Electroweak symmetry breaking | fermion masses

- We need masses for quarks and leptons without breaking gauge symmetry

$\Rightarrow$ Introduce Yukawa interactions:

$$\mathcal{L}_\mathrm{Y} = -\lambda_d \begin{pmatrix}\overline{u}_L & \overline{d}_L\end{pmatrix} \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} d_R - \lambda_u \begin{pmatrix}\overline{u}_L & \overline{d}_L\end{pmatrix} \begin{pmatrix}\phi^{0*} \\ -\phi^-\end{pmatrix} u_R$$

$$- \lambda_\ell \begin{pmatrix}\overline{\nu}_L & \overline{\ell}_L\end{pmatrix} \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \ell_R - \lambda_\nu \begin{pmatrix}\overline{\nu}_L & \overline{\ell}_L\end{pmatrix} \begin{pmatrix}\phi^{0*} \\ -\phi^-\end{pmatrix} \nu_R \quad + \mathrm{h.c.}$$

where $\quad \Phi^c \equiv i\sigma_2\Phi^* = \begin{pmatrix}\phi^{0*} \\ -\phi^-\end{pmatrix}$ transforms under SU(2) like $\quad \Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$

$\Rightarrow$ After EW SSB, fermions acquire masses ($\overline{f}f = \overline{f_L}f_R + \overline{f_R}f_L$):

$$\mathcal{L}_\mathrm{Y} \supset -\frac{1}{\sqrt{2}}(v+H)\left\{\lambda_d\,\overline{d}d + \lambda_u\,\overline{u}u + \lambda_\ell\,\overline{\ell}\ell + \lambda_\nu\,\overline{\nu}\nu\right\} \quad \Rightarrow \quad m_f = \lambda_f \frac{v}{\sqrt{2}}$$

Additional generations — Yukawa matrices

- There are 3 generations of quarks and leptons in Nature. They are identical copies with the same properties under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ differing only in their masses

$\Rightarrow$ Take a general case of n_G generations and let u_i^I, d_i^I, ν_i^I, ℓ_i^I be the members of family i ($i = 1, \ldots, n_G$). Superindex I (interaction basis) was omitted so far

$\Rightarrow$ General gauge invariant Yukawa Lagrangian:

$$\mathcal{L}_{\mathrm{Y}} = -\sum_{ij} \left\{ \begin{pmatrix} \overline{u}_{iL}^I & \overline{d}_{iL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{ij}^{(d)}\, d_{jR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{ij}^{(u)}\, u_{jR}^I \right] \right.$$
$$\left. + \begin{pmatrix} \overline{\nu}_{iL}^I & \overline{\ell}_{iL}^I \end{pmatrix} \left[\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \lambda_{ij}^{(\ell)}\, \ell_{jR}^I + \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \lambda_{ij}^{(\nu)}\, \nu_{jR}^I \right] \right\} + \text{h.c.}$$

where $\lambda_{ij}^{(d)}, \lambda_{ij}^{(u)}, \lambda_{ij}^{(\ell)}, \lambda_{ij}^{(\nu)}$ are arbitrary Yukawa matrices

Additional generations | mass matrices

- After EW SSB, in n_G-dimensional matrix form:

$$\mathcal{L}_{\rm Y} \supset -\left(1+\frac{H}{v}\right)\left\{\overline{\mathbf{d}}^I_L\,\mathbf{M}_d\,\mathbf{d}^I_R + \overline{\mathbf{u}}^I_L\,\mathbf{M}_u\,\mathbf{u}^I_R + \overline{\mathbf{l}}^I_L\,\mathbf{M}_\ell\,\mathbf{l}^I_R + \overline{\boldsymbol{\nu}}^I_L\,\mathbf{M}_\nu\,\boldsymbol{\nu}^I_R + \text{h.c.}\right\}$$

with mass matrices

$$(\mathbf{M}_d)_{ij} \equiv \lambda^{(d)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_u)_{ij} \equiv \lambda^{(u)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\ell)_{ij} \equiv \lambda^{(\ell)}_{ij}\frac{v}{\sqrt{2}} \quad (\mathbf{M}_\nu)_{ij} \equiv \lambda^{(\nu)}_{ij}\frac{v}{\sqrt{2}}$$

$\Rightarrow$ Diagonalization determines mass eigenstates d_j, u_j, ℓ_j, ν_j in terms of interaction states d^I_j, u^I_j, ℓ^I_j, ν^I_j, respectively

$\Rightarrow$ Each $\mathbf{M}_f$ can be written as

$$\mathbf{M}_f = \mathbf{H}_f\,\mathcal{U}_f = \mathbf{S}^\dagger_f\,\mathcal{M}_f\,\mathbf{S}_f\,\mathcal{U}_f \quad \Longleftrightarrow \quad \mathbf{M}_f\mathbf{M}^\dagger_f = \mathbf{H}^2_f = \mathbf{S}^\dagger_f\,\mathcal{M}^2_f\,\mathbf{S}_f$$

with $\mathbf{H}_f \equiv \sqrt{\mathbf{M}_f\mathbf{M}^\dagger_f}$ a Hermitian positive definite matrix and $\mathcal{U}_f$ unitary

– Every $\mathbf{H}_f$ can be diagonalized by a unitary matrix $\mathbf{S}_f$

– The resulting $\mathcal{M}_f$ is diagonal and positive definite

Additional generations **fermion masses and mixings**

- In terms of diagonal mass matrices (mass eigenstate basis):

$$\mathcal{M}_d = \text{diag}(m_d, m_s, m_b, \ldots)\ , \quad \mathcal{M}_u = \text{diag}(m_u, m_c, m_t, \ldots)$$
$$\mathcal{M}_\ell = \text{diag}(m_e, m_\mu, m_\tau, \ldots)\ , \quad \mathcal{M}_\nu = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, \ldots)$$

$$\mathcal{L}_\text{Y} \supset -\left(1 + \frac{H}{v}\right)\left\{\overline{\mathbf{d}}\,\mathcal{M}_d\,\mathbf{d} + \overline{\mathbf{u}}\,\mathcal{M}_u\,\mathbf{u} + \overline{\mathbf{l}}\,\mathcal{M}_\ell\,\mathbf{l} + \overline{\boldsymbol{\nu}}\,\mathcal{M}_\nu\,\boldsymbol{\nu}\right\}$$

where fermion couplings to Higgs are proportional to masses and

$$\mathbf{d}_L \equiv \mathbf{S}_d\,\mathbf{d}_L^I \qquad \mathbf{u}_L \equiv \mathbf{S}_u\,\mathbf{u}_L^I \qquad \mathbf{l}_L \equiv \mathbf{S}_\ell\,\mathbf{l}_L^I \qquad \boldsymbol{\nu}_L \equiv \mathbf{S}_\nu\,\boldsymbol{\nu}_L^I$$
$$\mathbf{d}_R \equiv \mathbf{S}_d\mathcal{U}_d\,\mathbf{d}_R^I \qquad \mathbf{u}_R \equiv \mathbf{S}_u\mathcal{U}_u\,\mathbf{u}_R^I \qquad \mathbf{l}_R \equiv \mathbf{S}_\ell\mathcal{U}_\ell\,\mathbf{l}_R^I \qquad \boldsymbol{\nu}_R \equiv \mathbf{S}_\nu\mathcal{U}_\nu\,\boldsymbol{\nu}_R^I$$

$\Rightarrow$ Neutral Currents preserve chirality
$\overline{\mathbf{f}}_L^I\,\mathbf{f}_L^I = \overline{\mathbf{f}}_L\,\mathbf{f}_L$ and $\overline{\mathbf{f}}_R^I\,\mathbf{f}_R^I = \overline{\mathbf{f}}_R\,\mathbf{f}_R$
$\Bigg\}\Rightarrow \mathcal{L}_\text{NC}$ does not change flavor

$\Rightarrow$ GIM mechanism [Glashow, Iliopoulos, Maiani '70]

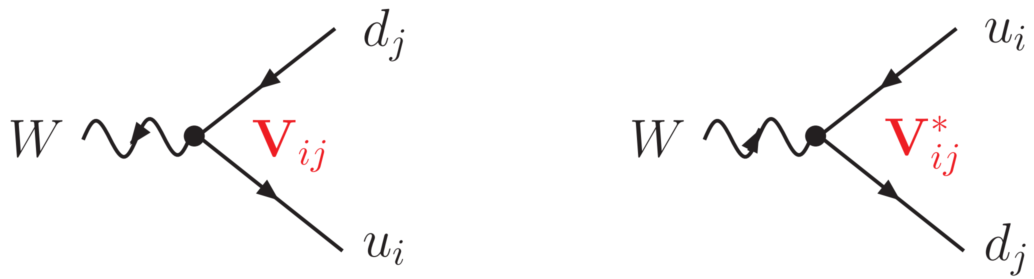
Additional generations — quark sector

- However, in Charged Currents (also chirality preserving and only LH):

$$\overline{\mathbf{u}}_L^I\, \mathbf{d}_L^I = \overline{\mathbf{u}}_L\, \mathbf{S}_u\, \mathbf{S}_d^\dagger\, \mathbf{d}_L = \overline{\mathbf{u}}_L \mathbf{V} \mathbf{d}_L$$

with $\mathbf{V} \equiv \mathbf{S}_u\, \mathbf{S}_d^\dagger$ the (unitary) CKM mixing matrix [Cabibbo '63; Kobayashi, Maskawa '73]

$$\Rightarrow \quad \mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} \sum_{ij} \overline{u}_i \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j\, W_\mu^\dagger + \text{h.c.}$$



$\Rightarrow$ If u_i or d_j had degenerate masses one could choose $\mathbf{S}_u = \mathbf{S}_d$ (field redefinition) and flavor would be conserved in the quark sector. But they are not degenerate

$\Rightarrow$ $\mathbf{S}_u$ and $\mathbf{S}_d$ are not observable. Just masses and CKM mixings are observable

Additional generations | quark sector

- How many physical parameters in this sector?
  - Quark masses and CKM mixings determined by mass (or Yukawa) matrices
  - A general $n_G \times n_G$ unitary matrix, like the CKM, is given by

$$n_G^2 \text{ real parameters} = n_G(n_G-1)/2 \text{ moduli} + n_G(n_G+1)/2 \text{ phases}$$

Some phases are unphysical since they can be absorbed by field redefinitions:

$$u_i \to \mathrm{e}^{\mathrm{i}\phi_i}\, u_i\,, \quad d_j \to \mathrm{e}^{\mathrm{i}\theta_j}\, d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to \mathbf{V}_{ij}\, \mathrm{e}^{\mathrm{i}(\theta_j-\phi_i)}$$

Therefore $2n_G - 1$ unphysical phases and the physical parameters are:

$$\boxed{(n_G-1)^2 = n_G(n_G-1)/2 \text{ moduli } + (n_G-1)(n_G-2)/2 \text{ phases}}$$

Additional generations — quark sector

⇒ Case of $n_G = 2$ generations: 1 parameter, the Cabibbo angle θ_C:

$$\mathbf{V} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

⇒ Case of $n_G = 3$ generations: 3 angles + 1 phase. In the standard parameterization:

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_{ud} & \mathbf{V}_{us} & \mathbf{V}_{ub} \\ \mathbf{V}_{cd} & \mathbf{V}_{cs} & \mathbf{V}_{cb} \\ \mathbf{V}_{td} & \mathbf{V}_{ts} & \mathbf{V}_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

⇒ δ_{13} only source of CP violation in the SM !

with $c_{ij} \equiv \cos\theta_{ij} \geq 0$, $\quad s_{ij} \equiv \sin\theta_{ij} \geq 0 \quad (i < j = 1, 2, 3)$ and $0 \leq \delta_{13} \leq 2\pi$

Additional generations — lepton sector

- If neutrinos were massless we could redefine the (LH) fields $\Rightarrow$ no lepton mixing
  But they have (tiny) masses because there are neutrino oscillations!

- Neutrinos are special:
  they *may* be their own antiparticle (Majorana) since they are neutral

- *If* they are Majorana:
  - Mass terms are different to Dirac case
    (neutrino and antineutrino *may* mix)
  - Intergenerational mixings are richer (more CP phases)

⋆ lepton sector

- About Majorana fermions

  – A Dirac fermion field is a spinor with 4 independent components: 2 LH+2 RH (left/right-handed particles and antiparticles)

  $$\psi_L = P_L\psi\ , \quad \psi_R = P_R\psi\ , \quad \psi_L^c \equiv (\psi_L)^c = P_R\psi^c\ , \quad \psi_R^c \equiv (\psi_R)^c = P_L\psi^c$$

  where $\psi^c \equiv C\overline{\psi}^{\mathsf{T}} = \mathrm{i}\gamma^2\psi^*$ (charge conjugate) with $C = \mathrm{i}\gamma^2\gamma^0$, $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$

  – A Majorana fermion field has just 2 independent components since $\psi^c \equiv \eta^*\psi$:

  $$\psi_L = \eta\psi_R^c\ , \quad \psi_R = \eta\psi_L^c$$

  where $\eta = -\mathrm{i}\eta_{\mathrm{CP}}$ (CP parity) with $|\eta|^2 = 1$. Only possible if neutral

⋆ lepton sector

- About mass terms

$$\overline{\psi_R}\psi_L = \overline{\psi_L^c}\psi_R^c \quad , \quad \overline{\psi_L}\psi_R = \overline{\psi_R^c}\psi_L^c \qquad (\Delta F = 0)$$

$$\left.\begin{array}{lcl} \overline{\psi_L^c}\psi_L & , & \overline{\psi_L}\psi_L^c \\ \overline{\psi_R}\psi_R^c & , & \overline{\psi_R^c}\psi_R \end{array}\right\} \qquad (|\Delta F| = 2)$$

$$\Rightarrow \quad -\mathcal{L}_m = \underbrace{m_D\,\overline{\psi_R}\psi_L}_{\text{Dirac term}} + \underbrace{\frac{1}{2}m_L\,\overline{\psi_L^c}\psi_L + \frac{1}{2}m_R\,\overline{\psi_R}\psi_R^c}_{\text{Majorana terms}} \quad + \text{h.c.}$$

- A Dirac fermion can only have Dirac mass term
- A Majorana fermion can have both Dirac and Majorana mass terms

⇒ In the SM:
- ∗ m_D from Yukawa coupling after EW SSB $\qquad (m_D = \lambda_\nu\, v/\sqrt{2})$
- ∗ m_L forbidden by gauge symmetry
- ∗ m_R compatible with gauge symmetry!

⋆ lepton sector

- About mass terms (a more transparent parameterization)

  Rewrite previous mass terms introducing an array of two Majorana fermions:

$$\chi^0 = \chi^{0c} = \chi_L^0 + \chi_L^{0c} \equiv \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix} , \qquad \begin{array}{l} \chi_1^0 = \chi_1^{0c} = \chi_{1L}^0 + \chi_{1L}^{0c} \equiv \psi_L + \psi_L^c \\ \chi_2^0 = \chi_2^{0c} = \chi_{2L}^0 + \chi_{2L}^{0c} \equiv \psi_R^c + \psi_R \end{array}$$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2}\,\overline{\chi_L^{0c}}\,\mathbf{M}\,\chi_L^0 + \text{h.c.} \quad \text{with} \quad \mathbf{M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}$$

M is a square symmetric matrix ⇒ diagonalizable by a unitary matrix $\widetilde{\mathcal{U}}$:

$$\widetilde{\mathcal{U}}^{\mathsf{T}}\mathbf{M}\,\widetilde{\mathcal{U}} = \mathcal{M} = \text{diag}(m_1', m_2') \,, \quad \chi_L^0 = \widetilde{\mathcal{U}}\chi_L \quad (\chi_L^{0c} = \widetilde{\mathcal{U}}^*\chi_L^c)$$

To get real and positive eigenvalues $m_i = \eta_i m_i'$ (physical masses) take $\chi_L^0 = \mathcal{U}\xi_L$:

$$\mathcal{U} = \widetilde{\mathcal{U}}\text{diag}(\sqrt{\eta_1}, \sqrt{\eta_2}) \,, \qquad \begin{array}{l} \xi_1 = \chi_{1L} + \eta_1\chi_{1L}^c \\ \xi_2 = \chi_{2L} + \eta_2\chi_{2L}^c \end{array} \quad \text{(physical fields)} \quad \eta_i = \text{CP parities}$$

⋆ lepton sector

- About mass terms (a more transparent parameterization)
  - Case of only Dirac term ($m_L = m_R = 0$)

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix} \quad \Rightarrow \quad \widetilde{\mathcal{U}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} , \quad m_1' = -m_D \, , \quad m_2' = m_D$$

Eigenstates $\Rightarrow$ Physical states

$$\chi_{1L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} - \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L - \psi^c_R)$$

$$\chi_{2L} = \frac{1}{\sqrt{2}}(\chi^0_{1L} + \chi^0_{2L}) = \frac{1}{\sqrt{2}}(\psi_L + \psi^c_R)$$

$$\xi_1 = \chi_{1L} + \eta_1 \chi^c_{1L} \quad [\eta_1 = -1]$$

$$\xi_2 = \chi_{2L} + \eta_2 \chi^c_{2L} \quad [\eta_2 = +1]$$

with masses $m_1 = m_2 = m_D$

$$\Rightarrow \quad -\mathcal{L}_m = \frac{1}{2} m_D (-\overline{\chi}_1 \chi_1 + \overline{\chi}_2 \chi_2) = \frac{1}{2} m_D (\overline{\xi}_1 \xi_1 + \overline{\xi}_2 \xi_2) = m_D (\overline{\psi_L} \psi_R + \overline{\psi_R} \psi_L)$$

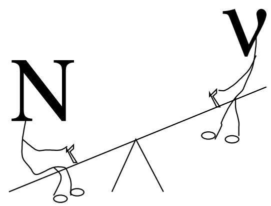
One Dirac fermion = two Majorana of equal mass and opposite CP parities

★ **lepton sector**

- About mass terms (a more transparent parameterization)
  - Case of seesaw (type I) [Yanagida '79; Gell-Mann, Ramond, Slansky '79; Mohapatra, Senjanovic '80]
    $(m_L = 0,\ m_D \ll m_R)$

$$\mathbf{M} = \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \Rightarrow \widetilde{\mathcal{U}} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \theta \simeq \frac{m_D}{m_R} \simeq \sqrt{\frac{m_\nu}{m_N}} \text{ (negligible)}$$

$$m_1 \equiv m_\nu \simeq \frac{m_D^2}{m_R} \ll m_2 \equiv m_N \simeq m_R$$



$$\begin{aligned} \xi_1 &\equiv \nu = \psi_L + \eta_1 \psi_L^c \quad [\eta_1 = -1] \\ \xi_2 &\equiv N = \psi_R^c + \eta_2 \psi_R \quad [\eta_2 = +1] \end{aligned} \Rightarrow -\mathcal{L}_m = \frac{1}{2} m_\nu \overline{\nu_L^c} \nu_L + \frac{1}{2} m_N \overline{N_R^c} N_R + \text{h.c.}$$

> *Perhaps* the observed neutrino ν_L is the LH component of a light Majorana ν (then $\overline{\nu}$ = RH) and light because of a very heavy Majorana neutrino N

e.g. $m_D \sim v \simeq 246$ GeV , $m_R \sim m_N \sim 10^{15}$ GeV $\Rightarrow$ $m_\nu \sim 0.1$ eV ✓

Additional generations **lepton sector**

- Lepton mixings
  - From Z lineshape: there are $n_G = 3$ generations of ν_L [ν_i $(i = 1, \ldots, n_G)$] (but we do not know (*yet*) if neutrinos are Dirac or Majorana fermions)
  - From neutrino oscillations: neutrinos are light, non degenerate and mix

$$|\nu_\alpha\rangle = \sum_i \mathbf{U}_{\alpha i}|\nu_i\rangle \quad \Longleftrightarrow \quad |\nu_i\rangle = \sum_\alpha \mathbf{U}^*_{\alpha i}|\nu_\alpha\rangle$$

mass eigenstates ν_i $(i = 1, 2, 3)$ / interaction states ν_α $(\alpha = e, \mu, \tau)$

⇒ **U** matrix is unitary (negligible mixing with heavy neutrinos) and analogous to $\mathbf{S}_u$, $\mathbf{S}_d$, $\mathbf{S}_\ell$ defined for quarks and charged leptons except for:

  - ν fields have both chiralities
  - *If* neutrinos are Majorana, **U** *may* contain two additional physical (Majorana) phases (irrelevant and therefore not measurable in oscillation experiments) that cannot be absorbed since then field phases are fixed by $\nu_i = \eta_i \nu_i^c$

Additional generations | lepton sector

- Lepton mixings

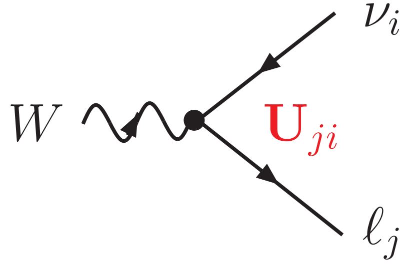
  The so called PMNS matrix **U** [Pontecorvo '57; Maki, Nakagawa, Sakata '62; Pontecorvo '68]

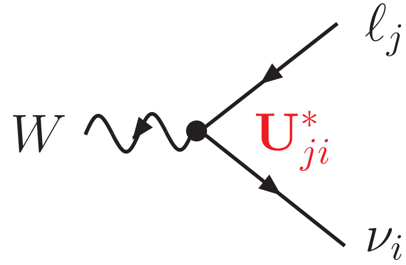
  – does not change Neutral Currents (unitarity), but

  – introduces intergenerational mixings in Charged Currents:

$$\mathcal{L}_{\rm CC} = \frac{g}{2\sqrt{2}} \sum_{\alpha i} \bar{\ell}_\alpha \, \gamma^\mu (1-\gamma_5) \, \mathbf{U}_{\alpha i} \, \nu_i \, W_\mu \, + \text{h.c.}$$

(basis where charged leptons are diagonal)





Additional generations **lepton sector**

⇒ Standard parameterization of the PMNS matrix:

$$\mathbf{U} = \begin{pmatrix} \mathbf{U}_{e1} & \mathbf{U}_{e2} & \mathbf{U}_{e3} \\ \mathbf{U}_{\mu1} & \mathbf{U}_{\mu2} & \mathbf{U}_{\mu3} \\ \mathbf{U}_{\tau1} & \mathbf{U}_{\tau2} & \mathbf{U}_{\tau3} \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,\mathrm{e}^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,\mathrm{e}^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,\mathrm{e}^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,\mathrm{e}^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,\mathrm{e}^{i\delta_{13}} & c_{23}\,c_{13} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{i\alpha_1} & 0 & 0 \\ 0 & \mathrm{e}^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(different values than in CKM) (Majorana phases)

[$\theta_{13} \equiv \theta_{\odot}$, $\theta_{23} \equiv \theta_{\mathrm{atm}}$ and θ_{13} (not yet δ_{13}) measured in oscillations]

Complete SM Lagrangian — fields and interactions

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_{\rm YM} + \mathcal{L}_\Phi + \mathcal{L}_{\rm Y} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP}$$

- Fields: [F] fermions; [S] scalars (Higgs and unphysical Goldstones); [V] vector bosons; [U] unphysical ghosts

- Interactions: [FFV] [FFS] [SSV] [SVV] [SSVV]
  [VVV] [VVVV] [SSS] [SSSS]
  [SUU] [UUVV]

Complete SM Lagrangian **Feynman rules**

- Feynman rules for generic couplings normalized to e (all momenta incoming):

$(\mathrm{i}\mathcal{L})$			
	[FFV$_\mu$]	$\mathrm{i}e\gamma^\mu(g_V - g_A\gamma_5) = \mathrm{i}e\gamma^\mu(g_L P_L + g_R P_R)$	
	[FFS]	$\mathrm{i}e(g_S - g_P\gamma_5) = \mathrm{i}e(c_L P_L + c_R P_R)$	
	[SV$_\mu$V$_\nu$]	$\mathrm{i}eKg_{\mu\nu}$	
	[S(p_1)S(p_2)V$_\mu$]	$\mathrm{i}eG(p_1 - p_2)_\mu$	
	[V$_\mu$(k_1)V$_\nu$(k_2)V$_\rho$(k_3)]	$\mathrm{i}eJ\left[g_{\mu\nu}(k_2-k_1)_\rho + g_{\nu\rho}(k_3-k_2)_\mu + g_{\mu\rho}(k_1-k_3)_\nu\right]$	
	[V$_\mu$(k_1)V$_\nu$(k_2)V$_\rho$(k_3)V$_\sigma$(k_4)]	$\mathrm{i}e^2C\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right]$	
	[SSV$_\mu$V$_\nu$]	$\mathrm{i}e^2C_2g_{\mu\nu}$	also [UUVV]
	[SSS]	$\mathrm{i}eC_3$	also [SUU]
	[SSSS]	$\mathrm{i}e^2C_4$	

Note: $g_{L,R} = g_V \pm g_A$

$c_{L,R} = g_S \pm g_P$

Attention to symmetry factors!

Complete SM Lagrangian | Feynman rules ('t Hooft-Feynman gauge)

FFV	$\bar{f}_i f_j \gamma$	$\bar{f}_i f_j Z$	$\bar{u}_i d_j W^+$	$\bar{d}_j u_i W^-$	$\bar{\nu}_i \ell_j W^+$	$\bar{\ell}_j \nu_i W^-$
g_L	$-Q_f \delta_{ij}$	$g_+^f \delta_{ij}$	$\frac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}$	$\frac{1}{\sqrt{2} s_W} \mathbf{V}_{ij}^*$	$\frac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}^*$	$\frac{1}{\sqrt{2} s_W} \mathbf{U}_{ji}$
g_R	$-Q_f \delta_{ij}$	$g_-^f \delta_{ij}$	0	0	0	0

$$g_\pm^f \equiv v_f \pm a_f \qquad v_f = \frac{T_3^{f_L} - 2Q_f s_W^2}{2 s_W c_W} \qquad a_f = \frac{T_3^{f_L}}{2 s_W c_W}$$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

FFS	$\bar{f}_i f_j H$	$\bar{f}_i f_j \chi$	$\bar{u}_i d_j \phi^+$	$\bar{d}_j u_i \phi^-$
c_L	$-\frac{1}{2s_W}\frac{m_{f_i}}{M_W}\delta_{ij}$	$-\frac{\mathrm{i}}{2s_W}2T_3^{f_L}\frac{m_{f_i}}{M_W}\delta_{ij}$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{u_i}}{M_W}\mathbf{V}_{ij}$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{d_j}}{M_W}\mathbf{V}_{ij}^*$
c_R	$-\frac{1}{2s_W}\frac{m_{f_i}}{M_W}\delta_{ij}$	$+\frac{\mathrm{i}}{2s_W}2T_3^{f_L}\frac{m_{f_i}}{M_W}\delta_{ij}$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{d_j}}{M_W}\mathbf{V}_{ij}$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{u_j}}{M_W}\mathbf{V}_{ij}^*$

FFS	$\bar{\nu}_i \ell_j \phi^+$	$\bar{\ell}_j \nu_i \phi^-$
c_L	$+\frac{1}{\sqrt{2}s_W}\frac{m_{\nu_i}}{M_W}\mathbf{U}_{ji}^*$	$-\frac{1}{\sqrt{2}s_W}\frac{m_{\ell_j}}{M_W}\mathbf{U}_{ji}$
c_R	$-\frac{1}{\sqrt{2}s_W}\frac{m_{\ell_j}}{M_W}\mathbf{U}_{ji}^*$	$+\frac{1}{\sqrt{2}s_W}\frac{m_{\nu_i}}{M_W}\mathbf{U}_{ji}$

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

SVV	HZZ	HW^+W^-	$\phi^\pm W^\mp\gamma$	$\phi^\pm W^\mp Z$
K	$M_W/s_W c_W^2$	M_W/s_W	$-M_W$	$-M_W s_W/c_W$

SSV	χHZ	$\phi^\pm\phi^\mp\gamma$	$\phi^\pm\phi^\mp Z$	$\phi^\mp HW^\pm$	$\phi^\mp\chi W^\pm$
G	$-\frac{\mathrm{i}}{2s_W c_W}$	∓ 1	$\pm\frac{c_W^2 - s_W^2}{2s_W c_W}$	$\mp\frac{1}{2s_W}$	$-\frac{\mathrm{i}}{2s_W}$

VVV	γW^+W^-	ZW^+W^-
J	-1	c_W/s_W

Complete SM Lagrangian **Feynman rules** ('t Hooft-Feynman gauge)

VVVV	$W^+W^+W^-W^-$	W^+W^-ZZ	$W^+W^-\gamma Z$	$W^+W^-\gamma\gamma$
C	$\frac{1}{s_W^2}$	$-\frac{c_W^2}{s_W^2}$	$\frac{c_W}{s_W}$	-1

SSVV	HHW^-W^+	$HHZZ$
C_2	$\frac{1}{2s_W^2}$	$\frac{1}{2s_W^2c_W^2}$

SSS	HHH
C_3	$-\frac{3M_H^2}{2M_W s_W}$

SSSS	$HHHH$
C_4	$-\frac{3M_H^2}{4M_W^2 s_W^2}$

– Would-be Goldstone bosons in [SSVV], [SSS] and [SSSS] omitted

– Faddeev-Popov ghosts in [UUVV] and [SUU] omitted

– All Feynman rules from FeynArts (same conventions):

http://www.ugr.es/local/jillana/SM/FeynmanRulesSM.pdf