OPE of Green functions in the odd sector of QCD

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What is it about?

- What do we do?
  - We study QCD at low energies using Green functions.
- What do we need?
  - Chiral perturbation theory (χPT) and Resonance chiral theory (RχT).
- What is it?
  - Effective description low-energy QCD.
  - χPT for $E \leq M_\rho$.
    - Spontaneous breaking of the chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry down to $\mathrm{SU}(3)_V$ in QCD leads to the presence of Goldstone bosons.
    - We identify them with the octet of pseudoscalar mesons (π, K, η) as the lightest hadronic observable states.
  - RχT for $M_\rho \leq E \leq 2\,\mathrm{GeV}$.
    - RχT increases the number of degrees of freedom of χPT by including massive $\mathrm{U}(3)$ multiplets of vector $V(1^{--})$, axial-vector $A(1^{++})$, scalar $S(0^{++})$ and pseudoscalar $P(0^{-+})$ resonances.
- What is it good for?
  - To study important theoretical and phenomenological aspects of QCD.

Green functions of chiral currents

- QCD introduces an octet of noether currents:
  - vector and axial-vector currents:

$$V_\mu^a = \overline{q}(x)\gamma_\mu T^a q(x)\,, \qquad A_\mu^a = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)\,,$$

  - scalar and pseudoscalar densities:

$$S^a = \overline{q}(x) T^a q(x)\,, \qquad P^a = i\overline{q}(x)\gamma_5 T^a q(x)\,.$$

- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the time ordered products of quantum fields (the group and Lorentz indices are suppresed):

$$\int \mathrm{d}^4 x_1 \int \mathrm{d}^4 x_2\, e^{i(p_1 x_1 + p_2 x_2)} \left\langle 0 \middle| \mathrm{T}\left[\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(0)\right] \middle| 0 \right\rangle .$$

- Only five nontrivial Green functions in the odd-intrinsic parity sector of QCD.
  - VVP, VAS, AAP, VVA and AAA.

How to calculate Green functions?

- We assume the saturation of dynamics with the lightest resonances.
- We restrict ourselves only to the three-point Green functions at tree level.
- Ingredients at the LO:

$$\mathcal{L}^{(2)}_{\chi\mathrm{PT}} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle\,,$$

$$\mathcal{L}^{(4)}_{R} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle$$
$$+ c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle + i d_m\langle P\chi_-\rangle + \frac{i d_{m0}}{N_F}\langle P\rangle\langle\chi_-\rangle\,.$$

- At the NLO, relevant Lagrangian in the odd-intrinsic parity sector, was formulated for the first time in [K. Kampf and J. Novotný '11]

$$\mathcal{L}^{(6)}_{R} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}\,.$$

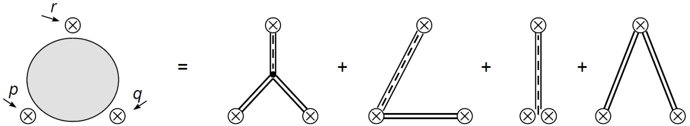
  - X stands for the single-resonance fields V, A, S, P, double-resonance fields $VV, AA, SA, SV, VA, PA, PV$ and triple-resonance fields VVP, VAS, AAP.

How to calculate Green functions?

- Example: a set of operators with one vector resonance field:

i	$\widehat{\mathcal{O}}^{V}_{i\,\mu\nu\alpha\beta}$	i	$\widehat{\mathcal{O}}^{V}_{i\,\mu\nu\alpha\beta}$
1	$i\langle V^{\mu\nu}(h^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma h^{\alpha\sigma})\rangle$	10	$\langle V^{\mu\nu}u^\alpha \chi_- u^\beta\rangle$
2	$i\langle V^{\mu\nu}(u_\sigma h^{\alpha\sigma}u^\beta - u^\beta h^{\alpha\sigma}u_\sigma)\rangle$	11	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, f_-^{\beta\sigma}\}\rangle g_{\rho\sigma}$
3	$i\langle V^{\mu\nu}(u_\sigma u^\beta h^{\alpha\sigma} - h^{\alpha\sigma}u^\beta u_\sigma)\rangle$	12	$\langle V^{\mu\nu}\{f_+^{\alpha\rho}, h^{\beta\sigma}\}\rangle g_{\rho\sigma}$
4	$i\langle [V^{\mu\nu}, \nabla^\alpha \chi_+]u^\beta\rangle$	13	$i\langle V^{\mu\nu}f_+^{\alpha\beta}\rangle\langle\chi_-\rangle$
5	$i\langle V^{\mu\nu}[f_-^{\alpha\beta}, u_\sigma u^\sigma]\rangle$	14	$i\langle V^{\mu\nu}\{f_+^{\alpha\beta}, \chi_-\}\rangle$
6	$i\langle V^{\mu\nu}(f_-^{\alpha\sigma}u^\beta u_\sigma - u_\sigma u^\beta f_-^{\alpha\sigma})\rangle$	15	$i\langle V^{\mu\nu}[f_-^{\alpha\beta}, \chi_+]\rangle$
7	$i\langle V^{\mu\nu}(u_\sigma f_-^{\alpha\sigma}u^\beta - u^\beta f_-^{\alpha\sigma}u_\sigma)\rangle$	16	$\langle V^{\mu\nu}\{\nabla^\alpha f_+^{\beta\sigma}, u_\sigma\}\rangle$
8	$i\langle V^{\mu\nu}(f_-^{\alpha\sigma}u_\sigma u^\beta - u^\beta u_\sigma f_-^{\alpha\sigma})\rangle$	17	$\langle V^{\mu\nu}\{\nabla_\sigma f_+^{\alpha\sigma}, u^\beta\}\rangle$
9	$\langle V^{\mu\nu}\{\chi_-, u^\alpha u^\beta\}\rangle$	18	$\langle V^{\mu\nu}u^\alpha u^\beta\rangle\langle\chi_-\rangle$

- Topology of the Feynman diagrams (the crossing is implicitly assumed):



Determination of the couplings κ_i^X

- Reminder:

$$\mathcal{L}_R^{(6)} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta} \varepsilon^{\mu\nu\alpha\beta} \,.$$

- $\mathcal{L}_R^{(6)}$: 67 operators and 67 corresponding unknown couplings κ_i^X in total.
  - Three-point Green functions contain 32 couplings κ_i^X together.
  - Not every coupling can be determined.
- How to determine the coupling constants?
  - High-energy behavior of Green functions.
  - Brodsky-Lepage behavior of the transition formfactor.
  - Matching calculations in RχT with χPT.
  - Experiments.

Operator Product Expansion (OPE)

- OPE is a framework to study short-distance behaviour of Green functions.
  - Formalism independent - purely mathematical property (no Lagrangians, no Feynman diagrams etc.).
  - The OPE is equivalent to an assumption that at large external momentum p, the two-point Green function of the operators above can be rewritten in the form

$$A(x)B(y) = \sum_{i=0}^{\infty} C_i(x-y)\mathcal{O}_i\left(\frac{x-y}{2}\right),$$

$$i\int \mathrm{d}^4x\, e^{ipx}\langle 0\big|\mathrm{T}\big[A(x)B(0)\big]\big|0\rangle = \sum_n C_n^{AB}(p^2)\langle 0|\mathcal{O}_n|0\rangle\,.$$

- QCD condensates $\mathcal{O}_D$ with dimension $D \leq 6$:

$$\begin{aligned}
\mathcal{O}_0 &= 1\,, & \mathcal{O}_5 &= \langle 0|\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle\,,\\
\mathcal{O}_3 &= \langle 0|\overline{q}q|0\rangle\,, & \mathcal{O}_6^q &= \langle 0|(\overline{q}\Gamma q)(\overline{q}\Gamma q)|0\rangle\,,\\
\mathcal{O}_4 &= \langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle\,, & \mathcal{O}_6^G &= \langle 0|G_{\mu\nu}G_\sigma^\nu G^{\sigma\mu}|0\rangle\,.
\end{aligned}$$

- Γ stands for a combination of $\{\mathbb{1}_4, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}\}$ and $\{\mathbb{1}_3, T^a\}$.

OPE for three-point Green functions: Example on VVP

- Vector currents $V_\mu^a(p)$, $V_\nu^b(q)$ and pseudoscalar density $P^c(r)$, $r = -p - q$.
- Ward identities, P, C and Lorentz invariance gives the structure

$$\left(\Pi_{VVP}(p,q;r)\right)_{\mu\nu}^{abc} = \Pi_{VVP}(p^2,q^2,r^2)d^{abc}\varepsilon_{\mu\nu(p)(q)}\,.$$

- All three momenta are considered to be large.
- OPE is easily obtained with two contractions as the lowest order contribution (only the quark condensate contributes):

$\langle 0|$  $|0\rangle$.

- The result: $\Pi_{VVP}(p^2,q^2,r^2)$ at high energies behaves as

$$\Pi_{VVP}^{\mathrm{OPE}}\left((\lambda p)^2,(\lambda q)^2;(\lambda r)^2\right) = \frac{B_0F^2}{2\lambda^4}\,\frac{p^2+q^2+r^2}{p^2q^2r^2} + \mathcal{O}\left(\frac{1}{\lambda^6}\right) \text{ for } \lambda \to \infty\,.$$

- In a condensed notation:

$$\Pi_{VVP}^{\mathrm{OPE}} \sim (+,+,+)\,,\qquad \Pi_{VAS}^{\mathrm{OPE}} \sim (+,-,-)\,,\qquad \Pi_{AAP}^{\mathrm{OPE}} \sim (+,+,-)\,.$$

VVP Green function

- Reminder:

$$\left(\Pi_{VVP}(p,q;r)\right)^{abc}_{\mu\nu} = \Pi_{VVP}(p^2,q^2,r^2)d^{abc}\varepsilon_{\mu\nu(p)(q)}\,.$$

- Our task is to calculate $\Pi_{VVP}(p^2,q^2,r^2)$ using our Lagrangian $\mathcal{L}_R^6$ and determine the couplings κ_i^X .
- χPT
  - Comparison with the calculation in χPT leads to an isolation of two low-energy constants C_7^W and C_{22}^W in terms of κ_i^X couplings.
- OPE
  - High-energy behaviour dictates the coupling constants constraints:

$$\kappa_{14}^V = \frac{N_c}{256\sqrt{2}\pi^2 F_V}\,, \quad \kappa_{16}^V + 2\kappa_{12}^V = -\frac{N_c}{32\sqrt{2}\pi^2 F_V}\,, \quad \kappa_{17}^V = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}\,,$$
$$\kappa_2^{VV} = \frac{F^2 + 16\sqrt{2}d_m F_V \kappa_3^{PV}}{32F_V^2} - \frac{N_c M_V^2}{512\pi^2 F_V^2}\,, \quad 8\kappa_2^{VV} - \kappa_3^{VV} = \frac{F^2}{8F_V^2}\,.$$

  - $\Pi_{VVP}^{R\chi T}(p^2,q^2,r^2)$: substituing the constraints back into $\Pi_{VVP}(p^2,q^2,r^2)$.

VVP Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor

- The transition $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor:

$$\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(p^2,q^2,r^2) = \frac{2r^2}{3B_0F}\Pi^{\mathrm{R}\chi\mathrm{T}}_{VVP}(p^2,q^2,r^2)\,.$$

- The Brodsky-Lepage behaviour for large momentum [G. P. Lepage and S. J. Brodsky '80, '81]:

$$\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(0,-Q^2,m_\pi^2) \sim -\frac{1}{Q^2} \text{ for } Q^2\to\infty\,.$$

- B-L behaviour leads to the constraint

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_mF_V}\,.$$

- BABAR measurement shows phenomenological disagreement with this condition that leads to the deviation with $\delta_{\mathrm{BL}} = -0,055 \pm 0.025\,,$

$$\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_mF_V}(1+\delta_{\mathrm{BL}})\,.$$

VVP Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor

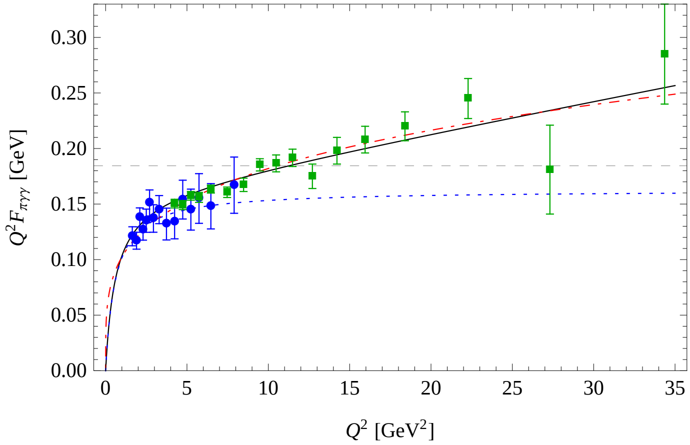


Figure: A plot of BABAR (green) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified Brodsky-Lepage condition. The full black line represents fit with $\delta_{\mathrm{BL}} = -0.055$, and blue dotted line is a fit with standard $\delta_{\mathrm{BL}} = 0$.

VVP Green function: Decays of $\pi(1300)$

- Two decay channels studied: $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$.

$$\mathcal{A}^{\mathrm{R}\chi\mathrm{T}}_{\pi(1300)\to\gamma\gamma} = e^2 \frac{8\sqrt{2}F_V}{3} \left(\frac{2\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^4} \right),$$

$$\mathcal{A}^{\mathrm{R}\chi\mathrm{T}}_{\pi(1300)\to\rho\gamma} = -e \frac{4\sqrt{2}}{3M_V} \left(\frac{\sqrt{2}\kappa_3^{PV} M_V^2 - F_V \kappa^{VVP}}{M_V^2} \right).$$

- Belle collaboration [K. Abe et al. '06] gives $\Gamma_{\pi(1300)\to\gamma\gamma} < 72\,\mathrm{eV}$ which leads to the estimate $\kappa^{VVP} \approx (-0.57 \pm 0.13)\,\mathrm{GeV}$.

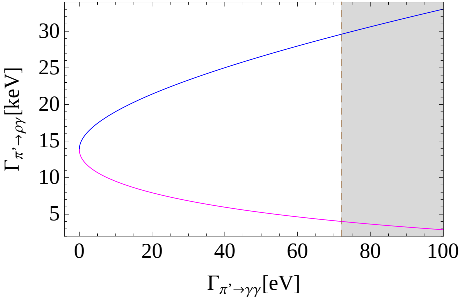


Figure: The connection of decay widths for $\pi(1300) \to \gamma\gamma$ and $\pi(1300) \to \rho\gamma$.

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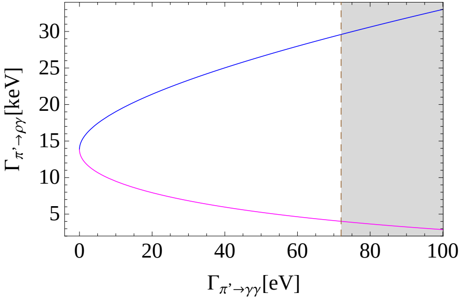

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VVP Green function: The muon $g-2$ factor

- Hadronic contributions: hadronic light-by-light scattering.
  - The main source of theoretical error in the SM prediction.
- The four point Green function $\langle VVVV \rangle$ can be simplified into:
  - $\pi^{\pm}$ and $K^{\pm}$ loops,
  - π^0, η, η' exchanges: the $\langle VVP \rangle$ case etc.
- Using the fully off-shell $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(p^2, q^2, r^2)$ formfactor we get:

$$a_\mu^{\mathrm{LbyL},\pi^0} = (65.8 \pm 1.2) \cdot 10^{-11}\,.$$

- The updated result [P. Roig, A. Guevara and G. L. Castro '14]:

$$a_\mu^{\pi^0} = (66.6 \pm 2.1) \cdot 10^{-11}\,.$$

VVA Green function

- The Ward identities restrict the general decomposition of the tensor part of VVA into four terms

$$\left(\Pi_{VVA}(p,q;r)\right)^{abc}_{\mu\nu\rho} = d^{abc}\Pi_{\mu\nu\rho}(p,q;r)\,,$$
$$\Pi_{\mu\nu\rho}(p,q;r) = w_L \varepsilon_{\mu\nu(p)(q)} r_\rho + w_T^{(1)}\Pi^{(1)}_{\mu\nu\rho} + w_T^{(2)}\Pi^{(2)}_{\mu\nu\rho} + w_T^{(3)}\Pi^{(3)}_{\mu\nu\rho}\,.$$

- The tensor part is nontrivial [M. Knecht, S. Peris, M. Perrottet and E. de Rafael '04]

$$\Pi^{(1)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} - q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho + \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(p-q)}\,,$$
$$\Pi^{(2)}_{\mu\nu\rho} = \varepsilon_{\mu\nu(p)(q)}(p-q)_\rho + \frac{p^2-q^2}{r^2}\varepsilon_{\mu\nu(p)(q)} r_\rho\,,$$
$$\Pi^{(3)}_{\mu\nu\rho} = p_\nu \varepsilon_{\mu\rho(p)(q)} + q_\mu \varepsilon_{\nu\rho(p)(q)} - \frac{p^2+q^2-r^2}{2}\varepsilon_{\mu\nu\rho(r)}\,.$$

VVA Green function

- Extracted formfactors [T. Kadavý, K. Kampf and J. Novotný '16]:

$$
\begin{aligned}
w_L &= \frac{N_c}{8\pi^2 r^2}\,, \\
w_T^{(1)} &= -\frac{2\sqrt{2}F_V\left[\kappa_{17}^V(p^2+q^2-2M_V^2)-\sqrt{2}F_V\kappa_3^{VV}\right]}{(p^2-M_V^2)(q^2-M_V^2)}\,, \\
w_T^{(2)} &= -\frac{2\sqrt{2}F_V(p^2-q^2)(2\kappa_{12}^V+\kappa_{16}^V-\kappa_{17}^V)}{(p^2-M_V^2)(q^2-M_V^2)}\,, \\
w_T^{(3)} &= \frac{2\sqrt{2}F_V(p^2-q^2)}{(p^2-M_V^2)(q^2-M_V^2)}\left(2\kappa_{11}^V+2\kappa_{12}^V-\kappa_{17}^V-\frac{\sqrt{2}F_A\kappa_5^{VA}}{r^2-M_A^2}\right).
\end{aligned}
$$

- Phenomenologically important formfactor $w_T(Q^2)$:

$$
\begin{aligned}
w_T(Q^2) &= -16\pi^2\left[w_T^{(1)}(-Q^2,0,-Q^2)+w_T^{(3)}(-Q^2,0,-Q^2)\right], \\
&= \frac{N_c}{M_V^2}+\frac{64\pi^2F_V}{M_V^2(Q^2+M_V^2)}\left[Q^2\left(\sqrt{2}(\kappa_{11}^V+\kappa_{12}^V)+\frac{F_A\kappa_5^{VA}}{Q^2+M_A^2}\right)-F_V\kappa_3^{VV}\right].
\end{aligned}
$$

VVA Green function: Coupling constants constraints

- Expand $w_T(Q^2)$ in terms of Q^2 up to $\mathcal{O}(\frac{1}{Q^8})$.
- Why?
    - Soft-wall AdS/QCD and OPE [J. J. Sanz-Cillero '12] and [P. Colangelo, F. De Fazio, J. J. Sanz-Cillero, F. Giannuzzi and S. Nicotri '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle\bar{q}q\rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

    - Two large momenta only!
- Comparison leads to a system of equations:

$$\frac{N_c}{64\pi^2 F_V} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = 0\,,$$

$$\frac{F_V\kappa_3^{VV} - F_A\kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) = -\frac{N_c}{64\pi^2 F_V}\,,$$

$$\frac{F_V\kappa_3^{VV} - F_A\kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A\kappa_5^{VA}\frac{M_A^2}{M_V^4} = 0\,,$$

$$\frac{F_V\kappa_3^{VV} - F_A\kappa_5^{VA}}{M_V^2} + \sqrt{2}(\kappa_{11}^V + \kappa_{12}^V) - F_A\kappa_5^{VA}\frac{M_A^2}{M_V^4}\left(1 + \frac{M_A^2}{M_V^2}\right) = -\frac{2\pi\alpha_s\chi\langle\bar{q}q\rangle^2}{9F_V M_V^4}\,.$$

VVA Green function: Coupling constants constraints

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VVA Green function: Coupling constants constraints

- It is possible to extract the following coupling constants constraints:

$$\kappa_{11}^{V} + \kappa_{12}^{V} = -\frac{N_c}{64\sqrt{2}\pi^2 F_V}\,, \quad \kappa_3^{VV} = -\frac{N_c M_V^4}{64\pi^2 M_A^2 F_V^2}\,, \quad \kappa_5^{VA} = \kappa_3^{VV}\frac{F_V}{F_A}\,.$$

  - Since it is not possible to solve the system of equations completely, the relevance of the constraints should be taken carefully!
- Determination of κ_5^{VA}:
  - Numerically: $\kappa_5^{VA} = -0.086$.
  - From the decay $f_1(1285) \to \rho\gamma$: $\kappa_5^{VA} = -0.062 \pm 0.030$.
- Using the constraints for VVP we can also determine:

$$\kappa_2^{VV} = \frac{1}{64F_V^2}\left(F^2 - \frac{N_c M_V^4}{8\pi^2 M_A^2}\right), \quad \kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}\left[1 + \frac{N_c M_V^2}{8\pi^2 F^2}\left(\frac{M_V^2}{M_A^2} - 1\right)\right].$$

  - Reminder: BABAR dictates $\kappa_3^{PV} = -\frac{F^2}{32\sqrt{2}d_m F_V}(1 + \delta_{\rm BL})$ with the value $\delta_{\rm BL} = -0,055 \pm 0.025$ from VVP.
- However, our prediction from VVA gives $\delta_{\rm BL} = -1.342\,$.

VVA Green function: $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}$ formfactor revisited

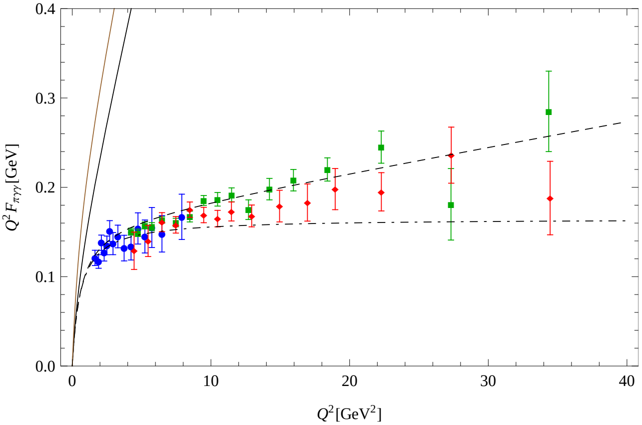


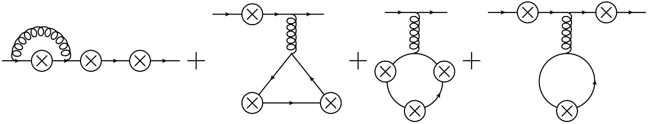
Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0, -Q^2; 0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{\mathrm{BL}} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

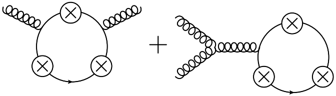
VVA Green functions: OPE

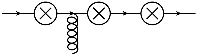
- VVA does not have the LO contribution to the OPE with all three momenta large, i.e.

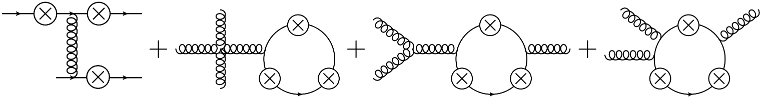
$\langle 0|$  $|0\rangle = 0\,.$

- Therefore, one needs to include other contributions from QCD condensates [T. Kadavý, K. Kampf and J. Novotný '16]:

$D = 3:$ 

$D = 4:$ 

$D = 5:$ 

$D = 6:$ 

Conclusion

- Some properties of low-energy QCD and Green functions were summarized.
- We study Green functions in the odd-intrinsic parity sector, calculated in the NLO, i.e. up to $\mathcal{O}(p^6)$.
- Properties of Green functions and their coupling constants are studied by:
  - High-energy behavior of Green functions.
  - Brodsky-Lepage behavior of the transition formfactor.
  - Matching calculations in RχT with χPT.
  - Experiments.
- Two specific correlators were shown:
  - VVP
    - Transition form factor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\to\gamma\gamma}(p^2,q^2,r^2)$, decays of $\pi(1300)$ and the contribution to the $g-2$ factor were shown.
  - VVA
    - Newest results were presented.
    - OPE with two large momenta is obviously inconsistent with reality, OPE with all three large momenta is needed (and in progress).

Thank you for your attention!

Questions?