Relativistic Stars in dRGT Massive Gravity

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Introduction

- About dRGT Massive Gravity
- Our Motivations

About dRGT Massive Gravity/ Our motivations

- It is the theory of ghost-free massive spin-2 field.
  [de Rham, Gabadadze, Tolley (2010)]

  → The theory can be describe accelerated universe.
  The mass terms behave like cosmological constant.
  [A. Gümrükçüoğ, C. Lin, S. Mukohyama(2010)]

- Relativistic stars' structure, especially maximum mass,
  can be modified by non-linear effects,
  strong-field structure of the theory.

- Massive neutron star, whose mass is 2 solar mass,
  may be explained by the theory.

- We calculate the hydrostatic equilibrium
  and maximum mass numerically.

Action of dRGT Massive Gravity

$$S_{\mathrm{dRGT}} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\det(g)}\left[R - 2m_0^2\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)\right] + S_{\mathrm{matter}}$$

- $g_{\mu\nu},\ f_{\mu\nu}$: Dynamical and Reference metric
- $\kappa^2 = 8\pi G$: Gravitational coupling
- $m_0,\ \beta_n$: Parameters of the theory

$$\left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\rho}\left(\sqrt{g^{-1}f}\right)^{\rho}{}_{\nu} = g^{\mu\rho}f_{\rho\nu}$$

$$[\mathbf{X}] := \mathrm{Tr}(\mathbf{X}), \quad e_0(\mathbf{X}) = 1\,, \quad e_1(\mathbf{X}) = [\mathbf{X}]$$

$$e_2(\mathbf{X}) = \frac{1}{2}\left([\mathbf{X}]^2 - [\mathbf{X}^2]\right), \quad e_3(\mathbf{X}) = \frac{1}{6}\left([\mathbf{X}]^3 - 3[\mathbf{X}][\mathbf{X}^2] + 2[\mathbf{X}^3]\right)\,,$$

$$e_4(\mathbf{X}) = \frac{1}{24}\left([\mathbf{X}]^4 - 6[\mathbf{X}]^2[\mathbf{X}^2] + 3[\mathbf{X}^2]^2 + 8[\mathbf{X}][\mathbf{X}^3] - 6[\mathbf{X}^4]\right) = \det(\mathbf{X})\,,$$

$$e_k(\mathbf{X}) = 0 \quad \text{for} \quad k > 4$$

Analysis on modified TOV equations

- A new constraint from mass terms
- Modified TOV equations

Our model / Assumptions

- minimal model : $\beta_0 = 3,\ \ \beta_1 = -1,\ \ \beta_2 = 0,\ \ \beta_3 = 0$
- graviton mass as cosmological constant : $m_0 = \frac{\hbar}{c}\sqrt{\Lambda} \sim 10^{-66}\mathrm{g}$
- Spherical and static dynamical metric

  & flat reference metric

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{2\phi}dt^2 + e^{2\sigma}d\rho^2 + D^2(\rho)(d\theta^2 + \sin^2\theta d\varphi^2)$$
$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + d\rho^2 + \rho^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$e^{-2\lambda(r)} \equiv 1 - \frac{2GM(r)}{r} \quad \Big\downarrow \quad D(\rho) \equiv r,\ \rho \equiv \chi(r),\ e^{2\sigma}(\chi'(r))^2 \equiv e^{2\lambda}$$

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{2\phi}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$
$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + (\chi'(r))^2dr^2 + \chi^2(r)(d\theta^2 + \sin^2\theta d\varphi^2)$$

A new constraint from mass terms

- Perfect fluid's energy-momentum tensor
  & their conservation laws

$$T_{\mu\nu} = \mathrm{diag}\left(e^{2\phi}\rho, e^{2\lambda}P, r^2 P, r^2\sin^2\theta P\right), \;\; \nabla^\mu T_{\mu\nu} = 0$$

- Abstract structure of equation of motions in our models.

$$G_{\mu\nu} + m_0^2 I_{\mu\nu} = \kappa^2 T_{\mu\nu}, \;\; I_{\mu\nu} = \sum_{n=0}^{3} (-1)^n \beta_n g_{\mu\lambda} Y^{\lambda}_{(n)\nu}(\sqrt{g^{-1}f})$$

$\nabla^\mu \times$

$$Y^{\lambda}_{(n)\nu}(\mathbf{X}) = \sum_{r=0}^{n} (-1)^r \left(X^{n-r}\right)^{\lambda}{}_{\nu} e_r(\mathbf{X})$$

- $\nabla^\mu T_{\mu\nu} = 0$ means a additional constraint.

$$\nabla^\mu I_{\mu\nu} = 0 \Leftrightarrow \left(\frac{2}{r} - (P+\rho)^{-1} P'\right)\left(1 - \frac{2GM(r)}{r}\right)^{1/2} - \frac{2}{r} = 0$$

Modified TOV equations

- For numerical calculations, equations should be written by dimensionless variables.

$$r_g \equiv GM_\odot,\ r \to r r_g,\ M \to m M_\odot,\ m_0 \to \alpha M_\odot,$$
$$\rho \to \tilde{\rho}(M_\odot/r_g^3),\ P \to p(M_\odot/r_g^3)$$

- $m(r)$ can be eliminated by the new constraint.

$$\nabla_\mu I^{\mu\nu} = 0 \Leftrightarrow \left(1 - \frac{2m(r)}{r}\right)^{1/2} = \left(1 - \frac{1}{2}\frac{p'(r)}{p(r)+\tilde{\rho}(r)} r\right)^{-1}$$

- The hydrostatic equation become a higher order differential equation of $\rho(r)$, $p(\rho(r))$.

Modified TOV equations

- To solve the system, one more equation is needed.
  A equation of state (EoS) $p(r) = p(\tilde{\rho}(r))$,
  which is defined by hadron physics, should be used.

- After using EOS for eliminating $p(r)$,
  TOV equations become very complicated forms.
  I show the abstract structures.

  - A 3rd order ODE of $\tilde{\rho}(r)$.

  - An algebraic equation.

$$m = m(r, \tilde{\rho}(r), \tilde{\rho}'(r))$$

Used boundary conditions

- To compare results with
  the same central density and radius of GR case,
  boundary conditions become as follows.

- $r = 0 : P'(r = 0) = 0$ and $\rho(r = 0) = \rho_c$.
  (2 boundary conditions exist.)

- $r = R$ s.t. $P(r = R) = 0$:
  The radius coincides with that in GR result
  of the same central density.
  (1 boundary condition exist.)

- The 3 boundary conditions are sufficient
  to solve the 3rd order ODE.

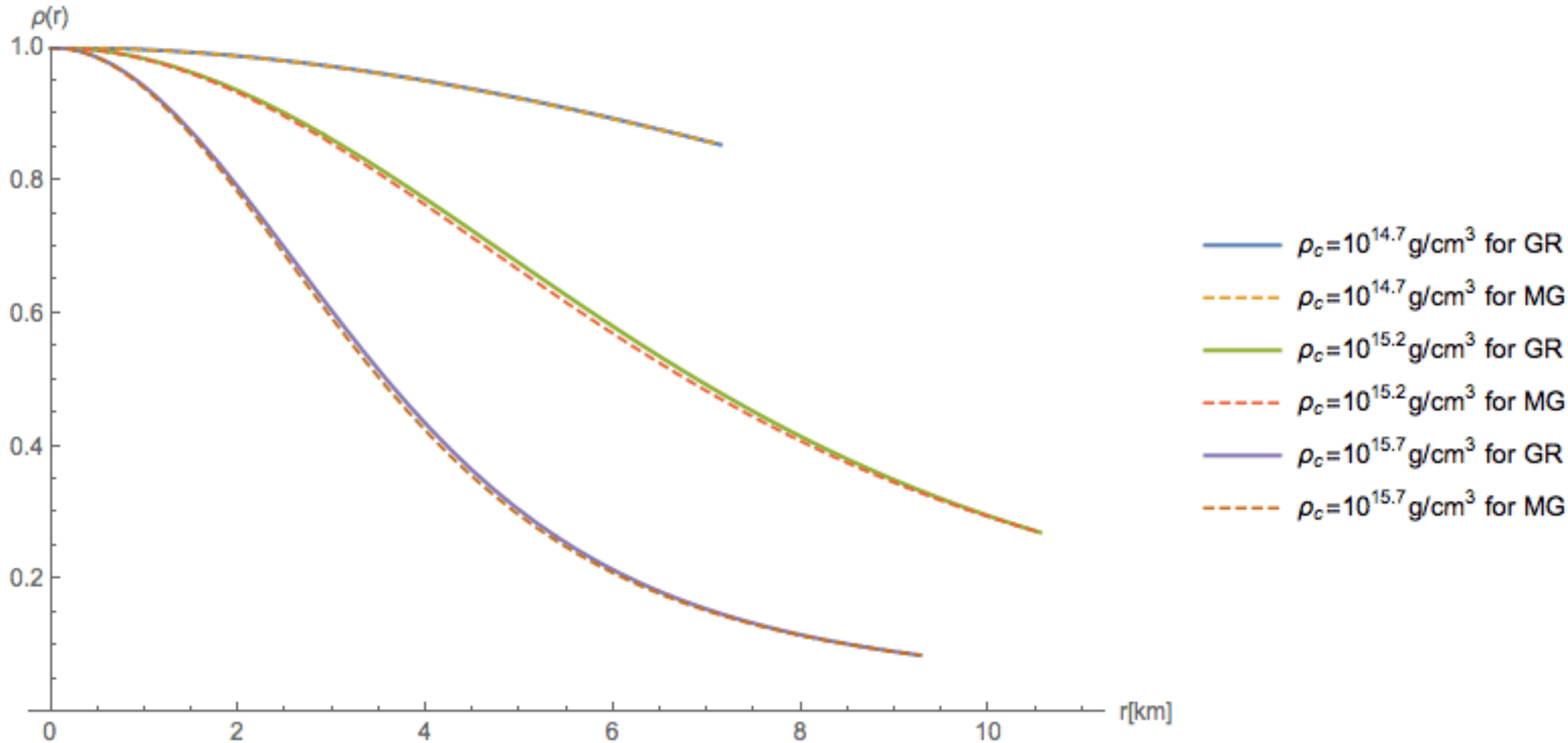
Numerical Results

- Quark stars (MIT bag model)
- Neutron stars (SLy Model)

The case of quark stars (MIT bag model)

- MIT bag model is used.

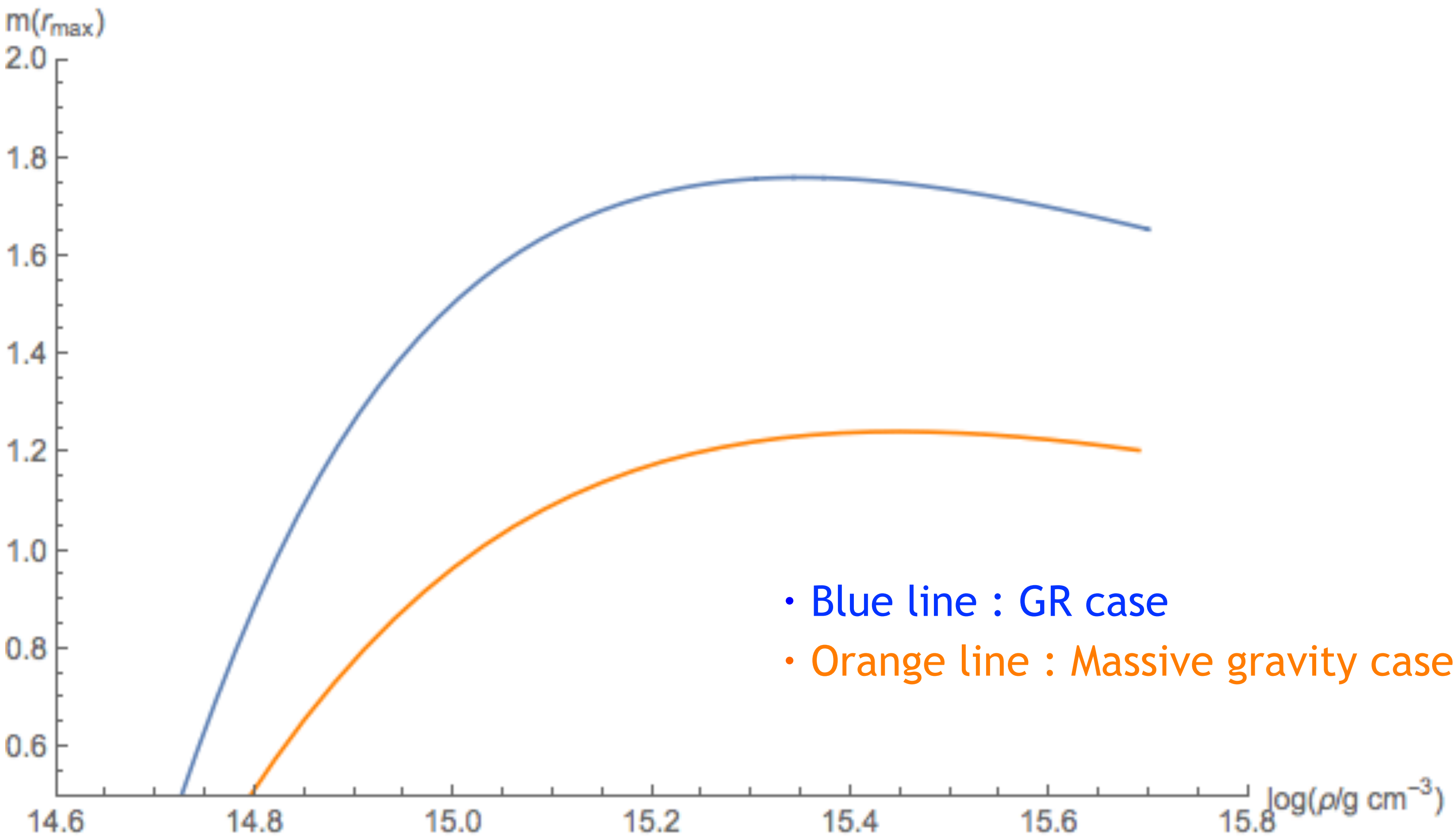
$$P(\rho(r)) = c(\rho(r) - 4B),\ c = 0.28,\ B = 60\mathrm{MeV/fm}^3$$



Because graviton mass is very tiny, the distribution is nearly the same.

Mass-central density relations of quark stars

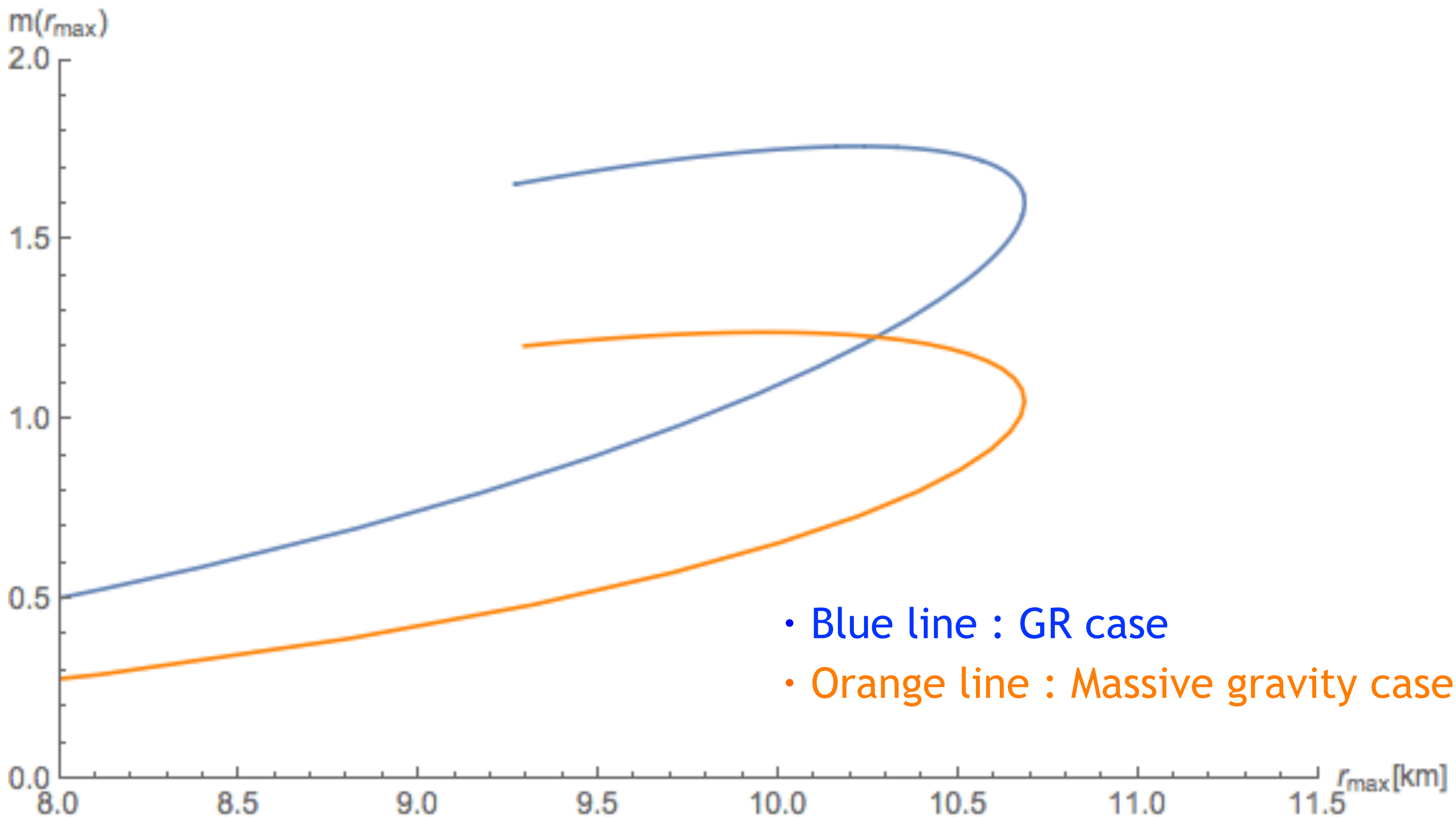
The masses are drastically changed by the new constraint.



- Blue line : GR case
- Orange line : Massive gravity case

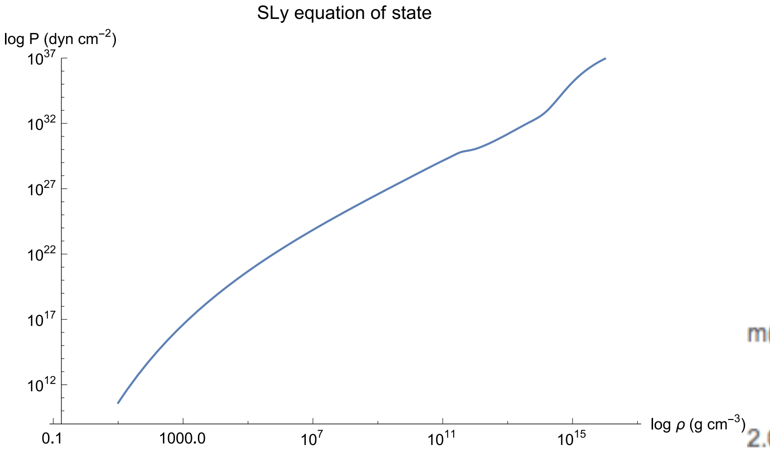
Mass-radius relations of quark stars

The masses are drastically changed by the new constraint.

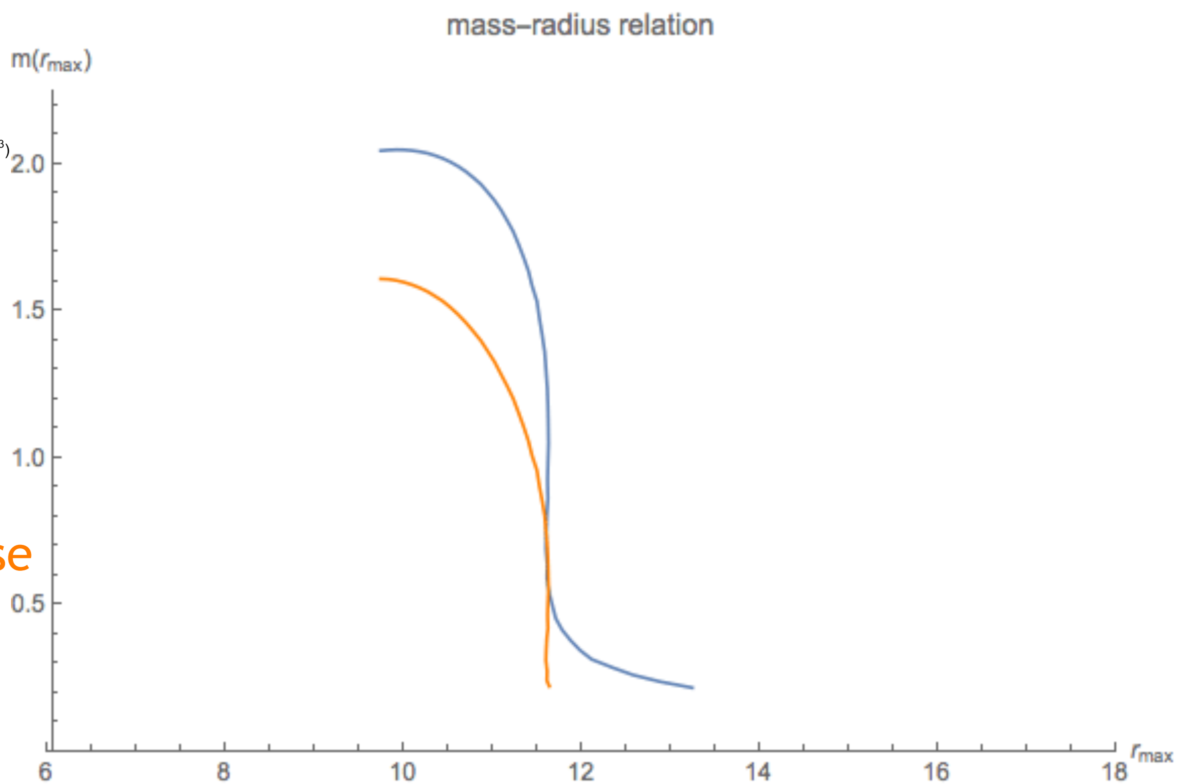


- Blue line : GR case
- Orange line : Massive gravity case

The case of neutron stars (SLy model)



← Log-Log plots of EoS of SLy model.



- Blue line : GR case
- Orange line : Massive gravity case

Summary / Future works

- The new constraint appears from divergence of mass terms
- Modified TOV equations are 3rd order ODE and the new constraint.
- Numerical Results
  - The density distributions are mostly not changed because of very tiny mass.
  - The masses are drastically changed because of the new constraint.
- The same analysis of other parameters are needed. That of bigravity are also needed.