

## TASI Lectures on Flavor Physics

Zoltan Ligeti

*Ernest Orlando Lawrence Berkeley National Laboratory,  
University of California, Berkeley, CA 94720*

*\*E-mail: ligeti@berkeley.edu*

These notes overlap with lectures given at the TASI summer schools in 2014 and 2011, as well as at the European School of High Energy Physics in 2013. This is primarily an attempt at transcribing my hand-written notes, with emphasis on topics and ideas discussed in the lectures. It is not a comprehensive introduction or review of the field, nor does it include a complete list of references. I hope, however, that some may find it useful to better understand the reasons for excitement about recent progress and future opportunities in flavor physics.

### Preface

There are many books and reviews on flavor physics (e.g., Refs. [1; 2; 3; 4; 5; 6; 7; 8; 9]). The main points I would like to explain in these lectures are:

- $CP$  violation and flavor-changing neutral currents (FCNC) are sensitive probes of short-distance physics, both in the standard model (SM) and in beyond standard model (BSM) scenarios.
- The data taught us a lot about not directly seen physics in the past, and are likely crucial to understand LHC new physics (NP) signals.
- In most FCNC processes  $BSM/SM \sim \mathcal{O}(20\%)$  is still allowed today, the sensitivity will improve to the few percent level in the future.
- Measurements are sensitive to very high scales, and might find unambiguous signals of BSM physics, even outside the LHC reach.
- There is a healthy and fun interplay of theoretical and experimental progress, with many open questions and important problems.

Flavor physics is interesting because there is a lot we do not understand yet. The “standard model flavor puzzle” refers to our lack of understanding of

why and how the 6 quark and 6 lepton flavors differ, why masses and quark mixing are hierarchical, but lepton mixing is not. The “new physics flavor puzzle” is the tension between the relatively low scale required to solve the fine tuning problem (also suggested by the WIMP paradigm), and the high scale that is seemingly required to suppress the non-SM contributions to flavor changing processes. If there is NP at the TeV scale, we need to understand why and in what way its flavor structure is non-generic.

The key questions and prospects that make the future interesting are [7]

- What is the achievable experimental precision?  
The LHCb, Belle II, NA62, KOTO,  $\mu \rightarrow e\gamma$ ,  $\mu 2e$ , etc., experiments will improve the sensitivity in many modes by orders of magnitude.
- What are the theoretical uncertainties?  
In many key measurements, the theory uncertainty is well below future experimental sensitivity; while in some cases theoretical improvements are needed (so you can make an impact!).
- How large deviations from SM can we expect due to TeV-scale NP?  
New physics with generic flavor structure is ruled out; observable effects near current bounds are possible, many models predict some.
- What will the measurements teach us?  
In all scenarios there is complementarity with high- $p_T$  measurements, and synergy in understanding the structure of any NP seen.

Another simple way to get a sense of (a lower bound on) the next 10–15 years of  $B$  physics progress is to consider the expected increase in data,

$$\frac{(\text{LHCb upgrade})}{(\text{LHCb } 1 \text{ fb}^{-1})} \sim \frac{(\text{Belle II data set})}{(\text{Belle data set})} \sim \frac{(\text{2009 BaBar data set})}{(\text{1999 CLEO data set})} \sim 50.$$

This will yield a  $\sqrt[4]{50} \sim 2.5$  increase in sensitivity to higher mass scales, even just by redoing existing measurements. More data has always motivated new theory ideas, yielding even faster progress. This is a comparable increase in reach as going from LHC7–8  $\rightarrow$  LHC13–14.

**Outline** The topics these lectures will cover include a brief introduction to flavor physics in the SM, testing the flavor structure in neutral meson mixing and  $CP$  violation, and examples of how to get theoretically clean information on short-distance physics. After a glimpse at the ingredients of the SM CKM fit, we discuss how sizable new physics contributions are still allowed in neutral meson mixing, and how this will improve in the future. Then we explain some implications of the heavy quark limit, tidbits of heavy quark symmetry, the operator product expansion and inclusive decays, to

try to give an impression of what makes some hadronic physics tractable. The last lecture discusses some topics in TeV-scale flavor physics, top quark physics, Higgs flavor physics, bits of the interplay between searches for supersymmetry and flavor, and comments on minimal flavor violation. Some questions one may enjoy thinking about are in the footnotes.

## 1. Introduction to Flavor Physics and $CP$ Violation

Most of the experimentally observed particle physics phenomena are consistent with the standard model (SM). Evidence that the minimal SM is incomplete comes from the lack of a dark matter candidate, the baryon asymmetry of the Universe, its accelerating expansion, and nonzero neutrino masses. The baryon asymmetry and neutrino mixing are certainly connected to  $CP$  violation and flavor physics, and so may be dark matter. The hierarchy problem and seeking to identify the particle nature of dark matter strongly motivate TeV-scale new physics.

Studying flavor physics and  $CP$  violation provides a rich program to probe the SM and search for NP, with sensitivity to the  $1 - 10^5$  TeV scales, depending on details of the models. As we shall see, the sensitivity to BSM contributions to the dimension-6 four-quark operators mediating  $K$ ,  $D$ ,  $B_d$ , and  $B_s$  mixing, when parametrized by coefficients  $1/\Lambda^2$ , corresponds to scales  $\Lambda \sim 10^2 - 10^5$  TeV (see Table 1 and the related discussion below).

Understanding the origin of this sensitivity and how it can be improved, requires going into the details of a variety of flavor physics measurements.

**Baryon asymmetry requires  $CP$  violation beyond SM** The baryon asymmetry of the Universe is the measurement of

$$\frac{n_B - n_{\bar{B}}}{s} \approx 10^{-10}, \quad (1)$$

where  $n_B$  ( $n_{\bar{B}}$ ) is the number density of (anti-)baryons and  $s$  is the entropy density. This means that  $10^{-6}$  seconds after the Big Bang, when the temperature was  $T > 1$  GeV, and quarks and antiquarks were in thermal equilibrium, there was a corresponding asymmetry between quarks and antiquarks. Sakharov pointed out [10] that for a theory to generate such an asymmetry in the course of its evolution from a hot Big Bang (assuming inflation washed out any possible prior asymmetry), it must contain:

- (1) baryon number violating interactions;
- (2)  $C$  and  $CP$  violation;
- (3) deviation from thermal equilibrium.

Interestingly, the SM contains 1–2–3, but (i)  $CP$  violation is too small, and (ii) the deviation from thermal equilibrium is too small at the electroweak phase transition. The SM expectation is many orders of magnitude below the observation, due to the suppression of  $CP$  violation by

$$[\prod_{u_i \neq u_j} (m_{u_i}^2 - m_{u_j}^2)] [\prod_{d_i \neq d_j} (m_{d_i}^2 - m_{d_j}^2)] / m_W^{12}, \quad (2)$$

and  $m_W$  indicates a typical weak interaction scale here.<sup>a</sup>

Therefore,  $CP$  violation beyond the SM must exist. While this argument does not tell us the scale of the corresponding new physics, it motivates searching for new sources of  $CP$  violation. (It may occur only in flavor-diagonal processes, such as EDMs, or only in the lepton sector, as in leptogenesis.) In any case, we want to understand the microscopic origin of  $CP$  violation, and how precisely we can test those  $CP$ -violating processes that we can measure.

Equally important is that almost all TeV-scale new physics models contain new sources of  $CP$  violation. Baryogenesis at the electroweak scale may still be viable, and the LHC will probe the remaining parameter space.

**The SM and flavor** The SM is defined by the gauge interactions,

$$SU(3)_c \times SU(2)_L \times U(1)_Y, \quad (3)$$

the particle content, i.e., three generations of the fermion representations,

$$Q_L(3, 2)_{1/6}, \quad u_R(3, 1)_{2/3}, \quad d_R(3, 1)_{-1/3}, \quad L_L(1, 2)_{-1/2}, \quad \ell_R(1, 1)_{-1}, \quad (4)$$

and electroweak symmetry breaking. A condensate  $\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$  breaks  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ , the dynamics of which we now know is well approximated by a seemingly elementary SM-like scalar Higgs field.

The kinetic terms in the SM Lagrangian are

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} \sum_{\text{groups}} (F_{\mu\nu}^a)^2 + \sum_{\text{rep's}} \bar{\psi} i \not{D} \psi. \quad (5)$$

These are always  $CP$  conserving, as long as we neglect a possible  $F\tilde{F}$  term. The “strong  $CP$  problem” [11] is the issue of why the coefficient of the  $F\tilde{F}$  term for QCD is tiny. Its solution is an open question; however, we know that it is negligible for flavor-changing processes. The Higgs terms,

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \quad (6)$$

<sup>a</sup>Why is this suppression a product of all up and down quark mass differences, while fewer factors of mass splittings suppress  $CP$  violation in hadron decays and meson mixings?

are  $CP$  conserving in the SM, but can be  $CP$  violating with an extended Higgs sector (already with two Higgs doublets; three are needed if natural flavor conservation is imposed [12]). Finally, the Yukawa couplings are,

$$\mathcal{L}_Y = -Y_{ij}^d \overline{Q_{Li}^I} \phi d_{Rj}^I - Y_{ij}^u \overline{Q_{Li}^I} \tilde{\phi} u_{Rj}^I - Y_{ij}^\ell \overline{L_{Li}^I} \phi \ell_{Rj}^I + \text{h.c.} \quad (7)$$

The  $Y_{u,d}^{ij}$  are  $3 \times 3$  complex matrices,  $i, j$  are generation indices,  $\tilde{\phi} = i\sigma_2 \phi^*$ .

After electroweak symmetry breaking, Eq. (7) gives quark mass terms,

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\overline{d_{Li}^I} (M_d)_{ij} d_{Rj}^I - \overline{u_{Li}^I} (M_u)_{ij} u_{Rj}^I + \text{h.c.} \\ &= -(\overline{d_L^I} V_{dL}^\dagger) (V_{dL} M_d V_{dR}^\dagger) (V_{dR} d_R^I) \\ &\quad - (\overline{u_L^I} V_{uL}^\dagger) (V_{uL} M_u V_{uR}^\dagger) (V_{uR} u_R^I) + \text{h.c.}, \end{aligned} \quad (8)$$

where  $M_f = (v/\sqrt{2}) Y^f$ . The last two lines show the diagonalization of the mass matrices necessary to obtain the physical mass eigenstates,

$$M_f^{\text{diag}} \equiv V_{fL} M_f V_{fR}^\dagger, \quad f_{Li} \equiv V_{fL}^{ij} f_{Lj}^I, \quad f_{Ri} \equiv V_{fR}^{ij} f_{Rj}^I, \quad (9)$$

where  $f = u, d$  denote up- and down-type quarks. The diagonalization is different for  $u_{Li}$  and  $d_{Li}$ , which are in the same  $SU(2)_L$  doublet,

$$\begin{pmatrix} u_{Li}^I \\ d_{Li}^I \end{pmatrix} = (V_{uL}^\dagger)_{ij} \begin{pmatrix} u_{Lj} \\ (V_{uL} V_{dL}^\dagger)_{jk} d_{Lk} \end{pmatrix}. \quad (10)$$

The ‘‘misalignment’’ between these two transformations,

$$V_{\text{CKM}} \equiv V_{uL} V_{dL}^\dagger, \quad (11)$$

is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. By virtue of Eq. (11), it is unitary.

Eq. (10) shows that the charged current weak interactions, which arise from the  $\overline{\psi} i \not{D} \psi$  terms in Eq. (5), become non-diagonal in the mass basis

$$-\frac{g}{2} \overline{Q_{Li}^I} \gamma^\mu W_\mu^a \tau^a Q_{Li}^I + \text{h.c.} \Rightarrow -\frac{g}{\sqrt{2}} (\overline{u_L}, \overline{c_L}, \overline{t_L}) \gamma^\mu W_\mu^+ V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}, \quad (12)$$

where  $W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$ . Thus, charged-current weak interactions change flavor, and this is the only flavor-changing interaction in the SM.

In the absence of Yukawa couplings, the SM has a global  $[U(3)]^5$  symmetry ( $[U(3)]^3$  in the quark and  $[U(3)]^2$  in the lepton sector), rotating the 3 generations of the 5 fields in Eq. (4). This is broken by the Yukawa interactions in Eq. (7). In the quark sector the breaking is

$$U(3)_Q \times U(3)_u \times U(3)_d \rightarrow U(1)_B, \quad (13)$$

In the lepton sector, we do not yet know if  $U(3)_L \times U(3)_\ell$  is fully broken.

**Flavor and  $CP$  violation in the SM** Since the  $Z$  couples flavor diagonally,<sup>b</sup> there are no tree-level flavor-changing neutral currents, such as  $K_L \rightarrow \mu^+ \mu^-$ . This led GIM [13] to predict the existence of the charm quark. Similarly,  $K^0 - \bar{K}^0$  mixing vanishes at tree-level, which allowed the prediction of  $m_c$  [14; 15] before the discovery of the charm quark. In the previous examples, because of the unitarity of the CKM matrix,

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0. \quad (14)$$

Expanding the loop functions, e.g., in a FCNC kaon decay amplitude,

$$V_{ud} V_{us}^* f(m_u) + V_{cd} V_{cs}^* f(m_c) + V_{td} V_{ts}^* f(m_t), \quad (15)$$

the result is always proportional to the up-quark mass-squared differences,

$$\frac{m_i^2 - m_j^2}{m_W^2}. \quad (16)$$

So FCNCs probe directly the differences between the generations.

One can also see that  $CP$  violation is related to irremovable phases of Yukawa couplings. Starting from a term in Eq. (7),

$$Y_{ij} \overline{\psi_{Li}} \phi \psi_{Rj} + Y_{ij}^* \overline{\psi_{Rj}} \phi^\dagger \psi_{Li} \xrightarrow{CP} Y_{ij} \overline{\psi_{Rj}} \phi^\dagger \psi_{Li} + Y_{ij}^* \overline{\psi_{Li}} \phi \psi_{Rj}. \quad (17)$$

The two expressions are identical if and only if a basis for the quark fields can be chosen such that  $Y_{ij} = Y_{ij}^*$ , i.e., that  $Y_{ij}$  are real.

**Counting flavor parameters** Most parameters of the SM (and also of many of its extensions) are related to flavor. In the CKM matrix, due to unitarity, 9 complex elements depend on 9 real parameters. Of these 5 phases can be absorbed by redefining the quark fields, leaving 4 physical parameters, 3 mixing angles and 1  $CP$  violating phase. This is the only source of  $CP$  violation in flavor-changing transitions in the SM.

A more general way to account for all flavor parameters is to consider that the two Yukawa matrices,  $Y_{i,j}^{u,d}$  in Eq. (7), contain 18 real and 18 imaginary parameters. They break the global  $[U(3)]^3 \rightarrow U(1)_B$ , see Eq. (13), so there are 26 broken generators (9 real and 17 imaginary). This leaves 10 physical quark flavor parameters: 9 real ones (the 6 quark masses and 3 mixing angles) and 1 complex  $CP$  violating phase.<sup>c</sup>

<sup>b</sup>Show that there are no tree-level flavor-changing  $Z$  couplings in the SM. What if, besides doublets, there were a left-handed  $SU(2)$  singlet quark field as well?

<sup>c</sup>Show that for  $N$  generations, the CKM matrix depends on  $N(N-1)/2$  mixing angles and  $(N-1)(N-2)/2$   $CP$  violating phases. So the 2-generation SM conserves  $CP$ .

**Neutrino masses** How does lepton flavor differ? With the particle content in Eq. (4), it is not possible to write down a renormalizable mass term for neutrinos. It would require introducing a  $\nu_R(1, 1)_0$  field, a singlet under all SM gauge groups, to be light, which is unexpected. Such a particle is sometimes called a sterile neutrino, as it has no SM interactions. Whether there are such fields can only be decided experimentally.

Viewing the SM as a low energy effective theory, there is a single type of dimension-5 gauge invariant term made of SM fields,

$$\mathcal{L}_Y = -\frac{Y_\nu^{ij}}{\Lambda_{\text{NP}}} L_{Li}^I L_{Lj}^I \phi \phi. \quad (18)$$

This term gives rise to neutrino masses and also violates lepton number. Its suppression cannot be the electroweak scale,  $1/v$  (instead of  $1/\Lambda_{\text{NP}}$ ), because such a term in the Lagrangian cannot be generated from SM fields at arbitrary loop level, or even nonperturbatively. [Eq. (18) violates  $B - L$ , which is an accidental symmetry of the SM that is not anomalous.] The above mass term is called a Majorana mass, as it couples  $\bar{\nu}_L$  to  $(\nu_L)^c$  instead of  $\nu_R$  [the latter occurs for Dirac mass terms, see Eq. (8)]. The key distinction is whether lepton number is violated or conserved. In the presence of Eq. (18) and the charged lepton Yukawa coupling in the last term in Eq. (7), the global  $U(3)_L \times U(3)_e$  symmetry is completely broken, and the counting of lepton flavor parameters is<sup>d</sup>

$$(12 + 18 \text{ couplings}) - (18 \text{ broken sym.}) \Rightarrow 12 \text{ physical parameters.} \quad (19)$$

These are the 6 masses, 3 mixing angles, and 3  $CP$  violating phases, of which one is the analog of the CKM phase measurable in oscillation experiments, while two additional “Majorana phases” only contribute to lepton number violating processes, such as neutrinoless double beta decay.<sup>e</sup>

**The CKM matrix** Quark mixing is observed to be approximately flavor diagonal. The Wolfenstein parametrization conveniently exhibits this,

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \dots, \quad (20)$$

<sup>d</sup>Show that the Yukawa matrix in Eq. (18) is symmetric,  $Y_\nu^{ij} = Y_\nu^{ji}$ . Derive that for  $N$  such generations there are  $N(N-1)/2$   $CP$  violating phases.

<sup>e</sup>Can you think of ways to get sensitivity to another linear combination of the two  $CP$  violating Majorana phases, besides the one that enters neutrinoless double beta decay?

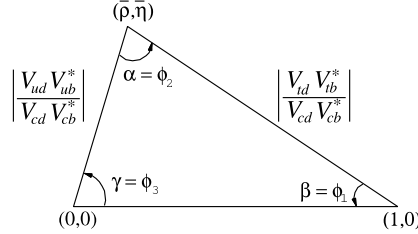


Fig. 1. The unitarity triangle.

where  $\lambda \simeq 0.23$  may be viewed as an “expansion parameter”. It is a useful book-keeping of the magnitudes of the CKM matrix elements, but it hides which combination of CKM elements are phase-convention independent. Sometimes it can be useful to think of  $V_{ub}$  and  $V_{td}$  as the ones with  $\mathcal{O}(1)$   $CP$  violating phases, but it is important that any  $CP$  violating observable in the SM must depend on at least four CKM elements.<sup>f</sup>

In any case, the interesting question is not primarily measuring CKM elements, but testing how precisely the SM description of flavor and  $CP$  violation holds. This can be done by “redundant” measurements, which in the SM relate to some combination of flavor parameters, but are sensitive to different BSM physics, thus testing for (in)consistency. Since there are many experimental constraints, a simple way to compare different measurements can be very useful. Recall that CKM unitarity implies

$$\sum_k V_{ik}V_{jk}^* = \sum_k V_{ki}V_{kj}^* = \delta_{ij}, \tag{21}$$

and the 6 vanishing relations can be represented as triangles in a complex plane. The most often used such “unitarity triangle” (shown in Fig. 1) arises from the scalar product of the 1st and 3rd columns,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{22}$$

(Unitarity triangles constructed from neighboring columns or rows are “squashed”.) We define the  $\alpha, \beta, \gamma$  angles of this triangle, and two more,

$$\begin{aligned} \alpha &\equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), & \beta &\equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), & \gamma &\equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \\ \beta_s &\equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right), & \beta_K &\equiv \arg\left(-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right). \end{aligned} \tag{23}$$

<sup>f</sup>Prove this statement. Are there constraints on which four?



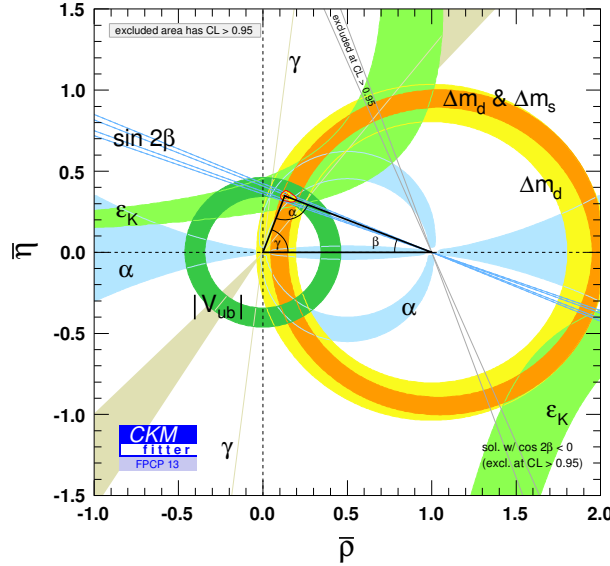


Fig. 2. The SM CKM fit, and individual constraints (colored regions show 95% CL).

On different continents the  $\phi_1 = \beta$ ,  $\phi_2 = \alpha$ ,  $\phi_3 = \gamma$ , and/or the  $\phi_s = -2\beta_s$  notations are used. Here  $\beta_s$  ( $\beta_K$ ), of order  $\lambda^2$  ( $\lambda^4$ ), is the small angle of a “squashed” unitarity triangle obtained by multiplying the 2nd column of the CKM matrix with the 3rd (1st) column.

The magnitudes of CKM elements determine the sides of the unitarity triangle. They are mainly extracted from semileptonic and leptonic  $K$  and  $B$  decays, and  $B_{d,s}$  mixing. Any constraint which renders the area of the unitarity triangle nonzero, such as angles, has to measure  $CP$  violation. Some of the most important constraints are shown in Fig. 2, together with the CKM fit in the SM. (Using  $\bar{\rho}$ ,  $\bar{\eta}$  instead of  $\rho$ ,  $\eta$  simply corresponds to a small modification of the parametrization, to keep unitarity exact.)

**The low energy effective field theory (EFT) viewpoint** At the few GeV scale, relevant for  $B$ ,  $D$ , and some  $K$  decays, all flavor changing processes (both tree and loop level) are mediated by dozens of higher dimension local operators. They arise from integrating out heavy particles,  $W$  and  $Z$  bosons and the  $t$  quark in the SM, or not yet observed heavy states (see Fig. 3). Since the coefficients of a large number of operators depend on just a few parameters in the SM, there are many correlations between decays of

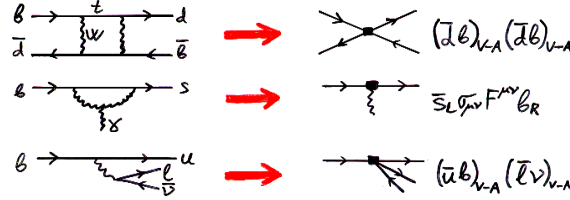


Fig. 3. Diagrams at the electroweak scale (left) and operators at the scale  $m_b$  (right).

hadrons containing  $s, c, b$  quarks, which NP may violate. From this point of view there is no difference between flavor-changing neutral currents and  $\Delta F = 1$  processes, as all flavor-changing processes are due to heavy particles with masses  $\gg m_{s,c,b}$ . Thus, one can test the SM in many ways by asking (i) does NP modify the coefficients of dimension-6 operators? (ii) does NP generate operators absent in the SM (e.g., right-handed couplings)?

**Neutral meson mixing** Let us first sketch a back-of-an-envelope estimate of the mass difference in  $K^0 - \bar{K}^0$  mixing. In the SM,

$$\Delta m_K \sim \alpha_w^2 |V_{cs}V_{cd}|^2 \frac{m_c^2 - m_u^2}{m_W^4} f_K^2 m_K. \quad (24)$$

The result is suppressed by CKM angles, a loop factor, the weak coupling, and the GIM mechanism. If a heavy particle,  $X$ , contributes  $\mathcal{O}(1)$  to  $\Delta m_K$ ,

$$\left| \frac{\Delta m_K^{(X)}}{\Delta m_K^{(\text{exp})}} \right| \sim \left| \frac{g^2 \Lambda_{\text{QCD}}^3}{M_X^2 \Delta m_K^{(\text{exp})}} \right| \Rightarrow \frac{M_X}{g} \gtrsim 2 \times 10^3 \text{ TeV}. \quad (25)$$

So even TeV-scale particles with loop-suppressed couplings [ $g \sim \mathcal{O}(10^{-3})$ ] can give observable effects. This illustrates that flavor physics measurements indeed probe the TeV scale if NP has SM-like flavor structure, and much higher scales if the NP flavor structure is generic.

A more careful evaluation of the bounds in all four neutral meson systems is shown in Table 1. (See Sec. 2 for the definitions of the observables in the  $B$  meson systems.) If  $\Lambda = \mathcal{O}(1 \text{ TeV})$  then  $C \ll 1$ , and if  $C = \mathcal{O}(1)$  then  $\Lambda \gg 1 \text{ TeV}$ . The bounds are weakest for  $B_{(s)}$  mesons, as mixing is the least suppressed in the SM in that case. The bounds on many NP models are the strongest from  $\Delta m_K$  and  $\epsilon_K$ , since so are the SM suppressions. These are built into NP models since the 1970s, otherwise the models are immediately excluded. In the SM, larger FCNCs and  $CP$  violating effects occur in  $B$  mesons, which can be measured precisely. In many BSM models

Table 1. Bounds on some  $\Delta F = 2$  operators,  $(C/\Lambda^2)\mathcal{O}$ , with  $\mathcal{O}$  given in the first column. The bounds on  $\Lambda$  assume  $C = 1$ , the bounds on  $C$  assume  $\Lambda = 1$  TeV. (From Ref. [19].)

Operator	Bound on $\Lambda$ [TeV] ( $C = 1$ )		Bound on $C$ ( $\Lambda = 1$ TeV)		Observables
	Re	Im	Re	Im	
$(\bar{s}_L \gamma^\mu d_L)^2$	$9.8 \times 10^2$	$1.6 \times 10^4$	$9.0 \times 10^{-7}$	$3.4 \times 10^{-9}$	$\Delta m_K; \epsilon_K$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$1.8 \times 10^4$	$3.2 \times 10^5$	$6.9 \times 10^{-9}$	$2.6 \times 10^{-11}$	$\Delta m_K; \epsilon_K$
$(\bar{c}_L \gamma^\mu u_L)^2$	$1.2 \times 10^3$	$2.9 \times 10^3$	$5.6 \times 10^{-7}$	$1.0 \times 10^{-7}$	$\Delta m_D;  q/p , \phi_D$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$6.2 \times 10^3$	$1.5 \times 10^4$	$5.7 \times 10^{-8}$	$1.1 \times 10^{-8}$	$\Delta m_D;  q/p , \phi_D$
$(\bar{b}_L \gamma^\mu d_L)^2$	$6.6 \times 10^2$	$9.3 \times 10^2$	$2.3 \times 10^{-6}$	$1.1 \times 10^{-6}$	$\Delta m_{B_d}; S_{\psi K_S}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$2.5 \times 10^3$	$3.6 \times 10^3$	$3.9 \times 10^{-7}$	$1.9 \times 10^{-7}$	$\Delta m_{B_d}; S_{\psi K_S}$
$(\bar{b}_L \gamma^\mu s_L)^2$	$1.4 \times 10^2$	$2.5 \times 10^2$	$5.0 \times 10^{-5}$	$1.7 \times 10^{-5}$	$\Delta m_{B_s}; S_{\psi\phi}$
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	$4.8 \times 10^2$	$8.3 \times 10^2$	$8.8 \times 10^{-6}$	$2.9 \times 10^{-6}$	$\Delta m_{B_s}; S_{\psi\phi}$

the 3rd generation is significantly different than the first two, motivated by the large top Yukawa, and may give larger signals in the  $B$  sector.

**A few more words on kaons** With recent lattice QCD progress on  $B_K$  and  $f_K$  [16],  $\epsilon_K$  has become a fairly precise constraint on the SM. However,  $\epsilon'_K$  is notoriously hard to calculate, involving cancellation between two comparable terms, each with sizable uncertainties. (Lattice QCD calculations of the hadronic matrix elements for  $\epsilon'_K$  may be reliably computed in the future.) At present, we cannot prove nor rule out that a large part of the observed value of  $\epsilon'_K$  is due to BSM. Thus, to test  $CP$  violation, one had to consider other systems; it was realized in the 1980s that many precise measurements of  $CP$  violation are possible in  $B$  decays.

In the kaon sector, precise calculations of rare decays involving neutrinos (see Fig. 4) are possible, and the SM predictions are [17]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11}, \quad \mathcal{B}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}. \tag{26}$$

The  $K_L^0$  decay is  $CP$  violating, and therefore it is under especially good theoretical control, since it is determined by the top quark loop contribution, and the  $CP$  conserving charm quark contribution is absent (which enters  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , and is subject to some hadronic uncertainty).

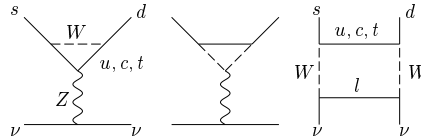


Fig. 4. Diagrams contributing to  $K \rightarrow \pi \nu \bar{\nu}$  decay.

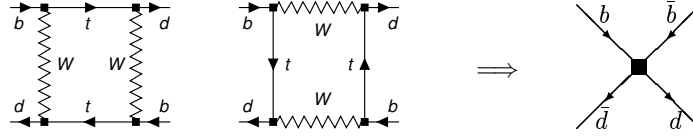


Fig. 5. Left: box diagrams that give rise to the  $B^0 - \bar{B}^0$  mass difference; Right: operator in the effective theory below  $m_W$  whose  $B$  meson matrix element determines  $\Delta m_B$ .

The E787/E949 measurement is  $\mathcal{B}(K \rightarrow \pi^+ \nu \bar{\nu}) = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$  [18], whereas in the  $K_L$  mode the experimental upper bound is still many times the SM rate. NA62 at CERN aims to measure the  $K^+$  rate with 10% uncertainty, and will start to have dozens of events in 2015. The  $K_L$  mode will probably be first observed by the KOTO experiment at J-PARC.

**2. Theory of Some Important  $B$  Decays**

Studying FCNC and  $CP$  violation is particularly interesting in  $B$  meson decays, because many measurements are possible with clean interpretations.

The main theoretical reasons are: (i)  $t$  quark loops are neither GIM nor CKM suppressed; (ii) large  $CP$  violating effects are possible; (iii) some of the hadronic physics is understandable model independently ( $m_b \gg \Lambda_{\text{QCD}}$ ).

The main experimental reasons are: (i) the long  $B$  lifetime (small  $|V_{cb}|$ ); (ii) the  $\Upsilon(4S)$  is a clean source of  $B$  mesons at  $e^+e^-$  colliders; (iii) for  $B_d$ , the ratio  $\Delta m/\Gamma = \mathcal{O}(1)$ .

**Neutral meson mixing formalism** Similar to neutral kaons, there are two neutral  $B^0$  meson flavor eigenstates,

$$|B^0\rangle = |\bar{b}d\rangle, \quad |\bar{B}^0\rangle = |b\bar{d}\rangle. \tag{27}$$

They mix in the SM due to weak interactions (see Fig. 5). The time evolutions of the two states are described by the Schrödinger equation,

$$i \frac{d}{dt} \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \left( M - \frac{i}{2} \Gamma \right) \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix}, \tag{28}$$

where the mass ( $M$ ) and the decay ( $\Gamma$ ) mixing matrices are  $2 \times 2$  Hermitian matrices.  $CPT$  invariance implies  $M_{11} = M_{22}$  and  $\Gamma_{11} = \Gamma_{22}$ . The heavier and lighter mass eigenstates are the eigenvectors of  $M - i\Gamma/2$ ,

$$|B_{H,L}\rangle = p |B^0\rangle \mp q |\bar{B}^0\rangle, \tag{29}$$

and their time dependence is

$$|B_{H,L}(t)\rangle = e^{-(im_{H,L} + \Gamma_{H,L}/2)t} |B_{H,L}\rangle. \quad (30)$$

Here  $\Delta m \equiv m_H - m_L$  and  $\Delta\Gamma = \Gamma_L - \Gamma_H$  are the mass and width differences. This defines  $\Delta m$  to be positive, but the sign of  $\Delta\Gamma$  is physical. Note that  $m_{H,L}$  ( $\Gamma_{H,L}$ ) are not the eigenvalues of  $M$  ( $\Gamma$ ).<sup>§</sup> The off-diagonal elements,  $M_{12}$  and  $\Gamma_{12}$ , arise from virtual and on-shell intermediate states, respectively. In the SM,  $M_{12}$  is dominated by the top-quark box diagrams in Fig. 5. Thus,  $M_{12}$  is determined by short-distance physics, it is calculable with good accuracy, and is sensitive to high scales. (This is the complication for  $D$  mixing: the  $W$  can always be shrunk to a point, but the  $d$  and  $s$  quarks in the box diagrams cannot, so long-distance effects are important.) The width difference  $\Gamma_{12}$  is determined by on-shell states to which both  $B^0$  and  $\bar{B}^0$  can decay, corresponding to  $c$  and  $u$  quarks in the box diagrams.

The solution of the eigenvalue equation is

$$\begin{aligned} (\Delta m)^2 - \frac{(\Delta\Gamma)^2}{4} &= 4|M_{12}|^2 - |\Gamma_{12}|^2, & \Delta m \Delta\Gamma &= -4 \operatorname{Re}(M_{12}\Gamma_{12}^*), \\ \frac{q}{p} &= -\frac{\Delta m + i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m + i\Delta\Gamma/2}. \end{aligned} \quad (33)$$

The physical observables that are measurable in neutral meson mixing are

$$x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta\Gamma}{2\Gamma}, \quad \left| \frac{q}{p} \right| = 1. \quad (34)$$

The orders of magnitudes of the SM predictions are shown in Table 2. That  $x \neq 0$  is established in the  $K$ ,  $B$ , and  $B_s$  mixing;  $y \neq 0$  in the  $K$ ,  $D$ , and  $B_s$  mixing;  $|q/p| \neq 1$  in  $K$  mixing. The significance of  $x_D \neq 0$  is  $\sim 2\sigma$ , and in  $B_{d,s}$  mixing there is an unconfirmed  $D\bar{O}$  signal for  $|q/p| \neq 1$ ; more below.

Simpler approximate solutions can be obtained expanding about the limit  $|\Gamma_{12}| \ll |M_{12}|$ . This is a good approximation in both  $B_d$  and  $B_s$  systems.  $|\Gamma_{12}| < \Gamma$  always holds, because  $\Gamma_{12}$  arises from decays to final

<sup>§</sup>Derive that the time evolutions of mesons that are  $B^0$  and  $\bar{B}^0$  at  $t = 0$  are given by

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle, \quad |\bar{B}^0(t)\rangle = \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle, \quad (31)$$

where, denoting  $m = (m_H + m_L)/2$  and  $\Gamma = (\Gamma_H + \Gamma_L)/2$ ,

$$\begin{aligned} g_+(t) &= e^{-it(m-i\Gamma/2)} \left( \cosh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta m t}{2} - i \sinh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta m t}{2} \right), \\ g_-(t) &= e^{-it(m-i\Gamma/2)} \left( -\sinh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta m t}{2} + i \cosh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta m t}{2} \right). \end{aligned} \quad (32)$$

Table 2. Orders of magnitudes of the SM predictions for mixing parameters. The uncertainty of  $(|q/p| - 1)_D$  is especially large.

meson	$x = \Delta m/\Gamma$	$y = \Delta\Gamma/(2\Gamma)$	$ q/p  - 1$
$K$	1	1	$10^{-3}$
$D$	$10^{-2}$	$10^{-2}$	$10^{-3}$
$B_d$	1	$10^{-2}$	$10^{-4}$
$B_s$	$10^1$	$10^{-1}$	$10^{-5}$

states common to  $B^0$  and  $\bar{B}^0$ . For  $B_s$  mixing the world average is  $\Delta\Gamma_s/\Gamma_s = 0.138 \pm 0.012$  [20], while  $\Delta\Gamma_d$  is expected to be  $\sim 20$  times smaller and is not yet measured. Up to higher order terms in  $|\Gamma_{12}/M_{12}|$ , Eqs. (33) become

$$\Delta m = 2|M_{12}|, \quad \Delta\Gamma = -2 \frac{\text{Re}(M_{12}\Gamma_{12}^*)}{|M_{12}|},$$

$$\frac{q}{p} = -\frac{M_{12}^*}{|M_{12}|} \left( 1 - \frac{1}{2} \text{Im} \frac{\Gamma_{12}}{M_{12}} \right), \quad (35)$$

where we kept the second term in  $q/p$ , as it will be needed later.

**$CP$  violation in decay** This is any form of  $CP$  violation that cannot be absorbed in a neutral meson mixing amplitude (also called direct  $CP$  violation). It can occur in any hadron decay, as opposed to those specific to neutral mesons discussed below. For a given final state,  $f$ , the  $B \rightarrow f$  and  $\bar{B} \rightarrow \bar{f}$  decay amplitudes can, in general, receive several contributions

$$A_f = \langle f | \mathcal{H} | B \rangle = \sum_k A_k e^{i\delta_k} e^{i\phi_k}, \quad \bar{A}_{\bar{f}} = \langle \bar{f} | \mathcal{H} | \bar{B} \rangle = \sum_k A_k e^{i\delta_k} e^{-i\phi_k}. \quad (36)$$

There are two types of complex phases. Complex parameters in the Lagrangian which enter a decay amplitude also enter the  $CP$  conjugate amplitude but in complex conjugate form. In the SM such “weak phases”,  $\phi_k$ , only occur in the CKM matrix. Another type of phase is due to absorptive parts of decay amplitudes, and gives rise to  $CP$  conserving “strong phases”,  $\delta_k$ . These phases arise from on-shell intermediate states rescattering into the desired final state, and they are the same for an amplitude and its  $CP$  conjugate. The individual phases  $\delta_k$  and  $\phi_k$  are convention dependent, but the phase differences,  $\delta_i - \delta_j$  and  $\phi_i - \phi_j$ , and therefore  $|\bar{A}_{\bar{f}}|$  and  $|A_f|$ , are physical. Clearly, if  $|\bar{A}_{\bar{f}}| \neq |A_f|$  then  $CP$  is violated; this is called  $CP$  violation in decay, or direct  $CP$  violation.<sup>h</sup>

<sup>h</sup>Derive that direct  $CP$  violation requires interference of at least two contributing amplitudes with different strong and weak phases,  $|\bar{A}|^2 - |A|^2 = 4A_1A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)$ .

There are many measurements of direct  $CP$  violation. While some give strong constraints on NP models which evade the SM suppressions (e.g.,  $\epsilon'_K$ , the first direct  $CP$  violation measured with high significance), at present no single direct  $CP$  violation measurement gives a precise test of the SM, due to the lack of reliable calculations of relevant strong phases. For all observations of direct  $CP$  violation in a single decay mode, viewed in isolation [see the caveat near Eq. (42)], it is possible that, say, half of the measured value is from BSM. For  $\epsilon'_K$ , lattice QCD may yield progress in the future. In certain  $B$  decays we may better understand the implications of the heavy quark limit; so far  $A_{K^+\pi^0} - A_{K^+\pi^-} = 0.12 \pm 0.02$  [20], the “ $K\pi$  puzzle”, is poorly understood.

**$CP$  violation in mixing** If  $CP$  were conserved, the mass and  $CP$  eigenstates would coincide, and the mass eigenstates would be proportional to  $|B^0\rangle \pm |\bar{B}^0\rangle$ , up to phases; i.e.,  $|q/p| = 1$  and  $\arg(M_{12}/\Gamma_{12}) = 0$ . If  $|q/p| \neq 1$ , then  $CP$  is violated. This is called  $CP$  violation in mixing. It follows from Eq. (29) that  $\langle B_H | B_L \rangle = |p|^2 - |q|^2$ , so if  $CP$  is violated in mixing, the physical states are not orthogonal. (This illustrates again that  $CP$  violation is a quantum mechanical effect, impossible in a classical system.) The simplest example is the  $CP$  asymmetry in semileptonic decay of neutral mesons to “wrong sign” leptons (Fig. 6 summarizes the data),

$$A_{\text{SL}}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow \ell^+ X) - \Gamma(B^0(t) \rightarrow \ell^- X)}{\Gamma(\bar{B}^0(t) \rightarrow \ell^+ X) + \Gamma(B^0(t) \rightarrow \ell^- X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \simeq \text{Im} \frac{\Gamma_{12}}{M_{12}}. \quad (37)$$

To obtain the right-hand side, use Eqs. (31) and (32) for the time evolution, and Eq. (35) for  $|q/p|$ . In kaon decays this asymmetry is measured [21], in agreement with the SM prediction,  $4 \text{Re} \epsilon_K$ . In  $B_d$  and  $B_s$  decays the asymmetry is expected to be [22]

$$A_{\text{SL}}^d \approx -4 \times 10^{-4}, \quad A_{\text{SL}}^s \approx 2 \times 10^{-5}. \quad (38)$$

The calculation of  $\text{Im}(\Gamma_{12}/M_{12})$  requires calculating inclusive nonleptonic decay rates, which can be addressed using an operator product expansion in the  $m_b \gg \Lambda_{\text{QCD}}$  limit. Such a calculation has sizable hadronic uncertainties, the details of which would lead to a long discussion. The constraints on new physics are significant nevertheless [23], as the  $m_c^2/m_b^2$  suppression of  $A_{\text{SL}}$  in the SM can be avoided in the presence of new physics.

### **$CP$ violation in the interference of decay with and without mixing**

A third type of  $CP$  violation is possible when both  $B^0$  and  $\bar{B}^0$  can decay

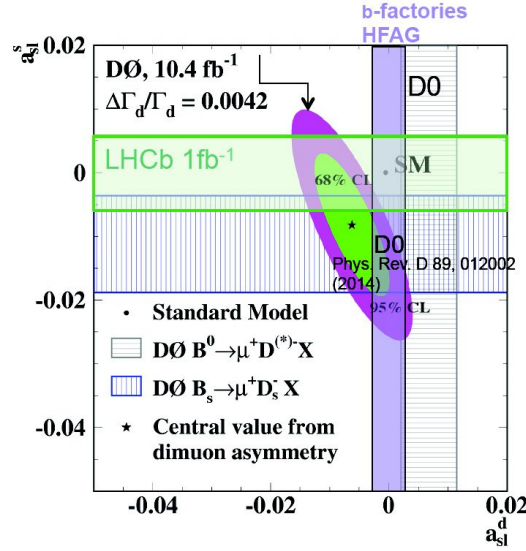


Fig. 6. Status of  $A_{SL}$  measurements (from M. Artuso, talk at FPCP 2014). The  $D\bar{0}$  result is in a  $3.6\sigma$  tension with the SM expectation.

to a final state,  $f$ . In the simplest cases, when  $f$  is a  $CP$  eigenstate, define

$$\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (39)$$

If there is no direct  $CP$  violation in a given mode, then  $\bar{A}_f = \eta_f \bar{A}_{\bar{f}}$ , where  $\eta_f = \pm 1$  is the  $CP$  eigenvalue of  $f$  [ $+1$  ( $-1$ ) for  $CP$ -even ( $-$ odd) states]. This is useful, because  $A_f$  and  $\bar{A}_{\bar{f}}$  are related by a  $CP$  transformation. If  $CP$  were conserved, then not only  $|q/p| = 1$  and  $|\bar{A}_{\bar{f}}/A_f| = 1$ , but the relative phase between  $q/p$  and  $\bar{A}_{\bar{f}}/A_f$  also vanishes, hence  $\lambda_f = \pm 1$ .

The experimentally measurable  $CP$  violating observable is<sup>1</sup>

$$\begin{aligned} a_f &= \frac{\Gamma[\bar{B}^0(t) \rightarrow f] - \Gamma[B^0(t) \rightarrow f]}{\Gamma[\bar{B}^0(t) \rightarrow f] + \Gamma[B^0(t) \rightarrow f]} \\ &= -\frac{(1 - |\lambda_f|^2) \cos(\Delta m t) - 2 \text{Im} \lambda_f \sin(\Delta m t)}{1 + |\lambda_f|^2} \\ &\equiv S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \end{aligned} \quad (40)$$

where we have neglected  $\Delta\Gamma$  (it is important in the  $B_s$  system). The last line defines the  $S$  and  $C$  coefficients, which are fit to the experimental data

<sup>1</sup>Derive the  $CP$  asymmetry in Eq. (40) using Eq. (31)). For extra credit, keep  $\Delta\Gamma \neq 0$ .



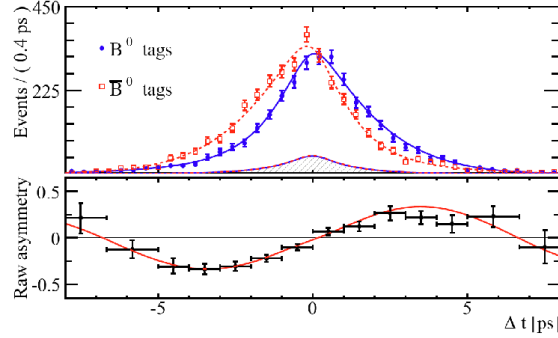


Fig. 7. Time dependence of tagged  $B \rightarrow \psi K$  decays (top);  $CP$  asymmetry (below) [24].

(see Fig. 7). If  $\text{Im}\lambda_f \neq 0$ , then  $CP$  violation arises in the interference between the decay  $B^0 \rightarrow f$ , and mixing followed by decay,  $B^0 \rightarrow \bar{B}^0 \rightarrow f$ .

This asymmetry can be nonzero if any type of  $CP$  violation occurs. In particular, in both the  $B_d$  and  $B_s$  systems  $||q/p| - 1| < \mathcal{O}(10^{-2})$  model independently, and it is much smaller in the SM [see, Eq. (38)]. If, in addition, amplitudes with a single weak phase dominate a decay, then  $|\bar{A}_f/A_f| \simeq 1$ , and  $\arg(\bar{A}_f/A_f)$  is just (twice) the weak phase, determined by short-distance physics. It is then possible that  $\text{Im}\lambda_f \neq 0$ ,  $|\lambda_f| \simeq 1$ , and although we cannot compute the decay amplitude, we can extract the weak phase difference between  $B^0 \rightarrow f$  and  $B^0 \rightarrow \bar{B}^0 \rightarrow f$  in a theoretically clean way from the measurement of

$$a_f = \text{Im}\lambda_f \sin(\Delta m t). \tag{41}$$

There is an interesting subtlety. Consider two final states,  $f_{1,2}$ . It is possible that direct  $CP$  violation in each channel,  $|\lambda_{f_1}| - 1$  and  $|\lambda_{f_2}| - 1$ , is unmeasurably small, but direct  $CP$  violation is detectable nevertheless. If

$$\eta_{f_1} \text{Im}(\lambda_{f_1}) \neq \eta_{f_2} \text{Im}(\lambda_{f_2}), \tag{42}$$

then  $CP$  violation must occur outside the mixing amplitude, even though it may be invisible in the data on any one final state.

**$\sin 2\beta$  from  $B \rightarrow \psi K_{S,L}$**  This is one of the cleanest examples of  $CP$  violation in the interference between decay with and without mixing, and one of the theoretically cleanest measurements of a CKM parameter.

There are “tree” and “penguin” contributions to  $B \rightarrow \psi K_{S,L}$ , with different weak and strong phases (see Fig. 8). The tree contribution is

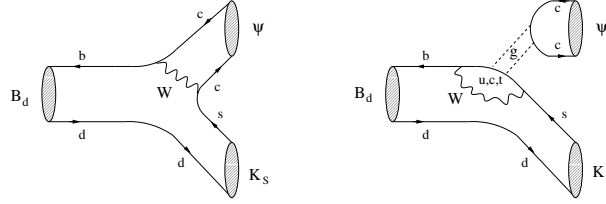


Fig. 8. “Tree” (left) and “penguin” (right) contributions to  $B \rightarrow \psi K_S$  (from Ref. [25]).

dominated by the  $b \rightarrow c\bar{c}s$  transition, while there are penguin contributions with three different combinations of CKM elements,

$$\bar{A}_T = V_{cb}V_{cs}^* T_{c\bar{c}s}, \quad \bar{A}_P = V_{tb}V_{ts}^* P_t + V_{cb}V_{cs}^* P_c + V_{ub}V_{us}^* P_u. \quad (43)$$

( $P_u$  can be defined to absorb the  $V_{ub}V_{us}^* T_{u\bar{u}s}$  “tree” contribution.) We can rewrite the decay amplitude using  $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$  to obtain

$$\begin{aligned} \bar{A} &= V_{cb}V_{cs}^* (T_{c\bar{c}s} + P_c - P_t) + V_{ub}V_{us}^* (P_u - P_t) \\ &\equiv V_{cb}V_{cs}^* T + V_{ub}V_{us}^* P, \end{aligned} \quad (44)$$

where the second line defines  $T$  and  $P$ . Since  $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \approx 0.02$ , the  $T$  amplitude with  $V_{cb}V_{cs}^*$  weak phase dominates. Thus,

$$\lambda_{\psi K_{S,L}} = \mp \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \right) \left( \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left( \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \right) = \mp e^{-2i\beta}, \quad (45)$$

and so  $\text{Im}\lambda_{\psi K_{S,L}} = \pm \sin 2\beta$ . The first term is the SM value of  $q/p$  in  $B_d$  mixing, the second is  $\bar{A}/A$ , the last one is  $p/q$  in the  $K^0$  system, and  $\eta_{\psi K_{S,L}} = \mp 1$ . Note that without  $K^0 - \bar{K}^0$  mixing there would be no interference between  $\bar{B}^0 \rightarrow \psi \bar{K}^0$  and  $B^0 \rightarrow \psi K^0$ . The accuracy of the relation between  $\lambda_{\psi K_{S,L}}$  and  $\sin 2\beta$  depends on model dependent estimates of  $|P/T|$ , which are below unity, so one expects it to be of order

$$\left| \frac{V_{ub}V_{us}^* P}{V_{cb}V_{cs}^* T} \right| \lesssim 10^{-2}. \quad (46)$$

The absence of detectable direct  $CP$  violation does not in itself bound this. To fully utilize future LHCb and Belle II data, better estimates are needed.

The first evidence for  $CP$  violation outside the kaon sector was the BaBar and Belle measurements of  $S_{\psi K}$ . The current world average is [20]

$$\sin 2\beta = 0.682 \pm 0.019. \quad (47)$$

This is consistent with other constraints, and shows that  $CP$  violation in quark mixing is an  $\mathcal{O}(1)$  effect, which is simply suppressed in  $K$  decays by small flavor violation suppressing the third generation’s contributions.

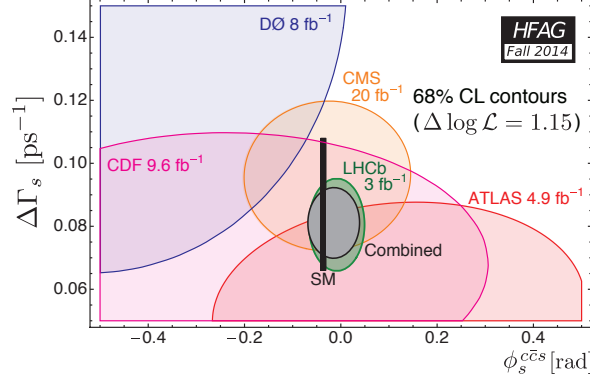


Fig. 9. Measurements of  $CP$  violation in  $B_s \rightarrow \psi\phi$  and  $\Delta\Gamma_s$  (from Ref. [20]).

$\phi_s \equiv -2\beta_s$  from  $B_s \rightarrow \psi\phi$  The analogous  $CP$  asymmetry in  $B_s$  decay, sensitive to BSM contributions to  $B_s - \bar{B}_s$  mixing, is  $B_s \rightarrow \psi\phi$ . Since the final state consists of two vector mesons, it is a combination of  $CP$ -even ( $L = 0, 2$ ) and  $CP$ -odd ( $L = 1$ ) partial waves. What is actually measured is the time-dependent  $CP$  asymmetry for each  $CP$  component of the  $\psi K^+ K^-$  and  $\psi\pi^+\pi^-$  final states. The SM prediction is suppressed compared to  $\beta$  by  $\lambda^2$ , and is rather precise,  $\beta_s = 0.0182_{-0.0006}^{+0.0007}$  [26]. The latest LHCb result using  $3 \text{ fb}^{-1}$  data is [27] (Fig. 9 shows all measurements)

$$\phi_s \equiv -2\beta_s = -0.010 \pm 0.039, \quad (48)$$

which has an uncertainty approaching that of  $2\beta$ , suggesting that the “room for new physics” in  $B_s$  mixing is no longer larger than in  $B_d$  (more below).

“Penguin-dominated” measurements of  $\beta_{(s)}$  Time dependent  $CP$  violation in  $b \rightarrow s$  dominated decays is a sensitive probe of new physics. Tree-level contributions to  $b \rightarrow s\bar{s}s$  transitions are expected to be small, and the penguin contributions to  $B \rightarrow \phi K_S$  (left diagram in Fig. 10) are

$$\bar{A}_P = V_{cb}V_{cs}^* (P_c - P_t) + V_{ub}V_{us}^* (P_u - P_t). \quad (49)$$

Due to  $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \approx 0.02$  and expecting  $|P_c - P_t|/|P_u - P_t| = \mathcal{O}(1)$ , the  $B \rightarrow \phi K_S$  amplitude is also dominated by a single weak phase,  $V_{cb}V_{cs}^*$ . Therefore, the theory uncertainty relating  $S_{\phi K_S}$  to  $\sin 2\beta$  is small, although larger than in  $B \rightarrow \psi K_S$ . There is also a “tree” contribution from  $b \rightarrow u\bar{u}s$  followed by  $u\bar{u} \rightarrow s\bar{s}$  rescattering (right diagram in Fig. 10). This amplitude is proportional to the suppressed CKM combination,  $V_{ub}V_{us}^*$ , and

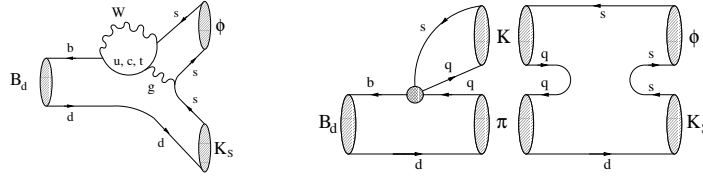


Fig. 10. “Penguin” (left) and “tree” (right) contributions to  $B \rightarrow \phi K_S$  (from Ref. [25]).

it is actually not separable from  $P_u - P_t$ . Unless its matrix element is largely enhanced, it should not upset the  $\text{Im}\lambda_{\phi K_S} = \sin 2\beta + \mathcal{O}(\lambda^2)$  expectation in the SM. Similar reasons make many other modes, such as  $B \rightarrow \eta^{(\prime)} K_S$ ,  $B_s \rightarrow \phi\phi$ , etc., interesting and promising to study.

**The determinations of  $\gamma$  and  $\alpha$**  By virtue of Eq. (23),  $\gamma$  does not depend on CKM elements involving the top quark, so it can be measured in tree-level  $B$  decays. This is an important distinction from  $\alpha$  and  $\beta$ , and implies that  $\gamma$  is less likely to be affected by BSM physics.

Most measurements of  $\gamma$  utilize the fact that interference of  $B^- \rightarrow D^0 K^-$  ( $b \rightarrow c\bar{u}s$ ) and  $B^- \rightarrow \bar{D}^0 K^-$  ( $b \rightarrow u\bar{c}s$ ) transitions can be studied in final states accessible in both  $D^0$  and  $\bar{D}^0$  decays [28]. (A notable exception is the measurement from the four time-dependent  $\bar{B}_s$  and  $B_s \rightarrow D_s^\pm K^\mp$  rates, which is possible at LHCb.) It is possible to measure the  $B$  and  $D$  decay amplitudes, their relative strong phases, and the weak phase  $\gamma$  from the data. There are many variants, based on different  $D$  decay channels [29; 30; 31; 32; 33; 34]. The best current measurement comes from  $D^0, \bar{D}^0 \rightarrow K_S \pi^+ \pi^-$  [33; 34], in which case both amplitudes are Cabibbo allowed, and the analysis can be optimized by studying the Dalitz plot dependence of the interference. The world average of all  $\gamma$  measurements is [26]

$$\gamma = (73.2^{+6.3}_{-7.0})^\circ. \tag{50}$$

Most importantly, the theory uncertainty in the SM measurement is smaller than the accuracy of any planned or imaginable future experiment.

The measurements usually referred to as determining  $\alpha$ , measure  $\pi - \beta - \gamma$ , the third angle of the unitarity triangle in any model in which the unitarity of the  $3 \times 3$  CKM matrix is maintained. These measurements are in time-dependent  $CP$  asymmetries in  $B \rightarrow \pi\pi$ ,  $\rho\rho$ , and  $\rho\pi$  decays. In these decays the  $b \rightarrow u\bar{u}d$  “tree” amplitudes are not much larger than the  $b \rightarrow \sum_q q\bar{q}d$  “penguin” contributions, which have different weak phases.<sup>j</sup> The

<sup>j</sup>Show that if the “tree” amplitudes dominated these decays then  $\lambda_{\pi\pi}^{(\text{tree})} = e^{2i\alpha}$ .

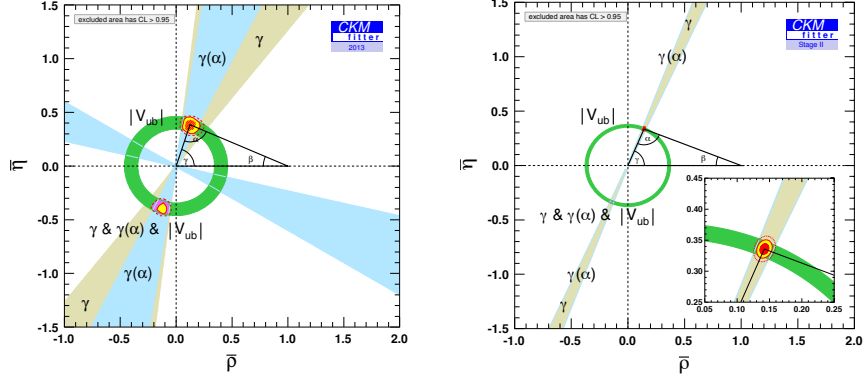


Fig. 11. Constraints on  $\bar{\rho}-\bar{\eta}$ , allowing new physics in the  $B_{d,s}$  mixing amplitudes. Left plot shows the current constraints, right plot is the expectation using  $50 \text{ ab}^{-1}$  Belle II and  $50 \text{ fb}^{-1}$  LHCb data. Colored regions show 95% CL, as in Fig. 2. (From Ref. [39].)

tree contributions change isospin by  $\Delta I = 3/2$  or  $1/2$ , while the penguin contribution is  $\Delta I = 1/2$  only. It is possible to use isospin symmetry of the strong interaction to isolate  $CP$  violation in the  $\Delta I = 3/2$  channel, eliminating the penguin contributions [35; 36; 37], yielding [26]

$$\alpha = (87.7^{+3.5}_{-3.3})^\circ. \quad (51)$$

Thus, the measurements of  $\alpha$  are sensitive to new physics in  $B^0-\bar{B}^0$  mixing and via possible  $\Delta I = 3/2$  (or  $\Delta I = 5/2$ ) contributions [38].

**New physics in  $B_d$  and  $B_s$  mixing** Although the SM CKM fit in Fig. 2 shows impressive and nontrivial consistency, the implications of the level of agreement are often overstated. Allowing new physics contributions, there are a larger number of parameters related to  $CP$  and flavor violation, and the fits become less constraining. This is shown in the left plot in Fig. 11 where the allowed region is indeed significantly larger than in Fig. 2 (the 95% CL combined fit regions are indicated on both plots).

It has been known for decades that the mixing of neutral mesons is particularly sensitive to new physics, and probes some of the highest scales. In a large class of models, NP has a negligible impact on tree-level SM transitions, and the  $3 \times 3$  CKM matrix remains unitary. (In such models  $\alpha+\beta+\gamma = \pi$  is maintained, and independent measurements of  $\pi-\beta-\alpha$  and  $\gamma$  can be averaged.) We can parametrize the NP contributions to neutral meson mixing as

$$M_{12} = M_{12}^{\text{SM}}(1 + h_q e^{2i\sigma_q}), \quad q = d, s. \quad (52)$$

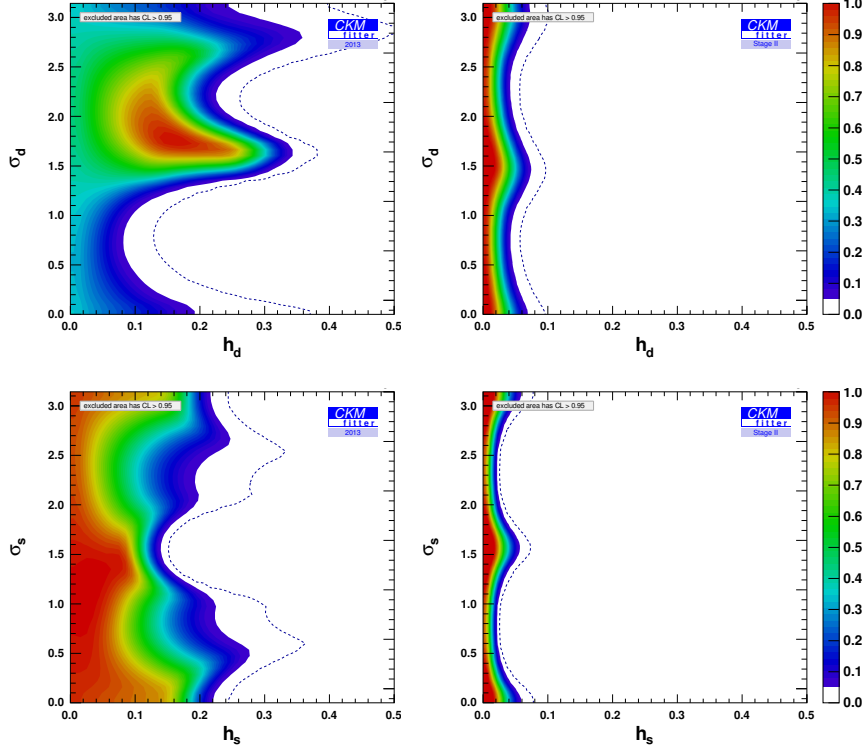


Fig. 12. Constraints on the  $h_d - \sigma_d$  (top row) and  $h_s - \sigma_s$  parameters (bottom row). Left plots show the current constraints, right plots show those estimated to be achievable using  $50 \text{ ab}^{-1}$  Belle II and  $50 \text{ fb}^{-1}$  LHCb data. Colored regions show  $2\sigma$  limits with the colors indicating CL as shown, while the dashed lines show  $3\sigma$  limits. (From Ref. [39].)

The constraints on  $h_q$  and  $\sigma_q$  in the  $B_d^0$  and  $B_s^0$  systems are shown in the top and bottom rows of Fig. 12, respectively.

For example, if NP modifies the SM operator describing  $B$  mixing, by

$$\frac{C_q^2}{\Lambda^2} (\bar{b}_L \gamma^\mu q_L)^2, \quad (53)$$

then one finds

$$h_q \simeq \frac{|C_q|^2}{|V_{tb}^* V_{tq}|^2} \left( \frac{4.5 \text{ TeV}}{\Lambda} \right)^2. \quad (54)$$

We can then translate the plots in Fig. 12 to the scale of new physics probed. The summary of expected sensitivities are shown in Table 3. The

Table 3. The scale of the operator in Eq. (53) probed by  $B_d^0$  and  $B_s^0$  mixings with  $50 \text{ ab}^{-1}$  Belle II and  $50 \text{ fb}^{-1}$  LHCb data. The differences due to CKM-like hierarchy of couplings and/or loop suppression is indicated. (From Ref. [39].)

Couplings	NP loop order	Scales (TeV) probed by	
		$B_d$ mixing	$B_s$ mixing
$ C_q  =  V_{tb}V_{tq}^* $ (CKM-like)	tree level	17	19
	one loop	1.4	1.5
$ C_q  = 1$ (no hierarchy)	tree level	$2 \times 10^3$	$5 \times 10^2$
	one loop	$2 \times 10^2$	40

sensitivities, even with SM-like loop- and CKM-suppressed coefficients, are comparable to the scales probed by the LHC.

### 3. Some Implications of the Heavy Quark Limit

We have not directly discussed so far that most quark flavor physics processes (other than top quark decays) involve strong interactions in a regime where perturbation theory is not (or not necessarily) reliable. The running of the QCD coupling at lowest order is

$$\alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 + \frac{\alpha_s}{2\pi} \beta_0 \ln \frac{\mu}{\Lambda}}, \quad (55)$$

where  $\beta_0 = 11 - 2n_f/3$  and  $n_f$  is the number of light quark flavors. Even in  $B$  decays, the typical energy scale of certain processes can be a fraction of  $m_b$ , possibly around or below a GeV. The ways I know how to deal with this in a tractable way are (i) symmetries of QCD, exact, or approximate in some limits (CP invariance, heavy quark symmetry, chiral symmetry); (ii) the operator product expansion (for inclusive decays); (iii) lattice QCD (for certain hadronic matrix elements). An example of (i) is the determination of  $\sin 2\beta$  from  $B \rightarrow \psi K_S$ , see Eq. (46). So is the determination of  $|V_{cb}|$  from  $B \rightarrow D^* \ell \bar{\nu}$ , see Eq. (73) below. An example of (ii) is the analysis of inclusive  $B \rightarrow X_s \gamma$  decay rates discussed below, which provides some of the strongest constraints on many TeV-scale BSM scenarios.

The role of (strong interaction) model-independent measurements cannot be overstated. To establish that a discrepancy between experiment and theory is a sign of new physics, model-independent predictions are crucial. Results that rely on modeling nonperturbative strong interaction effects will

not disprove the SM. Most model-independent predictions are of the form,

$$\text{Observable} = (\text{calculable terms}) \times \left\{ 1 + \sum_{i,k} [(\text{small parameters})_i]^k \right\}, \quad (56)$$

where the small parameters can be  $\Lambda_{\text{QCD}}/m_b$ ,  $m_s/\Lambda_{\chi\text{SB}}$ ,  $\alpha_s(m_b)$ , etc. For the purpose of these lectures, strong-interaction model-independent means that the theoretical uncertainty is suppressed by small parameters, so that theorists argue about  $\mathcal{O}(1) \times (\text{small numbers})$  instead of  $\mathcal{O}(1)$  effects. There are always theoretical uncertainties suppressed by some (small parameter)<sup>n</sup>, which cannot be calculated from first principles. If the goal is to test the SM, one must assign  $\mathcal{O}(1)$  uncertainties in such terms.

In addition, besides formal suppressions of certain corrections in some limits, experimental guidance is always needed to establish how well an expansion works; for example,  $f_\pi$ ,  $m_\rho$ , and  $m_K^2/m_s$  are all of order  $\Lambda_{\text{QCD}}$ , but their numerical values span an order of magnitude.

**Heavy quark symmetry (HQS)** In hadrons composed of heavy quarks the dynamics of QCD simplifies. Mesons containing a heavy quark – heavy antiquark pair,  $Q\bar{Q}$ , form positronium-type bound states, which become perturbative in the limit  $m_Q \gg \Lambda_{\text{QCD}}$  [40]. In mesons composed of a heavy quark,  $Q$ , and a light antiquark,  $\bar{q}$  (and gluons and  $q\bar{q}$  pairs), the heavy quark acts as a static color source with fixed four-velocity,  $v^\mu$ , and the wave function of the light degrees of freedom (the “brown muck”) become insensitive to the spin and mass (flavor) of the heavy quark, resulting in heavy quark spin-flavor symmetries [41].

The physical picture is similar to atomic physics, where simplifications occur due to the fact that the electron mass,  $m_e$ , is much smaller than the nucleon mass,  $m_N$ . The analog of flavor symmetry is that isotopes have similar chemistry, because the electrons’ wave functions become independent of  $m_N$  in the  $m_N \gg m_e$  limit. The analog of spin symmetry is that hyperfine levels are almost degenerate, because the interaction of the electron and nucleon spin diminishes in the  $m_N \gg m_e$  limit.

**Spectroscopy of heavy-light mesons** The spectroscopy of heavy hadrons simplifies due to heavy quark symmetry. We can write the angular momentum of a heavy-light meson as  $J = \vec{s}_Q + \vec{s}_l$ , where  $\vec{s}_l$  is the total angular momentum of the light degrees of freedom. Angular momentum conservation,  $[\vec{J}, \mathcal{H}] = 0$ , and heavy quark symmetry,  $[\vec{s}_Q, \mathcal{H}] = 0$ , imply  $[\vec{s}_l, \mathcal{H}] = 0$ . In the  $m_Q \gg \Lambda_{\text{QCD}}$  limit, the spin of the heavy quark and the



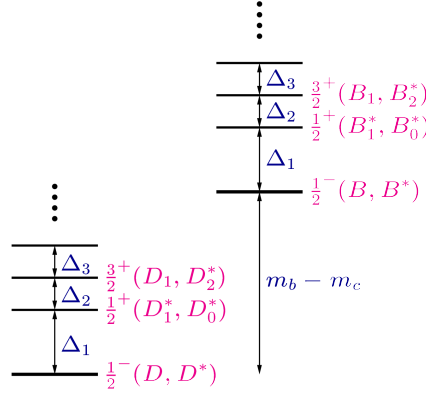


Fig. 13. Spectroscopy of  $B$  and  $D$  mesons. For each doublet level, the spin-parity of the light degrees of freedom,  $s_l^{\pi_l}$ , and the names of the physical states are indicated.

total angular momentum of light degrees of freedom are separately conserved, modified only by subleading interactions suppressed by  $\Lambda_{\text{QCD}}/m_Q$ .

Thus, hadrons containing a single heavy quark can be labeled with  $s_l$ , and for any value of  $s_l$  there are two (almost) degenerate states with total angular momentum  $J_{\pm} = s_l \pm \frac{1}{2}$ . (An exception occurs for the lightest baryons containing a heavy quark, when  $s_l = 0$ , and there is a single state with  $J = \frac{1}{2}$ , the  $\Lambda_b$  and  $\Lambda_c$ .) The ground state mesons with  $Q\bar{q}$  flavor quantum numbers contain light degrees of freedom with spin-parity  $s_l^{\pi_l} = \frac{1}{2}^-$ , giving a doublet containing a spin zero and spin one meson. For  $Q = c$  these are the  $D$  and  $D^*$ , while  $Q = b$  gives the  $B$  and  $B^*$  mesons.

The mass splittings between the doublets,  $\Delta_i$ , are of order  $\Lambda_{\text{QCD}}$ , and are the same in the  $B$  and  $D$  sectors at leading order in  $\Lambda_{\text{QCD}}/m_Q$ , as illustrated in Fig. 13. The mass splittings within each doublet are of order  $\Lambda_{\text{QCD}}^2/m_Q$ . This is supported by experimental data; e.g., for the  $s_l^{\pi_l} = \frac{1}{2}^-$  ground state doublets  $m_{D^*} - m_D \approx 140$  MeV while  $m_{B^*} - m_B \approx 45$  MeV, and their ratio, 0.3, is consistent with  $m_c/m_b$ .

Let us mention a puzzle. The mass splitting of the lightest vector and pseudoscalar mesons being  $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_Q)$  implies that  $m_V^2 - m_P^2$  is approximately constant. This argument relies on  $m_Q \gg \Lambda_{\text{QCD}}$ . The data are

$$\begin{aligned}
 m_{B^*}^2 - m_B^2 &= 0.49 \text{ GeV}^2, & m_{B_s^*}^2 - m_{B_s}^2 &= 0.50 \text{ GeV}^2, \\
 m_{D^*}^2 - m_D^2 &= 0.54 \text{ GeV}^2, & m_{D_s^*}^2 - m_{D_s}^2 &= 0.58 \text{ GeV}^2, \\
 m_{\rho}^2 - m_{\pi}^2 &= 0.57 \text{ GeV}^2, & m_{K^*}^2 - m_K^2 &= 0.55 \text{ GeV}^2.
 \end{aligned} \tag{57}$$

It is not understood why the light meson mass splittings in the last line are

so close numerically. (It is expected in the nonrelativistic constituent quark model, which fails to account for several properties of these mesons.) There must be something more going on than heavy quark symmetry, and if this were its only prediction, we could not say that there is strong evidence that it is useful. So in general, to understand a theory, it is not only important how well it works, but also how it breaks down outside its range of validity.

**Heavy quark effective theory (HQET)** The consequences of heavy quark symmetry and the corrections to the symmetry limit can be studied by constructing an effective theory which makes the consequences of heavy quark symmetry explicit. The heavy quark in a heavy-light meson is almost on-shell, so we can expand its momentum as  $p_Q^\mu = m_Q v^\mu + k^\mu$ , where  $|k| = \mathcal{O}(\Lambda_{\text{QCD}})$  and  $v^2 = 1$ . Expanding the heavy quark propagator,

$$\frac{i}{\not{p} - m_Q} = \frac{i(\not{p} + m_Q)}{p^2 - m_Q^2} = \frac{i(m_Q \not{v} + \not{k} + m_Q)}{2m_Q v \cdot k + k^2} = \frac{i}{v \cdot k} \frac{1 + \not{v}}{2} + \dots \quad (58)$$

it becomes independent of the heavy quark mass, a manifestation of heavy quark flavor symmetry. Hence the Feynman rules simplify,

$$\overline{\hspace{1.5cm}} \longrightarrow \overline{\hspace{1.5cm}} \quad (59)$$

$$\frac{i}{\not{p} - m_Q} \longrightarrow \frac{i}{v \cdot k} P_+(v),$$

where  $P_\pm = (1 \pm \not{v})/2$  are projection operators, and the double line denotes the heavy quark propagator. In the rest frame of the heavy quark,  $P_+ = (1 + \gamma^0)/2$  projects onto the heavy quark (rather than anti-quark) components. The coupling of a heavy quark to gluons simplifies due to

$$P_+ \gamma^\mu P_+ = P_+ v^\mu P_+ = v^\mu P_+, \quad (60)$$

hence we can replace

$$\begin{array}{ccc} \text{---} & \longrightarrow & \text{= =} \\ \text{---} & & \text{---} \\ \text{---} & & \text{---} \end{array} \quad (61)$$

$$ig\gamma^\mu \frac{\lambda^a}{2} \longrightarrow igv^\mu \frac{\lambda^a}{2}.$$

The lack of any  $\gamma$  matrix is a manifestation of heavy quark spin symmetry.

To derive the effective Lagrangian of HQET, it is convenient to decompose the four-component Dirac spinor as

$$Q(x) = e^{-im_Q v \cdot x} [Q_v(x) + \mathcal{Q}_v(x)], \quad (62)$$

where

$$Q_v(x) = e^{im_Q v \cdot x} P_+(v) Q(x), \quad \bar{Q}_v(x) = e^{im_Q v \cdot x} P_-(v) Q(x). \quad (63)$$

The  $e^{im_Q v \cdot x}$  factor subtracts  $m_Q v$  from the heavy quark momentum. At leading order only  $Q_v$  contributes, and the effects of  $\bar{Q}_v$  are suppressed by powers of  $\Lambda_{\text{QCD}}/m_Q$ . The heavy quark velocity,  $v$ , acts as a label of the heavy quark fields [42], because  $v$  cannot be changed by soft interactions. In terms of these fields the QCD Lagrangian simplifies,

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q = \bar{Q}_v i\not{D} Q_v + \dots = \bar{Q}_v (iv \cdot D) Q_v + \dots, \quad (64)$$

where the ellipses denote terms suppressed by powers of  $\Lambda_{\text{QCD}}/m_Q$ . The absence of any Dirac matrix is a consequence of heavy quark symmetry, which implies that the heavy quark's propagator and its coupling to gluons are independent of the heavy quark spin. This effective theory provides a framework to calculate perturbative  $\mathcal{O}(\alpha_s)$  corrections and to parametrize nonperturbative  $\mathcal{O}(\Lambda_{\text{QCD}}/m_Q)$  terms.

**Semileptonic  $B \rightarrow D^{(*)} \ell \bar{\nu}$  decays and  $|V_{cb}|$**  Heavy quark symmetry is particularly predictive for these decays. In the  $m_{b,c} \gg \Lambda_{\text{QCD}}$  limit, the configuration of the brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin. So when the weak current changes suddenly (on a time scale  $\ll \Lambda_{\text{QCD}}^{-1}$ ) the flavor  $b \rightarrow c$ , the momentum  $\vec{p}_b \rightarrow \vec{p}_c$ , and possibly flips the spin,  $\vec{s}_b \rightarrow \vec{s}_c$ , the brown muck only feels that the four-velocity of the static color source changed,  $v_b \rightarrow v_c$ . Therefore, the matrix elements that describe the transition probabilities from the initial to the final state are independent of the Dirac structure of weak current, and can only depend on a scalar quantity,  $w \equiv v_b \cdot v_c$ .

The ground-state pseudoscalar and vector mesons for each heavy quark flavor (the spin symmetry doublets  $D^{(*)}$  and  $B^{(*)}$ ) can be represented by a "superfield", combining fields with different spins, that has the right transformation property under heavy quark and Lorentz symmetry,

$$\mathcal{M}_v^{(Q)} = \frac{1 + \not{v}}{2} \left[ \gamma^\mu M_\mu^{*(Q)}(v, \varepsilon) - i\gamma_5 M^{(Q)}(v) \right]. \quad (65)$$

The  $B^{(*)} \rightarrow D^{(*)}$  matrix element of any current can be parametrized as

$$\langle M^{(c)}(v') | \bar{c}_{v'} \Gamma b_v | M^{(b)}(v) \rangle = \text{Tr} \left[ F(v, v') \bar{\mathcal{M}}_{v'}^{(c)} \Gamma \mathcal{M}_v^{(b)} \right]. \quad (66)$$

Because of heavy quark symmetry, there cannot be other Dirac matrices between the  $\bar{\mathcal{M}}_{v'}^{(c)}$  and  $\mathcal{M}_v^{(b)}$  fields. The most general form of  $F$  is

$$F(v, v') = f_1(w) + f_2(w)\not{v}' + f_3(w)\not{v}' + f_4(w)\not{v}'\not{v}'. \quad (67)$$

As stated above,  $w \equiv v \cdot v'$  is the only possible scalar, simply related to  $q^2 = (p_B - p_{D^{(*)}})^2 = m_B^2 + m_{D^{(*)}}^2 - 2m_B m_{D^{(*)}} w$ . Using  $\mathcal{M}_v^{(Q)} = P_+(v) \mathcal{M}_v^{(Q)} P_-(v)$ , we can write

$$\begin{aligned} F &\doteq P_-(v) F P_-(v') = [f_1(w) - f_2(w) - f_3(w) + f_4(w)] P_-(v) P_-(v') \\ &= \xi(w) P_-(v) P_-(v') \doteq \xi(w). \end{aligned} \tag{68}$$

This defines the Isgur-Wise function,  $\xi(w)$ , and  $\doteq$  denotes relations valid when evaluated inside the trace in Eq. (66).

Since only weak interactions change  $b$ -quark number, the matrix element of  $\bar{b}\gamma_0 b$ , the  $b$ -quark number current, is  $\langle B(v) | \bar{b}\gamma_0 b | B(v) \rangle = 2m_B v_0$ . Comparing it with the result obtained using Eq. (66),

$$\langle B(v) | \bar{b}\gamma_\mu b | B(v) \rangle = 2m_B v_\mu \xi(1), \tag{69}$$

implies that  $\xi(1) = 1$ . That is, at  $w = 1$ , the ‘‘zero recoil’’ point, when the  $D^{(*)}$  is at rest in the rest-frame of the decaying  $B$  meson, the configuration of the brown muck does not change at all, and heavy quark symmetry determines the hadronic matrix element (see Fig. 14). Moreover, the six form factors that describe semileptonic  $B \rightarrow D^{(*)} \ell \bar{\nu}$  decays are related to this universal function, which contains all the low energy nonperturbative hadronic physics relevant for these decays.<sup>k</sup>

The determination of  $|V_{cb}|$  from  $B \rightarrow D^{(*)} \ell \bar{\nu}$  decays use fits to the decay distributions to measure the rates near zero recoil,  $w = 1$ . The rates can be schematically written as

$$\frac{d\Gamma(B \rightarrow D^{(*)} \ell \bar{\nu})}{dw} = (\text{calculable}) |V_{cb}|^2 \begin{cases} (w^2 - 1)^{1/2} \mathcal{F}_*^2(w), & \text{for } B \rightarrow D^*, \\ (w^2 - 1)^{3/2} \mathcal{F}^2(w), & \text{for } B \rightarrow D. \end{cases} \tag{72}$$

Both  $\mathcal{F}(w)$  and  $\mathcal{F}_*(w)$  are equal to the Isgur-Wise function in the  $m_Q \rightarrow \infty$  limit, and  $\mathcal{F}_{(*)}(1) = 1$  is the basis for a model-independent determination

<sup>k</sup>Using only Lorentz invariance, six form factors parametrize  $B \rightarrow D^{(*)} \ell \bar{\nu}$  decay,

$$\begin{aligned} \langle D(v') | V_\nu | B(v) \rangle &= \sqrt{m_B m_D} [h_+(v + v')_\nu + h_-(v - v')_\nu], \\ \langle D^*(v') | V_\nu | B(v) \rangle &= i\sqrt{m_B m_{D^*}} h_V \epsilon_{\nu\alpha\beta\gamma} \epsilon^{*\alpha} v'^\beta v^\gamma, \\ \langle D(v') | A_\nu | B(v) \rangle &= 0, \\ \langle D^*(v') | A_\nu | B(v) \rangle &= \sqrt{m_B m_{D^*}} [h_{A_1}(w + 1)\epsilon_\nu^* - h_{A_2}(\epsilon^* \cdot v)v_\nu - h_{A_3}(\epsilon^* \cdot v)v'_\nu], \end{aligned} \tag{70}$$

where  $V_\nu = \bar{c}\gamma_\nu b$ ,  $A_\nu = \bar{c}\gamma_\nu\gamma_5 b$ , and  $h_i$  are functions of  $w$ . Show that this is indeed the most general form of these matrix elements, and that at leading order in  $\Lambda_{\text{QCD}}/m_Q$ ,

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w), \quad h_-(w) = h_{A_2}(w) = 0. \tag{71}$$

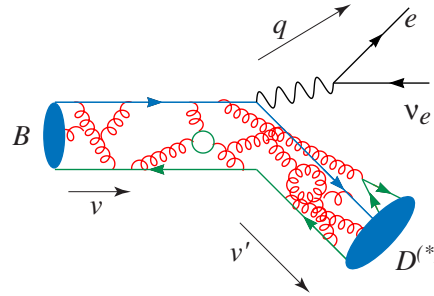


Fig. 14. Illustration of strong interactions parametrized by the Isgur-Wise function.

of  $|V_{cb}|$ . There are calculable corrections in powers of  $\alpha_s(m_{c,b})$ , as well as terms suppressed by  $\Lambda_{\text{QCD}}/m_{c,b}$ , which can only be parametrized, and that is where hadronic uncertainties enter. Schematically,

$$\begin{aligned} \mathcal{F}_*(1) &= 1_{(\text{Isgur-Wise})} + c_A(\alpha_s) + \frac{0_{(\text{Luke})}}{m_{c,b}} + \frac{(\text{lattice or models})}{m_{c,b}^2} + \dots, \\ \mathcal{F}(1) &= 1_{(\text{Isgur-Wise})} + c_V(\alpha_s) + \frac{(\text{lattice or models})}{m_{c,b}} + \dots \end{aligned} \quad (73)$$

The absence of the  $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b})$  term for  $B \rightarrow D^* \ell \bar{\nu}$  at zero recoil is a consequence of Luke's theorem [43]. Calculating corrections to the heavy quark limit in these decays is a vast subject. Heavy quark symmetry also makes model-independent predictions for  $B$  decays to excited  $D$  mesons [44]. It is due to heavy quark symmetry that the SM predictions for the recently observed anomalies in the  $B \rightarrow D^{(*)} \tau \bar{\nu}$  branching ratios [45] are under good theoretical control.

**Inclusive semileptonic decays and  $B \rightarrow X_s \gamma$**  Instead of identifying all final-state particles in a decay, sometimes it is useful to sum over final-state hadrons that can be produced by the strong interaction, subject to constraints determined by short-distance physics, e.g., the energy of a photon or a charged lepton. Although hadronization is nonperturbative, it occurs on much longer distance (and time) scales than the underlying weak decay. Typically we are interested in a quark-level transition, such as  $b \rightarrow c \ell \bar{\nu}$ ,  $b \rightarrow s \gamma$ , etc., and we would like to extract from the data short distance parameters,  $|V_{cb}|$ ,  $C_7(m_b)$ , etc. To do this, we need to relate the quark-level operators to the measurable decay rates.

For example, consider inclusive semileptonic  $b \rightarrow c$  decay mediated by

$$O_{sl} = -\frac{4G_F}{\sqrt{2}} V_{cb} (J_{bc})^\alpha (J_{\ell\nu})_\alpha, \tag{74}$$

where  $J_{bc}^\alpha = \bar{c} \gamma^\alpha P_L b$  and  $J_{\ell\nu}^\beta = \bar{\ell} \gamma^\beta P_L \nu$ . The decay rate is given by the square of the matrix element, integrated over phase space, and summed over final states,

$$\Gamma(B \rightarrow X_c \ell \bar{\nu}) \sim \sum_{X_c} \int d[\text{PS}] |\langle X_c \ell \bar{\nu} | O_{sl} | B \rangle|^2. \tag{75}$$

Since leptons have no strong interaction, the squared matrix element and phase space factorize into  $B \rightarrow X_c W^*$  and a perturbatively calculable leptonic part,  $W^* \rightarrow \ell \bar{\nu}$ . The nontrivial part is the hadronic tensor,

$$\begin{aligned} W^{\mu\nu} &= \sum_{X_c} (2\pi)^3 \delta^4(p_B - q - p_X) \langle B | J_{bc}^{\mu\dagger} | X_c \rangle \langle X_c | J_{bc}^\nu | B \rangle \\ &= \frac{1}{\pi} \text{Im} \int d^4x e^{-iq \cdot x} \langle B | T \{ J_{bc}^{\mu\dagger}(x) J_{bc}^\nu(0) \} | B \rangle, \end{aligned} \tag{76}$$

where the second line is obtained using the optical theorem, and  $T$  denotes here the time-ordered product of the operators. It is this time-ordered product that can be expanded in an operator product expansion (OPE) [46; 47; 48; 49]. In the  $m_b \gg \Lambda_{\text{QCD}}$  limit, the time-ordered product is dominated by short distances,  $x \ll \Lambda_{\text{QCD}}^{-1}$ , and one can express the hadronic tensor  $W^{\mu\nu}$  as a sum of matrix elements of local operators. Schematically,

$$= \text{[Diagrammatic expansion]} + \frac{0}{m_b} \text{[Diagram]} + \frac{1}{m_b^2} \text{[Diagram]} + \dots \tag{77}$$

This is analogous to the multipole expansion. At leading order in  $\Lambda_{\text{QCD}}/m_b$  the lowest dimension operator is  $\bar{b} \Gamma b$ , where  $\Gamma$  is some (process-dependent) Dirac matrix. Its matrix element is determined by the  $b$  quark content of the initial state using Eqs. (66) and (69); therefore, inclusive  $B$  decay rates in the  $m_b \gg \Lambda_{\text{QCD}}$  limit are equal to the  $b$  quark decay rates. Subleading effects are parametrized by matrix elements of operators with increasing number of derivatives, which are sensitive to the distribution of chromomagnetic and chromoelectric fields. There are no  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$  corrections, because the  $B$  meson matrix element of any dimension-4 operator vanishes,  $\langle B(v) | \bar{Q}_v^{(b)} iD_\alpha \Gamma Q_v^{(b)} | B(v) \rangle = 0$ . The leading nonperturbative effects, suppressed by  $\Lambda_{\text{QCD}}^2/m_b^2$ , are parametrized by two HQET matrix elements,

denoted by  $\lambda_{1,2}$ . This is the basis of the model-independent determinations of  $m_b$  and  $|V_{cb}|$  from inclusive semileptonic  $B$  decays.

Some important applications, such as  $B \rightarrow X_s \gamma$  [50] or  $B \rightarrow X_u \ell \bar{\nu}$ , are more complicated. Near boundaries of phase space, the energy release to the hadronic final state may not be large. One can think of the OPE as an expansion in the residual momentum of the  $b$  quark,  $k$ , shown in Eq. (77),

$$\frac{1}{(m_b v + k - q)^2 - m_q^2} = \frac{1}{[(m_b v - q)^2 - m_q^2] + [2k \cdot (m_b v - q)] + k^2}. \quad (78)$$

For the expansion in  $k$  to converge, the final state phase space can only be restricted in a way that allows hadronic final states,  $X$ , to contribute with

$$m_X^2 - m_q^2 \gg E_X \Lambda_{\text{QCD}} \gg \Lambda_{\text{QCD}}^2. \quad (79)$$

In  $B \rightarrow X_s \gamma$  when an experimental lower cut is imposed on  $E_\gamma$  to reject backgrounds, the left-most inequality can be violated. The same occurs in  $B \rightarrow X_u \ell \bar{\nu}$  when experimental cuts are used to suppress  $B \rightarrow X_c \ell \bar{\nu}$  backgrounds. If the right-most inequality in Eq. (79) is satisfied, a more complicated OPE in terms of nonlocal operators is still possible [51; 52].

#### 4. Top, Higgs, and New Physics Flavor

**The scale of new physics** In the absence of direct observation of BSM particles so far, viewing the standard model as a low energy effective theory, the search for new physics amounts to seeking evidence for higher dimension operators invariant under the SM gauge symmetries.

Possible dimension-6 operators include baryon and lepton number violating operators, such as  $\frac{1}{\Lambda^2} QQQ L$ . Limits on the proton lifetime imply  $\Lambda \gtrsim 10^{16}$  GeV. Non-SM flavor and  $CP$  violation could arise from  $\frac{1}{\Lambda^2} Q\bar{Q}Q\bar{Q}$ . The bounds on the scale of such operators are  $\Lambda \gtrsim 10^{4\dots 7}$  GeV, depending on the generation index of the quark fields. Precision electroweak measurements constrain operators of the form  $\frac{1}{\Lambda^2} (\phi D_\mu \phi)^2$  to have  $\Lambda \gtrsim 10^{3\dots 4}$  GeV. These constraints are remarkable, because flavor,  $CP$ , and custodial symmetry are broken by the SM itself, so it is unlikely for new physics to have a symmetry reason to avoid introducing additional contributions.

As mentioned earlier, there is a single type of gauge invariant dimension-5 operators made of SM fields, which give rise to neutrino masses, see Eq. (18). The observed neutrino mass square differences hint at scales  $\Lambda > 10^{10}$  GeV for these  $\frac{1}{\Lambda} (L\phi)^2$  type operators (in many models  $\Lambda \sim 10^{15}$  GeV). Such mass terms violate lepton number. It is an experimental question to determine the nature of neutrino masses, which is what makes the search

for neutrinoless double beta decay (and determining the neutrino mass hierarchy) so important.

**Charged lepton flavor violation (CLFV)** The SM with vanishing neutrino masses would have predicted lepton flavor conservation. We now know that this is not the case, hence there is no reason to impose it on possible new physics scenarios. In particular, if there are TeV-scale new particles that carry lepton number (e.g., sleptons), then they have their own mixing matrices, which could give rise to CLFV signals. While the one-loop SM contributions to processes such as  $\mu \rightarrow e\gamma$  are suppressed by the neutrino mass-squared differences<sup>1</sup>, the NP contributions have a-priori no such suppressions, other than the somewhat heavier scales and being generated at one-loop in most BSM scenarios.

Within the next decade, the CLFV sensitivity will improve by about 4 orders of magnitude, corresponding to an increase in the new physics scale probed by an order of magnitude, possibly the largest such gain in sensitivity achievable soon. If any CLFV signal is discovered, we would want to measure many processes to map out the underlying patterns, including  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ ,  $\tau \rightarrow e\gamma$ ,  $\tau \rightarrow 3e$ ,  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow 3\mu$ , etc.

**Electric dipole moments (EDM)** The experimental bound on the neutron EDM implies that a possible dimension-4 term in the SM Lagrangian,  $\theta_{\text{QCD}} F\tilde{F}/(16\pi^2)$ , has a coefficient  $\theta_{\text{QCD}} \lesssim 10^{-10}$ . While there are plausible explanations [11], we do not yet know the resolution with certainty. Neglecting this term,  $CP$  violation in the CKM matrix only gives rise to quark EDMs at three-loop order, and lepton EDMs at four-loop level, resulting in EDMs below near future experimental sensitivities. On the other hand, new physics (e.g., supersymmetry) could generate both quark and lepton EDMs at the one-loop level, so even if the scale of new physics is 10–100 TeV, observable effects could arise.

**Top quark flavor physics** Well before the LHC turned on, it was already certain that it was going to be a top quark factory; the HL-LHC is expected to produce a few times  $10^9$   $t\bar{t}$  pairs. In the SM, top quarks almost exclusively decay to  $Wb$ , as  $||V_{tb}| - 1| \approx 10^{-3}$ . The current bounds on FCNC top decays are at the  $10^{-3}$  level, and the ultimate LHC sensitivity is expected to reach the  $10^{-5}$  to  $10^{-6}$  level, depending on the decay mode.

<sup>1</sup>Estimate the  $\mu \rightarrow e\gamma$  rate in the SM.



The SM rates are much smaller<sup>m</sup>, so observation of any FCNC top decay signal would be clear evidence for new physics.

There is obvious complementarity between FCNC searches in the top sector and low energy flavor physics bounds. Since  $t_L$  is in the same  $SU(2)$  doublet as  $b_L$ , several operators have correlated effects in  $t$  and  $b$  decays. For some operators, mainly those involving left-handed quark fields, the low energy constraints already exclude a detectable LHC signal, whereas other operators may still have large enough coefficients to yield detectable effects in top FCNCs at the LHC (see, e.g., Ref. [53]).

The  $t\bar{t}$  forward-backward asymmetry provided a clear example recently of the interplay between flavor physics and anomalies in the high energy collider data (even those that may seem little to do with flavor at first). The CDF measurement in 2011,  $A_{t\bar{t}}^{\text{FB}}(m_{t\bar{t}} > 450 \text{ GeV}) = 0.475 \pm 0.114$  [54], was stated to be  $3.4\sigma$  above the NLO SM prediction. At the LHC, the same underlying physics would produce a rapidity asymmetry.<sup>n</sup> It became quickly apparent that models that could account for this signal faced severe flavor constraints. This provides an example (with hundreds of papers in the literature) that flavor physics will likely be crucial to understand what the explanation of a high- $p_T$  LHC anomaly can be, and also what it cannot be. By now this excitement has subsided, because the significance of the Tevatron anomaly decreased and because the LHC has not seen any anomalies in the top production data predicted by most models (see, e.g., Ref. [55]) built to explain the Tevatron signal.

**Higgs flavor physics** With the discovery of a SM-like Higgs boson at the LHC, it is now clear that the LHC is also a Higgs factory. Understanding the properties of this particle entails both the precision measurements of its observed (and not yet seen) couplings predicted by the SM, and the search for possible decays forbidden in the SM.

The source of Higgs flavor physics, obviously, is the same set of Yukawa couplings whose structure and consequences we also seek to understand in low energy flavor physics measurements. While in terms of SUSY model building  $m_h \approx 125 \text{ GeV}$  is challenging to understand, this mass allows experimentally probing many Higgs production and decay channels. The fact that ultimately the LHC will be able to probe Higgs production via (i) gluon fusion ( $gg \rightarrow h$ ), (ii) vector boson fusion ( $q\bar{q} \rightarrow q\bar{q}h$ ), (iii)  $W/Z$  associated

<sup>m</sup>Estimate the  $t \rightarrow cZ$  and  $t \rightarrow c\gamma$  branching ratios in the SM.

<sup>n</sup>Show that if in  $t\bar{t}$  production at the Tevatron more  $t$  goes in the  $p$  than in the  $\bar{p}$  direction, then at the LHC the mean magnitude of the  $t$  quark rapidity is greater than that of the  $\bar{t}$ .

production ( $q\bar{q} \rightarrow hZ$  or  $hW$ ), (iv)  $b/t$  associated production ( $gg \rightarrow hb\bar{b}$  or  $ht\bar{t}$ ) sensitively depend on the Yukawa couplings and  $m_h$ .<sup>o</sup>

If we allow new physics to contribute to Higgs-related processes, which is especially well motivated for loop-induced production (e.g., the dominant  $gg \rightarrow h$ ) and decay (e.g.,  $h \rightarrow \gamma\gamma$ ) channels, then the first evidence for non-universal Higgs couplings to fermions was the bound on  $h \rightarrow \mu^+\mu^-$  below  $10 \times$  (SM prediction), combined with the observations of  $h \rightarrow \tau^+\tau^-$  at the SM level, implicitly bounding  $\mathcal{B}(h \rightarrow \mu^+\mu^-)/\mathcal{B}(h \rightarrow \tau^+\tau^-) \lesssim 0.03$ .

There is an obvious interplay between the search for flavor non-diagonal Higgs decays and indirect bounds from flavor-changing quark transitions and bounds on CLFV in the lepton sector. For example,  $y_{e\mu} \neq 0$  would generate a one-loop contribution to  $\mu \rightarrow e\gamma$ ,  $y_{uc} \neq 0$  would generate  $D^0 - \bar{D}^0$  mixing, etc. [56]. In some cases the flavor physics constraints imply that there is no chance to detect a particular flavor-violating Higgs decay, while signals in some modes may be above future direct search sensitivities. The interplay between measurements and constraints on flavor-diagonal and flavor-changing Higgs decay modes can provide additional insight on which flavor models are viable (see, e.g., Ref. [57]).

**Supersymmetry and flavor** While I hope the LHC will discover something unexpected, of the known BSM scenarios, supersymmetry is particularly interesting, and its signals have been worked out in great detail. The minimal supersymmetric standard model (MSSM) contains 44  $CP$  violating phases and 80 other  $CP$  conserving flavor parameters [58].<sup>p</sup> It has long been known that flavor physics (neutral meson mixings,  $\epsilon'_K$ ,  $\mu \rightarrow e\gamma$ ,  $B \rightarrow X_s\gamma$ , etc.) imposes strong constraints on the SUSY parameter space. The MSSM also contains flavor-diagonal  $CP$  violation (in addition to  $\theta_{\text{QCD}}$ ), and the constraints from the bounds on electric dipole moments are fairly strong on these phases if the mass scale is near 1 TeV.

As an example, consider the  $K_L - K_S$  mass difference. The squark-gluino box contribution compared to the data contains terms, roughly,

$$\frac{\Delta m_K^{(\text{SUSY})}}{\Delta m_K^{(\text{exp})}} \sim 10^4 \left( \frac{1 \text{ TeV}}{\tilde{m}} \right)^2 \left( \frac{\Delta \tilde{m}^2}{\tilde{m}^2} \right)^2 \text{Re}[(K_L^d)_{12}(K_R^d)_{12}], \quad (80)$$

where  $K_L^d$  ( $K_R^d$ ) are the mixing matrices in the gluino couplings to left-handed (right-handed) down quarks and their scalar partners [3]. The constraint from  $\epsilon_K$  corresponds to replacing  $10^4 \text{Re}[(K_L^d)_{12}(K_R^d)_{12}]$  with

<sup>o</sup>How would Higgs production and decay change if  $m_t$  were, say, 50 GeV?

<sup>p</sup>Check this, using the counting of couplings and broken global symmetries.

$10^6 \text{Im}[(K_L^d)_{12}(K_R^d)_{12}]$ . The simplest supersymmetric frameworks with parameters in the ballpark of  $\tilde{m} = \mathcal{O}(1 \text{ TeV})$ ,  $\Delta\tilde{m}^2/\tilde{m}^2 = \mathcal{O}(0.1)$ , and  $(K_{L,R}^d)_{ij} = \mathcal{O}(1)$  are excluded by orders of magnitude.

There are several ways to address the supersymmetric flavor problems. There are classes of models that suppress each of the terms in Eq. (80): (i) heavy squarks, when  $\tilde{m} \gg 1 \text{ TeV}$  (e.g., split SUSY); (ii) universality, when  $\Delta\tilde{m}_{\tilde{Q},\tilde{D}}^2 \ll \tilde{m}^2$  (e.g., gauge mediation); (iii) alignment, when  $(K_{L,R}^d)_{12} \ll 1$  (e.g., horizontal symmetry). All viable models incorporate some of these ingredients in order not to violate the experimental bounds. Conversely, if SUSY is discovered, mapping out its flavor structure will help to answer important questions about even higher scales, e.g., the mechanism of SUSY breaking, how it is communicated to the MSSM, etc.

A special role in constraining SUSY models is played by  $D^0 - \bar{D}^0$  mixing, which was the first observed FCNC process in the up-quark sector. It is a special probe of BSM physics, because it is the only neutral meson system in which mixing is generated by intermediate down-type quarks in the SM, or intermediate up-type squarks in SUSY. The constraints are thus complementary to FCNC processes involving  $K$  and  $B$  mesons.  $D^0 - \bar{D}^0$  mixing and FCNC in the up-quark sector are particularly important in constraining scenarios utilizing quark-squark alignment [59; 60].

Another important implication for SUSY searches is that the LHC constraints on squark masses are sensitive to the level of (non-)degeneracy of squarks required to satisfy flavor constraints. Most SUSY searches assume that the first two generation squarks,  $\tilde{u}_{L,R}$ ,  $\tilde{d}_{L,R}$ ,  $\tilde{s}_{L,R}$ ,  $\tilde{c}_{L,R}$ , are all degenerate, which increases signal cross sections. Relaxing this assumption consistent with flavor bounds [60; 61], results in substantially weaker squark mass limits from Run 1, as low as around the 500 GeV scale [62].

It is apparent from the above discussion that there is a tight interplay between the implications of the non-observation of new physics at the LHC so far, and the non-observation of deviations from the SM in flavor physics. If there is new physics at the TeV scale, which we hope the LHC will discover in its next run, then we know already that its flavor structure must be rather non-generic to suppress FCNCs, and the combination of all data will contain plenty of additional information about the structure of new physics. The higher the scale of new physics, the less severe the flavor constraints are. If NP is beyond the reach of the LHC, flavor physics experiments may still observe robust deviations from the SM predictions, which would point to an upper bound on the next scale to probe.

**Minimal flavor violation (MFV)** The standard model without Yukawa couplings has a global  $[U(3)]^5$  symmetry ( $[U(3)]^3$  in the quark and  $[U(3)]^2$  in the lepton sector), rotating the 3 generations of the 5 fields in Eq. (4). This is broken by the Yukawa interactions in Eq. (7). One may view the Yukawa couplings as spurions, fields which transform under  $[U(3)]^5$  in a way that makes the Lagrangian invariant, and then the global flavor symmetry is broken by the background values of the Yukawas. BSM scenarios in which there are no new sources of flavor violation beyond the Yukawa matrices are called minimal flavor violation [63; 64; 65]. Since the SM breaks the  $[U(3)]^5$  flavor symmetry already, MFV gives a framework to characterize “minimal reasonable” deviations from the SM predictions.

Let us focus on the quark sector. Under  $U(3)_Q \times U(3)_u \times U(3)_d$  the transformation properties are

$$Q_L(3, 1, 1), \quad u_R(1, 3, 1), \quad d_R(1, 1, 3), \quad Y_u(3, \bar{3}, 1), \quad Y_d(3, 1, \bar{3}). \quad (81)$$

One can choose a basis in which

$$Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V_{\text{CKM}}^\dagger \text{diag}(y_u, y_c, y_t). \quad (82)$$

To generate a flavor-changing transition, requires constructing  $[U(3)]^3$  singlet terms that connect the required fields. For example, in the down-quark sector, the simplest terms are [65]

$$\bar{Q}_L Y_u Y_u^\dagger Q_L, \quad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger Q_L, \quad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger Y_d d_R. \quad (83)$$

A useful feature of this approach is that it allows EFT-like analyses.

Consider  $B \rightarrow X_s \gamma$  as an example. We are interested in the magnitude of a possible NP contribution to the Wilson coefficient of the operator  $\frac{X}{\Lambda} (\bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R)$ . A term  $\bar{Q}_L b_R$  is not invariant under  $[U(3)]^3$ . A term  $\bar{Q}_L Y_d d_R$  is  $[U(3)]^3$  invariant, but it is diagonal, so it only connects same generation fields. The first non-vanishing contribution comes from  $\bar{Q}_L Y_u Y_u^\dagger Y_d d_R$ , which has a  $V_{tb} V_{ts}^* y_t^2 y_b (\bar{s}_L b_R)$  component. We learn that in MFV models, in general,  $X \propto y_b V_{tb} V_{ts}^*$ , as is the case in the SM.

Thus, in MFV models, most flavor-changing operators “automatically” have their SM-like suppressions, proportional to the same CKM elements, quark masses from chirality flips, etc. Therefore, the scale of MFV models can be  $\mathcal{O}(1 \text{ TeV})$  without violating flavor physics bounds, thus solving the new physics flavor puzzle. Originally introduced for technicolor models [63], gauge-mediated supersymmetry breaking provides another well known scenario in which MFV is expected to be a good approximation.

MFV models have important implications for new particle searches, too. Since the only quark flavor-changing parameters are the CKM elements,

and the ones that couple the third generation to the lighter ones are very small, in MFV models new particles that decay to a single final quark (and other particles) decay to either a third generation quark or to quarks from the first two generations, but (to a good approximation) not to both [66].

The MFV ansatz can be incorporated into models that do not contain explicitly flavor breaking unrelated to Yukawa couplings. MFV is not expected to be an exact symmetry, but it may be a useful organizing principle to understand details of the new physics we soon hope to get a glimpse of.

## 5. Summary

An essential feature of flavor physics is its ability to probe very high scales, beyond the masses of particles that can be produced on-shell in colliders. Flavor physics can also teach us about properties of TeV-scale new physics, that cannot be learned from the direct production of new particles.

Some of the main points I tried to explain in these lectures were:

- Flavor-changing neutral currents and meson mixing probe scales well above the masses of particles colliders can produce, and provide strong constraints on TeV-scale new physics.
- $CP$  violation is always the result of interference phenomena, without a classical analog.
- The KM phase has been established as the dominant source of  $CP$  violation in flavor-changing processes.
- Tremendous progress will continue: Until  $\sim 10$  years ago, more than  $\mathcal{O}(1)$  deviations from the SM were possible; at present  $\mathcal{O}(20\%)$  corrections to most FCNC processes are still allowed; in the future, sensitivities of a few percent will be reached.
- The future goal is not measuring SM parameters better, but to search for corrections to the SM, and to learn about NP as much as possible.
- Direct information on new particles and their influence on flavor-changing processes will both be crucial to understand the underlying physics.
- The sensitivity of future experiments in a number of important processes is only limited by statistics, not theory.
- The interesting (and fun) interplay between theoretical and experimental developments in flavor physics will continue.

At present, both direct production and flavor physics experiments only give bounds on new physics. The constraints imply that if new physics is accessible at the LHC, it is likely to have flavor suppression factors similar

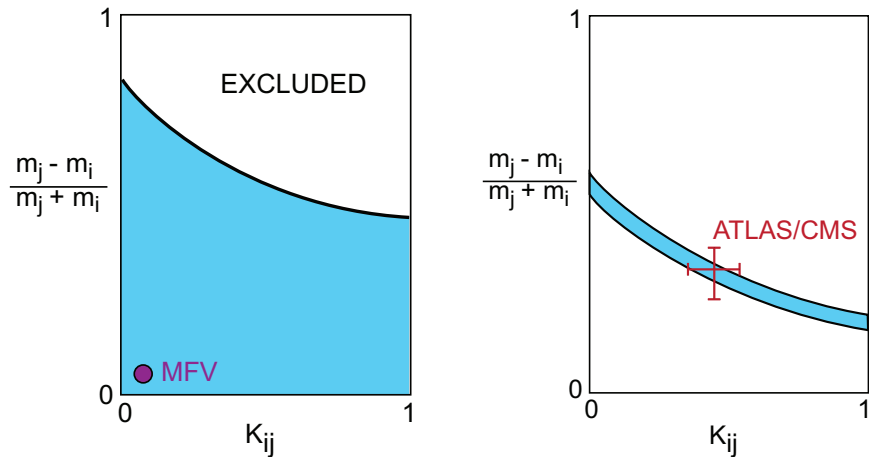


Fig. 15. Schematic description of the constraints on the mass splitting,  $(m_i - m_j)/(m_i + m_j)$ , and mixing angle,  $K_{ij}$ , between squarks (or sleptons). Left: typical constraint from not observing deviations from the SM. The fact that  $\mathcal{O}(1)$  splittings and mixings are excluded constitutes the new physics flavor puzzle. Right: possible future scenario where ATLAS/CMS measurements fit flavor physics signals of NP. (From Ref. [7].)

to the SM. In many models (e.g., the MSSM), measurements or bounds on FCNC transitions constrain the product of certain mass splittings times mixing parameters divided by the square of the new physics scale. If the LHC discovers new physics, then in principle the mass splittings and mixing parameters can be measured separately. If flavor physics experiments establish a deviation from the SM in a related process, the combination of LHC and flavor data can be very powerful to discriminate between models. The consistency of measurements could ultimately tell us that we understand the flavor structure of new physics and how the new physics flavor puzzle is solved. The present situation and an (optimistic) future scenario for supersymmetry are shown in Fig. 15. Let's hope that we shall have the privilege to think about such questions, motivated by data, in the coming years.

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