Vacuum currents in braneworlds on AdS bulk with compact dimensions

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Outline

General Introduction

Based on

Bellucci, Saharian, V. V., **JHEP 1511 (2015) 092**

See also

- Bellucci, Saharian, V. V., **Phys. Rev. D 93, 084011 (2016)**
- Bezerra de Mello, Saharian, V. V., **Phys.Lett. B741 (2015) 155-162**

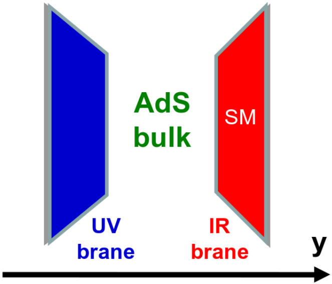
- We aim to consider combined effects of topology and gravity on the properties of quantum vacuum,
- Gravitational field is considered as a classical curved background,
- Back-reaction of quantum effects is described by Einstein equations with the expectation value of the energy-momentum tensor for quantum fields on the right-hand side,

General Introduction

- This hybrid but very useful scheme is an important intermediate step to the development of quantum gravity,
- We are interested in the effects of non-trivial topology and gravity on the vacuum expectation value (VEV) of the current:
  $j_\mu(x) = ie[\varphi^+(x)D_\mu\varphi(x) - (D_\mu\varphi^+(x))\varphi(x)]$
- Although the corresponding operator is local, due to the global nature of the vacuum, the VEV carries important information about the global properties of the background space-time
- As a background geometry we consider AdS spacetime.

Randall-Sundrum

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$



Randall-Sundrum

- Original Randall-Sundrum model (RS1) offers a solution to the hierarchy problem by postulating 5D AdS spacetime bounded by two (3+1)-dimensional branes
- Hierarchy problem between the gravitational and electroweak scales is solved for k × distance between branes = 40

The Geometry and Topology

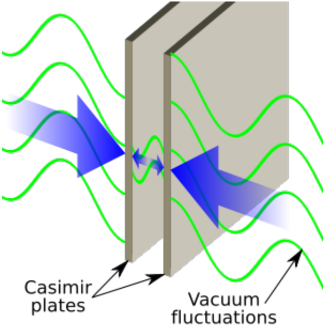
$D+1$-dimensional AdS Spacetime

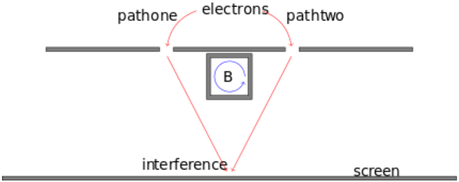
$$ds^2 = (a/z)^2(\eta_{ik}dx^i dx^k - dz^2)$$
$$0 \leq z < \infty$$
$$\eta_{ik} = diag(1, -1, ..., -1)$$

The Topology

$$R^p \times (S^1)^q, \text{ w/ } q + p = D - 1$$

Topological Casimir Effect and Aharonov-Bohm





Topological Casimir Effect and Aharonov-Bohm effect

- Periodic bnd conditions change the spectrum of the fields.
- Aharonov-Bohm like effect from the flux of the gauge field through the compact dim.

Field Content

Charged scalar field with general curvature coupling

$$(g^{\mu\nu} D_\mu D_\nu + m^2 + \xi R)\varphi(x) = 0, \; D_\mu = \nabla_\mu + ieA_\mu$$

Periodicity Conditions

$$\varphi(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \mathbf{x}_p, \mathbf{x}_q)$$

Special Cases

- untwisted fields: $\alpha_l = 0$
- twisted fields: $\alpha_l = \pi$

Classical Gauge Field

$$A_\mu = const$$

Sketch of the Strategy

We first evaluate the Hadamard function:

Hadamard Function

$$G^{(1)}(x,x') = \langle 0|\varphi(x)\varphi^{+}(x') + \varphi^{+}(x')\varphi(x)|0\rangle$$

Then:

Currents Density

$$\langle 0|j_\mu(x)|0\rangle = \langle j_\mu\rangle = \frac{i}{2} e \lim_{x'\to x}(\partial_\mu - \partial'_\mu)G^{(1)}(x,x')$$

Sum over modes:

Mode Sum

$$G^{(1)}(x,x') = \sum_\sigma \sum_{s=\pm} \varphi_\sigma^{(s)}(x)\varphi_\sigma^{(s)*}(x')$$

W/ $\left\{\varphi_\sigma^{(+)}(x), \varphi_\sigma^{(-)}(x)\right\}$ being the complete set of normalized positive- and negative-energy solutions to the field equation obeying the periodicity conditions.

Sketch of the Strategy

$$\varphi_\sigma^{(+/-)}(x) = z^{D/2} Z_\nu(\lambda z) e^{ik_r x^r \mp -i\omega t}$$

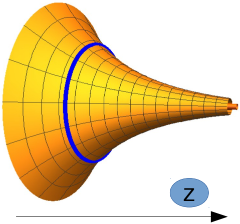
where

$$r = 1, ..., D-1$$
$$\nu = \sqrt{\frac{D^2}{4} - D(D+1)\xi + m^2 a^2}$$
$$\nu = \sqrt{\lambda^2 + k^2};\ k^2 = \sum_{l=1}^{D-1} k_l^2$$

1. $\nu \geq 0$
2. $\nu = 1/2$: Conformal Coupling
3. $\nu = D/2$: Minimal Coupling

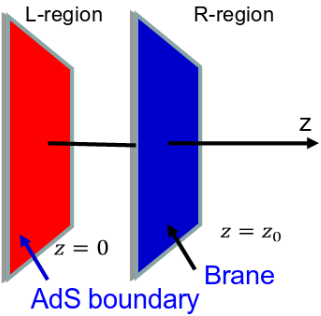
1. $\langle j^l \rangle = 0$ for l $\neq$compact
2. $\langle j^l \rangle \neq 0$ for l =compact

Bnd condition on brane



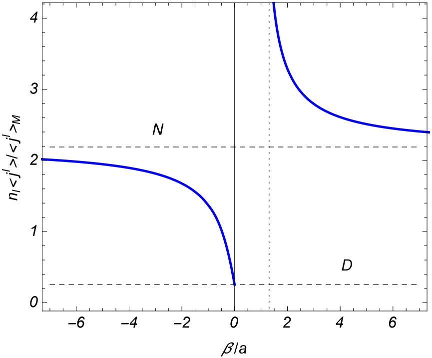
- Brane at $z = z_0$
- Robin bnd condition: $(1 + \beta n^{\mu} D_{\mu})\varphi(x) = 0$, at $z = z_0$
- Special cases: Dirichlet ($\beta = 0$) and Neumann ($\beta = \infty$)

Presence of the Brane

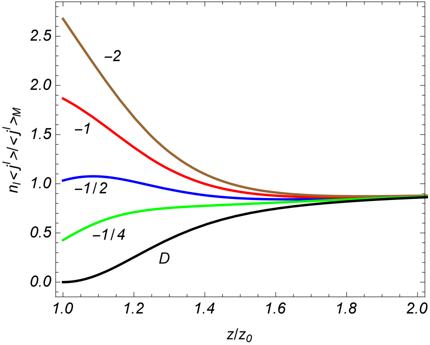


- Properties of the vacuum are different in L- and R-regions
- For both L- and R-regions the Hadamard function is decomposed into pure AdS and brane-induced contributions.
- Current density along the l-th compact dimension: $\langle j^l \rangle = \langle j^l \rangle_0 + \langle j^l \rangle_b, \; l = p + 1, ..., D - 1$

A representative result: Right region



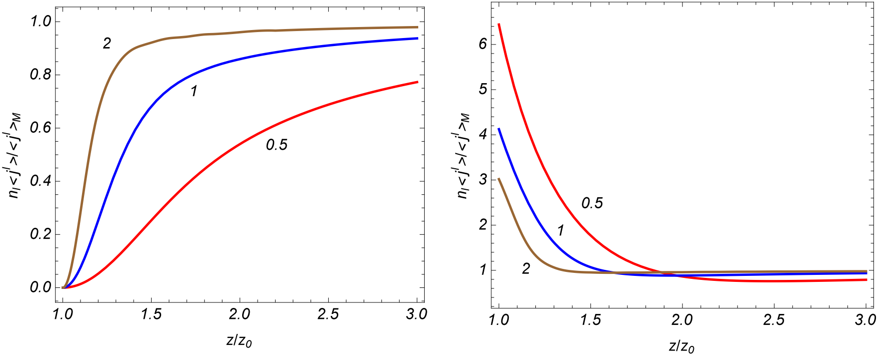
Region of unstable vacuum.



Robin bnd conditions.

The numbers on the curves are β/a.

A representative result: Right region

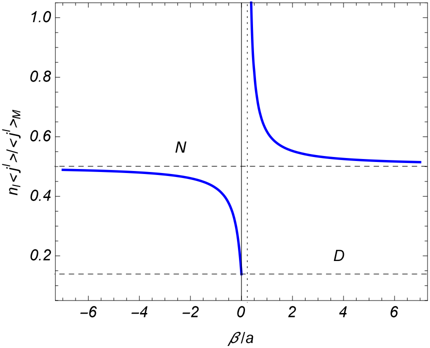


Dirichlet bnd condition.

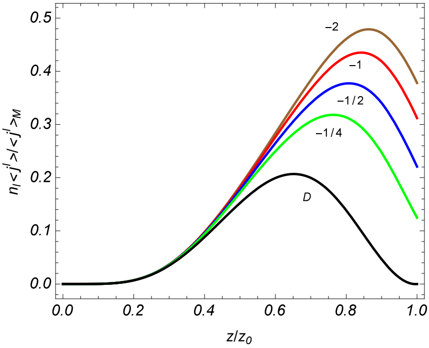
Neumann bnd condition.

The numbers on the curves represent z_0/L.

A representative result: Left region



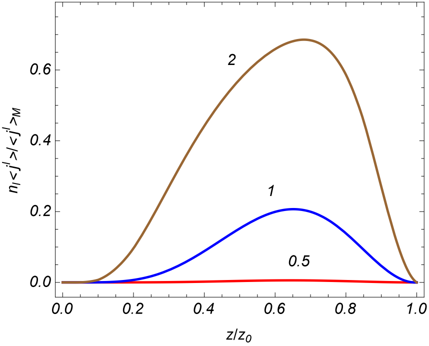
Region of unstable vacuum.



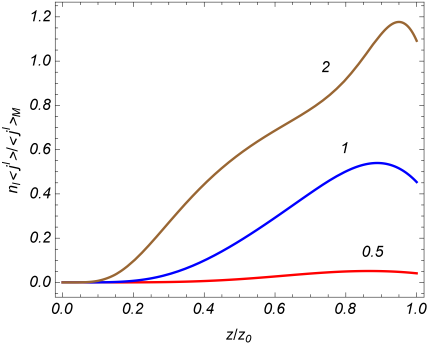
Robin bnd condition.

The numbers on the curves are β/α.

A representative result: Left region



Dirichlet bnd condition.



Neumann bnd condition.

The numbers on the curves represent z_0/L.

Results

- a) VEV of the current density for a massive scalar field is investigated in the background of AdS spacetime with spatial topology $R^p \times (S^1)^q$,
- b) Charge density and the components along the uncompactified dimensions vanish,
- c) Current density along compactified dimensions is a periodic function of the magnetic flux with the period of the flux quantum,
- d) Current density vanishes on the AdS boundary,
- e) Near the horizon the effects induced by the background curvature are small,
- f) In Kaluza-Klein-type models the current with the components along compact dimensions is a source of cosmological magnetic fields.

Extra Slides: Vacuum Currents Without Branes

$\langle j^l \rangle$ for l =compact

$$= \frac{4ea^{-1-D}L_l}{(2\pi)^{(D+1)/2}} \sum_{n_l=1}^{\infty} n_l \sin(\tilde{\alpha}_l n_l) \sum_{\mathbf{n}_{q-1}} \cos(\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1}) q_{\nu-1/2}^{(D+1)/2}(1 + g_{\mathbf{n}_q}^2 \frac{1}{2z^2})$$

where

1. $q_\alpha^\mu(x) = \frac{e^{-i\pi\mu} Q_\alpha^\mu(x)}{(x^2-1)^{\mu/2}}$
2. $g_{\mathbf{n}_q}^2 = (\sum_{i=p+1}^{D-1} n_i^2 L_i^2)^{1/2}$
3. $\mathbf{n}_{q-1} = (n_{p+1}, \ldots, n_{l-1}, n_{l+1}, \ldots, n_{D-1})$
4. $\tilde{\alpha}_{q-1} \cdot \mathbf{n}_{q-1} = \sum_{i=1,\neq l}^{D-1} \tilde{\alpha}_i n_i$

$\langle j^l \rangle = 0$ for l $\neq$compact

Extra Slides: Vacuum Currents - Brane Induced Part

Brane Induced Contribution to $\langle j^l \rangle$: right region

$$\langle j^l \rangle_b = -\frac{eC_p z^{D+2}}{2^{p-1} a^{D+1} V_q} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} dx\, x (x^2 - k_{(q)}^2)^{\frac{p-1}{2}} \frac{\bar{I}_\nu(z_0 x)}{\bar{K}_\nu(z_0 x)} K_\nu^2(zx)$$

where

1. $C_p = \frac{\pi^{-(p+1)/2}}{\Gamma((p+1)/2)}$

Brane Induced Contribution to $\langle j^l \rangle$: left region

$$\langle j^l \rangle_b = -\frac{eC_p z^{D+2}}{2^{p-1} a^{D+1} V_q} \sum_{\mathbf{n}_q} k_l \int_{k_{(q)}}^{\infty} dx\, x (x^2 - k_{(q)}^2)^{\frac{p-1}{2}} \frac{\bar{K}_\nu(z_0 x)}{\bar{I}_\nu(z_0 x)} I_\nu^2(zx)$$