Vacuum expectation value of the surface energy-momentum tensor in braneworlds for a complex scalar field

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Bulk + surface action

- In GR, when the underlying manifold has boundaries, correct variational problem is achieved by supplementing the action with **Gibbons-Hawking** surface term

$$S_{GR} = \frac{1}{2}\int d^{D+1}x\sqrt{|g|}R \longrightarrow S_{GH} = -\int d^D x\sqrt{|h|}\varepsilon K$$

- A complex scalar field on the background of a manifold with boundaries
  - The standard bulk action with coupling (to the background curvature) constant

$$S_b = \frac{1}{2}\int d^{D+1}x\sqrt{|g|}(\nabla_i\varphi\nabla^i\varphi^* - m^2\varphi\varphi^* - \xi R\varphi\varphi^*)$$

  - The surface action with the analog of Gibbons-Hawking term and surface mass term

$$S_s = -\int d^D x\sqrt{|h|}\varepsilon(\xi K\varphi\varphi^* + m_s\varphi\varphi^*)$$

$$h_{ik} = g_{ik} - \varepsilon n_i n_k$$

$$K_{ik} = h_i^l h_k^m \nabla_l n_m$$

Metric energy-momentum tensor

- The EMT yielded by variational problem with respect to metric: $T_{ik} = T_{ik}^{(vol)} + T_{ik}^{(surf)}$

$$T_{ik}^{(vol)} = \frac{1}{2}(\nabla_i\varphi\nabla_k\varphi^* + \nabla_k\varphi\nabla_i\varphi^*) + \left[\left(\xi - \frac{1}{4}\right) g_{ik}g^{lm}\nabla_l\nabla_m - \xi R_{ik} - \xi\nabla_i\nabla_k\right]\varphi\varphi^*$$

$$T_{ik}^{(surf)} = \delta(x;\partial M_s)\left[\xi\varphi\varphi^* K_{ik} - \left(\xi - \frac{1}{4}\right) h_{ik}n^l\nabla_l(\varphi\varphi^*)\right]$$

- Important notes
  - Equations of motion

$$g^{lm}\nabla_l\nabla_m\varphi + m^2\varphi + \xi R\varphi = 0$$

$$2\left(\xi K + m_s\right)\varphi + n^l\nabla_l\varphi = 0$$

Boundary condition of Robin type.

  - Surface EMT is orthogonal to the boundary
  - Integral conservation law for EMT

The model under discussion

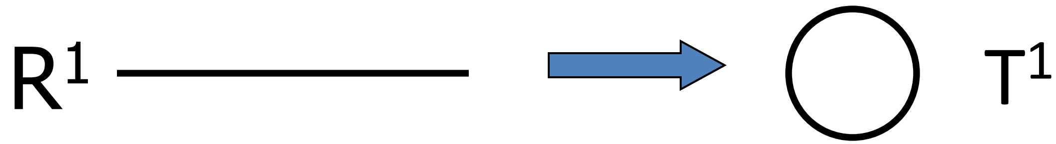
- Randall-Sundrum braneworlds
  - There are natural timelike boundaries (branes) in AdS spacetime
  - The background in the original RS models is 5D AdS spacetime and the branes are located perpendicular to the additional (timelike) dimension
  - In the 2-brane model the hierarchy between electroweak and Planck scales is resolved at large inter-brane distances
- The model at hand
  - D+1 dimensional spacetime
  - Local AdS geometry $ds^2 = e^{-2y/a}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$

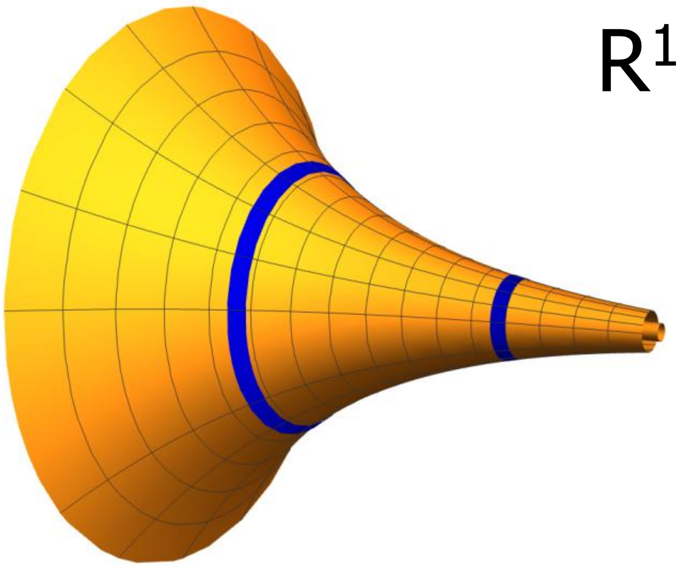
$$ds^2 = \left(\frac{a}{z}\right)^2 \left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

  - One or two D dimensional brane(s) located at y1 and y2

The model hand (continued)

- Globally the topology is different from AdS: $R^p \times T^q$





- The proper length of compact dimensions

$$L_{(p)i} = ({}^{a}/_{z})L_i = e^{-{}^{y}/_{a}}L_i$$

- Presence of constant background gauge field

- Quasi-periodicity condition

$$\varphi(t, x^1, \ldots, x^l + L_l, \ldots, y) = e^{i\alpha_l}\varphi(t, x^1, \ldots, x^l, \ldots, y), \quad l = p+1, \ldots, D-1$$

VEV of surface EMT

- The vacuum expectation value (VEV) of the surface energy-momentum tensor is proportional to the VEV of field squared

$$\langle 0| T^{(surf)}_{\mu\nu} |0\rangle = -\delta(x;\partial M_s) g_{\mu\nu} \left[\xi \frac{n^{(j)}}{a} - \frac{2\xi - \frac{1}{2}}{\beta^{(j)}} \right] \langle 0| \varphi\varphi^{\dagger} |0\rangle$$

- Cosmological constant type contribution on the brane

$$\langle 0| \tau_i^{(j)k} |0\rangle_{y=y_j} = -\frac{c_j}{a} \delta_i^k \langle 0| \varphi\varphi^{\dagger} |0\rangle_{y=y_j}$$

- In 1-brane model regularized quantities are obtained via zeta function regularization, with the help of extended Chowla-Selberg formula.

- In 2-brane model the zeta function is represented as sum of one brane induced and interference terms

$$\zeta_{j\boldsymbol{k}_q}(s) = \zeta^{(J)}_{j\boldsymbol{k}_q}(s) - \frac{B_j}{\Gamma\left(-\frac{s}{2}\right)\Gamma\left(\alpha_s + 1\right)\mu^{1+s}} \int_{k_q}^{\infty} \mathrm{d}\lambda\lambda \left(\lambda^2 - k_q^2\right)^{\alpha_s} \Omega_{j\nu}(\lambda z_1, \lambda z_2)$$

The interference term

- The energy density corresponding to the interference term is finite

$$\Delta\epsilon_j = \frac{2c_j B_j^2 z_j^D}{a^D (4\pi)^{\frac{p+1}{2}} V_q \Gamma\left(\frac{p+1}{2}\right)} \sum_{\boldsymbol{k}_q} \int_{k_q}^{\infty} d\lambda\lambda \left(\lambda^2 - k_q^2\right)^{\frac{p-1}{2}} \Omega_{j\nu}(\lambda z_1, \lambda z_2)$$

$$\Omega_{1\nu}(u,v) = \frac{\bar{K}_\nu^{(2)}(v)}{\bar{K}_\nu^{(1)}(u) G_\nu^{(1;2)}(u,v)}$$

$$\Omega_{2\nu}(u,v) = \frac{\bar{I}_\nu^{(1)}(u)}{\bar{I}_\nu^{(2)}(v) G_\nu^{(1;2)}(u,v)}$$

- Asymptotics for visible brane in the case of large inter-brane distance
  - Large KK modes: $k_q z_2 \gg 1$

$$\Delta\epsilon_{2,\boldsymbol{k}_q} \approx \frac{2\pi c_2 B_2^2 z_2^{D-\frac{p-1}{2}}}{a^D (4\pi)^{\frac{p+1}{2}} V_q} \frac{k_q^{\frac{p+3}{2}}}{[A_2 + B_2 k_q z_2]^2} \frac{\bar{I}_\nu^{(1)}(k_q z_1)}{\bar{K}_\nu^{(1)}(k_q z_1)} e^{-2k_q z_2}$$

  - Small KK modes: $k_q z_1 \ll 1$

$$\Delta\epsilon_{2,\boldsymbol{k_q}} = \frac{z_2^q}{a^D d_1(\nu) V_q} \left(\frac{z_1}{z_2}\right)^{2\nu} f_{2\nu}^{(k_q)}$$

$$f_{2\nu}^{(k_q)} = \frac{2^{2-2\nu} c_2 B_2^2}{(4\pi)^{\frac{p+1}{2}} \Gamma\left(\frac{p+1}{2}\right) \nu\Gamma^2(\nu)} \int_{k_q z_2}^{\infty} du \frac{u^{1+2\nu}\left(u^2 - k_q^2 z_2^2\right)^{\frac{p-1}{2}}}{\bar{I}_\nu^{(2)2}(u)}$$

Cosmological constant

- The cosmological constant due to the interference term on the visible brane, in the case of large inter-brane distances in Planck units

$$h_2 \approx (D-2)^{-\bar{\nu}}(aM_{D+1})^{\bar{\nu}-(p+1)}(V'_{2,q}M^q_{D+1})^{\bar{\nu}}\left(\frac{M_{D+1}}{M_{p+1;2}}\right)^{p+1+\bar{\nu}(p-1)}\frac{1}{d_1(\nu)}\sum_{\boldsymbol{k}_q} f^{(k_q)}_{2\nu}$$

- Observed cosmological constant (Plank units): ~ 10^{-123}

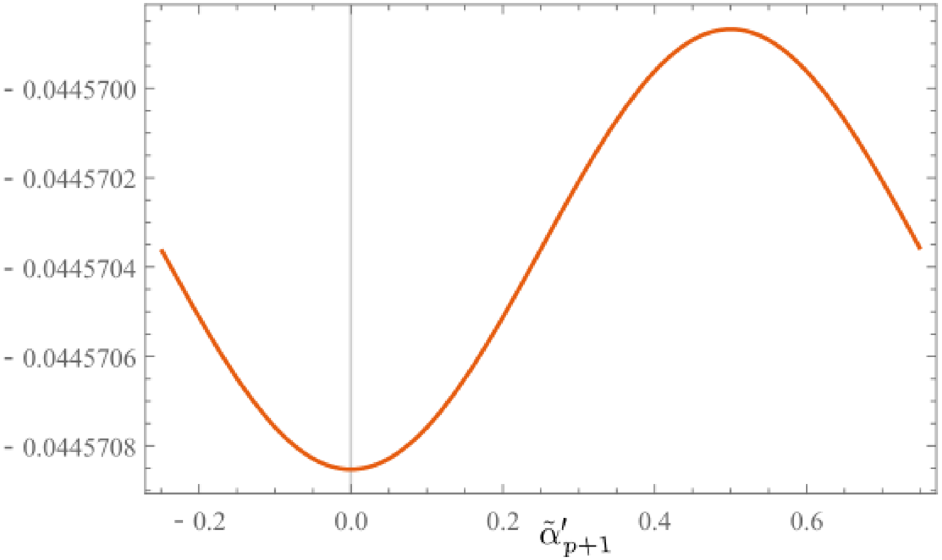
- When the hierarchy problem between electroweak and Planck scales is resolved
  - The fundamental scales in the problem are TeV: $1/a \sim M_{D+1} \sim 1\ \mathrm{TeV}$
  - Planck scale for the visible brane is 10^{16} TeV: $M_{p+1;2} = M_{Pl} \sim 10^{16}\ \mathrm{TeV}$
  - Proper volume of compact subspace is of Planck order: $V'_{2,q}M^q_{D+1} \sim 1$
  - The cosmological constant due to the interference term is as follows

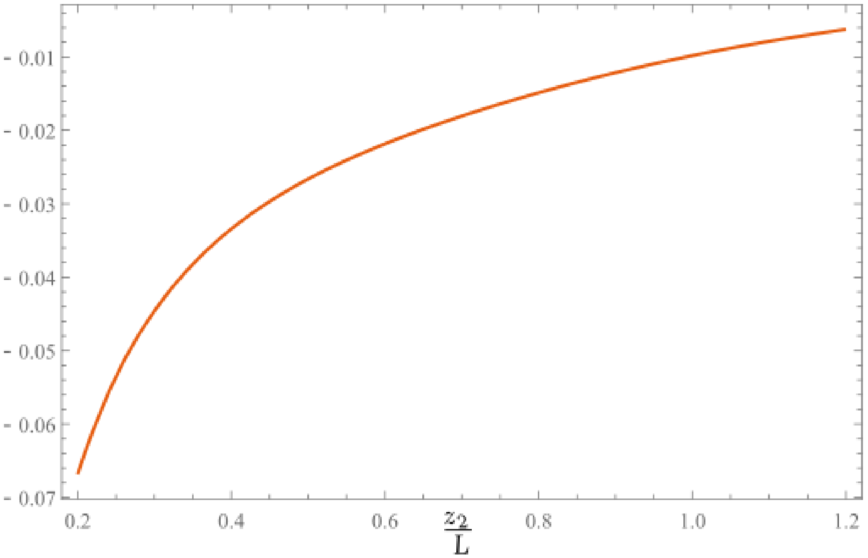
$$h_2 \approx 10^{-32(2+\bar{\nu})}\frac{(D-2)^{-\bar{\nu}}}{d_1(\nu)}\sum_{\boldsymbol{k}_q} f^{(k_q)}_{2\nu}$$

Cosmological constant

- E.g. for a massless, minimally coupled scalar field, and a single compact dimension the power of ten alone in the coefficient is 10^{-117}

- The rest of the expression for cosmological constant





- The sign of cosmological constant is not dependent of the gauge flux

Conclusion

- Surface terms for scalar field with coupling to the background curvature
  - Achieve correctly posed variational (with respect to metric tensor) problem in the presence of scalar field
  - Naturally arising Robin boundary conditions
  - Integral conservation of total energy density

- The cosmological constant problem
  - Natural mechanism for obtaining small cosmological constant from VEV
  - The cosmological constant problem is solved with the same configuration when the hierarchy problem is solved
  - Sign of cosmological constant on visible brane is determined by the boundary conditions on both branes

Thank you!