# Introduction to Many-body Theory III

Part III: Linear response and examples

- The 2-particle Green's function and optical spectra
- Hedin's equations
- Linear response
- Examples: Time-dependent screening in an electron gas

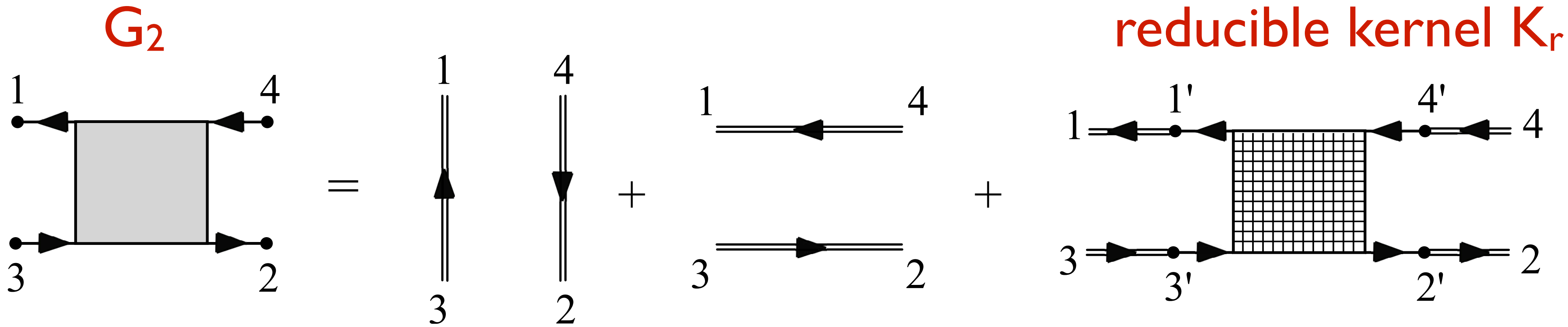
# The 2-particle Green's function

We can further expand the two-particle Green's function using Wick's theorem

$$G_2(a,b;c,d)$$

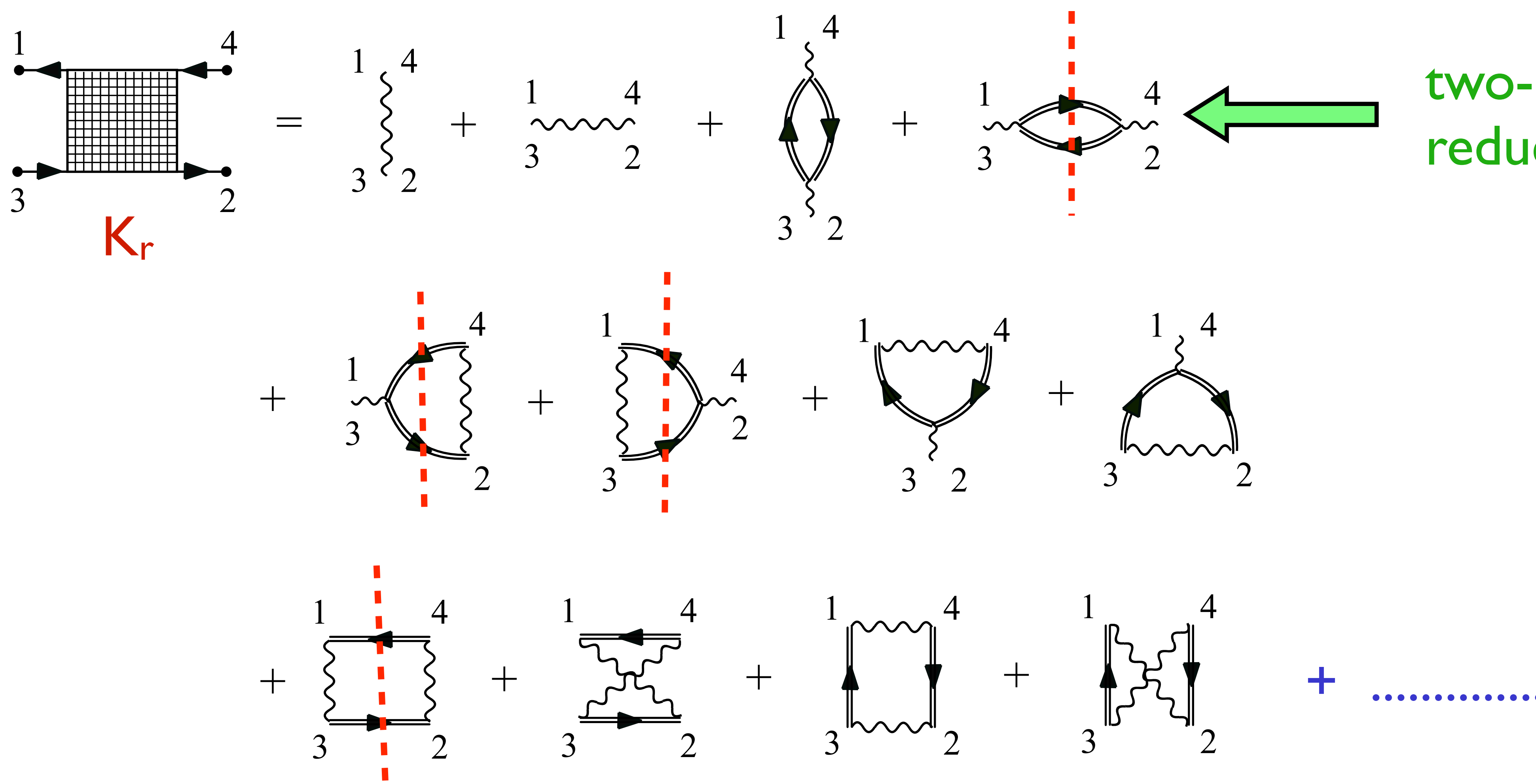
$$= \frac{\sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\mathrm{i}}{2}\right)^k \int v(1;1') \ldots v(k;k') \begin{vmatrix} G_0(a;c) & G_0(a;d) & \ldots & G_0(a;k'^+) \\ G_0(b;c) & G_0(b;d) & \ldots & G_0(b;k'^+) \\ \vdots & \vdots & \ddots & \vdots \\ G_0(k';c) & G_0(k';d) & \ldots & G_0(k';k'^+) \end{vmatrix}_\pm}{\sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\mathrm{i}}{2}\right)^k \int v(1;1') \ldots v(k;k') \begin{vmatrix} G_0(1;1^+) & G_0(1;1'^+) & \ldots & G_0(1;k'^+) \\ G_0(1';1^+) & G_0(1';1'^+) & \ldots & G_0(1';k'^+) \\ \vdots & \vdots & \ddots & \vdots \\ G_0(k';1^+) & G_0(k';1'^+) & \ldots & G_0(k';k'^+) \end{vmatrix}_\pm}$$

Again only connected diagrams contribute. In the same way as before non-connected diagrams cancel and we can expand in G-skeletons by removing self-energy insertions

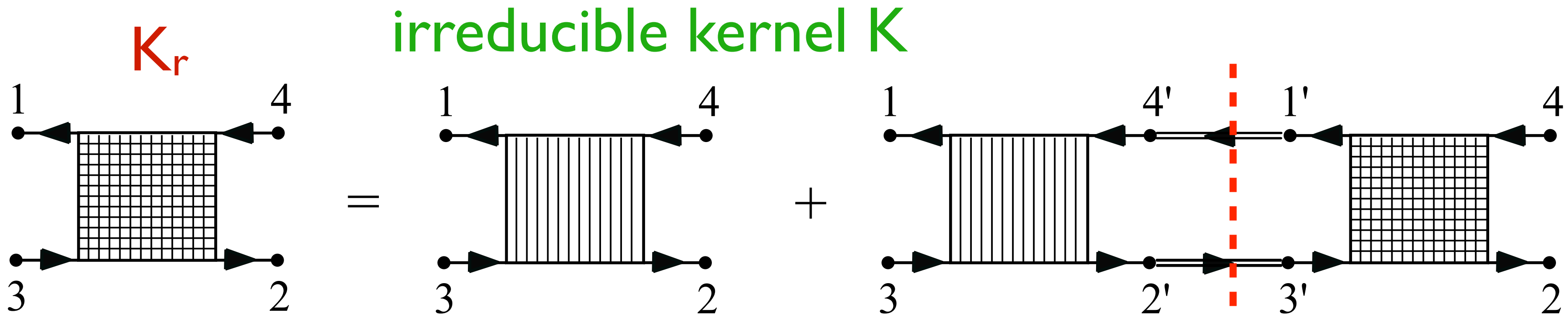



$$G_2(1,2;3,4) = G(1;3)G(2;4) \pm G(1;4)G(2;3) + \int G(1;1')G(3';3)K_r(1',2';3',4')G(4';4)G(2;2')$$

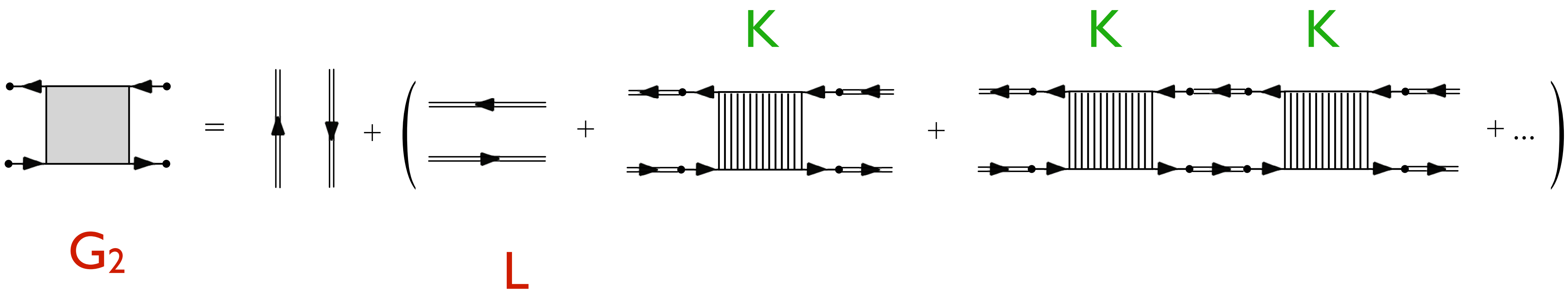
noninteracting form

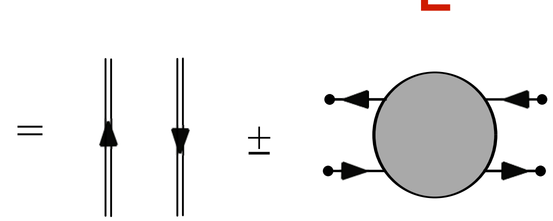



two-particle reducible

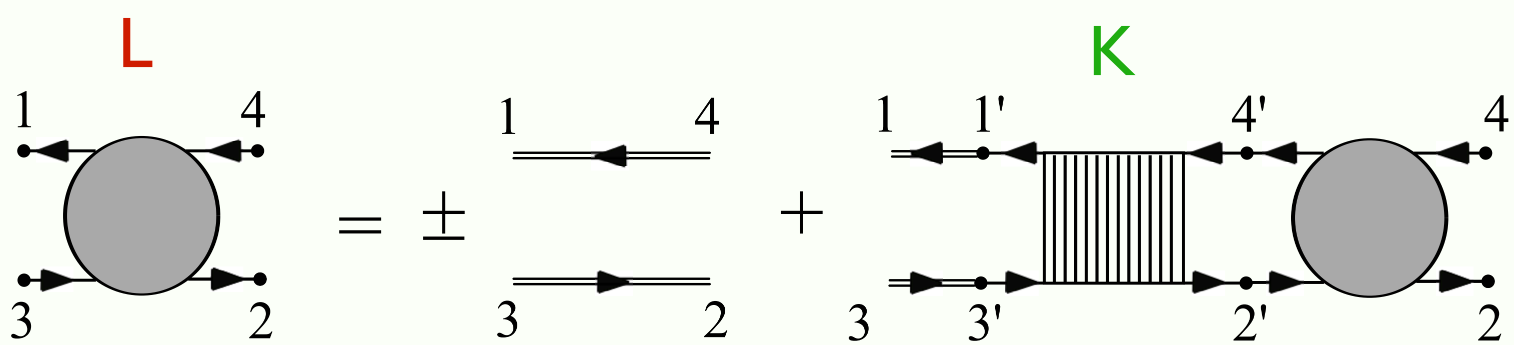



$$K_r(1,2;3,4) = K(1,2;3,4) \pm \int K(1,2';3,4')G(4';1')G(3';2')K_r(1',2;3',4)$$



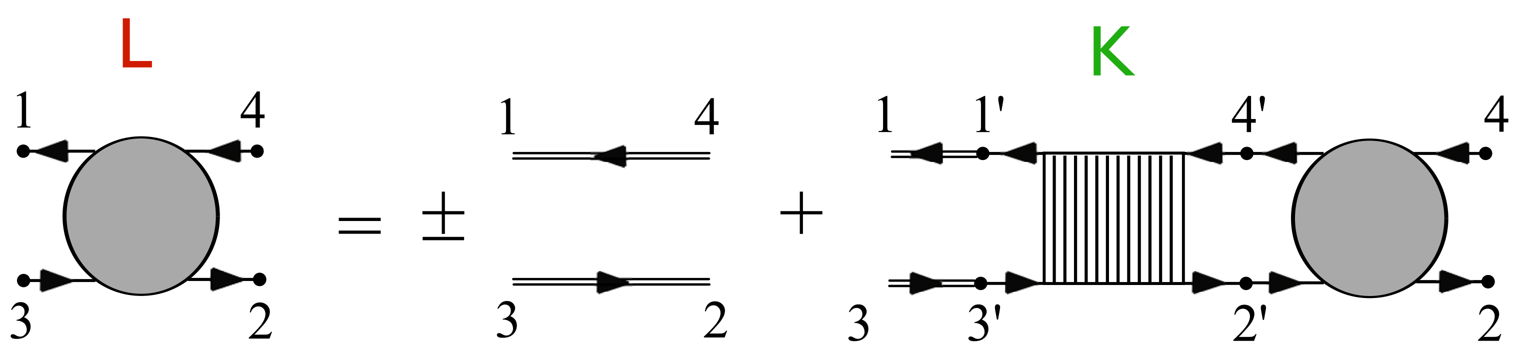

$$L(1,2;3,4) \equiv \pm\big[G_2(1,2;3,4) - G(1;3)G(2;4)\big]$$




Bethe-Salpeter equation

$$L(1,2;3,4) = G(1;4)G(2;3) \pm \int G(1;1')G(3';3)K(1',2';3',4')L(4',2;2',4)$$

To find the 2-particle Green's function we have to solve the Bethe-Salpeter equation




Bethe-Salpeter equation

$$L(1,2;3,4) = G(1;4)G(2;3) \pm \int G(1;1')G(3';3)K(1',2';3',4')L(4',2;2',4)$$

If we expand the self-energy in G-skeletonic diagrams then the following important relation is valid

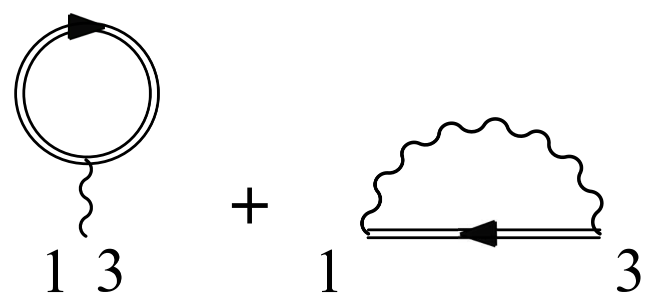
$$\boxed{K(1,2;3,4) = \pm \frac{\delta\Sigma(1;3)}{\delta G(4;2)}}$$

It is not hard to prove this diagrammatically

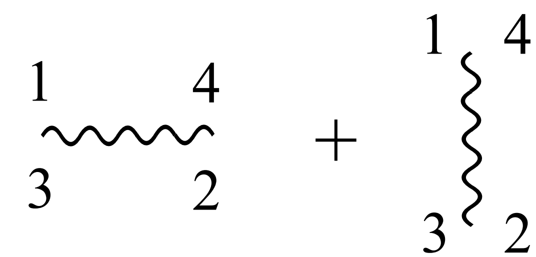
## Let us give some examples

K

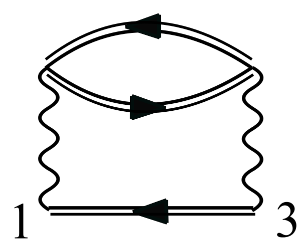
$\Sigma_{\mathrm{HF}}(1;3) =$



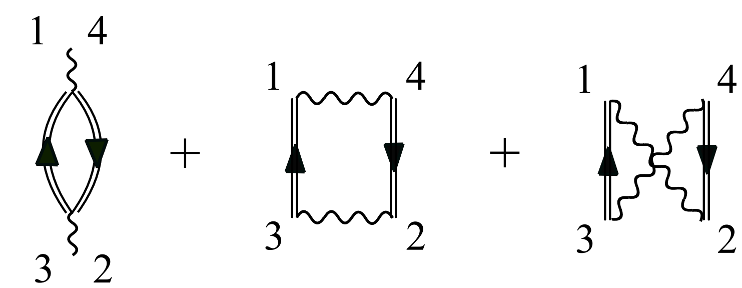

$\pm \dfrac{\delta \Sigma_{\mathrm{HF}}(1;3)}{\delta G(4;2)} =$



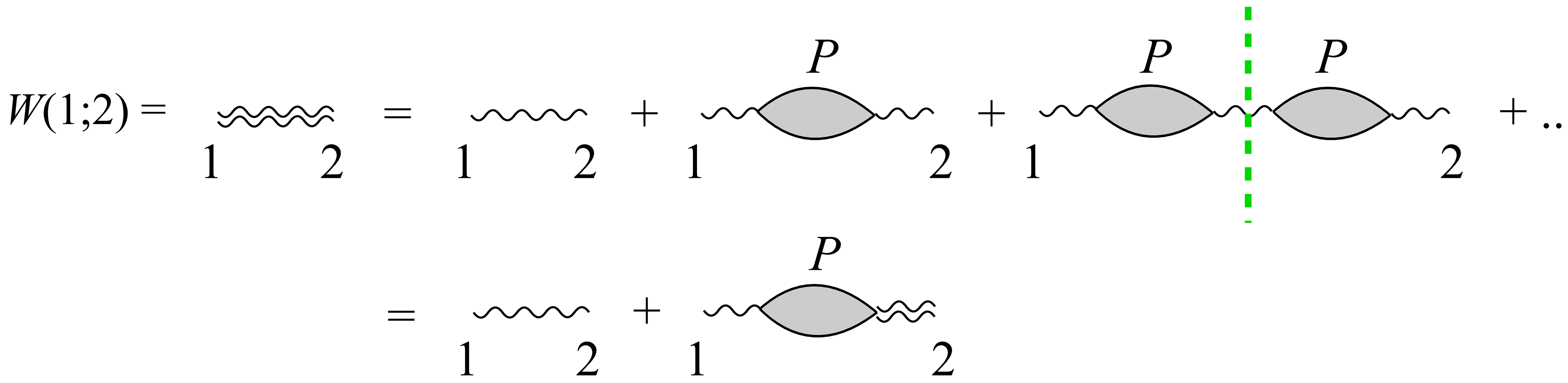

$\Sigma_{\mathrm{2B,bubble}}(1;3) =$




$\pm \dfrac{\delta \Sigma_{\mathrm{2B,bubble}}(1;3)}{\delta G(4;2)} =$

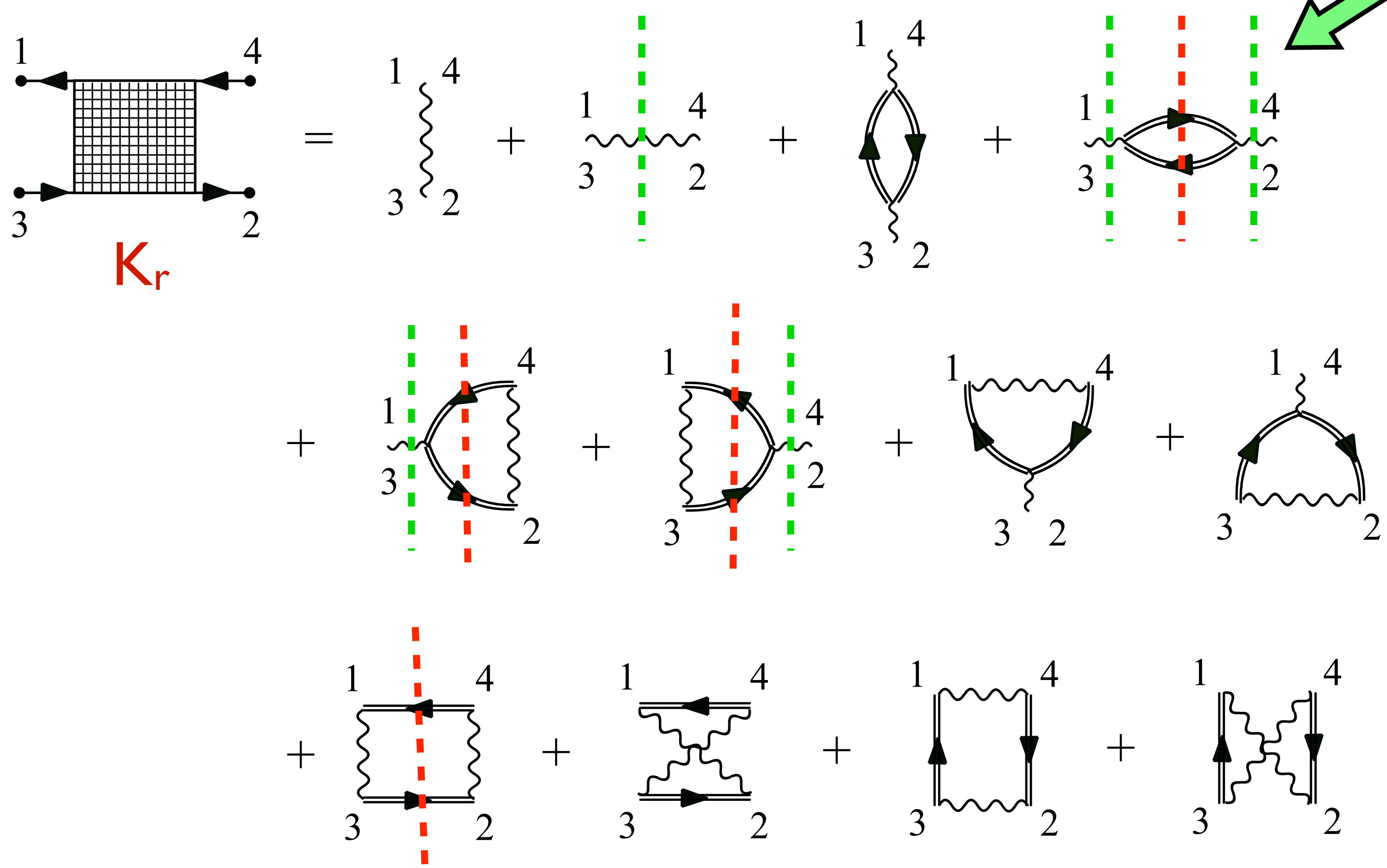


What about the W-skeletons? Remember that

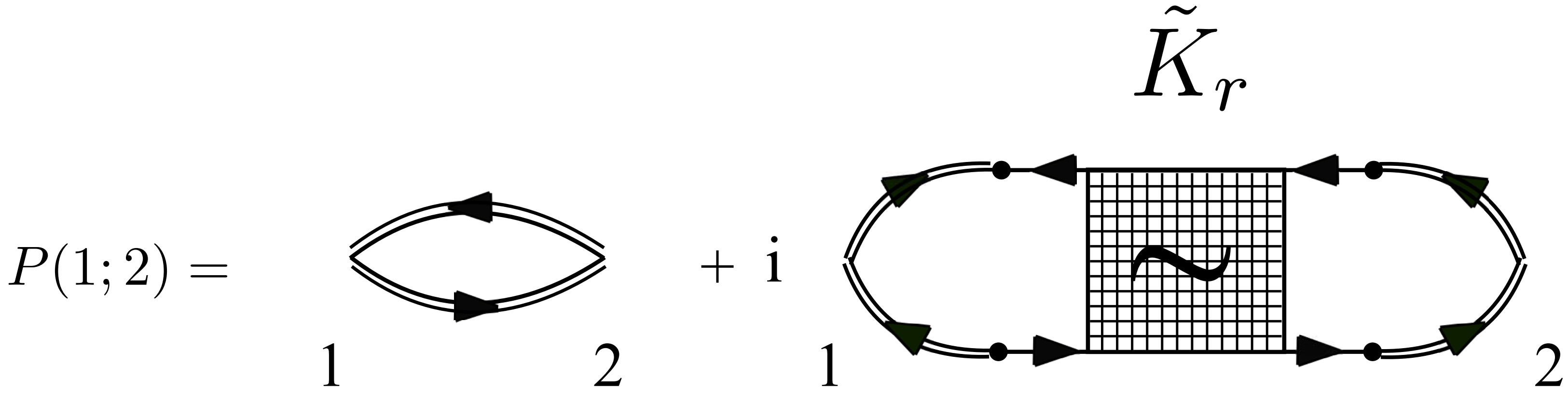



Let us look at the reducible kernel again

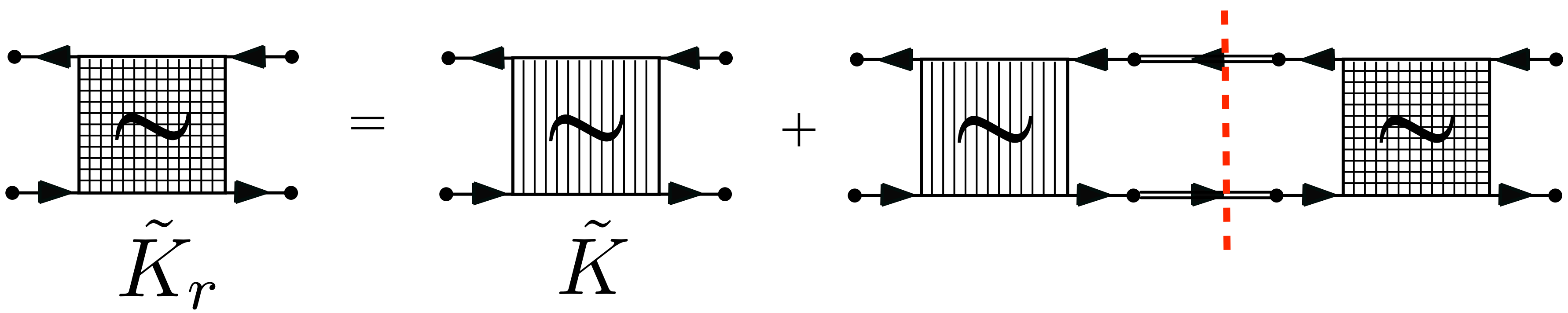
interaction line reducible

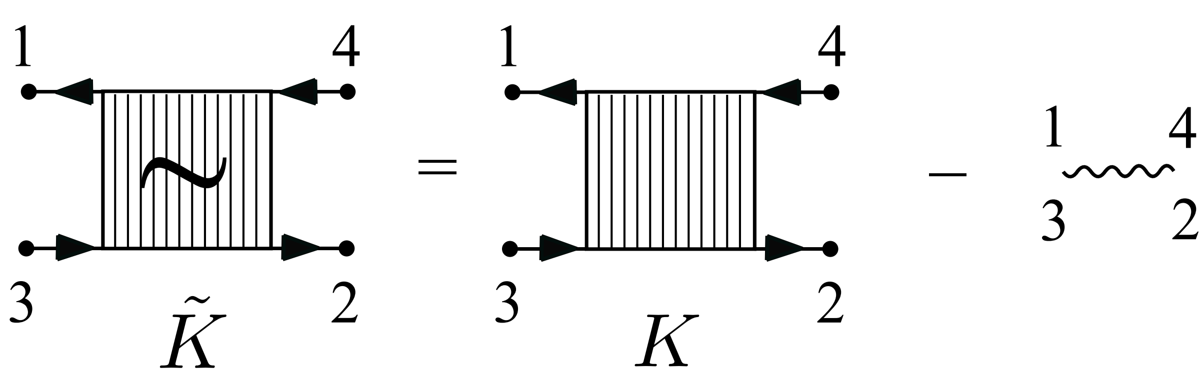


If we remove from $K_r$ all interaction line reducible we obtain a new kernel $\tilde{K}_r$



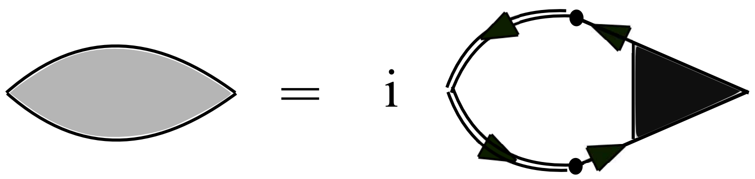

The reducible kernel can again be expanded in a G-line irreducible one



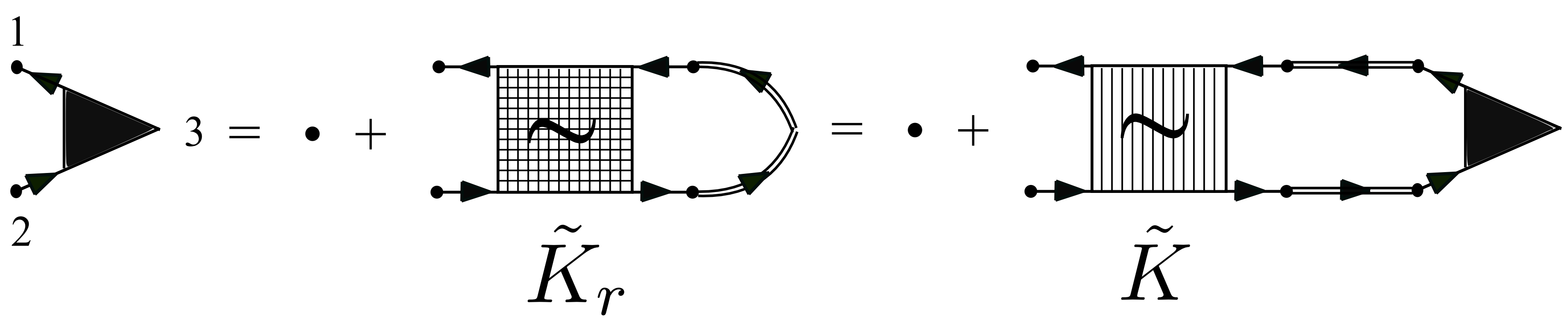

$$\tilde{K}(1,2;3,4) = K(1,2;3,4) - \mathrm{i}\,\delta(1;3)\delta(2;4)v(1;2).$$

The polarizability can then be expressed as




$$P(1,2) = -i \int d4\, G(1,3)\, G(4,1)\, \Gamma(3,4;2)$$

where we defined the vertex function as




$$\Gamma(1,2;3) = \delta(1,2^+)\delta(3,2) - \int d(4567)\, \tilde{K}(1,5;2,4)\, G(4,6)\, G(7,5)\, \Gamma(6,7;3)$$

For the kernel $\tilde{K}$ the following relation is valid

$$\boxed{\tilde{K}(1,2;3,4) = -\frac{\delta\Sigma_{\rm xc}(1,3)}{\delta G(4,2)}}$$

as follows from a diagrammatic proof

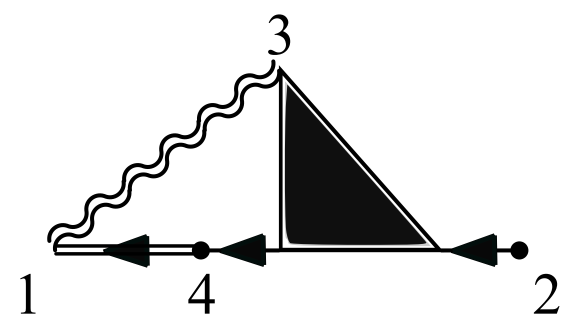
The equation for the vertex therefore becomes

$$\Gamma(1,2;3) = \delta(1;2^+)\delta(3;2) + \int d4d5d6d7\, \frac{\delta\Sigma_{\rm xc}(1;2)}{\delta G(4;5)}\, G(4;6)G(7;5)\Gamma(6,7;3)$$

If we can further express the vertex in terms of the vertex we have a closed set of functional differential equations

By a diagrammatic derivation we can also show that the self-energy can be expressed in terms of the vertex

$\Sigma(1;2) =$ 

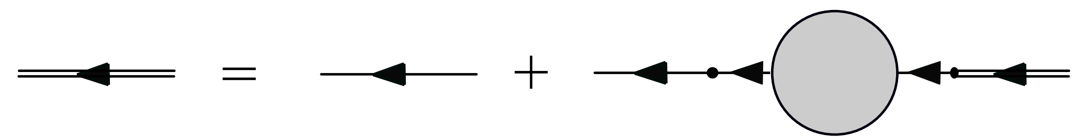
$+\,\mathrm{i}$ 


$$\Sigma(1;2) = \pm\mathrm{i}\,\delta(1;2)\int d3\,v(1;3)G(3;3^{+}) + \mathrm{i}\int d3d4\,W(1;3)G(1;4)\Gamma(4,2;3)$$

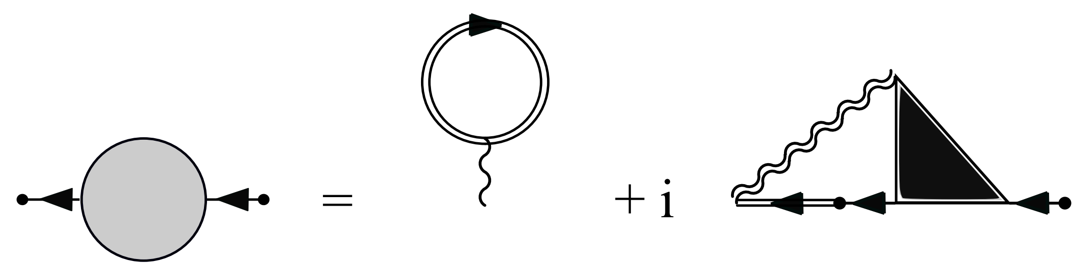
Let us collect all the equations that we derived

# Hedin's equations

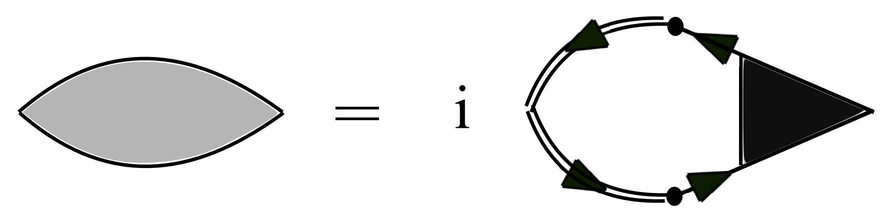
$$G = G_0 + G_0 \Sigma G$$



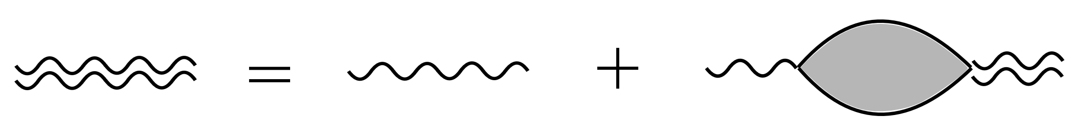
$$\Sigma = \Sigma_{\mathrm{H}} + \mathrm{i} G W \Gamma$$



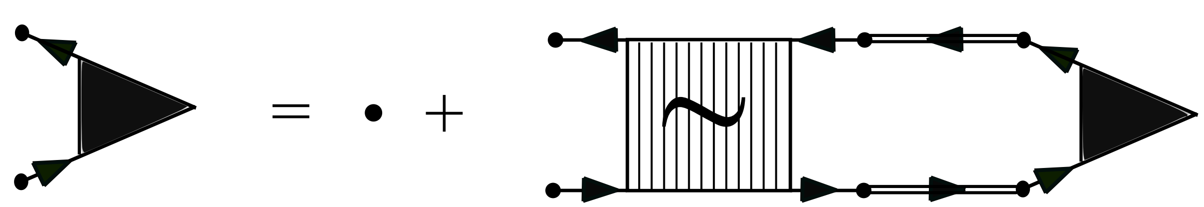
$$P = \pm \mathrm{i} G G \Gamma$$



$$W = v + v P W$$



$$\Gamma = \delta + \frac{\delta \Sigma_{\mathrm{xc}}}{\delta G} G G \Gamma$$



# 2-particle Green's function and Hedin's equations: Take home message

- For the 2-particle Green's function we can derive an equation with a reducible kernel, known as the Bethe-Salpeter equation.

- The reducible kernel is the functional derivative of the self-energy with respect to the Green's function.

- From the diagrammatic rules we can derive a set of functional differential equations relating the vertex, the Dyson and the Bethe-Salpeter equation. These equations are known as the Hedin equations.

- The Hedin equations can be iterated in various ways to generate different perturbation series. It is not known whether all skeleton diagrams are generated once by such a procedure.

# Linear response functions

$$\langle \hat{n}(\mathbf{x},t)\rangle = \frac{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\hat{n}(\mathbf{x},t)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}}$$

If we make the variation $\hat{H}(z) \to \hat{H}(z) + \delta\hat{V}(z)$ $\quad$ $\delta\hat{V}(z) = \int d\mathbf{x}\,\hat{n}(\mathbf{x})\,\delta v(\mathbf{x}z)$

then

$$\delta\langle \hat{n}(\mathbf{x},t)\rangle = -i\int_\gamma dz_1 \frac{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\hat{n}(\mathbf{x},t)\delta\hat{V}(z_1)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}} + i\langle \hat{n}(\mathbf{x},t)\rangle \int_\gamma dz_1 \frac{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\delta\hat{V}(z_1)\right\}}{\mathrm{Tr}\ \mathcal{T}\left\{ e^{-i\int_\gamma d\bar{z}\,\hat{H}(\bar{z})}\right\}}$$

which can be rewritten as

$$\delta n(1) = \int d2\,\chi(1,2)\,\delta v(2)$$

$$\chi(1,2) = -i\left[\,\langle \mathcal{T}\left\{\hat{n}_H(1)\hat{n}_H(2)\right\}\rangle - n(1)n(2)\,\right]$$

There is a close relation between the density response function and the Bethe-Salpeter equation. We have

$$
\begin{aligned}
L(1,2;1',2') &= -\left[G_2(1,2;1',2') - G(1,1')G(2,2')\right] \\
&= \langle \mathcal{T}\left\{\hat{\psi}_H(1)\hat{\psi}_H(2)\hat{\psi}^\dagger_H(2')\hat{\psi}^\dagger_H(1')\right\}\rangle - \langle \mathcal{T}\left\{\hat{\psi}_H(1)\hat{\psi}^\dagger_H(1')\right\}\rangle\langle \mathcal{T}\left\{\hat{\psi}_H(2)\hat{\psi}^\dagger_H(2')\right\}\rangle
\end{aligned}
$$

and therefore

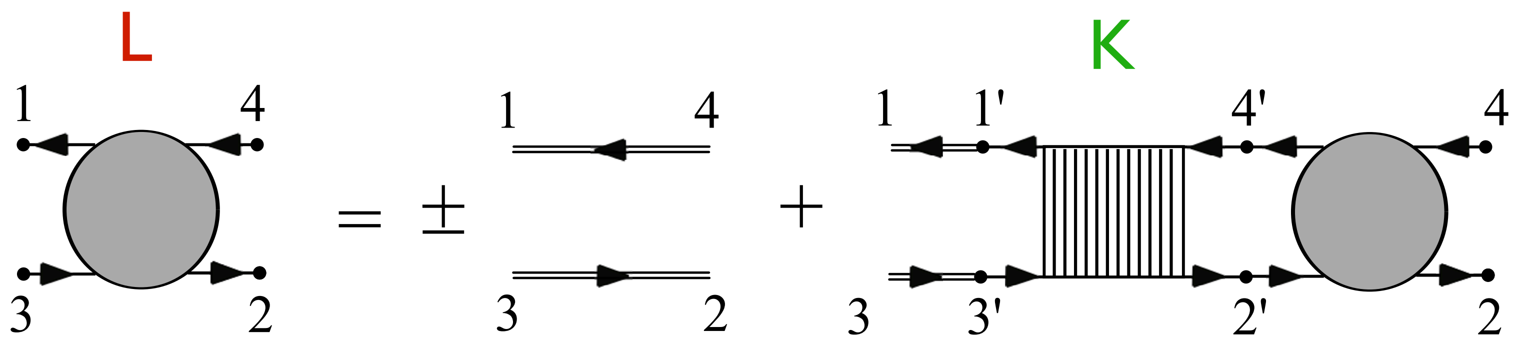
$$
\boxed{\chi(1,2) = -i\,[\,\langle \mathcal{T}\,\{\hat{n}_H(1)\hat{n}_H(2)\}\rangle - n(1)n(2)\,] = -i\,L(1,2;1^+,2^+)}
$$

In combination with the Bethe-Salpeter equation we can then further derive that

$$
\chi(1,2) = P(1,2) + \int d3d4\,P(1,3)\,w(3,4)\,\chi(4,2)
$$

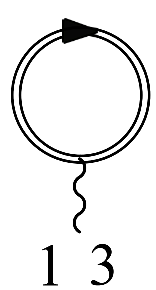
A diagrammatic expansion of the polarisability therefore directly gives an approximation for the density response function

# Random Phase Approximation and plasmons

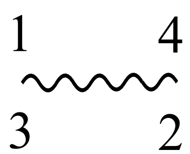



Bethe-Salpeter equation

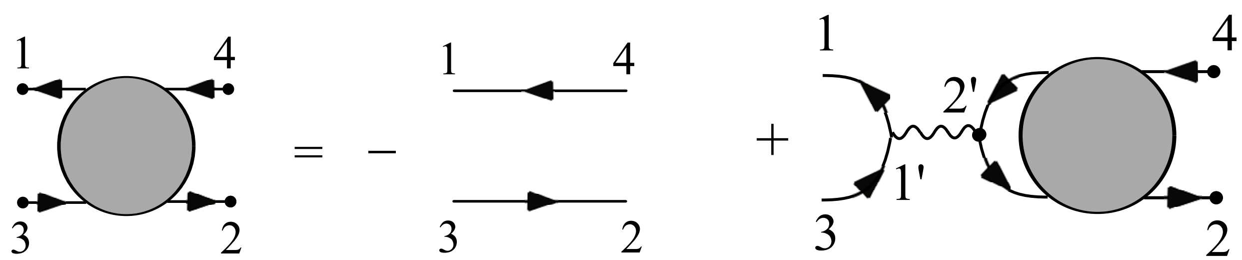
If we calculate the Bethe-Salpeter from the Hartree self-energy

$\Sigma_H =$ 

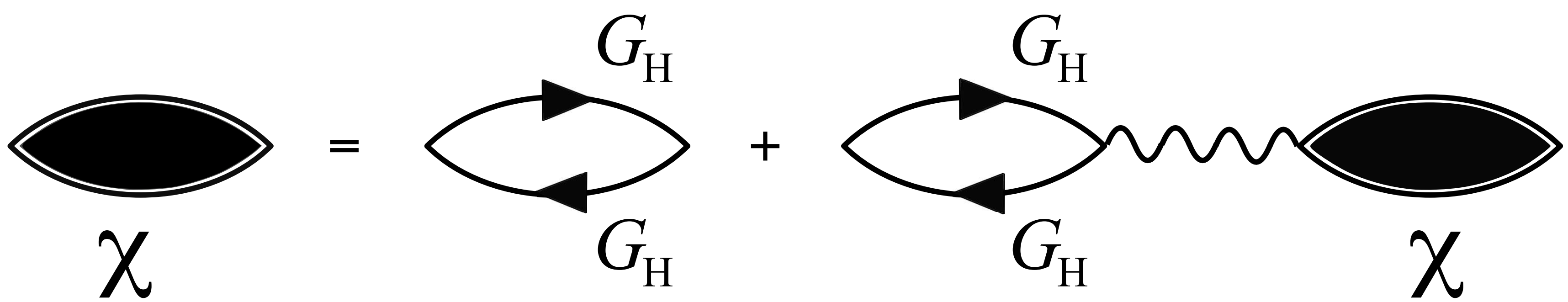

$$K_H(1,2;3,4) = -\frac{\delta \Sigma_H(1,3)}{\delta G(4,2)} =$$




then the Bethe-Salpeter equation becomes



From $\chi(1,2) = -i\, L(1,2;1^+,2^+)$ it then follows




if we take the retarded component of this expression and Fourier transform then we find

$$\chi^{\mathrm{R}}(\mathbf{x}_1,\mathbf{x}_2;\omega) = \chi_0^{\mathrm{R}}(\mathbf{x}_1,\mathbf{x}_2;\omega) + \int d\mathbf{x}_3 d\mathbf{x}_4\, \chi_0^{\mathrm{R}}(\mathbf{x}_1,\mathbf{x}_3;\omega) v(\mathbf{x}_3,\mathbf{x}_4) \chi^{\mathrm{R}}(\mathbf{x}_4,\mathbf{x}_2;\omega)$$

This approximation for the density response function is also known as the Random Phase Approximation (RPA).

A better name is the Time-Dependent Hartree Approximation (it amounts to TDDFT with zero xc-kernel)

Let us now take the case of the homogeneous electron gas. Since the system is translational invariant we can write

$$\sum_{\sigma\sigma'} \chi^{\mathrm{R}}(\mathbf{x},\mathbf{x}';\omega) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{\mathrm{i}\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} \chi^{\mathrm{R}}(\mathbf{p},\omega)$$

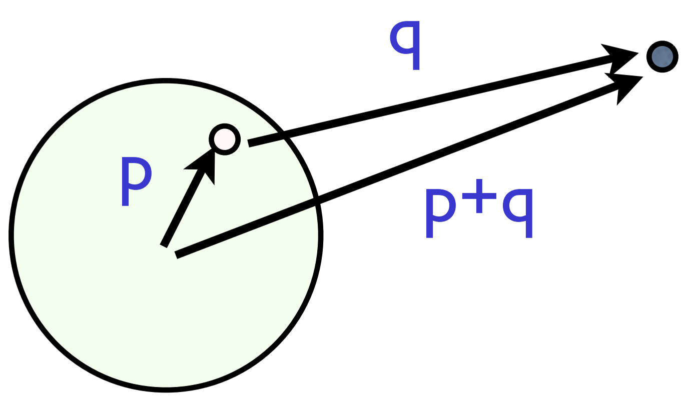
$$\chi^{\mathrm{R}}(\mathbf{q},\omega) = \frac{\chi_0^{\mathrm{R}}(\mathbf{q},\omega)}{1-\tilde{v}_{\mathbf{q}}\chi_0^{\mathrm{R}}(\mathbf{q},\omega)}, \qquad \tilde{v}_{\mathbf{q}} = \frac{4\pi}{q^2}$$

Fourier transform Coulomb potential

The RPA response function has poles at the poles of $\chi_0(\mathbf{q},\omega)$ and when

$$1-\tilde{v}_{\mathbf{q}}\,\chi_0(\mathbf{q},\omega) = 0$$

The extra pole corresponding to this condition is known as the plasmon and corresponds to a collective mode of the electron gas




Fermi sphere
with radius $p_F$

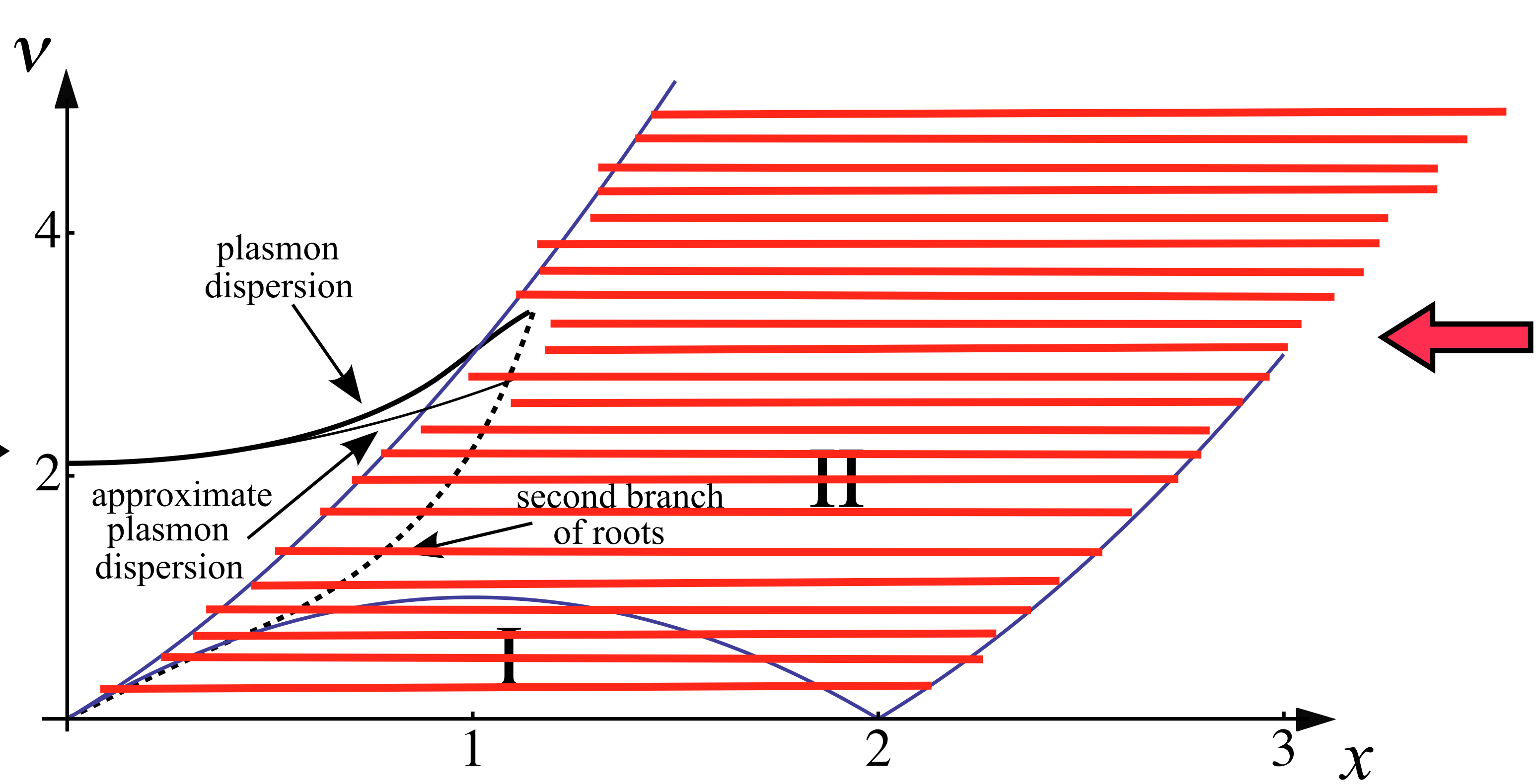
$$\epsilon = \frac{(\mathbf{p}+\mathbf{q})^2}{2} - \frac{\mathbf{p}^2}{2} = \frac{\mathbf{q}^2}{2} + |\mathbf{p}||\mathbf{q}|\cos\theta$$

$$\frac{q^2}{2} - q\,p_{\mathrm{F}} \;\le \epsilon \le \frac{q^2}{2} + q\,p_{\mathrm{F}} \qquad\qquad q = |\mathbf{q}|$$

The particle-hole excitations lie between two parabolas in the q-ω plane

$$\nu = \frac{2\omega}{p_{\mathrm{F}}^2}$$

$\omega_p$
plasma frequency
Long wavelength plasmon




particle-hole continuum

$$x = \frac{q}{p_{\mathrm{F}}}$$

Sudden creation of a positive charge (such as in the creation of a core-hole)

$$\delta V(\mathbf{x},t) = \theta(t)\frac{Q}{r} = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\omega}{2\pi} e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}-\mathrm{i}\omega t}\,\delta V(\mathbf{q},\omega)$$

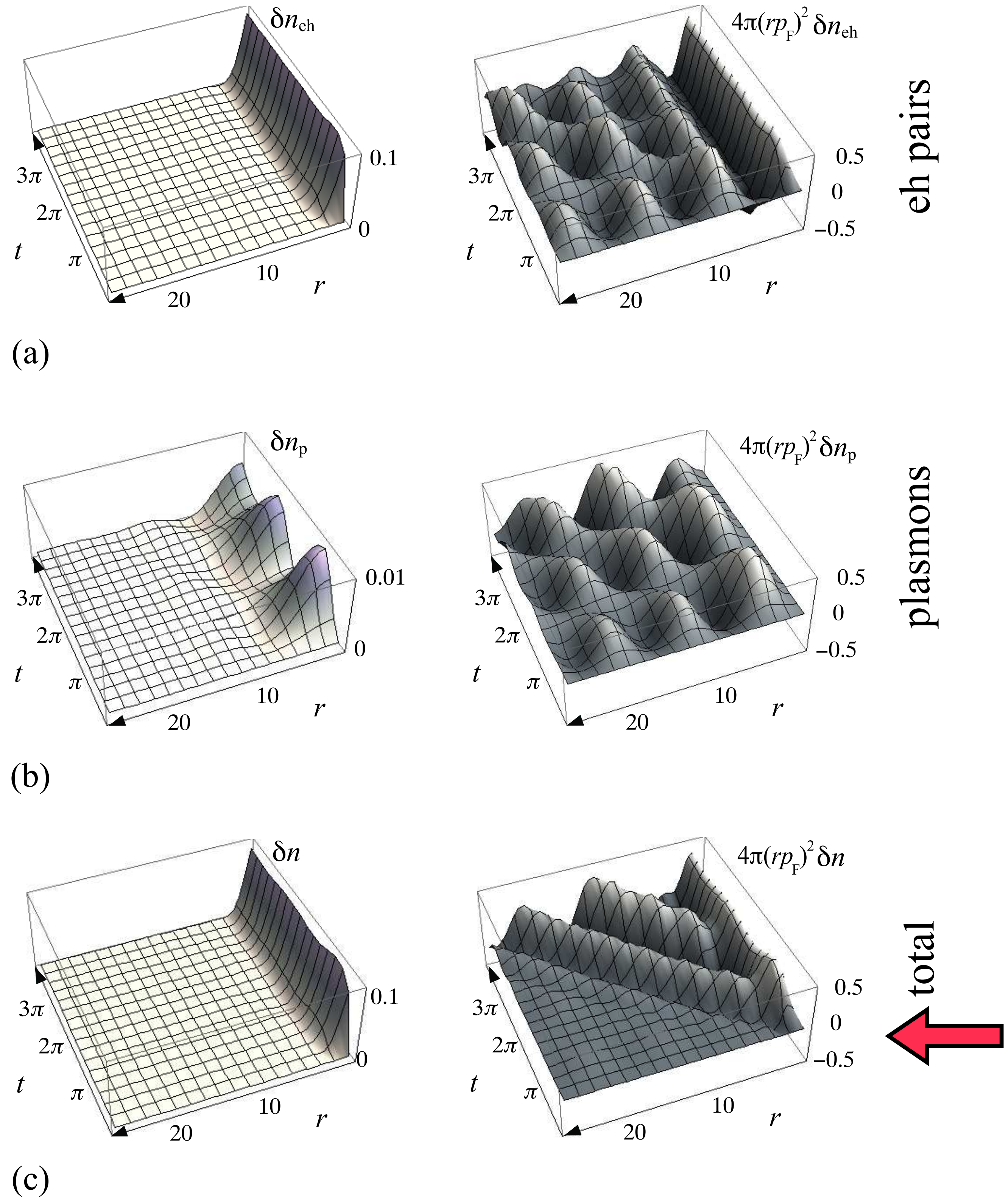
$$\delta V(\mathbf{q},\omega) = \frac{4\pi Q}{q^2}\frac{\mathrm{i}}{\omega+\mathrm{i}\eta} = \tilde{v}_{\mathbf{q}}\,Q\frac{\mathrm{i}}{\omega+\mathrm{i}\eta}.$$

We can calculate the induced density change from the RPA response function.
A few manipulations lead to

$$\delta n(\mathbf{r},t) = -\frac{16\pi Q}{(2\pi)^4}\frac{1}{r}\int_0^\infty dq\,q\,\sin qr\int_0^\infty \mathrm{Im}\chi^{\mathrm{RPA}}(q,\omega)\,\tilde{v}_{\mathbf{q}}\frac{1-\cos\omega t}{\omega}$$

$$\Downarrow$$

The integral can be split into a contribution from particle-hole excitations and a contribution from the plasmon




The positive charge is screened at a time-scale of the inverse plasmon frequency

Figure 15.7: This figure shows the 3D plot of the transient density in an electron gas with $r_s = 3$ induced by the sudden creation of a point-like positive charge $Q = 1$ in the origin at $t = 0$. The contribution due to the excitation of electron-hole pairs (a) and plasmons (b) is, for clarity, multiplied by $4\pi(rp_{\mathrm{F}})^2$ in the plots to the right. Panel (c) is simply the sum of the two contributions. Units: $r$ is in units of $1/p_{\mathrm{F}}$, $t$ is in units of $1/\omega_{\mathrm{p}}$ and all densities are in units of $p_{\mathrm{F}}^3$.

In the long time limit we have

$$\delta n_s(\mathbf{r}) \equiv \lim_{t\to\infty} \delta n(\mathbf{r},t) = -\frac{Q}{2\pi^2}\frac{1}{r}\int_0^\infty dq\, q \sin(qr)\tilde{v}_{\mathbf{q}}\, \chi^{\mathrm{R}}(\mathbf{q},0)$$

has spatial oscillations known as Friedel oscillations

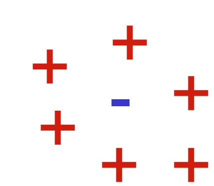
Suppose now that $Q = q = -1$ is the same a the electron charge. The total density change due to this test charge is

$$q\,\delta n_{\mathrm{tot}}(\mathbf{r}) = q[\delta(\mathbf{r}) + \delta n_s(\mathbf{r})]$$

The interaction energy between this charge and a generic electron is

$$e_{\mathrm{int}}(\mathbf{r}) = \int d\mathbf{r}' v(\mathbf{r},\mathbf{r}')\delta n_{\mathrm{tot}}(\mathbf{r}')$$

$$\begin{aligned} e_{\mathrm{int}}(\mathbf{r}) &= \int d\mathbf{r}' v(\mathbf{r},\mathbf{r}') \left[\delta(\mathbf{r}) + \int \frac{d\mathbf{q}}{(2\pi)^3}\, e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}'}\, \tilde{v}_{\mathbf{q}}\, \chi^{\mathrm{R}}(\mathbf{q},0)\right] \\ &= \int \frac{d\mathbf{q}}{(2\pi)^3}\, e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}} \left[\tilde{v}_{\mathbf{q}} + \tilde{v}_{\mathbf{q}}^2 \chi^{\mathrm{R}}(\mathbf{q},0)\right] \\ &= \int \frac{d\mathbf{q}}{(2\pi)^3}\, e^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}\, W^{\mathrm{R}}(\mathbf{q},0) \xrightarrow[r\to\infty]{} \frac{e^{-r/\lambda_{\mathrm{TF}}}}{r} \end{aligned}$$




In the static limit W describes the interaction between a test charge an an electron

## Linear response: Take home message

- We can derive a diagrammatic expansion for the linear response function from the diagrammatic rules for the 2-particle Green's function

- The linear response function gives direct information on neutral excitation spectra such as measured in optical absorption experiments

- The random phase approximation to the linear response function describes the phenomena of plasmon excitation in metallic systems

- The screening of a an added charge in the electron gas happens at a time-scale of the inverse plasmon frequency

# Spectral properties of an electron gas : GW

We have seen that the spectral function describes the energy distribution of excitations upon addition or removal of an electron. We therefore expect so see both plasmon and particle-hole excitations when we do a photo-emission experiment on an electron gas ( or electron gas like metals such a sodium )

Dyson equation

$$G^R(\mathbf{q},\omega) = g^R(\mathbf{q},\omega) + g^R(\mathbf{q},\omega)\Sigma^R(\mathbf{q},\omega)G^R(\mathbf{q},\omega)$$

$$g^R(\mathbf{q},\omega) = \frac{1}{\omega - \epsilon_{\mathbf{q}} + i\eta} \qquad \epsilon_{\mathbf{q}} = \frac{|\mathbf{q}|^2}{2}$$

$$\Rightarrow \qquad G^R(\mathbf{q},\omega) = \frac{g^R(\mathbf{q},\omega)}{1 - g^R(\mathbf{q},\omega)\Sigma^R(\mathbf{q},\omega)} = \frac{1}{\omega - \epsilon_{\mathbf{q}} - \Sigma^R(\mathbf{q},\omega)}$$

We calculate the self-energy in the GW approximation using noninteracting Green's function we find

$$\Sigma^{\lessgtr}(p,\omega) = \frac{\mathrm{i}}{(2\pi)^3\, p} \int d\omega' \int_0^\infty dk\, k\, G^{\lessgtr}(k,\omega') \int_{|k-p|}^{k+p} dq\, q\, W^{\gtrless}(q,\omega'-\omega)$$

The greater and lesser self-energies describe scattering rates for added or removed particles with energy ω and momentum p

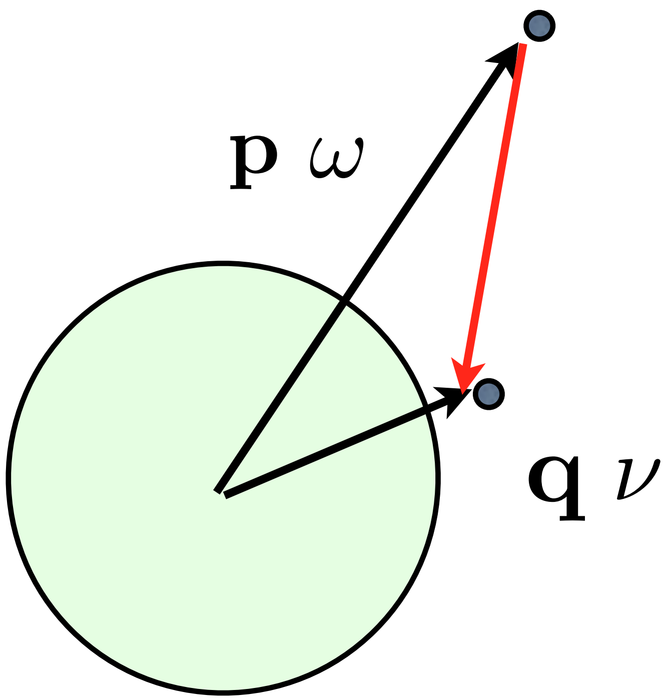
The self-energy vanishes when $\omega \to \mu$ due to the fact an added particle can maximally lose energy $\omega - \mu$ as states below the Fermi energy are occupied

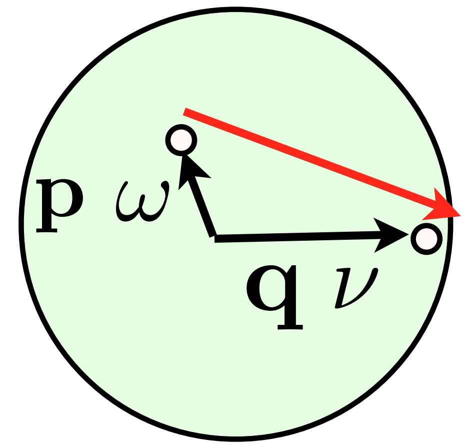
$$i(\Sigma^{>}(\mathbf{q},\omega) - \Sigma^{<}(\mathbf{q},\omega)) = -2\,\mathrm{Im}\,\Sigma^{R}(\mathbf{q},\omega) = \Gamma(\mathbf{q},\omega)$$

$$\lim_{\omega\to\mu} \mathrm{Im}\,\Sigma^{R}(\mathbf{q},\omega) = 0$$

$$\Sigma^{R}(\mathbf{q},\omega) = \Lambda(\mathbf{q},\omega) - \frac{i}{2}\Gamma(\mathbf{q},\omega)$$

## Scattering processes




Loss of energy by a particle.
Scattering rate given by $i\,\Sigma^{>}(\mathbf{p},\omega)$

Only relevant when $p \geq p_{\mathrm{F}}$

A plasmon can be excited only when $\omega \geq \mu + \omega_p$

Absorption of energy by a hole.
Scattering rate given by $-\,i\,\Sigma^{<}(\mathbf{p},\omega)$

Only relevant when $p \leq p_{\mathrm{F}}$

A plasmon can be absorbed only when $\omega \leq \mu - \omega_p$

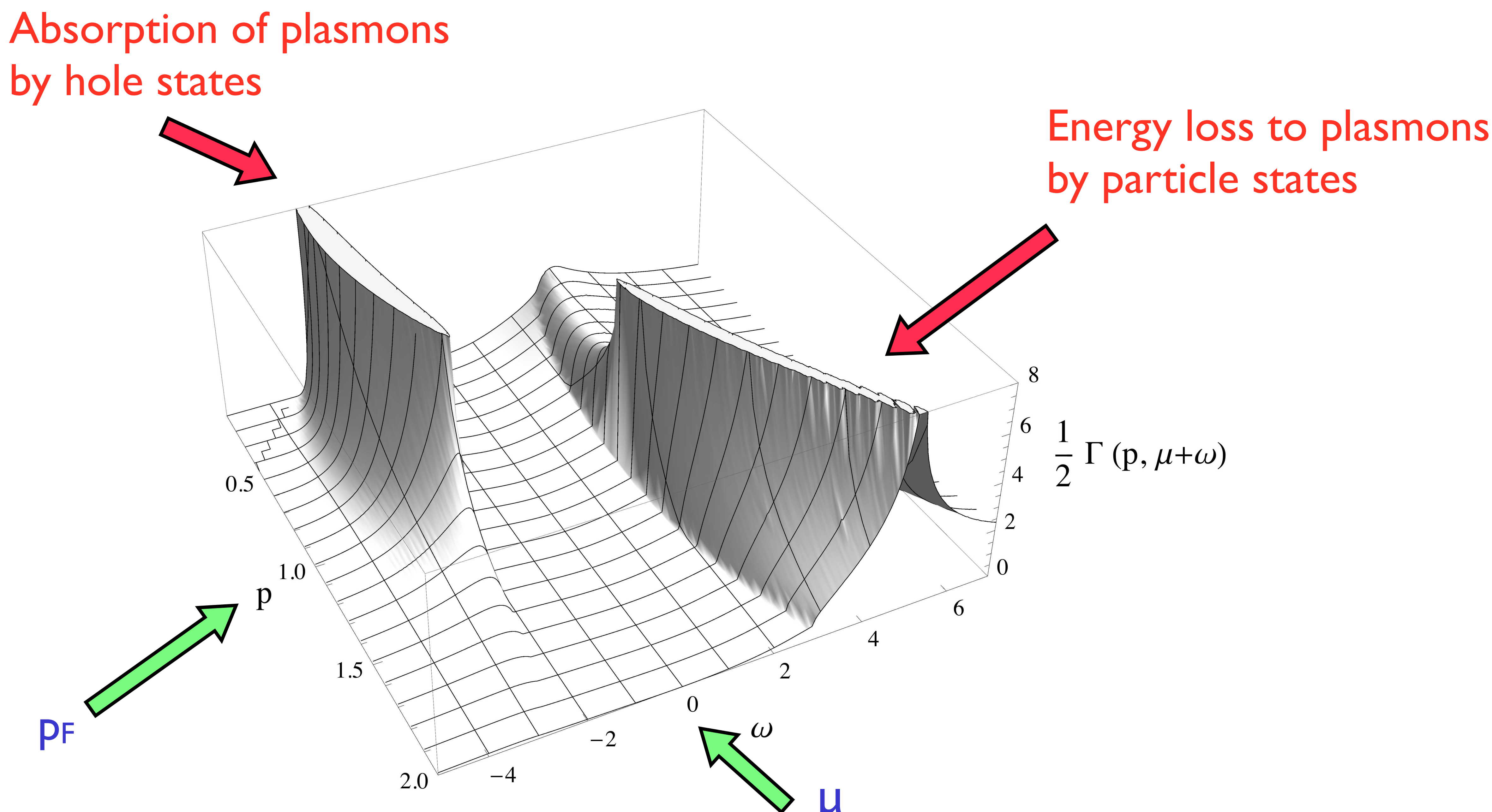



Figure 15.9: The imaginary part of the retarded self-energy $-\mathrm{Im}[\Sigma^{\mathrm{R}}(p,\omega+\mu)] = \Gamma(p,\omega+\mu)/2$ for an electron gas at $r_s = 4$ within the $G_0W_0$ approximation as a function of the momentum and energy. The momentum $p$ is measured in units of $p_{\mathrm{F}}$ and the energy $\omega$ and the self-energy in units of $\epsilon_{p_{\mathrm{F}}} = p_{\mathrm{F}}^2/2$.

For the spectral function this implies the following

$$A(\mathbf{q},\omega) = -2\,\mathrm{Im}\,G^R(\mathbf{q},\omega) = \frac{\Gamma(\mathbf{q},\omega)}{(\omega - \epsilon_{\mathbf{q}} - \Lambda(\mathbf{q},\omega))^2 + \left(\frac{\Gamma(\mathbf{q},\omega)}{2}\right)^2}$$

If $\Gamma(\mathbf{q},\omega)$ is small then the spectral function can only become large ( $\sim 1/\Gamma$ ) when

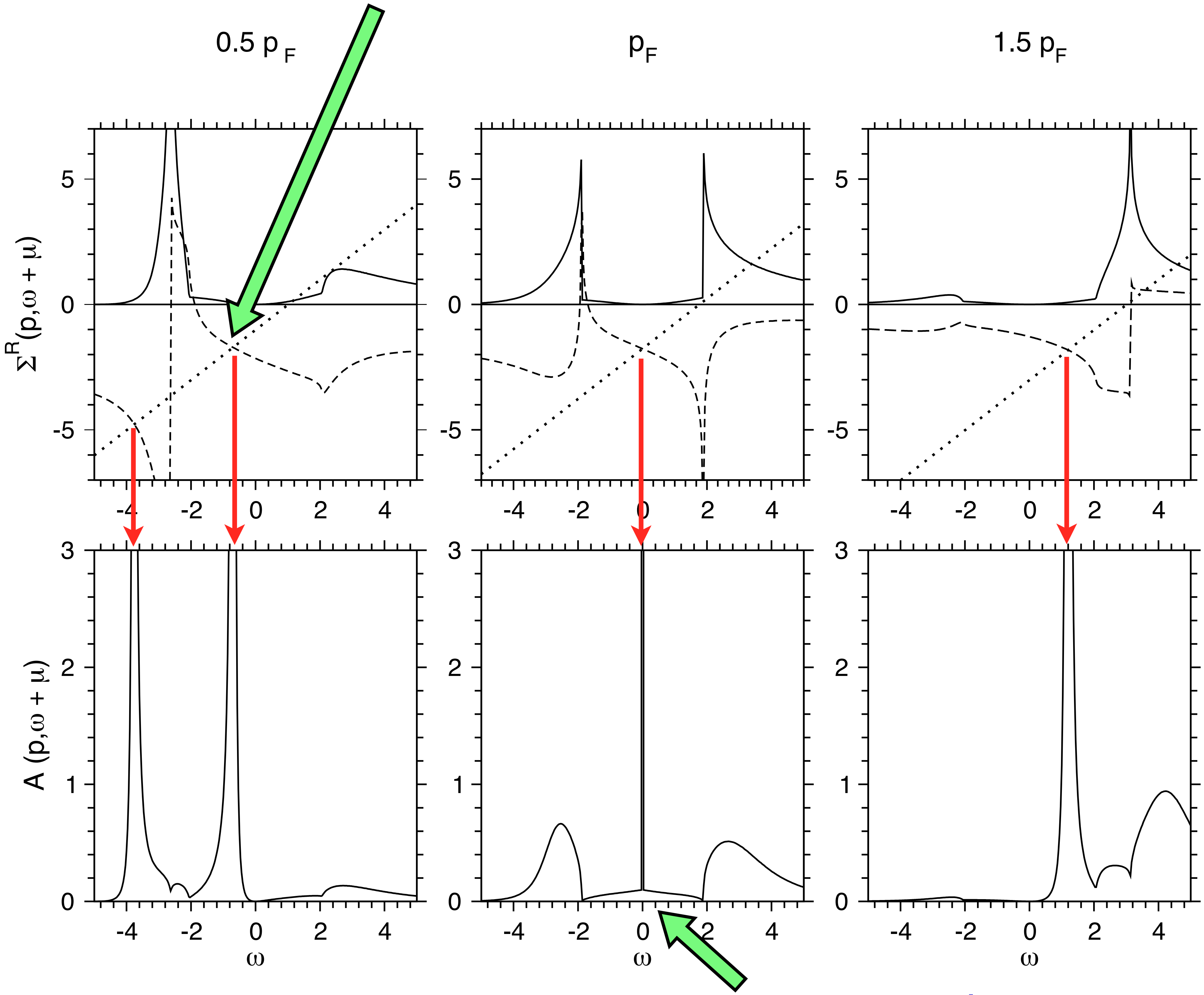
$$\omega - \epsilon_{\mathbf{q}} - \Lambda(\mathbf{q},\omega) = 0$$

The Luttinger-Ward theorem tells that this happens when $q = p_{\mathrm{F}}\ ,\ \omega = \mu$

$$\mu - \epsilon_{p_{\mathrm{F}}} - \Lambda(p_{\mathrm{F}},\mu) = 0$$

(not explained in these lectures, requires a derivation of the Luttinger-Ward functional, see G.Stefanucci, RvL, Nonequilibrium Many-Body Theory of Quantum Systems)

$$\omega - \epsilon_{\mathbf{p}} - \Lambda(\mathbf{p}, \omega) = 0$$



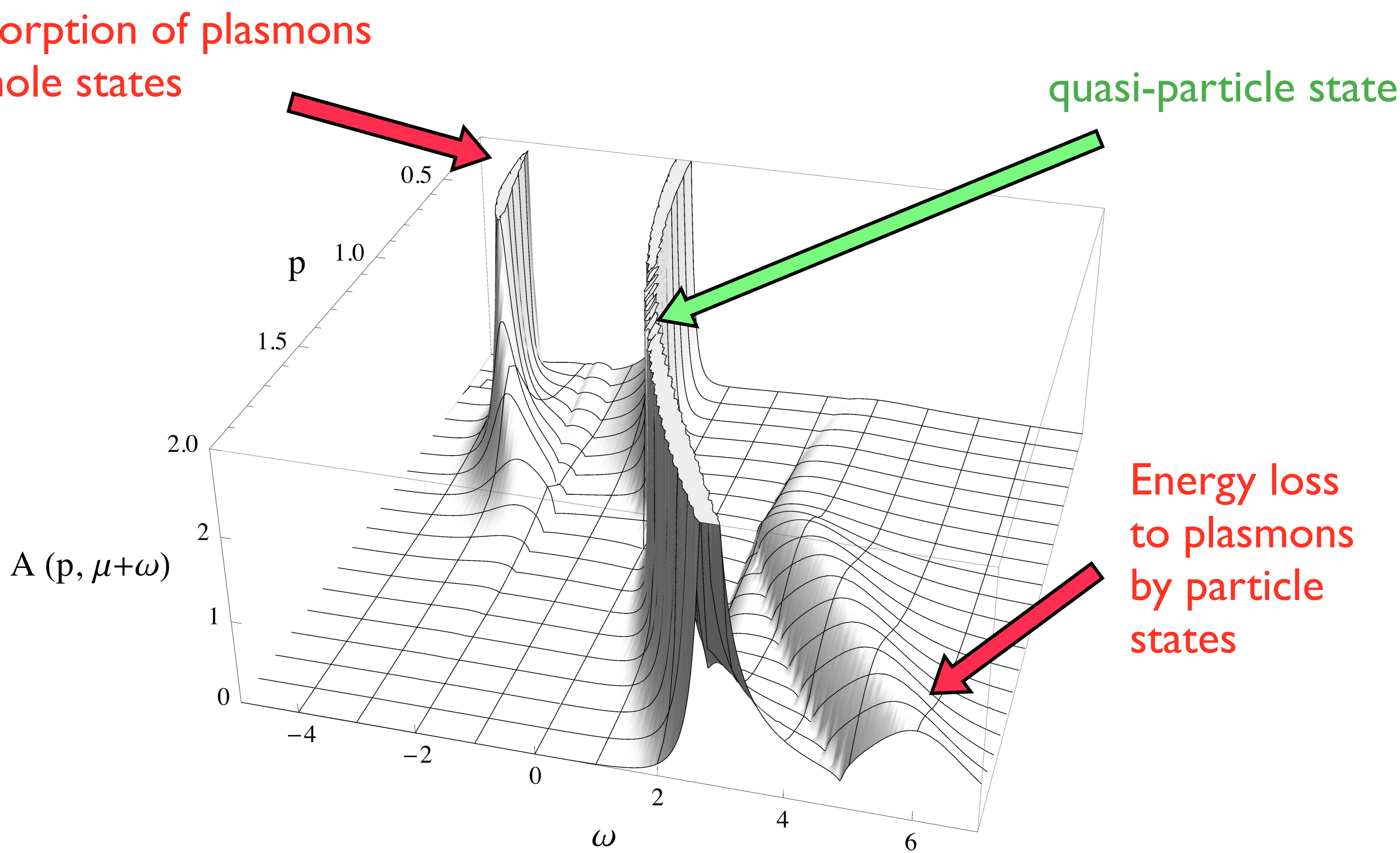
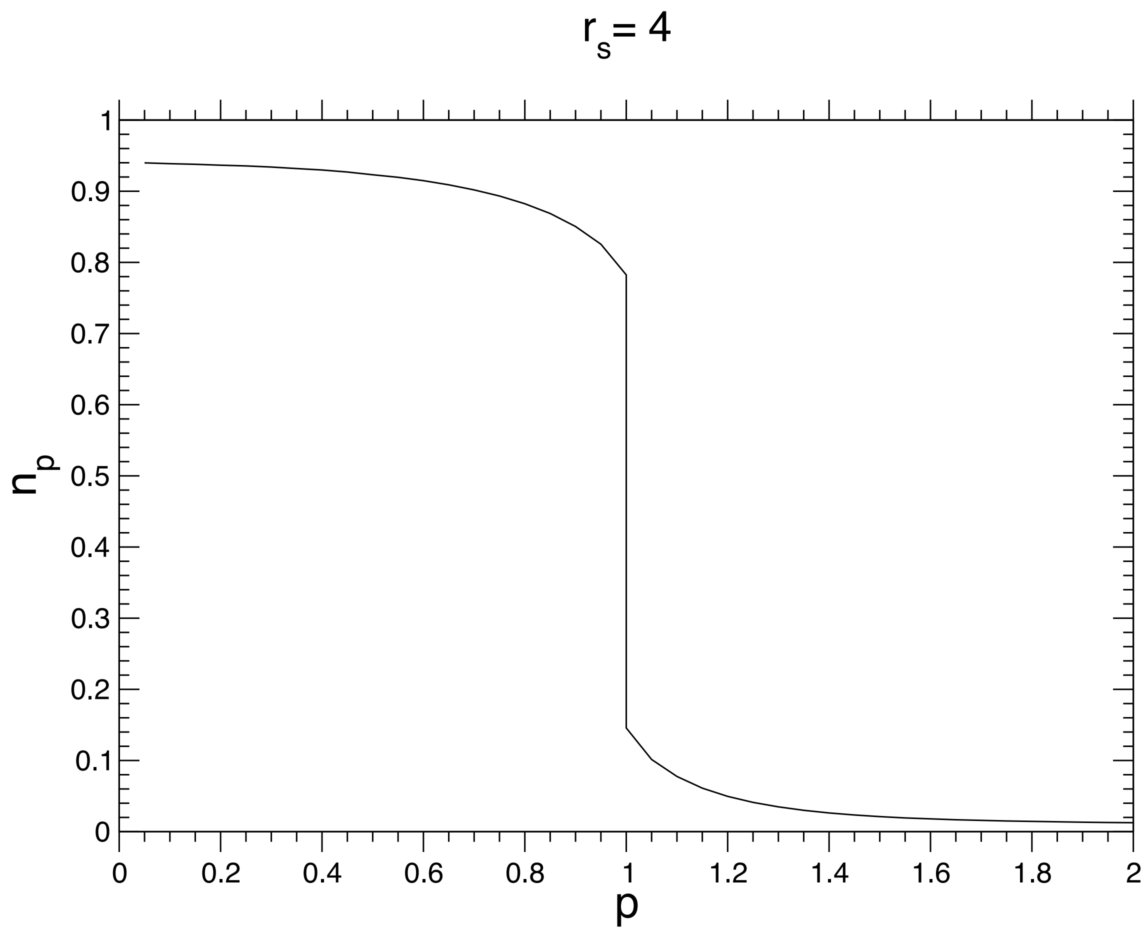



Figure 15.12: The spectral function $A(p, \mu{+}\omega)$ as a function of the momentum and energy for an electron gas at $r_s = 4$ within the $G_0W_0$ approximation. The momentum $p$ is measured in units of $p_{\mathrm{F}}$ and the energy $\omega$ and the spectral function in units of $\epsilon_{p_{\mathrm{F}}} = p_{\mathrm{F}}^2/2$.

The momentum distribution in the electron gas is given by

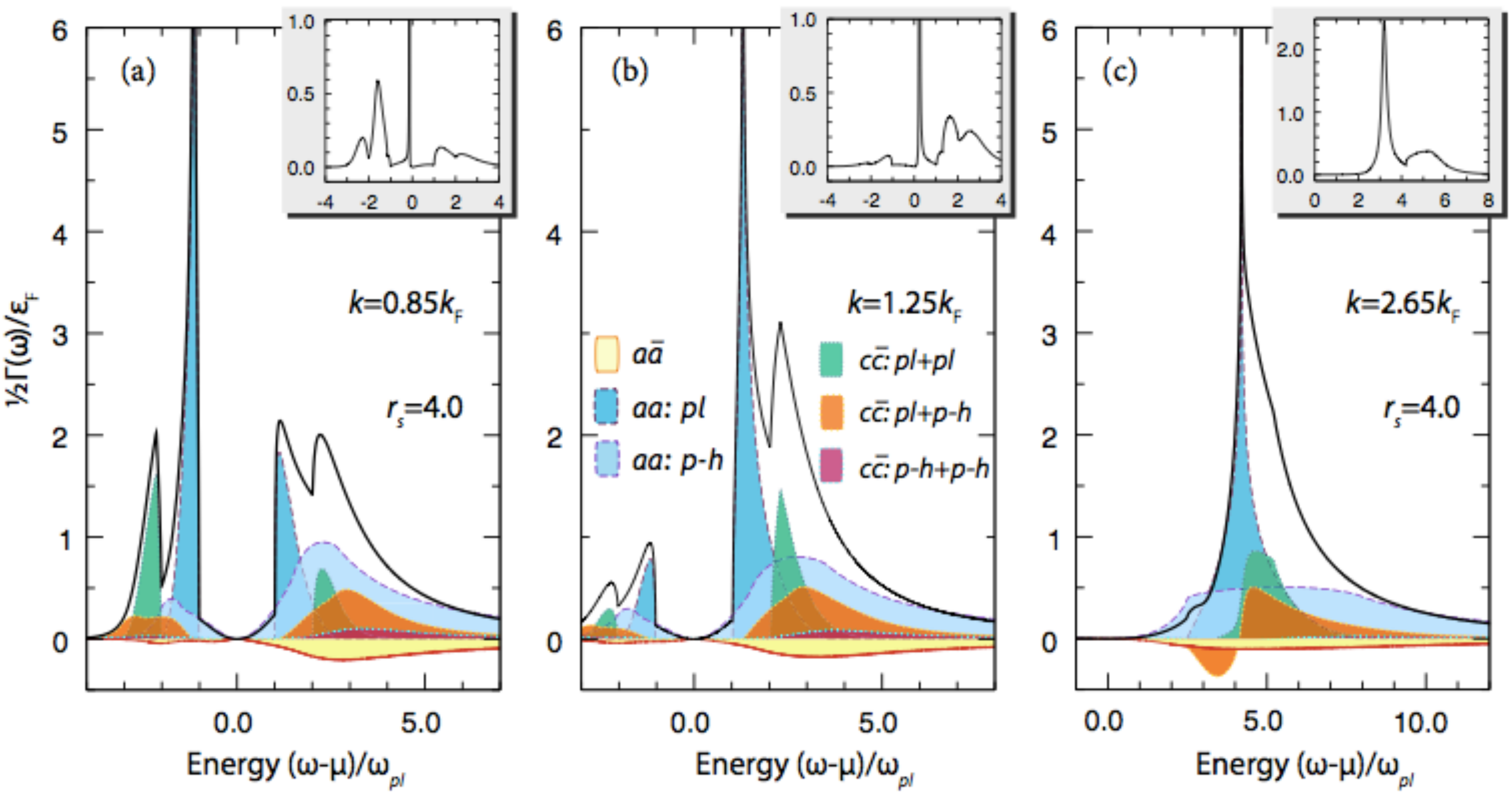
$$n_p = \int_{-\infty}^{\mu} \frac{d\omega}{2\pi} A(p,\omega)$$

Due to the appearance of a delta peak in the spectral function at the Fermi momentum $p_F$ the momentum distribution jumps discontinuously at the Fermi momentum. The jump is the strength of the quasi-particle peak.

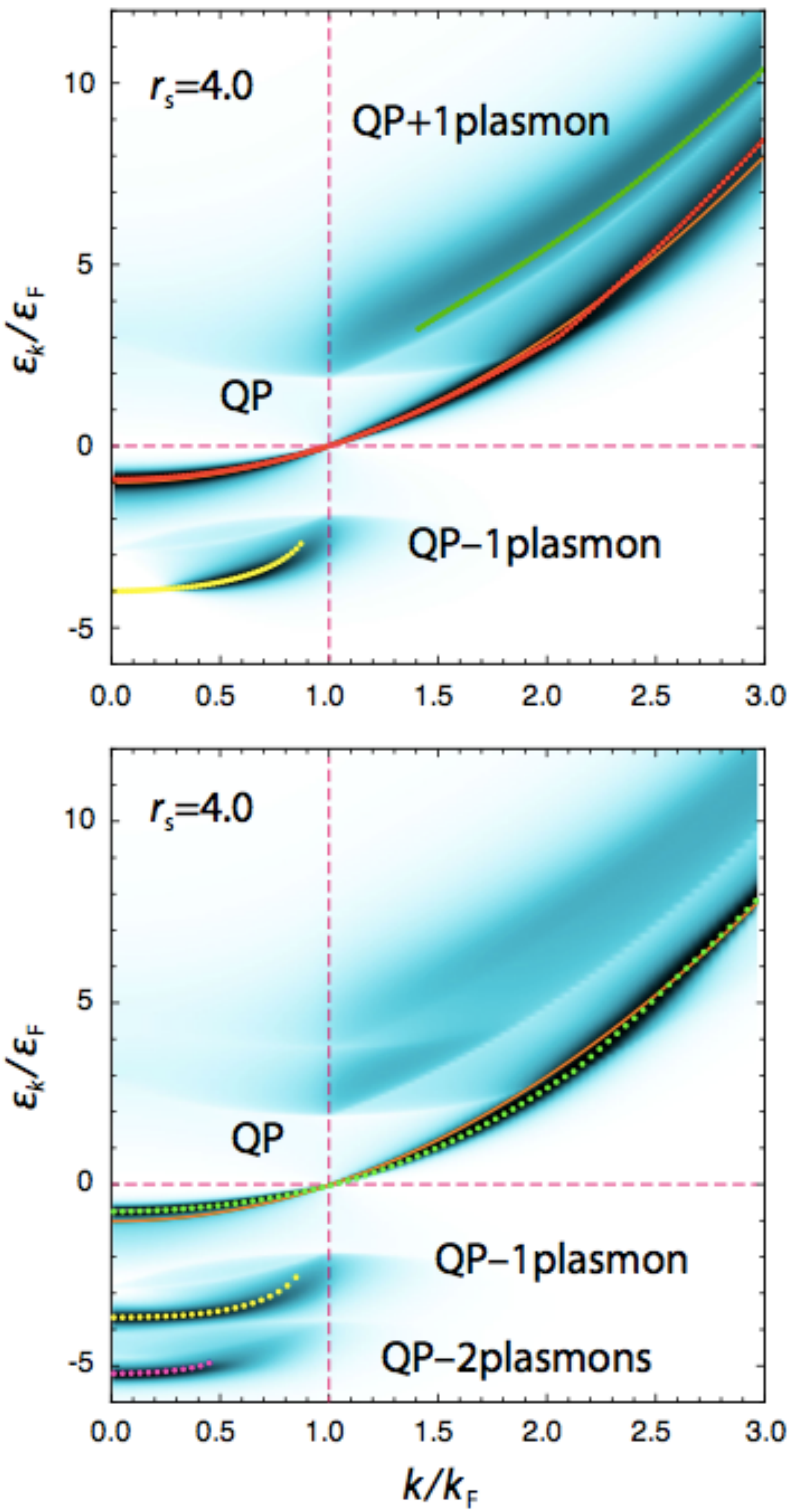


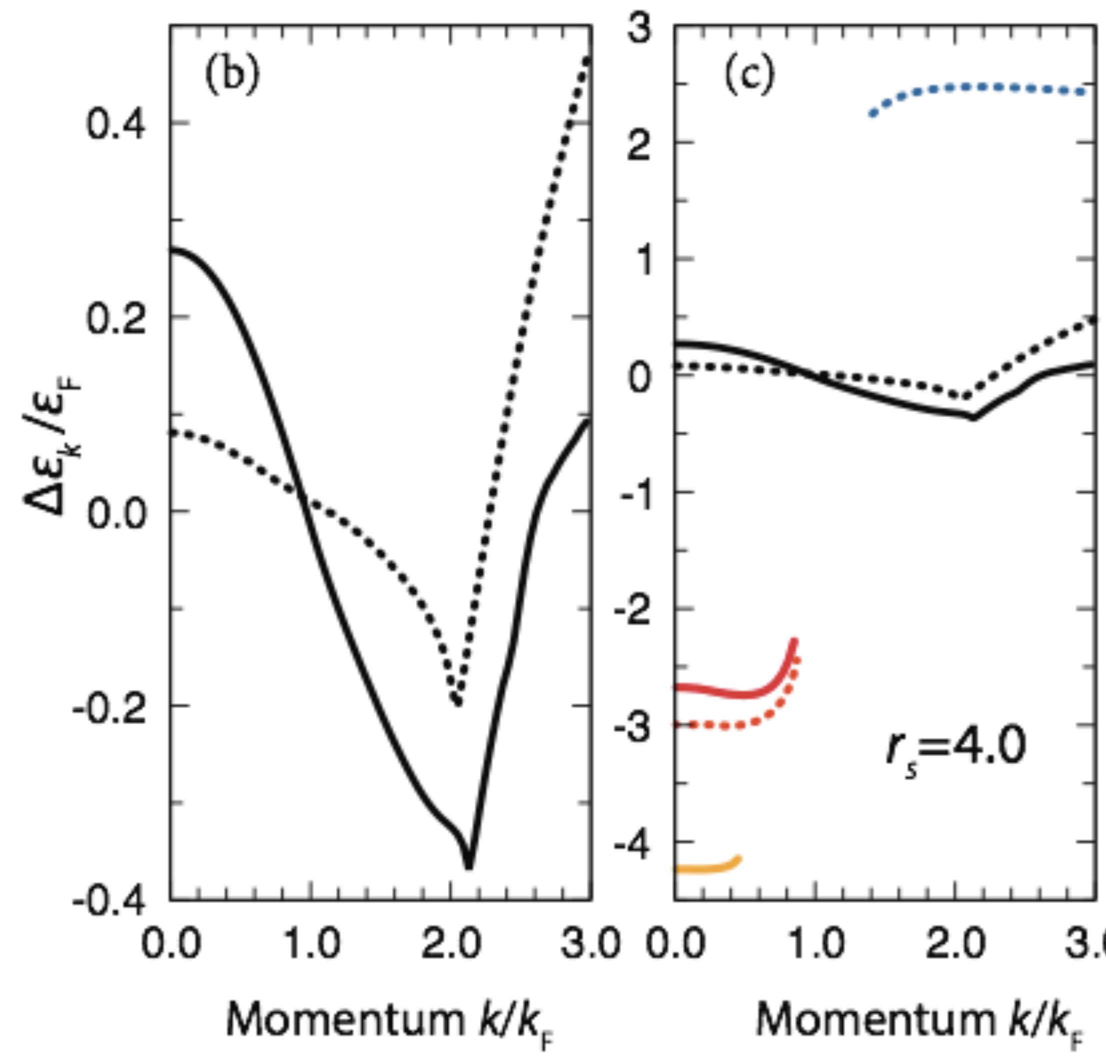
# Some recent results beyond GW

Y.Pavlyukh, A.-M. Uimonen, G.Stefanucci, RvL, arXiv. 1607.04309




Due to negative corrections around the chemical potential in the rate function, vertex corrections sharpen the quasi-particle peak as compared to $G_0W_0$




Vertex corrections:

- Reduce the band width by 27 percent ( sc GW increases by 20 percent
- Wash out the plasmon above the chemical potential
- Reduce the first plasmon energy

## Spectral properties of the electron gas: Take home message

- By addition or removal of an electron we create particle-hole and plasmon excitations

- The self-energy at the Fermi-surface vanishes due to phase-space restrictions. This has various consequences:

  1) The momentum distribution of the electron gas jumps discontinuously at the Fermi momentum

  2) Quasi-particles at the Fermi surface have an infinite life-time.

- The GW approximation gives extra plasmon structure in the spectral function due to plasmons

- Multiple-plasmons excitations (satellites) are beyond GW and require vertex corrections.

# Things we did not talk about

- Feynman diagrams for the grand canonical potential and the action. Luttinger-Ward functionals, variational principles

- Connections to TDDFT and TD current DFT :
  - Diagrammatic expansion of xc-kernel
  - Sham-Schlüter equation and TDOEP

- General initial states

- Bethe-Salpeter and excitons, Lehmann representation

- Non-equilibrium phenomena and the Kadanoff-Baym equations (quantum transport)

- Conserving approximations and Ward identities

- Open quantum systems, T-matrix, superconductivity, phonons, Bose condensates,.......etc.