Atomtronics, Benasque, 8-19 May 2017

SUPERFLUIDITY OF A SPIN-ORBIT COUPLED BEC GAS

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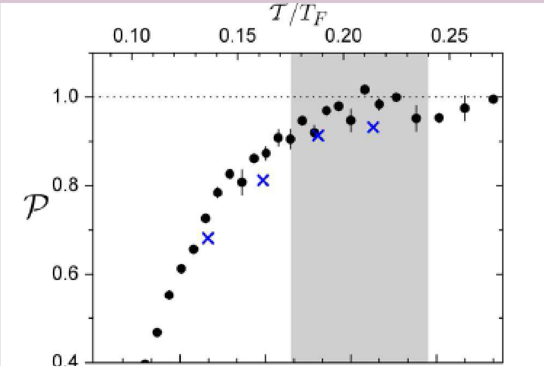
PROVINCIA AUTONOMA DI TRENTO

IRROTATIONALITY OF VELOCITY FIELD IS DISTINCTIVE FEATURE OF ROTATING SUPERFLUIDS

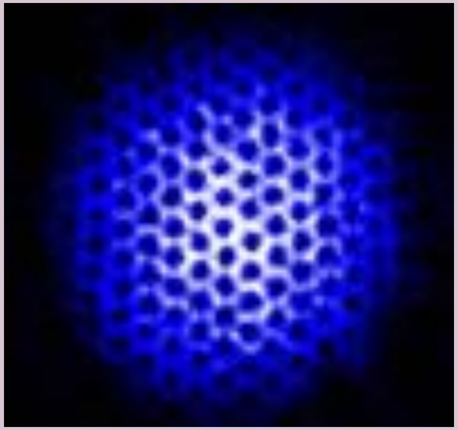
$$\vec{v}(\vec{r}) = \frac{\hbar}{m} \nabla \phi(\vec{r})$$

Role of the phase $\phi(\vec{r})$ of the order parameter

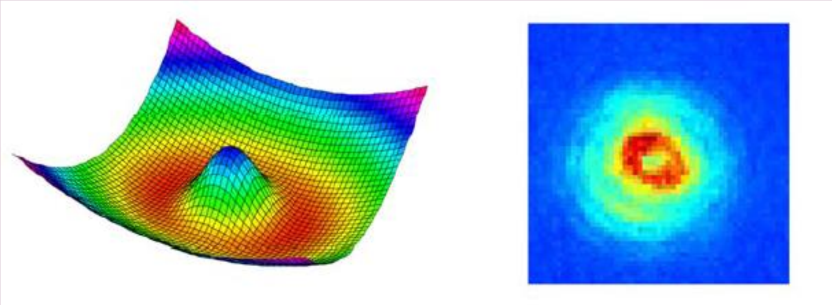


QUENCHING OF MOMENT OF INERTIA
(Innsbruck 2011)

- **Quantization of vortices**
  (ENS, JILA, MIT, 2000)



Persistent currents and Quantization of circulation



(Nist 2007)

Main message of this talk:

Spin-orbit coupling deeply affects the superfluid and rotational properties of Bose-Einstein condensed gases

Current-phase relation (yielding **irrotational** flow)

$$\vec{j}(\vec{r}) = \frac{\hbar}{m} n(\vec{r}) \vec{\nabla} \phi(\vec{r})$$

is violated by spin-orbit coupling, causing the emergence of diffused vorticity

$$\nabla \times \vec{v} \neq 0$$

Questions addressed in this talk

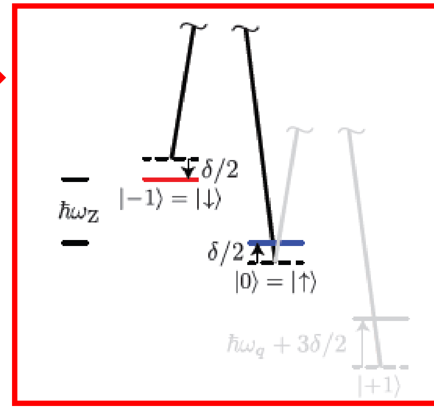
- Calculation of **superfluid density** in uniform matter and its quenching caused by spin-orbit coupling

- **Violation** of **irrotationality** constraint and emergence of **diffused vorticity:** consequences on **moment of inertia** and **quantum of circulation**

- **Anisotropic** expansion from **isotropic** trap

Raman induced 1D spin-orbit Hamiltonian Spielman (Nist 2009)

Two detuned and **polarized** laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian (assumed zero effective detuning)



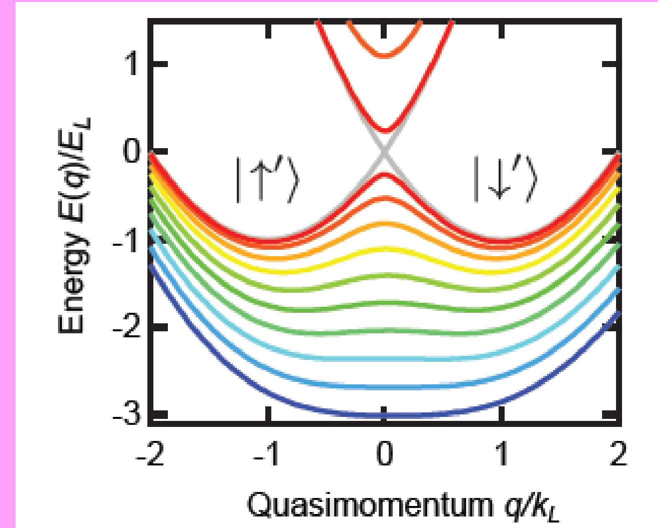
$$h_0 = \frac{1}{2m}[\left(p_x - \hbar k_0 \sigma_z\right)^2 + p_\perp^2] - \frac{\hbar}{2}\Omega\sigma_x$$

$p_x = -i\hbar\partial_x$ is canonical momentum

k_0 is laser wave vector difference

Ω Raman coupling, fixed by laser intensity

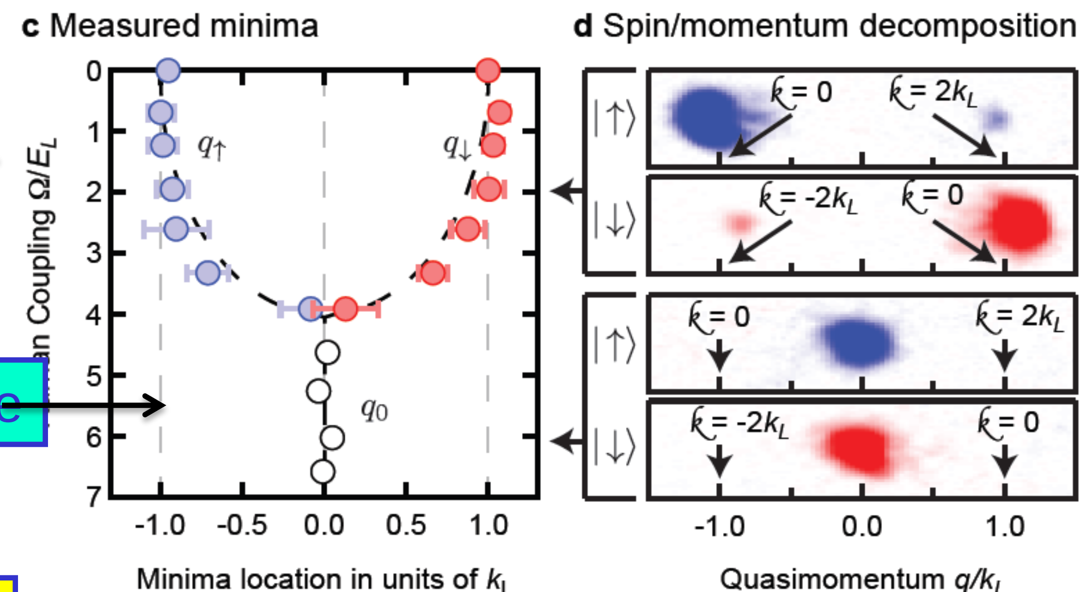
Single particle Hamiltonian gives rise to two band structure



- If $\Omega < 2\hbar k_0^2 / m$ lowest band exhibits **two degenerate minima** which can host a BEC
- If $\Omega > 2\hbar k_0^2 / m$ lowest band exhibits **single-minimum**

Plane wave phase

Single momentum phase



Second order phase transition at $\Omega = 2\hbar k_0^2 / m$

(Lin et al. Nature 2011)

Spin orbit Hamiltonian

$$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] - \frac{1}{2}\Omega\sigma_x$$

is translationally invariant.

However it breaks Galilean invariance, since physical momentum $(p_x - \hbar k_0 \sigma_z)$ **does not commute with** h_0 **.**

New SOC Hamiltonian yields result

$$j_x(\vec{r}) = \frac{\hbar}{m}[n(\vec{r})\nabla_x\phi(\vec{r}) - k_0 s_z(\vec{r})]$$

For the current and can yield violation of irrotational constraint $\vec{v}(\vec{r}) = \vec{j}(\vec{r})/n(\vec{r})$ **for velocity field.**

Violation of Galilean invariance raises the question of breakdown of superfluidity and of consequences on the rotational properties

Definition of superfluid density

Normal ($\rho_n = \rho - \rho_s$) density

$$\frac{\rho_n}{\rho} = Q^{-1} \lim_{q \to 0} \sum_{m,n} e^{-\beta E_m} \frac{|< m \,|\, J_x^T(q) \,|\, n >|^2}{E_n - E_m} + (q \to -q)$$

is defined by static response to transverse current operator

$J_x^T(q) = \sum_k P_{k,x} e^{iqy_k}$ (Baym 1969)

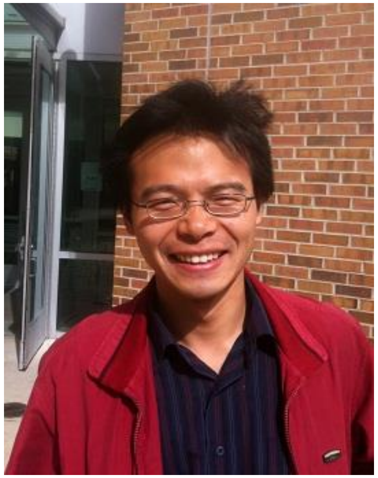
At T=0 normal density vanishes in Galilean invariant superfluids (liquid Helium, usual BEC and Fermi gases) and **system is fully superfluid** ($\rho_s / \rho = 1$)

Superfluid density differs from **BEC condensation**

For example condensate fraction n_0 / n in superfluid He4 is only 8% at T=0

Suppression of superfluidity in SOC Bose-Einstein condesed gases

Collaboration with Lev Pitaevskii (Trento)
and Shizhong Zhang + collaborators (Hong Kong)
Yi-Cai Zhang et al., Phys. Rev. A 94, 033635 (2016)



To calculate normal density $\rho_n = \rho - \rho_s$ at T=0

$$\frac{\rho_n}{\rho} = \frac{1}{N} \lim_{q \to 0} \sum_n \frac{|<0| J_x^T(q) | n>|^2}{E_n - E_0} + (q \to -q)$$

$$J_x^T(q) = \sum_k (p_{k,x} - \hbar k_0 \sigma_{k,z}) e^{iqy_k}$$

one needs knowledge of spectrum of elementary excitations

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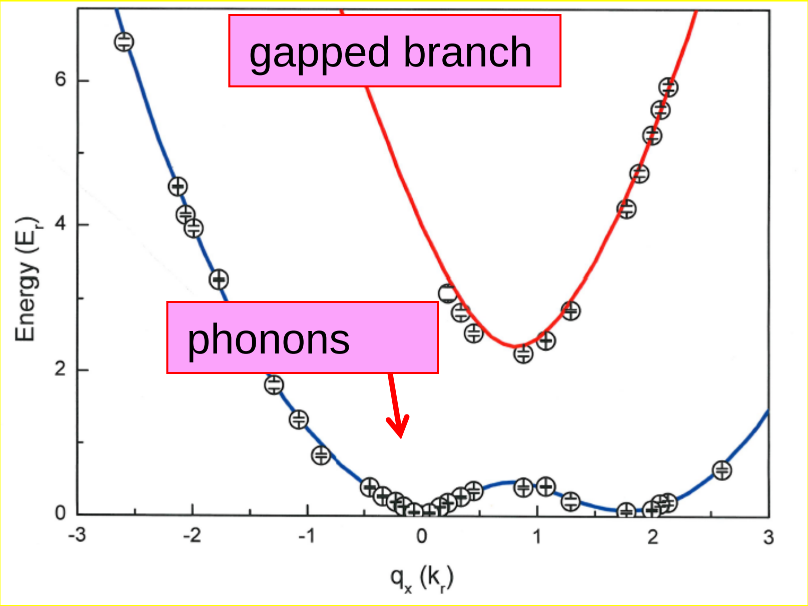
one needs knowledge of spectrum of elementary excitations

Spinor BEC's exhibit two branches in the excitation spectrum

Due to Raman coupling **only one branch is gapless** and exhibits phonon behavior at small q

Exp: Si-Cong Ji et al., PRL 2015; Khamehchi et al, PRA 2014
Theory: Martone et al., PRA 2012



Results for normal density in plane wave and single momentum phase

Plane wave phase

$$\Omega \le \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{\Omega^2}{\Omega_c^2}$$

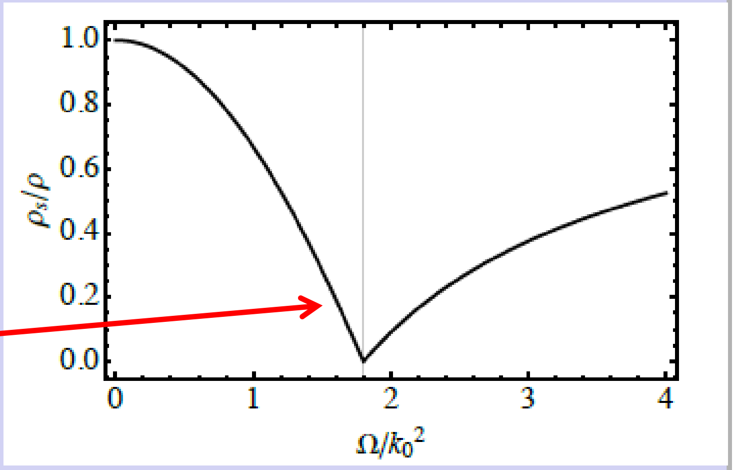
Single momentum phase

$$\Omega \ge \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{\Omega_c}{\Omega}$$

$$\Omega_C = 2\hbar k_0^2 / m$$

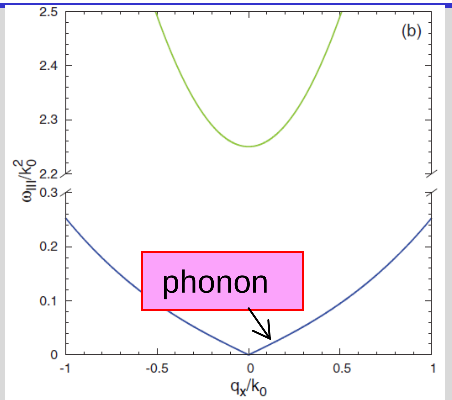
At the transition between the two phases one finds $\rho_n = \rho$ and superfluid density $\rho_S = \rho - \rho_n$ identically vanishes despite almost full BEC !!



parameters of Rb87

CAN WE MEASURE ρ_s ?

Differently from Galilean invariant systems,



f-sum rule

$$\hbar^2 \int d\omega \omega S(q,\omega) = \hbar^2 \frac{q^2}{m}$$

is **not exhausted** by phonon branch ($\omega = cq$).
Contribution of **phonon** branch fixed by superfluid fraction ρ_s / ρ

Phonon branch instead exhausts compressibility sum rule

$$\lim_{q \to 0} \int \frac{d\omega}{\omega} S(q,\omega) = \kappa$$

Macroscopic relationship

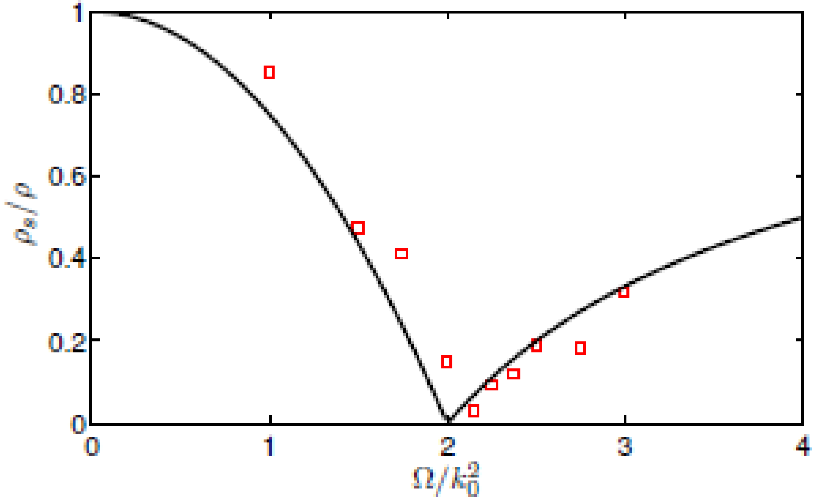
$$\rho_s = \rho m c^2 \kappa$$

holding at T=0

between **superfluid** density, **sound** velocity and **compressibility**.

- Equation of state (and hence compressibility) is not modified by SOC. **Velocity** of **sound** instead **exhibits strong suppression** (measured by Si-Cong ji et al., PRL 2015)

Superfluid density as a function of Raman coupling



$$\rho_s = \rho m c^2 \kappa$$

- Full line is prediction of theory

- Exp points are obtained using measured values of sound velocity

Consequences of reduced superfluidity on the rotational properties of a spin-orbit coupled BEC

Moment of inertia in the presence of spin-orbit coupling

(S.S., PRL 118 145302 (2017))

Moment of inertia is **linear response** to static **angular momentum** constraint

$$< L_z >_{\omega_{rot} \to 0} = \omega_{rot} \theta$$

$$H \to H - \omega_{rot} L_z$$

In Bose-Einstein condensates without **spin-orbit coupling** the rotational constraint induces irrotational velocity field of the form $\vec{v} \propto \vec{\nabla} xy$ yielding the **irrotational** value for the moment of inertia:

$$\theta_{irr} = \delta^2 \theta_{rig}$$

$\delta = \dfrac{<(x^2 - y^2)>}{<(x^2 + y^2)>}$ is deformation of the atomic cloud

$\theta_{rig} = N <(x^2 + y^2)>$ is rigid value of moment of inertia

For axi-symmetric trapping **moment of inertia vanishes at zero temperature** (effect of superfluidity)

To calculate moment of inertia in a SOC system a useful description is provided by **hydrodynamic formalism**

For small angular velocities the **relative phase** of the order parameter of the two spin components is **locked** because of the gap caused by Raman coupling

$$\phi_1(\vec{r},t)=\phi_2(\vec{r},t)\equiv\phi(\vec{r},t)$$

The order parameter of s=1/2 spinor takes the form

$$\Psi(\vec{r},t)=\begin{pmatrix}\sqrt{n_1(\vec{r},t)}\\ \sqrt{n_2(\vec{r},t)}\end{pmatrix}e^{i\phi(\vec{r},t)}$$

and one should look for equations for total density (n), spin density (s_z) and the phase (ϕ)

Equations of **spinor hydrodynamics** (Martone PRA12)

$$\frac{\partial}{\partial t}n+\frac{\hbar}{m}\nabla\cdot(n\nabla\phi)-\frac{\hbar}{m}k_0\nabla_x s_z=0$$

$$\hbar\frac{\partial}{\partial t}\phi+gn+V_{ho}-\mu=0$$

$$-\frac{\hbar}{m}k_0 n\nabla_x\phi+\Omega s_z/2=0$$

hold for low frequency dynamics

- **Equation of continuity affected** by SOC (new definition of the current: **spin** contribution

- **Equation for the phase** is not affected by SOC (EoS and density profiles are **unaffected** by SOC)

- **New relationhip** between **spin density** and **gradient** of the **phase**

In the presence of spin-orbit coupling angular momentum takes **additional spin** contribution

$$\mathrm{L_z} = \sum_k [x_k p_{k,y} - y(p_{k,x} - k_0 \sigma_{k,z})]$$

and HD equations at equilibrium take the form

$$\nabla \cdot [n(\nabla \phi - \vec{\omega}_{rot} \times \vec{r})] - k_0 \nabla_x s_z = 0$$

$$-\frac{\hbar k_0}{m} \nabla_x \phi + \frac{1}{2} \Omega \frac{s_z}{n} - \omega_{rot} k_0 y = 0$$

For isotropic trapping HD eqs. admit solution

$$\phi = \alpha xy$$

$$k_0 s_z = 2\alpha yn$$

$$\alpha = \frac{\omega_{rot} k_0^2}{\Omega - \Omega_{cr}/2}$$

where $\Omega_{cr} = 2\hbar \frac{k_0^2}{m}$

Behavior of velocity field

$$\vec{v} = \frac{\hbar}{m}\nabla\phi - \frac{\hbar}{m}\vec{k}_0 \frac{s_z}{n}$$

$$v_x = \frac{\hbar}{m}\nabla_x\phi - \frac{\hbar}{m}k_0 \frac{s_z}{n} = \frac{\hbar}{m}(\alpha y - 2\alpha y) = -\frac{\hbar}{m}\alpha y$$

$$v_y = \frac{\hbar}{m}\nabla_y\phi = \frac{\hbar}{m}\alpha x$$

Behavior of velocity field

$$\vec{v} = \frac{\hbar}{m}\nabla\phi - \frac{\hbar}{m}\vec{k}_0\frac{s_z}{n}$$

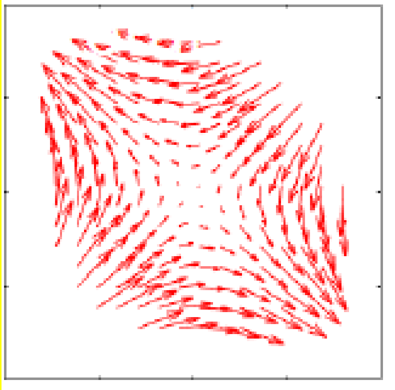
$$v_x = \frac{\hbar}{m}\nabla_x\phi - \frac{\hbar}{m}k_0\frac{s_z}{n} = \frac{\hbar}{m}(\alpha y - 2\alpha y) = -\frac{\hbar}{m}\alpha y$$

$$v_y = \frac{\hbar}{m}\nabla_y\phi \quad = \frac{\hbar}{m}\alpha x$$

$$\vec{v} = \vec{\omega}_{rot} \times \vec{r}$$

at $\Omega = \Omega_{cr}$

$$\vec{v} = \frac{\hbar}{m}\nabla\phi$$



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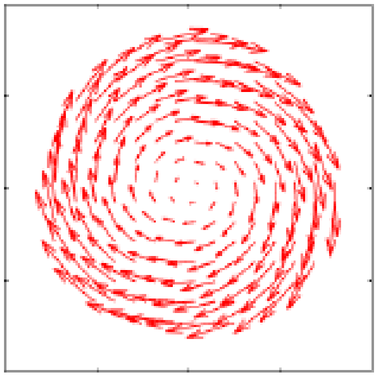
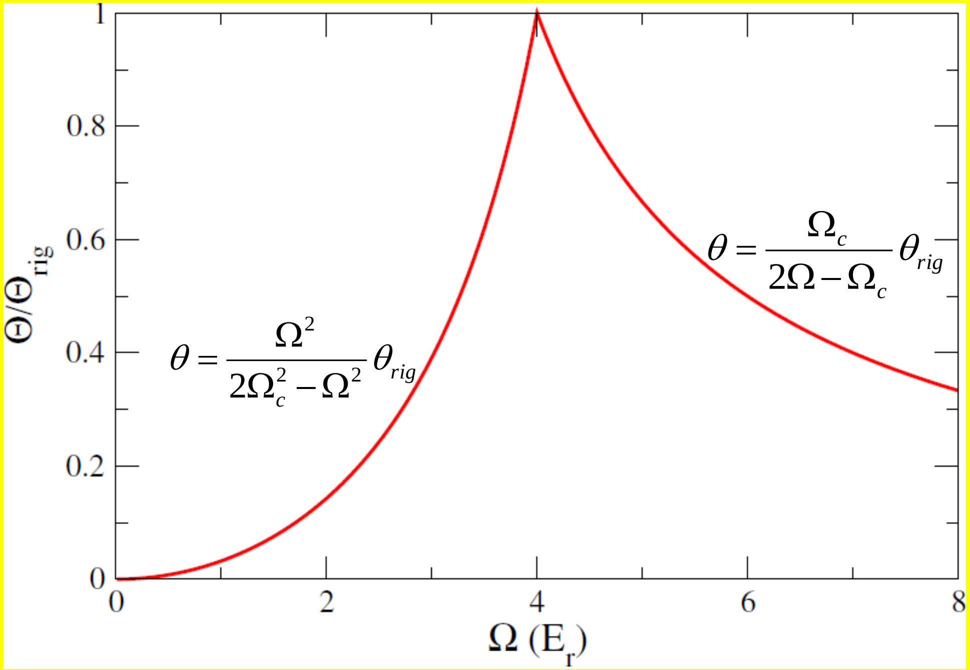


Behavior of moment of inertia

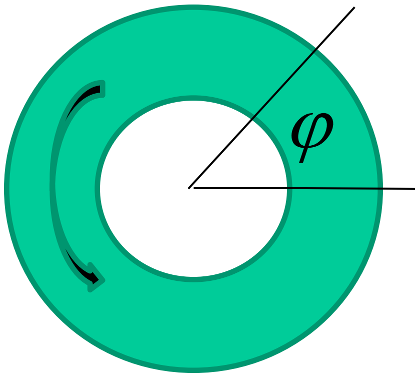


Rigid value $\theta = \theta_{rig}$ at the transition between plane wave and single momentum phase.

Dramatic consequence of SOC

Effect of spin-orbit coupling on the quantization of circulation in toroidal configurations

Using **Laguerre-Gauss laser beams** transferring angular momentum $\hbar l$ it is possible to generate **spin-orbit coupling** in **toroidal** configuration.



In the absence of SOC the quantum of circulation is determined in units of

$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{\mathrm{m}}$$

Persistent currents and quanta of circulation already **measured** in BECs confined in **toroidal traps** (Nist 2013)

SOC solution in the **ring** can be **mapped** into **1D spin orbit problem** with **periodic** boundary conditions

Relevant equation for the spin density

$$-n\frac{\hbar l}{mR^2}\partial_\varphi \phi + \frac{1}{2}\Omega s_z = 0$$

with the choice $\phi = \varphi$ (azimuthal angle)

yields new rule for **quantization of circulation**

$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{\mathrm{m}}\left(1 - \frac{\Omega_c}{\Omega}\right)$$

$$\Omega_c = 2\hbar l^2 / mR^2$$

SOC gives rise to reduction of quantum of circulation (reduction of angular momentum)

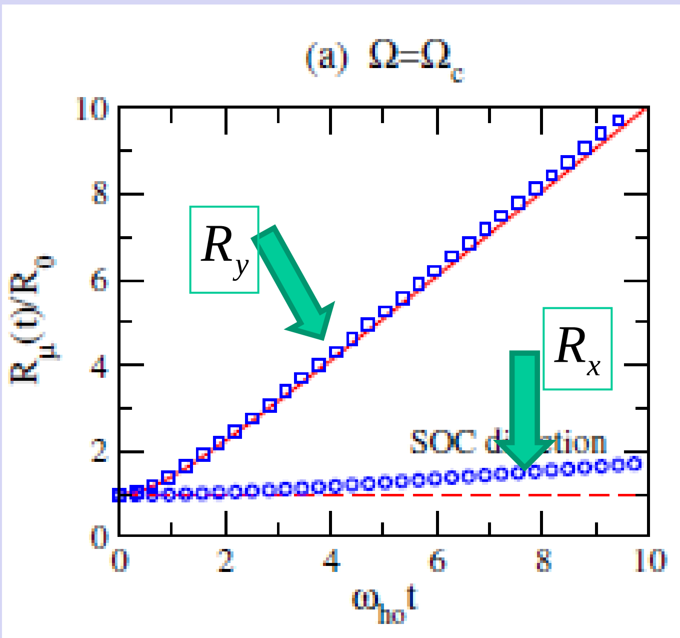
Anisotropic expansion from isotropic trap: consequence of quenched superfluidity

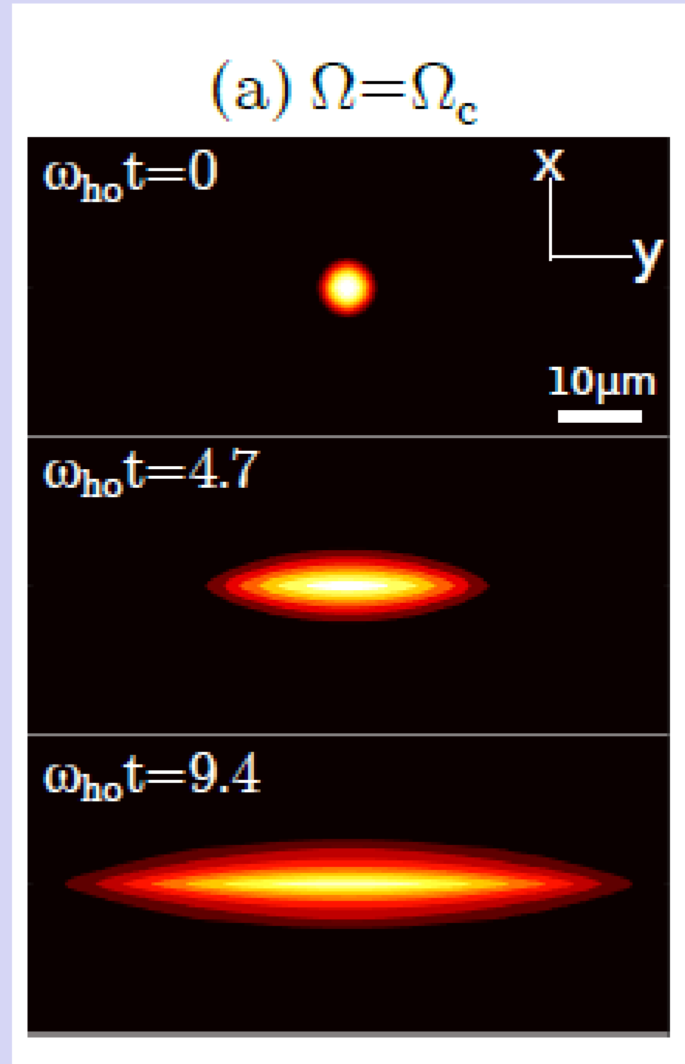
(Chunlei Qu, Lev Pitaevskii and SS. arXiv:1704.00677)

Quenching of superfluidity causes **slowing down of expansion** along the direction of spin-orbit coupling

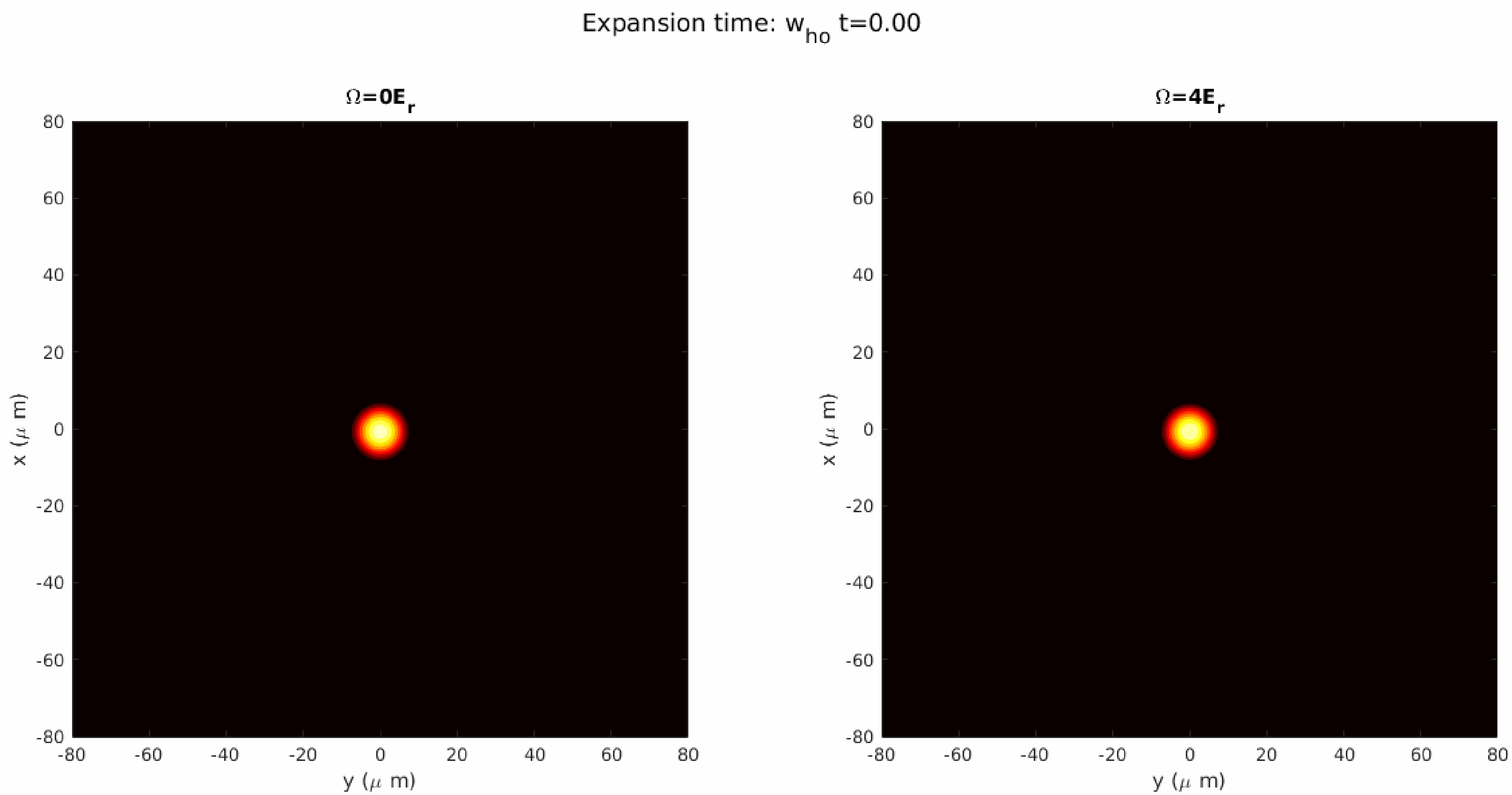
Behavior of anisotropic expansion at $\Omega = \Omega_{cr}$

Quenching of superfluid flow along the spin-orbit direction (x-axis)





Gross-Pitaevskii simulation

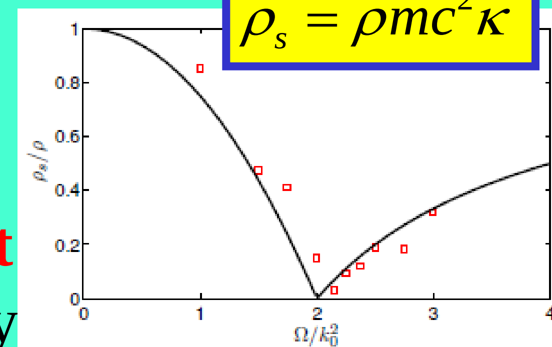


Expansion time: w
ho
t=0.00
Ω=0E
r
Ω=4E
r
80
60
40
20
0
-20
-40
-60
-80
x (μ m)
y (μ m)

MAIN CONCLUSIONS

- Spin-orbit coupling has deep consequence on **superfluid** behavior of a BEC (consequence of **breaking of Galilean** invariance)
  Strong quenching of superfluidity at $\Omega = \Omega_c$

$$\rho_s = \rho m c^2 \kappa$$



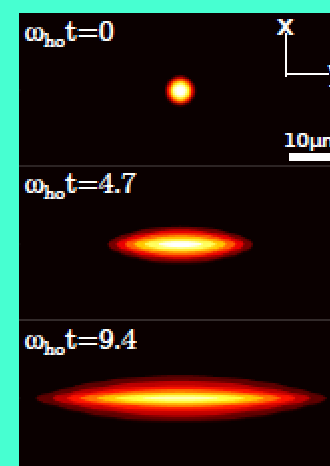
- superfluid density extracted from **measurement** of **sound velocity** : good **agreement** with theory

- Velocity field no longer constrained by irrotationality
  Moment of inertia acquires rigid value at $\Omega = \Omega_{cr}$

- **Quantization of circulation** is affected by SOC

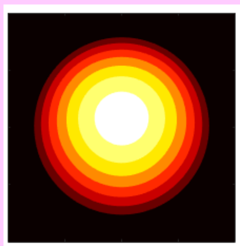
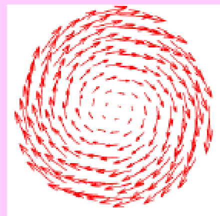
$$\oint d\vec{l} \cdot \vec{v}(\vec{r}) = \frac{h}{\mathrm{m}}\left(1 - \frac{\Omega_c}{\Omega}\right)$$

- **Anistropic expansion** from **isotropic trap**

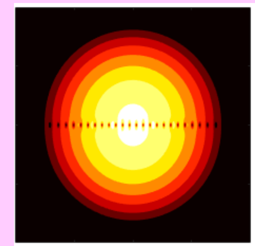
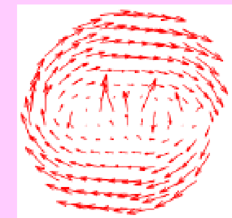


SOME RUNNING PROJECTS

- **Transition from rigid rotation to formation of quantized vortices (collaboration with Chunlei Qu)**

small angular velocity

larger angular velocity

- **Quantum and thermal fluctuations (phase and spin)**

- **Superfluidity of Rashba Hamiltonian (SOC with non Abelian gauge fields)**

$$h_0 = \frac{1}{2m}[\left(p_x - \hbar k_x \sigma_x\right)^2 + \left(p_y - \hbar k_y \sigma_y\right)^2] + \frac{1}{2m} p_z^2$$