

Technical Optics with Matter Waves

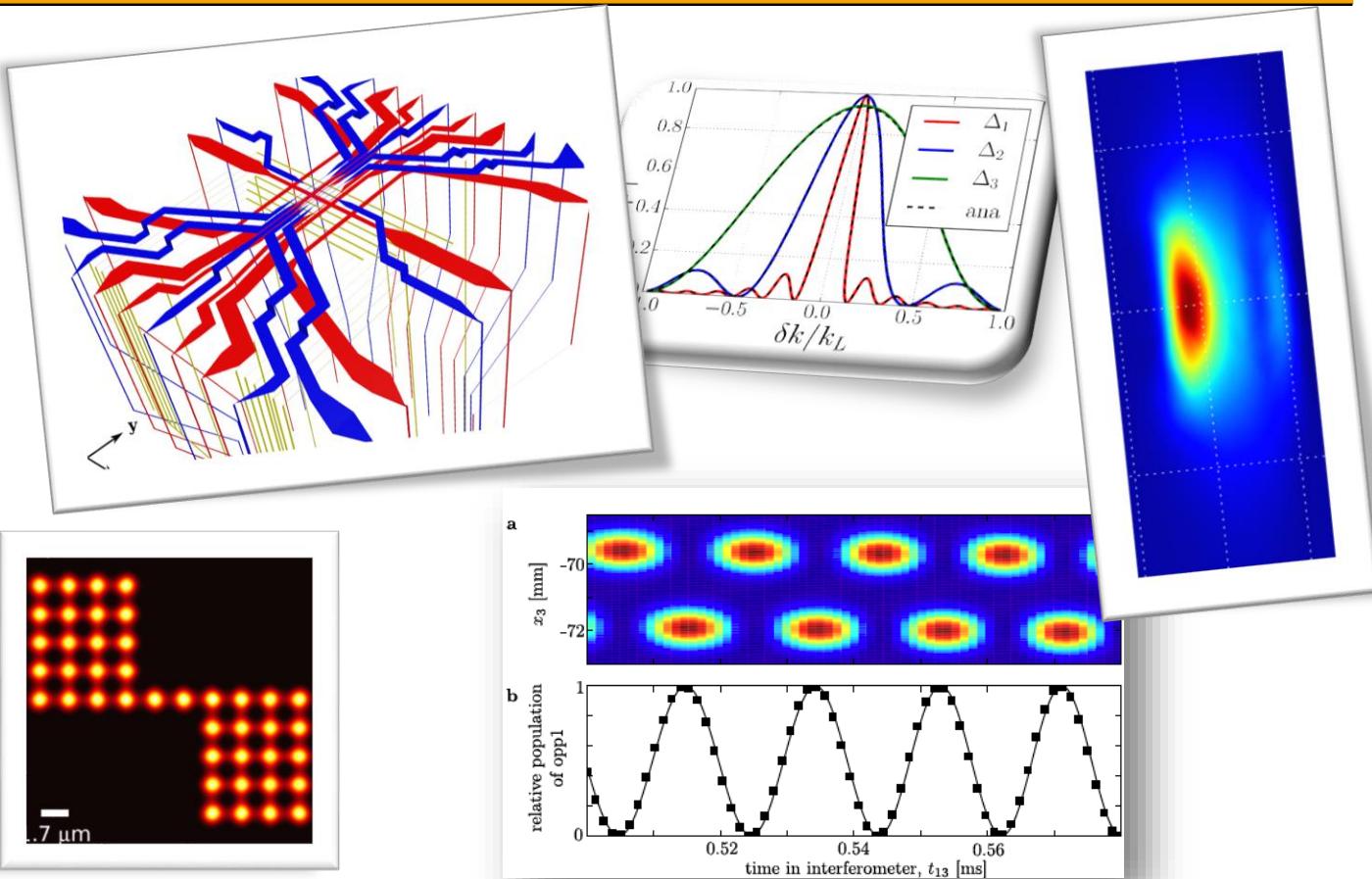
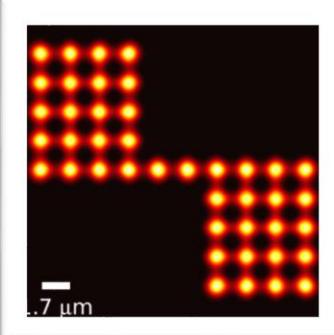
- geometric, thermal, coherent & quantum



M. Sturm
J. Teske
A. Neumann
J. Battenberg
R. Walser



Collaboration
M. Schlosser
G. Birk



History of success: transistor → IC

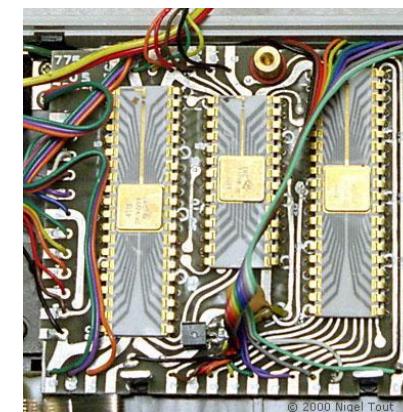


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1947 invention of **transistor**,
J. Bardeen, W. Brattain, W. Shockley
@ Bell Labs

1958 invention of **IC Jack Kilby**
working at **Texas Instruments**

- Nobelprize 2000 ½
- **Z. Alferov** ¼ invention of semiconductor heterostructures
- **H. Kroemer** ¼ sc-hs opto-electronics



- http://www.ti.com/corp/docs/webemail/2008/enewsltr/public-affairs/graphics/Jack_Kilby300.jpg
- http://www.vintagecalculators.com/assets/images/CanonPocketronic_1.JPG
- http://www.vintagecalculators.com/assets/images/CanonPocketronic_7.jpg

Atomic gas hardware \leftrightarrow oxymoron?



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Either: Experiments q-gases **model systems** in well controlled lab environments ($N > 0$ students)

Or: quantum technology (EU Flagship program),
q-manifesto (T. Calarco)

➤ **Applications:** q-sensing, q-metrology, q-computing

➤ **Robustness:** mechanical structures

- **atomic chips** (J. Schmiedmayer, R. Folman, C. Zimmermann, J. Fortagh, J. Reichel, T. Hänsch, M. Prentiss, P. Treutlein, E. Hinds) lithography, etching

**Fifteen years of cold matter on the atom chip: promise, realizations, and prospects*
M. Keil et al., JMO, 63, 1840 (2016)

- **micro-lens arrays** (G. Birkl) optical elements, e-beam lithography
- **3D printing** (H. Giessen, micro-lenses on q-dots)

➤ **Miniaturization, reproducability, reliability, cost (UHV)**

➤ **Hybridization: superconductivity, cryo, optical, rf, ...**



Cold gases in μ -g



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University of Hannover



Ernst M. Rasel
Wolfgang Ertmer

ZARM Bremen



Hansjörg Dittus
Claus Lämmerzahl

University of Berlin



Achim Peters

University of Hamburg



Klaus Sengstock



ulm university

universität



Wolfgang P. Schleich

Financial support by the

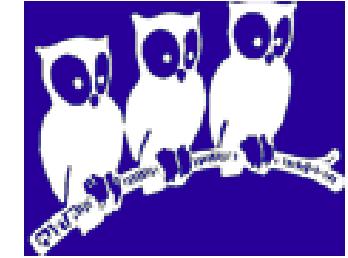


Ferdinand Braun Institute



German Space Agency

MPQ Munich and
Laboratoire Kastler Brossel



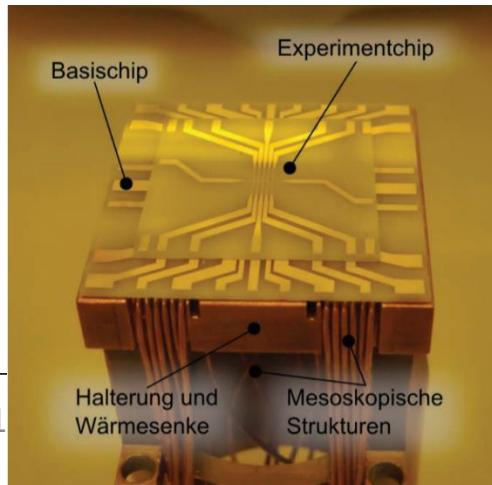
Theodor W. Hänsch
Jakob Reichel

First BEC in space

23.1.2017 BEC MAIUS (S. Seidel) rocket
launched Kiruna, reached 243km,
85 experiments



Lift off at Esrange



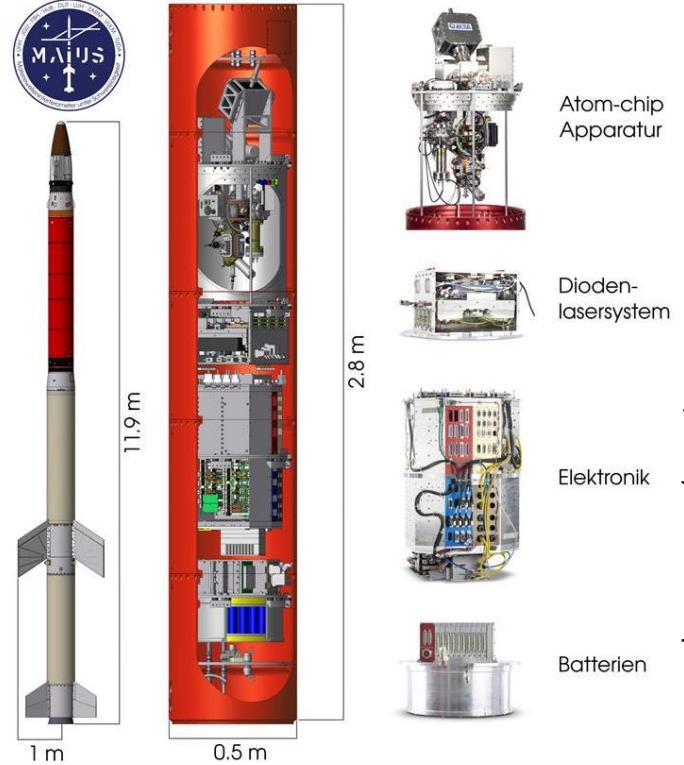
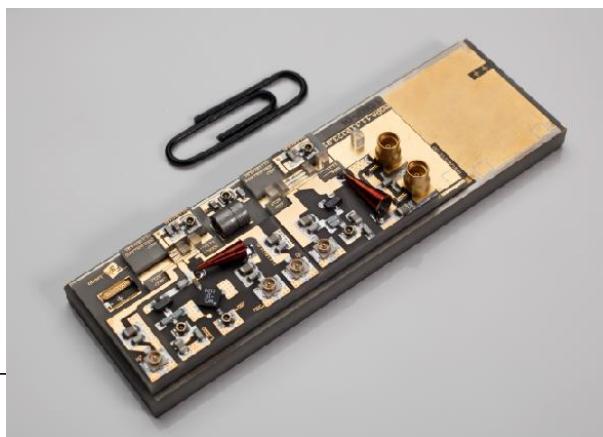
Atomic chip

17_05_1

Benasque

http://www.dlr.de/dlr/en/desktopdefault.aspx/tabcid-100081/151_read-20337/#/gallery/25194

MOPA, HU & Ferdinand-Braun
Institute, A. Peters, A. Wicht



5/41

Simulation of mw-devices



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- **Challenges:** 3D (dimension), large expansion times ($t > 2000\text{ms}$), and scales ($d > \text{mm}$), T (emperature) , \hbar (particle vs. waves), g (nonlinearity), (z^3) anharmonicity, time-dependence (dkc), noise

arXiv:1701.06789, G. Nandi et al., PRA, 76, 063617 (2007)

- **Toolbox mw-optics**

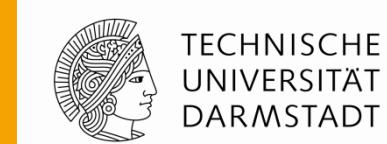
- Magnetic traps & lenses
- Designing quantum simulators with micro-lens arrays (M. Sturm, M. Schlosser, G. Birkl)
- Bragg beam-splitters

- **Methods & applications**

- Geometrical mw-optics: raytracing, aberrations
- Thermal mw-optics: 3D interferometry @ finite T
- Coherent mw-optics: delta-kick-collimation
- Quantum mw-optics: Josephson-Junction rings

mw-traps & lenses with magnetic chips

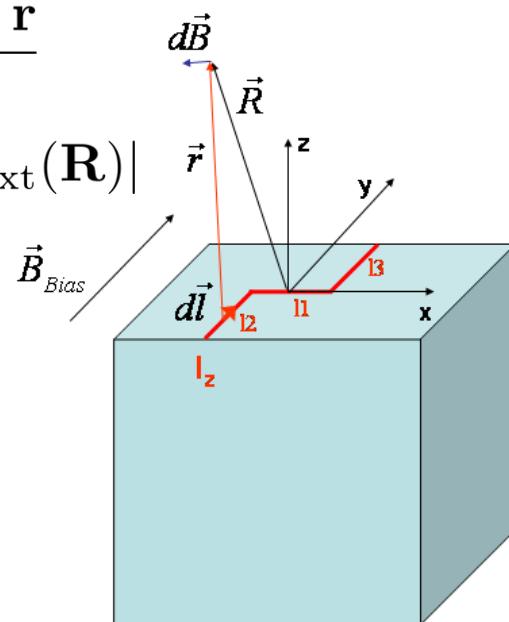
J. Battenberg



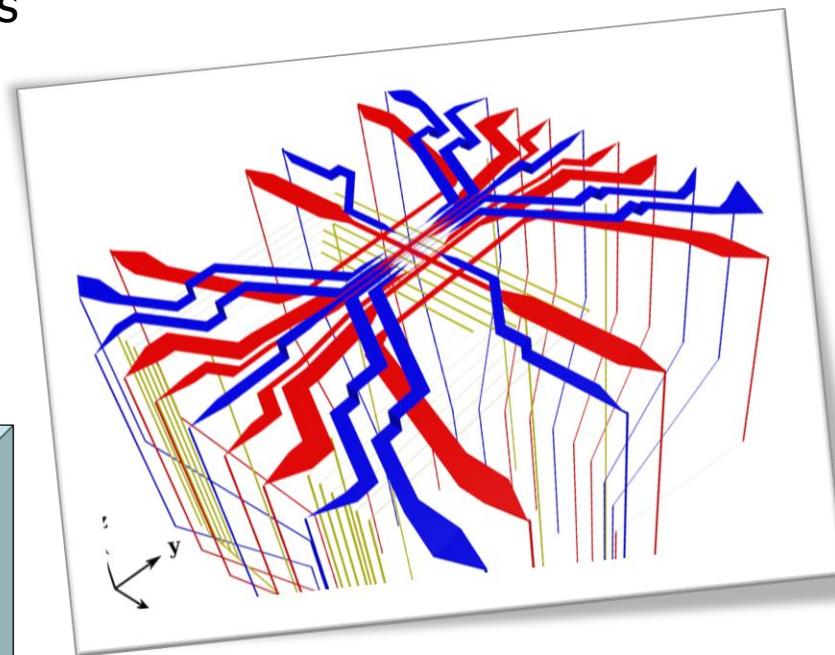
- geom. representation of 2D conducting strips
- Biot-Savart law

$$\mathbf{B}(\mathbf{R}) = \frac{\mu_0 I}{4\pi} \int_{\mathcal{C}} \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

$$V(\mathbf{R}) \propto |\mathbf{B}(\mathbf{R}) + \mathbf{B}_{\text{ext}}(\mathbf{R})|$$



- several active layers
- multi-dimensional
- current control

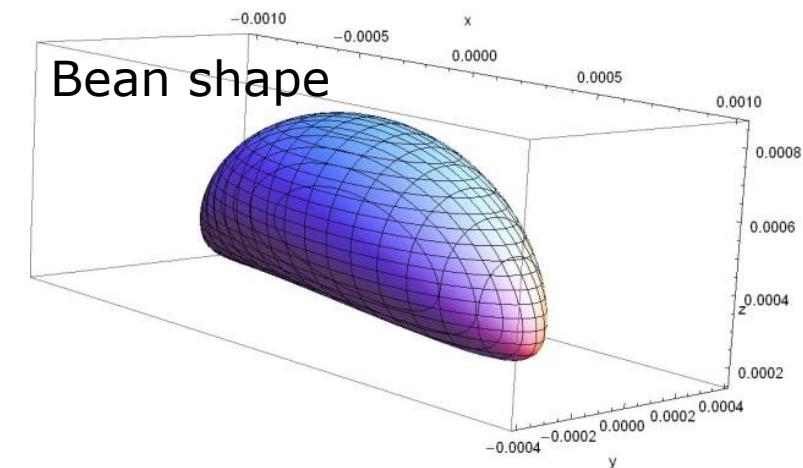
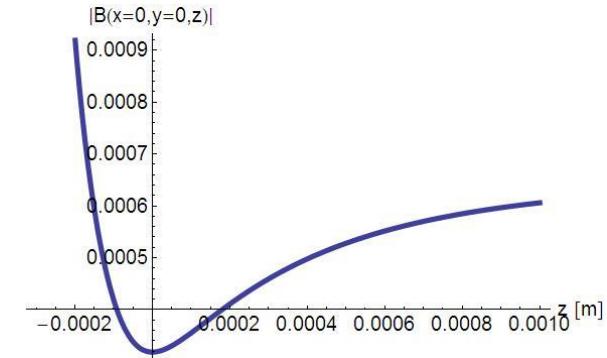
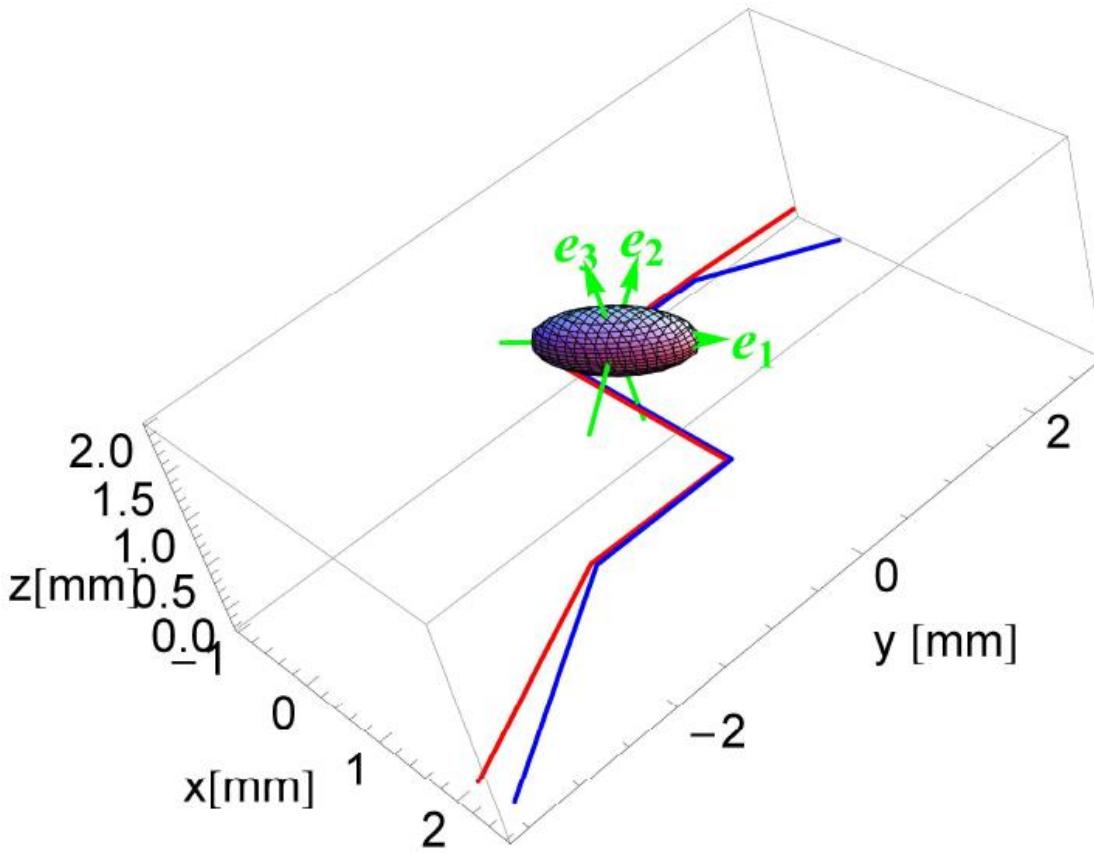


exp. chip design
W. Herr, LHU

Shape of magnetic potential



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QII frequency manifold



$\{(\nu_1, \nu_2, \nu_3) \in R^3 | (I_x, I_y, I_b, I_s) \in R^4 \text{ with feasible solution}\}$

Eigen-frequencies of potential Hess-matrix at potential minimum

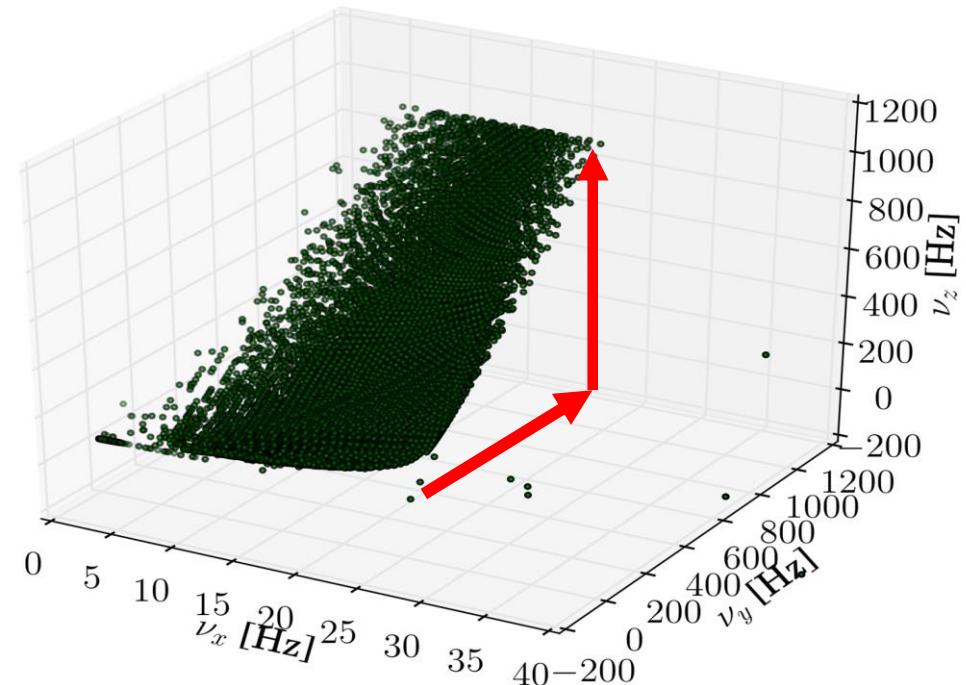
Feasible frequencies
2D planar manifold
Earnshaw theorem
magnetic shield

$$\mathbf{B}(\mathbf{R}) = -\nabla\phi_M$$

$$\Delta\phi_M = 0$$

$$\phi_M(\mathbf{r}) = \sum_{lm} \phi_{lm} R_{lm}(\mathbf{r})$$

Map of possible frequencies in current QII setup

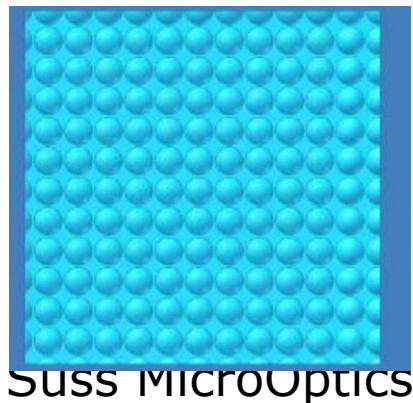


Adjustable optical microtraps arrays

Designing robust light fields
in the itinerant tunneling regime
M. Sturm, M. Schlosser, G. Birkl



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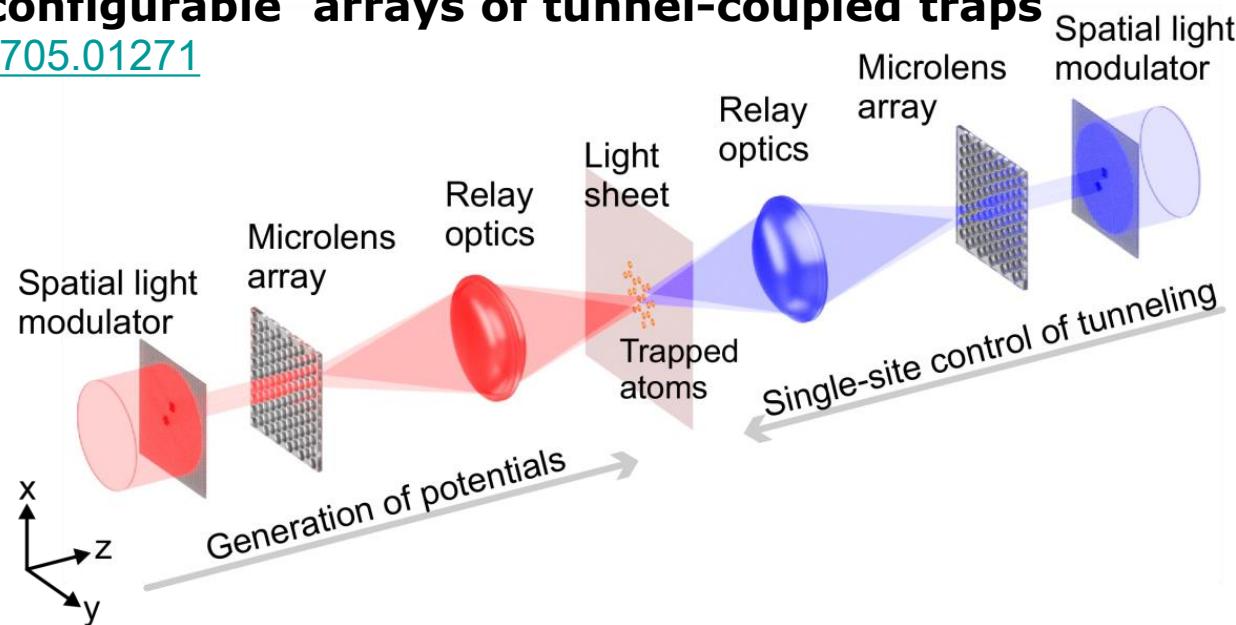


SUSS MICROOPTICS

Microlens arrays with spatial light modulators:
arbitrary arrays of microtraps

**Quantum simulators by design –many-body physics
in reconfigurable arrays of tunnel-coupled traps**

[arxiv:1705.01271](https://arxiv.org/abs/1705.01271)

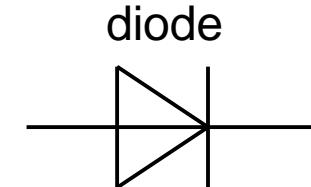
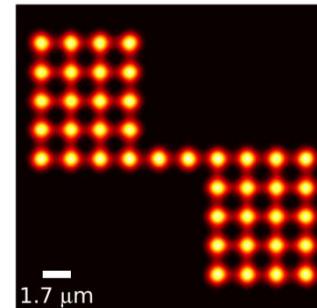


Configurations with tunneling

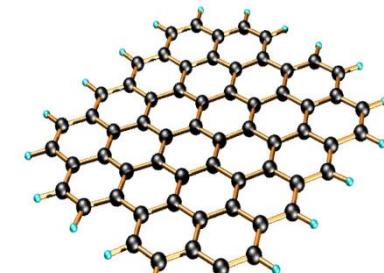
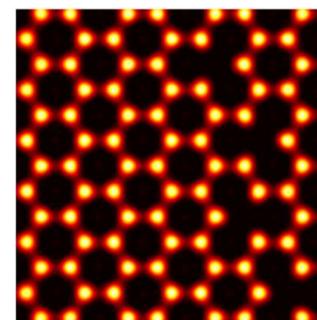


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- **Pinboard for atomtronics**
implement atomtronic devices
like diodes and transistors

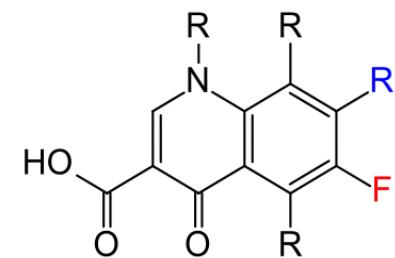
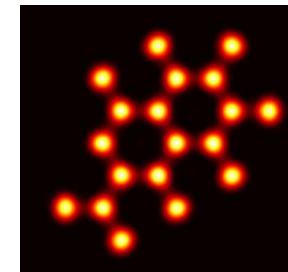


- **Designable lattices**
Exotic lattice geometries
quasi crystals
point/line defects
controllable disorder



Copyright: Chris Ewels

- **Molecular structures**
mimick electronic structure
of molecules



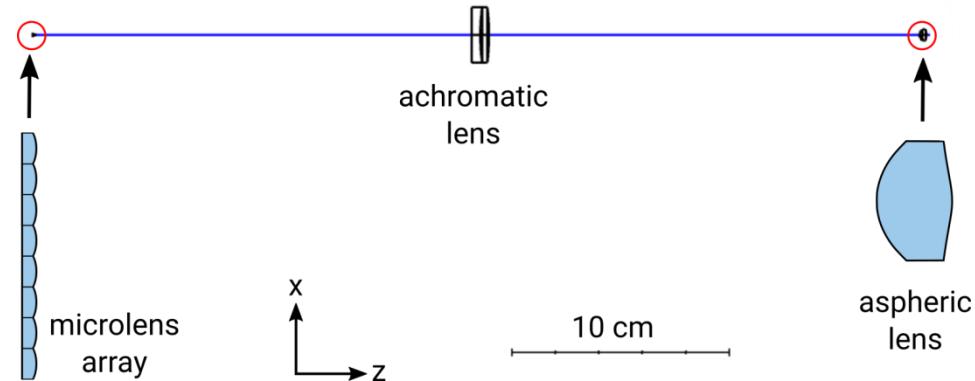
<https://en.wikipedia.org/wiki/Quinolone>

Simulation of the light field

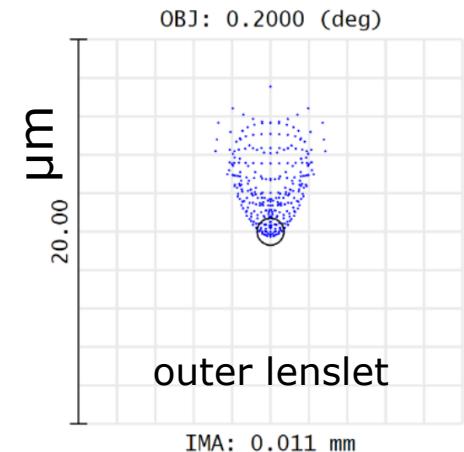
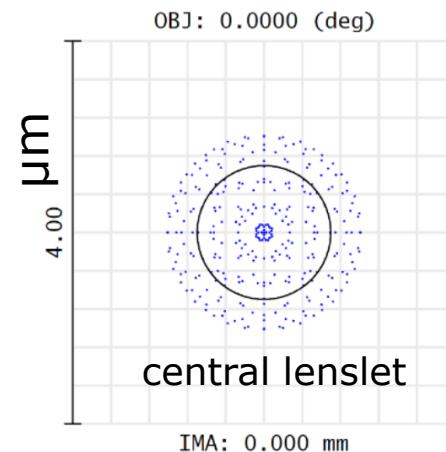
- Precise optical modeling using commercial software

- Microlens array
 - diameter of lenslets: 106 μm
 - $\sim 100 \times 100$ lenslets
 - ROC=2.65 mm

- Aspheric lens with NA=0.68
- Raytracing and wave optics



Spot size diagram



2D Optical potential +1D light sheet



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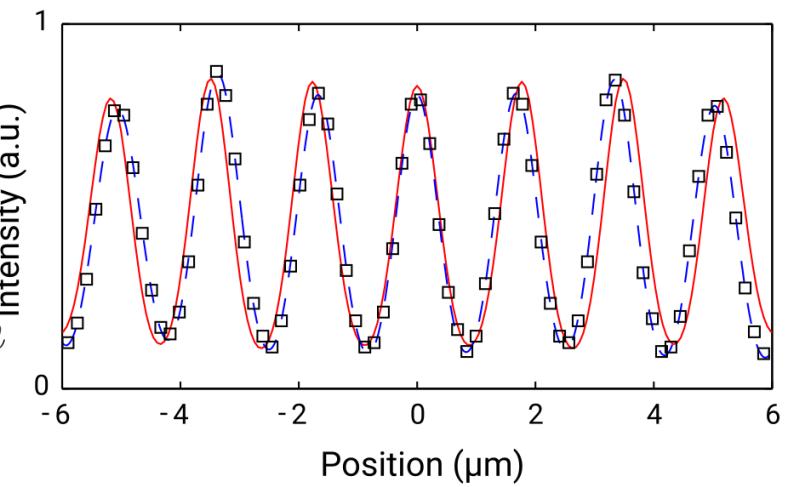
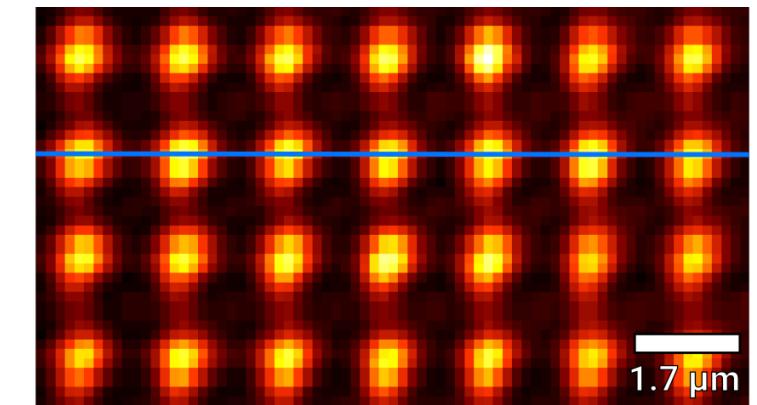
✓ Measured and simulated
intensity distributions agree

$$V(x, y, z) \approx V_{\perp}(x, y) + V_{\parallel}(z)$$

$$V_{\perp}(x, y) = - \sum_{\mathbf{R}_i} V_{0\perp}^{(i)} e^{-2 \frac{(x-X_i)^2 + (y-Y_i)^2}{w_{0\perp}^2}}$$

$$V_{\parallel}(z) = -V_{0\parallel} e^{-2z^2/w_{0\parallel}^2}$$

$$d = 1.7 \text{ } \mu m \quad w_{0\perp} = 0.7 \text{ } \mu m \quad w_{0\parallel} = 2.5 \text{ } \mu m$$



Measurement, Gaussian fit, simulation

Bose-Hubbard parameters



- Bose-Hubbard model using 2+1D Schrödinger equation

$$\hat{H} = U \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$

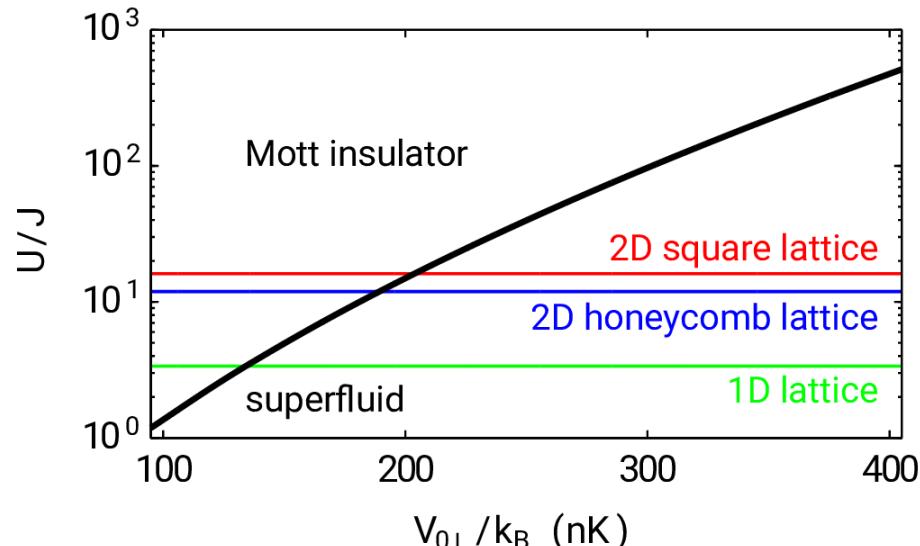
- Onsite interaction

$$U = \frac{4\pi a_s \hbar^2}{m} \int \varphi_i^4(x, y) \phi^4(z) d^3r$$

- Nearest-neighbour tunneling

$$J = \langle \varphi_i \phi | \hat{H}_1 | \varphi_j \phi \rangle$$

	Li 7	Na 23	K 41	Rb 87
J/h	18.6 Hz	3.5 Hz	2.3 Hz	1.5 Hz
U/h	186 Hz	35 Hz	23 Hz	15 Hz



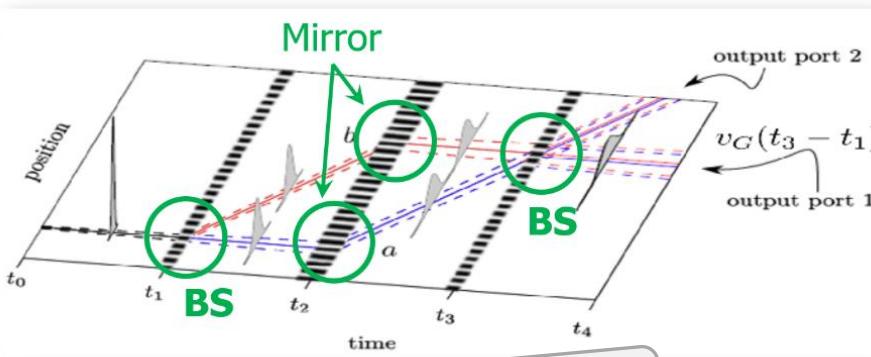
For Rubidium 87

mw-beam splitter

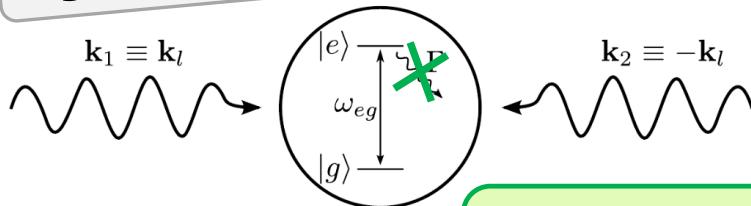
Velocity dispersion of optical Bragg beam splitter in 3D with temporal pulses,
spatial beams shapes, wavefront curvature
A. Neumann



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light-matter interaction



$$\omega_1 = \omega_2 \equiv \omega_l$$
$$\Delta = \omega_{eg} - \omega_l$$

large detuning:
 $\Delta = \omega_{eg} - \omega_l, |\Delta| \gg \Gamma$

BS goal:
coherently splitting motion
of atoms with **unit response**
and **wide momentum range**

BS: Bragg diffraction of atoms
by periodic grating
(optical standing wave)

$$\rightarrow i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$$

solve with split
operator method

Plane wave Bragg diffraction



$$\hat{H} = \frac{\hat{p}^2}{2M} \otimes \mathbb{1} + \frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{\hbar\Omega_0}{2}$$

$$\Psi(t) = e^{-i\hat{H}t}|\Psi(0)\rangle = \hat{U}|\Psi(0)\rangle$$

Loss into off resonant higher diffraction orders:

$$|\Psi(t)\rangle = \sum_{m=-N}^N g_m |g, m \cdot k_l\rangle + e_{m'} |e, m' \cdot k_l\rangle$$

m odd,
 m' even

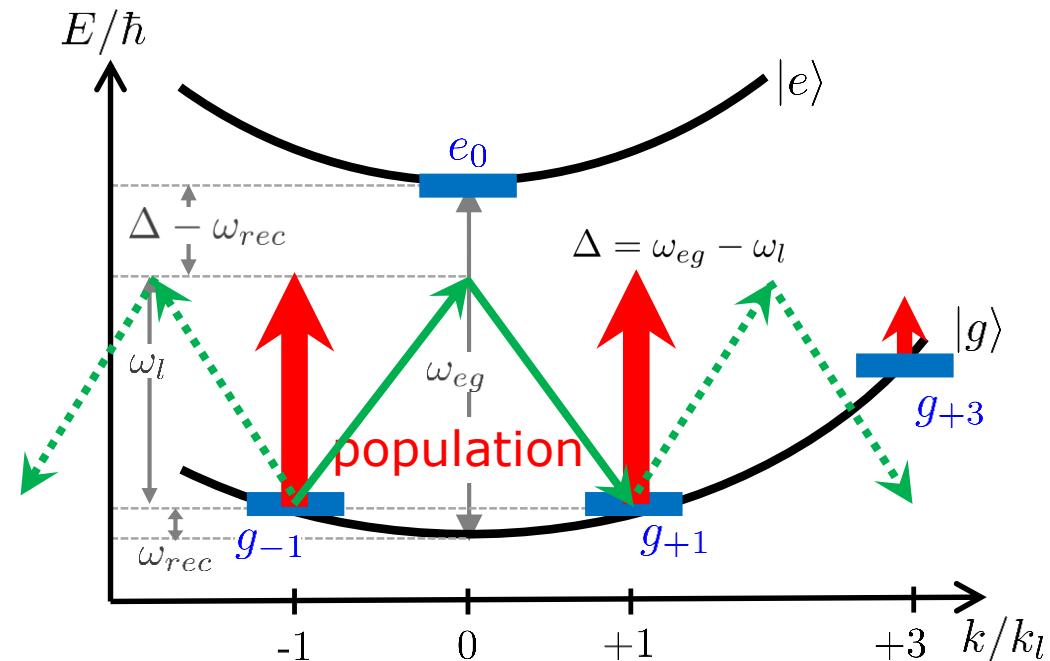
absorb / emit 1 photon:

$$e^{\pm i k_l \hat{x}} = \int dp |p \pm \hbar k_l\rangle \langle p|$$

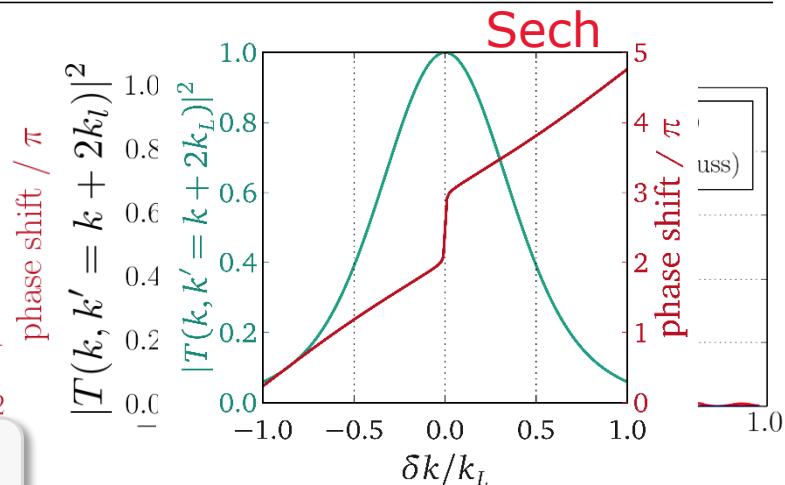
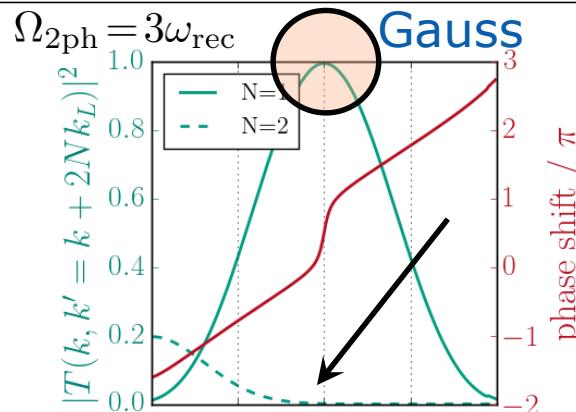
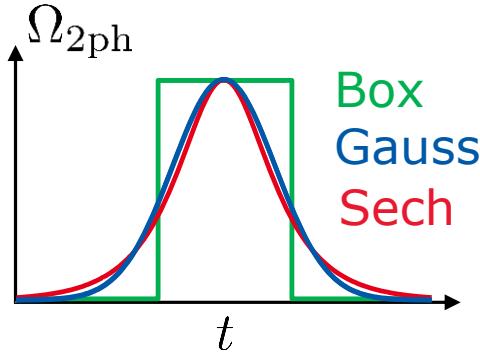
→ coupling:

$$|g, k\rangle \leftrightarrow |g, k \pm 2Nk_l\rangle$$

(Bragg order N)



Temporal envelopes



Analytic Demkov-Kunike model:
1D 1st order Bragg diffraction

$$\Omega_{2\text{ph}}(t) = \Omega_{2\text{ph}} \cdot \text{sech}\left(\sqrt{\frac{\pi}{2}} \frac{t}{\sigma}\right)$$

$$T_{kk'}^{\text{N}=1} = \frac{\sigma \Omega_{2\text{ph}} \sqrt{\text{sech}\left(\sqrt{\frac{\pi}{2}} \frac{t}{\sigma}\right)}}{i\sqrt{2\pi} - 8\omega_{\text{rec}}\delta k \sigma} \cdot {}_2F_1[a, b; c, z(t)]$$

$$\frac{a}{b} = 1 - \frac{\sigma \Omega_{2\text{ph}}}{\sqrt{2\pi}}, \quad c = \frac{3}{2} + 2i\sqrt{\frac{2}{\pi}}\omega_{\text{rec}}\delta k \sigma, \quad z(t) = \frac{1}{2} + \frac{\tanh\left(\sqrt{\frac{\pi}{2}} \frac{t}{\sigma}\right)}{2}$$

- ✓ suppressed side maxima
- ✓ insignificant population loss into higher diffraction orders
- ✓ Analytic model (DK) for 'sech'- pulses

Comparison of 1D simulation with experimental data*

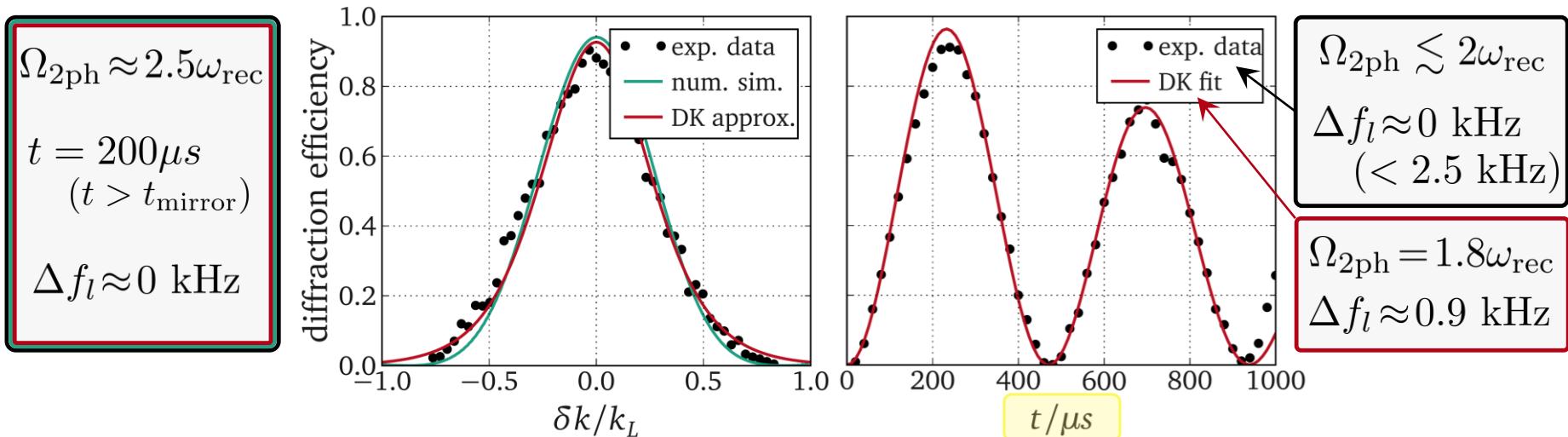


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- Laser:
- spatial dependence: \sim plane waves
 - temporal: Gaussian (+ fit with Demkov Kunike)
 - Laser frequency detuned to resonance Δf_l

* M.Gebbe (Universität Bremen, priv. com.)

Atoms: BEC @ 50 nK \sim Thomas Fermi (width in momentum space $\ll k_l$)



simulation + DK-approximation match experimental data

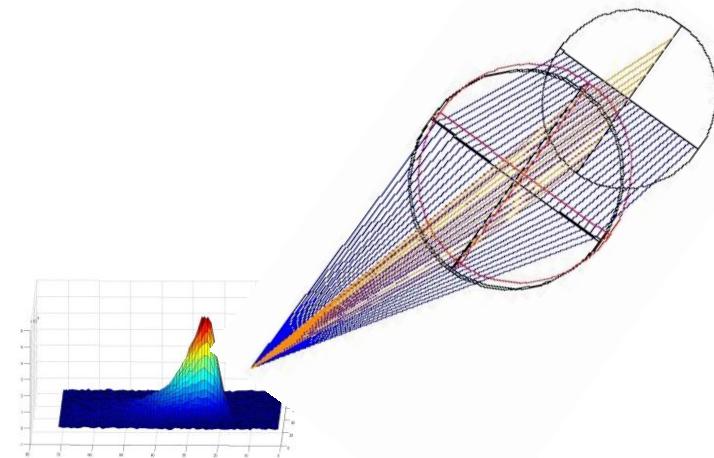
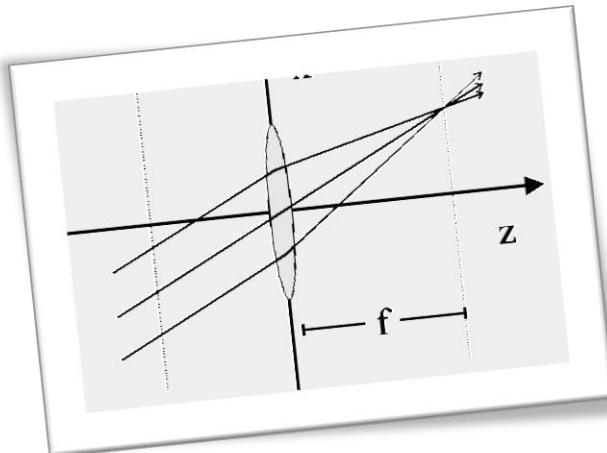
Application 1: delta-kick collimation

-geometric-, thermal-, coherent mw optics

B. Okhrimenko, J. Teske



Ray optics with light (2+1D) Matter wave optics (3+1D)



AMMANN, Hubert ; CHRISTENSEN, Nelson: Delta Kick Cooling:
A New Method for Cooling Atoms, *Phys. Rev. Lett.* 78,
2088 (1997)

Matter wave optics DKC-sequence:

$$t_{lens} = \frac{1}{\omega} \tan \left(\frac{1}{\omega t_1} \right)$$



DKC of thermal cloud: position

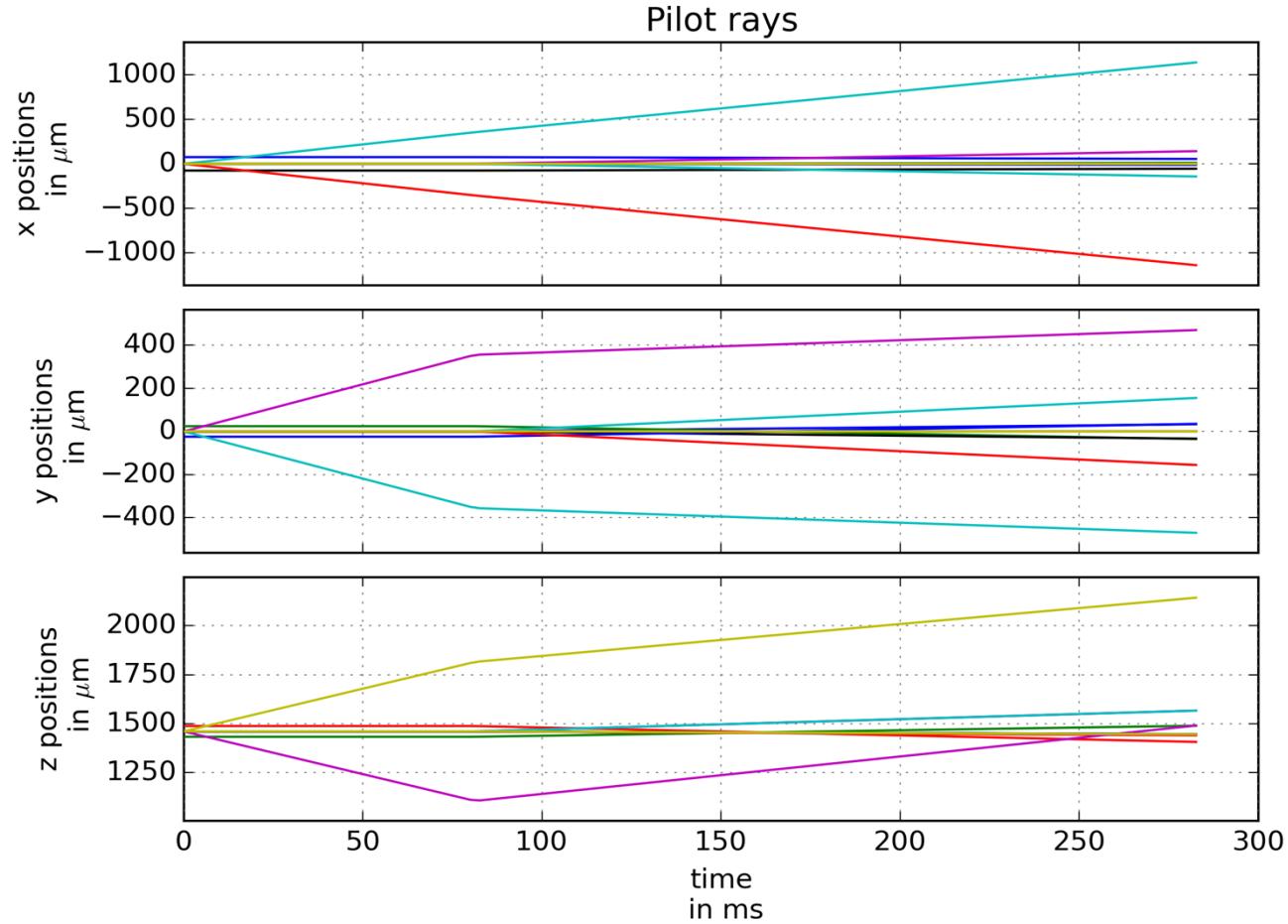
DKC sequence:

$t_{\text{preToF}} = 80 \text{ ms}$,

$t_{\text{lens}} = 2.64 \text{ ms}$,

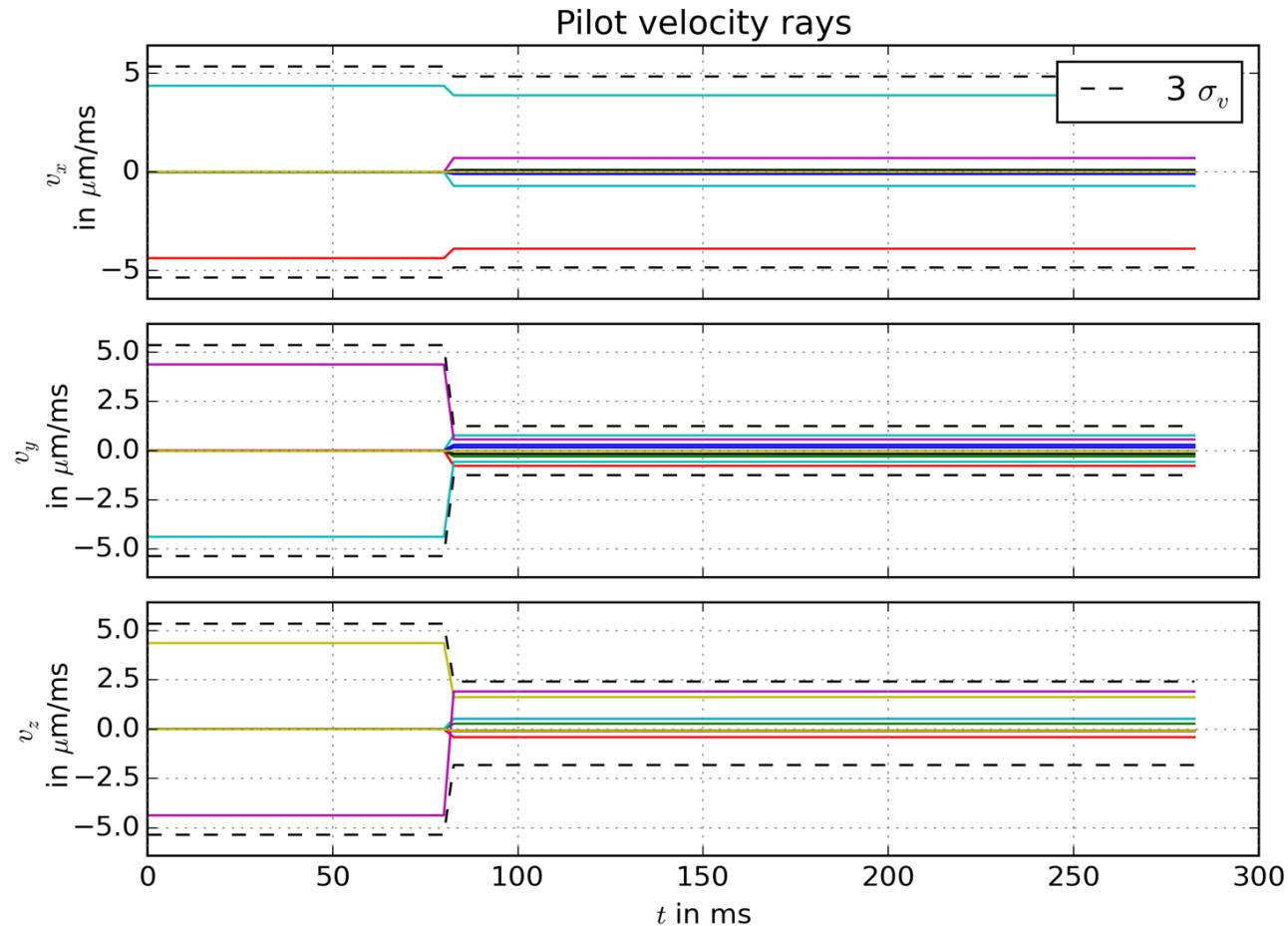
$t_{\text{ToF}} = 200 \text{ ms}$

Pilot rays: estimating final position and width provides integration grid



Velocity rays

measure
quality of
collimation



Iso-potential of release trap

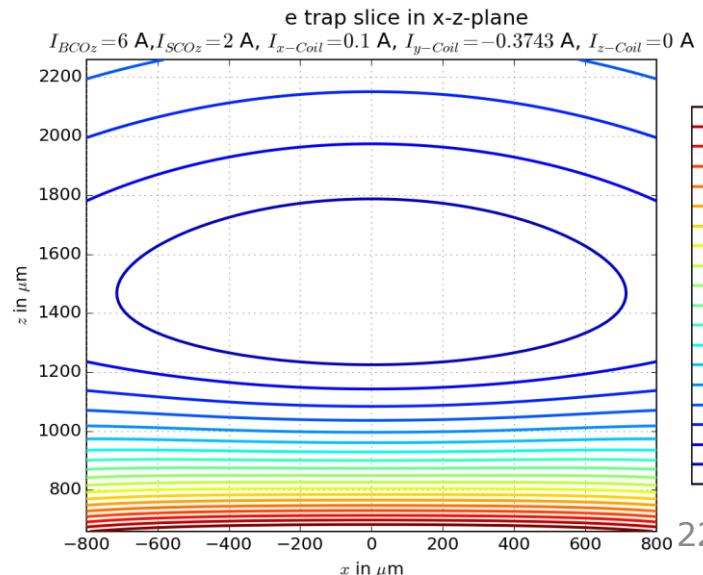
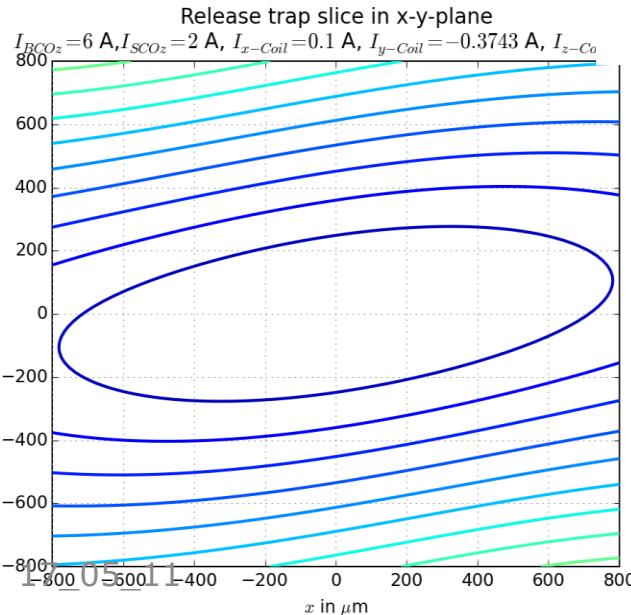
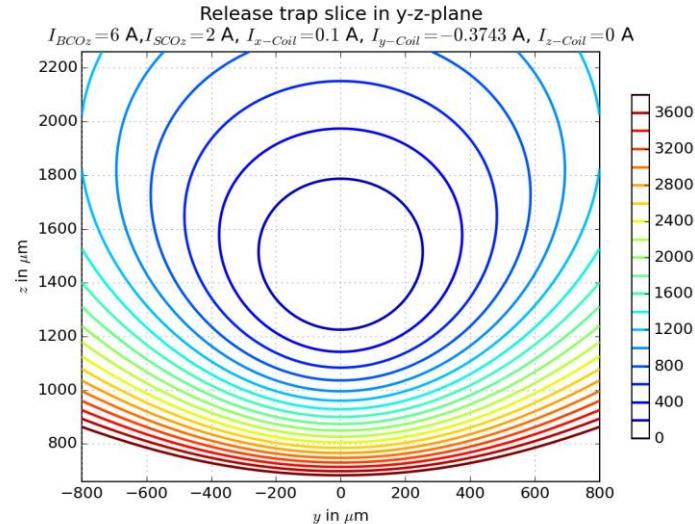


$$I_{BCOz} = 6 \text{ A}, I_{SCOz} = 2 \text{ A}$$

$$I_x = 0.1 \text{ A}, I_y = -0.3743 \text{ A}, I_z = 0 \text{ A}$$

$$f_1 = 9.209 \text{ Hz}, f_2 = 28.479 \text{ Hz}, f_3 = 25.301 \text{ Hz}$$

$$\mathbf{R}_{\min} = (-1.48e-04, -2.03e-05, 1459.77) \mu\text{m}$$

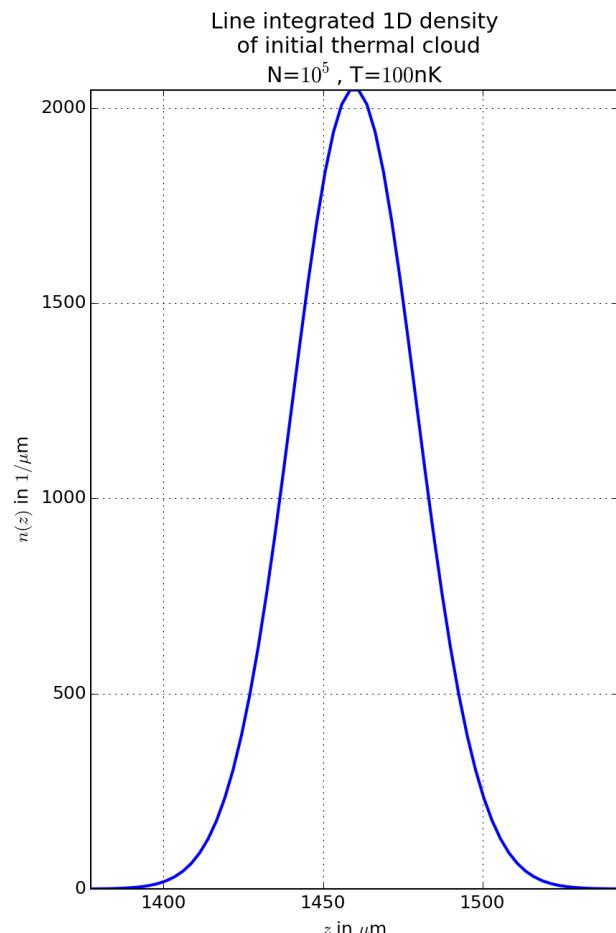
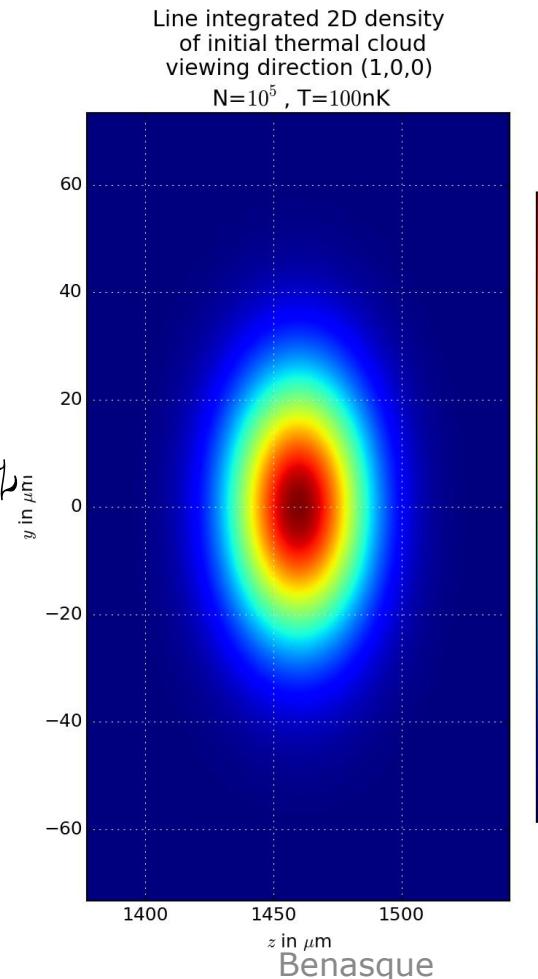
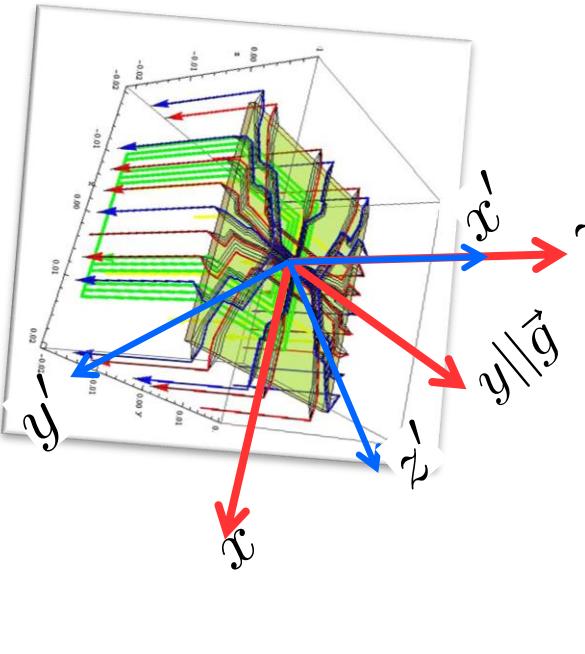


Benasque

Initial position density $n^{(2)}(y, z, t = 0)$



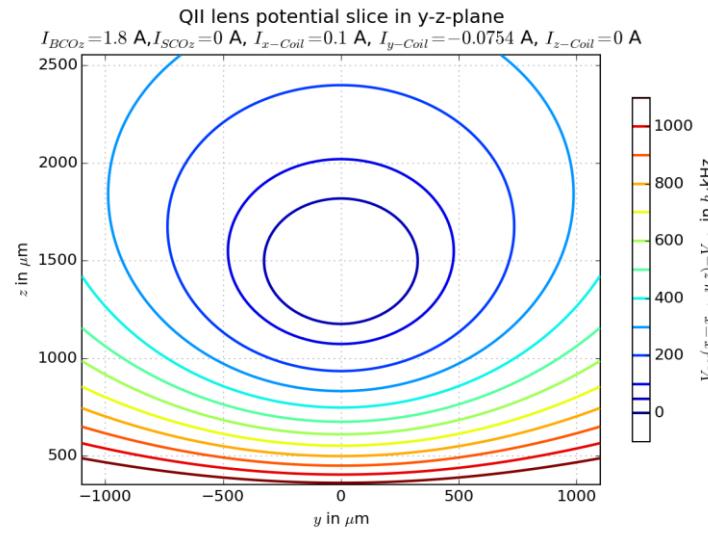
Initial thermal Wigner distribution: $N=10^5$ particles, $T=100\text{nK}$ (harmonic approximation)



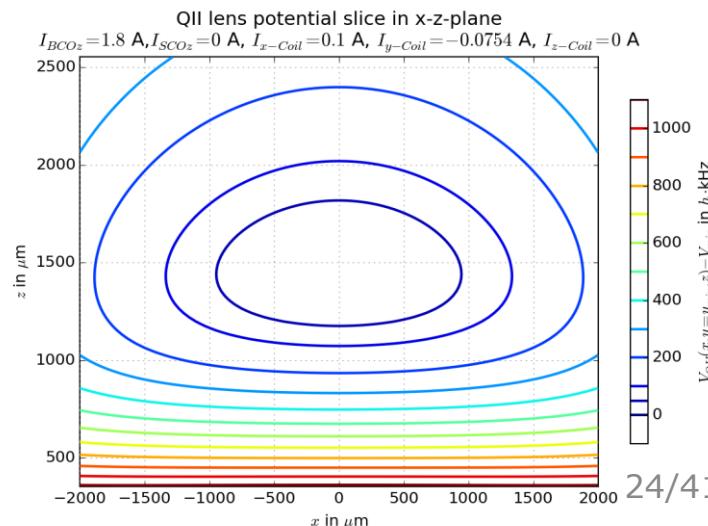
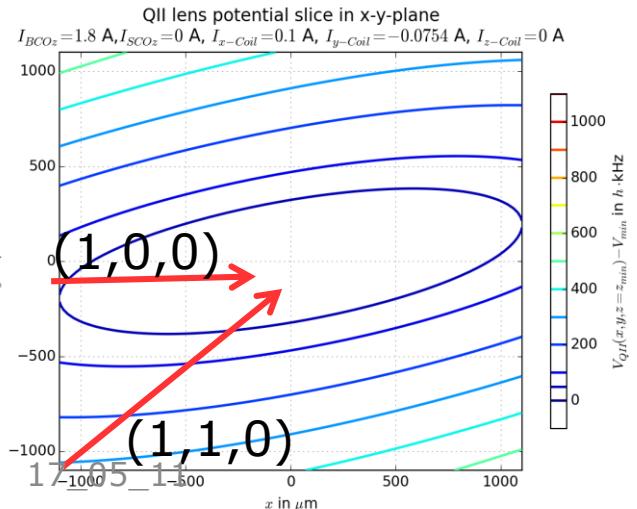
Isopotential of DKC lense



- Lense kick t=2.64 ms**
- Anharmonicity**
deforms gaussian cloud



$I_{BCOz} = 1.8 \text{ A}, I_{SCOz} = 0 \text{ A}$
 $I_x = 0.1 \text{ A}, I_y = -0.0754283 \text{ A}, I_z = 0 \text{ A}$
 $f_1 = 2.961 \text{ Hz}, f_2 = 11.039 \text{ Hz}, f_3 = 11.041 \text{ Hz}$
 $R_{min} = (-1.2e-03, -1.3e-03, 1453.92) \mu\text{m}$



Thermal density $n^{(2)}(y, z, t_f)$



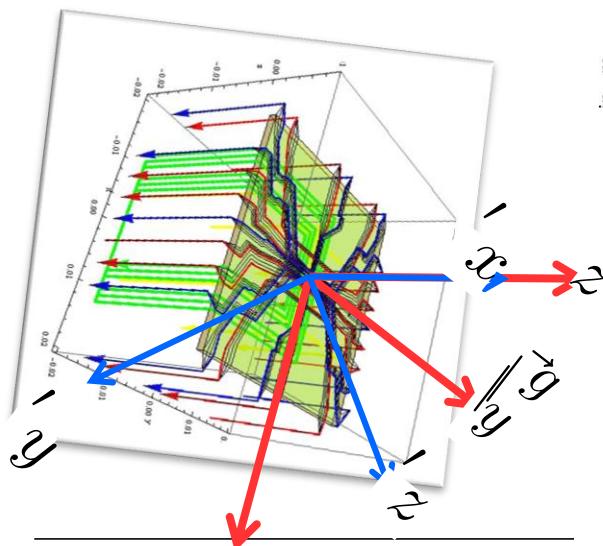
$t_{\text{ToF}} = 200 \text{ ms}$

T=100nK, N=10⁵

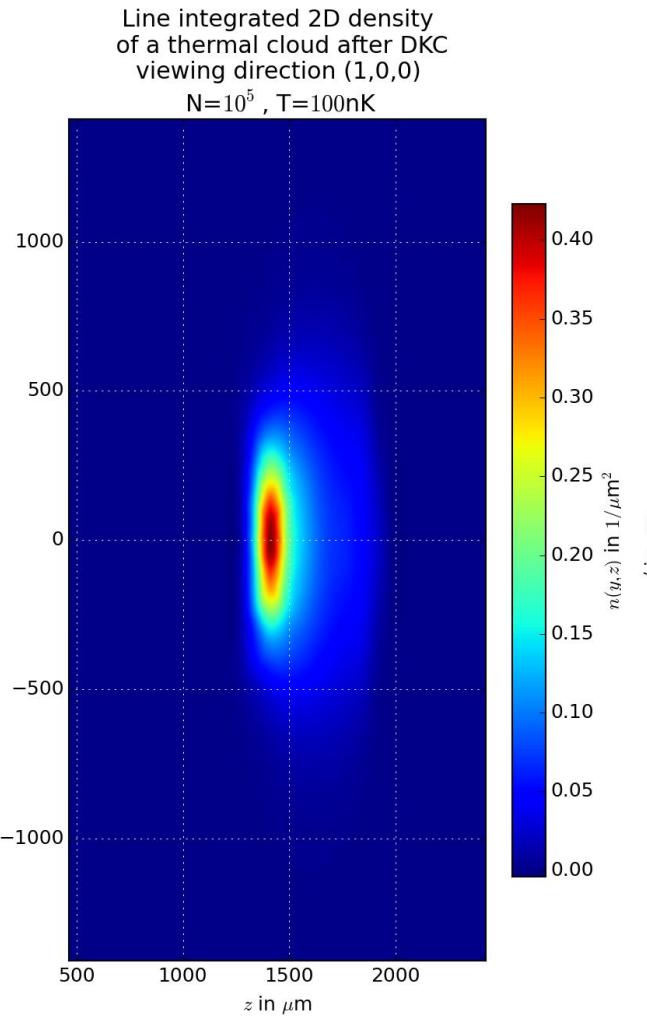
Axis of sight

(1,1,0) vs. (1,0,0)

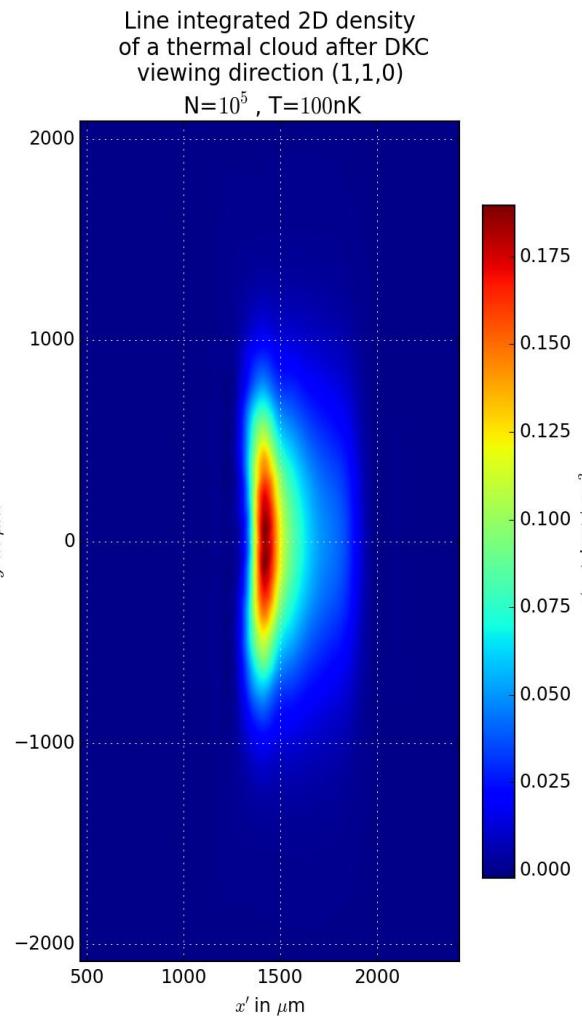
Change of 2D shape



17_05_11



Benasque



25/41

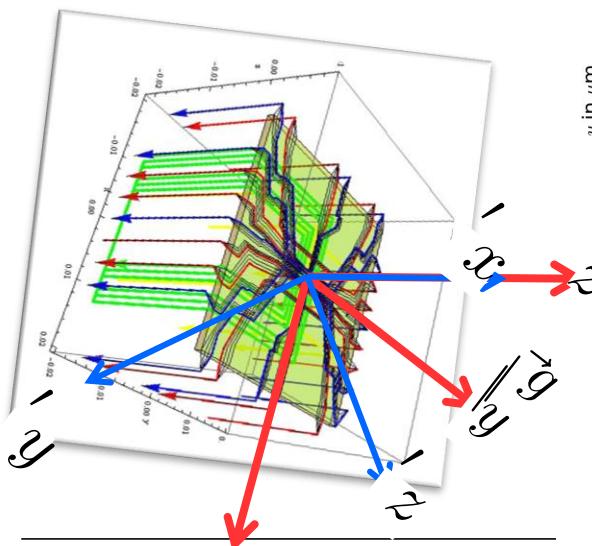
Thermal density $n^{(2)}(y, z, t_f)$



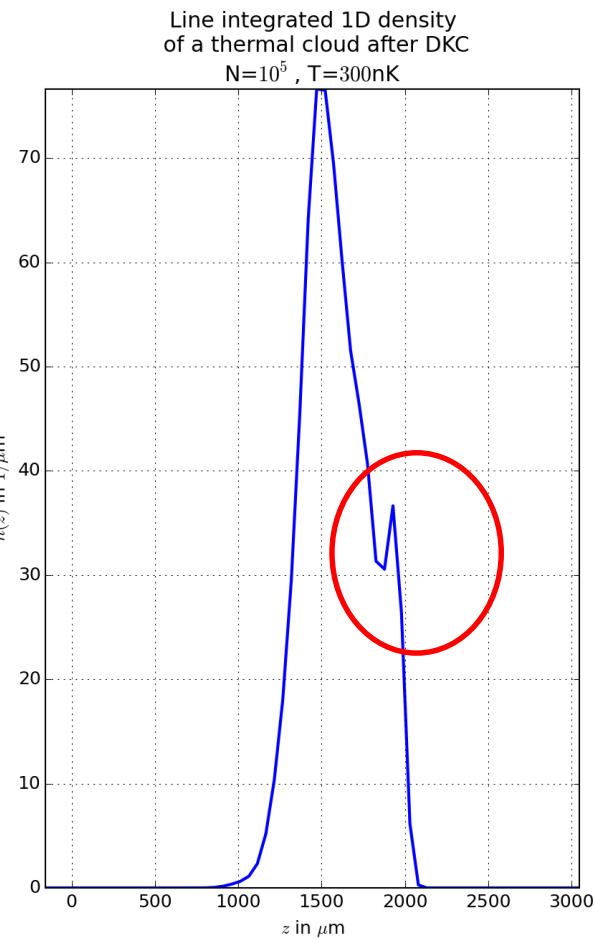
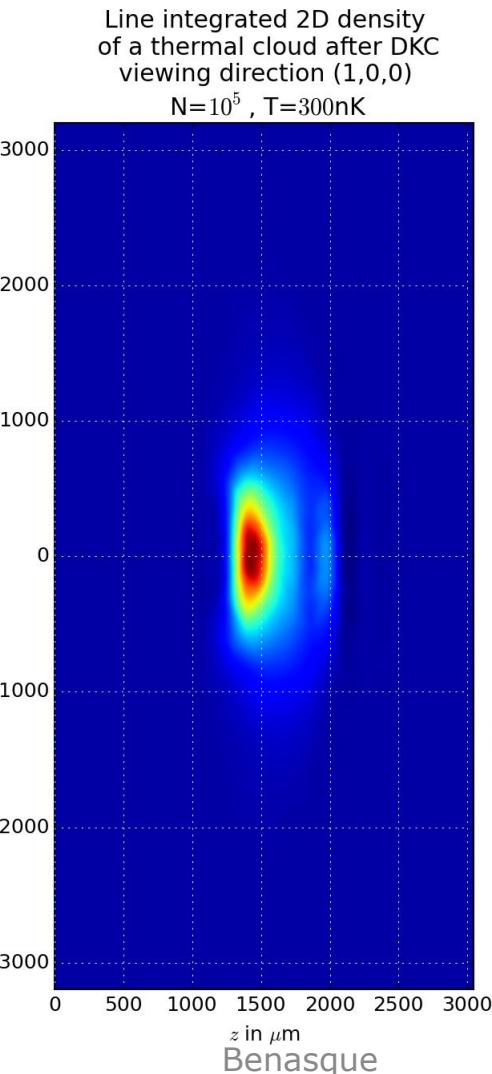
$t_{\text{ToF}} = 200 \text{ ms}$

T=300 nK, N=10⁵

Higher temperatures,
higher expansion rates
more unharmonicity



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Coherent density of BEC

$$n^{(2)}(y, z, t_f)$$



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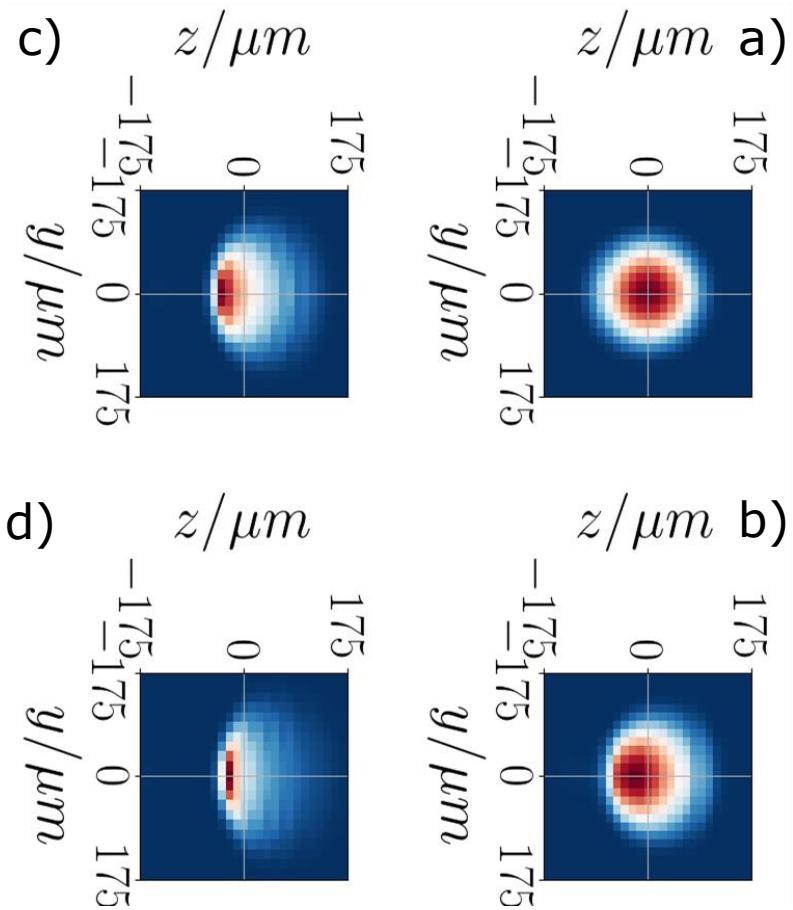
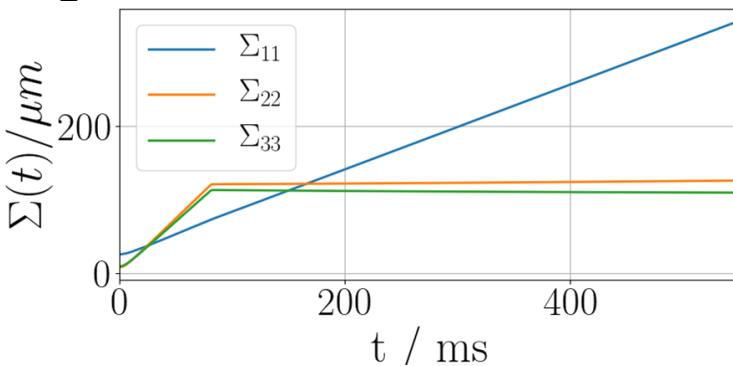
BEC: 3D time-dependent GP solution

2D column integrated densities

- a) 0 ms, b) 150 ms,
c) 300 ms, d) 450 ms
after lens

$N = 10^5$ particles

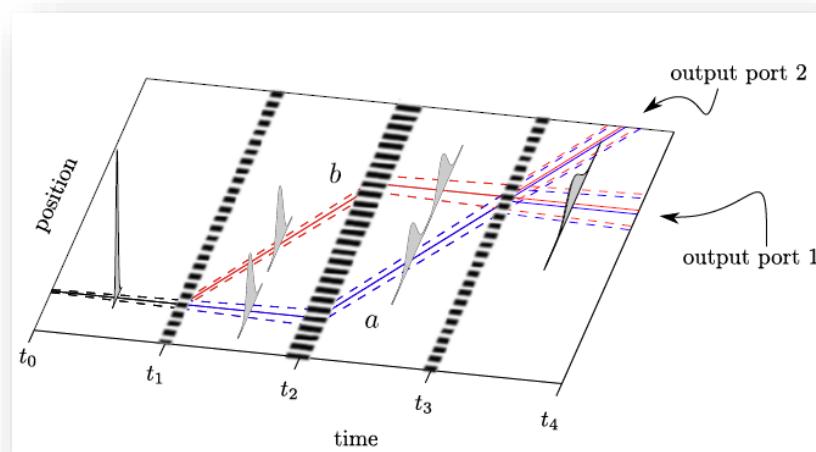
Scaling sizes:



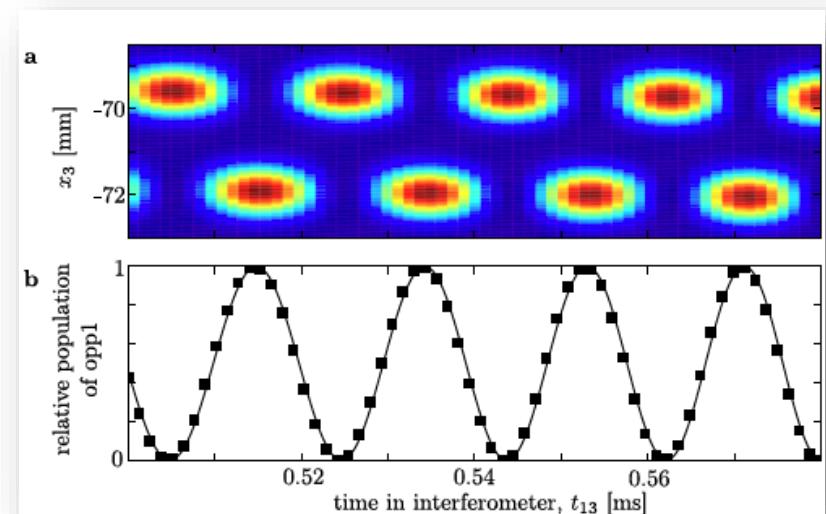
Application 2: Thermal Mach Zehnder-Interferometer

-thermal matter wave optics
M. Schneider

Mach-Zehnder
interferometer



Gravity: population
oscillation, acceleration
sensor



Ray-tracing with matter waves

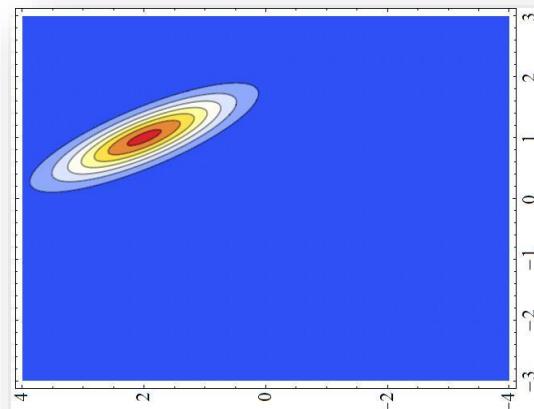


Simulation of interferometry with
classical transport & coherence creating devices

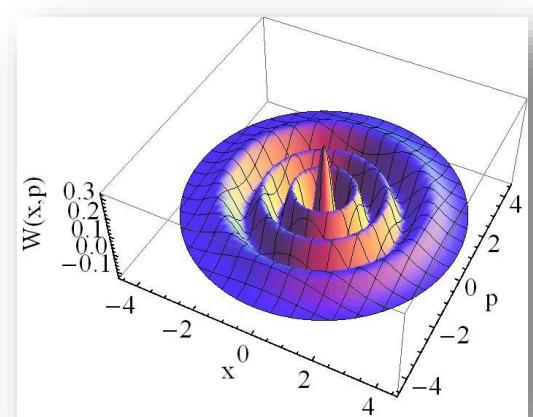
Wigner function: a quantum distribution in phase space

$$f(x, p) = \int_{-\infty}^{\infty} d\xi \frac{e^{-ip\xi/\hbar}}{2\pi\hbar} \langle x + \frac{\xi}{2} | \hat{\rho} | x - \frac{\xi}{2} \rangle$$

Coherent
squeezed
state



Number
state
 $n=6$



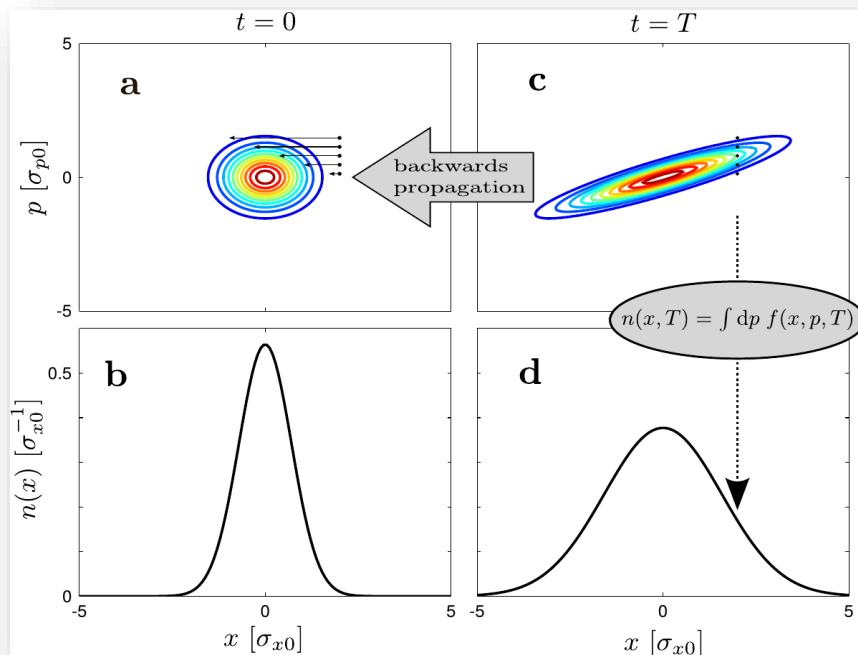
Classical transport in phase space



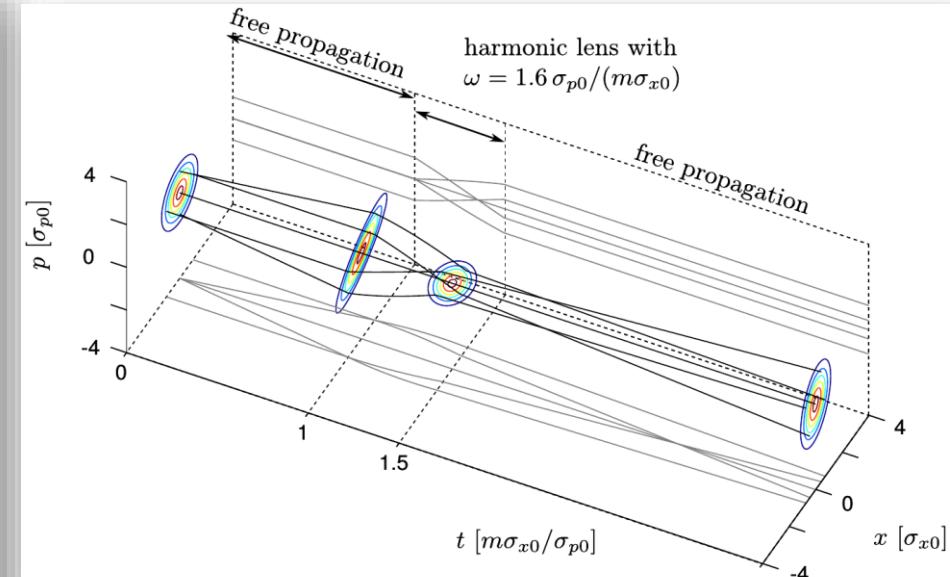
Transport: Hamiltonian evolution in phase space

$$\frac{\partial f}{\partial t} - \{\mathcal{H}, f\} = 0, \quad \text{if } \frac{\partial^l \mathcal{H}}{\partial x^l} = 0 \text{ for } l > 2 \text{ and odd}$$

Free expansion



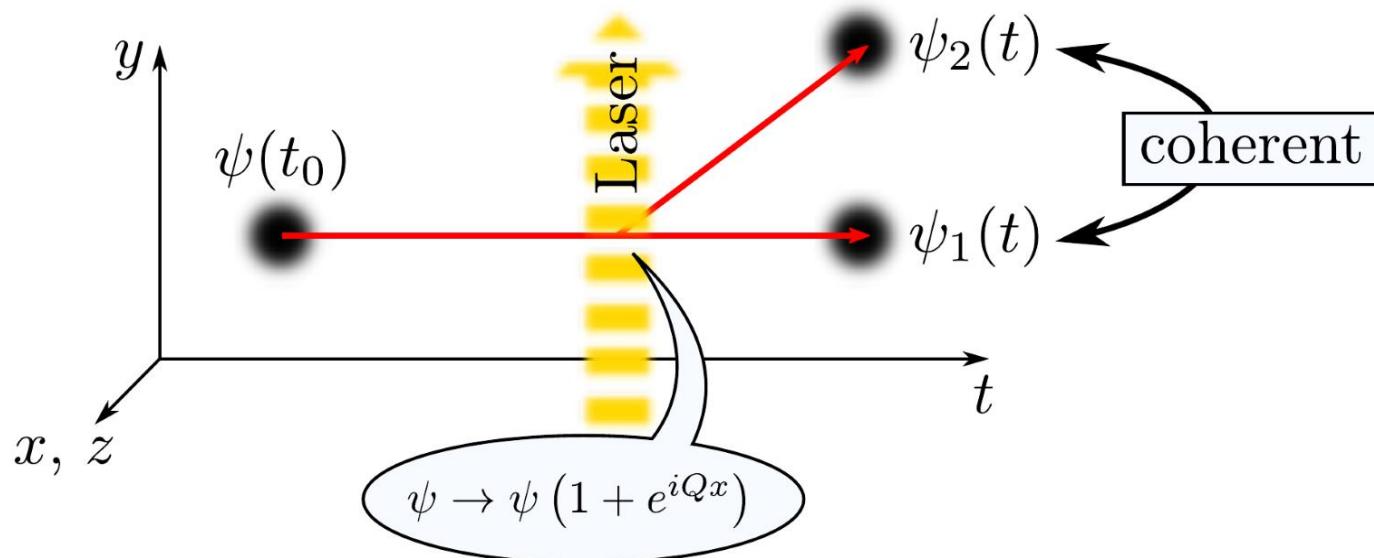
Harmonic lens



Coherence creating devices



- **Double slits or beam-splitters** in Hilbert-space
- E.g.: **Bragg scattering**, laser beam splits matter wave in coherent superposition, creation of coherence



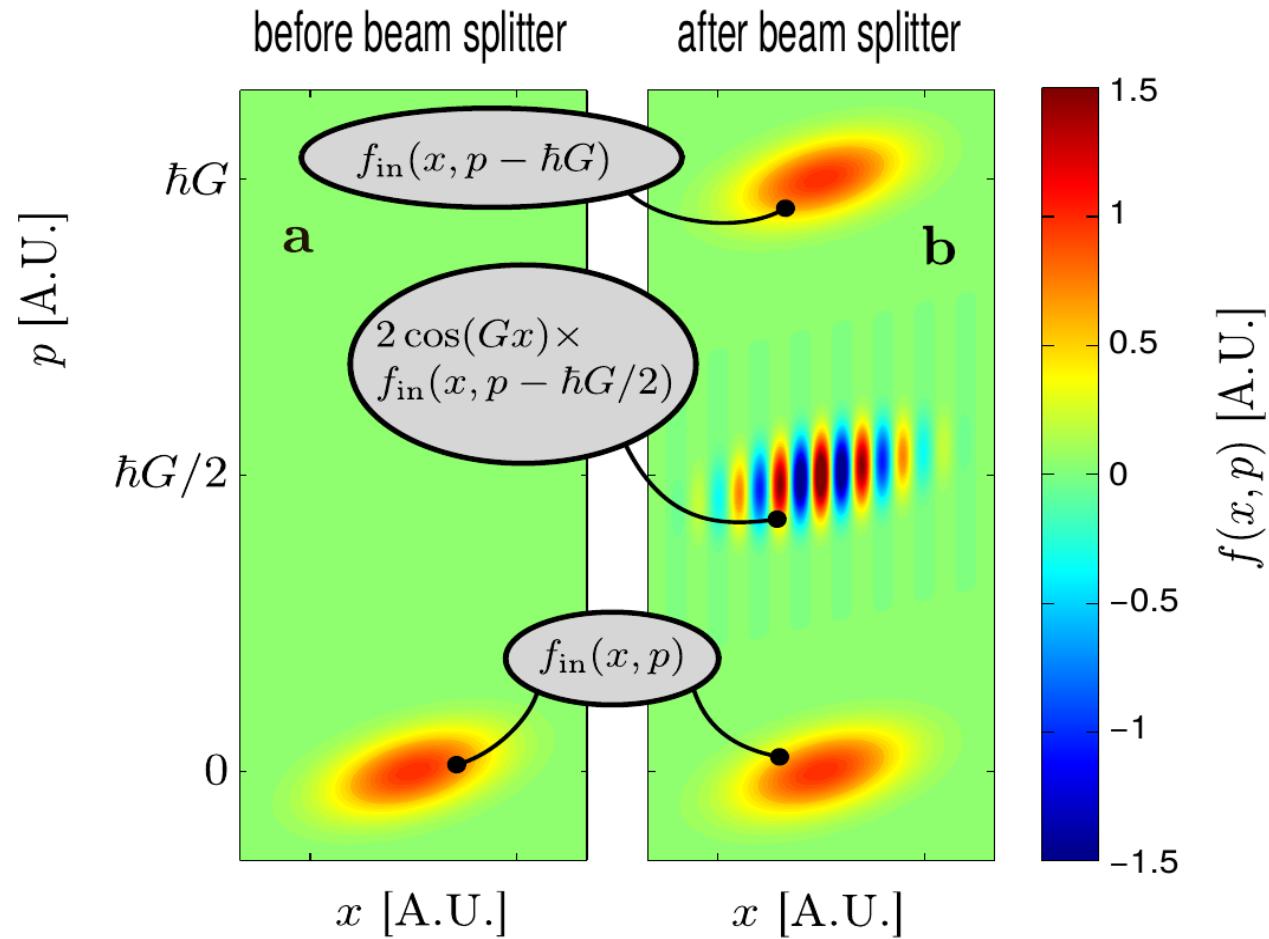
NOT described by classical transport

Beam-splitter in phase-space

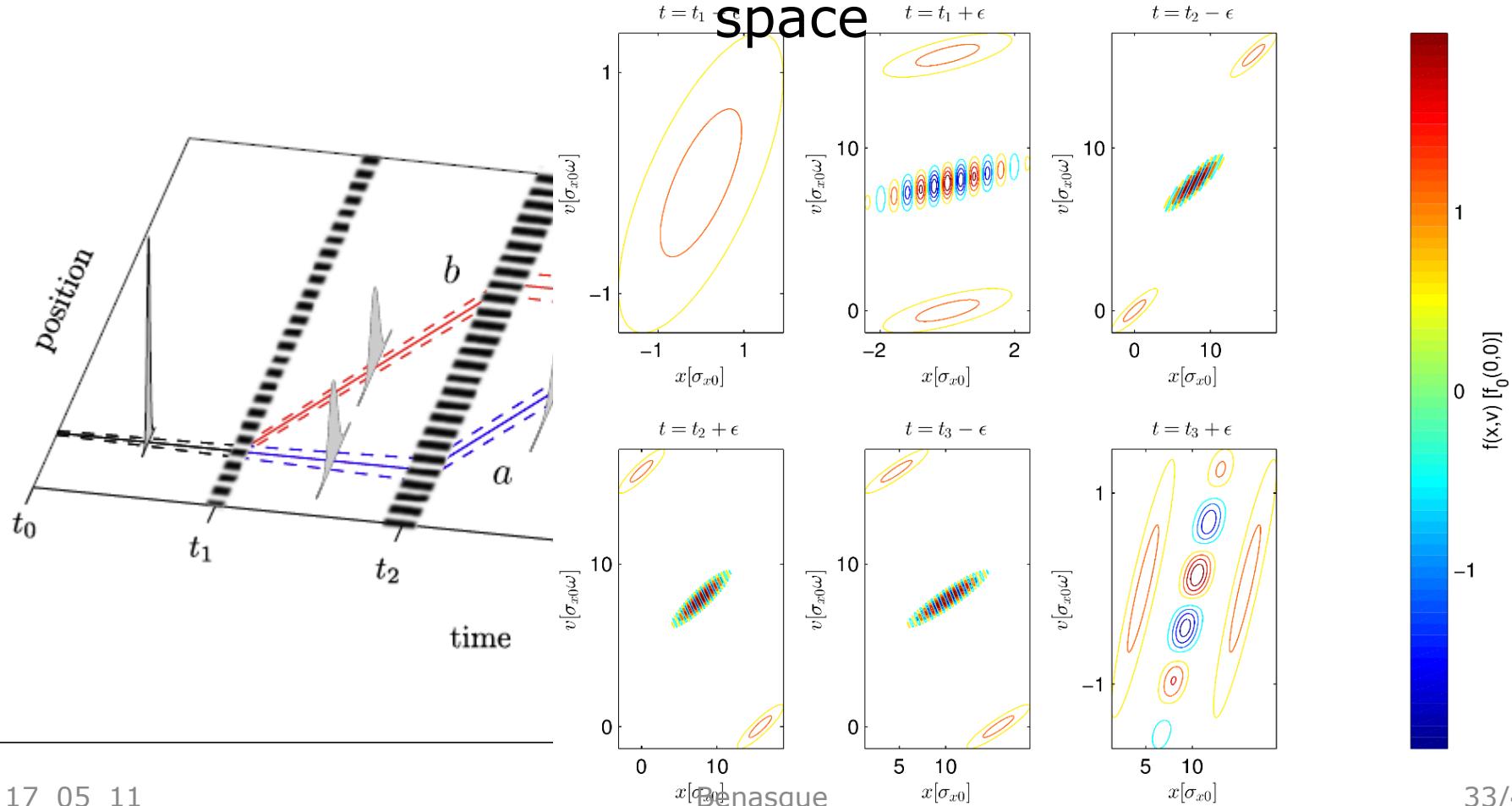


Wigner function
in phase space

Before
and
after
a beam splitter



MZ interferometer in time MZI sequence in phase-space



Free space: 3D asymmetric MZI



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Temperature
dependence:

2D density plots
1D cross sections
and
Fourier transforms
for three different
temperatures:
T=100nK
T=500nK
T=1000nK
after recombination

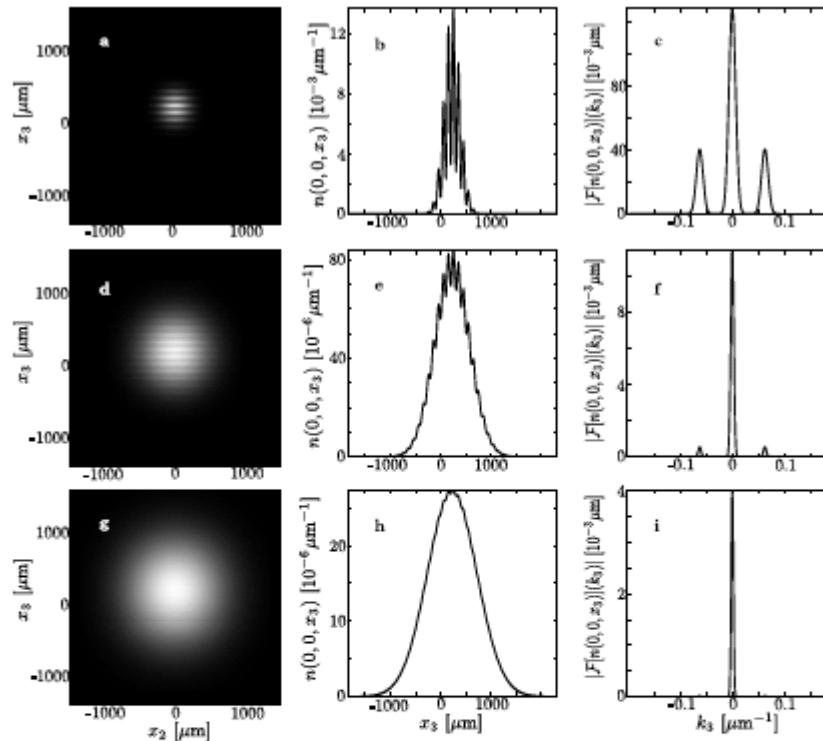


Figure 7.5.: Position densities after passing a MZI for various temperatures of the initial state. These are $T = 100\text{nK}$ (1st line, a, b, c), $T = 500\text{nK}$ (2nd line, d, e, f), and $T = 1000\text{nK}$ (3rd line, g, h, i). The left column (parts a, d, g) shows 2D slices in the x_2 - x_3 -plane with $x_1=0$ of the 3D density. The middle column (parts b, e, h) displays the densities along the x_3 axis with $x_1=x_2=0$. The right column (parts c, f, i) shows the absolute values of Fourier transforms of densities along the x_3 axis, depicted in the middle columns. Initial state parameters are $(\omega_1, \omega_2, \omega_3) = 2\pi(127.3, 127.3, 31.8)\text{Hz}$, $M = 87\text{amu}$, $N = 10^5$. MZI parameters are $t_{01} = 20\text{ms}$, $t_{12} = 10\text{ms}$, $t_{23} = 9.9\text{ms}$, $t_{34} = 10\text{ms}$, $\mathbf{G} = (0, 0, 31.42)\mu\text{m}^{-1}$.

Free space: 3D asymmetric MZI



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Time
dependence

2D density plots
1D cross
sections

and
Fourier
transforms

@ T=100nK for
three different
times after
recombination

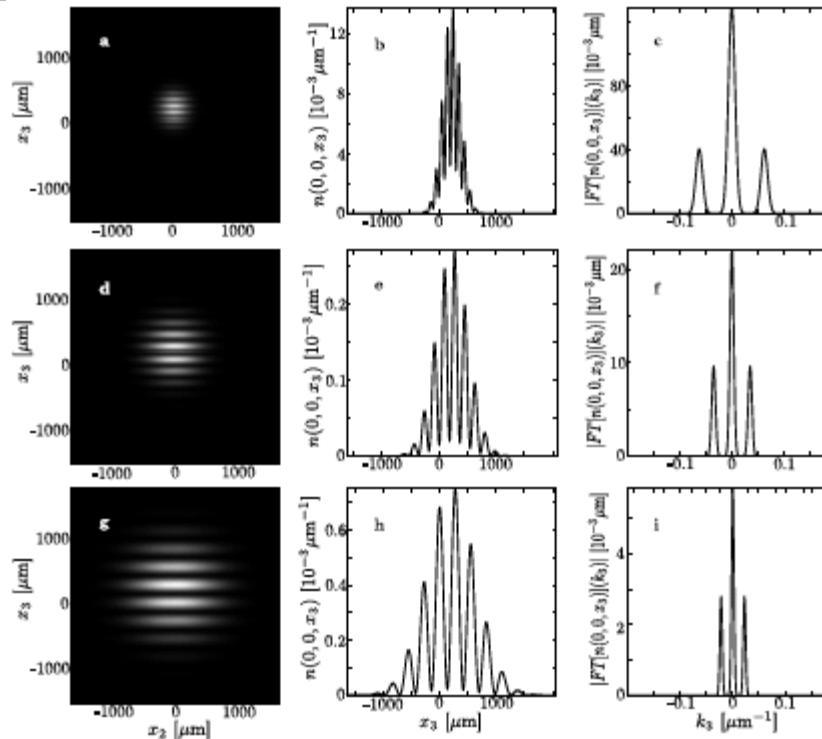


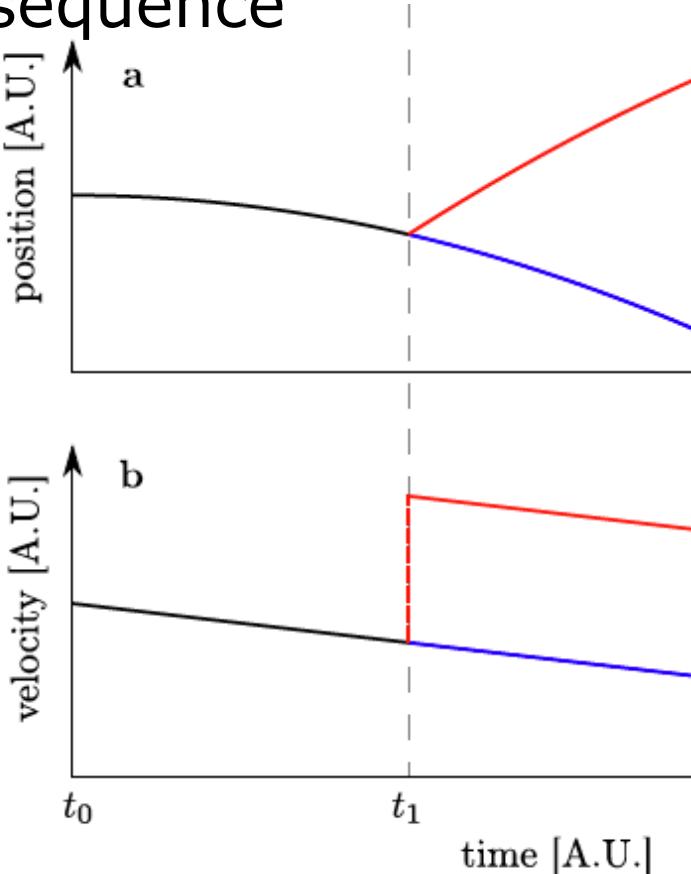
Figure 7.6.: Position densities after passing a MZI for various timespans between recombination and detection, t_{34} . These are $t_{34} = 10\text{ms}$ (1st line, a, b, c), $t_{34} = 50\text{ms}$ (2nd line, d, e, f), and $t_{34} = 100\text{ms}$ (3rd line, g, h, i). The left column (parts a, d, g) shows 2D slices in the x_2 - x_3 -plane with $x_1=0$ of the 3D density. The middle column (parts b, e, h) displays the density along the x_3 axis with $x_1 = x_2 = 0$. The right column (parts c, f, i) shows the absolute value of the Fourier transform of the density along the x_3 axis, depicted in the middle columns. Initial state parameters are $(\omega_1, \omega_2, \omega_3) = 2\pi(127.3, 127.3, 31.8)\text{Hz}$, $M = 87\text{amu}$, $T = 100\text{nK}$, $N = 10^3$. MZI parameters are $t_{01} = 20\text{ms}$, $t_{12} = 10\text{ms}$, $t_{23} = 9.9\text{ms}$, $\mathbf{G} = (0, 0, 31.42)\mu\text{m}^{-1}$.

Fringe
contrast =
relative
strength of
Fourier
components

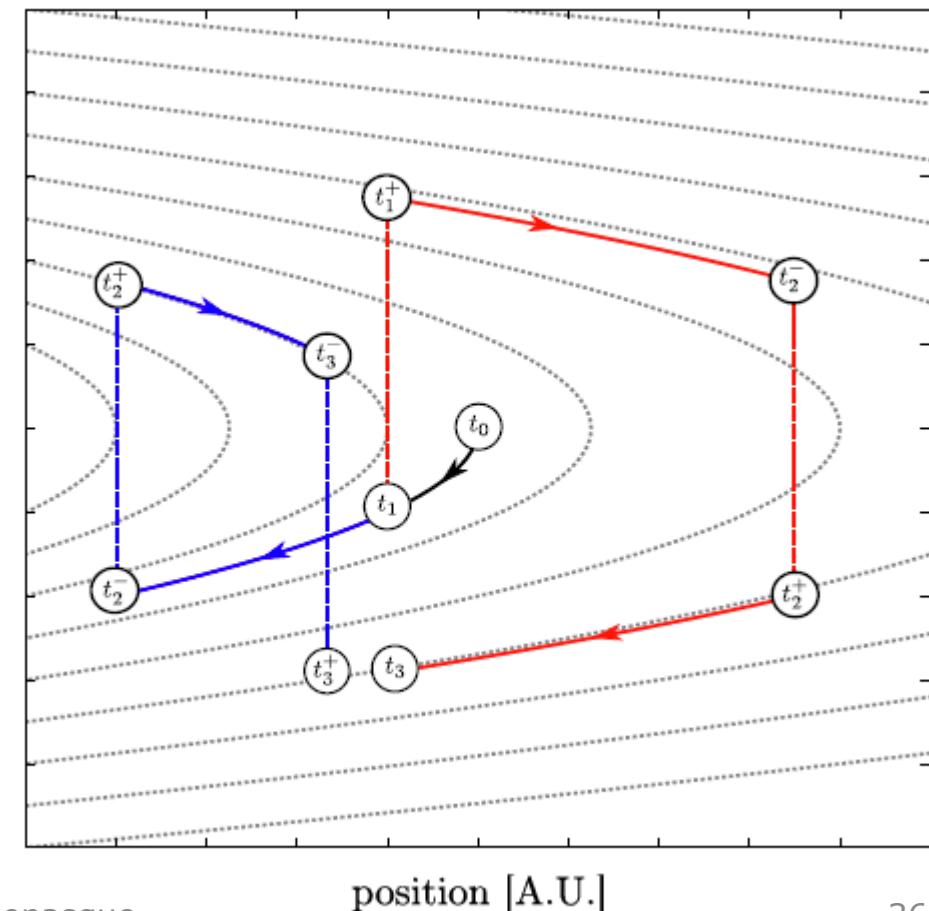
MZI in gravity



Real space MZI sequence



MZI in phase space



Acceleration sensor for gravity



Symmetric MZI

Population oscillations
between the
two output ports
1+2
of a symmetric MZI
($t_{23}=t_{12}$)

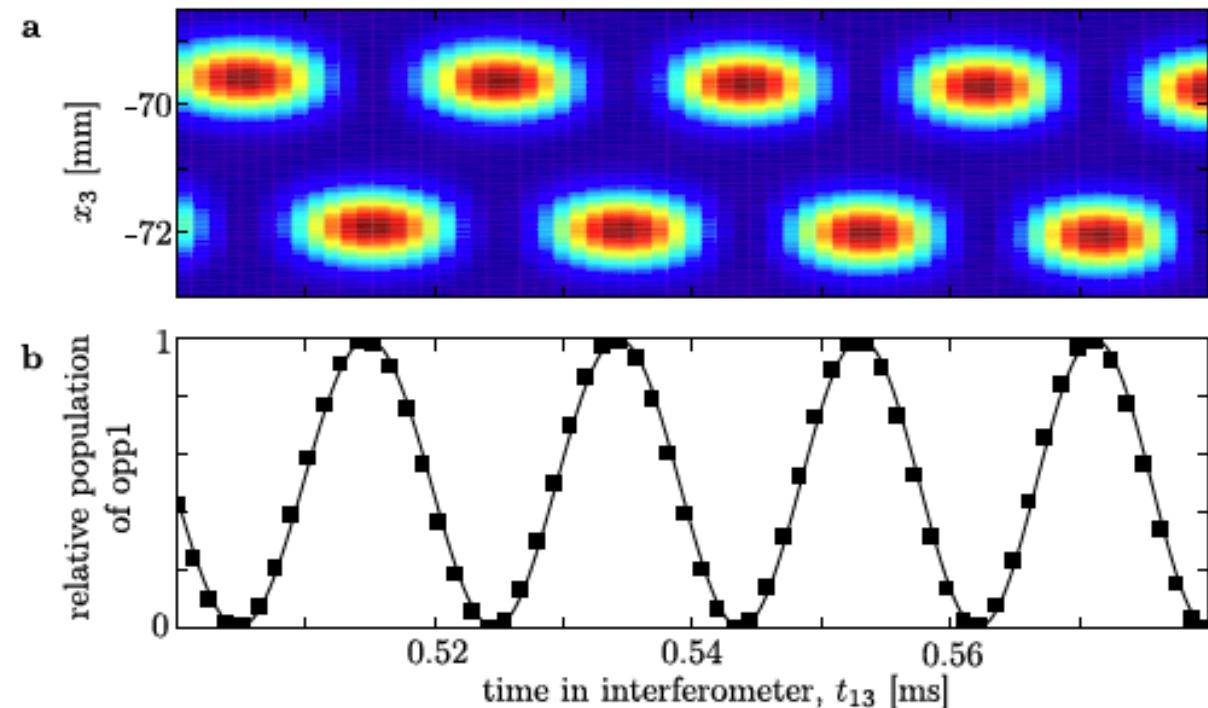


Figure 7.8.: Population of output ports under influence of constant gravity acceleration $\dot{x}_3 = -g = 9.81\mu\text{m}/\text{ms}^2$. Part a: False color representation of position density along vertical direction x_3 . Part b: Relative population of output port 1. Other parameters are $t_{01} = 20\text{ms}$, $t_{34} = 100\text{ms}$, $N = 10^5$, $M = 87\text{amu}$, $T = 100\text{nK}$, $G = 31.42\mu\text{m}^{-1}$. Initial trap is harmonic with $(\omega_1, \omega_2, \omega_3) = 2\pi(127.3, 127.3, 31.8)\text{Hz}$.

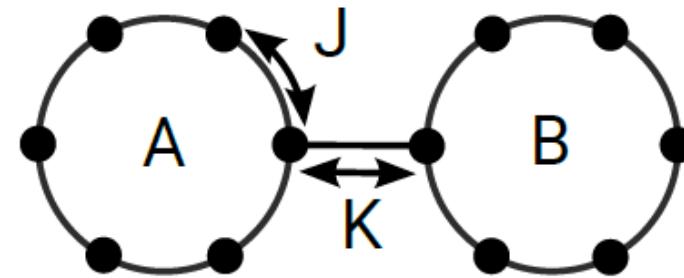
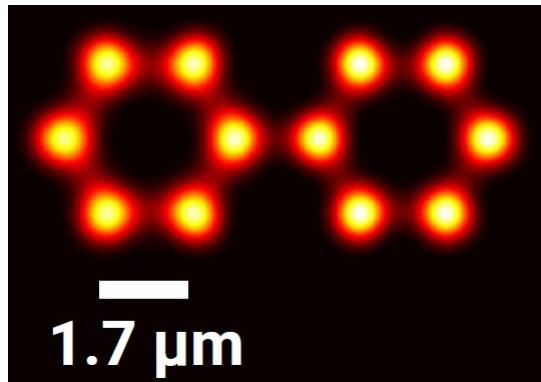
Application 3: Coupled Josephson-rings

-quantum mw-optics

M. Sturm



Bose-Hubbard system: M=12 sites, N=4 particles, onsite interaction U, intraring hopping J, interring hopping K<<J



$$\hat{H} = \hat{H}_A + \hat{H}_B - K(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0) \quad \hat{H}_A = \frac{U}{2} \sum_{i=1}^M \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j$$

Tunneling dynamics



- **Initial state:** all atoms in ground-state of ring A

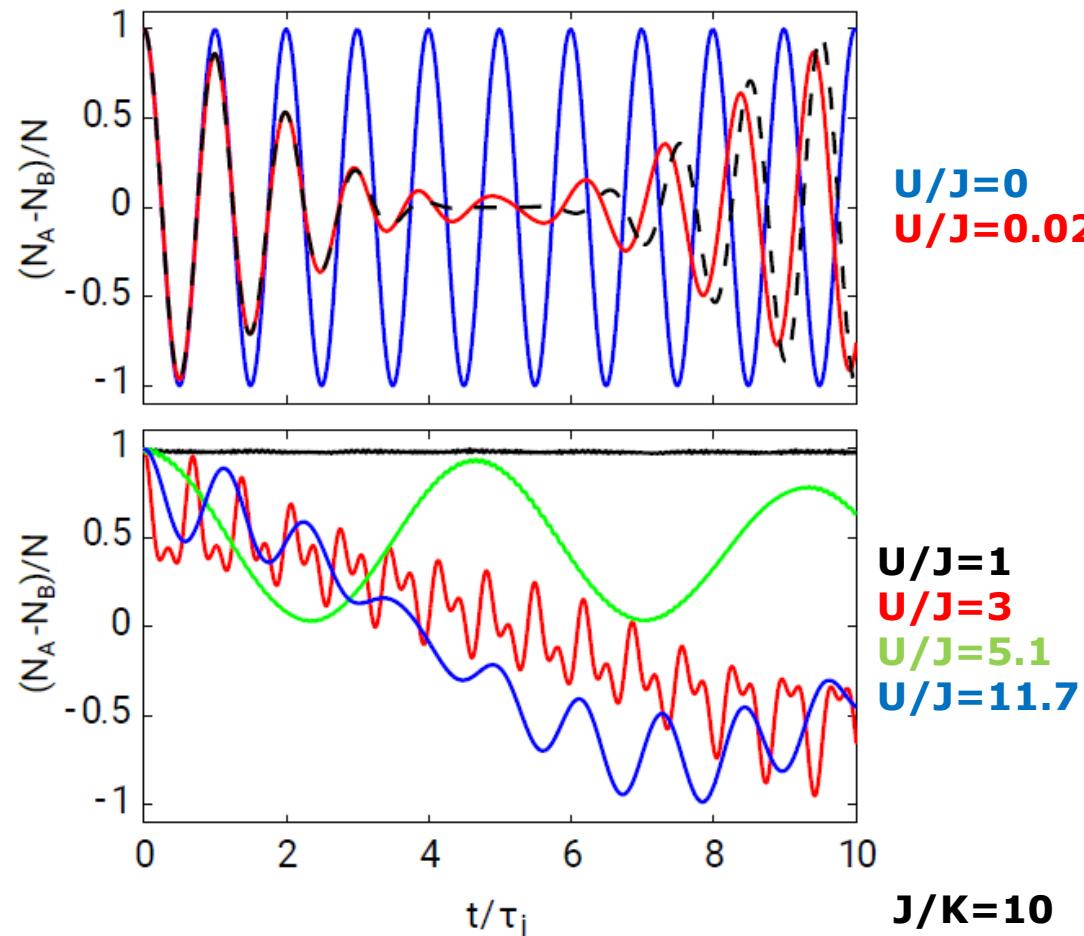
- **No interaction:** sinusoidal J-oscillations

$$\tau_J = h \frac{M}{K}$$

- **Weak interactions:** collapse and revival

$$\tau_C = \tau_R \sqrt{\frac{2}{\pi^2(N-1)}} \quad \tau_R = \frac{M}{U}$$

- **Strong interactions:** self-trapping, many-body tunneling resonances



Many-body resonances



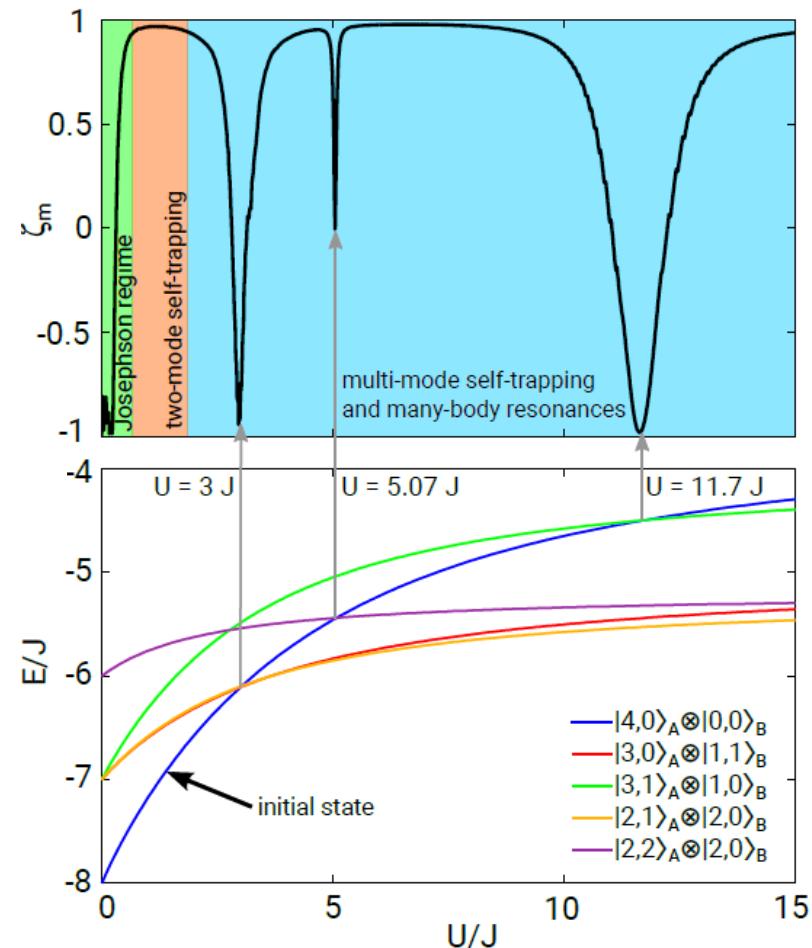
- Minimal population inversion

$$\zeta_m = \min_{0 \leq t \leq \tau} \frac{N_A - N_B}{N}$$

- Resonances explained by level crossing of mb-states of isolated rings

$$|N_A, q_A\rangle_A \otimes |N_B, q_B\rangle_B$$

- Similar effects: tilted 1D lattices
F.Meinert et al., PRL **116** 205301 (2016)
double-wells Juliá-Díaz et al. PRA **82**
063626 (2010)



Summary: technical mw-optics



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Software → hard ware → space



Toolbox mw-optics

- a. Magnetic traps & lenses
- b. Designing quantum simulators with micro lenses
(M. Sturm, M. Schlosser, G. Birkl)
- c. Bragg beam-splitters

Methods & applications

- a. geometrical mw-optics: raytracing, aberrations
- b. thermal mw-optics: 3D interferometry @ finite T
- c. coherent mw-optics: 2 s, delta-kick-collimation
- d. quantum mw-optics: JJ's manybody resonances

Thank you for the attention!

