3. Phenomenology

Input parameters

- Parameters:

17 +9 =	1	1	1	1	9 +3	4	6
formal:	g	g'	v	λ	λ_f	$\mathbf{V}_{\text{CKM}}$	$\mathbf{U}_{\text{PMNS}}$
practical:	α	M_W	M_Z	M_H	m_f		

where $g = \dfrac{e}{s_W}$ $g' = \dfrac{e}{c_W}$ and

$$\underbrace{\alpha = \frac{e^2}{4\pi} \qquad M_W = \frac{1}{2} g v \qquad M_Z = \frac{M_W}{c_W}}_{g,\ g',\ v} \qquad M_H = \sqrt{2\lambda}\, v \qquad m_f = \frac{v}{\sqrt{2}} \lambda_f$$

⇒ Many (more) experiments

⇒ After Higgs discovery, for the first time *all* parameters measured!

Input parameters

- Experimental values [Particle Data Group '15]
  - Fine structure constant:

    $\alpha^{-1} = 137.035\,999\,074\,(44)$ from Harvard cyclotron (g_e)

  - The SM predicts $M_W < M_Z$ in agreement with measurements:

    $M_Z = (91.1876 \pm 0.021)$ GeV from LEP1/SLD

    $M_W = (80.387 \pm 0.016)$ GeV from LEP2/Tevatron/LHC

  - Top quark mass:

    $m_t = (173.24 \pm 0.95)$ GeV from Tevatron/LHC

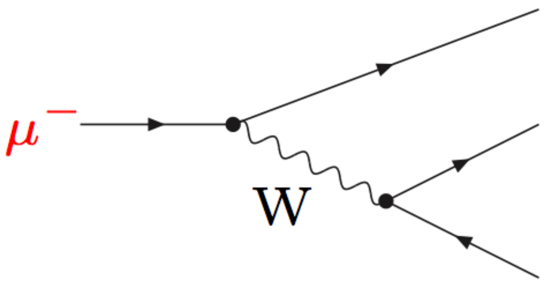
  - Higgs boson mass:

    $M_H = (125.6 \pm 0.4)$ GeV from LHC

  - ...

Observables and experiments

- Low energy observables ($Q^2 \ll M_Z^2$)
  - ν-nucleon (NuTeV) and νe (CERN) scattering:
    asymmetries CC/NC and $\nu/\bar{\nu}$ $\quad\Rightarrow \boxed{s_W^2}$
  - Atomic parity violation (Ce, Tl, Pb, Bi):
    asymmetries $\mathrm{e}_{R,L}N \to \mathrm{e}X$ due to Z-exchange between e and nucleus $\quad\Rightarrow \boxed{s_W^2}$
  - muon decay: $\mu \to e\,\overline{\nu}_e \nu_\mu$ (PSI)
    lifetime



$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\, f(m_e^2/m_\mu^2)$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad \Rightarrow \boxed{G_F}$$

$$= 0.99981295$$

$$\mathrm{i}\mathcal{M} = \left(\frac{\mathrm{i}e}{\sqrt{2}s_W}\right)^2 \bar{e}\gamma^\rho \nu_L\, \frac{-\mathrm{i}g_{\rho\delta}}{q^2 - M_W^2} \overline{\nu_L}\gamma^\delta \mu \equiv \overbrace{\mathrm{i}\frac{4G_F}{\sqrt{2}}\ (\bar{e}\gamma^\rho \nu_L)(\overline{\nu_L}\gamma_\rho \mu)}^{\text{Fermi theory } (-q^2 \ll M_W^2)}\ ;\ \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2}$$

Observables and experiments

- Low energy observables

⇒ Fermi constant provides the Higgs VEV (electroweak scale):

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \approx 246\,\text{GeV}$$

⇒ Consistency checks: e.g.

From muon lifetime:

$$G_F = 1.166\,378\,7(6) \times 10^{-5}\ \text{GeV}^{-2}$$

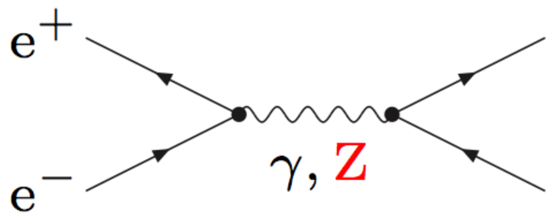
If one compares with (tree level result)

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}$$

using measurements of M_W, M_Z and α there is a discrepance that disappears when quantum corrections are included

Observables and experiments

- $e^+e^- \to \bar{f}f$ (PEP, PETRA, TRISTAN)



$$\frac{d\sigma}{d\Omega} = N_c^f \frac{\alpha^2}{4s}\beta_f \Big\{ \left[1+\cos^2\theta + (1-\beta_f^2)\sin^2\theta\right] G_1(s) + 2(\beta_f^2-1)\, G_2(s) + 2\beta_f \cos\theta\, G_3(s) \Big\}$$

$$G_1(s) = Q_e^2 Q_f^2 + 2Q_e Q_f v_e v_f \mathrm{Re}\chi_Z(s) + (v_e^2+a_e^2)(v_f^2+a_f^2)|\chi_Z(s)|^2$$

$$G_2(s) = (v_e^2+a_e^2)a_f^2|\chi_Z(s)|^2$$

$$G_3(s) = 2Q_e Q_f a_e a_f \mathrm{Re}\chi_Z(s) + 4v_e v_f a_e a_f|\chi_Z(s)|^2 \quad \Rightarrow \quad A_{FB}(s)$$

with $\chi_Z(s) \equiv \dfrac{s}{s - M_Z^2 + \mathrm{i}M_Z\Gamma_Z}$, $N_c^f = 1$ (3) for $f =$ lepton (quark), $\beta_f =$ velocity

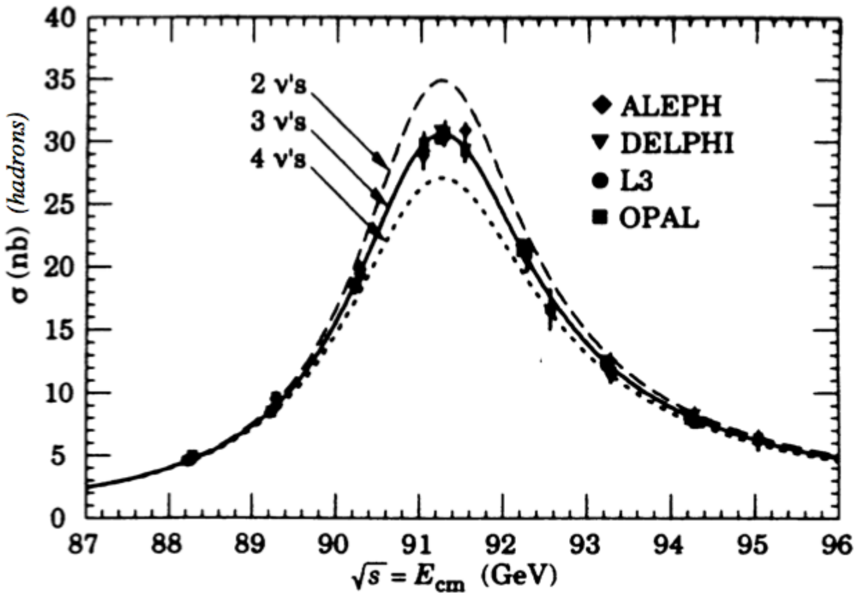
$$\sigma(s) = N_c^f \frac{2\pi\alpha^2}{3s}\beta_f \left[(3-\beta_f^2)G_1(s) - 3(1-\beta_f^2)G_2(s)\right] , \quad \beta_f = \sqrt{1-4m_f^2/s}$$

Observables and experiments

- Z production (LEP1/SLD)

  $M_Z, \Gamma_Z, \sigma_{\rm had}, A_{FB}, A_{LR}, R_b, R_c, R_\ell \quad \Rightarrow \boxed{M_Z, s_W^2}$

  from $e^+e^- \to \bar{f}f$ at the Z pole ($\gamma - Z$ interference vanishes). Neglecting m_f:



$$\sigma_{\rm had} = 12\pi \frac{\Gamma(e^+e^-)\Gamma(\text{had})}{M_Z^2\Gamma_Z^2}$$

$$R_b = \frac{\Gamma(b\bar{b})}{\Gamma(\text{had})} \quad R_c = \frac{\Gamma(c\bar{c})}{\Gamma(\text{had})} \quad R_\ell = \frac{\Gamma(\text{had})}{\Gamma(\ell^+\ell^-)}$$

$$\left[\Gamma(Z \to f\bar{f}) \equiv \Gamma(f\bar{f}) = N_c^f \frac{\alpha M_Z}{3}(v_f^2 + a_f^2)\right]$$

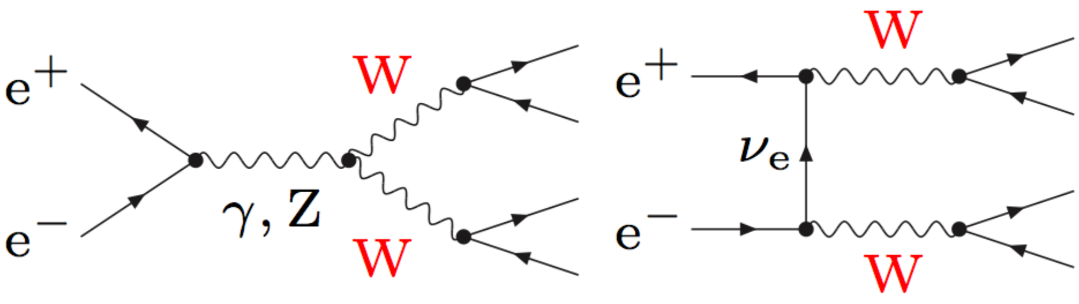
$$\Rightarrow N_\nu = 2.990 \pm 0.007$$

Forward-Backward and (if polarized e^-) Left-Right asymmetries due to Z:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_f \frac{A_e + P_e}{1 + P_e A_e} \qquad A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e P_e \quad \text{with } A_f \equiv \frac{2v_f a_f}{v_f^2 + a_f^2}$$

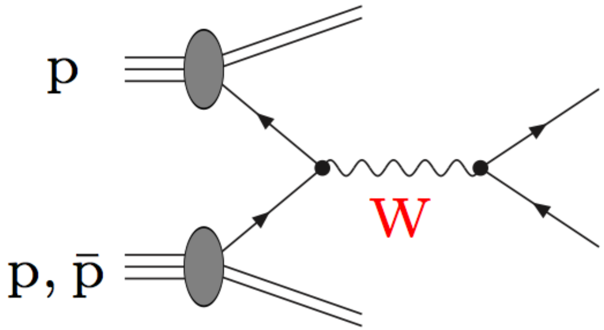
Observables and experiments

- W-pair production (LEP2)
  $e^+e^- \to WW \to 4\,f\ (+\gamma)$



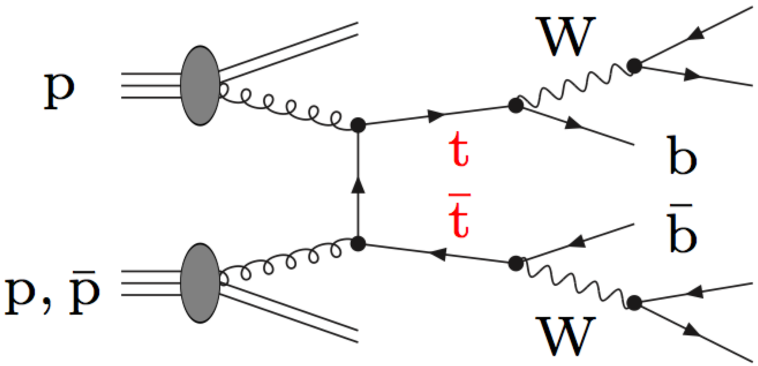
$\Rightarrow \boxed{M_W}$

- W production (Tevatron/LHC)
  $pp/p\bar{p} \to W \to \ell\nu_\ell\ (+\gamma)$



$\Rightarrow \boxed{M_W}$

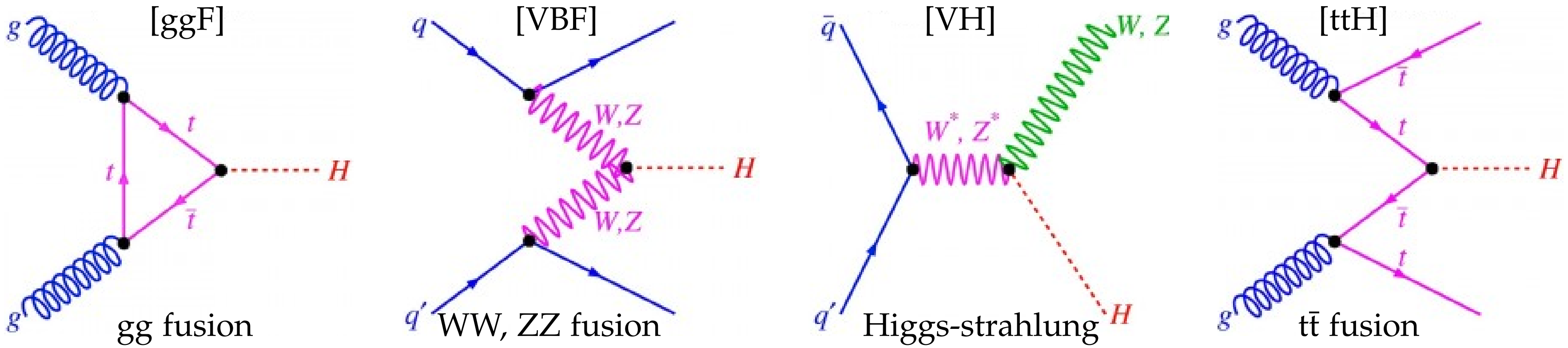
- Top-quark production (Tevatron/LHC)
  $pp/p\bar{p} \to t\bar{t} \to 6\,f$

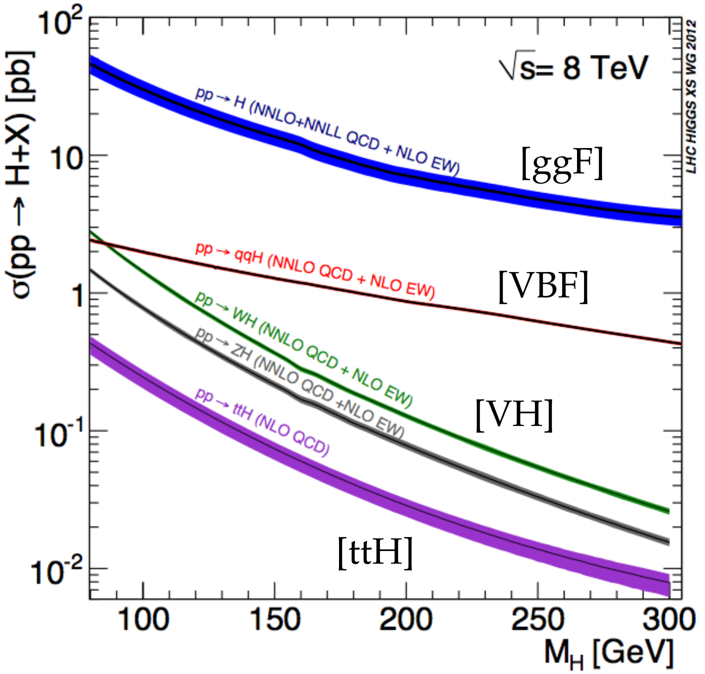


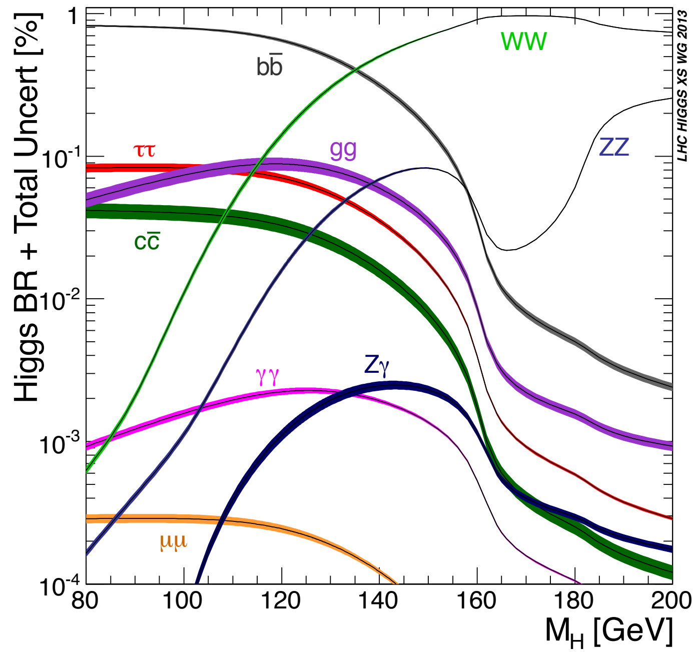
$\Rightarrow \boxed{m_t}$

Observables and experiments

- Higgs production (LHC)
  pp $\rightarrow H + X$ and H decays to different channels $\Rightarrow \boxed{M_H}$







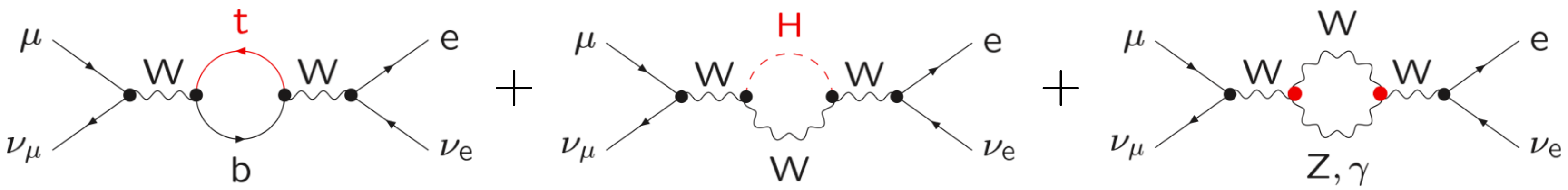
Precise determination of parameters

- Experimental precision requires accurate predictions $\Rightarrow$ quantum corrections (complication: loop calculations involve renormalization)

– Correction to G_F from muon lifetime:

$$\frac{G_F}{\sqrt{2}} \to \frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2(1 - M_W^2/M_Z^2)M_W^2}[1 + \Delta r(m_t, M_H)]$$

when loop corrections are included:



$+ \cdots + 2$ loops

Since muon lifetime is measured more precisely than M_W, it is traded for G_F:

$$\Rightarrow M_W^2(\alpha, G_F, M_Z, m_t, M_H) = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}[1 + \Delta r(m_t, M_H)]}\right)$$

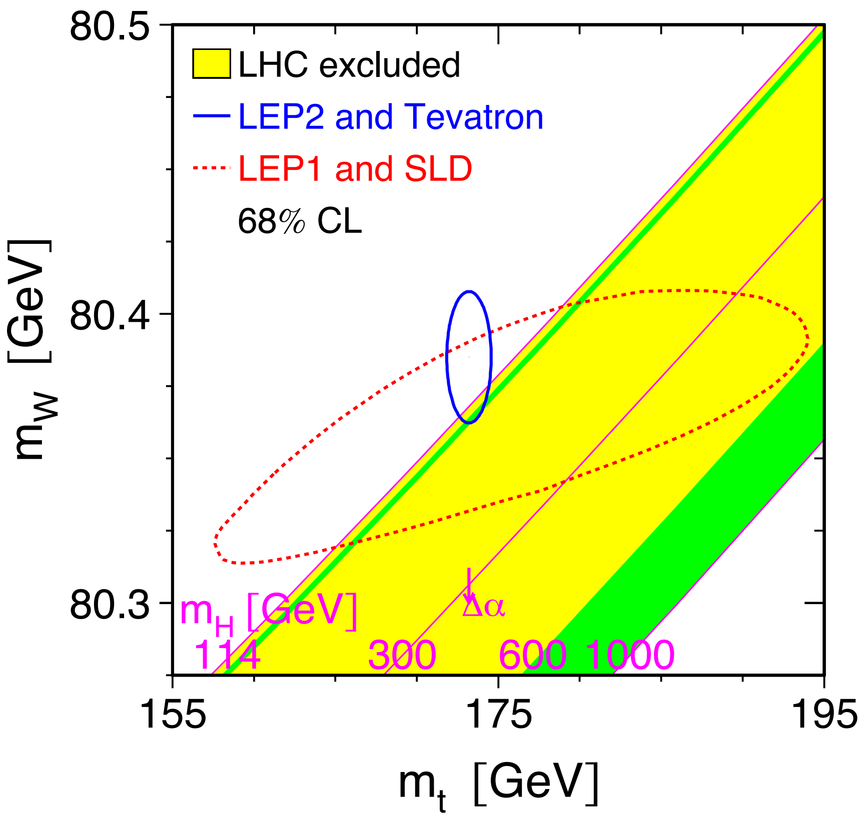
(correlation between M_W, m_t and M_H, given α, G_F and M_Z)

Precise determination of parameters

Indirect constraints from LEP1/SLD Direct measurements from LEP2/Tevatron

$M_H(M_W, m_t)$ Allowed regions for M_H allowed by direct searches LHC excluded

[LEPEWWG 2013]



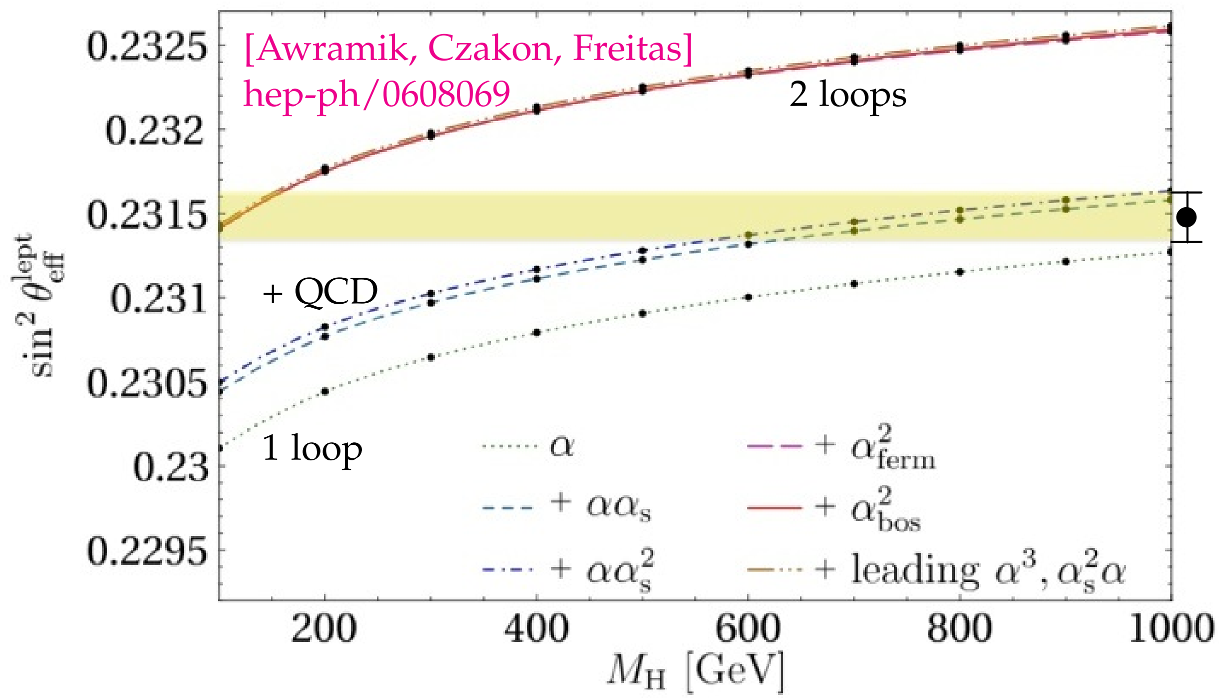
Precise determination of parameters

– Corrections to vector and axial couplings from Z pole observables:

$$v_f \to g_V^f = v_f + \Delta g_V^f \qquad a_f \to g_A^f = a_f + \Delta g_A^f$$

$$\Rightarrow \sin^2\theta_{\rm eff}^f = \frac{1}{4|Q_f|}\left[1 - {\rm Re}(g_V^f/g_A^f)\right] \equiv \overbrace{(1 - M_W^2/M_Z^2)}^{s_W^2}\ \kappa_Z^f$$

(Two) loop calculations are crucial and point to a light Higgs:



$s_W^2 = 0.22290 \pm 0.00029$ (tree)

$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23148 \pm 0.000017$ (exp)

Precise determination of parameters

- In addition, experiments and observables testing the flavor structure of the SM:
  flavor conserving: dipole moments, ... flavor changing: $b \to s\gamma$, ...

  $\Rightarrow$ very sensitive to new physics through loop corrections

  Extremely precise measurements are:

  – muon anomalous magnetic moment: $a_\mu = (g_\mu - 2)/2$

a_μ^{exp}	$= 116\,592\,089\,(63)$	$\times 10^{-11}$	[Brookhaven '06]
a_μ^{QED}	$= 116\,584\,718$	$\times 10^{-11}$	[QED: 5 loops]
a_μ^{EW}	$= 154$	$\times 10^{-11}$	[W, Z, H: 2 loops]
a_μ^{had}	$= 6\,930\,(48)$	$\times 10^{-11}$	[$e^+e^- \to$ had]
a_μ^{SM}	$= 116\,591\,802\,(49)$	$\times 10^{-11}$	

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 287\,(80) \times 10^{-11}$$

3.6σ !

  – electron magnetic moment (new physics suppressed by a factor of m_e^2/m_μ^2):

$$\left.\begin{array}{ll} \text{exp:} & g_e/2 = 1.001\,159\,652\,180\,76\,(27) \\ \text{theo:} & \text{QED (8 loops!)} \end{array}\right\} \Rightarrow \alpha^{-1} = 137.035\,999\,074\,(44)$$

Global fits

- Fit input data from a list of observables (EWPO):

$$M_H,\ M_W,\ \Gamma_W,\ M_Z,\ \Gamma_Z,\ \sigma_{\rm had},\ A_{FB}^{b,c,\ell},\ A_{b,c,\ell},\ R_{b,c,\ell},\ \sin^2\theta_{\rm eff}^{\rm lept},\ \ldots$$

finding the $\chi^2_{\rm min}$ for $n_{\rm dof} = 13\ (14)$ when M_H is included (excluded):

$$\underbrace{\alpha_s(M_Z)}_{1\ (\text{QCD})},\ \underbrace{\Delta\alpha_{\rm had}(M_Z),\ G_F,\ M_Z,\ 9\ \text{fermion masses},\ M_H}_{17-4=13\ (\text{CKM irrelevant})}$$

$$\alpha(M_Z) \equiv \frac{\alpha}{1-\Delta\alpha(M_Z)}$$

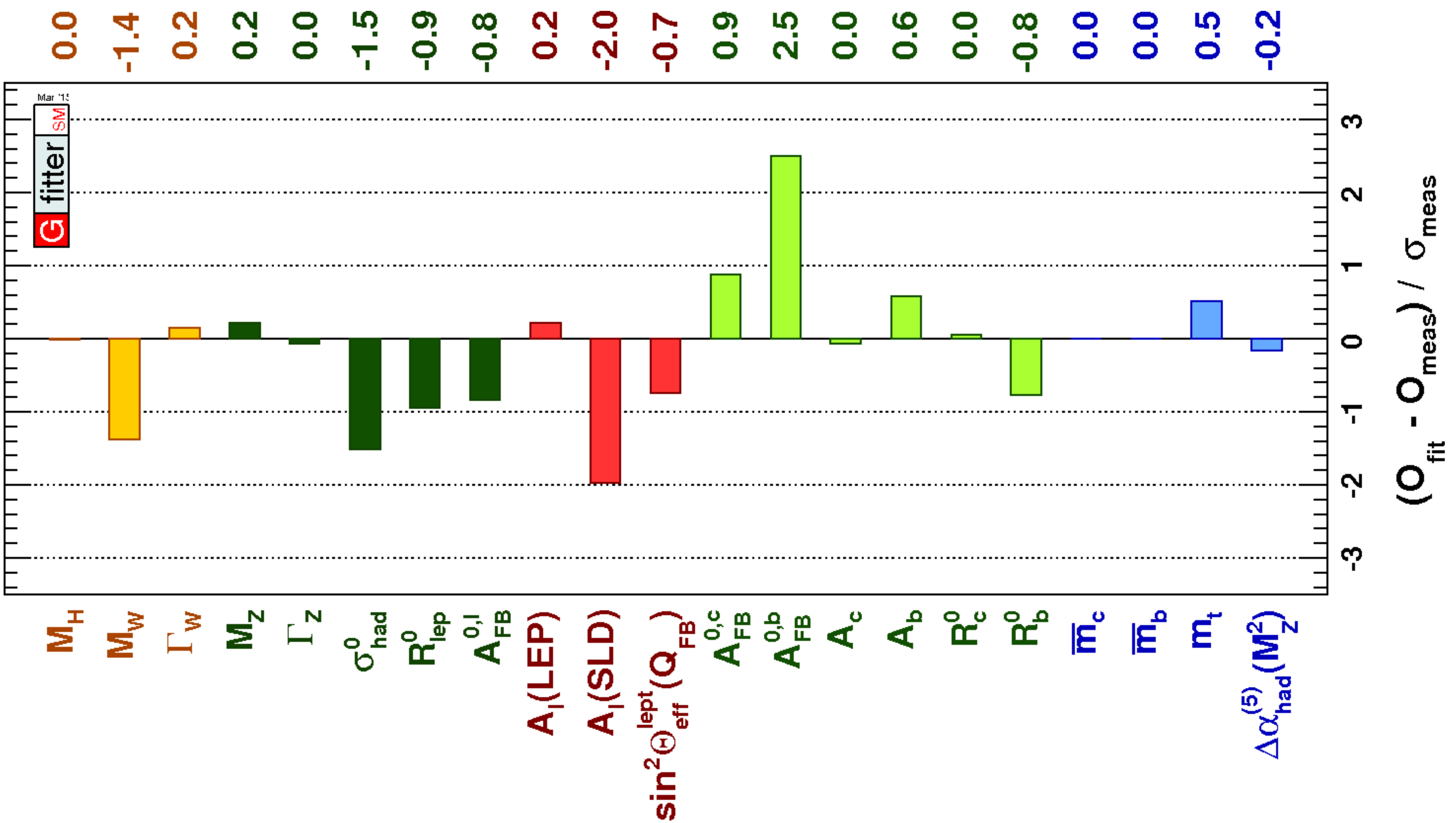
[Gfitter 2014: 1407.3792] http://gfitter.desy.de

$n_{\rm dof}$	$\chi^2_{\rm min}$	p-value
14	17.8	0.21

$\Rightarrow$ SM describes data to 0.8σ

Global fits (Comparisons)

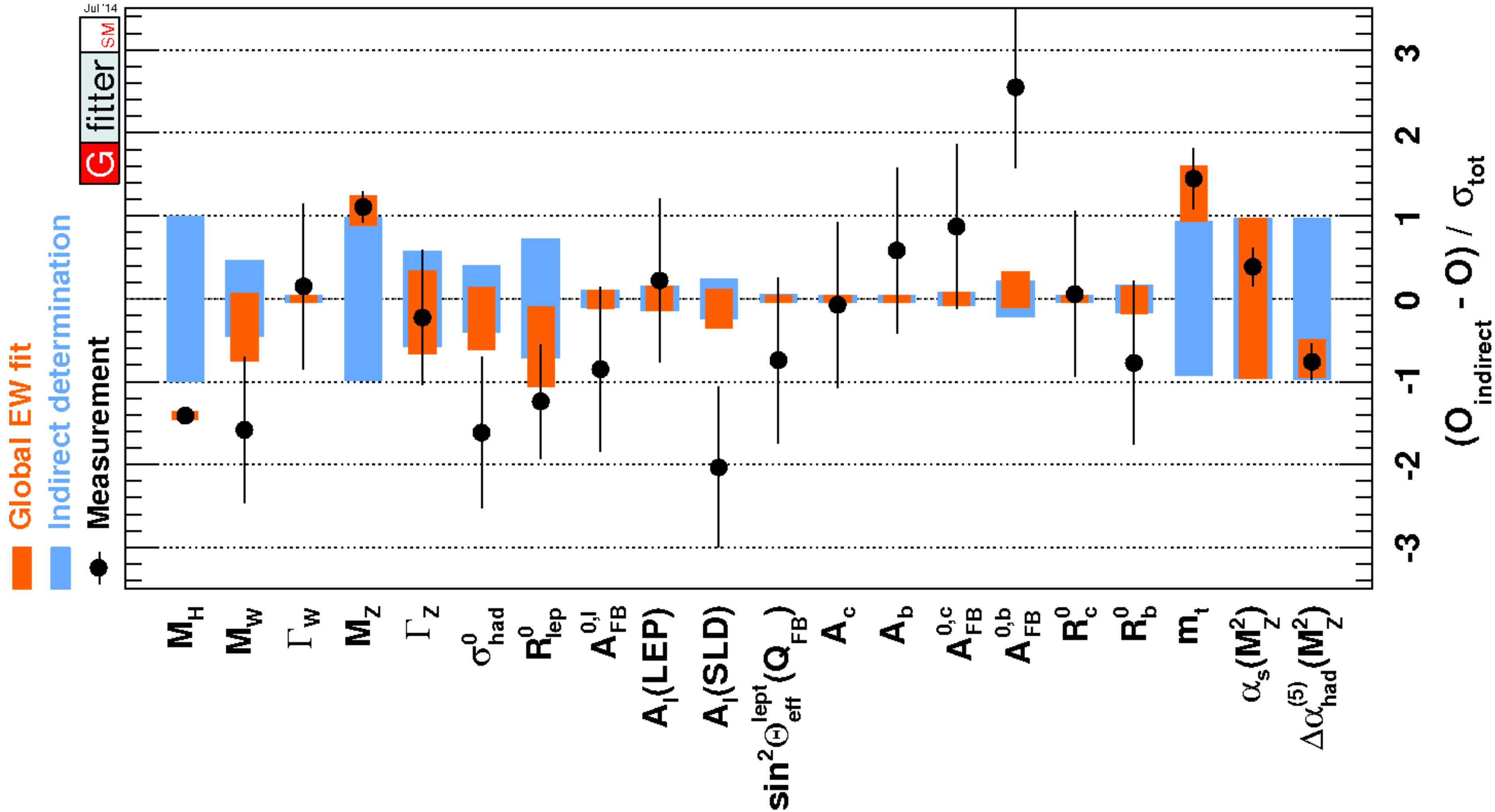
- Compare direct measurements of these observables with fit values:



⇒ some tensions (none above 3σ): A_ℓ(SLD), A^b_{FB}(LEP), R_b, ...

Global fits (Comparisons)

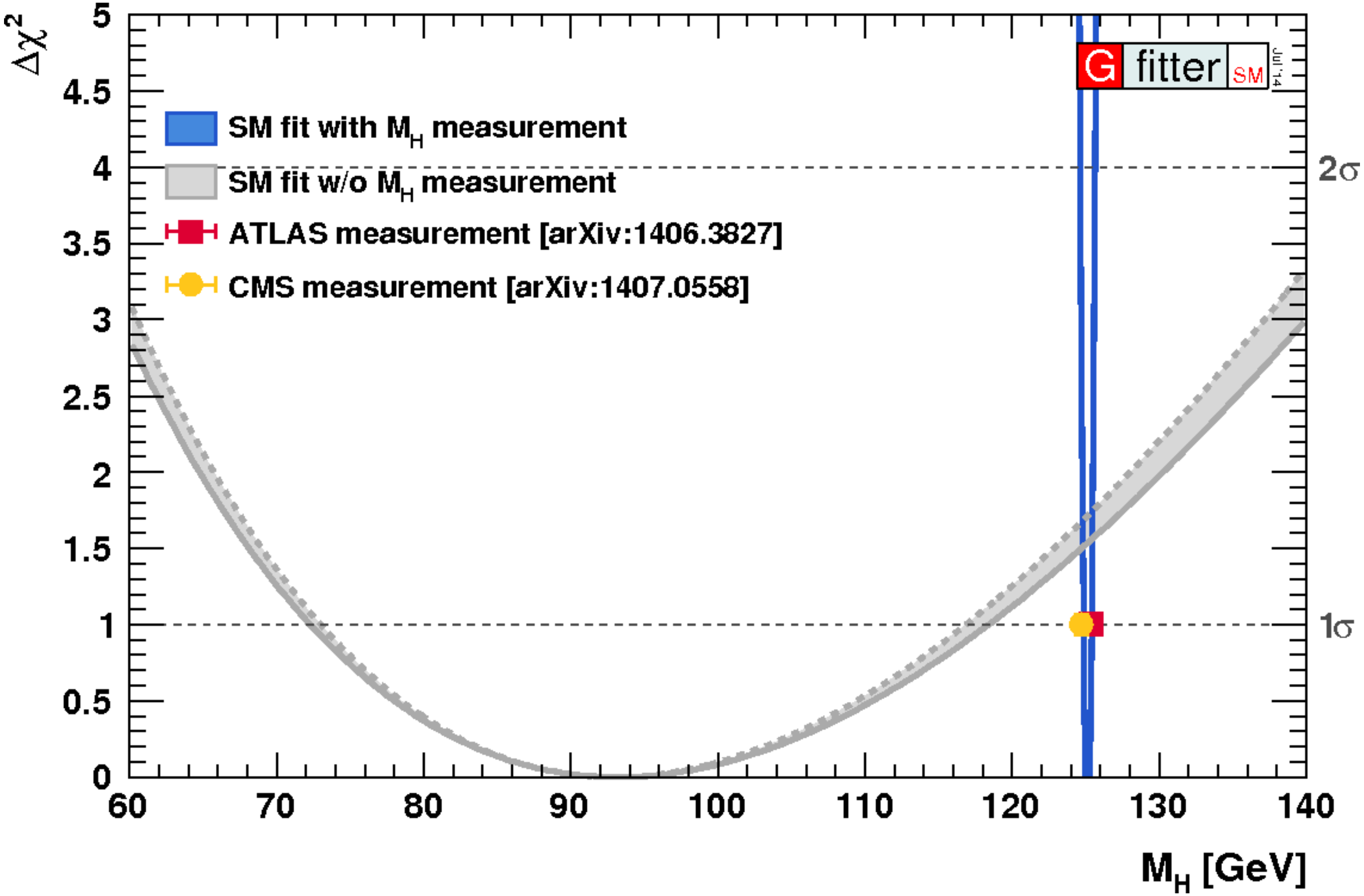
- Compare indirect determinations with fit values (error bars are direct measmts.):



[indirect determination means fit without using constraint from given direct measurement]

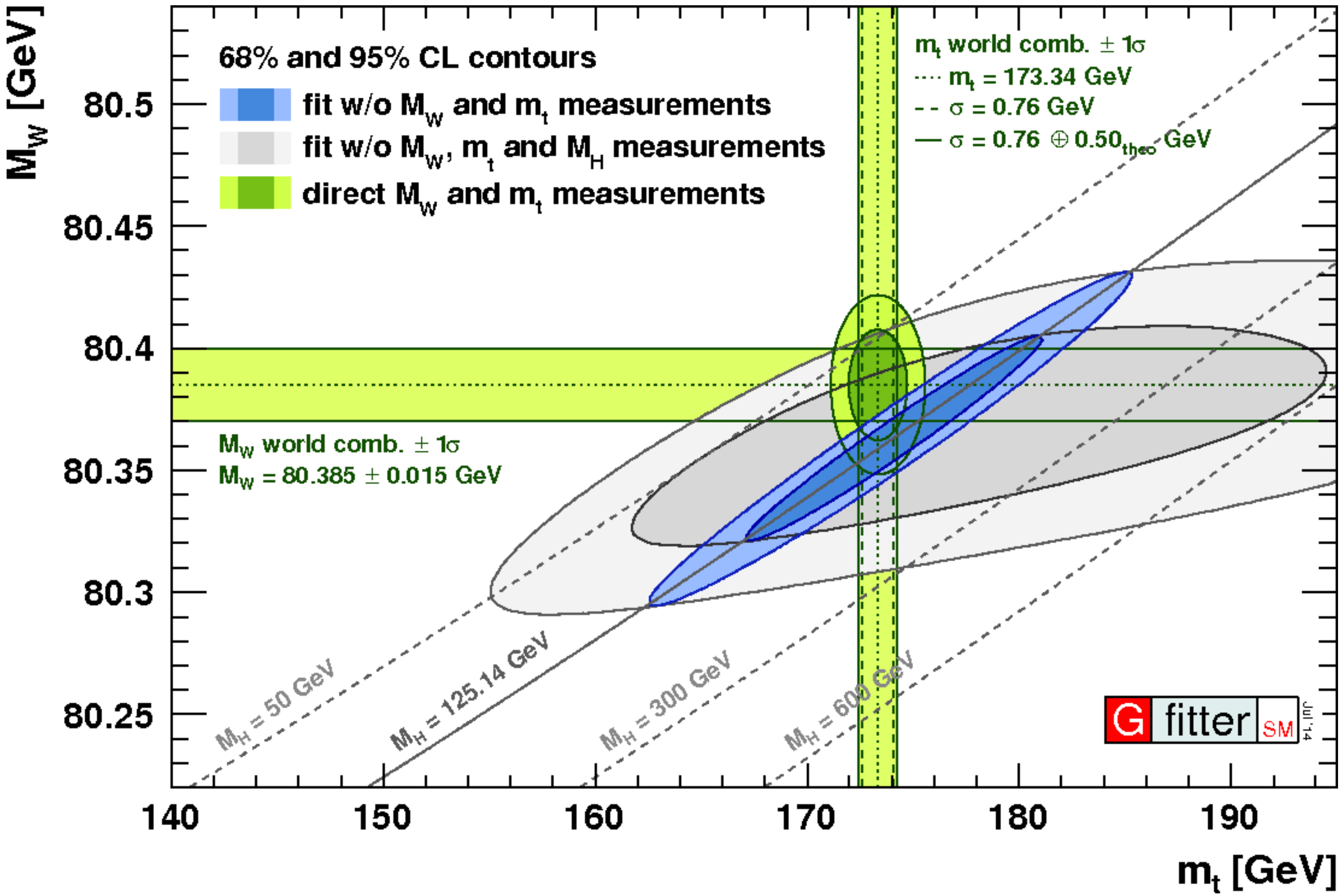
Global fits (Conclusions)

⇒ Fits prefer a somewhat lighter Higgs:



Global fits (Conclusions)

⇒ In general, impressive consistency of the SM, e.g.:



Summary

- The SM is a gauge theory with spontaneous symmetry breaking (renormalizable)
- Confirmed by many low and high energy experiments with remarkable accuracy, at the level of quantum corrections, with (almost) no significant deviations
- In spite of its tremendous success, it leaves fundamental questions unanswered:

  why 3 generations? why the observed pattern of fermion masses and mixings?

  and there are several hints for physics beyond:

  – phenomenological:
    * $(g_\mu - 2)$
    * neutrino masses
    * dark matter
    * baryogenesis
    * cosmological constant

  – conceptual:
    * gravity is not included
    * hierarchy problem

  $\Rightarrow$ The SM is an Effective Theory valid up to electroweak scale?


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
TASHAKKUR ATU
YAQHANYELAY
SUKSAMA
EKHMET
MEHRBANI
GRAZIE
BOLZÏN
MERCI
BİYAN
SHUKRIA
TINGKI
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
KOMAPSUMNIDA
MAAKE
PALDIES