Beyond Special Relativity and DSR theory

José Javier Relancio Martínez

Universidad de Zaragoza

Motivation

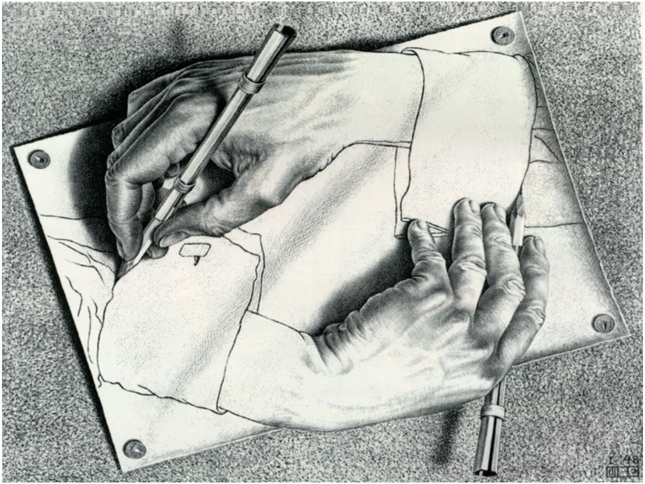
Space-time: the last frontier

- Any answer has to include matter and also the space-time structure $\rightarrow$ Gravity
- If fundamental constituents of matter exist, does it happen the same for space-time?
- Do space «atoms» exist?

QFT y GR: incompatibilities

- One of the challenges for the present physics is the unification of GR and QFT $\rightarrow$ QGT.
- In QFT, one asumes a given space-time and one studies in detail the properties and the movement of particles in it.
- In GR, one asumes that the properties of matter and radiation are given and one describes in detail the resultant space-time (curvature).

QFT y GR: incompatibilities



Why do we need a QGT?

- Study of the first moments of the universe.
- Black holes: Singularity, information?
- Answers$\to$ QGT.

Quantum Gravity Theories

- Approaches to QGT: string theory, quantum loop gravity, supergravity, causal set theory...
- There are no experimental evidences of a fundamental QGT.
- New approach: study the low energy theory of QGT. **Doubly Special Relativity (DSR)** $\rightarrow$posible experimental evidences.

Doubly Special Relativity (DSR)

- Two invariants in every inertial frame: speed of light c and Planck length l_P.
- We can obtain t_P, M_P y Λ

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1,6 \times 10^{-35}\,\mathrm{m}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5,4 \times 10^{-44}\,\mathrm{s}$$

$$\frac{\Lambda}{c^2} = M_P = \sqrt{\frac{\hbar c}{G}} = 2,2 \times 10^{-8}\,\mathrm{kg} = 1,2 \times 10^{19}\,\mathrm{GeV}/c^2$$

Quantum Gravity Phenomenology

- Planck energy $\rightarrow 10^{19}$ GeV
- Particle accelerators $\rightarrow 1.3\times 10^{4}$ GeV
- Cosmic rays$\rightarrow 10^{11}$ GeV.
- Phenomenology?$\rightarrow$ Amplifications at low energy.

Photon time delay

- A «foamy» structure of space-time could produce stochastic variations of the speed of particles.
- These deviations can be obtained through modified dispersion relations (MDR), that for $E \ll \Lambda$

$$E^2 - \vec{p}^2 - m^2 \simeq \xi_n E^2 \left(\frac{E}{\Lambda}\right)^n$$

- With the Hamiltonian concept of speed

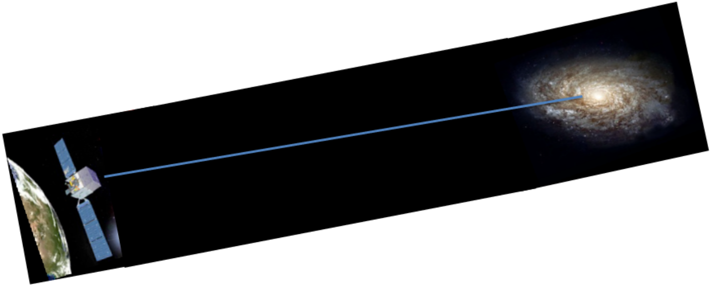
$$v = \frac{dE}{dp}$$

this causes a difference in the flight time

$$\Delta t \sim \left(\frac{d}{c}\right) \xi_n \left(\frac{E}{\Lambda}\right)^n$$

Photon time delay

- This delay can be measured for photons with different energies coming from a *gamma ray burst*.
- Recent experiments impose strong restrictions to deviations respect to SR at leading order ($n = 1$)



Relative locality

- When SR was formulated, space-time was flat.
- With the development of GR, Einstein introduces the concept of curved space-time.
- Born: why not to consider also a curved momentum space?
- This happens when one considers a modified composition law (MCL) of momenta

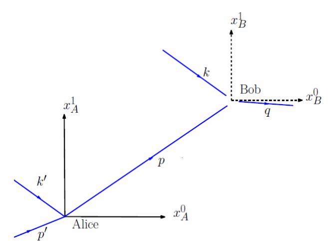
$$(p \oplus q)^\mu = p^\mu + q^\mu + \Gamma^\mu_{\nu\lambda} p^\nu q^\lambda + \ldots$$

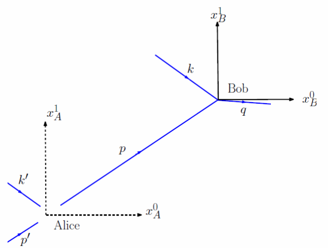
  where $\Gamma^\mu_{\nu\lambda}$ is the affine connection.
- This composition law is not neccesarely commutative

$$p \oplus q \neq q \oplus p$$

Relative locality

- Alice sees the production local and the detection non local, while Bob sees production non local and detection local.





First order model

- Most general MDR rotational invariant at first order

$$p_0^2 - \vec{p}^2 + \frac{\alpha_1}{\Lambda} p_0^3 + \frac{\alpha_2}{\Lambda} \vec{p}^2 p_0 = m^2$$

- Lorentz invariance violation (LIV) vs. M_P-Deformed Lorentz invariance (DSR)
- In DSR, the presence of an energy scale requires that the deformed transformations act non-linearly
- Then, a linear $(p_0, \vec{p})$-conservation law is not compatible with a relativity principle (RP) $\Longrightarrow$ MCL.

$$p_1 \oplus p_2|^0 = p_1^0 + p_2^0 + \frac{\beta_1}{\Lambda} p_1^0 p_2^0 + \frac{\beta_2}{\Lambda} \vec{p}_1 \vec{p}_2$$

$$\overrightarrow{p_1 \oplus p_2} = \vec{p}_1 + \vec{p}_2 + \frac{\gamma_1}{\Lambda} p_1^0 \vec{p}_2 + \frac{\gamma_2}{\Lambda} \vec{p}_1 p_2^0 + \frac{\gamma_3}{\Lambda} \vec{p}_1 \times \vec{p}_2$$

Lorentz Transformations

- Deformed infinitesimal Lorentz transformation (one particle sector):

$$\lambda_1 \, , \, \lambda_2 \, , \, \lambda_3$$

- Two particle sector

$$\eta_1^L \, , \, \eta_2^L \, , \, \eta_1^R \, , \, \eta_2^R$$

- Now we impose the Relativity Principle: all observers agree with conservation law.

Relations

- One can obtain the relations between the coefficients of MDR and MCL and those of the Lorentz transformation.

$$\alpha_1 = -2\left(\lambda_1 + \lambda_2 + 2\lambda_3\right) \qquad \alpha_2 = 2\left(\lambda_1 + 2\lambda_2 + 3\lambda_3\right)$$

$$\beta_1 = 2\left(\lambda_1 + \lambda_2 + 2\lambda_3\right) \qquad \beta_2 = -2\lambda_3 - \eta_1^L - \eta_1^R$$

$$\gamma_1 = \lambda_1 + 2\lambda_2 + 2\lambda_3 - \eta_1^L \qquad \gamma_2 = \lambda_1 + 2\lambda_2 + 2\lambda_3 - \eta_1^R$$

$$\gamma_3 = \eta_2^L - \eta_2^R$$

Relations

From the previous relations, one can obtain the relations (golden rules) between the MDR and the MCL:

$$\alpha_1 = -\beta_1 \qquad \alpha_2 = \gamma_1 + \gamma_2 - \beta_2$$

Not every coefficients satisfy a RP.

Modified Heisenberg algebra: Snyder algebra

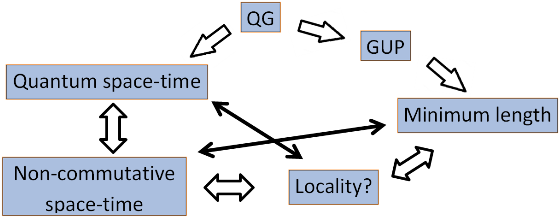
- The Snyder algebra is

$$[x_\mu, x_\nu] = -\frac{i}{\Lambda^2} J_{\mu\nu} \qquad [x_\mu, p_\nu] = i\left(\eta_{\mu\nu} - \frac{1}{\Lambda^2} p_\mu p_\nu\right)$$

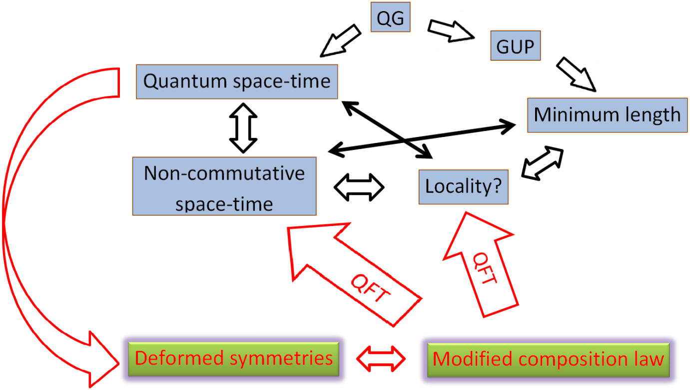
- This algebra is related to momentum space with the following MCL (at leading order in Λ^2)

$$(p \oplus q)^\mu = p^\mu + q^\mu - \frac{1}{\Lambda^2}(p.q)\, q^\mu - \frac{1}{2\Lambda^2}(p.q)\, p^\mu - \frac{1}{2\Lambda^2} p^2 q^\mu$$

Conclusions



Conclusions



Conclusions