can use this formalism to determine *joint* dependence of abundances on expansion rate as well as baryon-to-photon ratio

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} \propto \eta \sum_{+,-} Y \times Y \times \langle \sigma v \rangle_T \quad \text{and} \quad dT/dt \propto -T^3 \sqrt{g_\star} \quad \text{so:}$$

$$\frac{\mathrm{d}Y_i}{\mathrm{d}T} \propto -\frac{\eta}{g_\star^{1/2}} T^{-3} \sum_{+,-} Y \times Y \times \langle \sigma v \rangle_T \Rightarrow \log \eta - \frac{1}{2} \log g_\star = \mathrm{const}$$

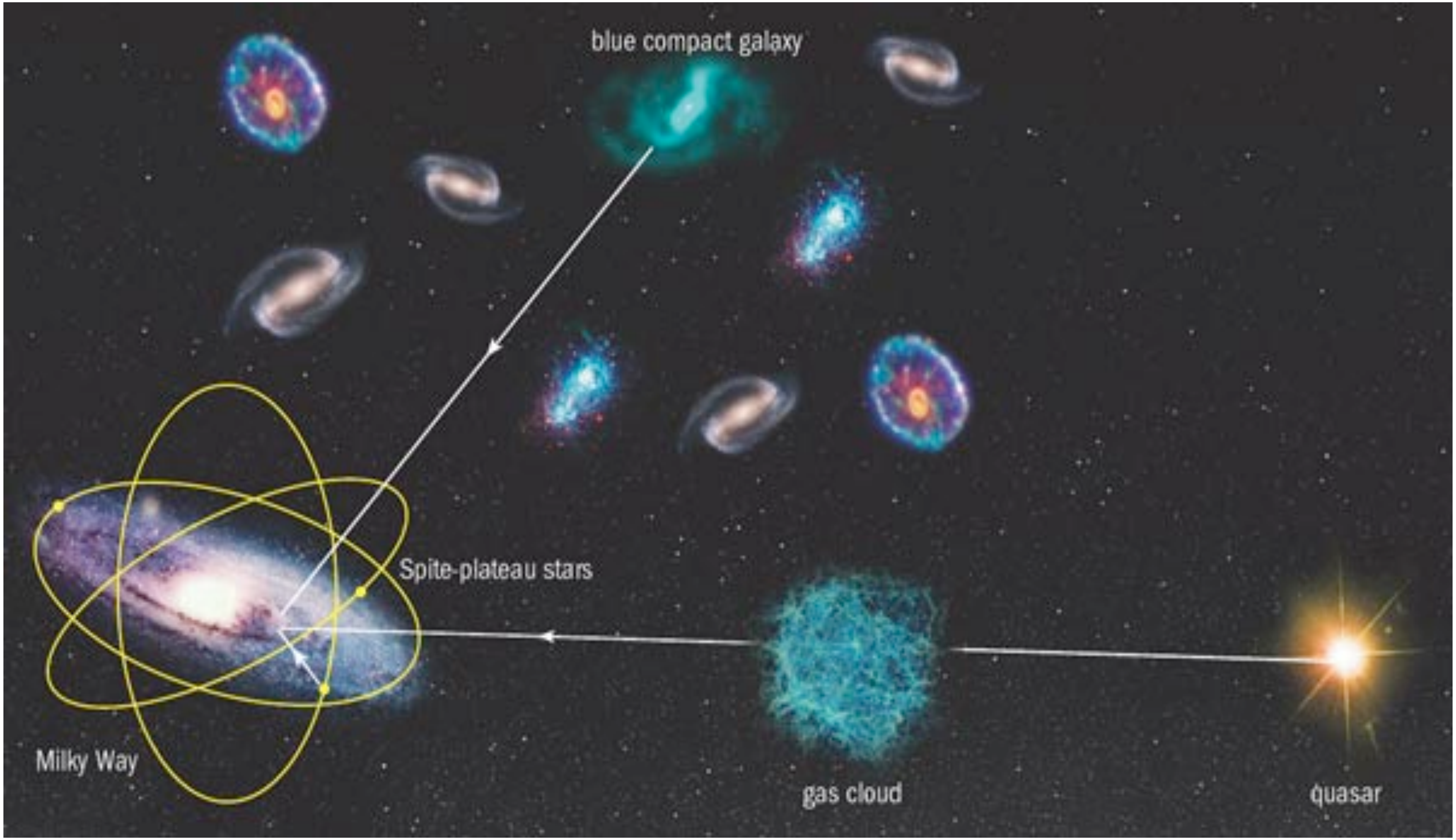
… can therefore employ simple χ^2 statistics to determine best-fit values and uncertainties (*faster* than Monte Carlo + Maximum Likelihood)

$$S_{ij}^2(\eta) = \sigma_{ij}^2(\eta) + \overline{\sigma_{ij}^2} \qquad \overline{\sigma_{ij}^2} = \delta_{ij} \overline{\sigma_i} \overline{\sigma_j} \qquad W_{ij}(\eta) = [S_{ij}^2(\eta)]^{-1}$$

$$\chi^2(\eta) = \sum_{ij} [Y_i(\eta) - \overline{Y_i}] W_{ij}(\eta) [Y_j(\eta) - \overline{Y_j}]$$

Lisi, Sarkar, Villante, Phys.Rev.D**59**:123520,1999

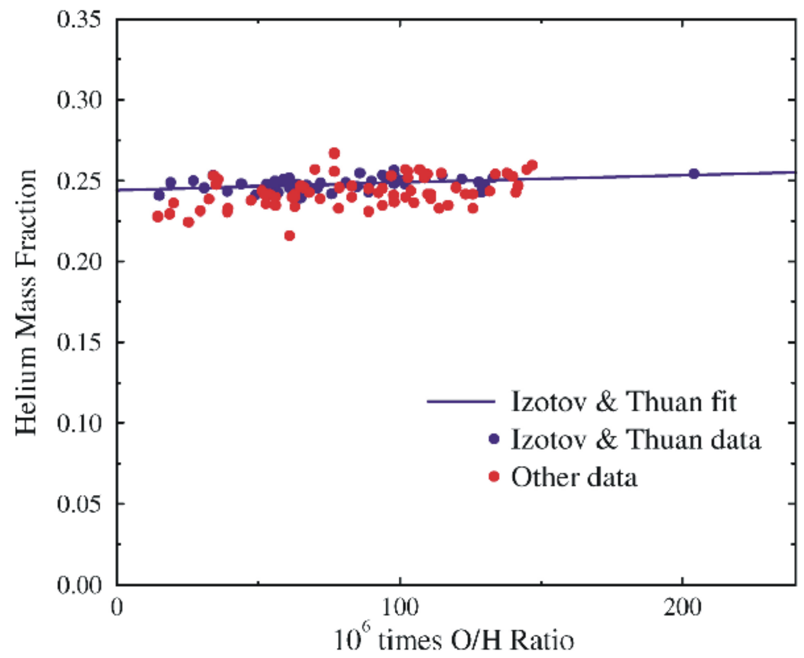
Inferring primordial abundances



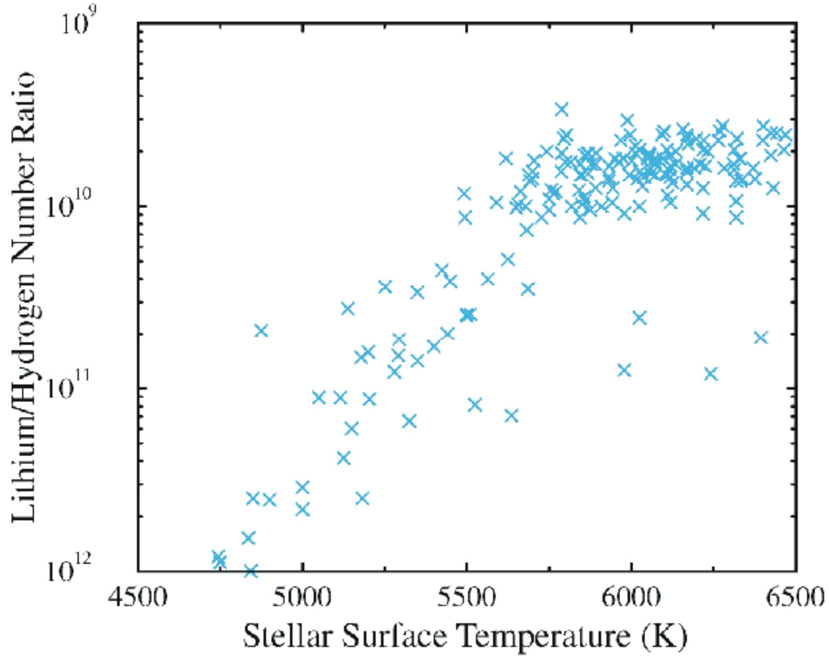
Observations of the light elements He and Li

- **Helium Abundance**
  - measured in extragalactic HII regions with lowest observed abundances of heavier elements such as Oxygen and Nitrogen

  (i.e. smallest levels of contamination from stellar nucleosynthesis)



- **Lithium Abundance**
  - measured in halo Pop II stars
  - Lithium is easily destroyed hence observe the transition from low mass stars (low surface temp) whose core material is well mixed by convection, to higher mass stars (higher surface temp) where mixing of core is not efficient



Look in **Q**uasar **A**bsorption **S**ystems - low density clouds of gas seen in absorption along the lines of sight to distant quasars (when universe was only ~10% of its present age)

The difference between H and D nuclei causes a *small* change in the energies of electron transitions, shifting their absorption lines apart and enabling D/H to be measured

$$E_{\text{Ly}-\alpha} \sim \alpha^2 \mu_{\text{reduced}}$$

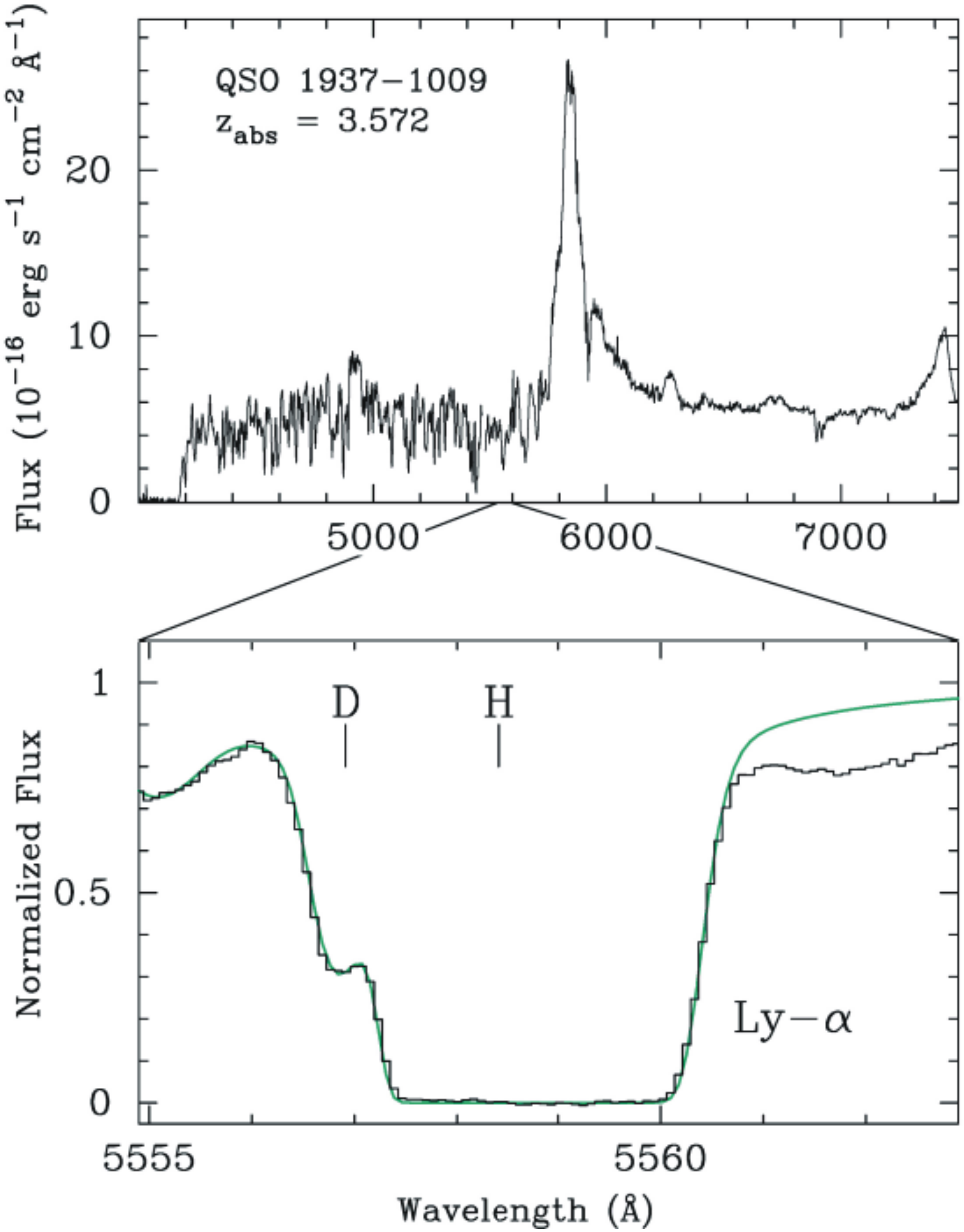
$$\frac{\delta\lambda_{\text{D}}}{\lambda_{\text{H}}} = -\frac{\delta\mu_{\text{D}}}{\mu_{\text{H}}} = -\frac{m_e}{2m_p}$$

$$c\delta z = 82 \text{ km/s}$$

But:

- Hard to find clean systems
- Do not resolve clouds
- Dispersion/systematics?

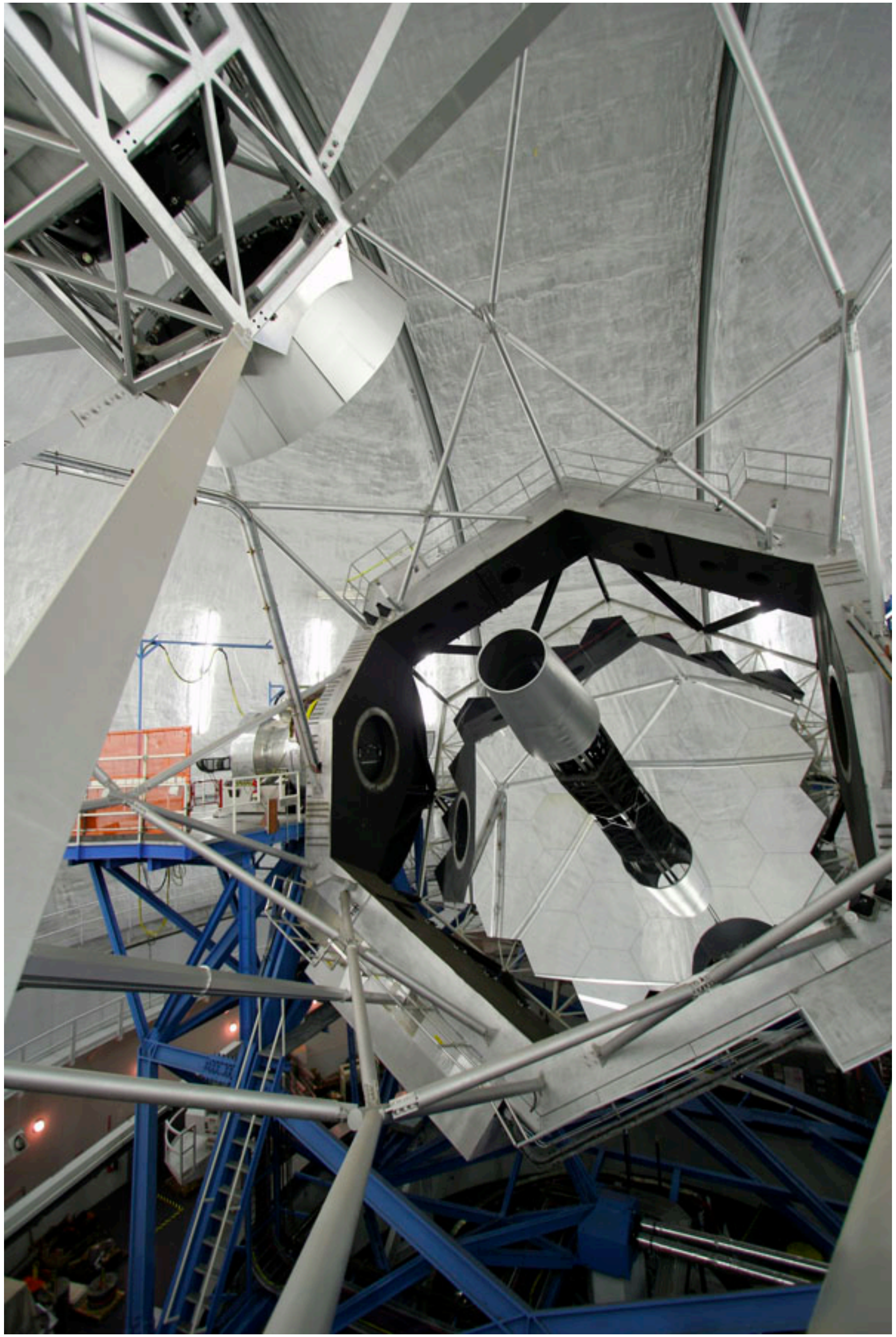
Primordial deuterium?



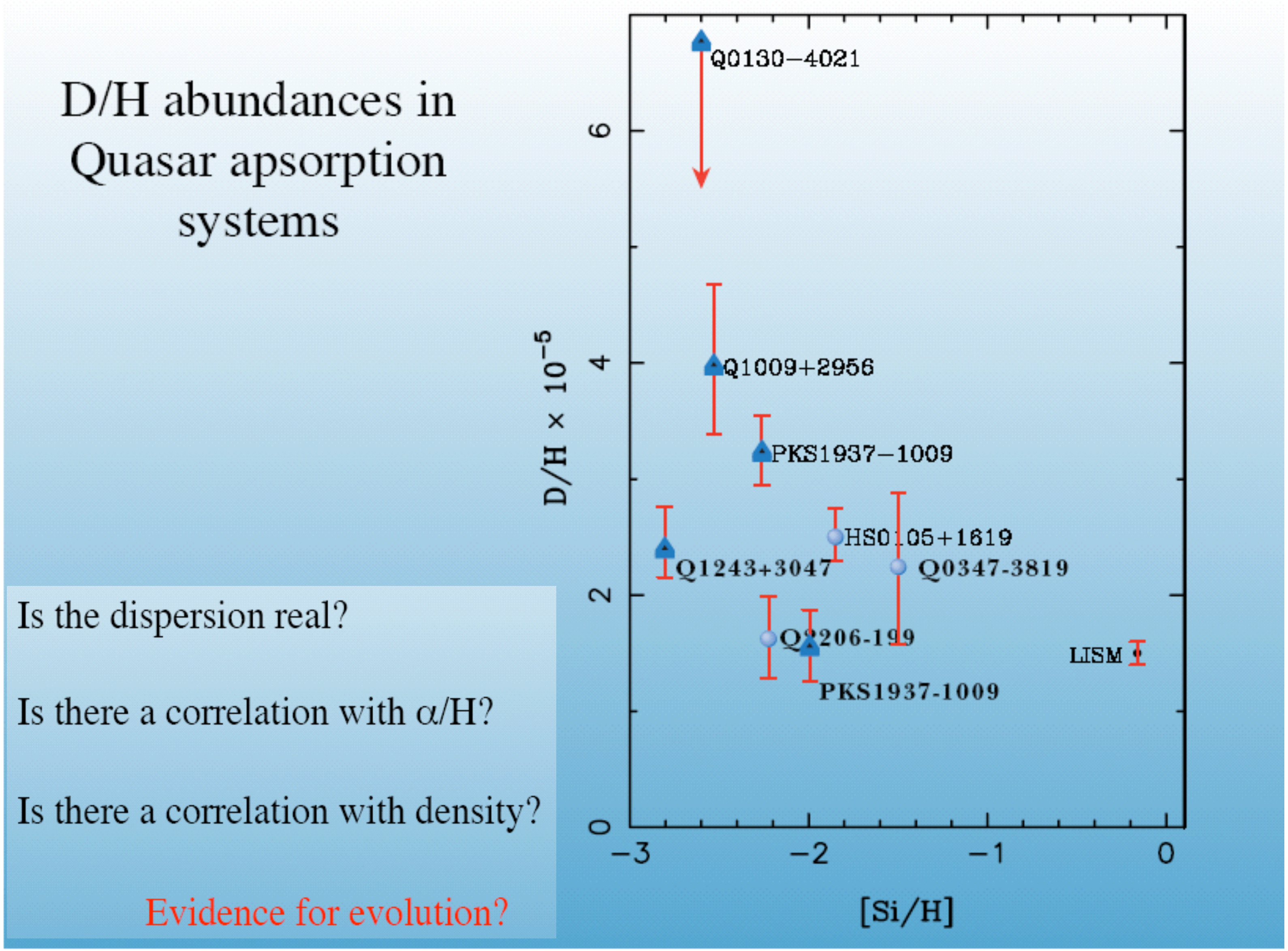
Kirkman, Tytler, Suzuki, O'Meara, Lubin, ApJS **149**:1,2003

W. M. Keck Observatory

Spectra with the necessary resolution for such distant objects *can* be obtained with 10m class telescopes … this has revolutionised the determination of the primordial D abundance



The observed scatter is *not* consistent with fluctuations about an average value!

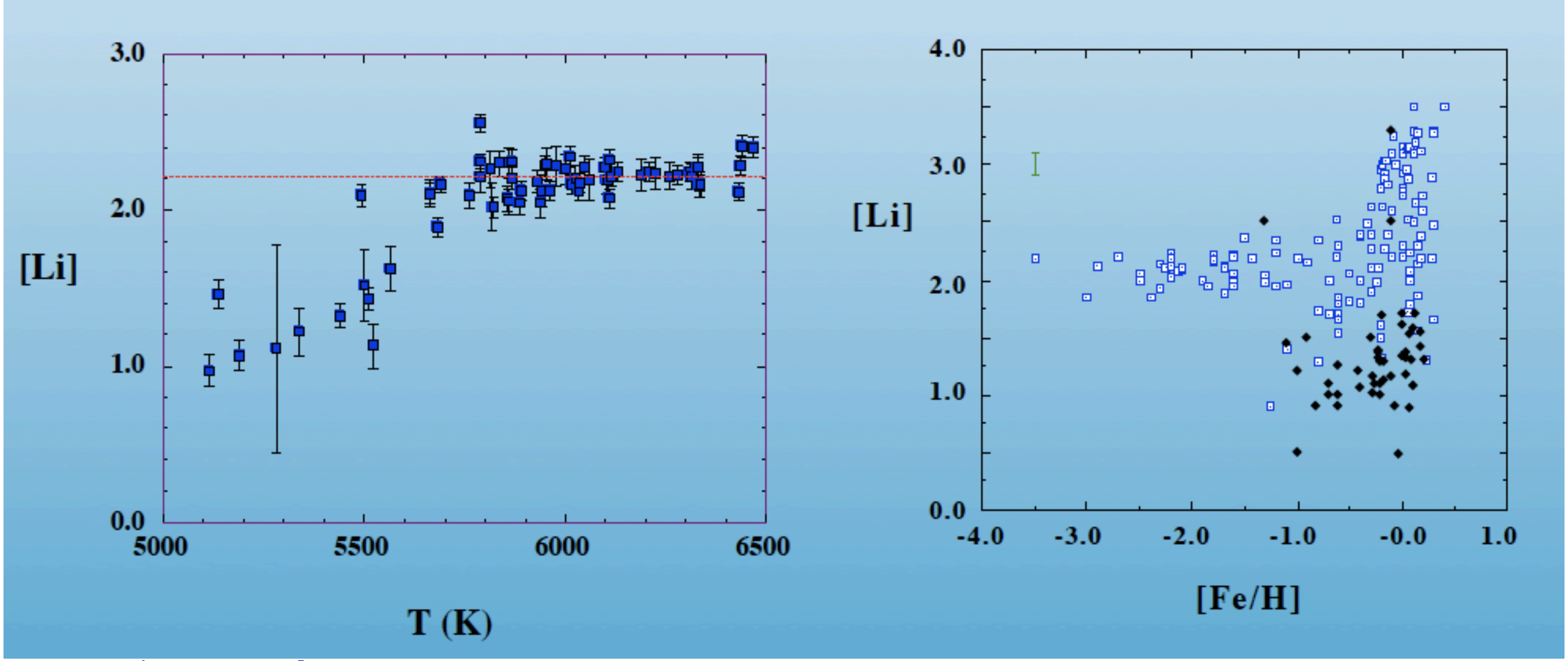


Progress made by looking at 'damped Ly-α' systems in which the H column density can be precisely measured and *many* resolved D I absorption lines are seen – leading to a precise determination of $\log(D/H) = -4.597 \pm 0.006$ (Cooke & Pettini, MNRAS **425**:1244,2012)

Primordial Lithium

Observe in primitive (Pop II) stars: (most abundant isotope is ^{7}Li)

- Li-Fe correlation ⇒ mild evolution
- Transition from low mass/surface temp stars (core well mixed by convection) to higher mass/temp stars (mixing of core is not efficient)



'Plateau' at low Fe (high T) ⇒ constant abundance at early epochs

… so *infer* observed '^{7}Li plateau' is primordial (Spite & Spite 1982)

Inferred primordial abundances

^{4}He observed in extragalactic HII regions:

$$Y_P = 0.2465 \pm 0.0097$$

^{2}H observed in quasar absorption systems (and ISM):

$$D/H|_P = (2.53 \pm 0.04) \times 10^{-5}$$

^{7}Li observed in atmospheres of dwarf halo stars:

$$Li/H|_P = (1.6 \pm 0.3) \times 10^{-10}$$

(^{3}He can be both created & destroyed in stars … so primordial abundance *cannot* be reliably estimated)

Systematic errors have been re-evaluated based on scatter in data
(see Particle Data Group, Chinese.Phys.C38:09001,2014)

BBN versus CMB

η_{BBN} is in agreement with η_{CMB}
allowing for large uncertainties in the
inferred elemental abundances

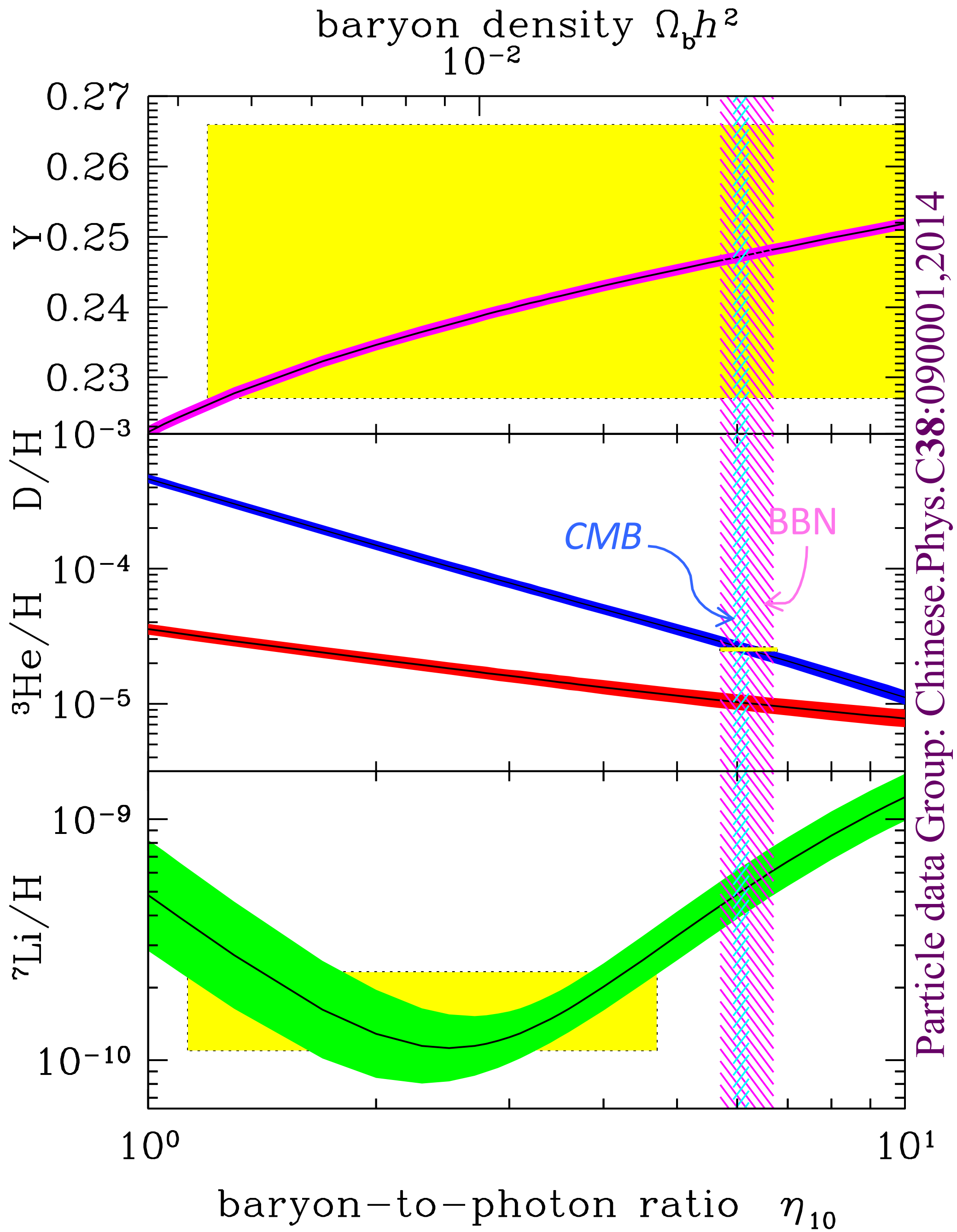
$5.7 < \eta_{10} < 6.7$ (95% CL)

Confirms and sharpens the case
for **(two kinds of) dark matter**

Baryonic Dark Matter:
warm-hot IGM, Ly-α , X-ray gas …
+
Non-baryonic dark matter: ?

Constrains the Hubble expansion rate
at $t \sim 1$ s $\Rightarrow$ bounds on new particles

There is a "lithium problem" *possibly*
indicative of non-standard physics



The Cosmic Microwave Background

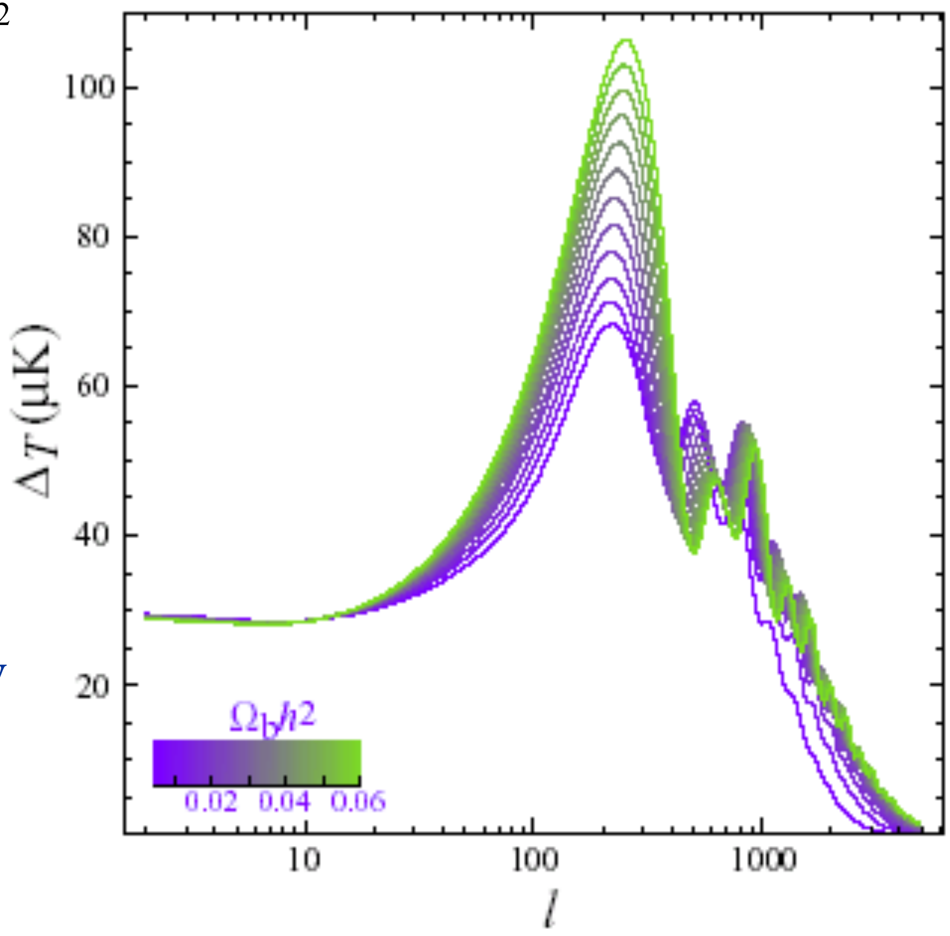
ΔT_ℓ provide *independent* measure of $\Omega_B h^2$

Acoustic oscillations in (coupled) photon-baryon fluids imprint features at small angles ($< 1^o$) in angular power spectrum

Detailed peak positions, heights, …
sensitive to cosmological parameters
e.g. 2nd/1st peak ratio ⇒ baryon density

e.g. WMAP-5 best-fit:

$$\Omega_B h^2 = 0.02273 \pm 0.00062$$

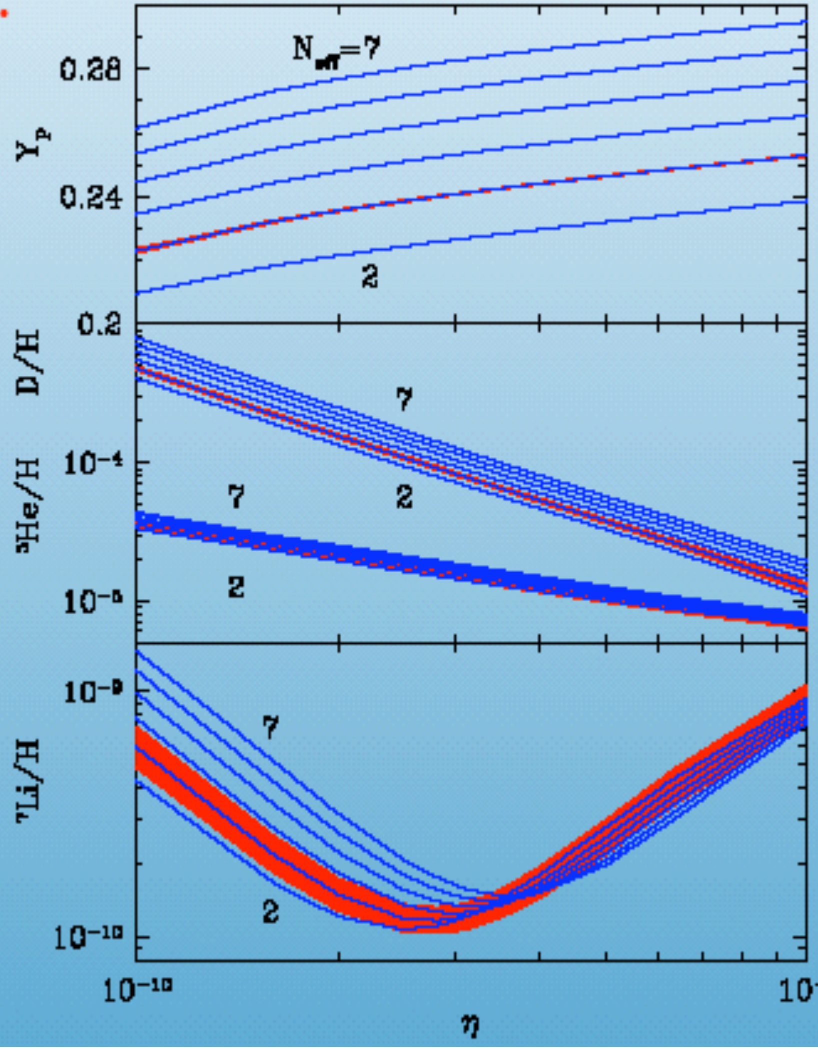


Bond & Efstathiou, ApJ **285**:L45,1984
Dodelson & Hu, ARAA **40**:171,2002

Limits on Particle Properties

- BBN Concordance rests on balance between interaction rates and expansion rate.

- Allows one to set constraints on:
  - Particle Types
  - Particle Interactions
  - Particle Masses
  - Fundamental Parameters



Courtesy: Keith Olive

Constraints from balance of weak rates vs Hubble rate

$$G_F^2 T^5 \sim \Gamma(T_f) \sim H(T_f) \sim \sqrt{G_N N} T_f^2$$

through He abunance

$\frac{n}{p} \sim e^{-\Delta m/T}$ fixed at freezeout $Y \sim \frac{2(n/p)}{1+(n/p)}$

Sets constraints on G_F, G_N, N, etc.

Note *n-p* mass difference is sensitive to both em and strong interactions, hence ^{4}He abundance is *exponentially* sensitive to *all* coupling strengths

Conversely obtain bound of less than few % on *any* additional contribution to energy density driving expansion e.g. gravitational waves, \`dark radiation', new particles …

E.g. rule out $\Lambda \sim H^2$ (since this just corresponds to a 'renormalisation' of G_N)

"Neutrino counting"

Light element abundances are sensitive to expansion history during BBN

$$H^2 \sim G\rho_{\mathrm{rel}}$$

$\Rightarrow$ observed values constrain the relativistic energy density at BBN

$$\rho_{\mathrm{rel}} \equiv \rho_{\mathrm{EM}} + N_{\nu,\mathrm{eff}}\, \rho_{\nu\bar{\nu}}$$

(Hoyle & Taylor 1964, Shvartsman 1969, Steigman, Schramm & Gunn 1977, …)

Pre-CMB:

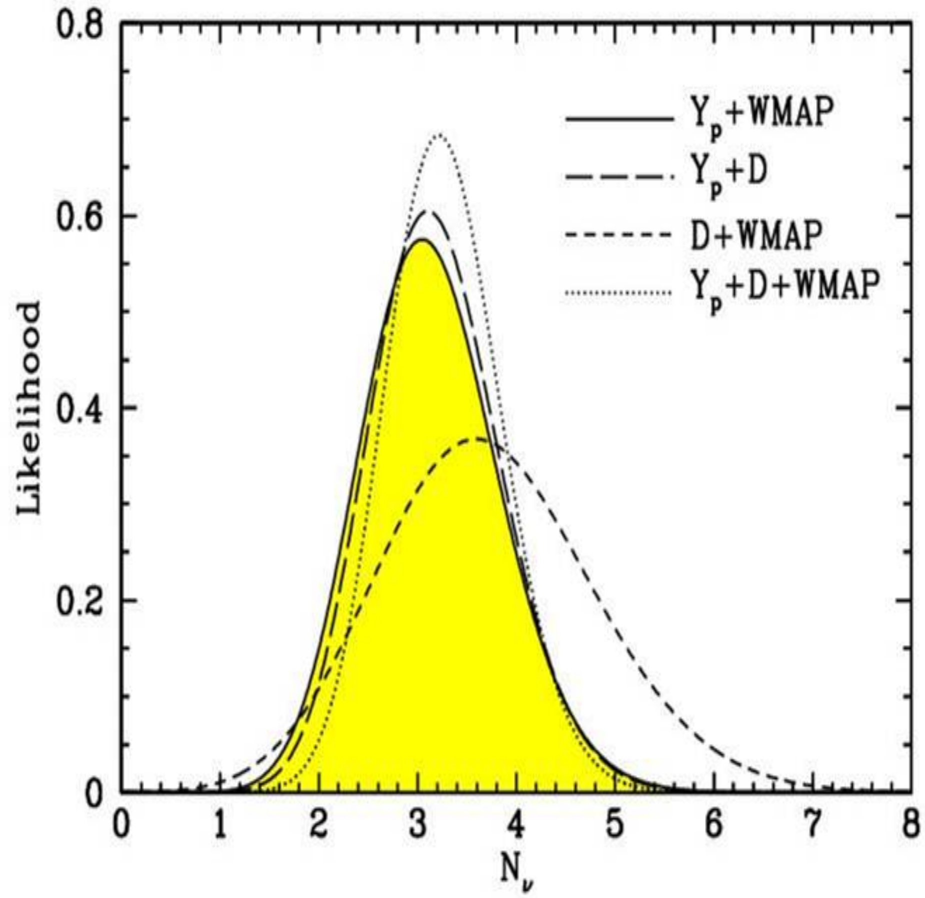
^{4}He as probe, other elements give η

With η from CMB:

All abundances can be used (*assuming* that η did not change between 1s and 10^5 yr)

$N_\nu = 3.28 \pm 0.28$

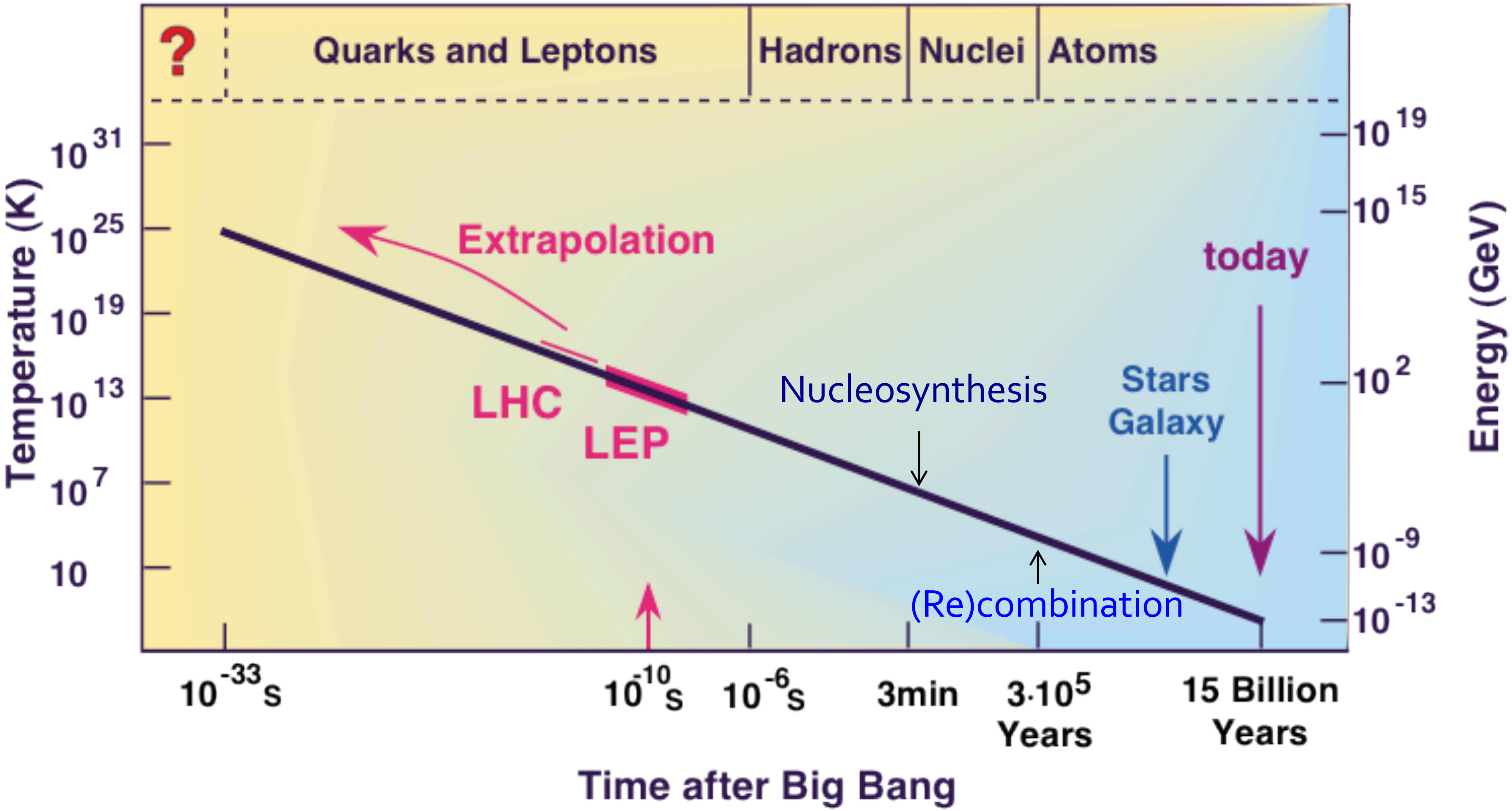
(Cooke *et al*, *ApJ* **781**:31,2014)



Cyburt, Fields, Olive, Skillman, AP 23:313,2005

This constrains sterile neutrinos (and other hypothetical particles) which do *not* couple to the Z^0 … *complementary* to laboratory bounds e.g from LEP

The blackbody temperature can be used as a clock (assuming adiabatic expansion: aT = constant), so our thermal history can be reconstructed



The furthest we 'see' directly is back to $t \sim 1$ s when light elements were synthesised
(but the baryon asymmetry, dark matter and fluctuations were generated *much* earlier)

Addressing the ‘big questions’

We have today a ‘standard’ model of both particle physics and cosmology which allows us to extrapolate back from the present day to the very first moments following the Big Bang

While successful in accounting for a wide range of observations, this has raised a new set of more fundamental questions concerning the universe

- ◆ *The origin of the baryon asymmetry*
- ◆ *The nature and origin of dark matter*
- ◆ *The origin of the primordial density perturbations that seeded structure*
- ◆ *The nature and origin of dark energy*

\+

- ➢ The initial singularity problem
- ➢ The cosmological constant problem
- ➢ The origin of space-time,
- ➢ …