Decays of neutral pions

Electromagnetic form factors and radiative corrections

Tomáš Husek

Institute of Particle and Nuclear Physics
Charles University, Prague

In collaboration with
K. Kampf, J. Novotný (*Charles University*), S. Leupold (*Uppsala University*)

Benasque
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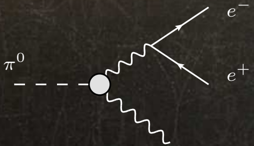
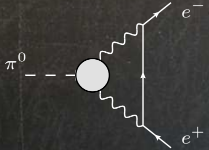
Introduction

Decay modes of neutral pion: $\pi^0 \to \gamma\gamma$, $\pi^0 \to e^+e^-\gamma$, $\pi^0 \to e^+e^+e^-e^-$, ...





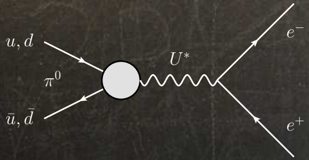
Rare decay $\pi^0 \to e^+e^-$

- precise measurements of branching ratio
  → KTeV experiment at Fermilab (*Abouzaid et al.*, PRD 75 (2007))

$$B^{\text{KTeV}}(\pi^0 \to e^+e^-(\gamma), x_\text{D} > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

- Standard Model theoretical prediction
  → $3.3\,\sigma$ disagreement (*Dorokhov and Ivanov*, PRD 75 (2007))
- discrepancy not satisfactorily explained yet

New physics?



- very fashionable to ascribe eventual discrepancies to effects of new physics

BUT

- first, look for more conventional solution (i.e. within SM)

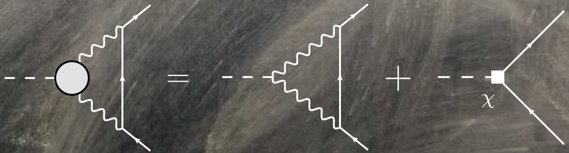
  → radiative corrections (usually very important)

  → form factor modeling

Let's clean the blackboard

Leading order

- pions are complicated composite objects
  → elementary interactions are not point-like
- electromagnetic pion transition form factor $F_{\pi^0\gamma^*\gamma^*}$ describes this complexity



LO contribution
in QED expansion

its representation
as the LO of χPT

- free parameter $\chi^{(\mathrm{r})}(\mu)$ appears in the finite part of the counter term

$$\chi = [\text{UV-divergent part}] + \chi^{(\mathrm{r})}(\mu)$$

→ unique for every form factor, e.g. $\chi^{(\mathrm{r})}_{\mathrm{KTeV}}(M_\rho) = 6.0 \pm 1.0$

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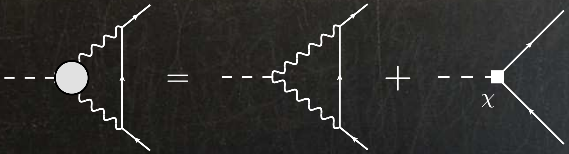

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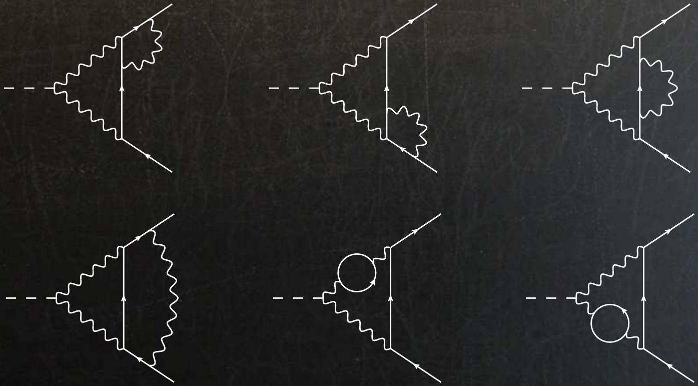
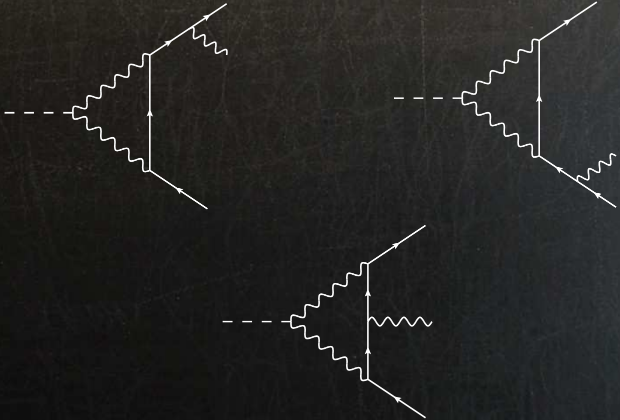
Two-loop virtual radiative corrections

- calculated by *Vaško and Novotný*, JHEP 1110 (2011)



Bremsstrahlung

- compensation of infrared divergences in 2-loop contributions
  → *TH, Kampf and Novotný*, EPJC 74 (2014)



Final results

Contents

Size of the radiative corrections (newly calculated)

$$\delta^{\text{NLO}}(0.95) \equiv \delta^{\text{virt.}} + \delta^{\text{BS}}(0.95) = (-5.5 \pm 0.2)\,\%$$

- can be thought as model-independent
- differs significantly from previous approximate calculations

Bergström, Z.Ph.C 20 (1983): $\delta(0.95) = -13.8\,\%$
Dorokhov et al., EPJC 55 (2008): $\delta(0.95) = -13.3\,\%$

- original KTeV vs. SM discrepancy reduced to the $2\,\sigma$ level or less

$\rightarrow \chi^{(\text{r})}_{\text{KTeV}}(M_\rho) = 4.5 \pm 1.0$

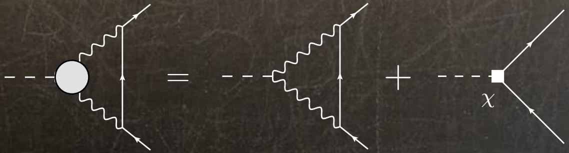
- LMD model (*Knecht et al.*, PRL 83 (1999))

$$\chi^{(\text{r})}_{\text{LMD}}(M_\rho) = 2.2 \pm 0.9$$

NLO radiative corrections in the QED sector did not solve the discrepancy

→ back to LO, but use different model

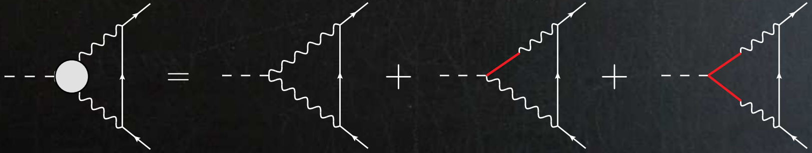
Resonances



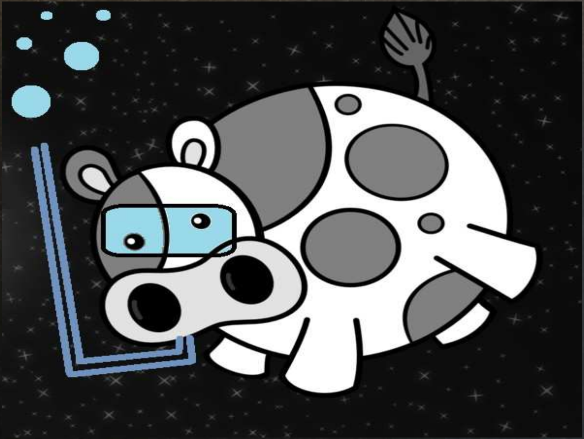
Chiral Perturbation Theory (χPT)

$\Updownarrow$

Resonance Chiral Theory (RχT)



Spherical cow in the vacuum



THS model for PVV correlator

1) Ansatz for Pseudoscalar-Vector-Vector (PVV) correlator

- Two-Hadron-Saturation (THS) - 2 meson multiplets per channel

$$\Pi^{\text{THS}}(r^2;p^2,q^2) \sim \frac{1}{r^2(r^2-M_P^2)}\frac{P(r^2;p^2,q^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}$$

- in numerator stands general polynomial symmetrical in p^2 and q^2
  - → correlator must drop at large momenta
  - → 22 free parameters

$$P(r^2;p^2,q^2) = c_0 p^2 q^2 + c_1[(p^2)^3 q^2 + (q^2)^3 p^2] + c_2 (r^2)^2 p^2 q^2 + \ldots$$

2) Use high- and low-energy limits to constrain the parameters

- Operator product expansion (OPE)
- Brodsky–Lepage (BL) quark counting rules
- chiral anomaly

THS and $\mathcal{F}_{\pi^0\gamma^*\gamma^*}$ form factor

Form factor is in general related to PVV correlator as

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) \sim \lim_{r^2\to 0} r^2\Pi(r^2;p^2,q^2)$$

→ in our case complicated, but with only one free parameter

$$\mathcal{F}^{\text{THS}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\right]$$
$$\times\left\{1+\frac{\kappa}{2N_c}\frac{p^2q^2}{(4\pi F)^4}-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^2 M_{V_2}^2}\left[6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right]\right\}$$

κ determined from fit to ω-π transition form factor measurements

$$\kappa = 21 \pm 3$$

$M_{V_1} \sim \rho, \omega$ vector-meson mass
$M_{V_2} \sim$ between physical masses of first and second vector-meson excitations

$$M_{V_2} \in [1400, 1740]\ \text{MeV}$$

VMD and LMD models

Examples of other approaches

- Vector-Meson Dominance (VMD)

$$\mathcal{F}^{\mathrm{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = -\frac{N_c}{12\pi^2 F}\left[\frac{M_{V_1}^4}{(p^2-M_{V_1}^2)(q^2-M_{V_1}^2)}\right]$$

$\rightarrow$ violates OPE: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q^2,q^2) \not\sim \frac{1}{q^2}\,,\; q^2 \rightarrow -\infty$

- Lowest-Meson Dominance (LMD)

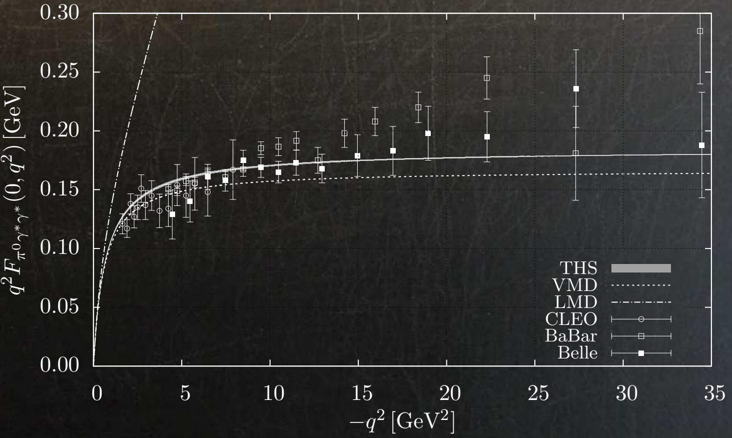
$$\mathcal{F}^{\mathrm{LMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2) = \mathcal{F}^{\mathrm{VMD}}_{\pi^0\gamma^*\gamma^*}(p^2,q^2)\left\{1-\frac{4\pi^2F^2(p^2+q^2)}{N_c M_{V_1}^4}\right\}$$

$\rightarrow$ violates BL: $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2) \not\sim \frac{1}{q^2}\,,\; q^2 \rightarrow -\infty$

- none of the models used two meson multiplets in both channels

  $\rightarrow$ vector and pseudoscalar

Form factor data



Results

Theoretical prediction within THS model

$$B^{\mathsf{THS}}(\pi^0 \to e^+e^-(\gamma),\, x_{\mathrm{D}} > 0.95) = (5.8 \pm 0.2) \times 10^{-8}$$

- recall experimental value: $B^{\mathsf{KTeV}} = (6.44 \pm 0.33) \times 10^{-8}$

  → disagreement at the level of only 1.8 σ
- matching on LO χPT gives $\chi^{(\mathsf{r})}_{\mathsf{THS}}(M_\rho) = 2.2 \pm 0.7$
- if KTeV result confirmed → two scenarios are conceivable:

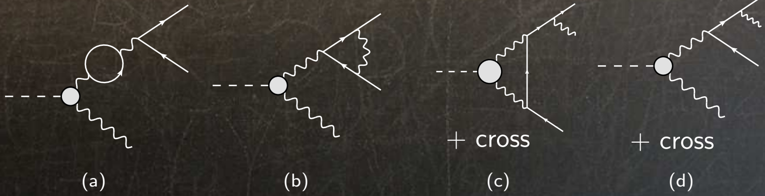
  a) some aspects of the THS approach not well-suited for $\pi^0 \to e^+e^-$
  b) beyond-Standard Model physics influences the rare pion decay significantly
- under the present circumstances the current discrepancy is inconclusive

Quantity really measured by KTeV

$$\left.\frac{\Gamma(\pi^0 \to e^+e^-(\gamma)\,,\; x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma(\gamma)\,,\; x > 0.2319)}\right|_{\mathsf{KTeV}} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}$$

→ Dalitz decay comes into play

Dalitz decay radiative corrections



- corrections to the Dalitz plot in the form of a table of values
  → *Mikaelian and Smith*, PRD 5 (1972)
- new calculations motivated by needs of NA48/NA62 experiments at CERN
  → measure the slope a of $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0,q^2)$
- unlike before no approximation was used
  → can be used also for related decays $\eta \to \ell^+\ell^-\gamma$ etc.
- C++ code returns the correction for any given x and y
  → propagated into simulation software of NA62 experiment
- *TH, Kampf and Novotný*, PRD 92 (2015)

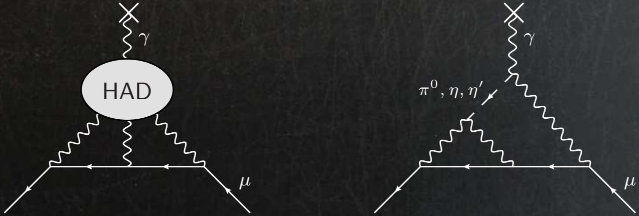
Outlook

Pseudoscalar decays

- $\chi^{(\mathrm{r})}$ universal for $P \to l^+l^-$ processes up to $\mathcal{O}(m_l^2/\Lambda_{\chi\mathrm{PT}}^2)$

Muon $g-2$: hadronic light-by-light scattering

- pseudoscalar meson exchange contribution requires hadron-physics input



(a) HLbL scattering general contribution

(b) Pseudoscalar meson exchange

Summary

All NLO QED radiative corrections for discussed processes are now available
→ can be taken into account in future experimental analyses

- $\pi^0 \to e^+e^-$
  Vaško and Novotný, JHEP 1110 (2011)
  TH, Kampf and Novotný, EPJC 74 (2014)
- $\pi^0 \to e^+e^-\gamma$
  TH, Kampf and Novotný, PRD 92 (2015)

THS model for $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2, q^2)$

- phenomenologically successful
- satisfies all main theoretical constraints
- *TH and S. Leupold,* EPJC 75 (2015)

Altogether, we get reasonable SM prediction
→ differs from KTeV by 1.8 σ

Goodbye

Thank you for your attention!