A simple inflationary quintessential model

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This talk is based in the manuscript:

Simple inflationary quintessential model II: Power law potentials J. de Haro, J. Amorós, S. Pan, [arXiv:1607.06726].

and in the previous papers:

1. Evolution and Dynamics of a Matter creation model S. Pan, J. de Haro, A. Paliathanasis, R. Jan Slagter, Mon. Not. Roy. Astron. Soc. 460 (2), 1445-1456 (2016), [arXiv:1601.03955].
2. Simple inflationary quintessential model J. de Haro, J. Amorós, S. Pan, Phys. Rev. D 93, 084018 (2016), [arXiv:1601.08175].
3. Inflation and late-time acceleration from a double-well potential with cosmological constant J. de Haro, E. Elizalde, Gen. Rel. Grav. 48, 77 (2016) [arXiv:1602.03433].

Outline

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The models and properties

We assume the following simple dynamics:

$$\dot{H} = \begin{cases} (-3H_E^2 + \Lambda)\left(\frac{H}{H_E}\right)^{\alpha} & \text{for} \quad H \geq H_E \\ -3H^2 + \Lambda & \text{for} \quad H \leq H_E, \end{cases}$$

where H_E is an specific value of the Hubble parameter, $\Lambda \ll H_E^2$ is a cosmological constant and $\alpha \in [0, 1]$ is the parameter, which defines the family of models.

Properties:

1. The model has an accelerated period at early times. Since $\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{H^{\alpha}}{3}\left(3H_E^{2-\alpha} - H^{2-\alpha}\right)$, one can see that for $H > 3^{\frac{1}{2-\alpha}} H_E$, the universe accelerates. It has a fixed point at $H = \sqrt{\frac{\Lambda}{3}}$ which depicts the current cosmic acceleration.
2. Phase transition to a kinetion (or deflationary) regime when $H = H_E$, to produce enough particles to reheat the universe.
3. The universe is nonsingular in cosmic time.

The models and properties

Effective Equation of State parameter

$$w_{eff} = -1 - \frac{2\dot{H}}{3H^2}$$

In our case

$$w_{eff} = \begin{cases} -1 + 2\left(1 - \frac{\Lambda}{3H_E^2}\right)\left(\frac{H}{H_E}\right)^{\alpha-2} & H \geq H_E \\ 1 - \frac{2\Lambda}{3H^2} & H \leq H_E. \end{cases}$$

Conclusion:

- For $H \gg H_E \Longrightarrow w_{eff} \cong -1$. Early time acceleration (Inflation).
- For $H \cong H_E \Longrightarrow w_{eff} \cong 1$. Kination regime.
- For $H \cong \sqrt{\frac{\Lambda}{3}} \Longrightarrow w_{eff} \cong -1$. Current acceleration.

The models and properties

Equation of State for $\Lambda \cong 0$

$$P = \begin{cases} -\rho + 2\left(\frac{\rho}{\rho_E}\right)^{\frac{\alpha-2}{2}}\rho & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E. \end{cases}$$

For $\alpha = 0$

$$P = \begin{cases} -\rho + 2\rho_E & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E. \end{cases}$$

For $\alpha = 1$

$$P = \begin{cases} -\rho + 2\sqrt{\rho\rho_E} & \rho \geq \rho_E \\ \rho & \rho \leq \rho_E. \end{cases}$$

The models and properties

Case $\alpha = 1$.

$$H(t) = \begin{cases} H_E e^{\frac{(-3H_E^2+\Lambda)t}{H_E}} & t \leq 0 \\ \sqrt{\frac{\Lambda}{3}} \frac{3H_E + \sqrt{3\Lambda}\tanh(\sqrt{3\Lambda}t)}{3H_E \tanh(\sqrt{3\Lambda}t) + \sqrt{3\Lambda}} & t \geq 0, \end{cases}$$

and since $\Lambda \cong 0$ one has

$$H(t) \cong \begin{cases} H_E e^{-3H_E t} & t \leq 0 \\ \frac{H_E}{3H_E t + 1} & t \gtrsim 0. \end{cases}$$

$$a(t) \cong \begin{cases} a_E e^{-\frac{1}{3}[e^{-3H_E t} - 1]} & t \leq 0 \\ a_E (3H_E t + 1)^{\frac{1}{3}} & t \gtrsim 0. \end{cases}$$

For $t \gtrsim 0$ the universe enters in a kination regime.

The models and properties

Case $0 \leq \alpha < 1$. $\Lambda \cong 0$

$$H(t) = \begin{cases} H_E \left(3(\alpha-1)H_E t + 1\right)^{\frac{1}{1-\alpha}} & t \leq 0 \\ \frac{H_E}{3H_E t+1} & t \gtrsim 0. \end{cases}$$

$$a(t) = \begin{cases} a_E e^{-\frac{1}{3(2-\alpha)}\left[(3(\alpha-1)H_E t+1)^{\frac{2-\alpha}{1-\alpha}}-1\right]} & t \leq 0 \\ a_E(3H_E t + 1)^{\frac{1}{3}} & t \gtrsim 0. \end{cases}$$

For $t \gtrsim 0$ the universe enters in a kinetion regime.

The potential (Reconstruction method)

Formula:

$$\varphi = M_{pl}\int\sqrt{-2\dot{H}}dt = -M_{pl}\int\sqrt{-\frac{2}{\dot{H}}}dH.$$

In our case:

$$\varphi = \begin{cases} -\frac{2\sqrt{2}}{\sqrt{3}(2-\alpha)}M_{pl}\left(\frac{H}{H_E}\right)^{\frac{2-\alpha}{2}}\frac{H_E}{\sqrt{H_E^2-\frac{\Lambda}{3}}} & H \geq H_E \\ -\sqrt{\frac{2}{3}}M_{pl}\left[\ln\left(\frac{H}{H_E}\right)+\frac{2}{2-\alpha}\right] & H \lesssim H_E. \end{cases}$$

The potential (Reconstruction method)

Conversely

$$H = \begin{cases} H_E \left(\frac{\varphi}{\varphi_E}\right)^{\frac{2}{2-\alpha}} & \varphi \leq \varphi_E \\ H_E e^{-\sqrt{\frac{3}{2}}\frac{\varphi}{M_{pl}} - \frac{2}{2-\alpha}} & \varphi \gtrsim \varphi_E, \end{cases}$$

where $\varphi_E \equiv -\frac{2\sqrt{2}}{\sqrt{3}(2-\alpha)} \frac{H_E}{\sqrt{H_E^2 - \frac{\Lambda}{3}}} M_{pl} \cong -\frac{2\sqrt{2}}{\sqrt{3}(2-\alpha)} M_{pl}$.

Formula

$$V(H) = 3H^2 M_{pl}^2 + \dot{H} M_{pl}^2 \Longrightarrow V(\varphi) = 3H^2(\varphi) M_{pl}^2 + \dot{H}(\varphi) M_{pl}^2.$$

The potential (Reconstruction method)

For our family of models:

$$V(H) = \begin{cases} 3H^{\alpha}\left(H^{2-\alpha} - \frac{H_E^2 - \frac{\Lambda}{3}}{H_E^{\alpha}}\right) M_{pl}^2 & H \geq H_E \\ \Lambda M_{pl}^2 & H \leq H_E. \end{cases}$$

$$V(\varphi) = \begin{cases} 3\left(\frac{H_E M_{pl}}{\varphi_E}\right)^2 \left(\frac{\varphi}{\varphi_E}\right)^{\frac{2\alpha}{2-\alpha}} \left[\varphi^2 - \varphi_E^2\left(1 - \frac{\Lambda}{3H_E^2}\right)\right] & \varphi \leq \varphi_E \\ \Lambda M_{pl}^2 & \varphi \geq \varphi_E. \end{cases}$$

For $\alpha = 0$ quadratic potential, for $\alpha = \frac{2}{3}$ cubic and for $\alpha = 1$ quartic.

Slow roll and spectral parameters

Slow roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}$$

Spectral index (n_s), its running (α_s), and the ratio of tensor to scalar perturbations (r)

$$n_s - 1 = -6\epsilon + 2\eta, \quad \alpha_s = \frac{H\dot{n}_s}{H^2 + \dot{H}}, \quad r = 16\epsilon.$$

Number of e-folds

$$N = \int_{t_*}^{t_{end}} H dt = -\int_{H_{end}}^{H_*} \frac{H}{\dot{H}} dH.$$

Slow roll and spectral parameters

For our family:

Spectral index (n_s), its running (α_s), the ratio of tensor to scalar perturbations (r), and the number of e-folds (N)

$$n_s - 1 = (\alpha - 4)\epsilon_*, \quad \alpha_s = \frac{(\alpha-4)(2-\alpha)\epsilon_*^2}{1-\epsilon_*}, \quad r = 16\epsilon_*.$$

$$N = \frac{1}{2-\alpha}\left(\frac{1}{\epsilon_*} - 1\right).$$

with

$$\epsilon_* = 3\left(\frac{H_E}{H_*}\right)^{2-\alpha},$$

where H_* is the value of the Hubble parameter when the pivot scale crosses the Hubble radius.

Slow roll and spectral parameters

REMARK:
For potentials of the form $V(\varphi) = \lambda\varphi^{\frac{4}{2-\alpha}}$, and using that

$$\epsilon \cong \frac{M_{pl}^2}{2}\left(\frac{V_\varphi}{V}\right)^2, \qquad \eta \cong M_{pl}^2 \frac{V_{\varphi\varphi}}{V}$$

one also obtains $n_s - 1 \cong (\alpha - 4)\epsilon_*$

This means that, our family of potentials, during the inflationary regime, are like power law potentials

Slow roll and spectral parameters

In terms of the number of e-folds:

Spectral index (n_s), its running (α_s) and the ratio of tensor to scalar perturbations (r)

$$n_s - 1 = \frac{\alpha - 4}{1 + (2-\alpha)N}, \quad r = \frac{16}{1 + (2-\alpha)N}.$$

$$\alpha_s = \frac{\alpha - 4}{N(1 + (2-\alpha)N}.$$

REMARK: From nucleosynthesis bounds, i.e., if one admits reheating temperatures from 10^9 GeV to 1 MeV, in quintessential inflation, the number of e-folds will range approximately between 65 and 75.

Comparison with Planck2013 data

From the formulas

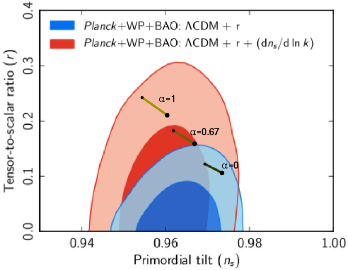
$$\mathcal{P}=\frac{H_*^2}{8\pi^2\epsilon_* M_{pl}^2}\sim 2\times 10^{-9},\quad H_*=\frac{H_E}{\left(\frac{\epsilon_*}{3}\right)^{\frac{1}{2-\alpha}}},\quad \epsilon_*=\frac{1-n_s}{4-\alpha}$$

One obtains

$$H_E\sim 7\times 10^{-4}\left(\frac{1-n_s}{3(4-\alpha)}\right)^{\frac{4-\alpha}{2(2-\alpha)}} M_{pl}$$

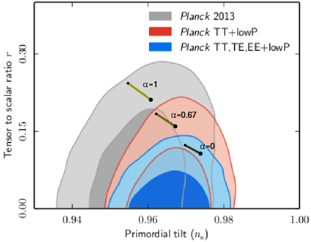
Taking, as usual, $n_s\cong 0.96$ one has the value of H_E for each value of the parameter α.
For $\alpha=0$, one has $H_E\sim 2\times 10^{-6}M_{pl}\sim 5\times 10^{12}$ GeV
For $\alpha=1$, one has $H_E\sim 10^{-7}M_{pl}\sim 2\times 10^{11}$ GeV

Comparison with Planck2013 data



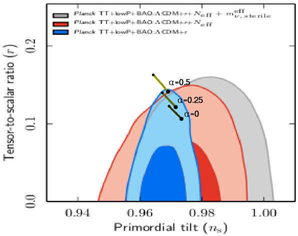
- Confidence region for the values of (n_s, r), with (red region) and without (blue region) allowing the running, according to Planck 2013.
- For indicated values of α, the interval of values of (n_s, r) yielded by our model from $N = 65$ (small circle) to $N = 75$ efolds (big circle) of expansion is superimposed on the Planck confidence regions.

Comparison with Planck2015 data



- Confidence region for the values of (n_s, r), with running, according to Planck 2015 and 2013.
- Planck2015 with running only allow values of $\alpha \in [0, \frac{2}{3}]$.

Comparison with Planck2015 data



- Confidence region for the values of (n_s, r), without running, according to Planck 2015 and 2013.
- Planck2015 without running only allow values of $\alpha \in [0, \frac{1}{2}]$.

Gravitational particle production

Massive quantum field conformally coupled with gravity

$$\chi_k'' + \omega_k^2(\tau)\chi_k = 0,$$

where $'$ denotes the derivative with respect the conformal time τ and $\omega_k(\tau) = \sqrt{k^2 + m^2 a^2(\tau)}$ is the frequency of the particle in the k-mode.
Adiabatic regime near the phase transition

$$H_E \ll m \Longrightarrow \omega_k'(\tau) \ll \omega_k^2(\tau),$$

means that one can use the first order WKB solution to define approximately the vacuum modes:

$$\chi_{1,k}^{WKB}(\tau) \equiv \sqrt{\frac{1}{2W_{1,k}(\tau)}} e^{-i\int^\tau W_{1,k}(\eta)d\eta},$$

where

$$W_{1,k} = \omega_k - \frac{1}{4}\frac{\omega_k''}{\omega_k^2} + \frac{3}{8}\frac{(\omega_k')^2}{\omega_k^3}.$$

Gravitational particle production

Before the phase transition the vacuum is depicted approximately by $\chi_{1,k}^{WKB}(\tau)$, but after it this mode becomes a mixure of positive and negative frequencies of the form $\alpha_k\chi_{1,k}^{WKB}(\tau)+\beta_k(\chi_{1,k}^{WKB})^*(\tau)$. The β_k-Bogoliubov coefficient could be obtained, as usual, matching both expressions at the transition time τ_E, obtaining

$$\beta_k=\frac{\mathcal{W}[\chi_{1,k}^{WKB}(\tau_E^-),\chi_{1,k}^{WKB}(\tau_E^+)]}{\mathcal{W}[(\chi_{1,k}^{WKB})^*(\tau_E^+),\chi_{1,k}^{WKB}(\tau_E^+)]}$$

$\mathcal{W}$ is the Wronskian.

The square modulus of the β_k-Bogoliubov coefficient will be given by

$$|\beta_k|^2\cong\frac{m^4a_E^{10}\left(\ddot{H}_E^+-\ddot{H}_E^-\right)^2}{256(k^2+m^2a_E^2)^5}=\frac{81(2-\alpha)^2m^4a_E^{10}H_E^6}{256(k^2+m^2a_E^2)^5}.$$

Gravitational particle production

number and energy density

$$n_\chi \equiv \frac{1}{2\pi^2 a^3}\int_0^\infty k^2|\beta_k|^2 dk, \quad \rho_\chi \equiv \frac{1}{2\pi^2 a^4}\int_0^\infty \omega_k k^2|\beta_k|^2 dk,$$

For our models

$$n_\chi \sim 3\times 10^{-3}(2-\alpha)^2\frac{H_E^6}{m^3}\left(\frac{a_E}{a}\right)^3, \quad \rho_\chi \sim m n_\chi.$$

Particles are far from being in thermal equilibrium and, at the beginning, their energy density scales as a^{-3}, eventually they will decay into lighter particles, which will interact through multiple scattering. At the end of these process, the universe becomes filled by a relativistic plasma in thermal equilibrium whose energy density decays as a^{-4}.

Thermalization and reheating

The thermalization process, where the cross section for $2\rightarrow 3$ scattering with gauge bosons exchange whose typical energy is $\rho_\chi^{\frac{1}{4}}(\tau_E)$, is given by $\sigma=\beta^3\rho_\chi^{-\frac{1}{2}}(\tau_E)$, with $\beta^2\sim 10^{-3}$. The thermalization rate is

$$\Gamma=\sigma n_\chi(\tau_E)\sim 5\times 10^{-2}(2-\alpha)\beta^3\left(\frac{H_E}{m}\right)^2 H_E.$$

Thermal equilibrium is reached when $\Gamma\sim H(t_{eq})\cong H_E\left(\frac{a_E}{a_{eq}}\right)^3$, which leads to the relation $\frac{a_E}{a_{eq}}\sim 4\times 10^{-1}(2-\alpha)^{1/3}\beta\left(\frac{H_E}{m}\right)^{2/3}$. Then, at the equilibrium one has

$$\rho_\chi(t_{eq})\sim 10^{-4}(2-\alpha)^3\beta^3\left(\frac{H_E}{m}\right)^4 H_E^4,$$

$$\rho(t_{eq})\sim 7\times 10^{-3}(2-\alpha)^2\beta^6\left(\frac{H_E}{m}\right)^4 H_E^2 M_{pl}^2.$$

Thermalization and reheating

After this thermalization, the relativistic plasma and the background evolve as

$$\rho_\chi(t)=\rho_\chi(t_{eq})\left(\frac{a_{eq}}{a}\right)^4, \quad \rho(t)=\rho(t_{eq})\left(\frac{a_{eq}}{a}\right)^6.$$

The reheating is obtained when both energy densities are of the same order, which happens when $\frac{a_{eq}}{a_R}\sim\sqrt{\frac{\rho_\chi(t_{eq})}{\rho(t_{eq})}}$, and thus, obtaining a reheating temperature of the order

$$T_R\sim\rho_\chi^{\frac{1}{4}}(t_{eq})\sqrt{\frac{\rho_\chi(t_{eq})}{\rho(t_{eq})}}\sim 10^{-1}\left(\frac{H_E}{M_{pl}}\right)^2\left(\frac{H_E}{m}\right)M_{pl}.$$

Since, $H_E\ll m$, if we consider masses of the order 10^2H_E one has

$$T_R\sim 10^{-3}\left(\frac{H_E}{M_{pl}}\right)^2 M_{pl}\sim 5\times 10^{-10}\left(\frac{1-n_s}{3(4-\alpha)}\right)^{\frac{4-\alpha}{2-\alpha}}M_{pl}.$$

Thermalization and reheating

- Quadratic potential: $\alpha = 0 \Longrightarrow T_R \sim 5 \times 10^{-15} M_{pl} \sim 10^4$ GeV.
- Cubic potential: $\alpha = \frac{2}{3} \Longrightarrow T_R \sim 5 \times 10^{-16} M_{pl} \sim 10^3$ GeV.
- Quartic potential: $\alpha = 1 \Longrightarrow T_R \sim 4 \times 10^{-17} M_{pl} \sim 10^2$ GeV.

So, one obtains a reheating temperature that preserves nucleosynthesis success.

Detailed calculation

Main formula

$$\frac{k_*}{a_0 H_0} = e^{-N_*}\frac{H_*}{H_0}\frac{a_{end}}{a_E}\frac{a_E}{a_R}\frac{a_R}{a_M}\frac{a_M}{a_0}$$
$$= e^{-N_*}\frac{H_*}{H_0}\frac{a_{end}}{a_E}\frac{\rho_R^{-1/12}\rho_M^{1/4}}{\rho_E^{1/6}}\frac{a_M}{a_0},$$

where, "end", R and M means at the end of inflation, the beginning of radiation era, the beginning of matter domination and the subindex 0 means the current time. We have used the relations

$$\left(\frac{a_E}{a_R}\right)^6 = \frac{\rho_R}{\rho_E}, \quad \left(\frac{a_R}{a_M}\right)^4 = \frac{\rho_M}{\rho_R}.$$

Detailed calculation

Taking as a pivot scale $k_* = 0.05$ Mpc^{-1}, and since the current horizon scale is $a_0 H_0 \cong 2\times 10^{-4}$ Mpc^{-1}, one obtains

Main formula

$$N_* = -5.52 + \ln\left(\frac{H_*}{H_0}\right) + \ln\left(\frac{a_{end}}{a_E}\right)$$
$$+\frac{1}{4}\ln\left(\frac{\rho_M}{\rho_R}\right) + \frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) + \ln\left(\frac{a_M}{a_0}\right).$$

Detailed calculation

We have to use the important and well-known formula $T_0 = \frac{a_M}{a_0} T_M$.
Moreover, $\rho_M \cong \frac{\pi^2}{15} g_M T_M^4$ and $\rho_R \cong \frac{\pi^2}{30} g_R T_R^4$

Main formula

$$N_* = -5.52 + \ln\left(\frac{H_*}{H_0}\right) + \ln\left(\frac{a_{end}}{a_E}\right)$$
$$+\frac{1}{4}\ln\left(\frac{2g_M}{g_R}\right) + \frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) + \ln\left(\frac{T_0}{T_R}\right).$$

Detailed calculation

Using that $H_0 \sim 6\times 10^{-61} M_{pl}$ and $\mathcal{P}=\frac{H_*^2}{8\pi^2\epsilon_* M_{pl}^2} \sim 2\times 10^{-9}$ one obtains

$$\ln\left(\frac{H_*}{H_0}\right) = 131.38 + \frac{1}{2}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right).$$

Using that $T_0 \cong 2.73$ K $\cong 2\times 10^{-13}$ GeV and $g_M = 3.36$ one has

$$\frac{1}{4}\ln\left(\frac{2g_M}{g_R}\right) + \ln\left(\frac{T_0}{T_R}\right) = -28.76 - \ln\left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}}\right)$$

From the value of the Hubble parameter at the transition time, one will obtain

$$\frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) = -26.16 - \frac{4-\alpha}{6(2-\alpha)}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right) + \frac{2}{3}\ln\left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}}\right)$$

Detailed calculation

Collecting all the terms one obtains

Main formula

$$N_* = 70.94 + \ln\left(\frac{a_{end}}{a_E}\right) + \frac{1-\alpha}{3(2-\alpha)}\ln\left(\frac{1-n_s}{3(4-\alpha)}\right) - \frac{1}{3}\ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\text{GeV}}\right)$$

On the other hands, a simple calculation leads to

$$\ln\left(\frac{a_{end}}{a_E}\right) = \int_{H_E}^{H_{end}} \frac{H}{\dot{H}}dH = -\frac{2}{3(2-\alpha)}.$$

Detailed calculation

Final formula

$$N_* = 70.94 - \frac{1}{3(2-\alpha)}\left[2-(1-\alpha)\ln\left(\frac{1-n_s}{3(4-\alpha)}\right)\right] - \frac{1}{3}\ln\left(\frac{g_R^{\frac{1}{4}} T_R}{\text{GeV}}\right)$$

Using that for our models $T_R \sim 10^8 \left(\frac{1-n_s}{3(4-\alpha)}\right)^{\frac{4-\alpha}{2-\alpha}}$ GeV, and the fact that $g_R = 107$ for $T_R \geq 175$ GeV, one gets

Final formula

$$N_* = 64.41 - \frac{1}{3(2-\alpha)}\left[2+3\ln\left(\frac{1-n_s}{3(4-\alpha)}\right)\right].$$

Detailed calculation

Taking as usual $n_s \cong 0.96$ one has

- $\alpha = 0 \Longrightarrow N_* = 67$.
- $\alpha = \frac{2}{3} \Longrightarrow N_* = 68$.
- $\alpha = 1 \Longrightarrow N_* = 69$.

THANK YOU VERY MUCH FOR YOUR ATTENTION!!!!