Scalar field dark energy with a minimal coupling in a spherically symmetric background

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Introduction I

Various observations in astronomy, e.g. Cosmic Microwave Background radiation (CMB), Baryon Acoustic Oscillations (BAO), and the observations of the Ia type supernovae (SNIa) imply that the accelerated expansion of the Universe is realized in the beginning of the Universe and in the present time.

Einstein's equation

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{Modified gravity}} = \underbrace{8\pi G T^{(m)}_{\mu\nu}}_{\text{Dark energy}}$$

Modified gravity Dark energy

Can we observe the modifications to the Einstein's gravity in the solar system?

Introduction II

Action

$$S = \int d^4x \left[\frac{R}{2\kappa^2} - K(\phi, X) \right] + S_{\mathrm{matter}},$$

$$X \equiv -\partial_\mu \phi \partial^\mu \phi / 2$$

- A generalization of the quintessence model.
- There is neither gravitational coupling nor matter coupling.

Equations I

A static metric with a spherical symmetry

$$ds^2 = -\mathrm{e}^{2\Phi(r)}dt^2 + \mathrm{e}^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

Einstein equations

$$\frac{1}{\kappa^2}\left(\frac{1}{r^2} - \frac{2\lambda'}{r}\right)\mathrm{e}^{-2\lambda} - \frac{1}{\kappa^2 r^2} = -K - \rho_{\mathrm{m}},$$

$$\frac{1}{\kappa^2}\left(\frac{1}{r^2} + \frac{2\Phi'}{r}\right)\mathrm{e}^{-2\lambda} - \frac{1}{\kappa^2 r^2} = -K - \mathrm{e}^{-2\lambda}\phi'^2 K_{,X} + p_{\mathrm{m}},$$

$$\frac{1}{\kappa^2}\left(\Phi'' + \Phi'^2 - \lambda'\Phi' + \frac{\Phi'}{r} - \frac{\lambda'}{r}\right)\mathrm{e}^{-2\lambda} = -K + p_{\mathrm{m}}.$$

Field equation

$$(K_{,X} - \mathrm{e}^{-2\lambda}K_{,XX}\phi'^2)\phi'' + \left(K_{,X\phi}\phi' + \frac{2}{r}K_{,X}\right)\phi' + \mathrm{e}^{2\lambda}K_{,\phi}$$

$$-(K_{,X} - \mathrm{e}^{-2\lambda}K_{,XX}\phi'^2)\phi'\lambda' + K_{,X}\phi'\Phi' = 0.$$

Equations II

$|\Phi|, |\lambda| \ll 1$ and $\rho_{\rm m} = p_{\rm m} = 0,$

$$\lambda' + \frac{1}{r}\lambda = \frac{\kappa^2 r}{2} K,$$

$$\Phi' - \frac{1}{r}\lambda = -\frac{\kappa^2 r}{2}\left[K + (1-2\lambda)\phi'^2 K_{,X}\right],$$

$$\Phi(r) = r_0/r + \delta\Phi(r),$$

$$\lambda(r) = -\Phi(r) + \delta\lambda(r)$$

$$\delta\lambda'(r) = -\frac{\kappa^2 r}{2}\phi'^2(r) K_{,X}(\phi(r), X(r)).$$

$$\delta\Phi(r) = \frac{1}{r}\int^r dl \left\{\delta\lambda(l) - \frac{\kappa^2 l^2}{2}\left[K(\phi(l), X(l)) + \phi'^2 K_{,X}(\phi(l), X(l))\right]\right\} + \frac{r_1}{r},$$

Equations III

$$|\Phi|, |\lambda| \ll 1 \text{ and } \rho_{\rm m} = p_{\rm m} = 0,$$

$$\Rightarrow (K_{,X} - K_{,XX}\phi'^2)\phi'' + \left(K_{,X}\frac{2}{r} + K_{,X\phi}\phi'\right)\phi' + K_{,\phi} = 0.$$

(Eq. 1)

A procedure to investigate the behavior of the scalar field

1. Solving Eq.1.
2. Substituting the solution of Eq.1 into δλ and δΦ.
3. Checking the conditions | δλ |<<1 and | δΦ |<<1.

The conditions | δλ |<<1 and | δΦ |<<1 inform us the applicable region of Eq.1. The behavior of the scalar field in the region Eq.1 is not applicable is clarified by numerical calculations. The boundary conditions for them are supplied from Eq.1.

Quintessence

$$K(\phi, X) = -X + V(\phi) = \frac{1}{2}\mathrm{e}^{-2\lambda}\phi'^2 + V(\phi),$$

Field equation

$$\phi'' + \frac{2}{r}\phi' - V_{,\phi} = 0$$

Metric functions

$$\delta\lambda'(r) = \frac{\kappa^2 r}{2}\phi'^2(r)$$

$$\delta\Phi(r) = \frac{1}{r}\int^r dl\left[\frac{\kappa^2 l^2}{4}(\phi'^2(l) - 2V(l)) + \delta\lambda(l)\right] + \frac{r_1}{r}$$

Exponential potential I

$$V(\phi) = M^4 e^{-\frac{\phi}{\phi_0}}. \qquad M^4 \sim M_{\rm pl}^2 H_0^2 \qquad \phi_0 \sim M_{\rm pl}$$

Field equation

$$\phi'' + \frac{2}{r}\phi' + \frac{1}{\phi_0} M^4 e^{-\frac{\phi}{\phi_0}} = 0.$$

$|\phi| \ll \phi_0$

$\Rightarrow$

$$\phi'' + \frac{2}{r}\phi' + \frac{1}{\phi_0} M^4 \simeq 0.$$

Solution

$$\boxed{\phi(r) = -\frac{M^4}{6\phi_0} r^2 + \frac{c_1}{r} + m_1,}$$

Exponential potential II

$$V(\phi) = M^4 \mathrm{e}^{-\frac{\phi}{\phi_0}}. \qquad M^4 \sim M_{\rm pl}^2 H_0^2 \quad \phi_0 \sim M_{\rm pl}$$

Solution

$$\boxed{\phi(r) = -\frac{M^4}{6\phi_0} r^2 + \frac{c_1}{r} + m_1,}$$

$|\phi| \ll \phi_0$ is now rewritten as $|c_1|/M_{\rm pl} \ll r \ll H_0^{-1}$.

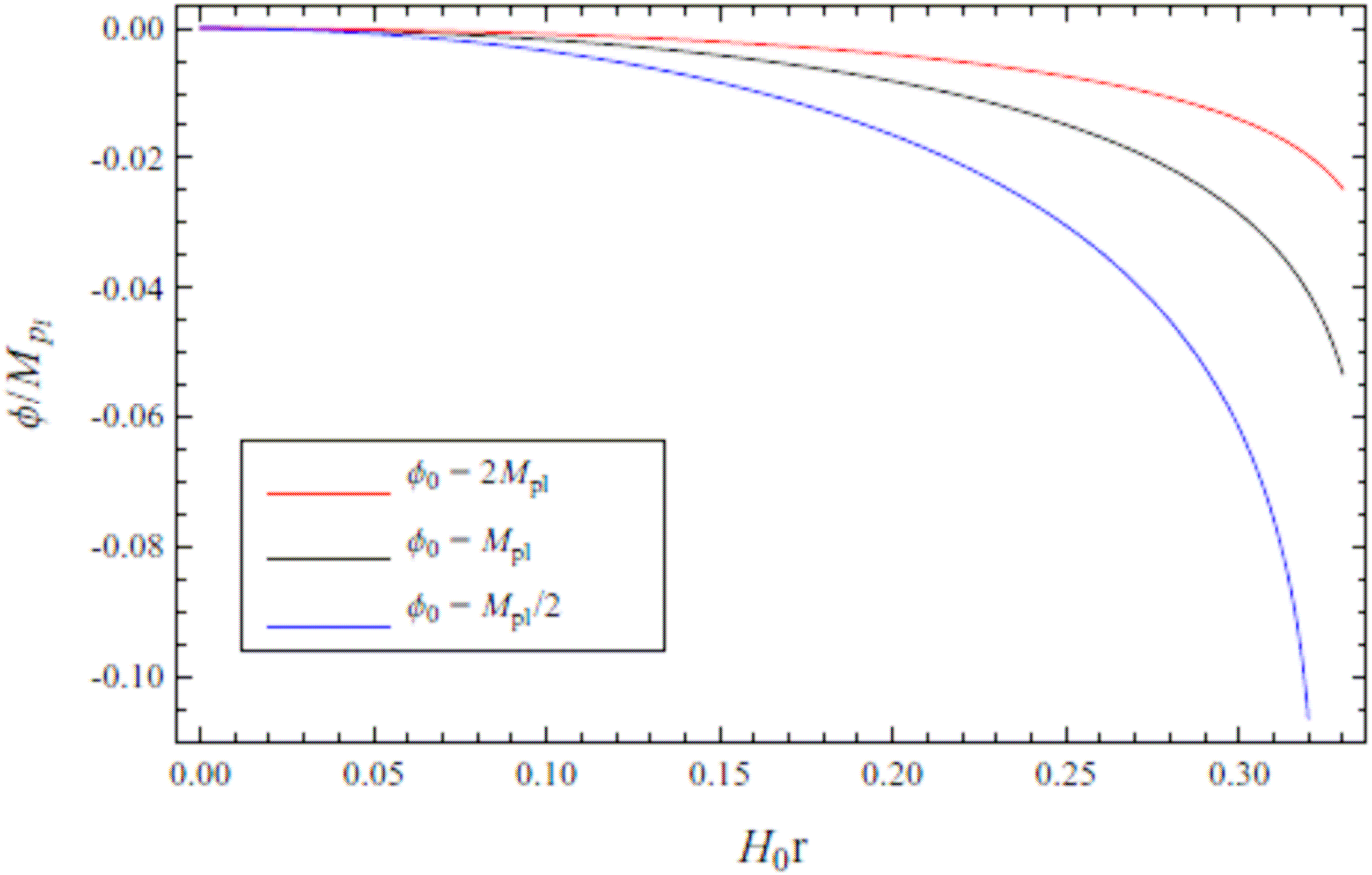
$$\delta\lambda(r) = \frac{\kappa^2 M^8}{72\phi_0^2} r^4 + \frac{c_1 \kappa^2 M^4}{3\phi_0} r + c_2 - \frac{c_1^2 \kappa^2}{4r^2},$$

$$\delta\Phi(r) \simeq \frac{\kappa^2 M^8}{120\phi_0^2} r^4 - \frac{\kappa^2 M^4}{6} r^2 + \frac{c_1 \kappa^2 M^4}{4\phi_0} r + c_2 + \frac{r_1}{r},$$

$|\delta\lambda| \ll 1$, $|\delta\Phi| \ll 1$ $\Rightarrow$ $\boxed{|c_1/M_{\rm pl}| \ll r \ll H_0^{-1}}$

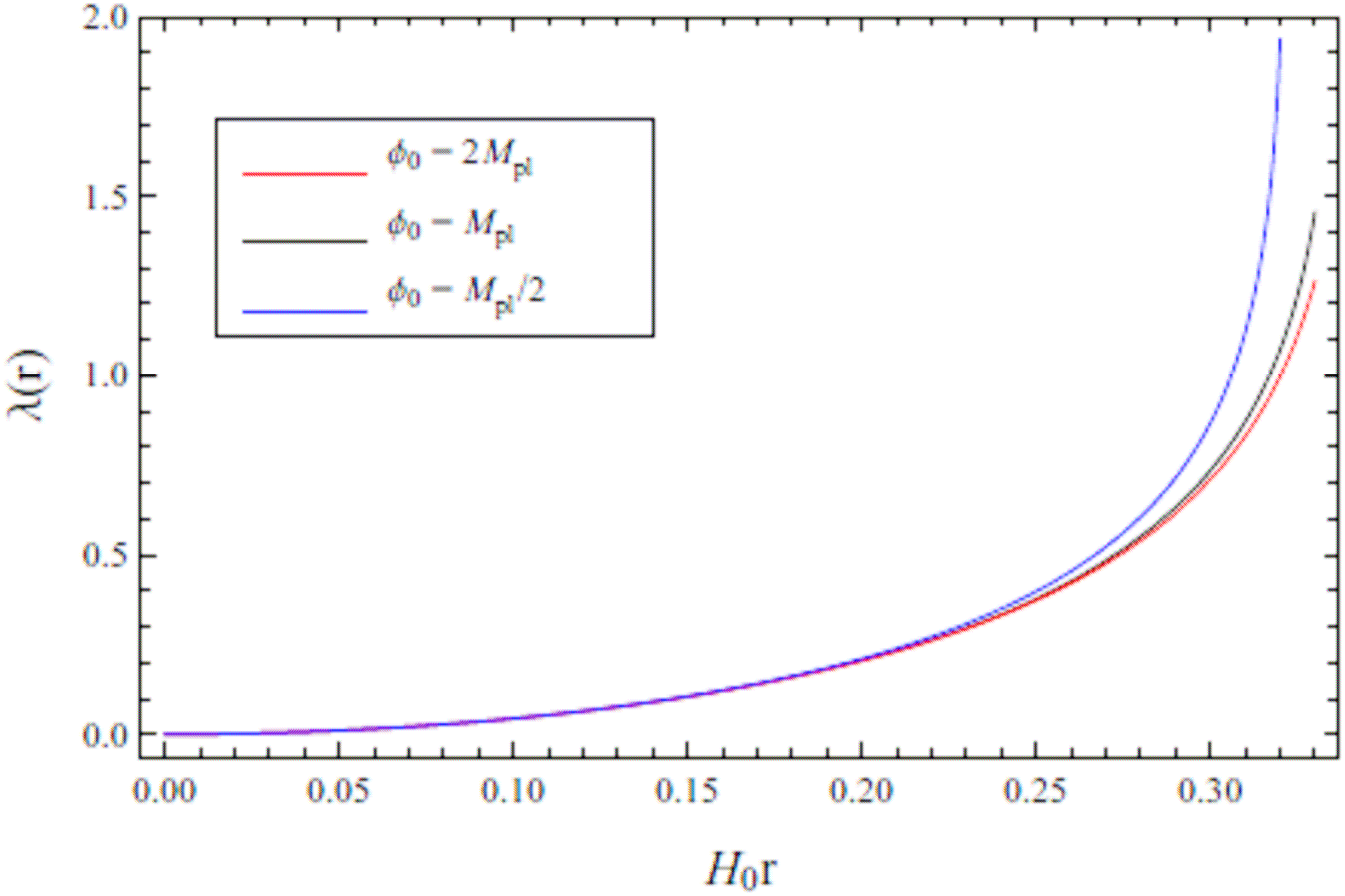
Exponential potential III

$$V(\phi) = M^4 e^{-\frac{\phi}{\phi_0}}, \quad M^4 = H_0^2 M_{\rm pl}^2.$$



Exponential potential IV

$$V(\phi) = M^4 e^{-\frac{\phi}{\phi_0}}, \;\; M^4 = H_0^2 M_{\rm pl}^2.$$



Negative power law potential I

$$V(\phi) = M^{4+n}\phi^{-n}, \qquad V(M_{\rm pl}) \sim H_0^2 M_{\rm pl}^2$$

$$M \sim 10^{-\frac{46-19n}{4+n}}\,\mathrm{GeV}.$$

Field equation

$$\phi'' + \frac{2}{r}\phi' + nM^{4+n}\phi^{-n-1} = 0.$$

A particular solution

$$\boxed{\phi(r) = \left[-\frac{2(4+n)}{n(2+n)^2 M^{4+n}}\right]^{-\frac{1}{2+n}} r^{\frac{2}{2+n}},}$$

n should be a positive odd number.

Negative power law potential II

Solution

$$\boxed{\phi(r) = \left[-\frac{2(4+n)}{n(2+n)^2 M^{4+n}}\right]^{-\frac{1}{2+n}} r^{\frac{2}{2+n}},}$$

$$\delta\lambda(r) = \frac{\kappa^2 A^2}{2(2+n)} r^{\frac{4}{2+n}} + C_1,$$

$$\delta\Phi(r) = \frac{4+n}{2n(2+n)(6+n)} \kappa^2 A^2 r^{\frac{4}{2+n}} + \frac{r_1}{r} + C_1,$$

$$A \equiv \left[-\frac{2(4+n)}{n(2+n)^2 M^{4+n}}\right]^{-\frac{1}{2+n}},$$

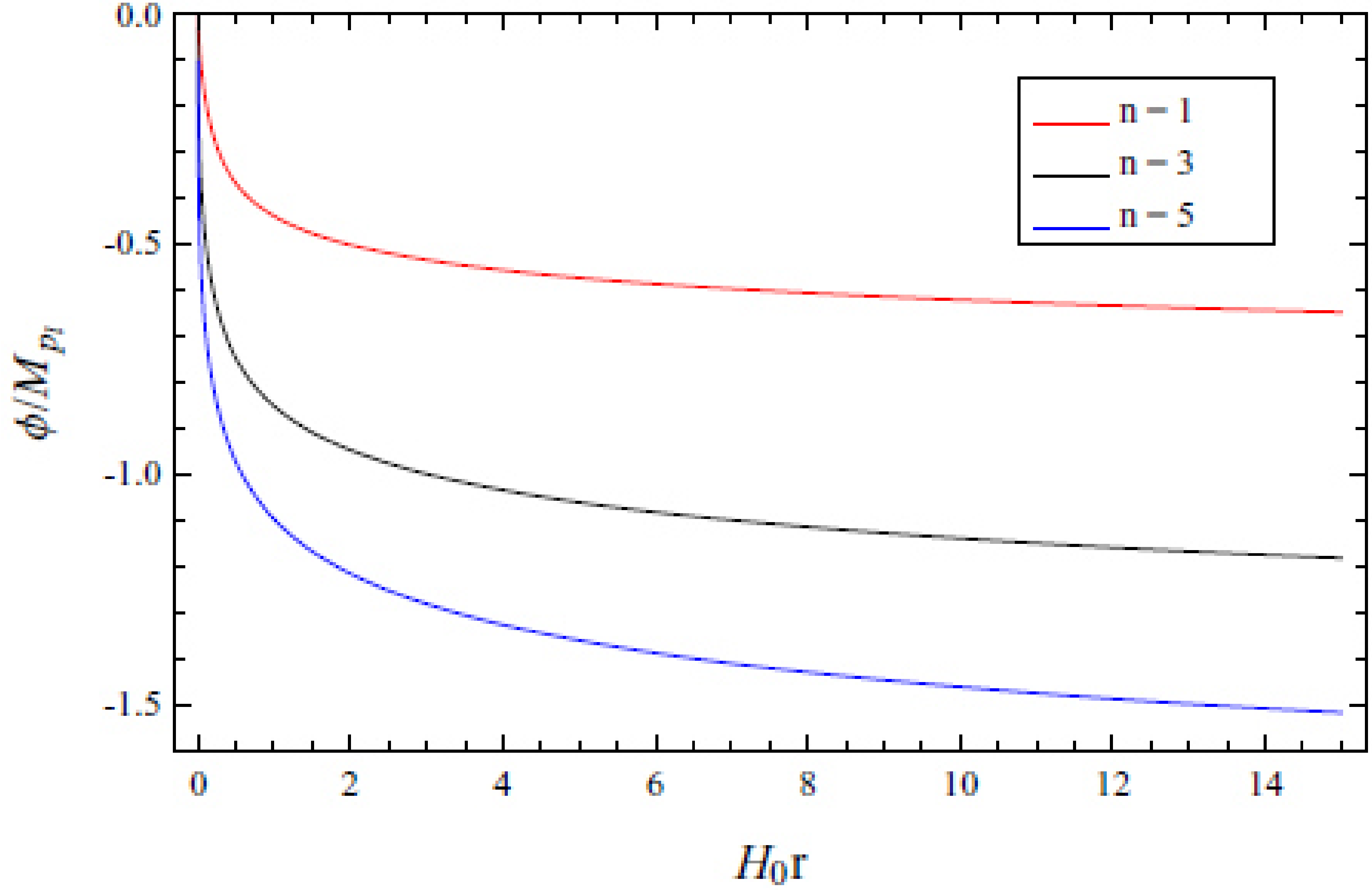
$|\delta\lambda| \ll 1$, $|\delta\Phi| \ll 1$ $\Rightarrow$ $\boxed{|r_1| \ll r \ll H_0^{-1}}$

$|\gamma - 1| = |\delta\lambda - \delta\Phi|/(GM/r) < 10^{-5}$ $\Rightarrow$ $\boxed{n \lesssim 5.}$

$M = M_\odot,\ r = 1.6 R_\odot$

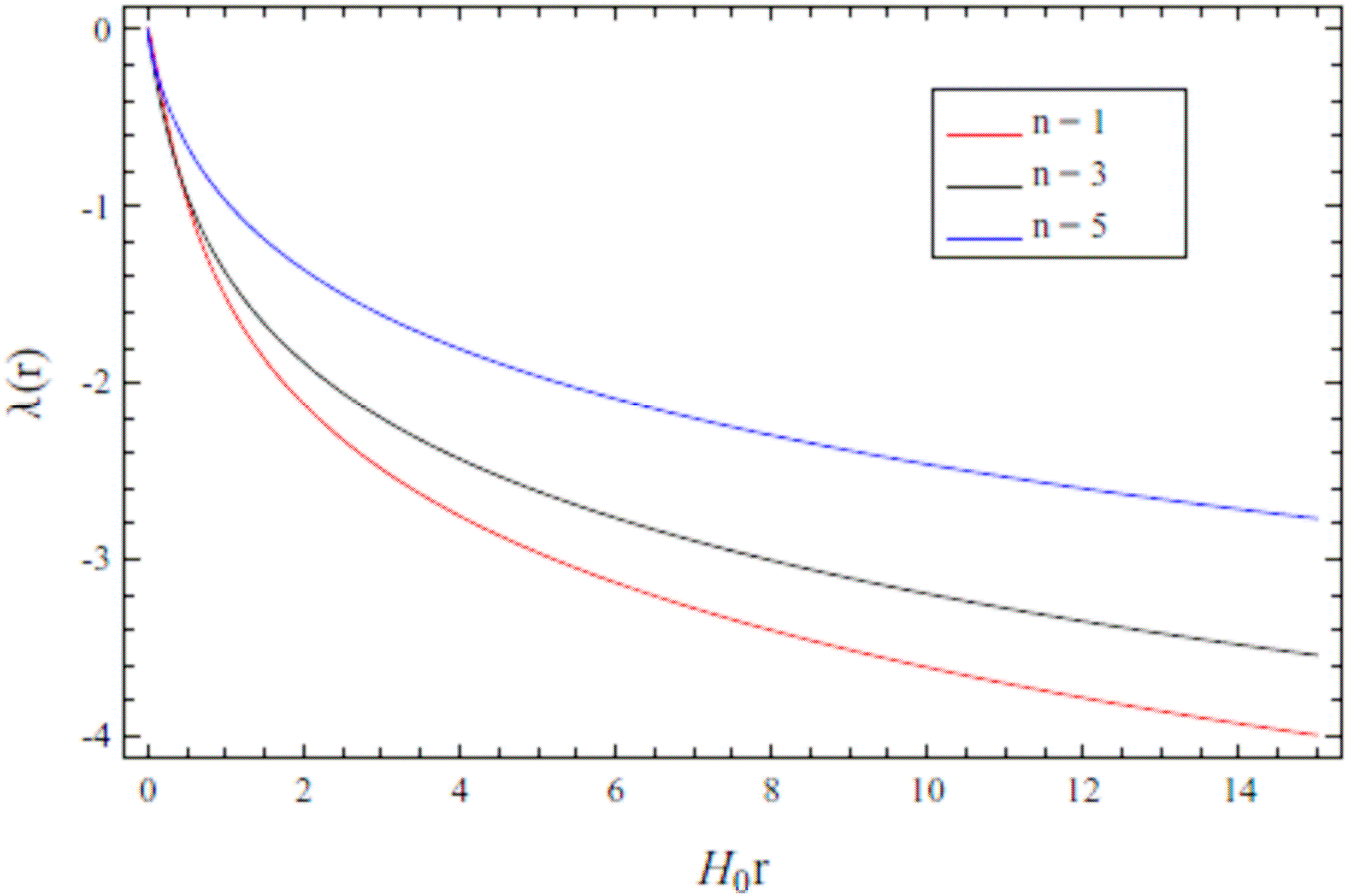
Negative power law potential III

$$V(\phi) = H_0^2 M_{\rm pl}^{2+n} \phi^{-n}$$



Negative power law potential IV

$$V(\phi) = H_0^2 M_{\rm pl}^{2+n} \phi^{-n}$$



Summary

We have considered the behavior of the scalar field, which cause the current accelerated expansion of the Universe, in a static and isotropic background.

The solution in quintessence model with an exponential potential:

$$\phi(r) = -\frac{M^4}{6\phi_0} r^2 + \frac{c_1}{r} + m_1, \quad |c_1/M_{\rm pl}| \ll r \ll H_0^{-1}.$$

The solution in quintessence model with a negative power law potential:

$$\phi(r) = \left[-\frac{2(4+n)}{n(2+n)^2 M^{4+n}}\right]^{-\frac{1}{2+n}} r^{\frac{2}{2+n}}, \quad |r_1| \ll r \ll H_0^{-1}.$$

n = 1, 3, 5.

The other cases are also investigated in arXiv: 1607.03003[gr-qc].