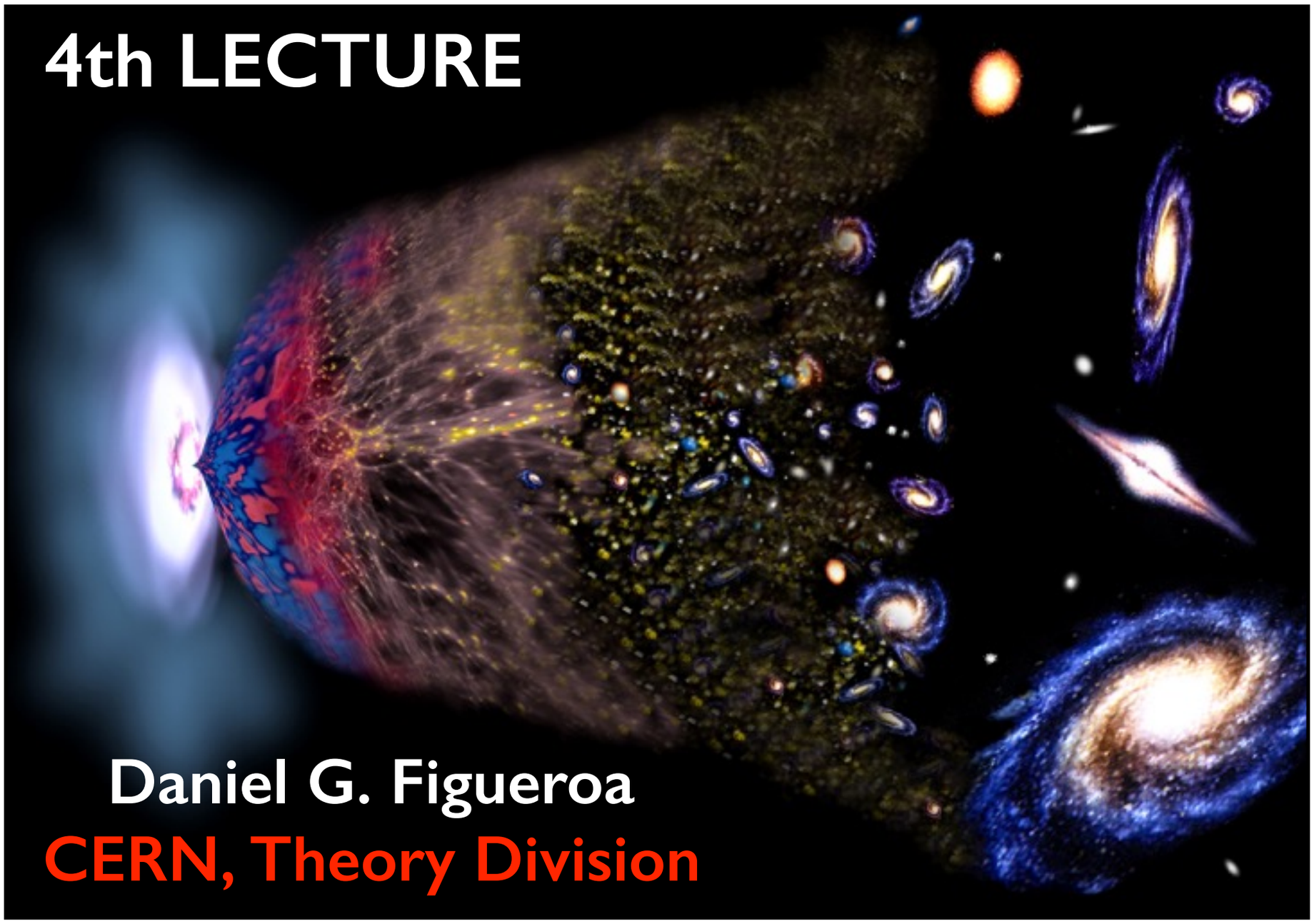
GRAVITATIONAL WAVES
PROBE OF THE EARLY UNIVERSE



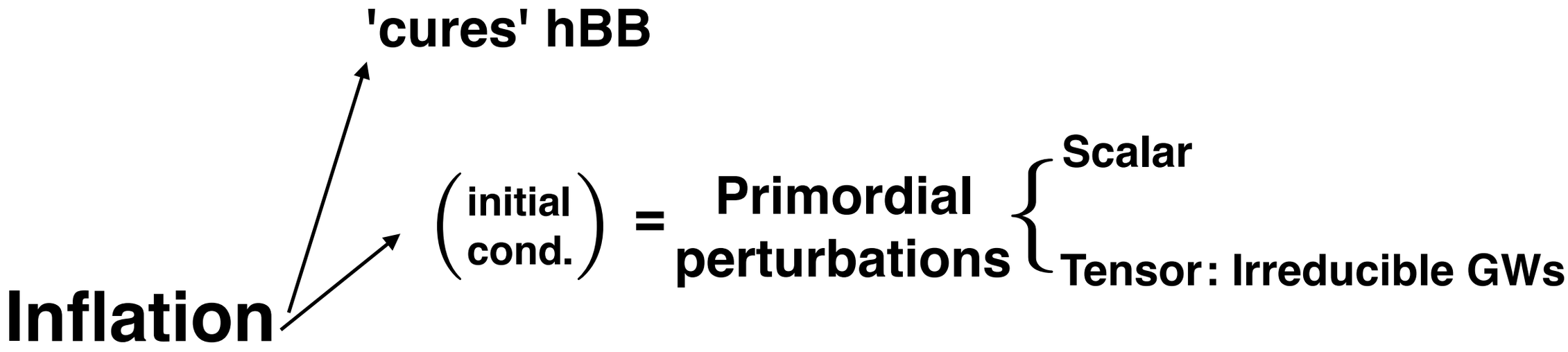
School on Gravitational Waves for Cosmology and Astrophysics, Benasque, May 28 - June 10, 2017

INFLATIONARY COSMOLOGY

'cures' hBB

Inflation

INFLATIONARY COSMOLOGY



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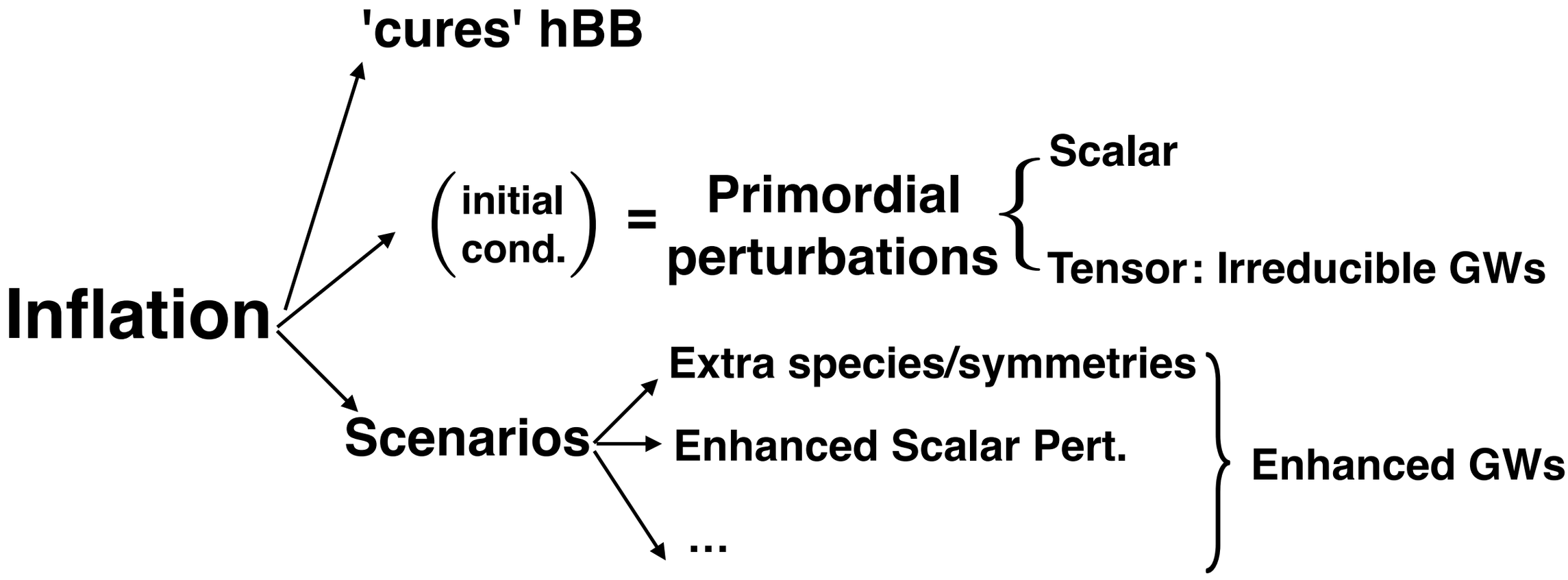

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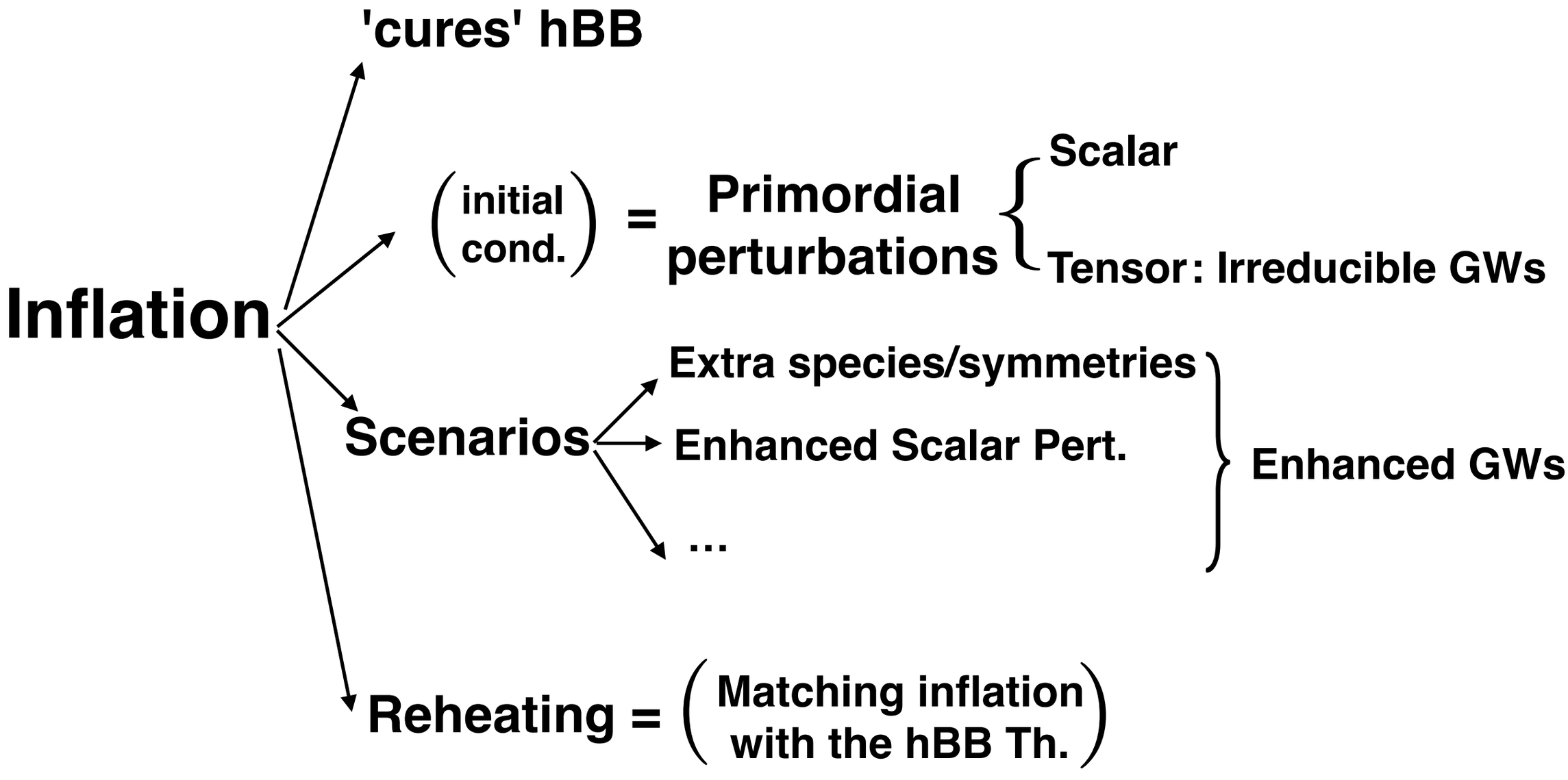

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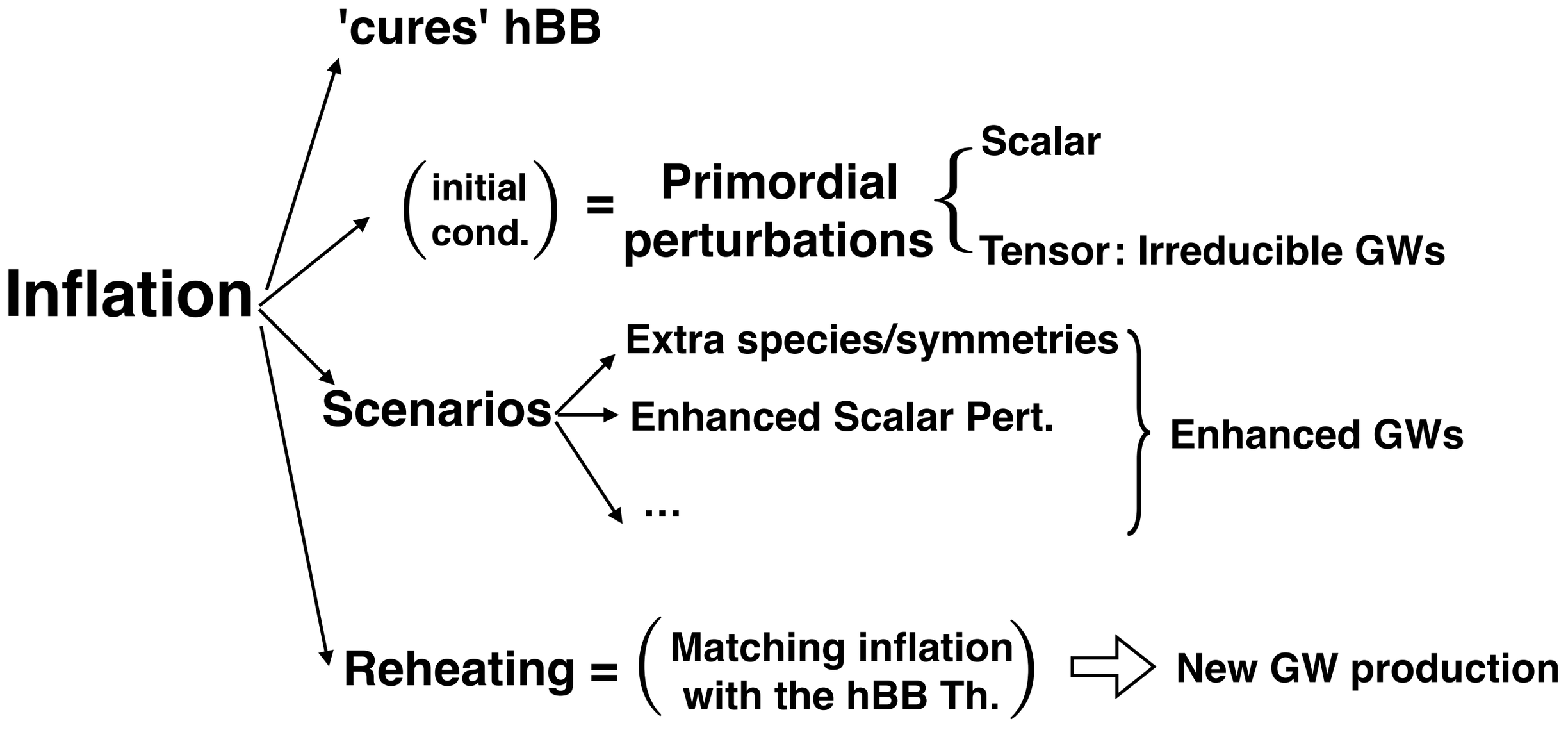

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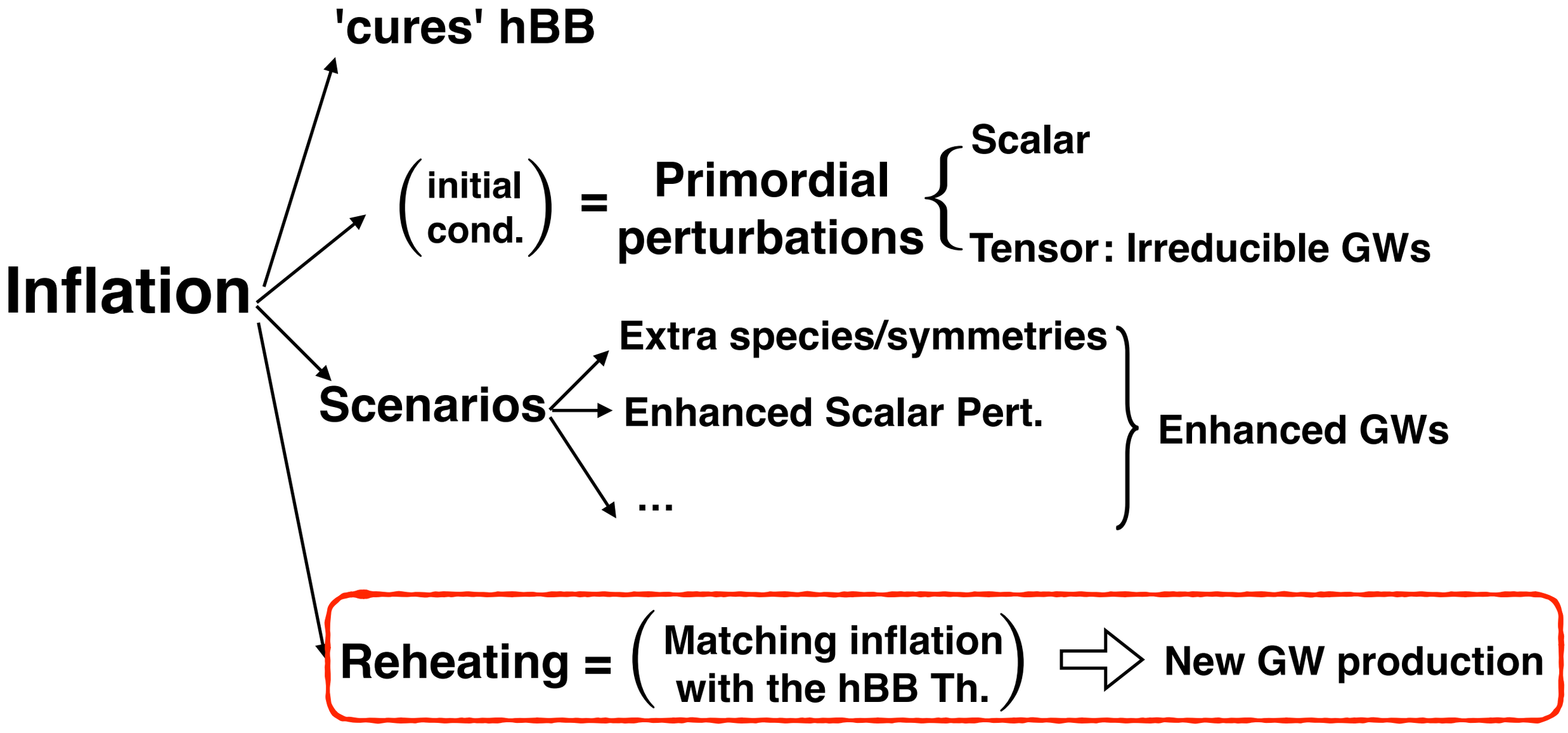
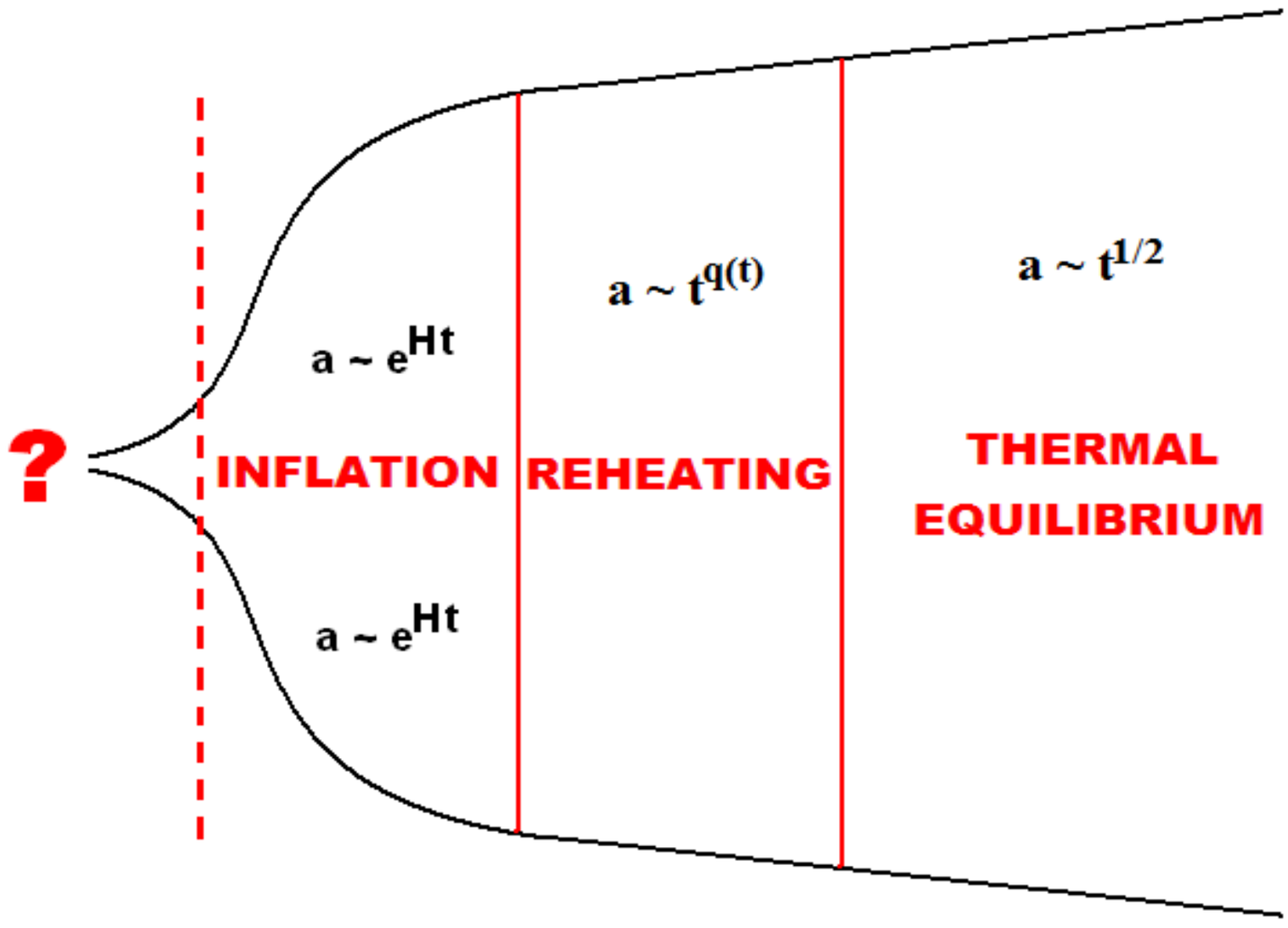


INFLATIONARY REHEATING

INFLATION $\longrightarrow$ REHEATING $\longrightarrow$ BIG BANG THEORY



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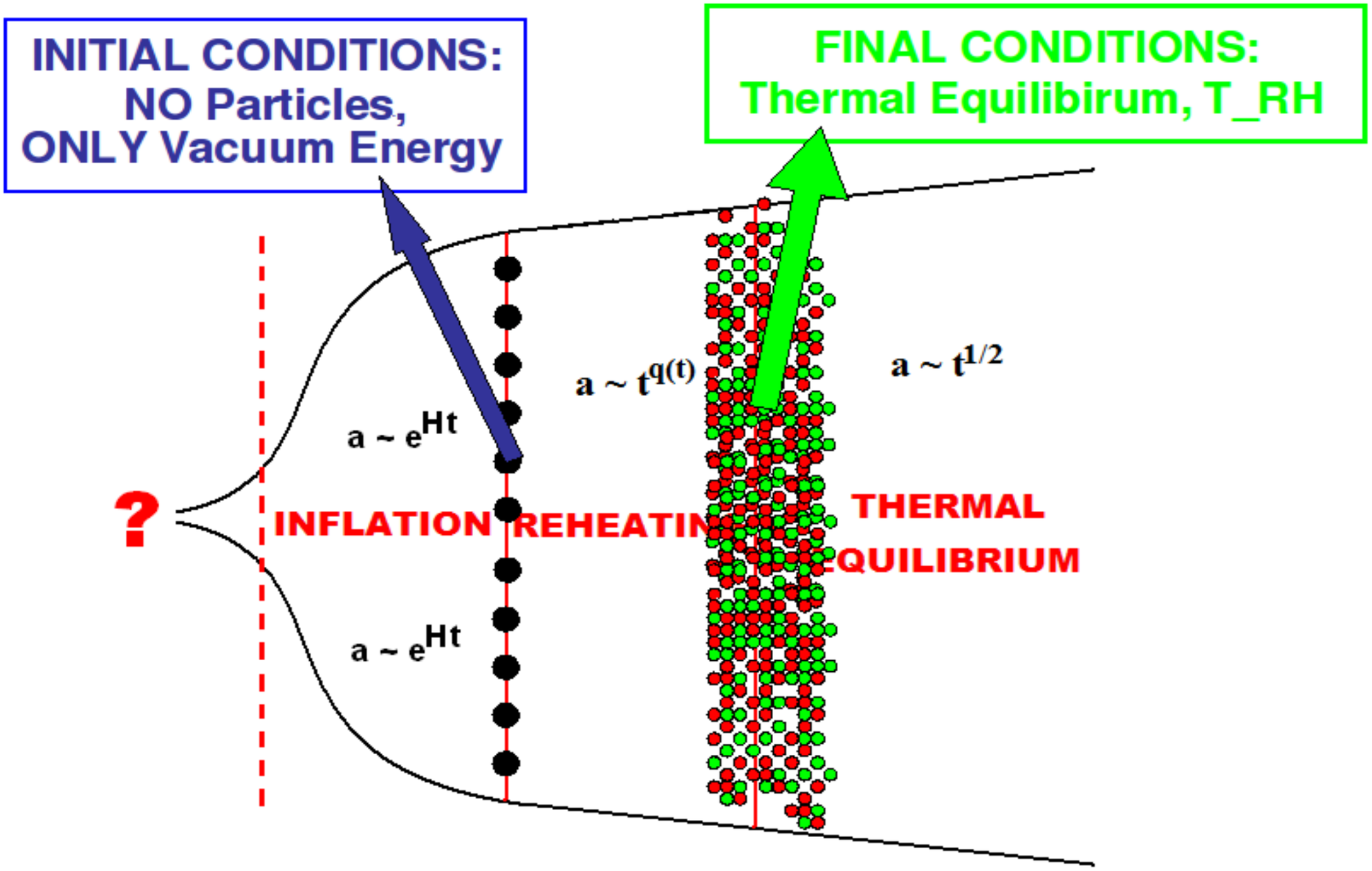

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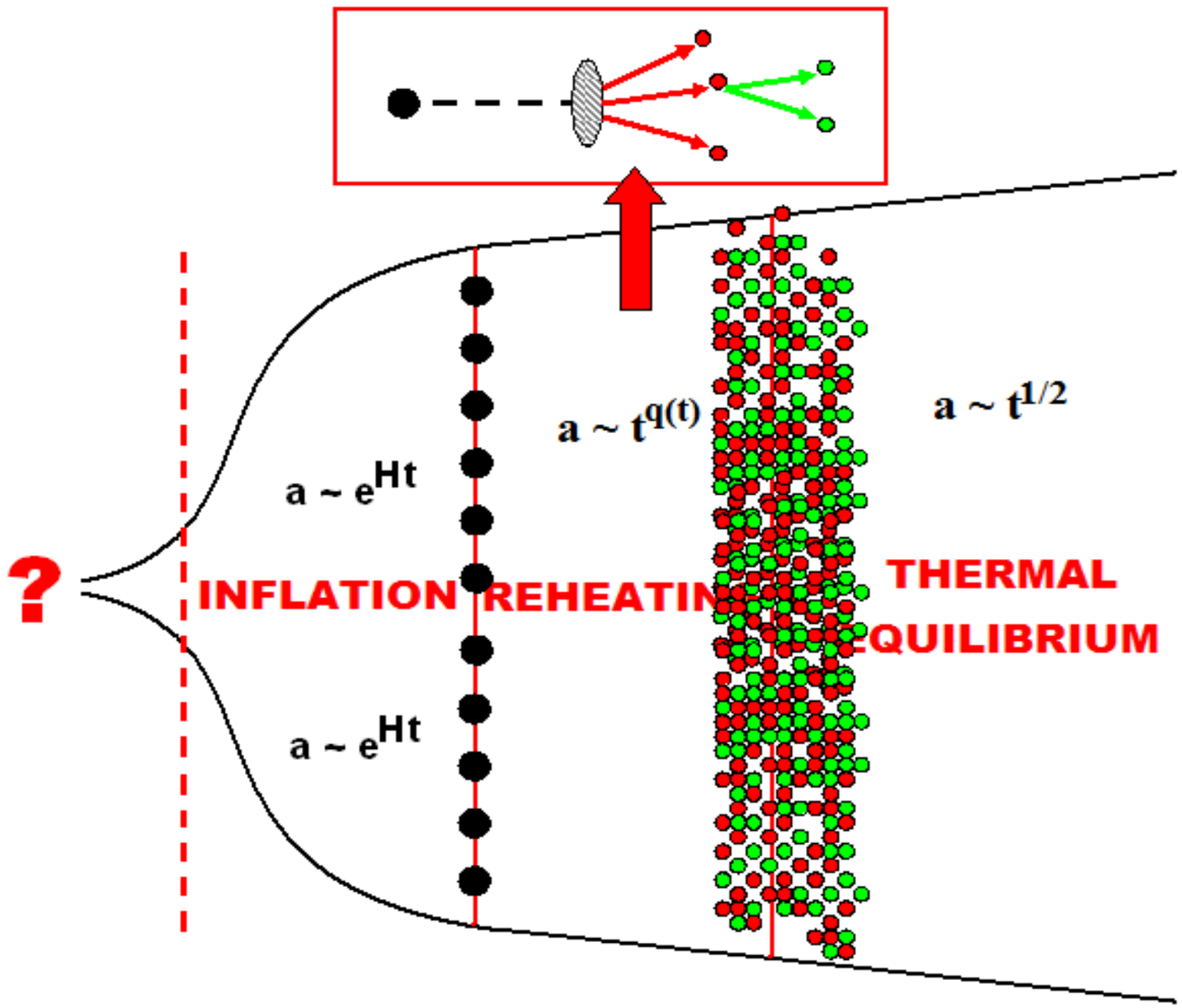

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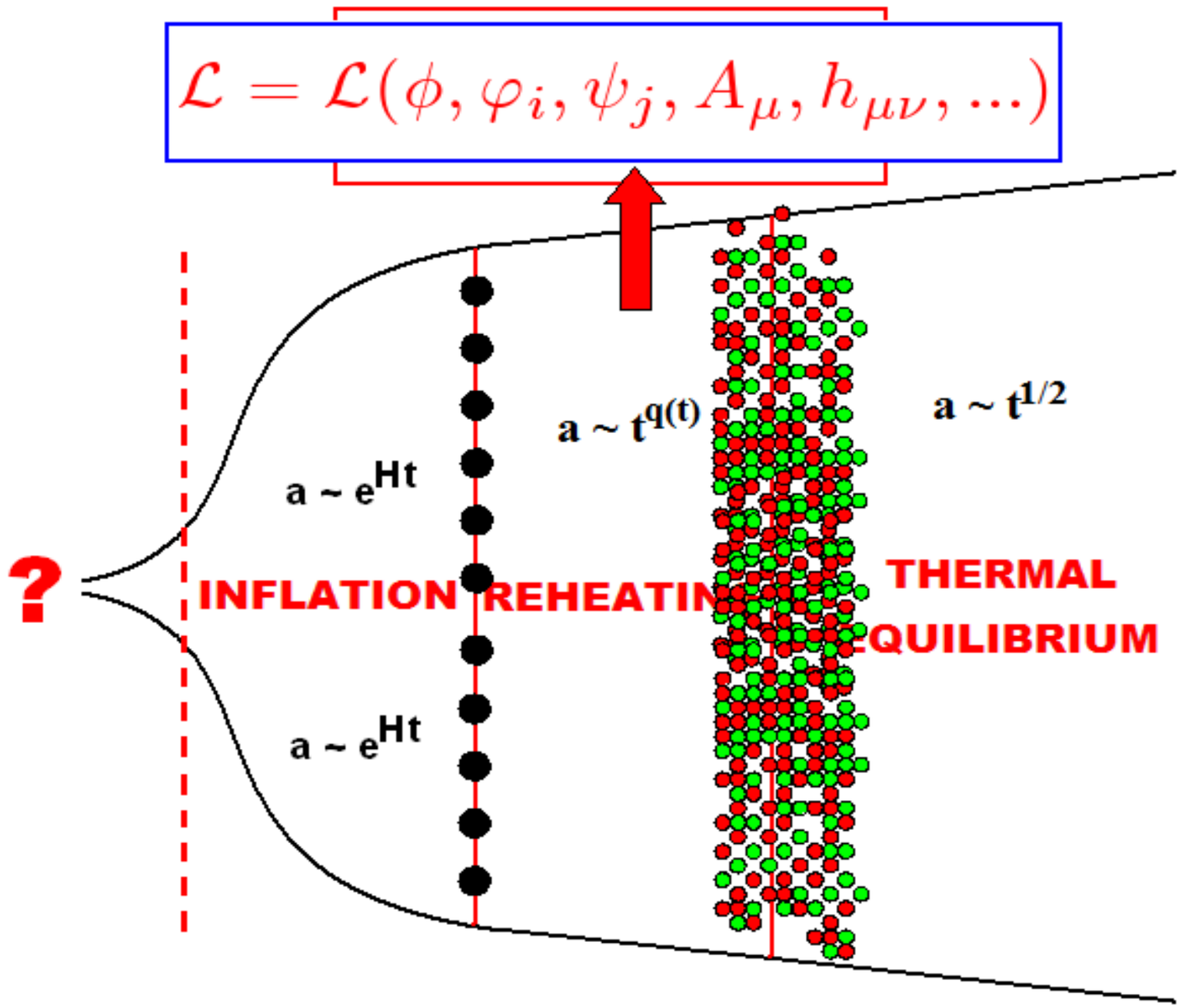

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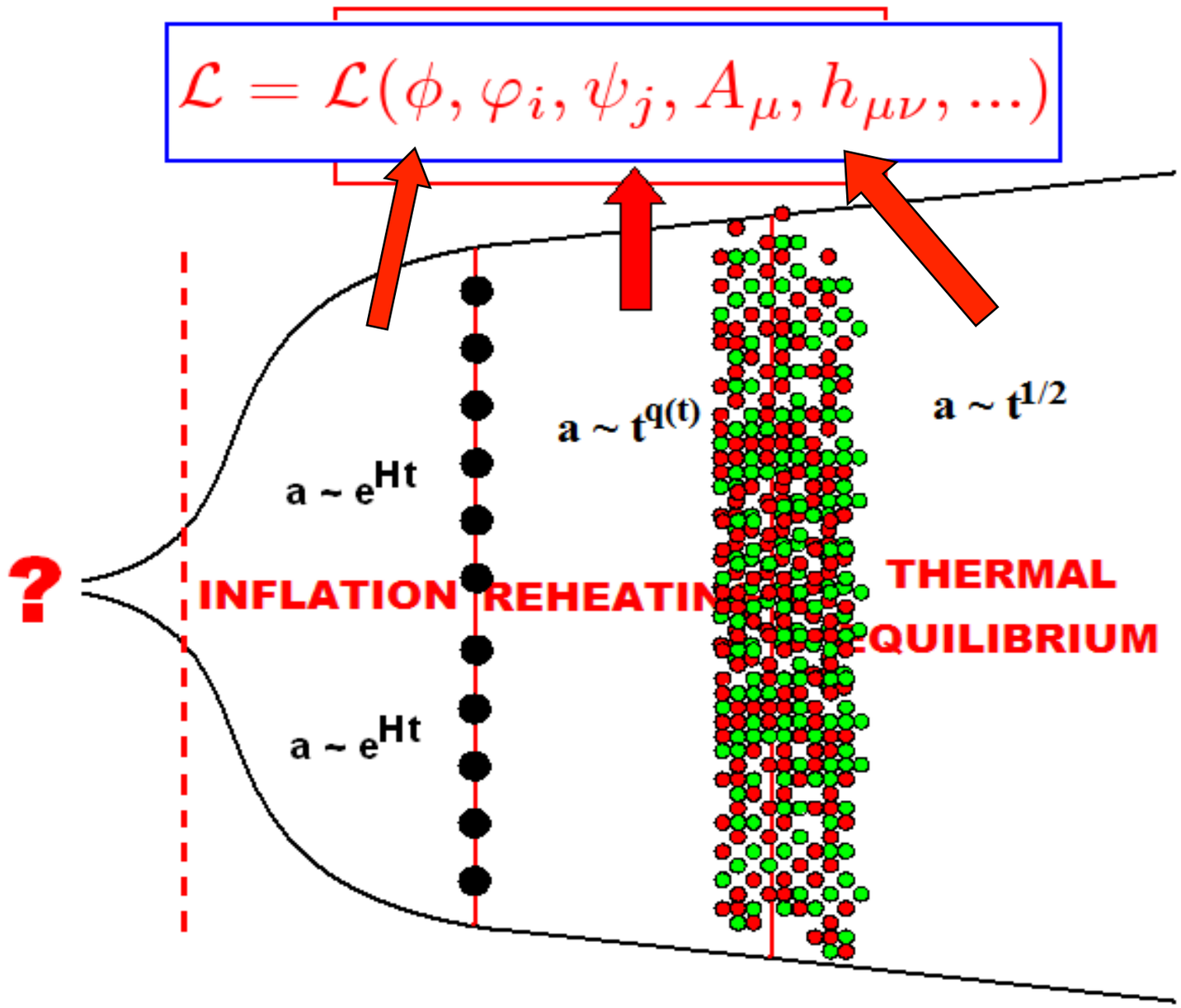

SCALAR REHEATING

1) $V(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Chaotic)

2) $V(\phi,\chi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\chi^2$ (Hybrid)

INFLATON **MATTER** **COUPLING**

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$$\begin{cases} \ddot{\phi}(t) + 3H\dot{\phi} + V'(\phi) = 0 \quad (\textbf{Inflaton Zero-Mode} : \text{Damped Oscillator}) \\ \Box\phi_k + F(\int dq\phi_q\chi_{|k-q|})\phi_k + ... = 0 \quad (\textbf{Inflaton Fluctuations}) \\ \Box\chi_k + F(\int dq\chi_q, \phi_{|k-q|})\chi_k + ... = 0 \quad (\textbf{Matter Fluctuations}) \end{cases}$$

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DYNAMICS:

Non-Linear, Non-Perturbative & Far-From-Equilibrium

$\mathbf{k}_i \pm \Delta\mathbf{k}_i \rightarrow \varphi_k(t), n_k(t) \sim exp\{\mu_k\, t\}$

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1) Chaotic Scenarios: PARAMETRIC RESONANCE

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MATTER FIELD FLUCTUATIONS

INFLATON

$$\left.\begin{array}{ll} \text{Massless : (n = 4)} & X_k''+(\kappa^2+\frac{g^2}{\lambda}cn^2(z))X_k=0 \quad \text{(Lamé Eq.)} \\ \text{Massive : (n = 2)} & X_k''+(A_k-2q\cos(2z))X_k=0 \quad \text{(Mathieu Eq.)} \end{array}\right\}$$

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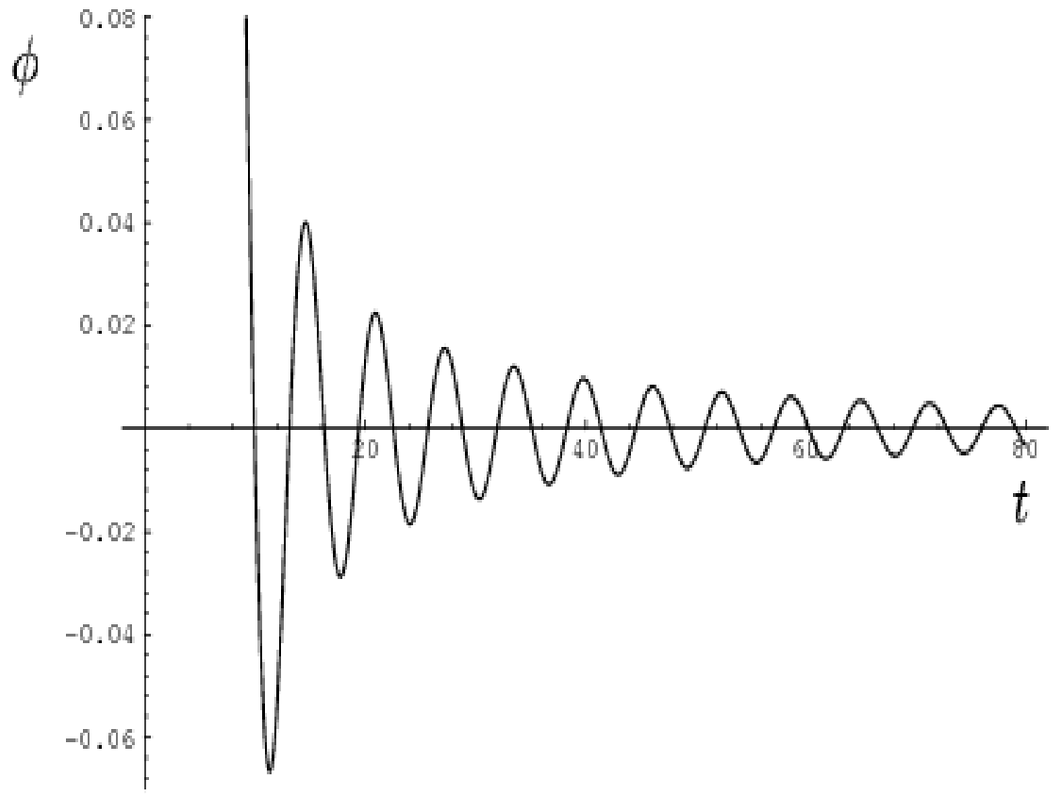

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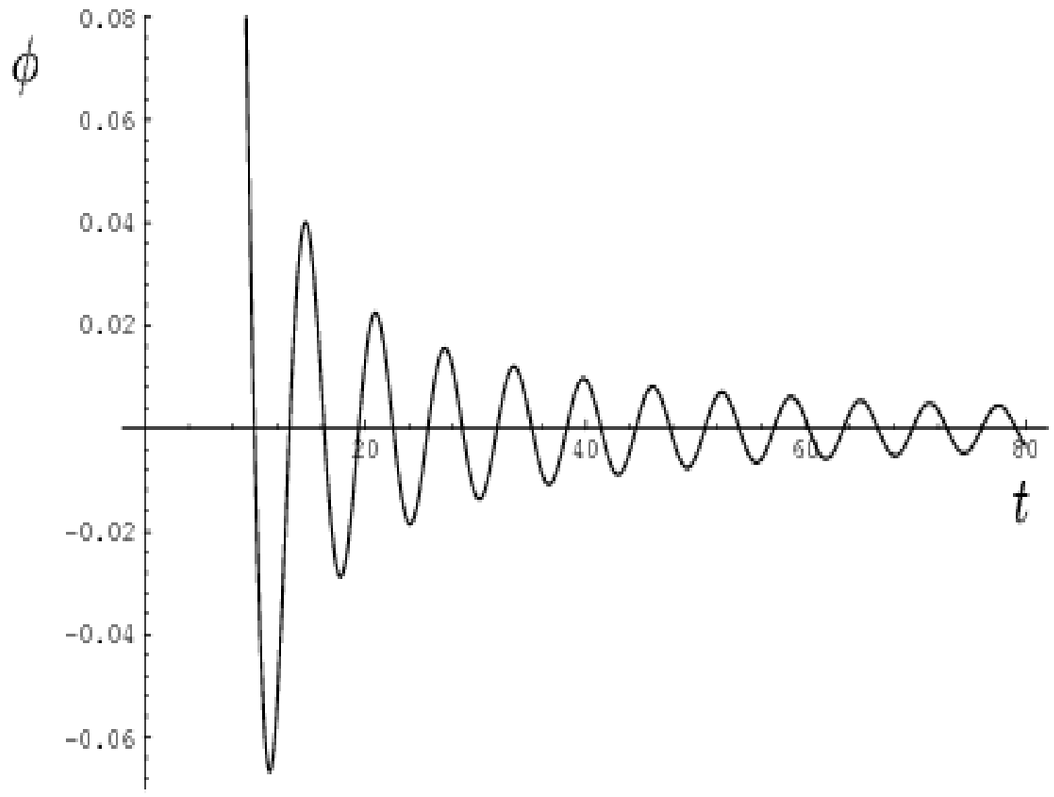

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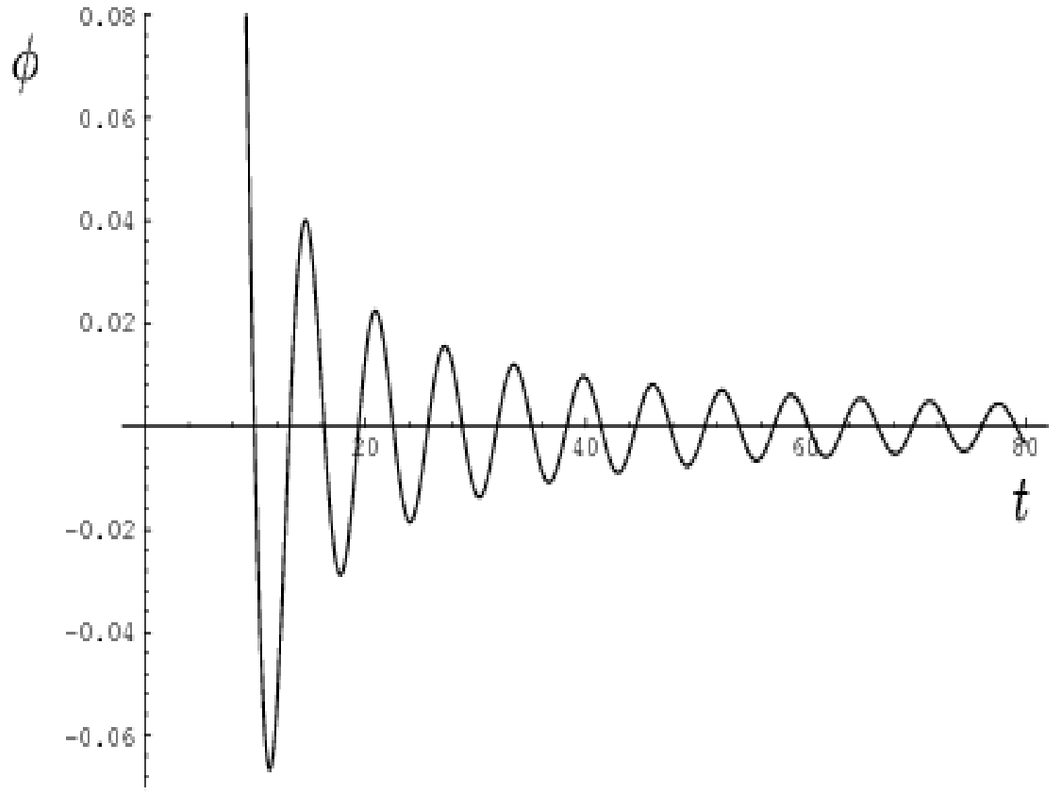

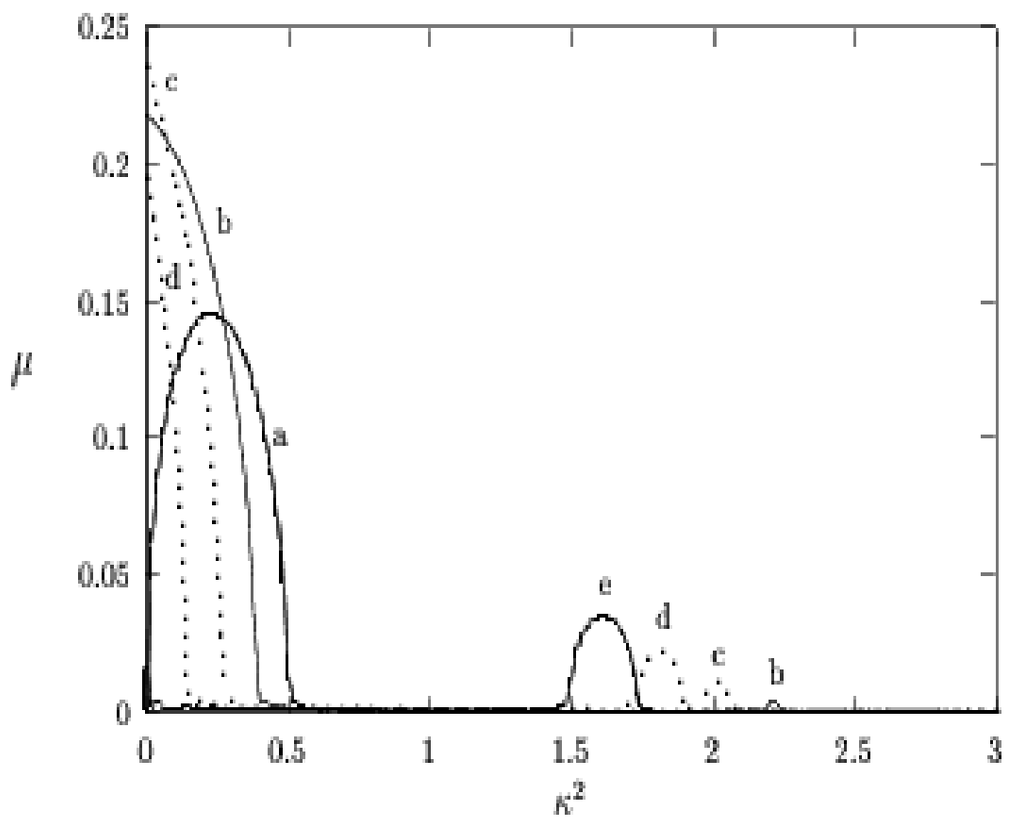

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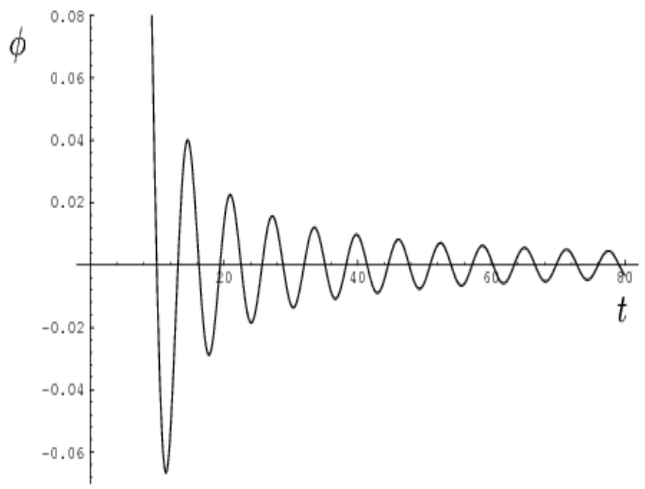

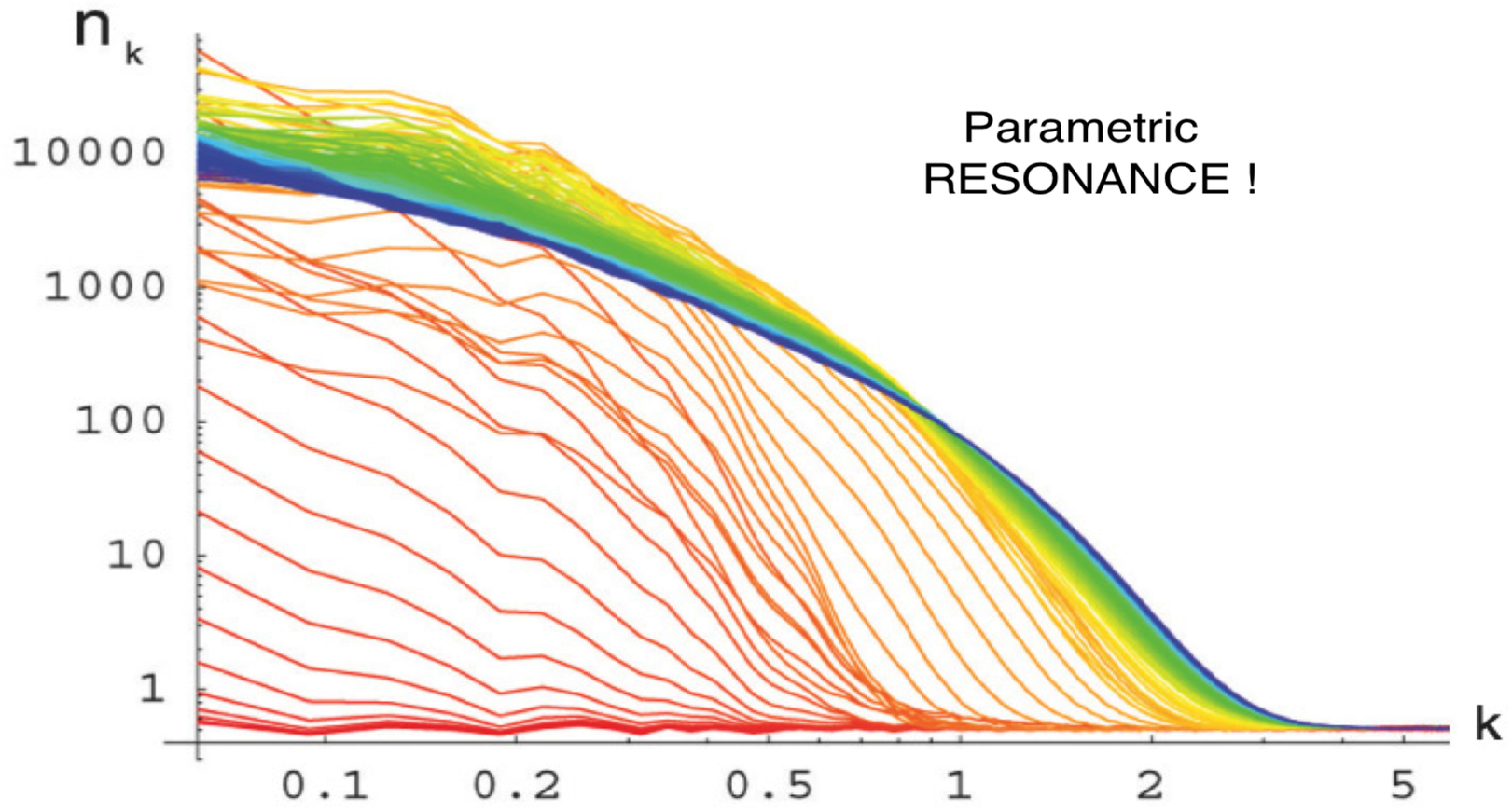

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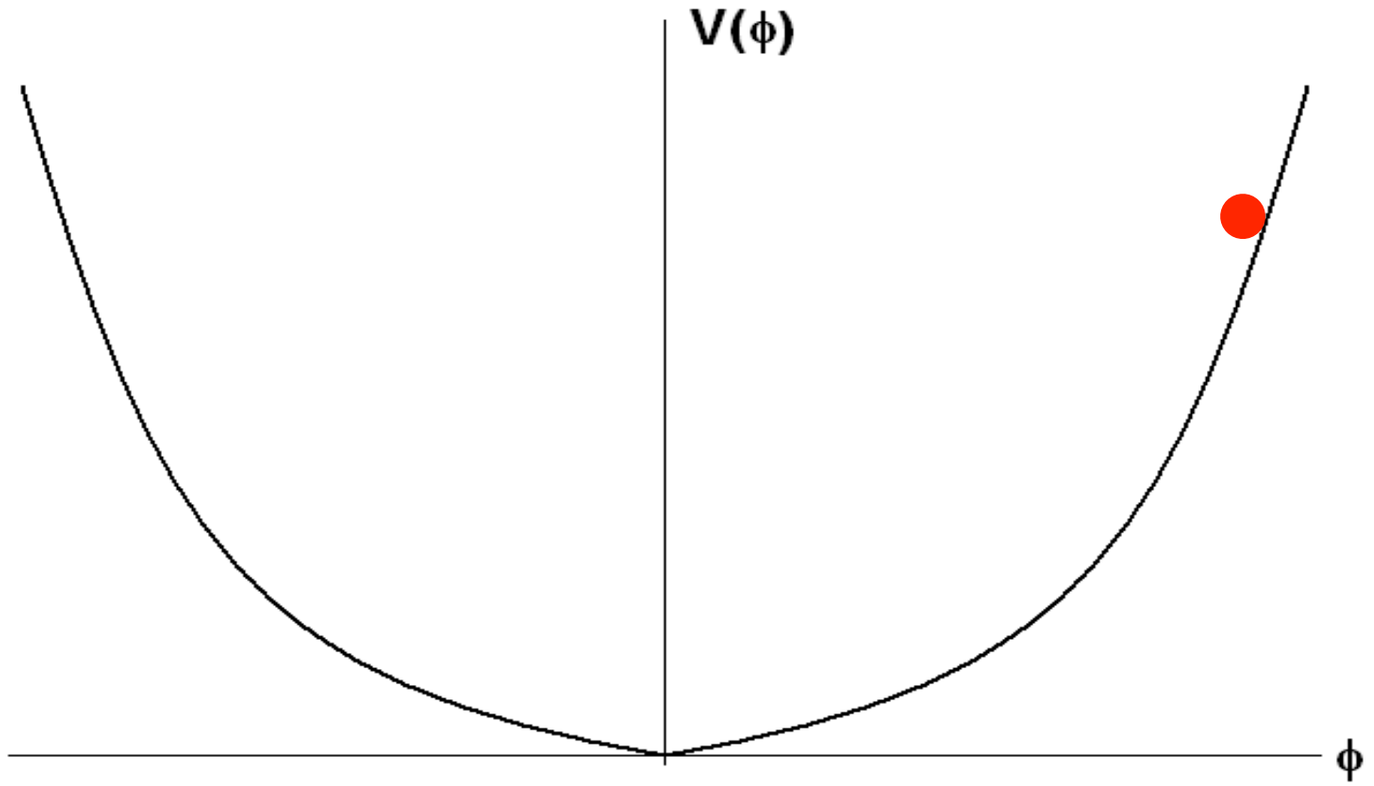

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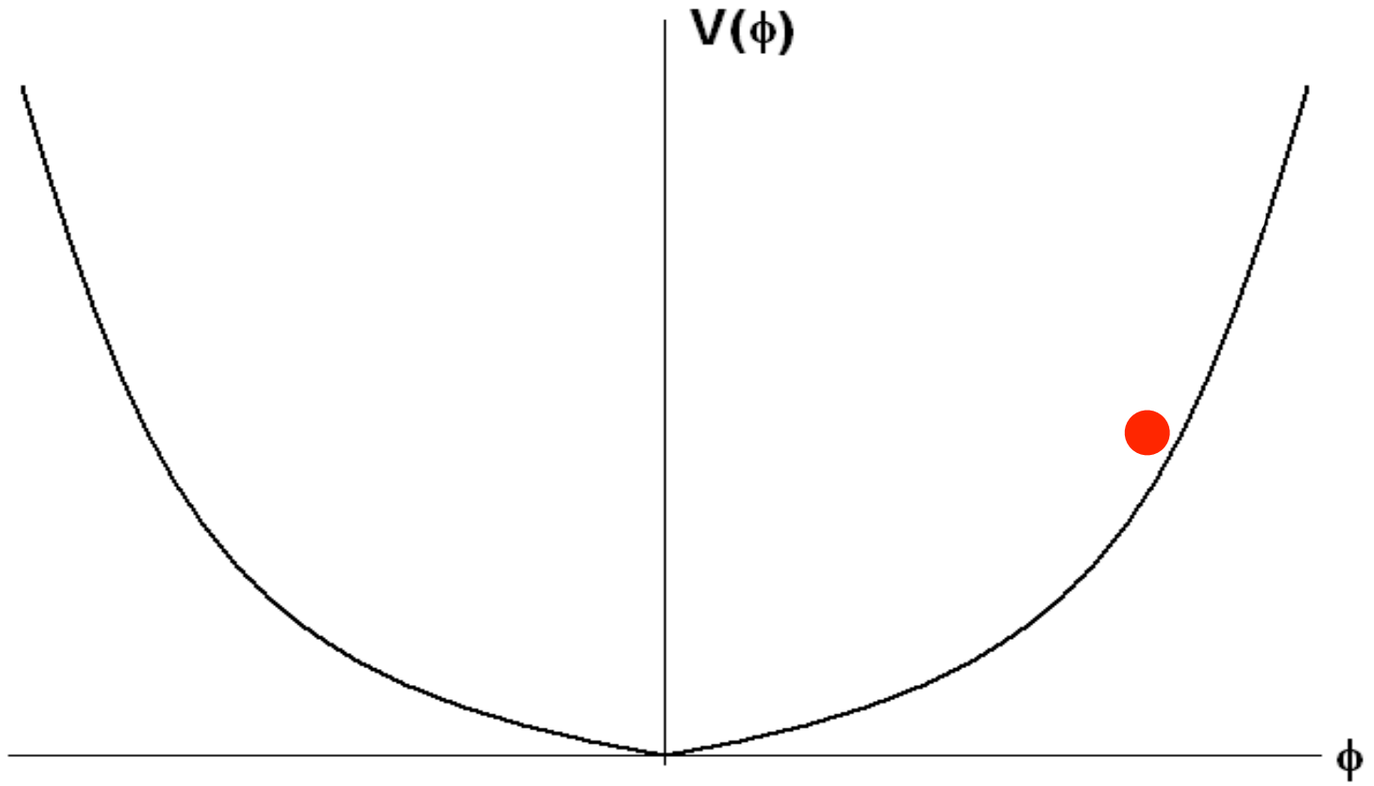

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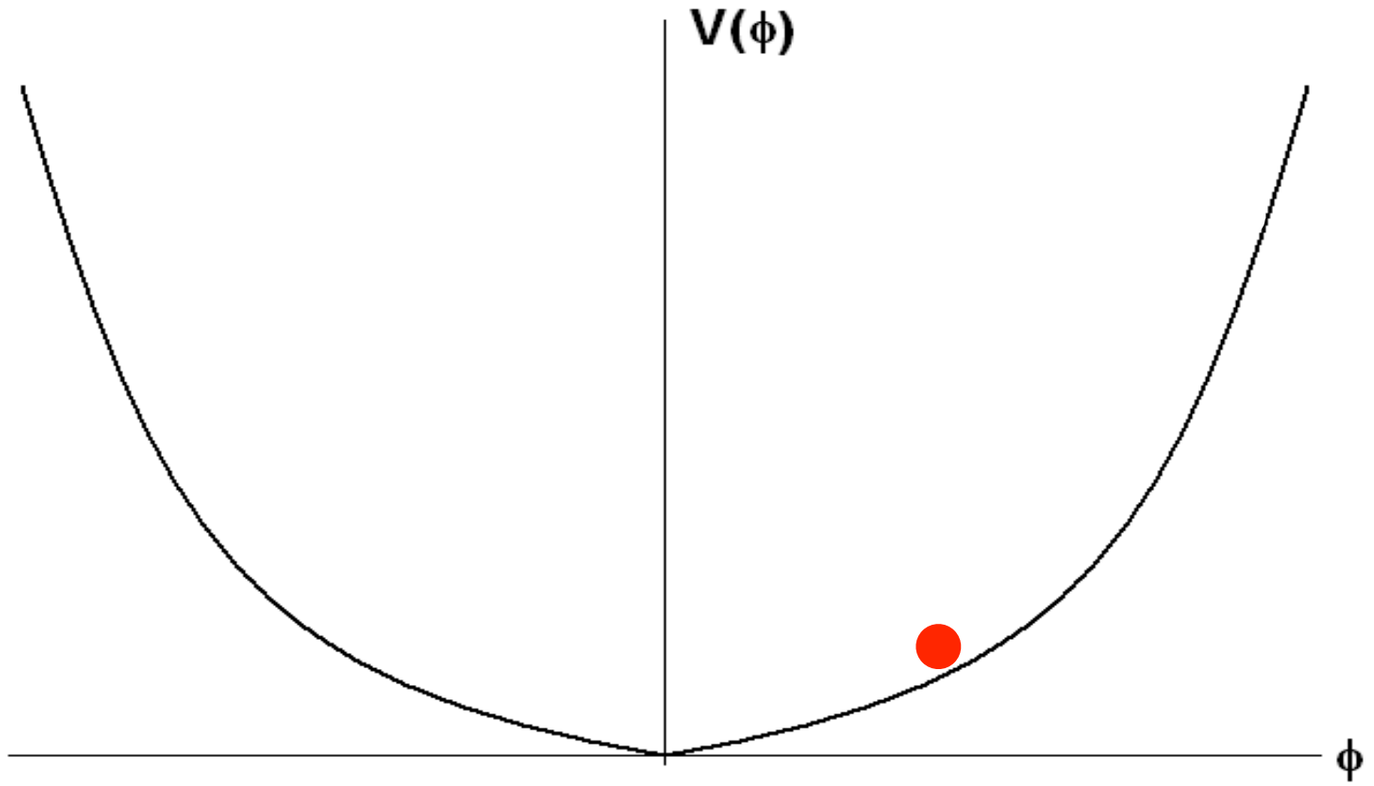

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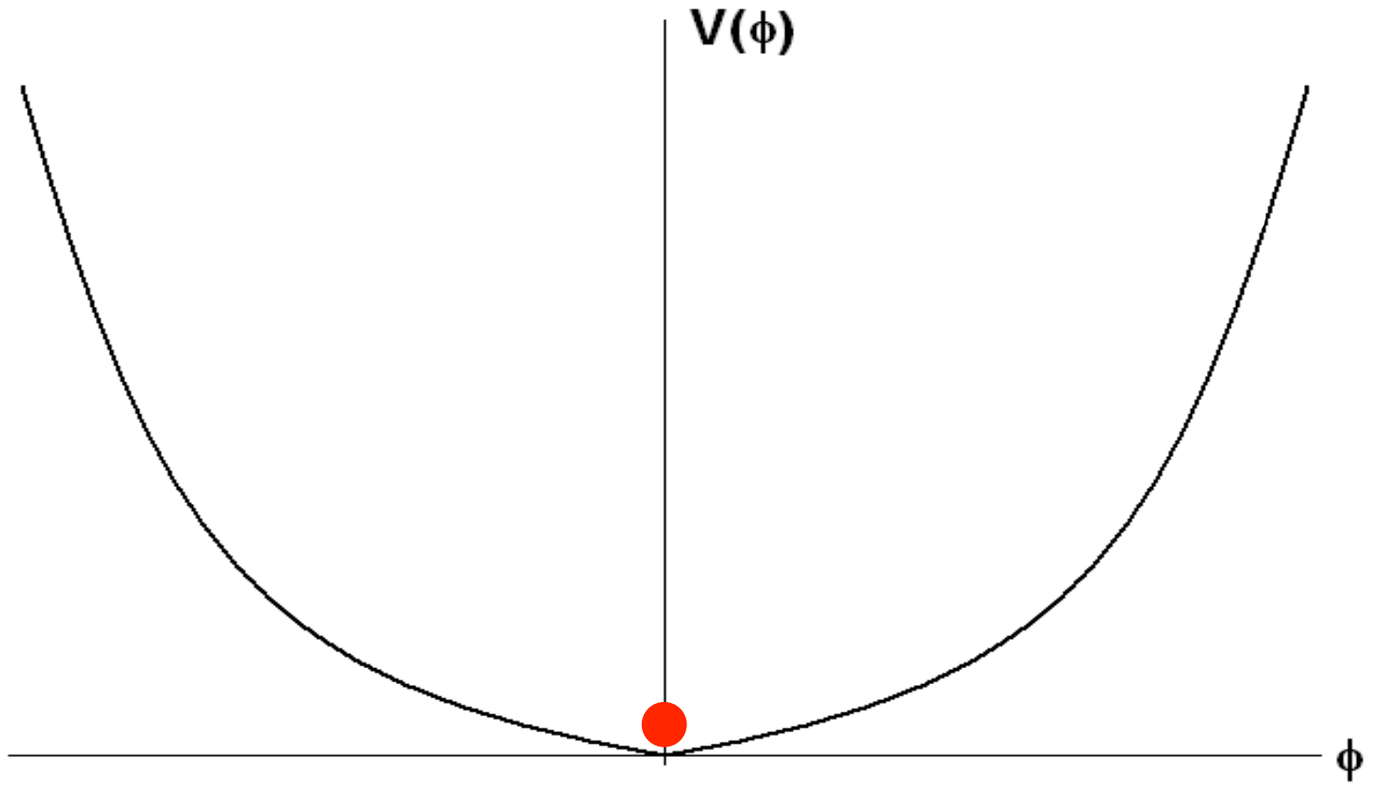

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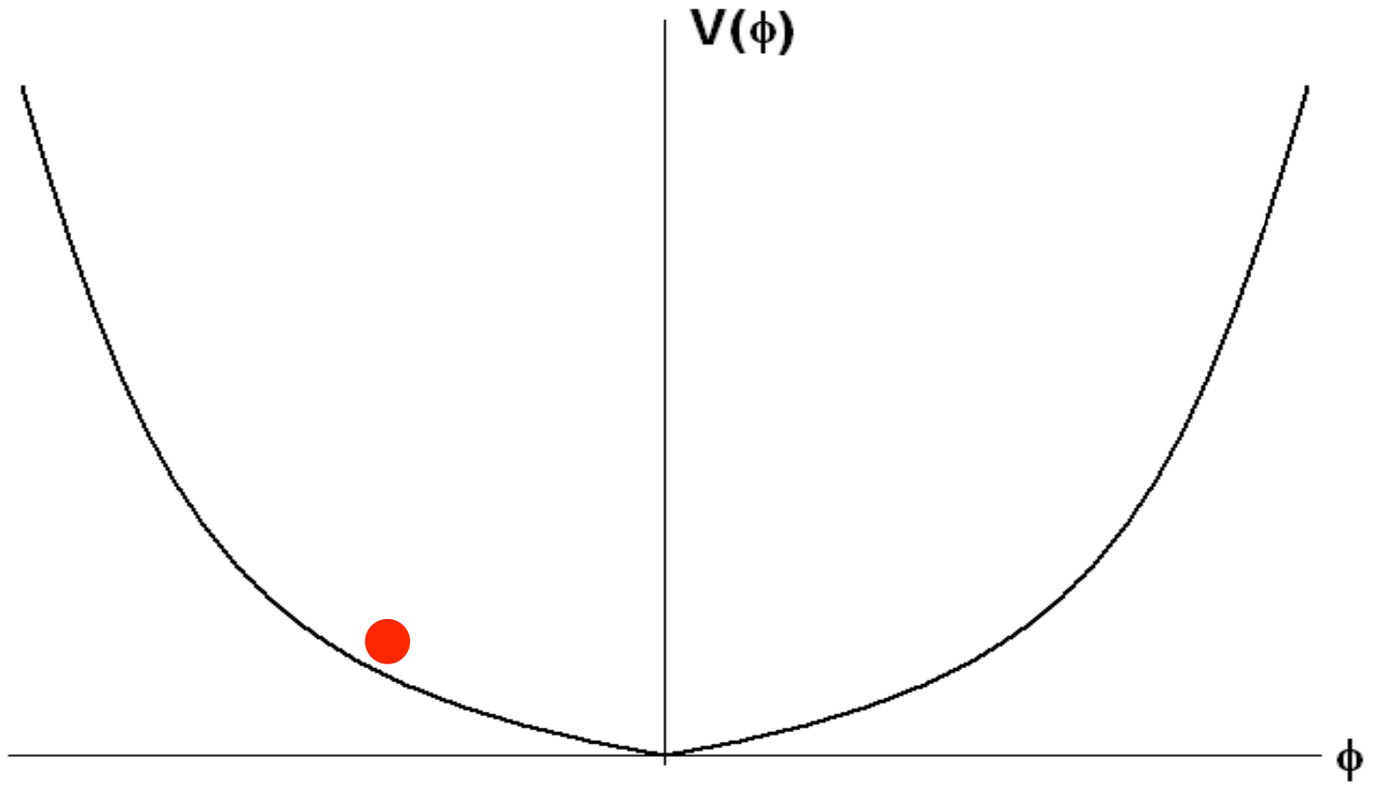

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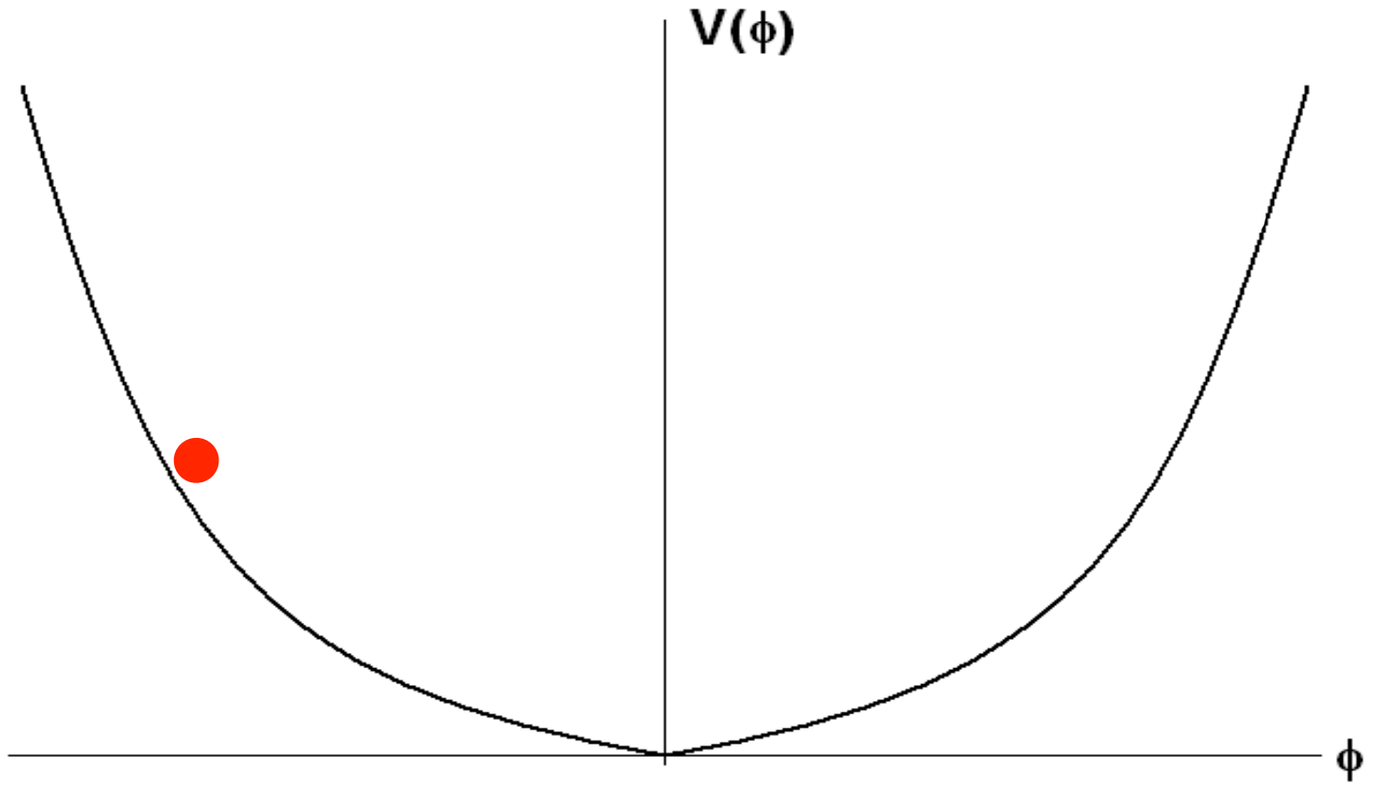

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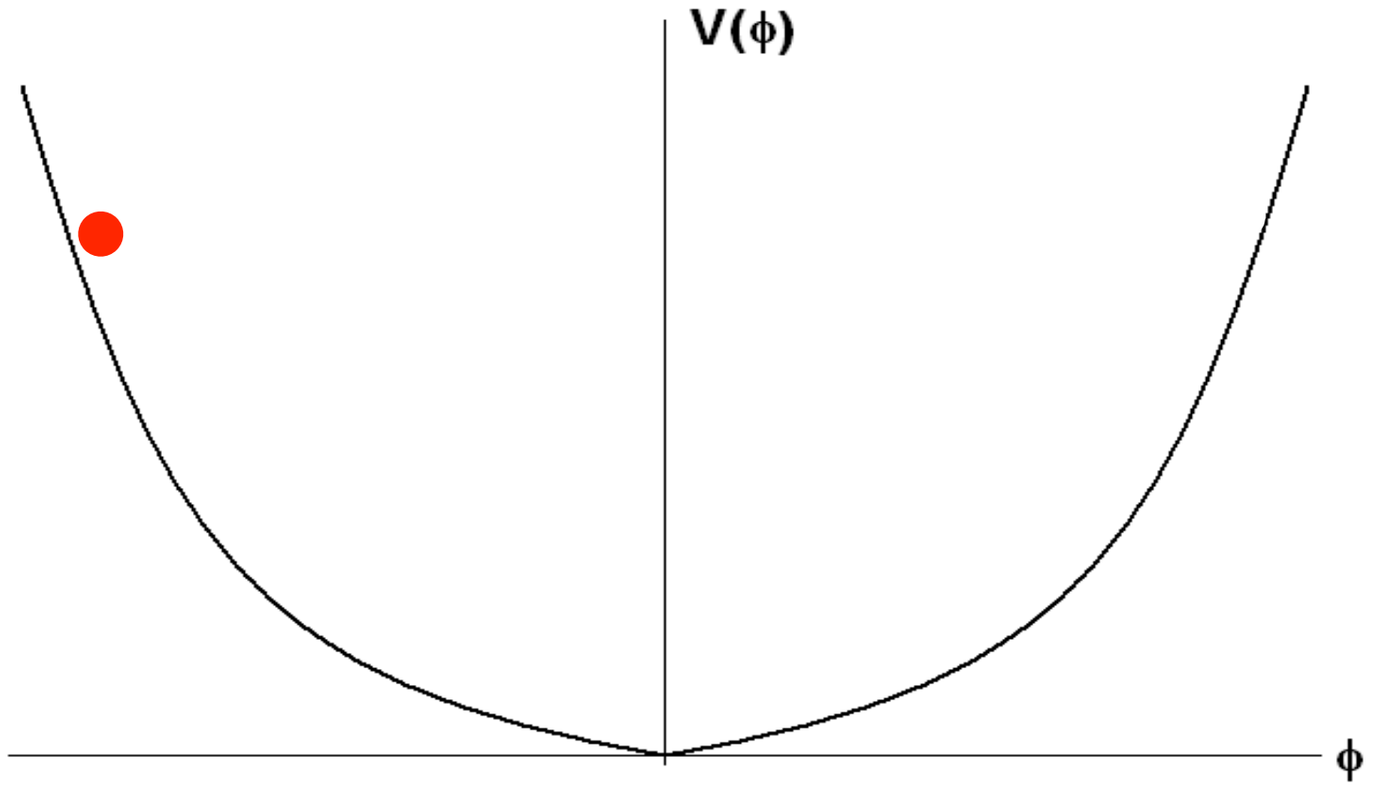

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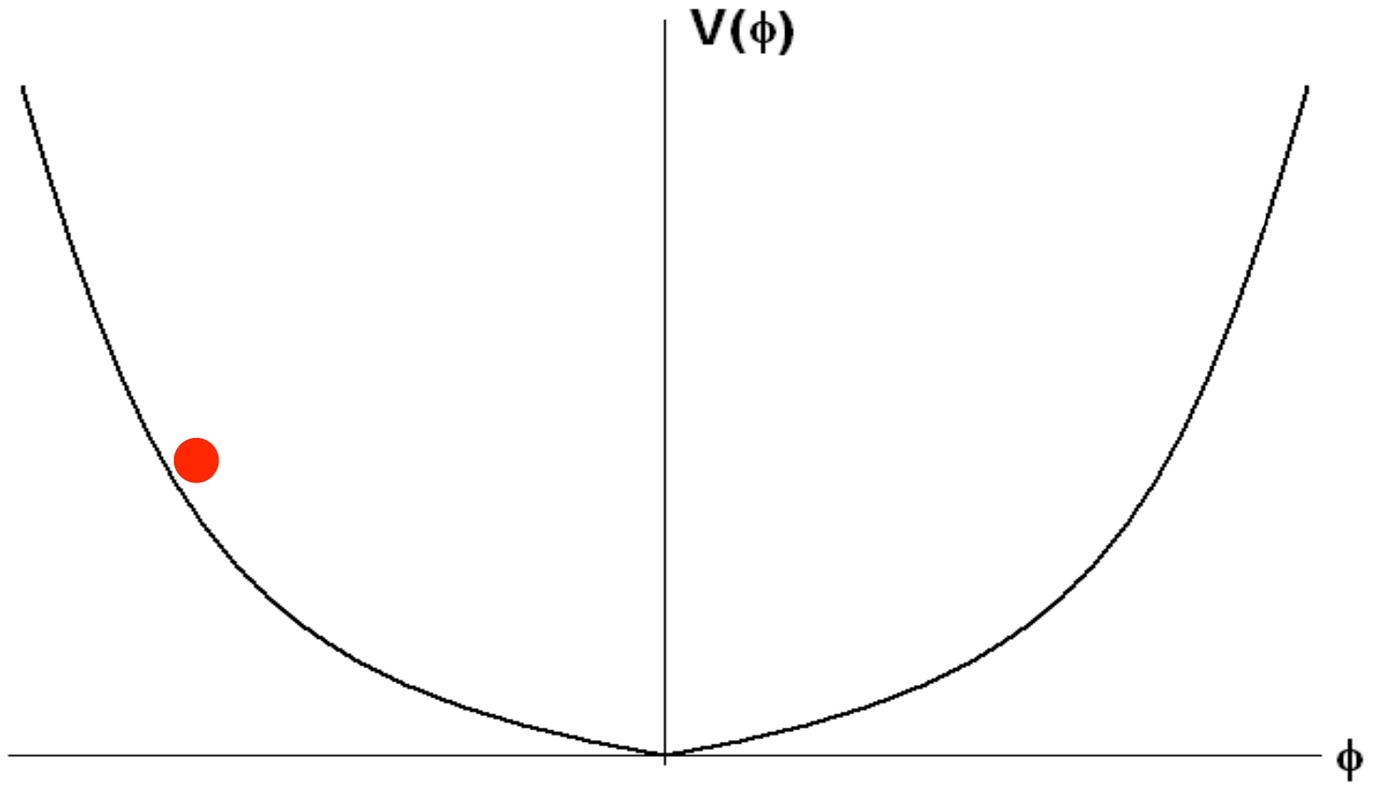

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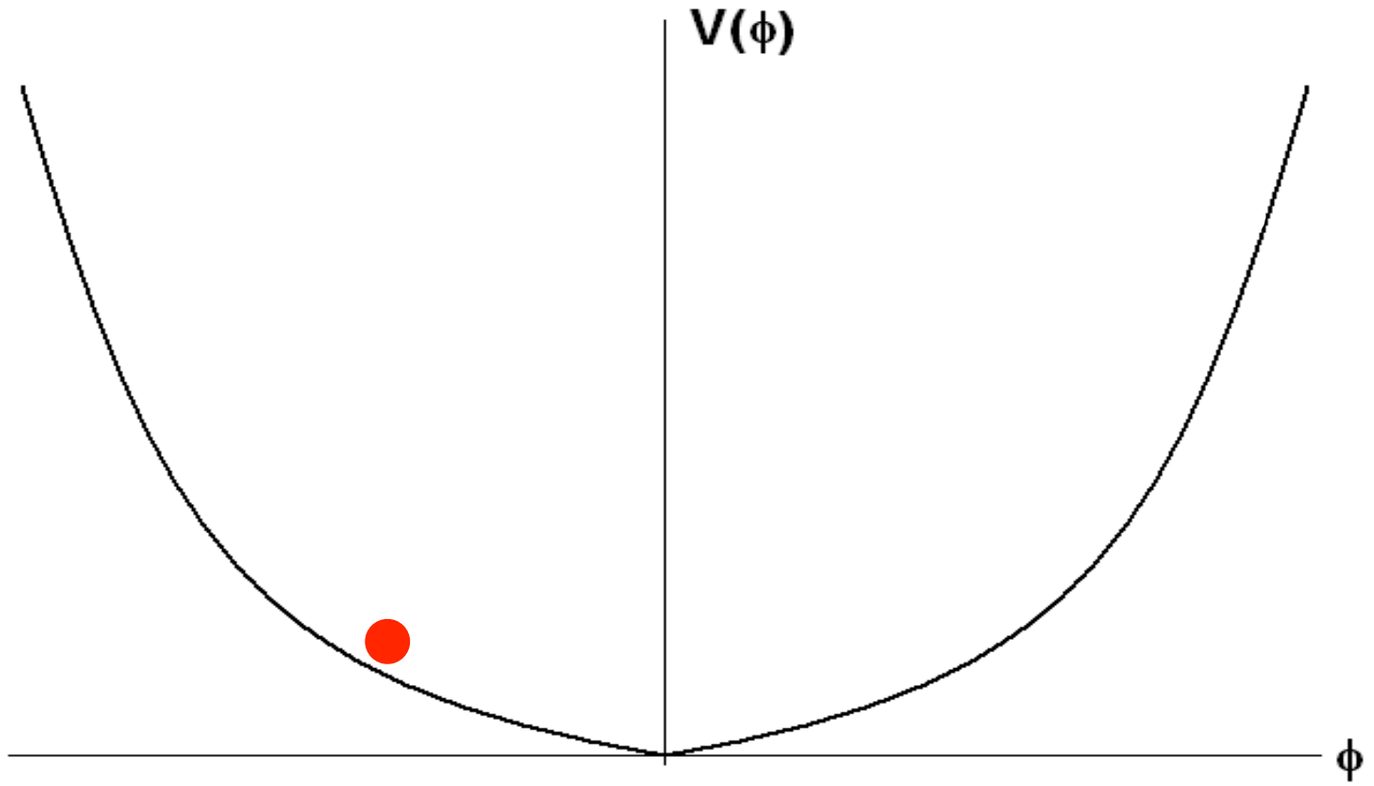

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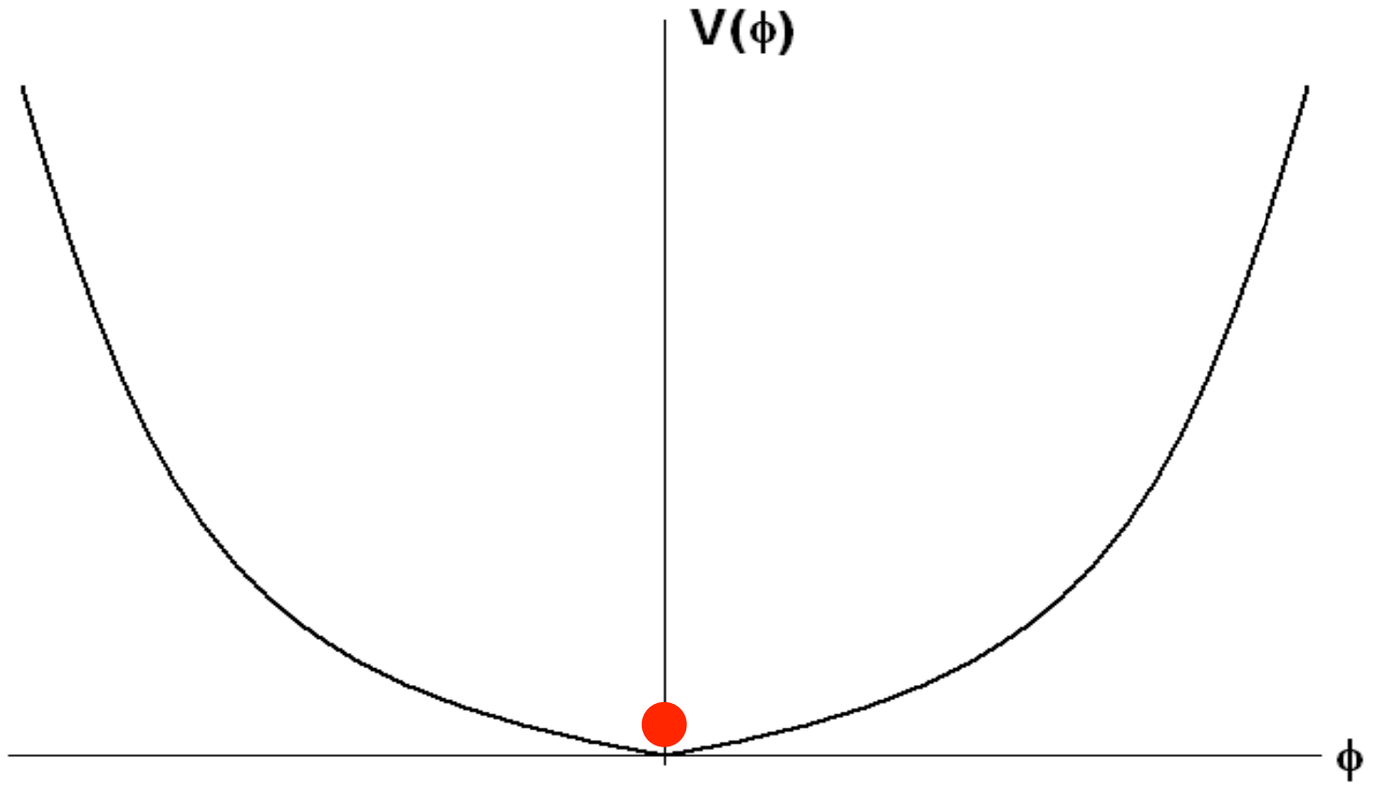

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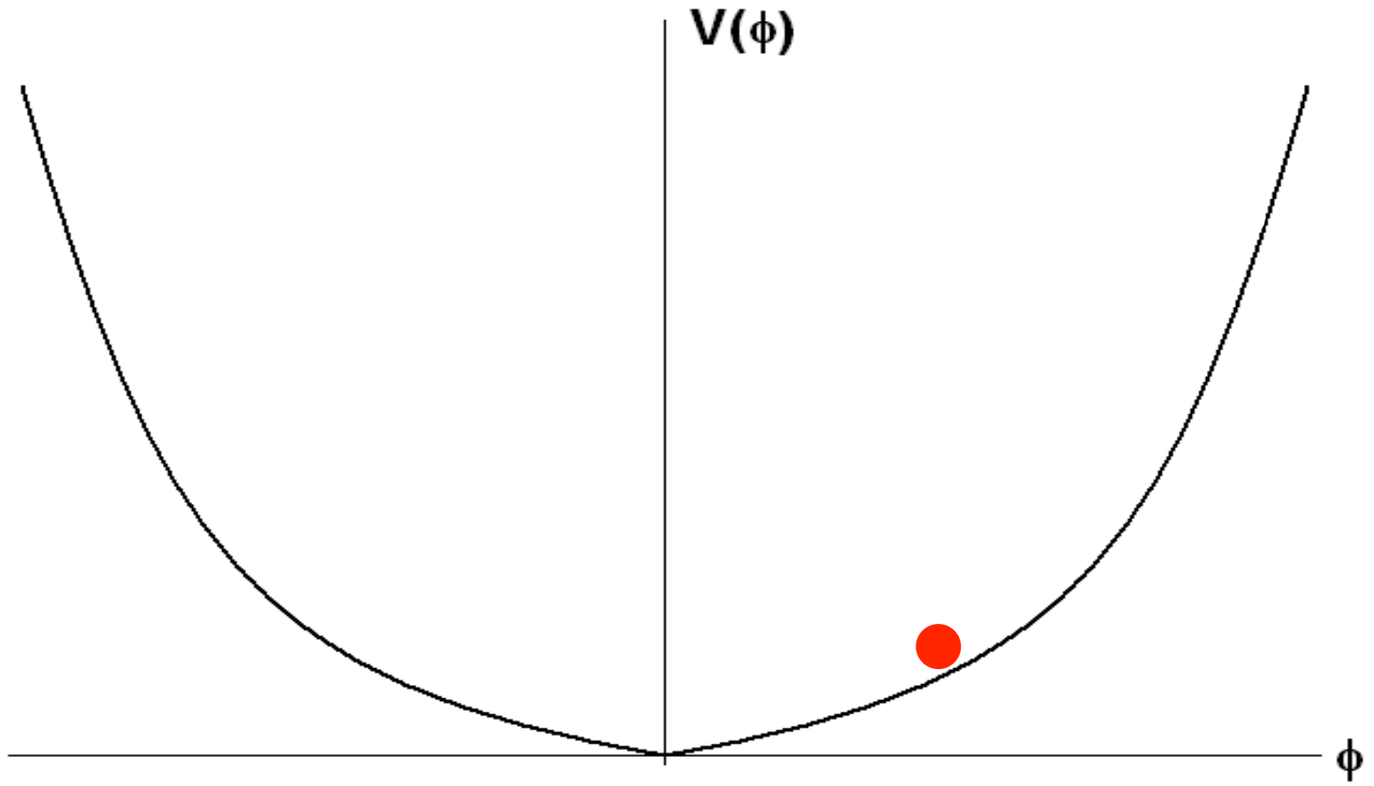

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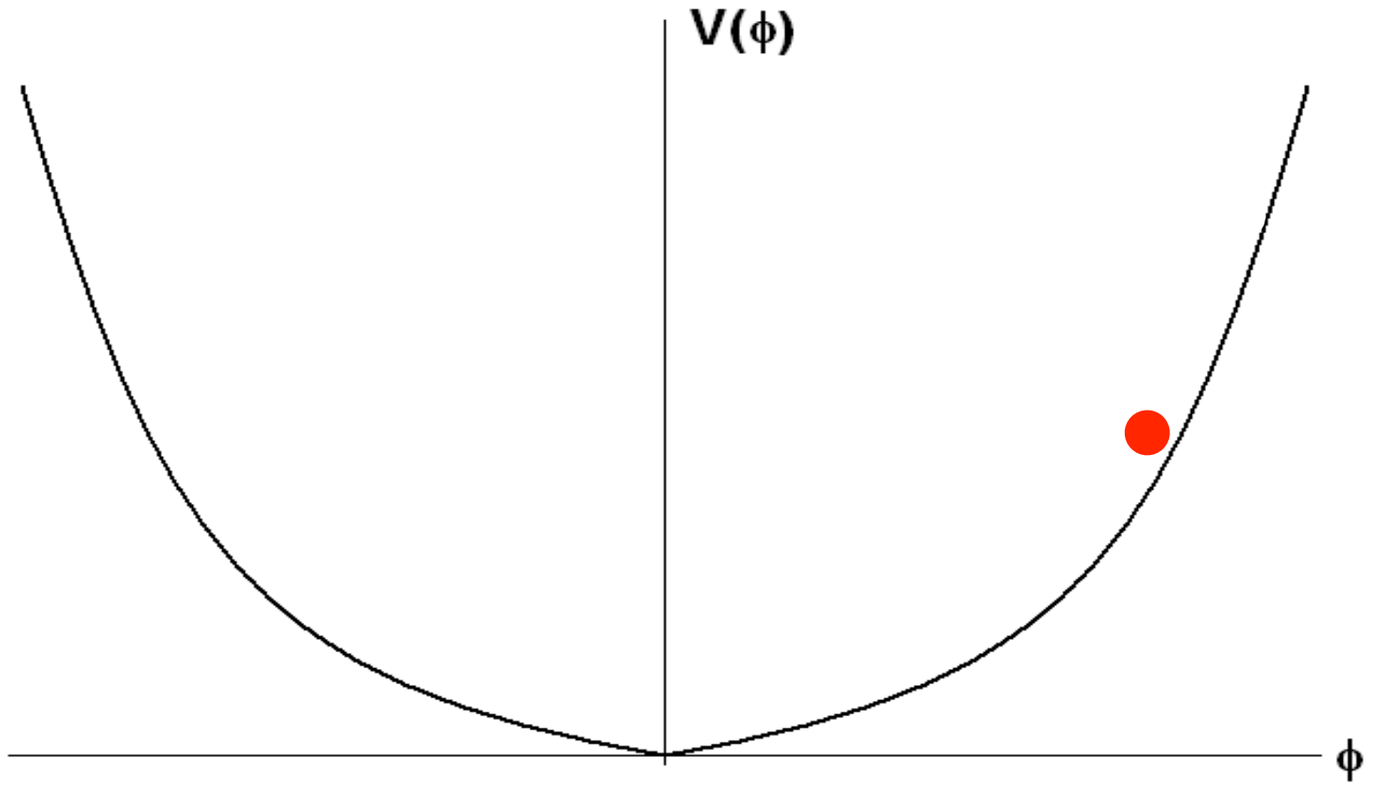

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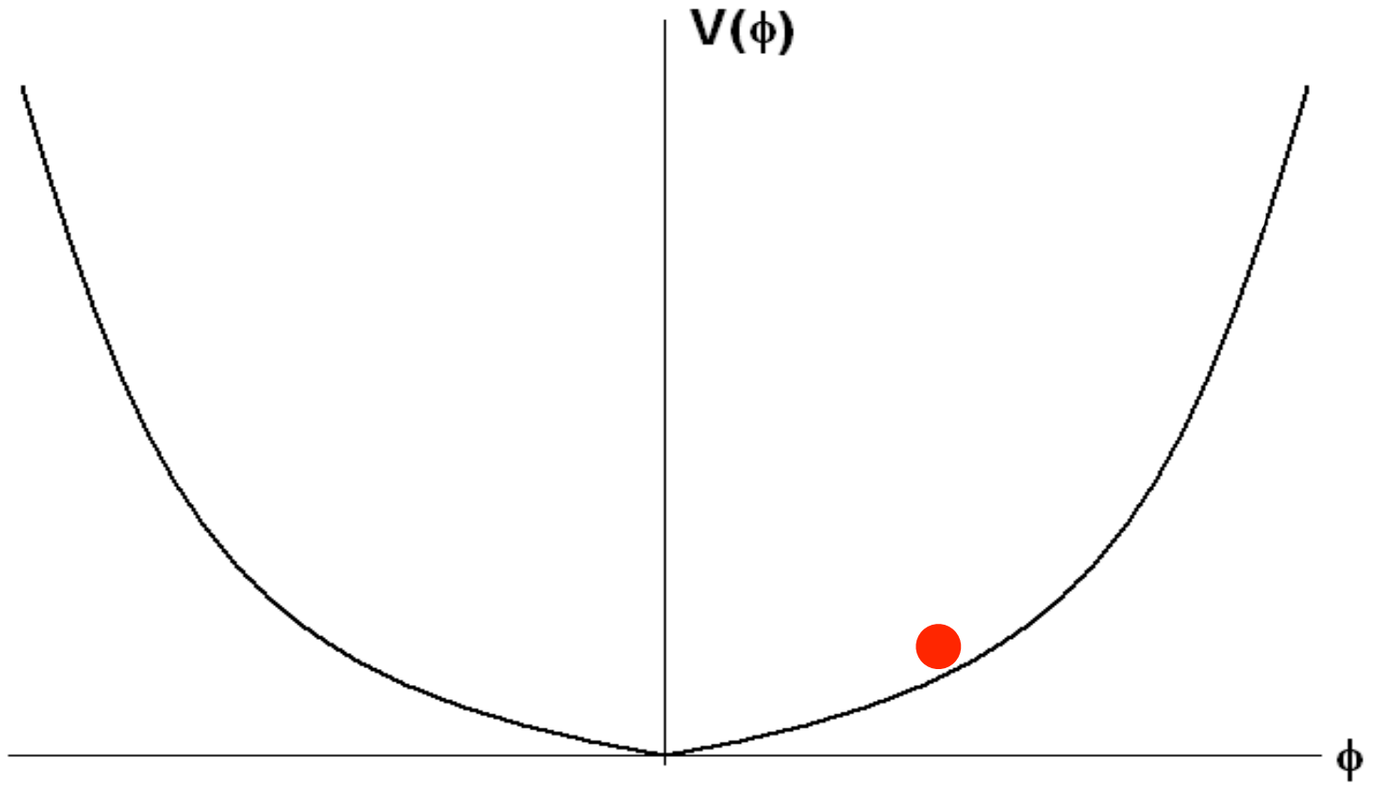

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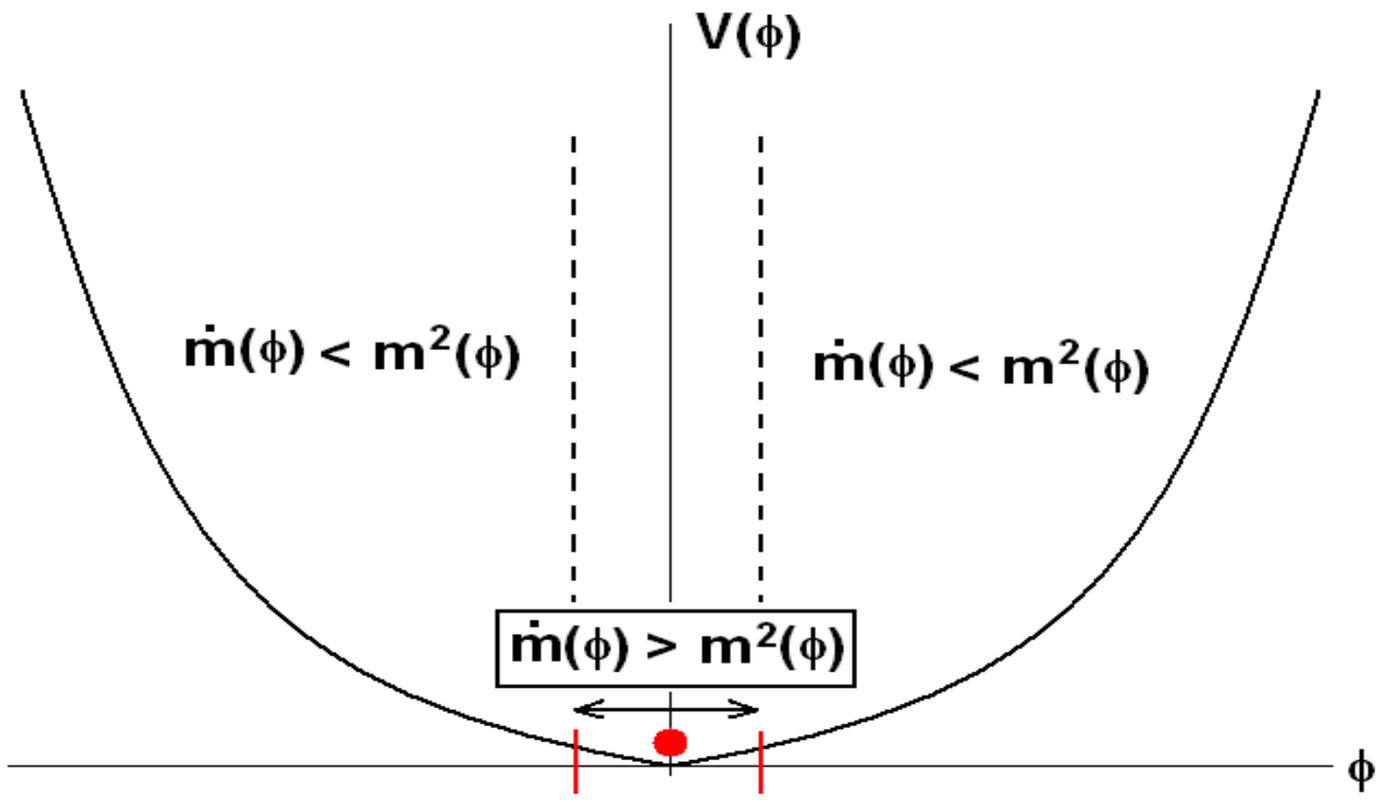

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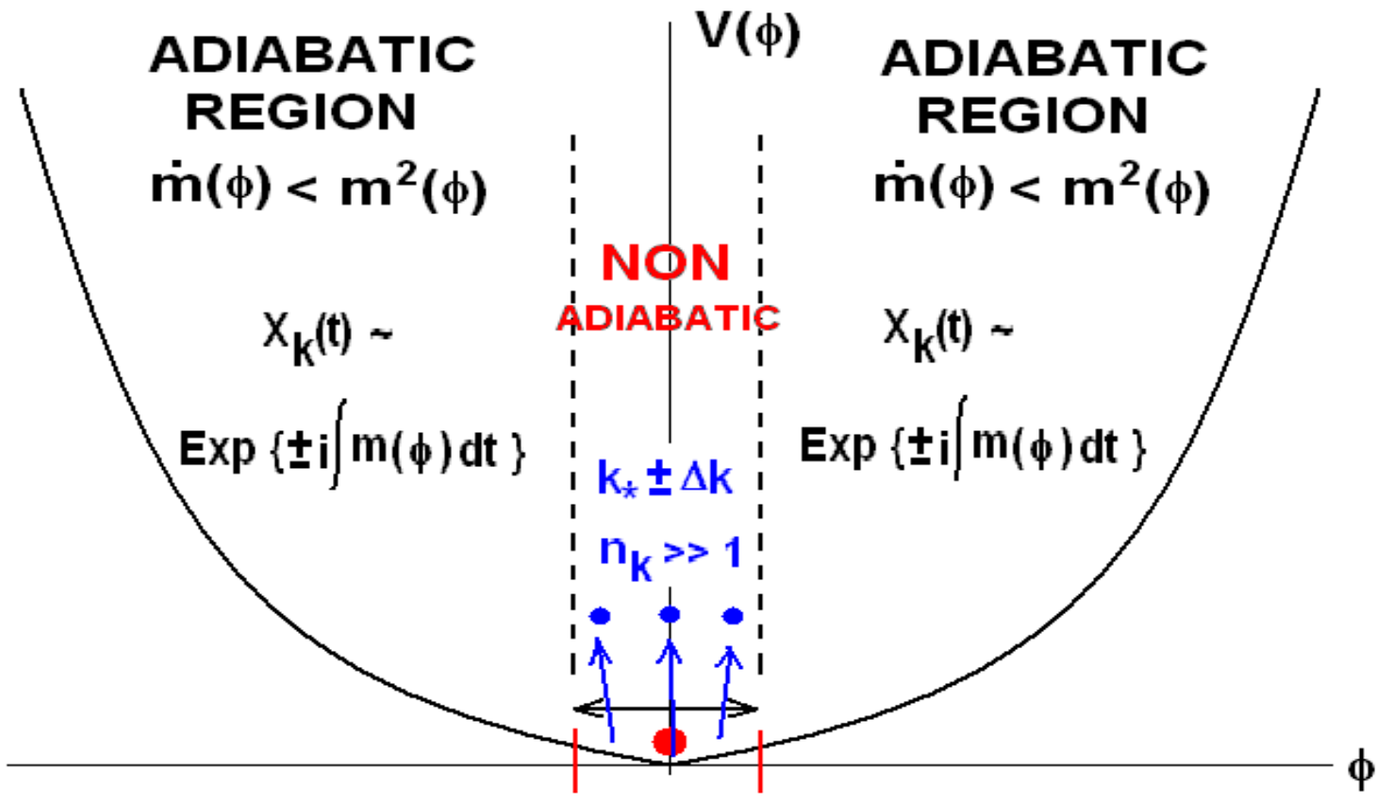

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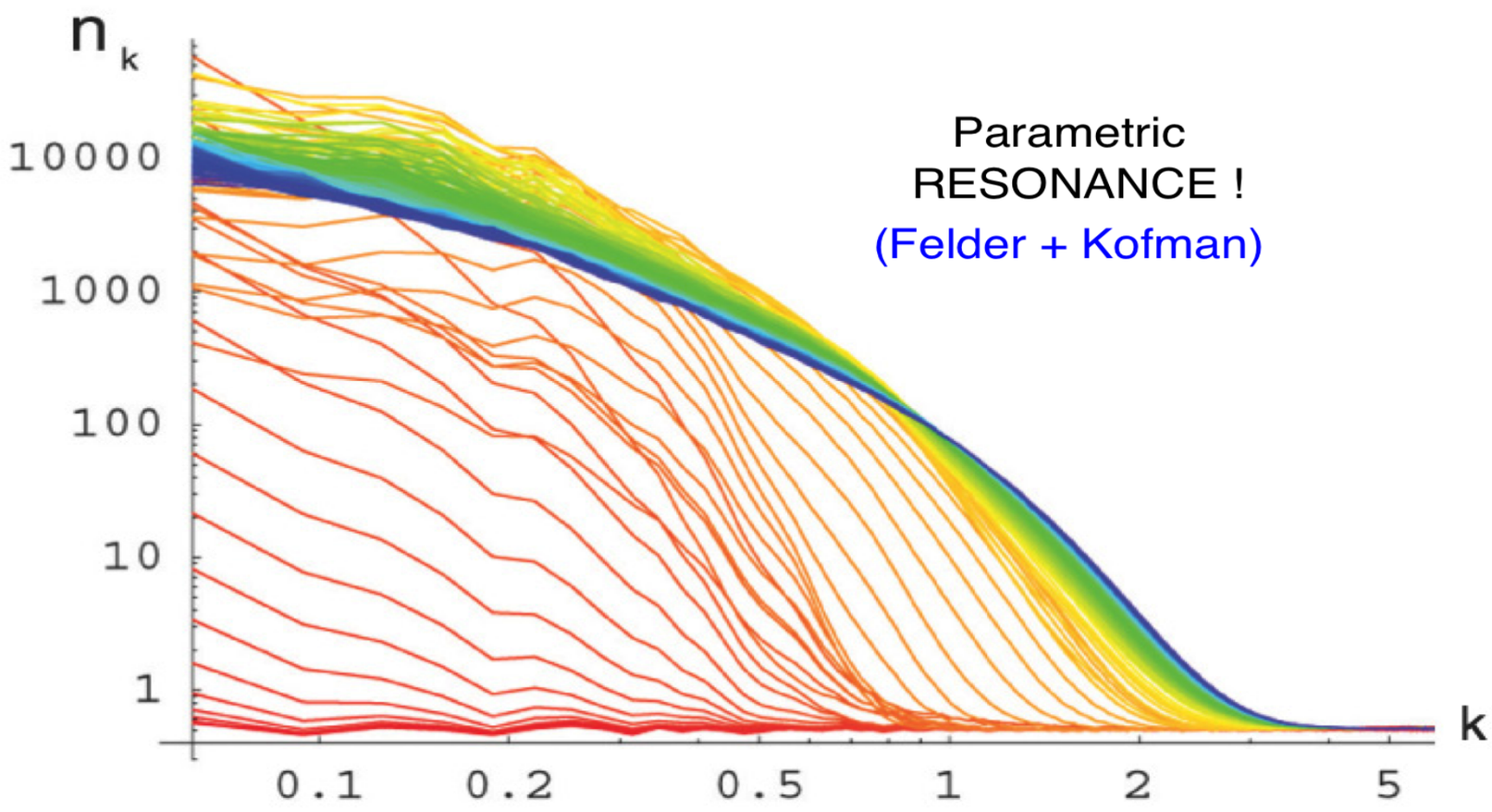

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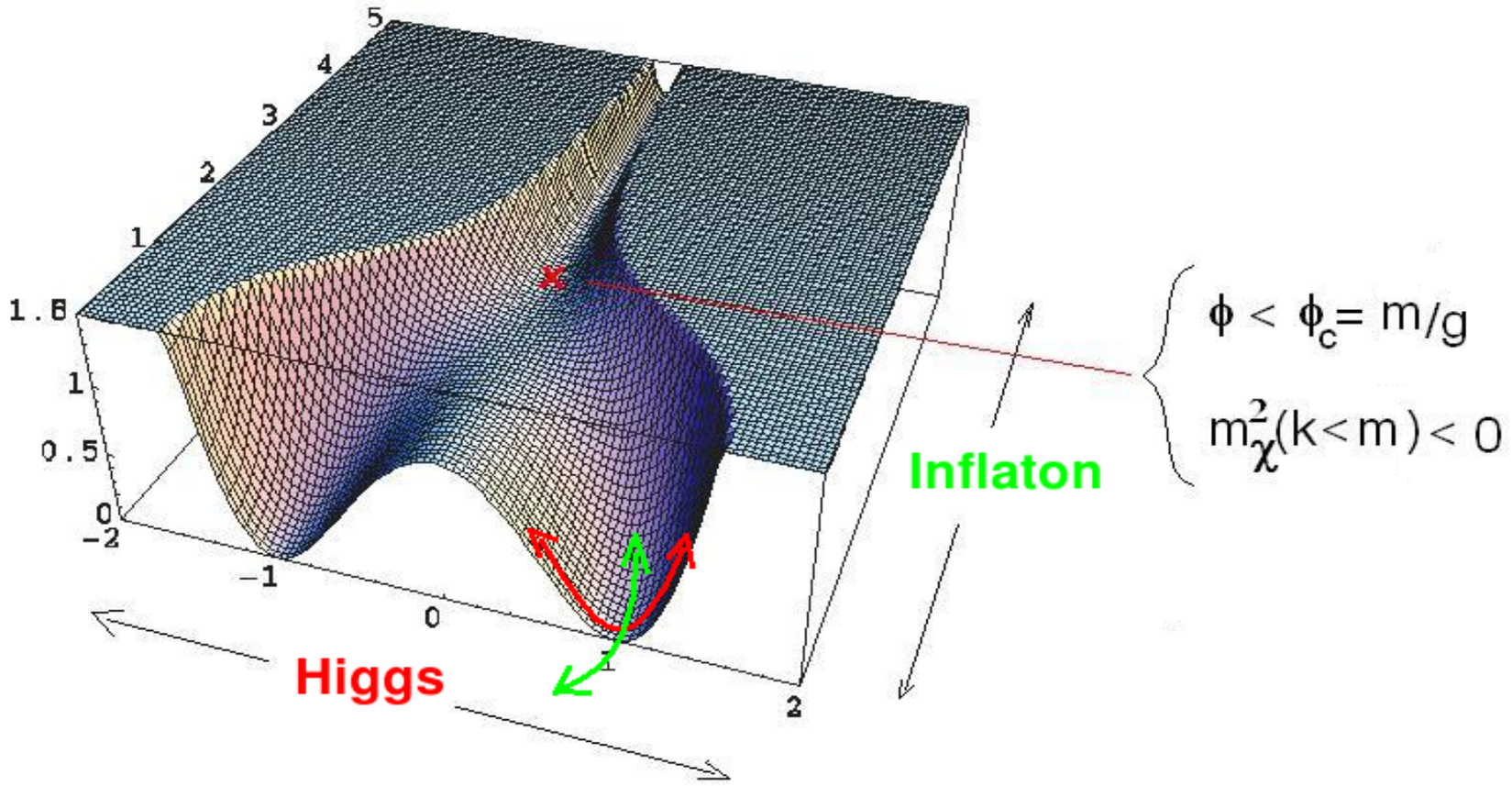

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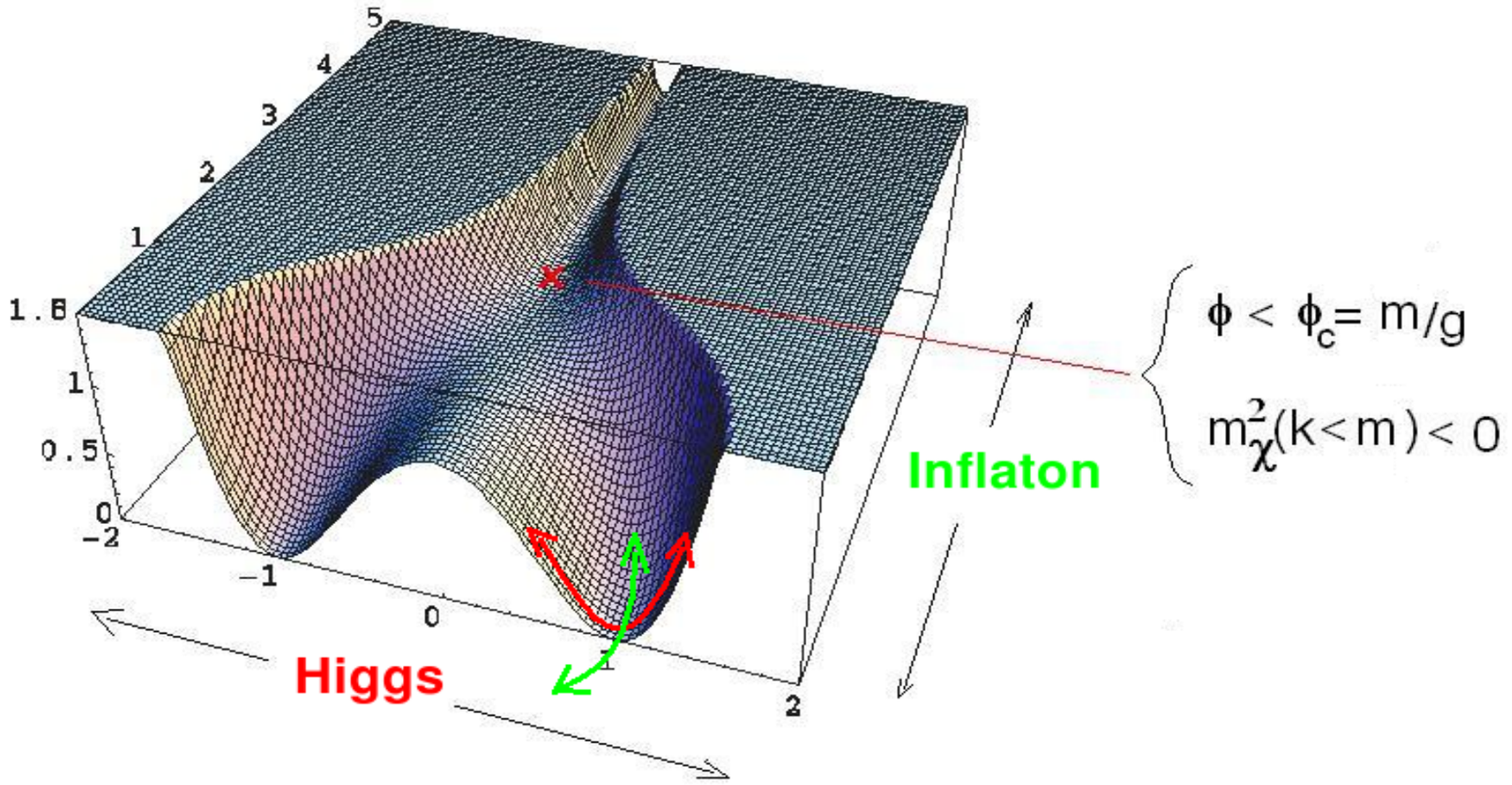

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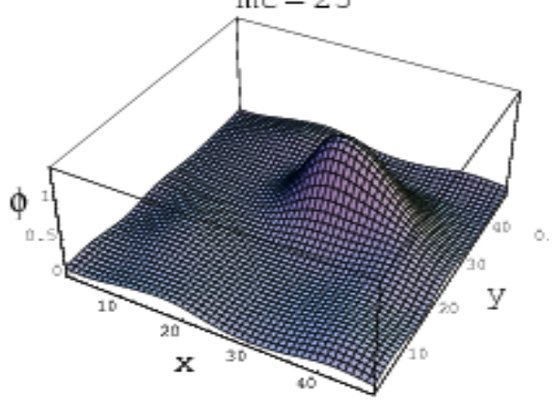

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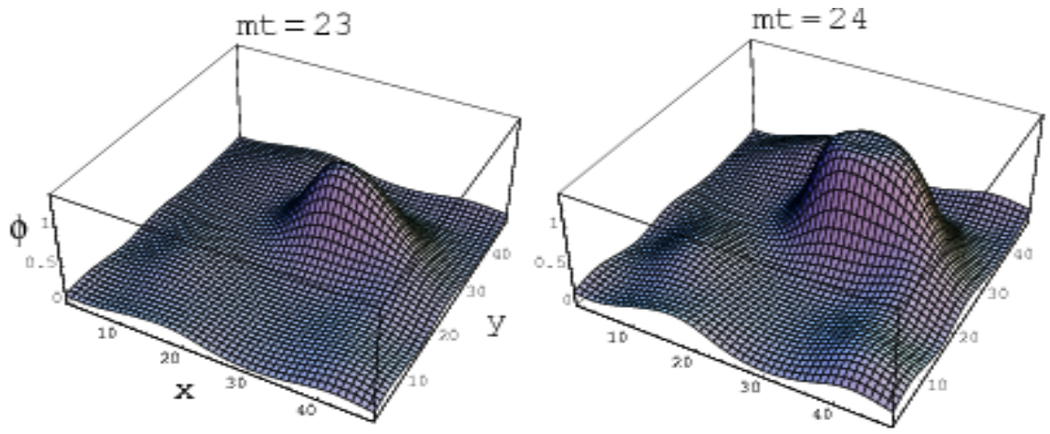

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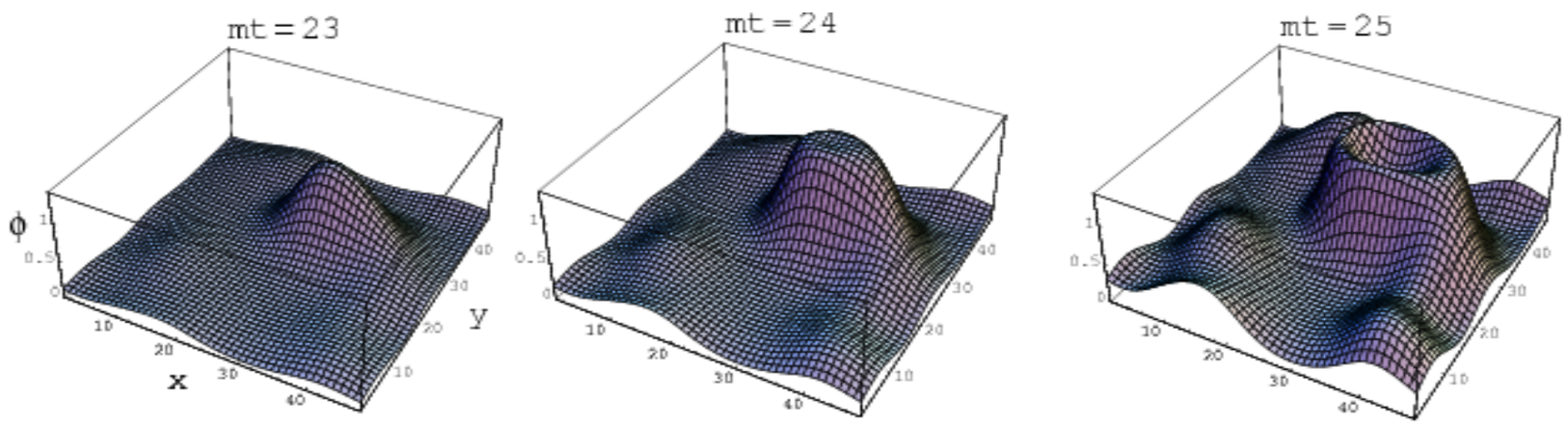

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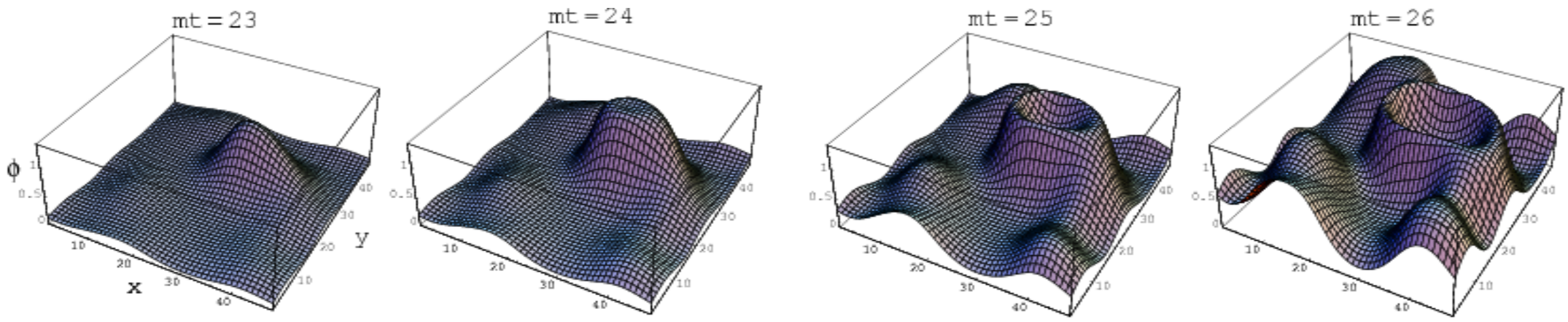

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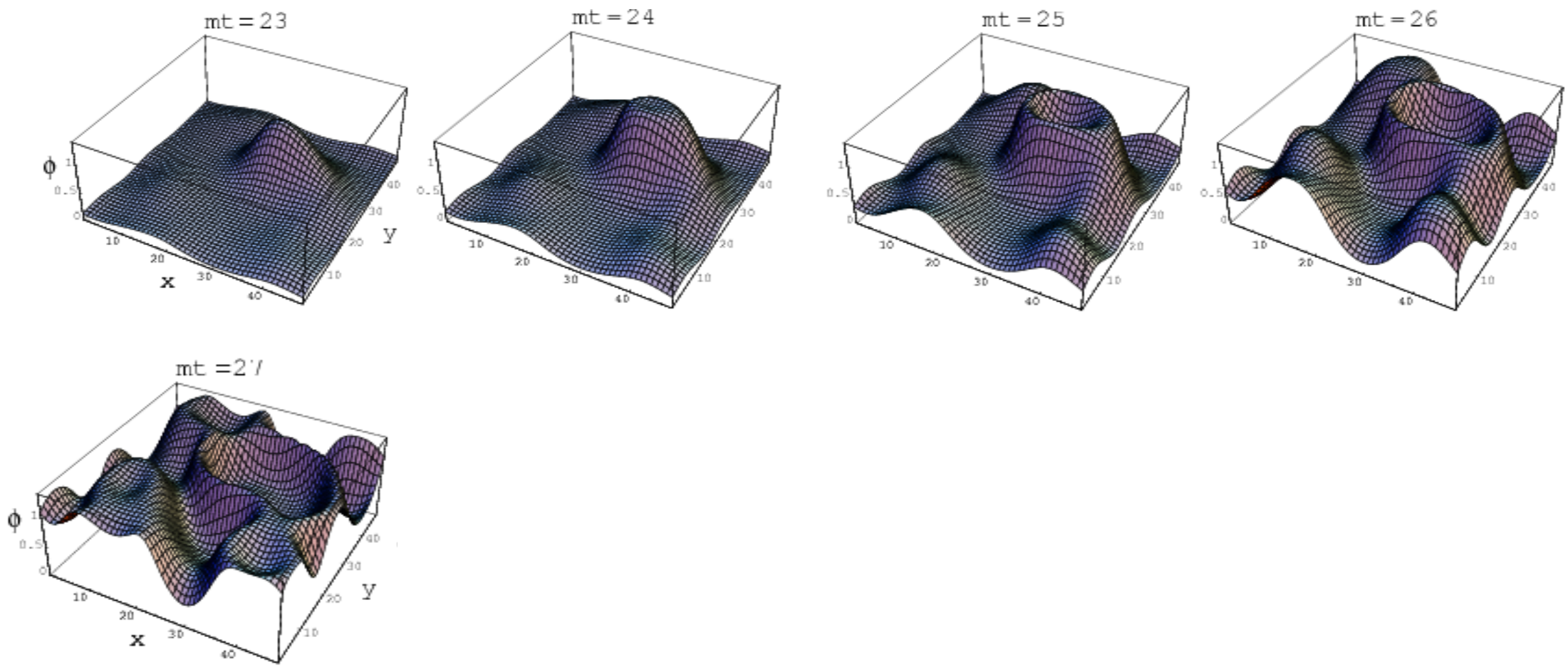

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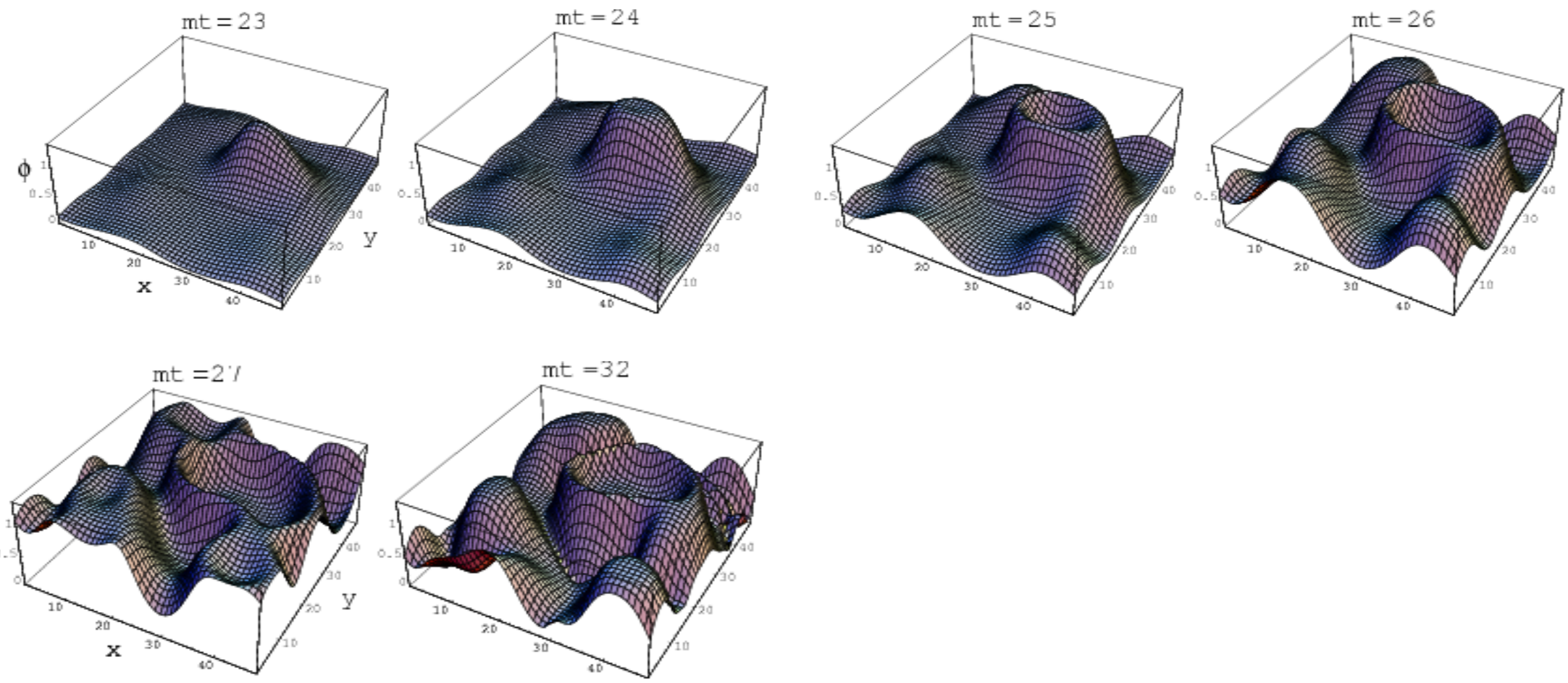

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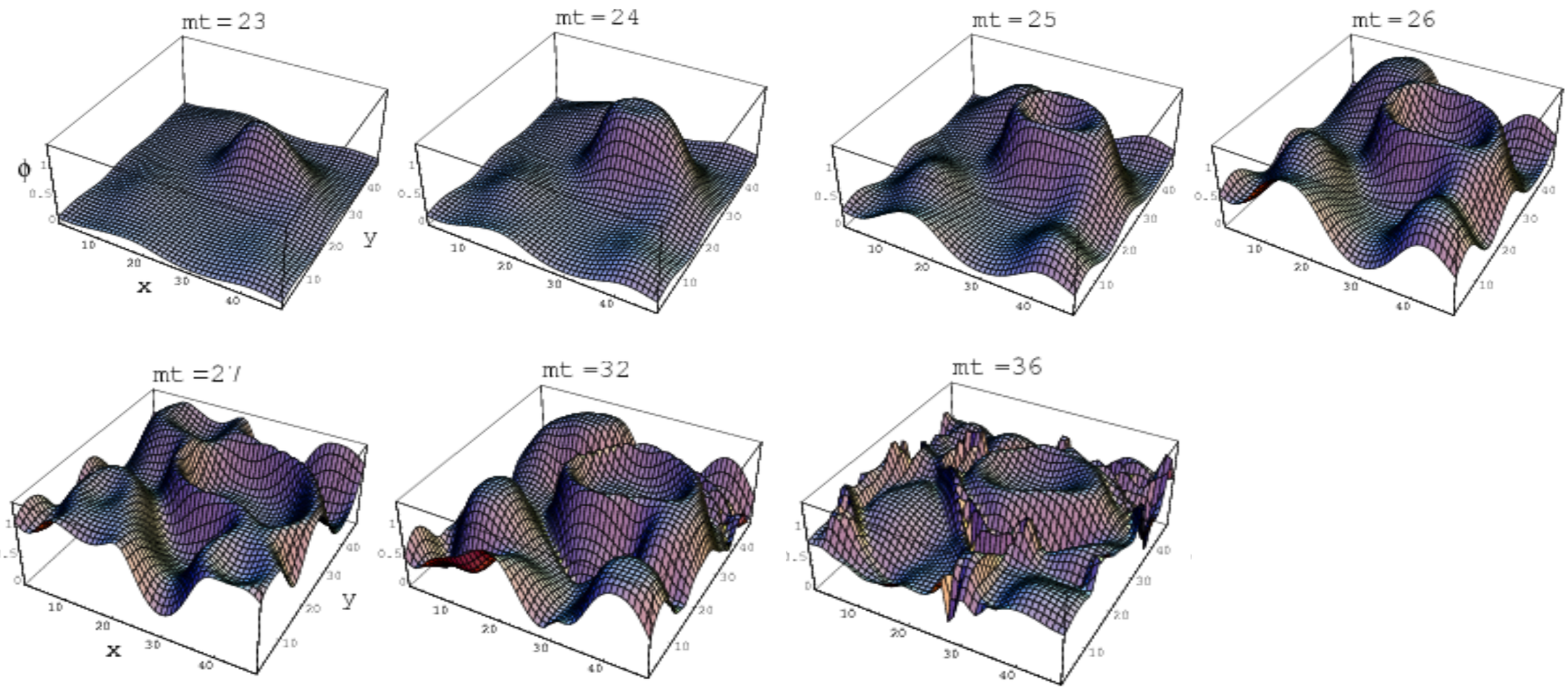

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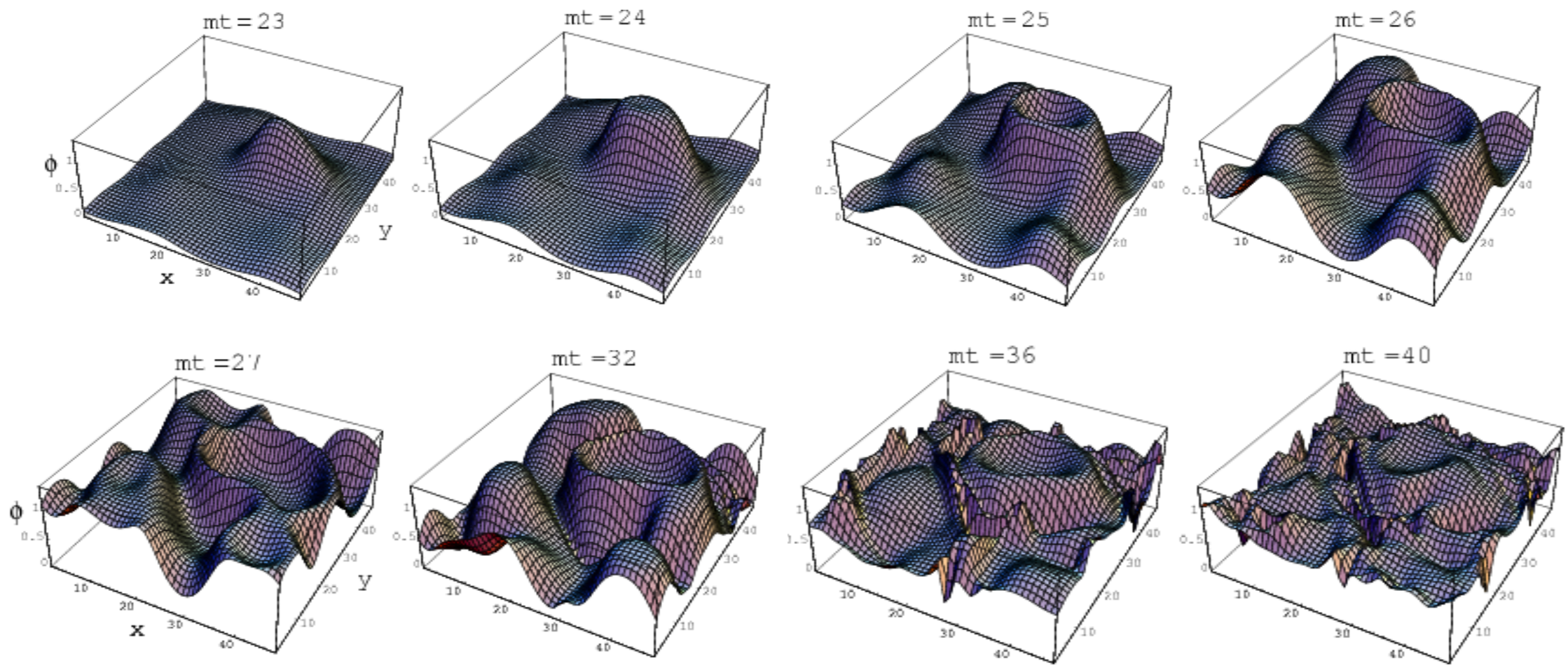

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INFLATIONARY REHEATING (RH)

Poor Understanding of RH:

ABSENCE: SM, DM, Thermalization,...
DIFFICULTIES: Many dof, Non-Lin, Non-Pert, Out-of-Eq.

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Probing RH: $\left(\begin{array}{c} \text{Phenomenological effects} \\ \text{with observable consequences} \end{array} \right) \rightarrow$ **GW**

INFLATIONARY REHEATING (RH)

Physics of (p)REHEATING: $\ddot{\varphi}_k + \omega^2(k,t)\varphi_k = 0$

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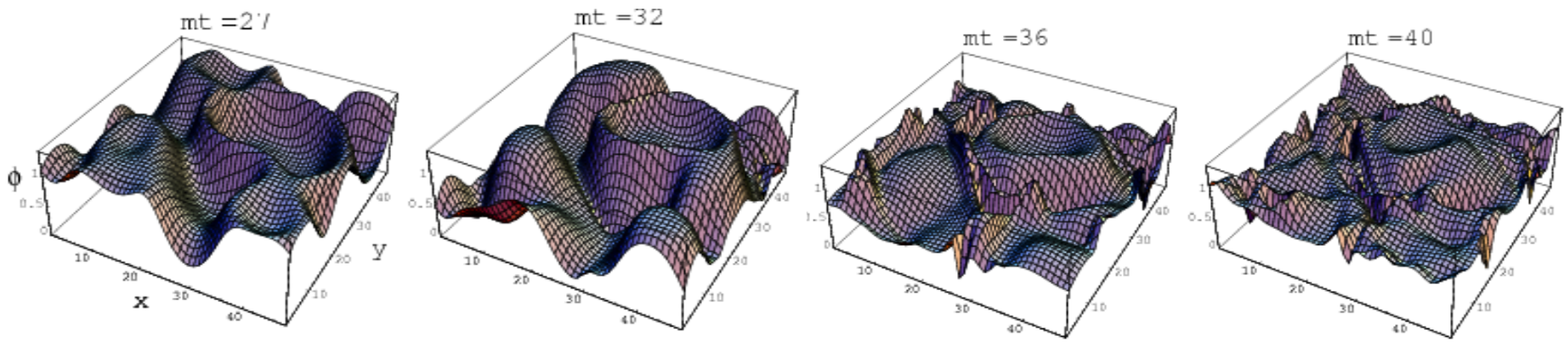

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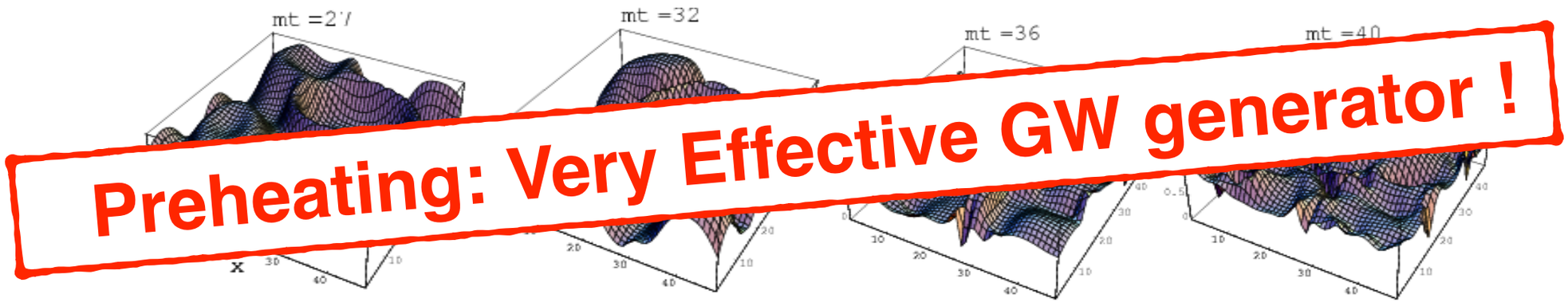

Preheating: Very Effective GW generator !

INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**

→ **out-Eq**

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Semi-classical regime $\pi_k \approx \kappa\phi_k + ...$ (**Squeezed States**)

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- GW: $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\rm TT}_{ij}, \qquad \Pi^{TT}_{ij} = \{\partial_i\chi^a \partial_j\chi^a\}^{TT}$

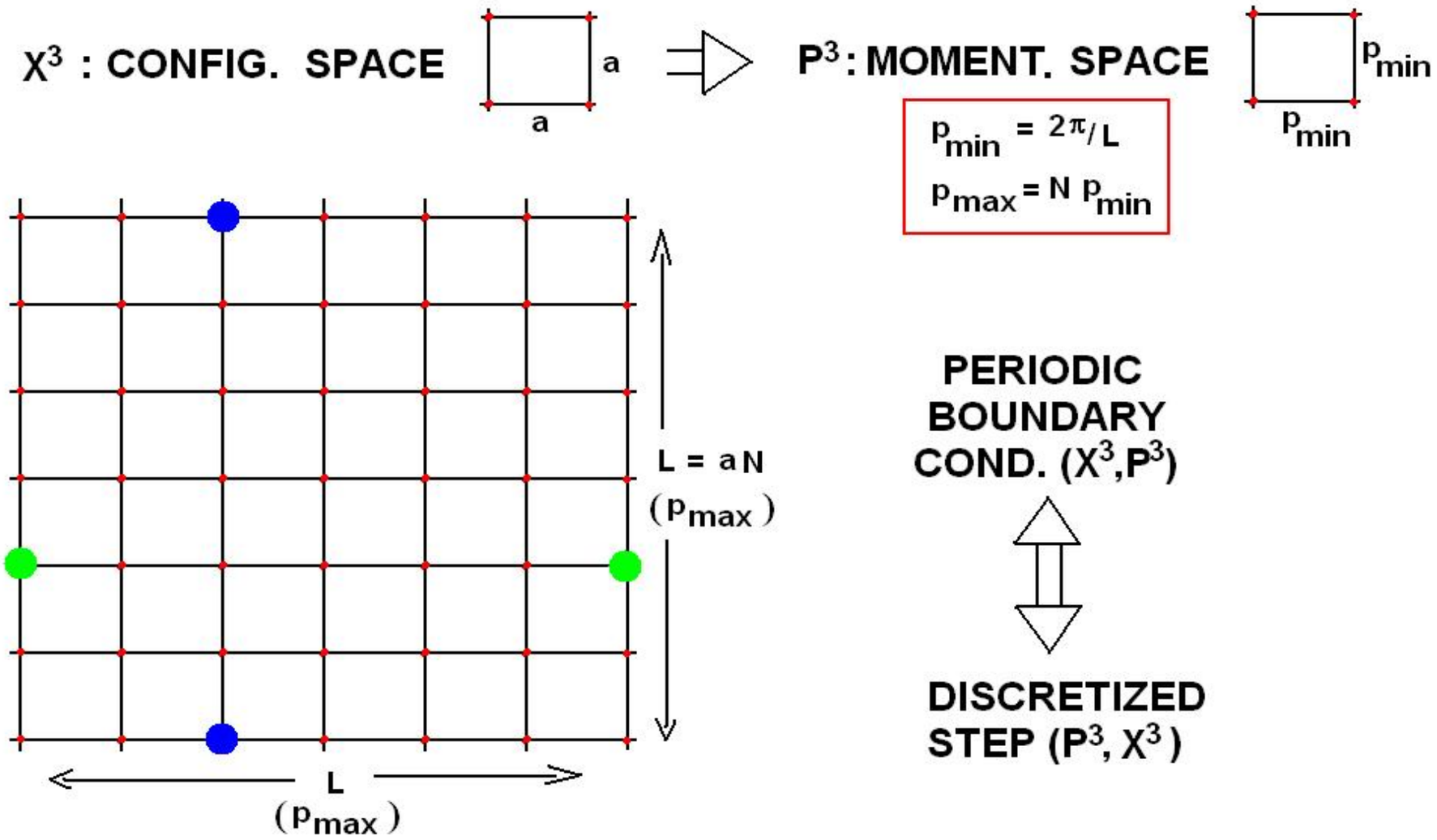
$ds^2 = a^2(-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \qquad {\rm TT}: \begin{cases} h_{ii} = 0 \\ h_{ij},_j = 0 \end{cases}$

INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

$$\partial_\mu O(x) \to (O(x+\mu) - O(x-\mu))/2a_\mu$$

$$\partial_\mu \partial_\mu O(x) \to (O(x+2\mu) + O(x-2\mu) - 2O(x))/4a_\mu^2$$



INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

Building the Solution:

$$\begin{cases} h_{ij}(\mathbf{k},t) = \Lambda_{ij,lm}(\hat{\mathbf{k}})u_{lm}(\mathbf{k},t) \\ \\ u_{lm}(\mathbf{k},t) = \int_{t_0}^{t} dt' G(t-t')\Pi^{\rm eff}_{lm}(\mathbf{k},t') \end{cases}$$

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3) Snapshots: $h_{ij}(t,\mathbf{x}) = (2\pi)^{-3/2}\int d^3k e^{-i\mathbf{kx}}\Lambda_{ij,lm}(\hat{\mathbf{k}})u_{lm}(t,\mathbf{k})$

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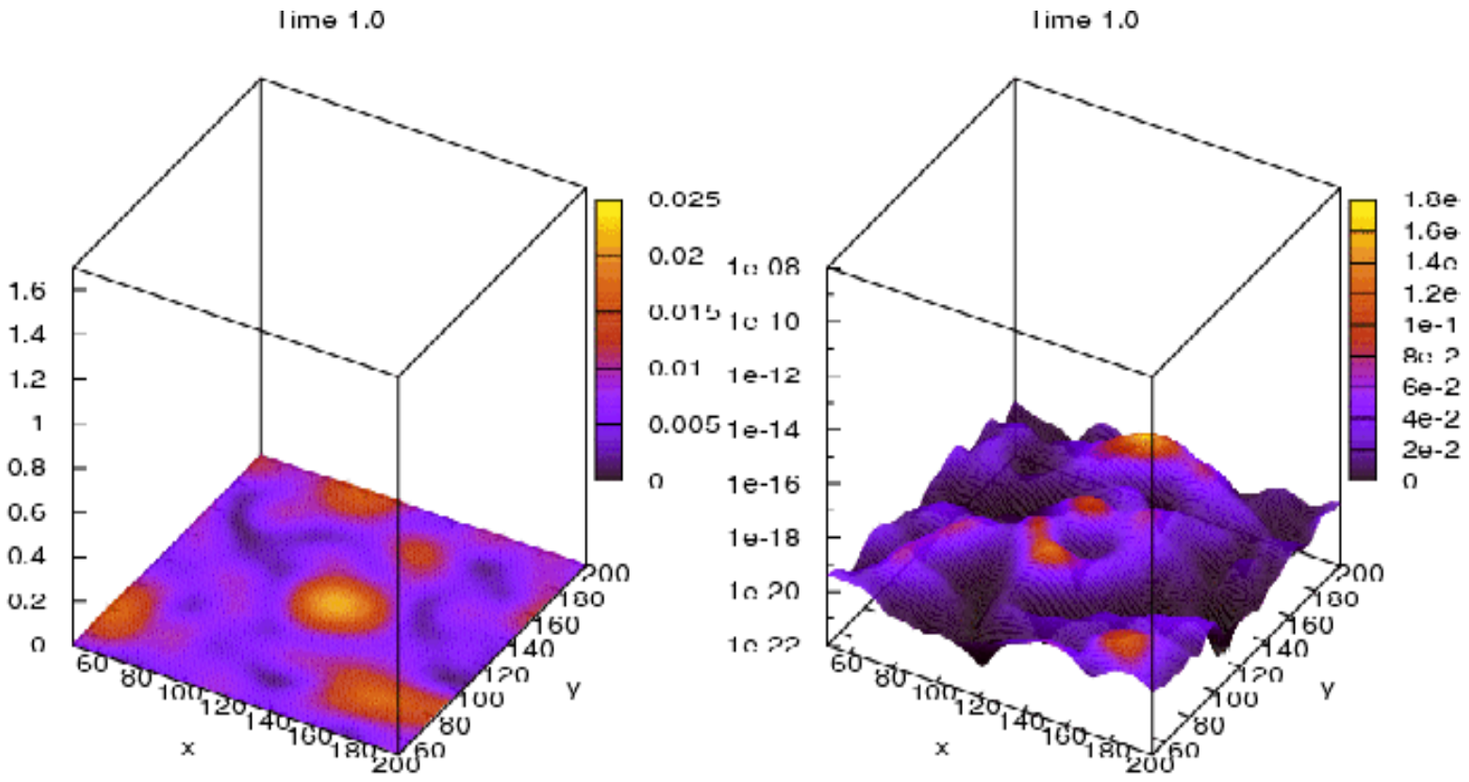
INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

Hybrid Preheating

$$V(\phi,\chi) = \tfrac{\lambda}{4}(|\chi|^2 - v^2)^2 + \tfrac{1}{2}|\chi|^2\phi^2 + V(\phi)$$

Animation by Alfonso Sastre



Higgs **GW (Energy density)**

INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

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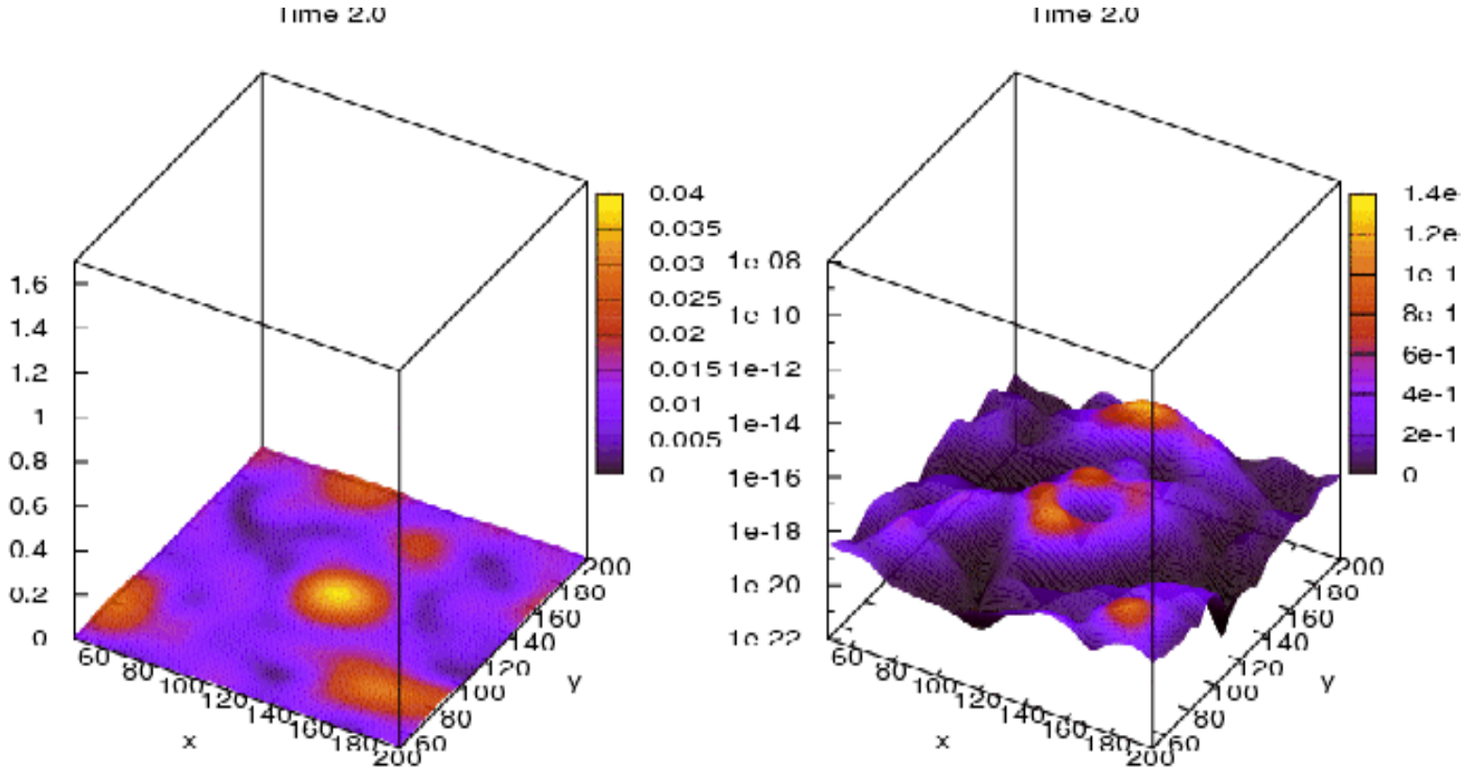

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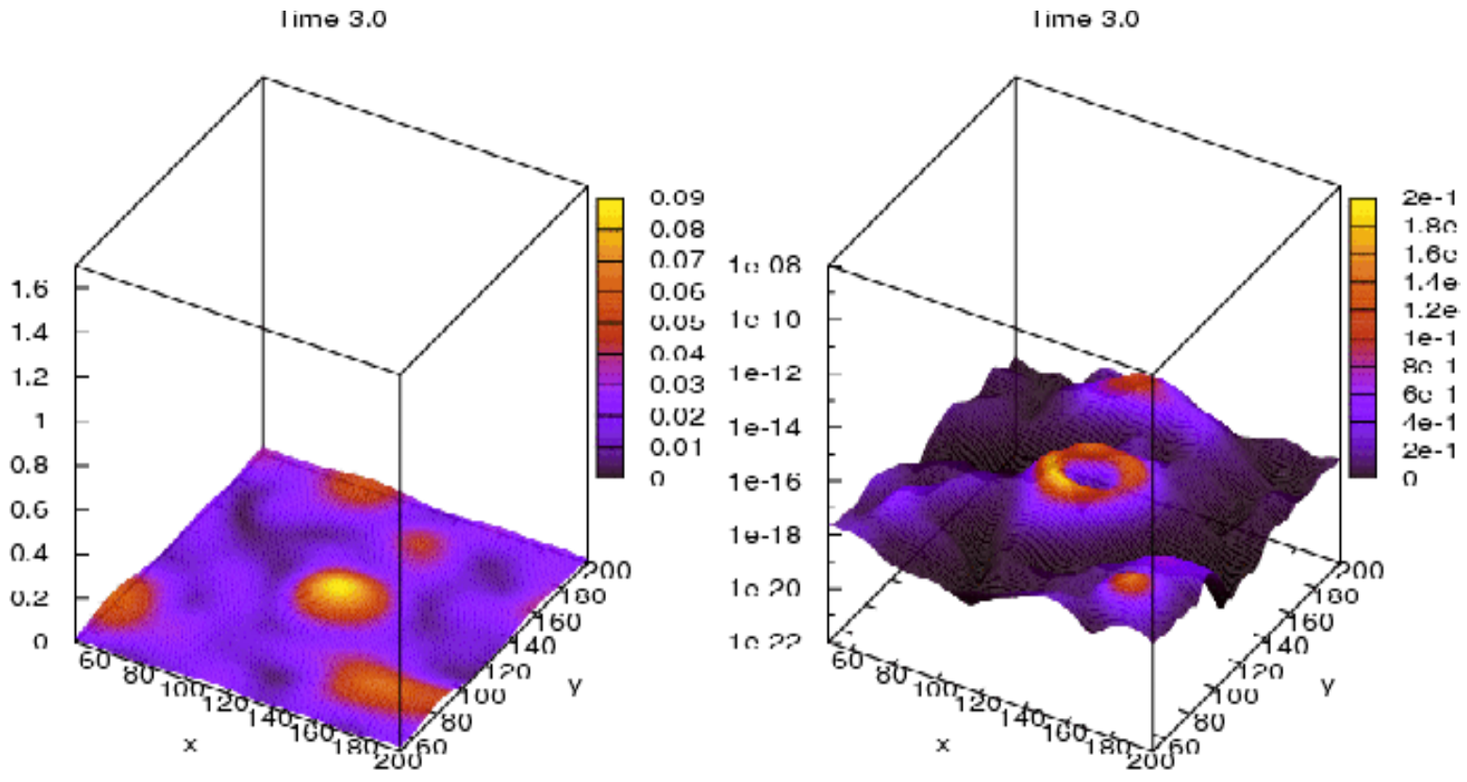

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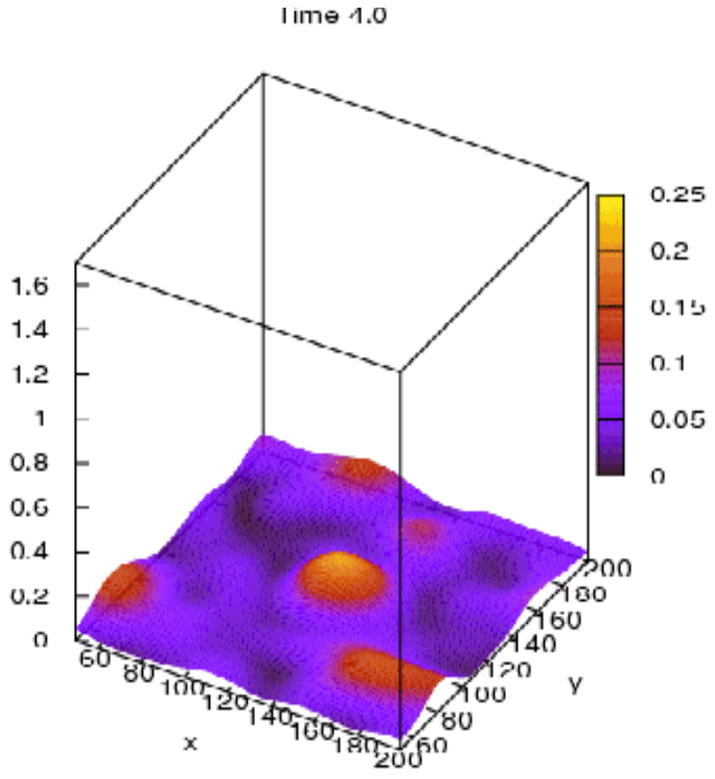

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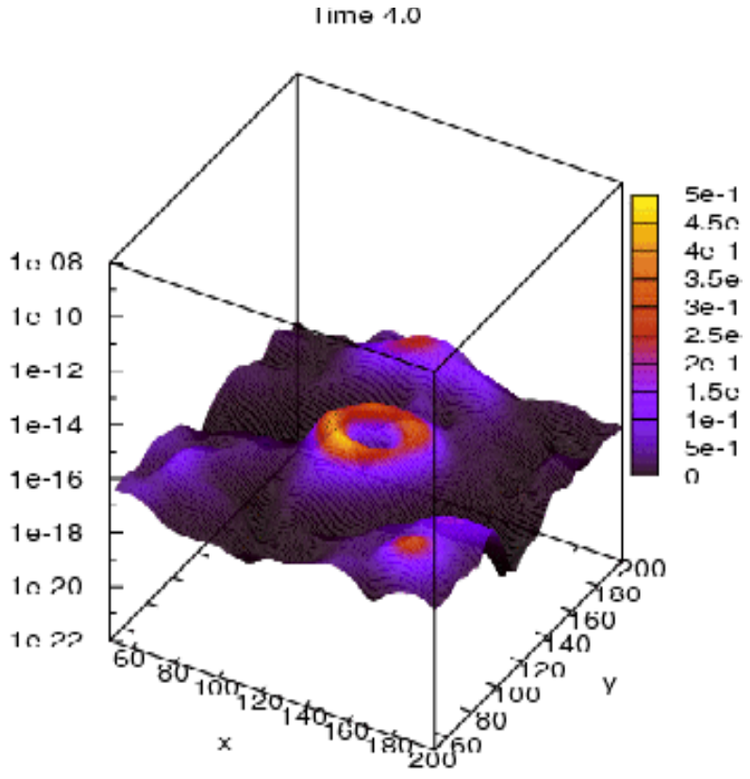

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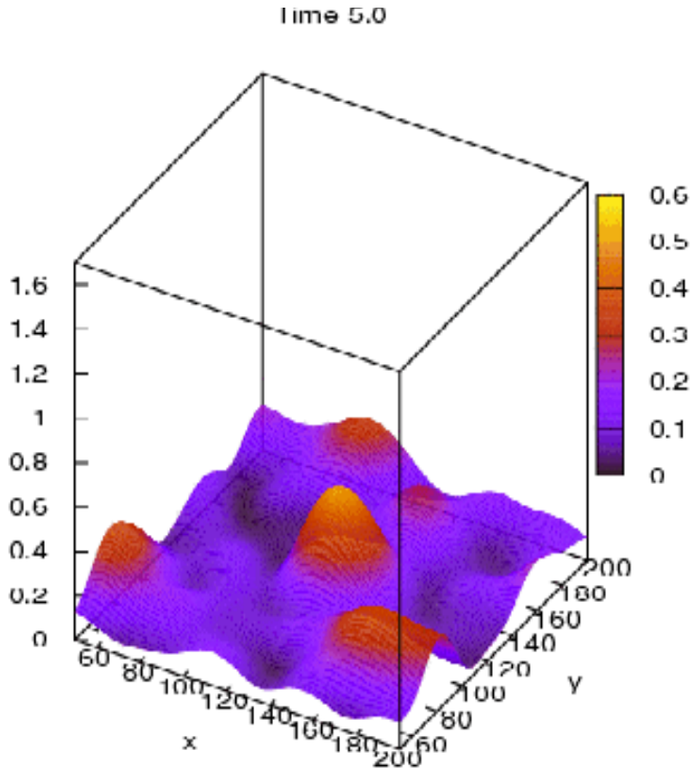

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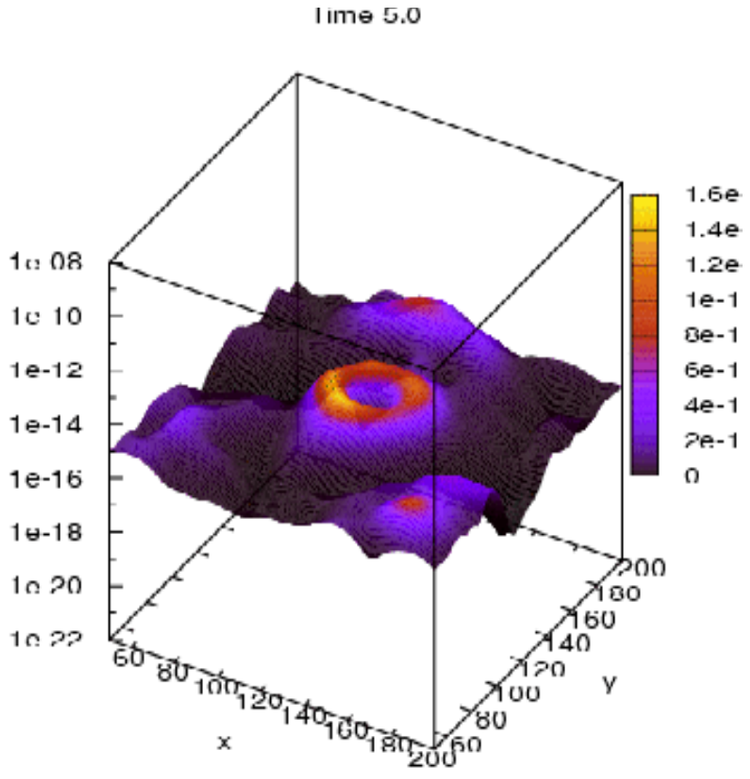

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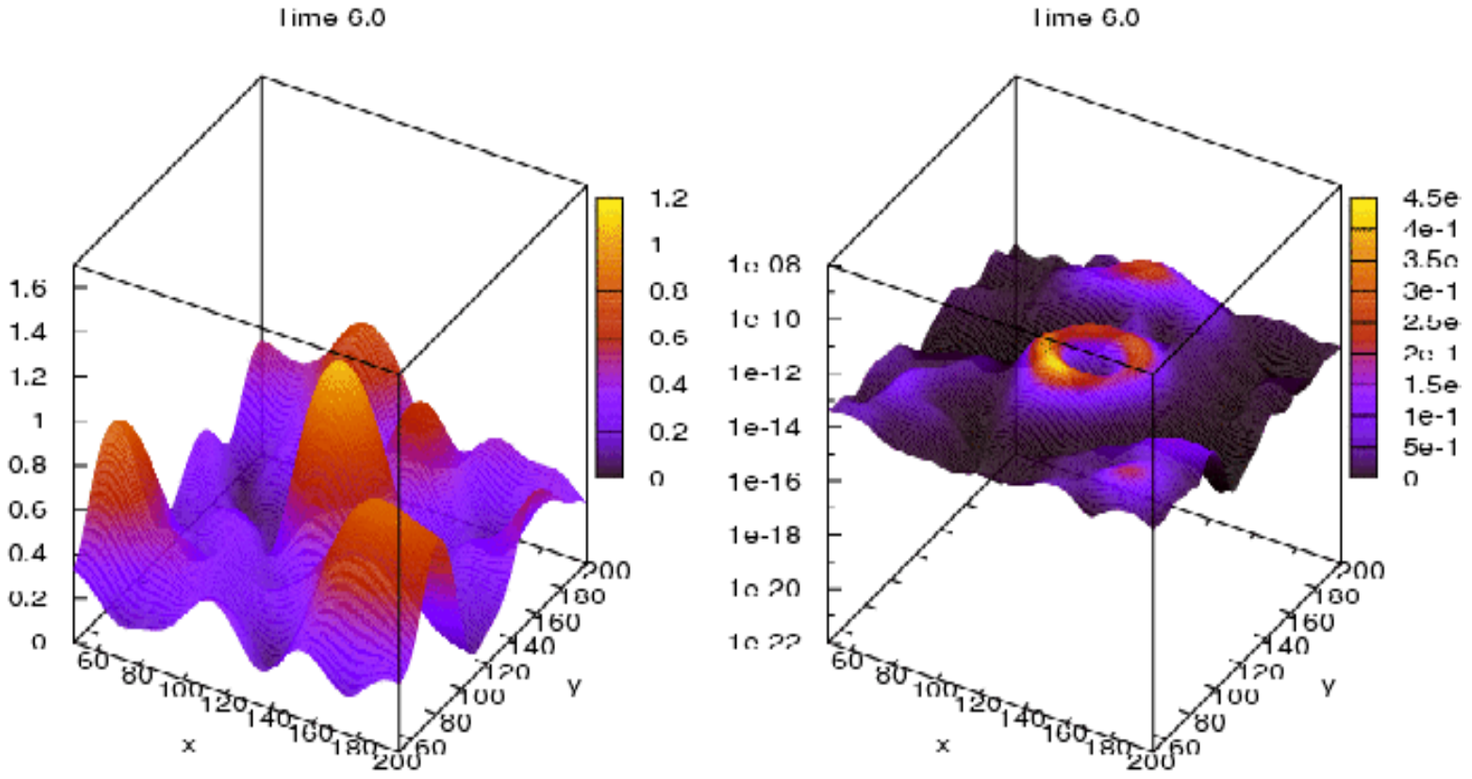

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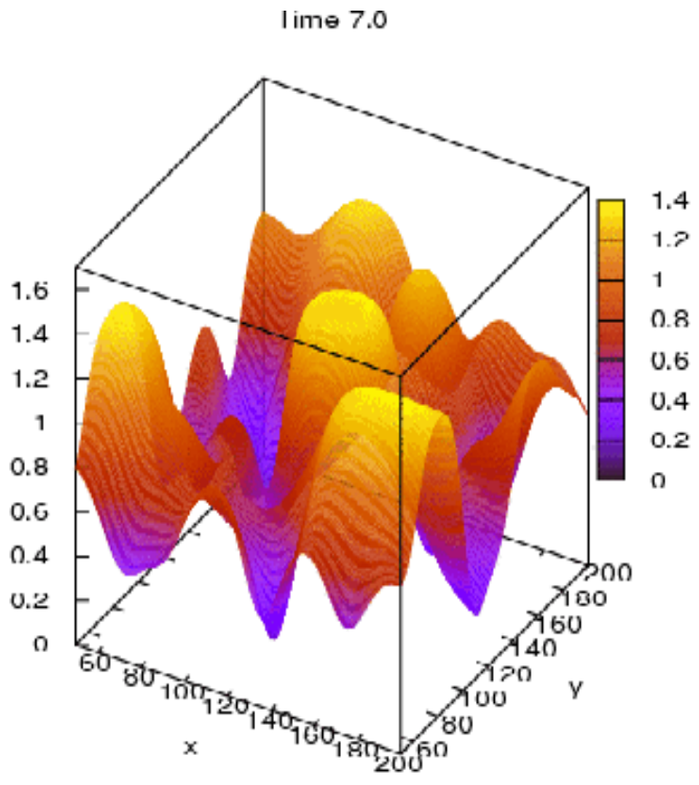

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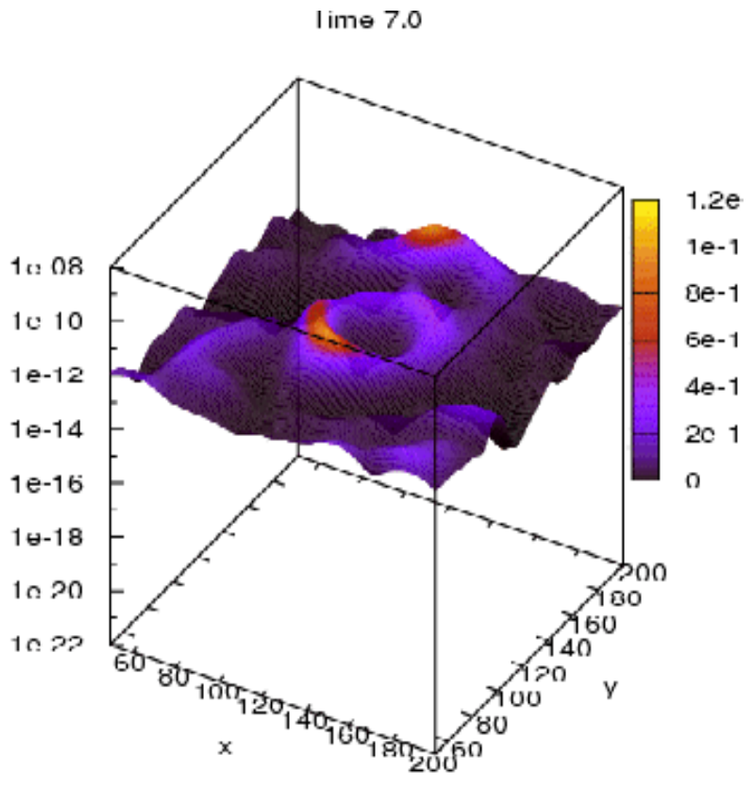

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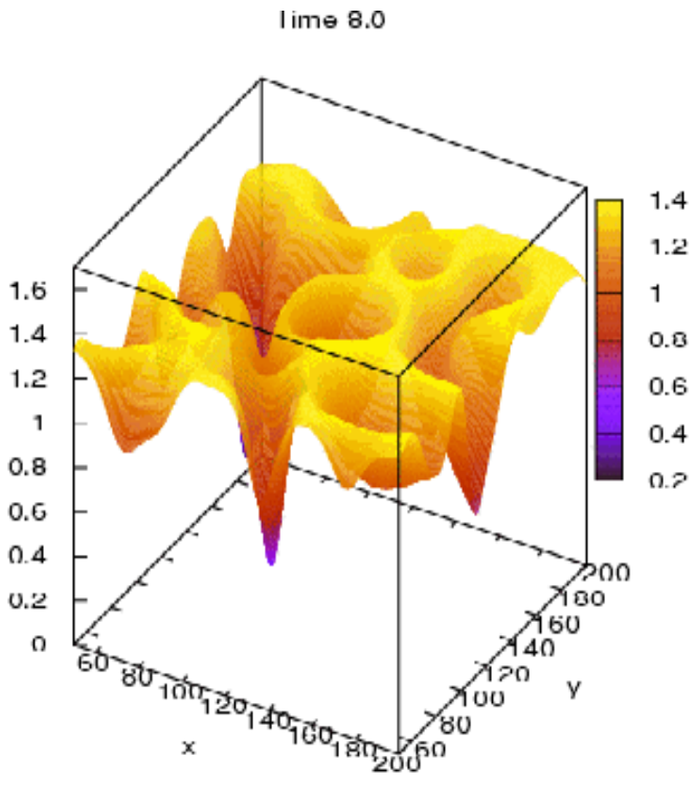

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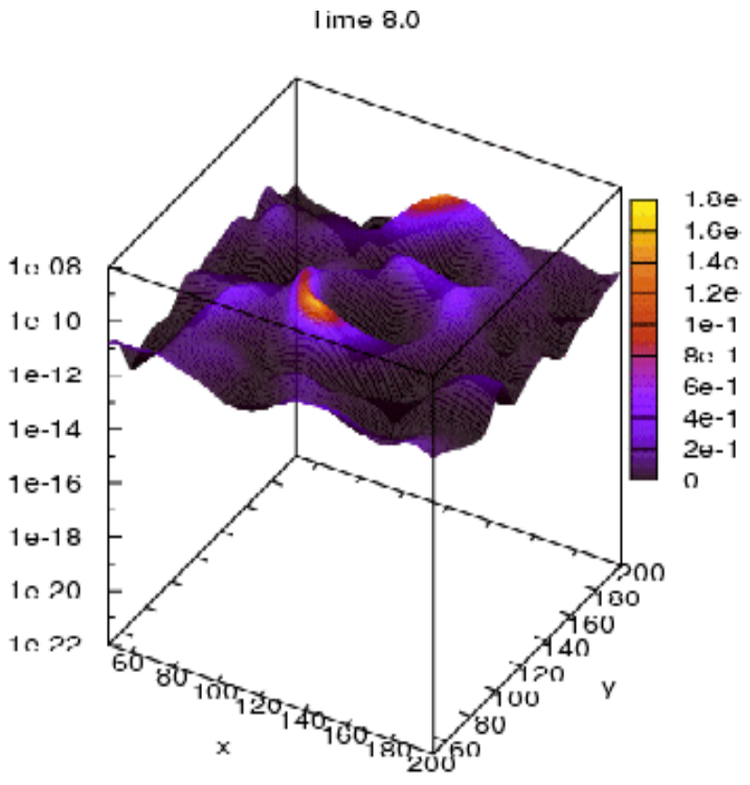

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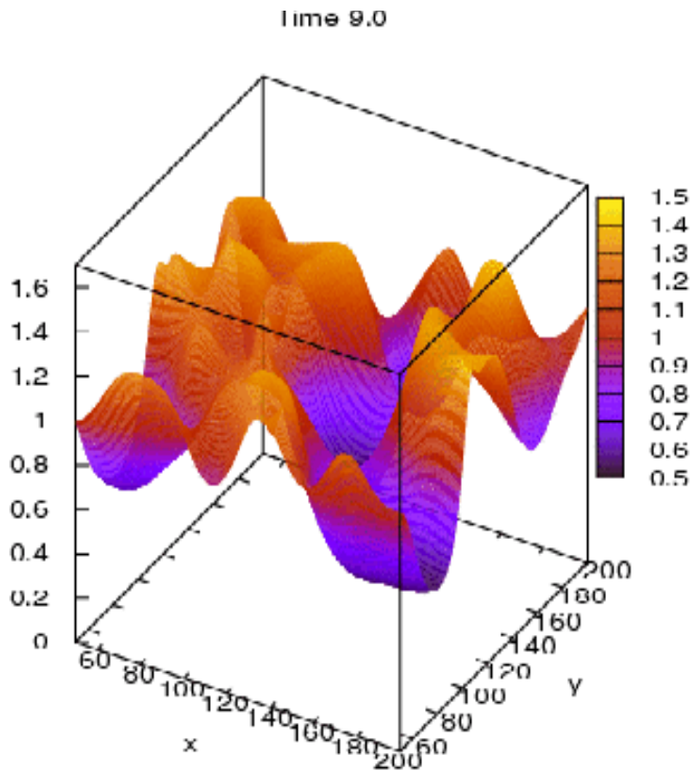

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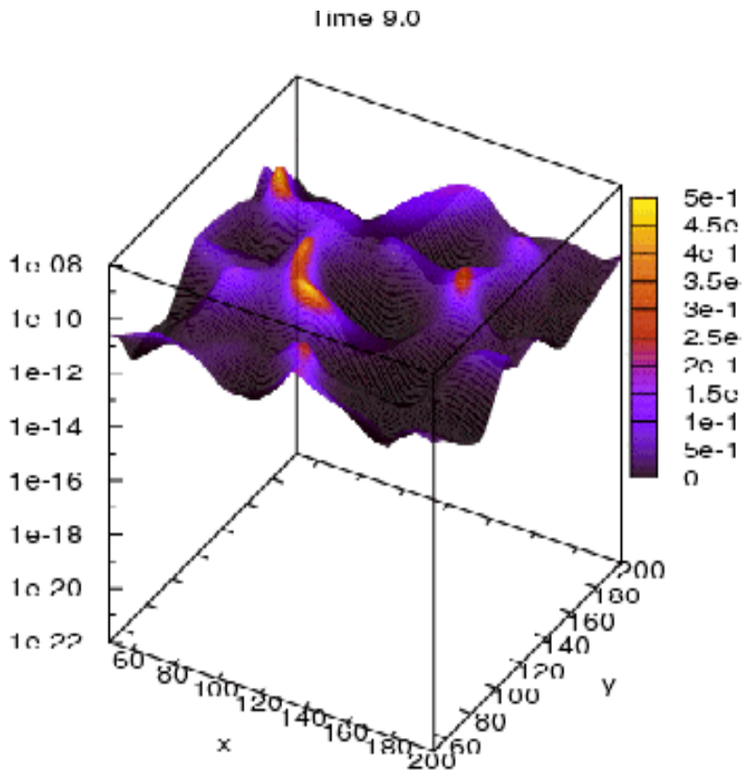

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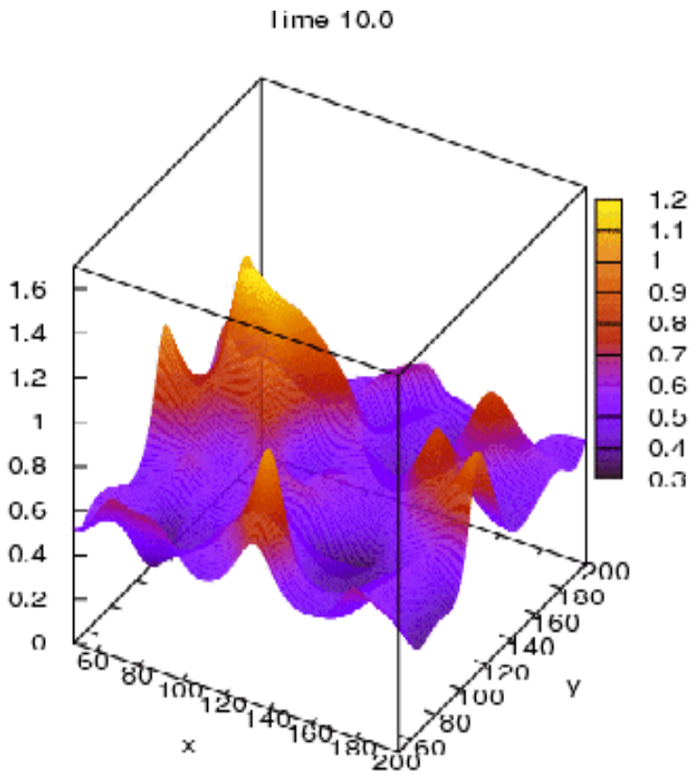

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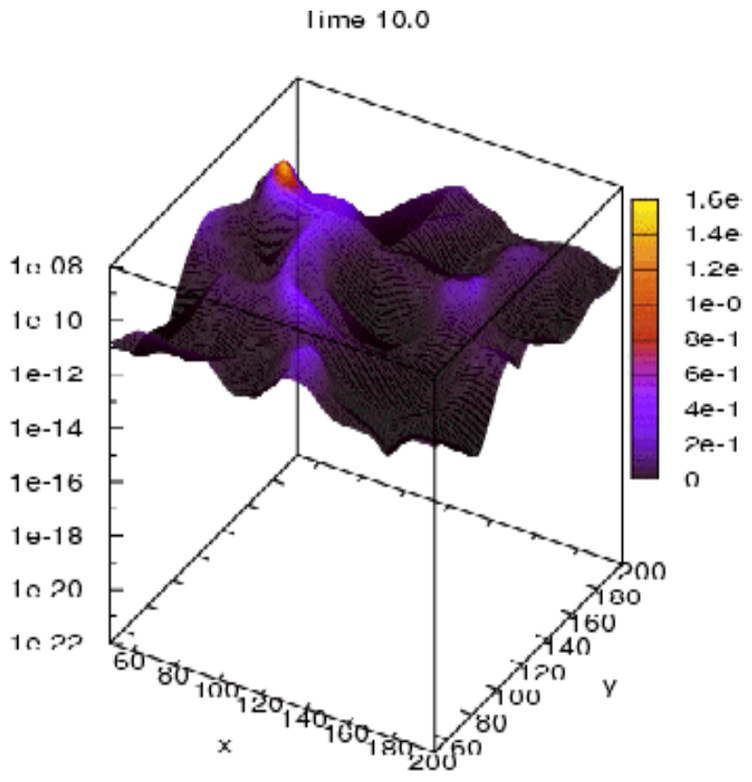

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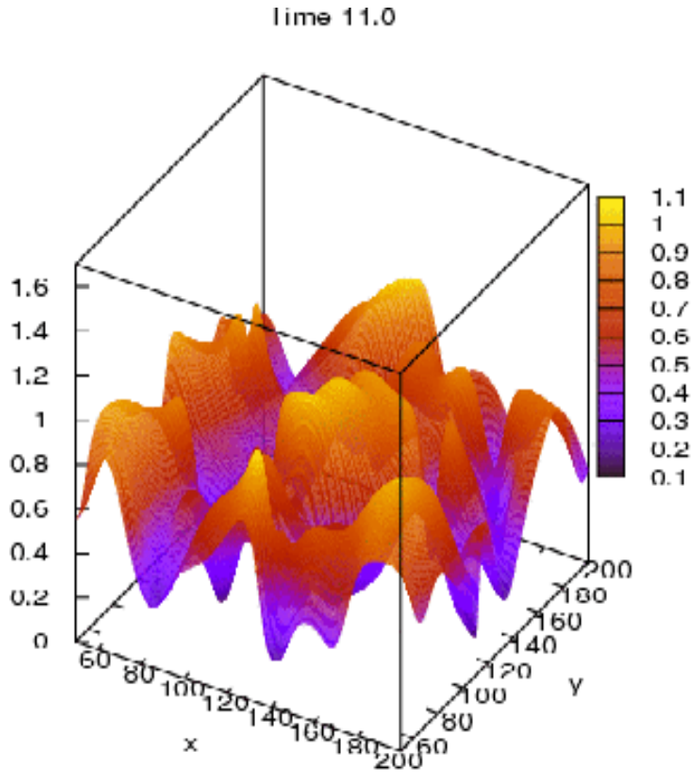

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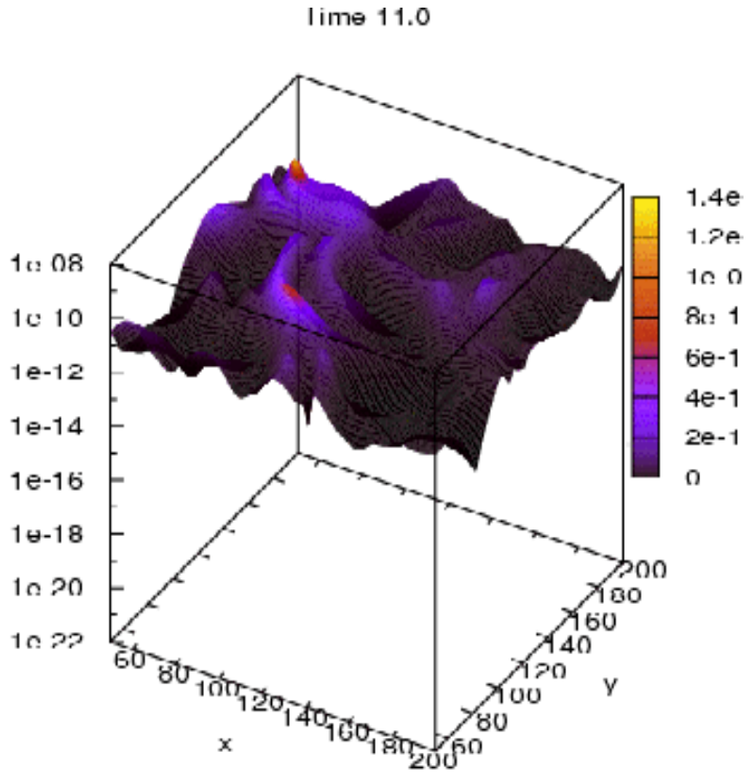

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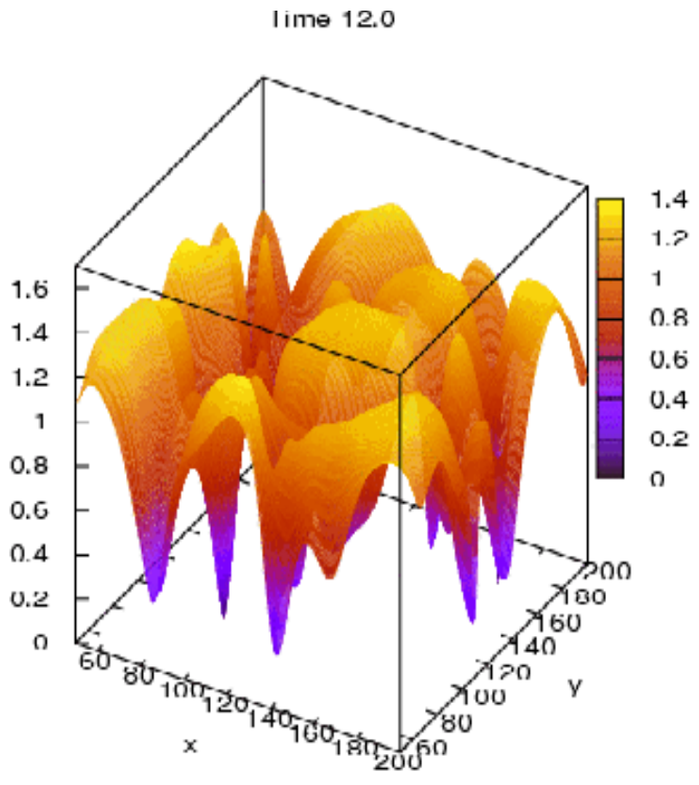

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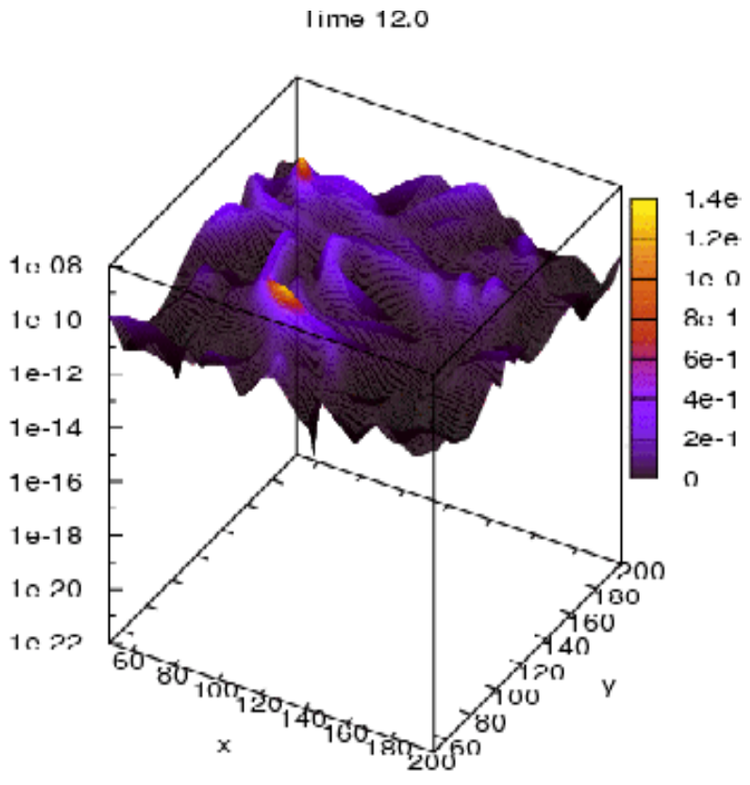

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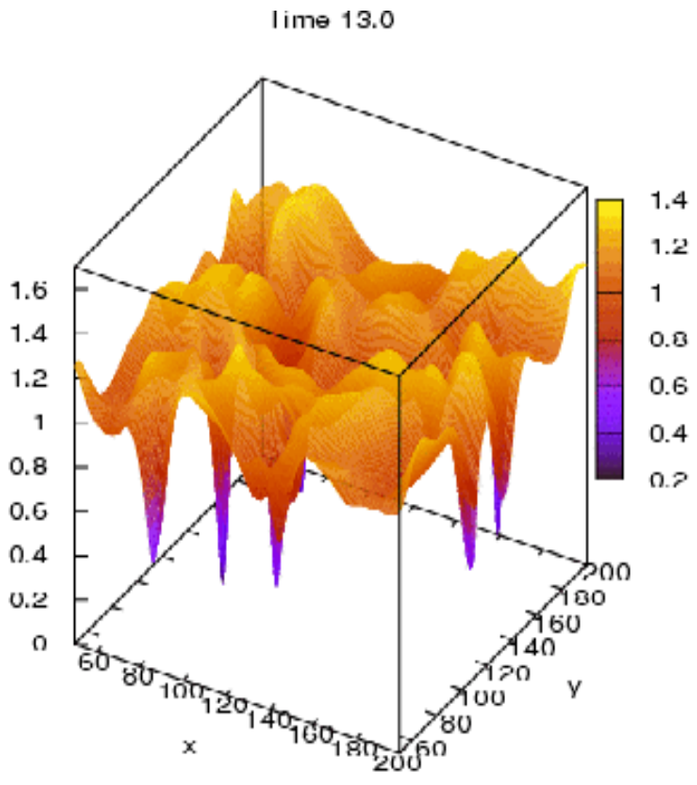
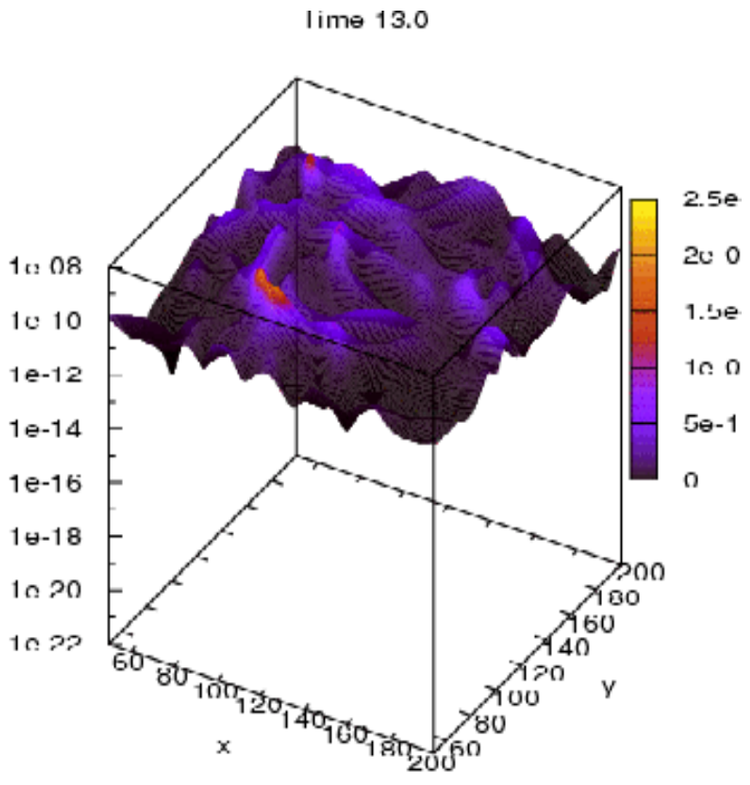


Higgs



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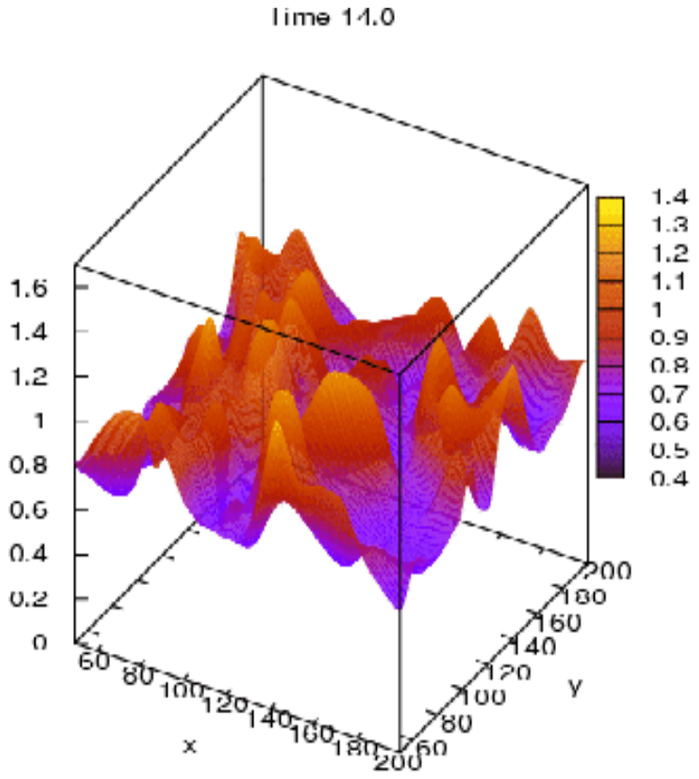

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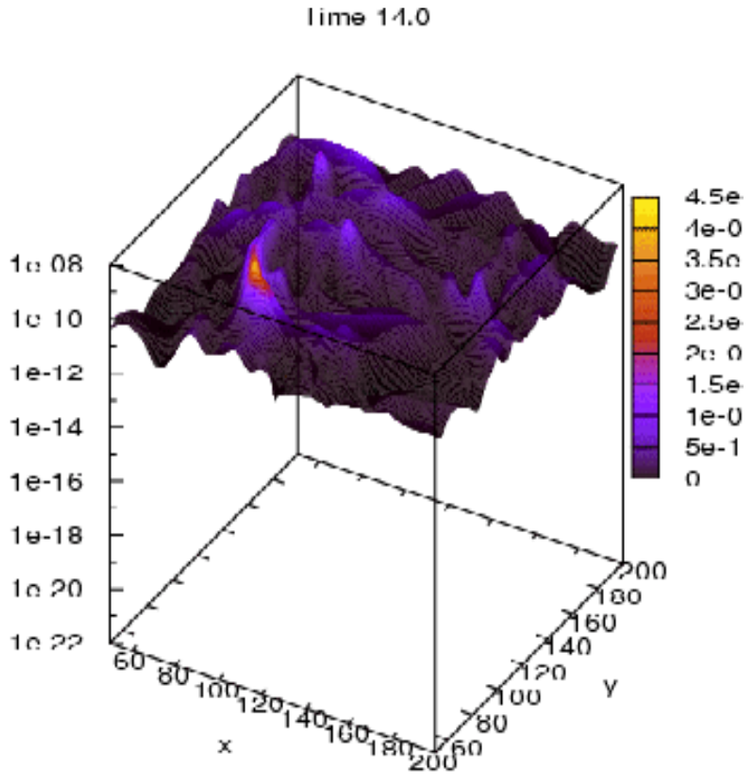

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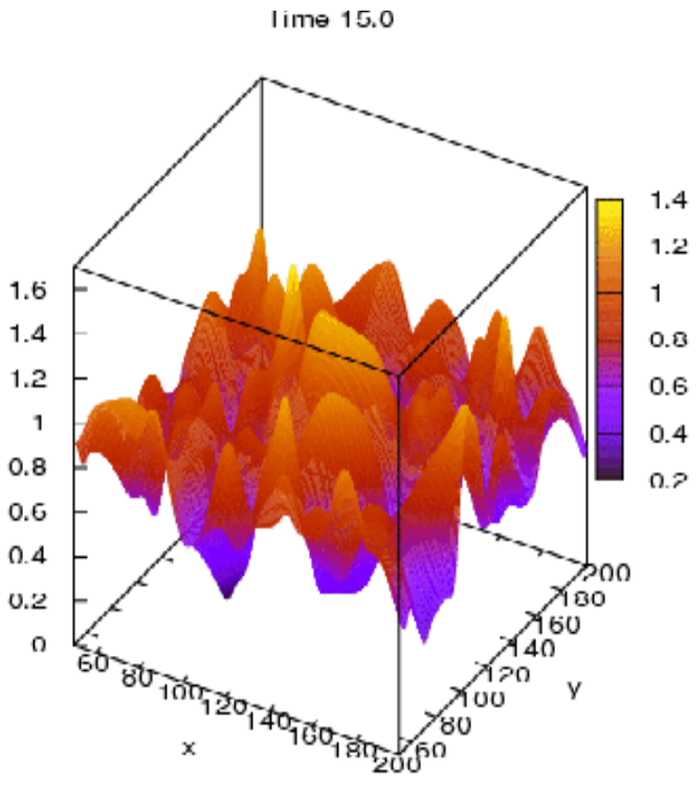

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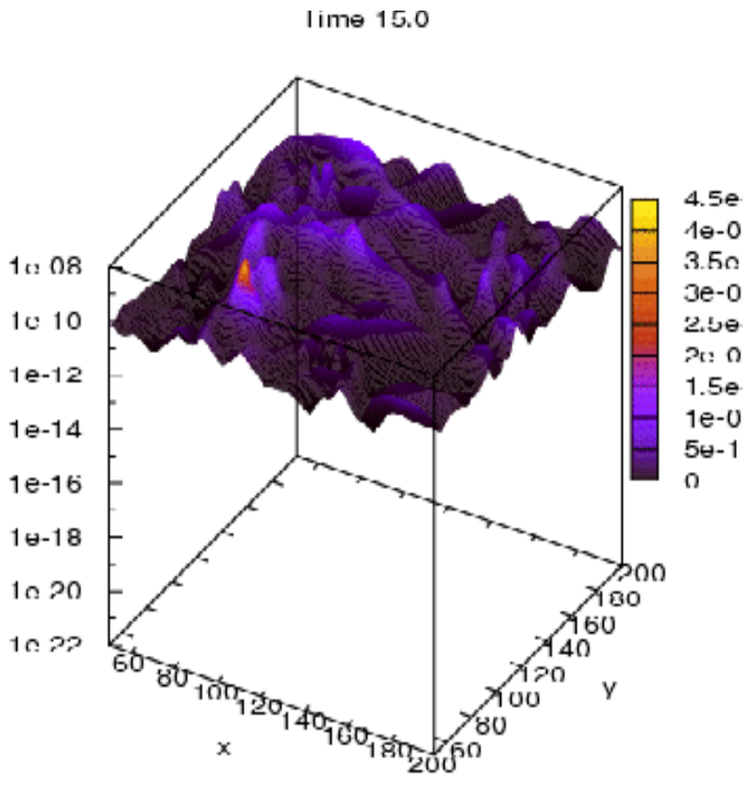

GW (Energy density)

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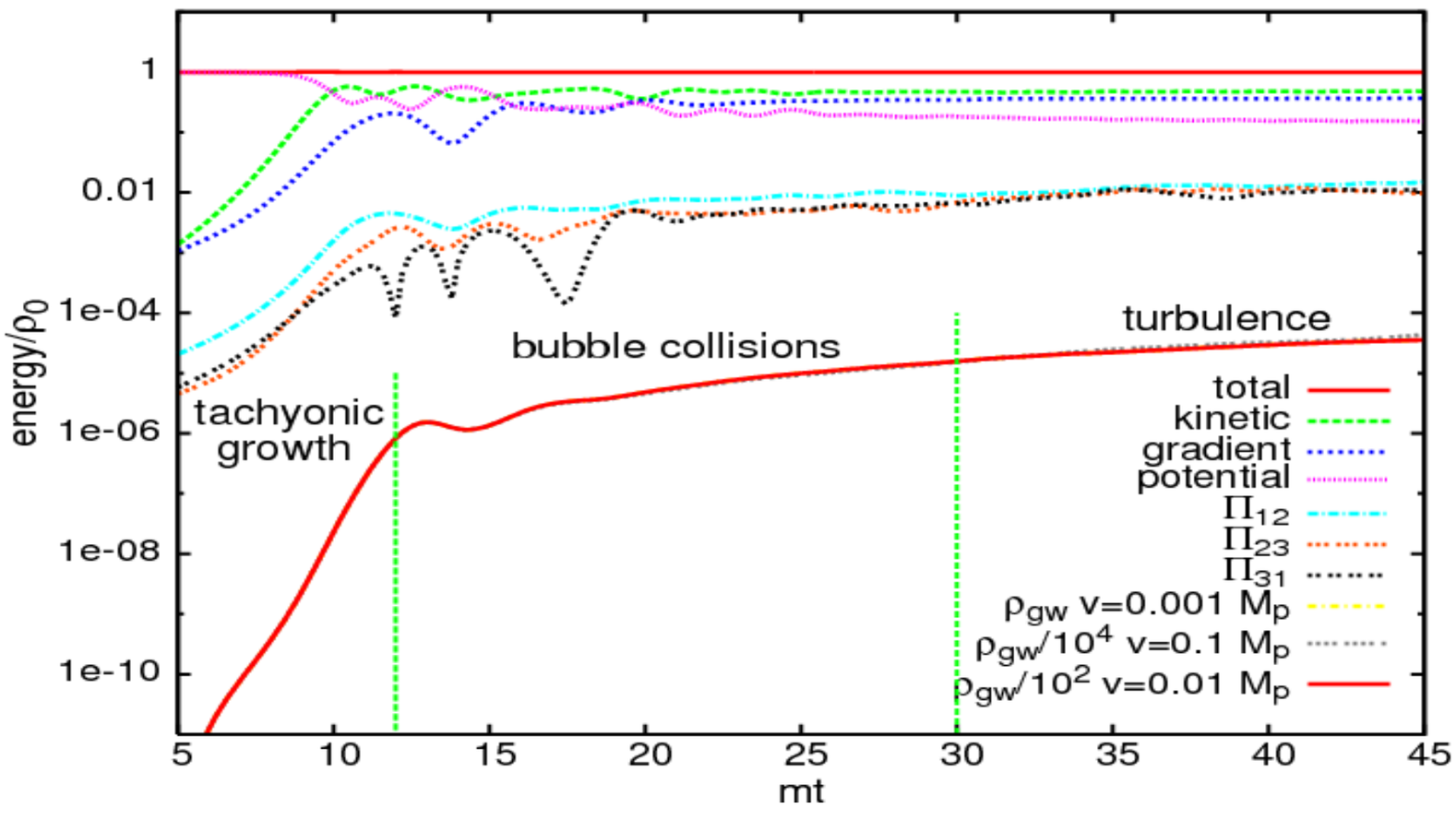
3 stages: **Exp. Instabilities** → **Bubble Collisions** → **Turbulence**



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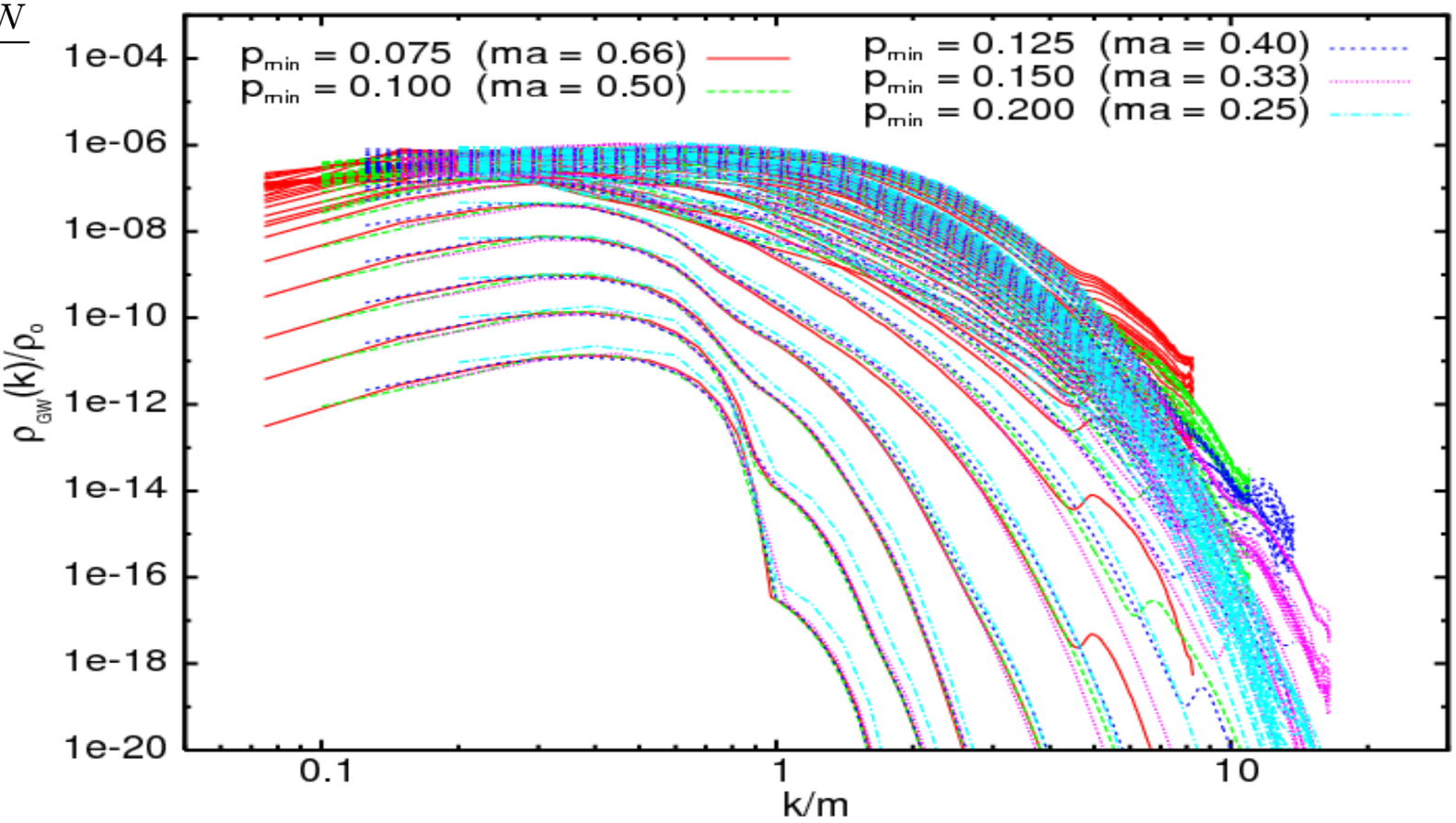
$$\frac{1}{\rho_{\rm tot}}\frac{d\log\rho_{\rm GW}}{d\log k}$$



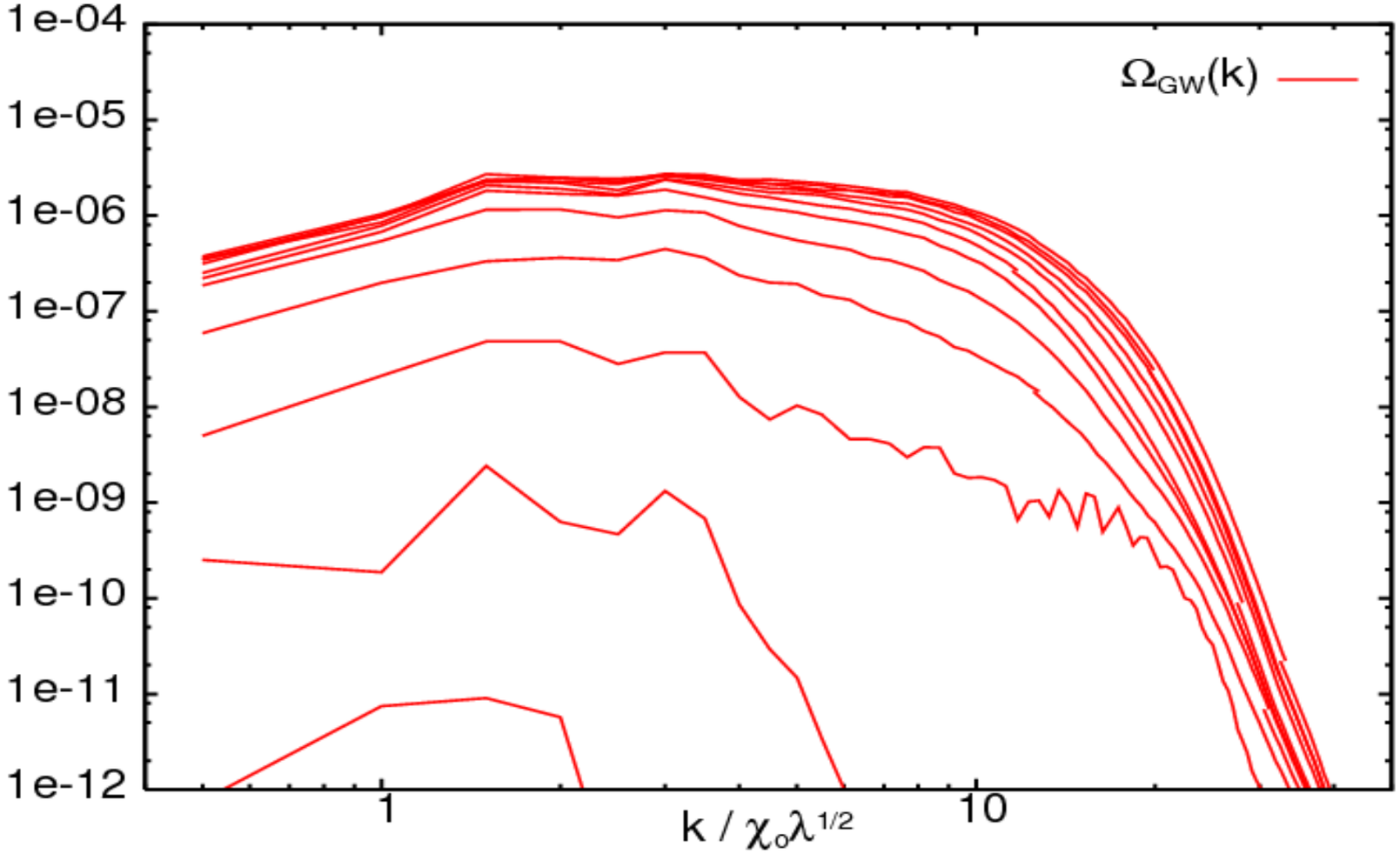
INFLATIONARY REHEATING (RH)

Lattice Simulations: Dynamics → **non-linear**, **out-Eq**

Chaotic Preheating

$$\lambda = 10^{-14},\ g^2/\lambda = 120\ \left(V = \tfrac{1}{4}\lambda\phi^4 + \tfrac{1}{2}g^2\phi^2\chi^2\right)$$

$$\frac{1}{\rho_{\rm tot}}\frac{d\log\rho_{\rm GW}}{d\log k}$$



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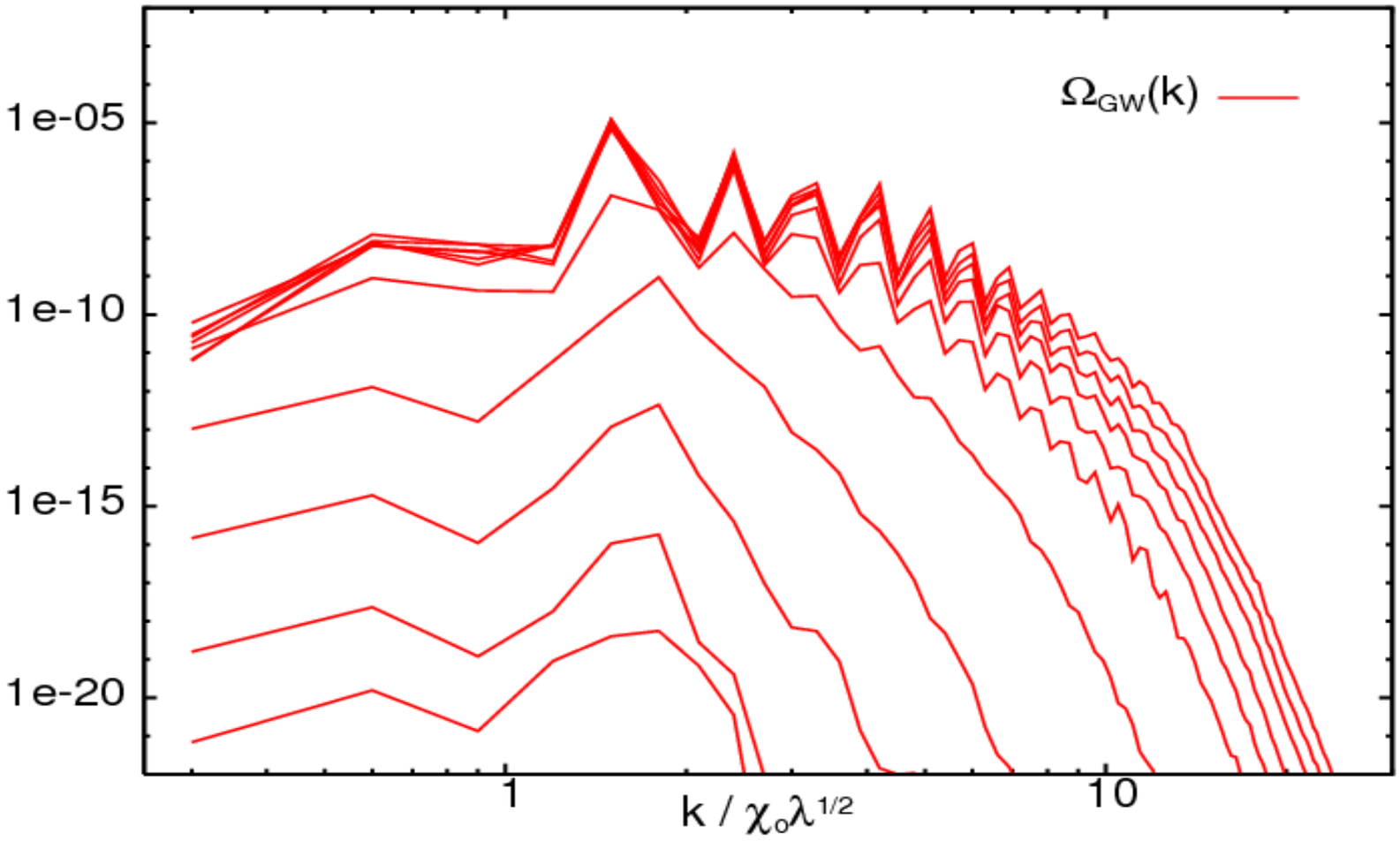
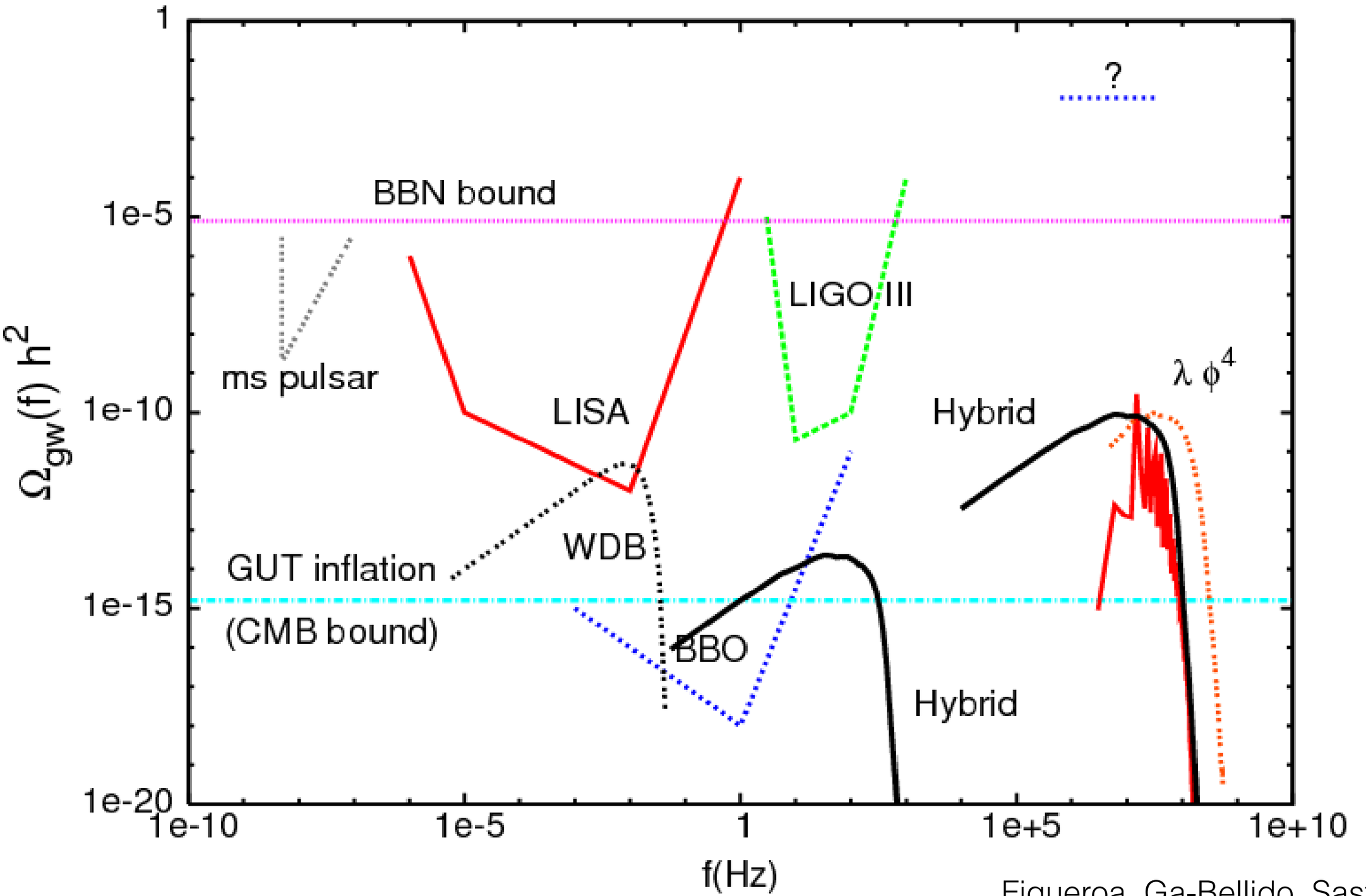


INFLATIONARY REHEATING (RH)

Today's Signal (GW RedShifted)



Figueroa, Ga-Bellido, Sastre '07

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim \frac{h^2\Omega_{\rm rad}}{8\pi^4}\left(\frac{g_o}{g_{\rm f}}\right)^{1/3} \times \epsilon_{\rm i}\, A^2 \frac{\omega^6}{\rho_I m_p^2}\, q^{-\frac{1}{2}+\delta}$

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

$$\omega_*^2 \equiv V''(\Phi_I)$$

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Initial energy

INFLATIONARY REHEATING (RH)

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$\omega_*^2 \equiv V''(\Phi_I)$

Initial energy

$q \equiv \dfrac{g^2\Phi_{\rm i}^2}{\omega_*^2}$

Resonance Parameter

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude)

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expansion history

Initial energy

Resonance Parameter

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numerics

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numerics

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numerics

$\omega_*^2 \equiv V''(\Phi_I)$

numerics

expansion history

Initial energy

$q \equiv \dfrac{g^2\Phi_{\rm i}^2}{\omega_*^2}$

Resonance Parameter

$$f_o \sim 5\cdot 10^{10}\left(\frac{\omega_*}{\rho_{\rm i}^{1/4}}\right)\epsilon_i^{\frac{1}{4}}\, q^{\frac{1}{4}}\ {\rm Hz}$$

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,,$

Large amplitude !

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad f_o \sim 10^8 - 10^9\ {\rm Hz}$

Large amplitude ! **... but at high Frequency !**

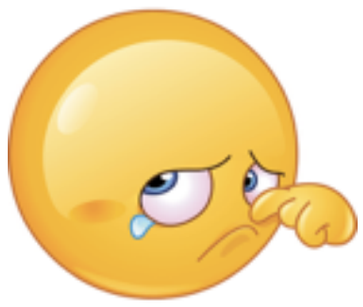
INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Chaotic Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad f_o \sim 10^8 - 10^9\ {\rm Hz}$

Large amplitude ! **... but at high Frequency !**

Very unfortunate... not detectors there !



INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Hybrid Models: $\Omega_{\rm GW}^{(o)} \propto \left(\frac{v}{m_p}\right)^2 \times f(\lambda, g^2) \quad , \qquad f_o \sim \lambda^{1/4} \times 10^9 \text{ Hz}$

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude**)**

Hybrid Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}$

Large amplitude !

(for $v \simeq 10^{16}$ GeV)

INFLATIONARY REHEATING (RH)

Parameter Dependence (Peak amplitude)

Hybrid Models: $\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \begin{cases} f_o \sim 10^8 - 10^9 \text{ Hz} \\ \boxed{\lambda \sim 0.1} \text{ (natural)} \end{cases}$

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Parameter Dependence (Peak amplitude)

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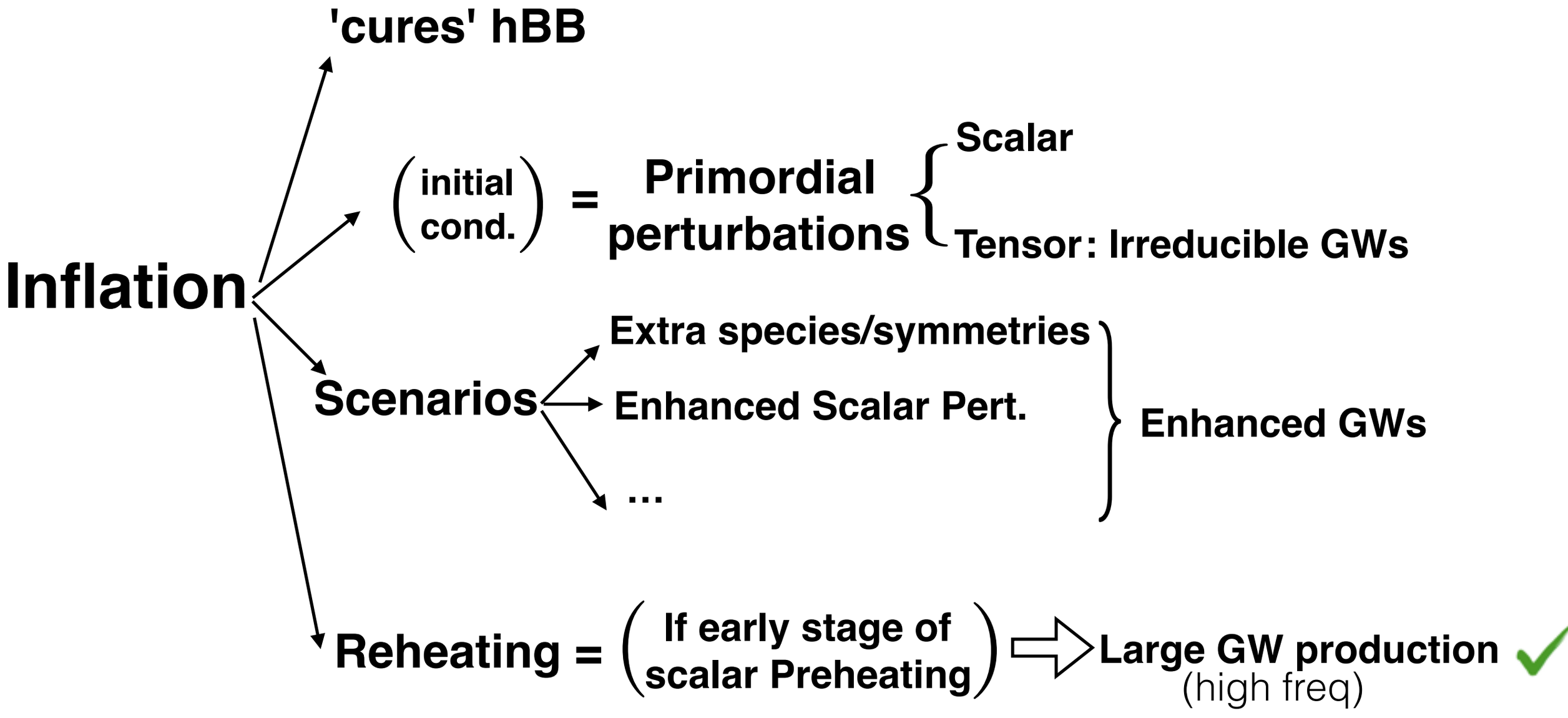
Large amplitude !

(for $v \simeq 10^{16}$ GeV)

realistically speaking ...

INFLATIONARY COSMOLOGY



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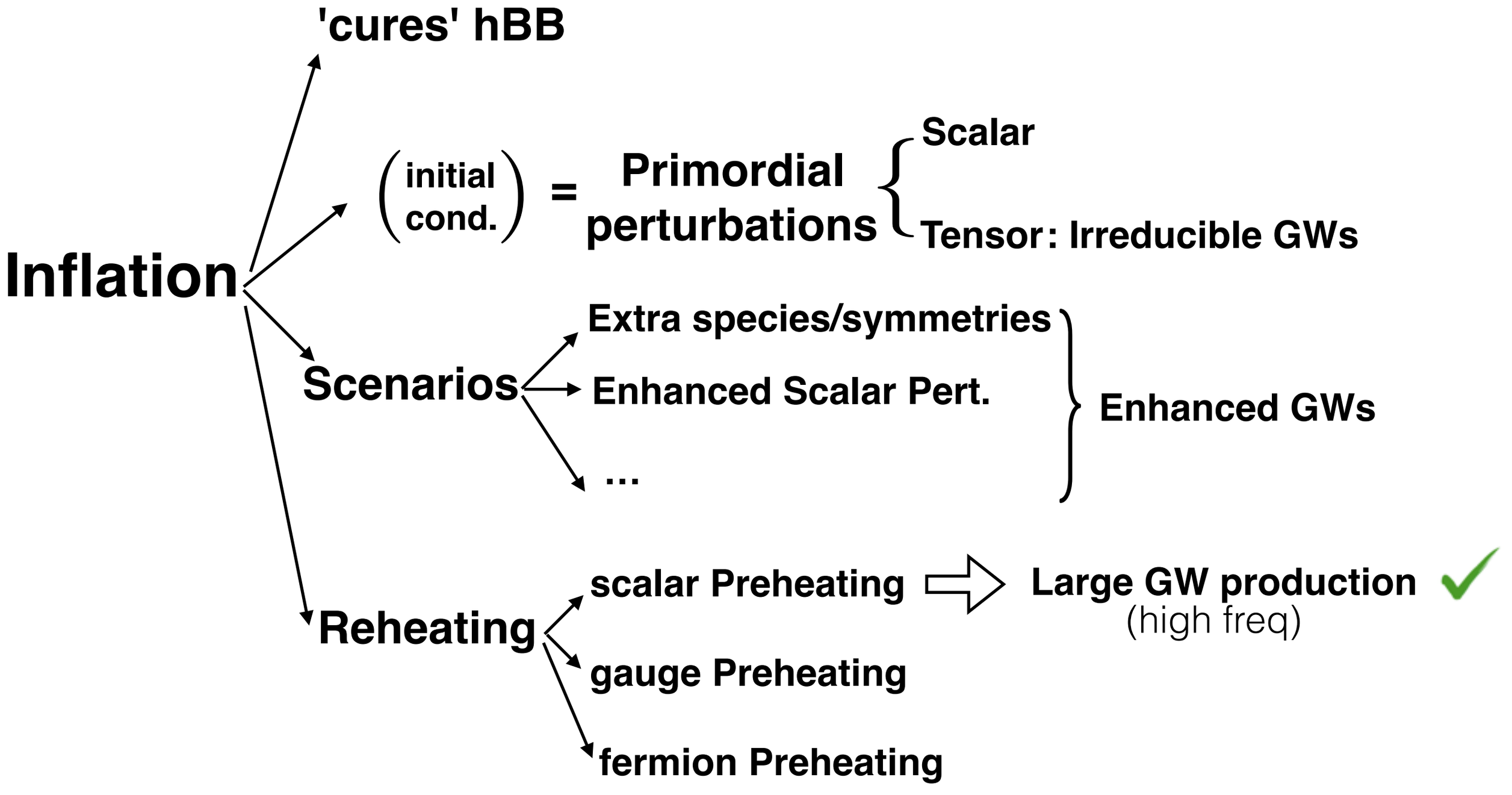

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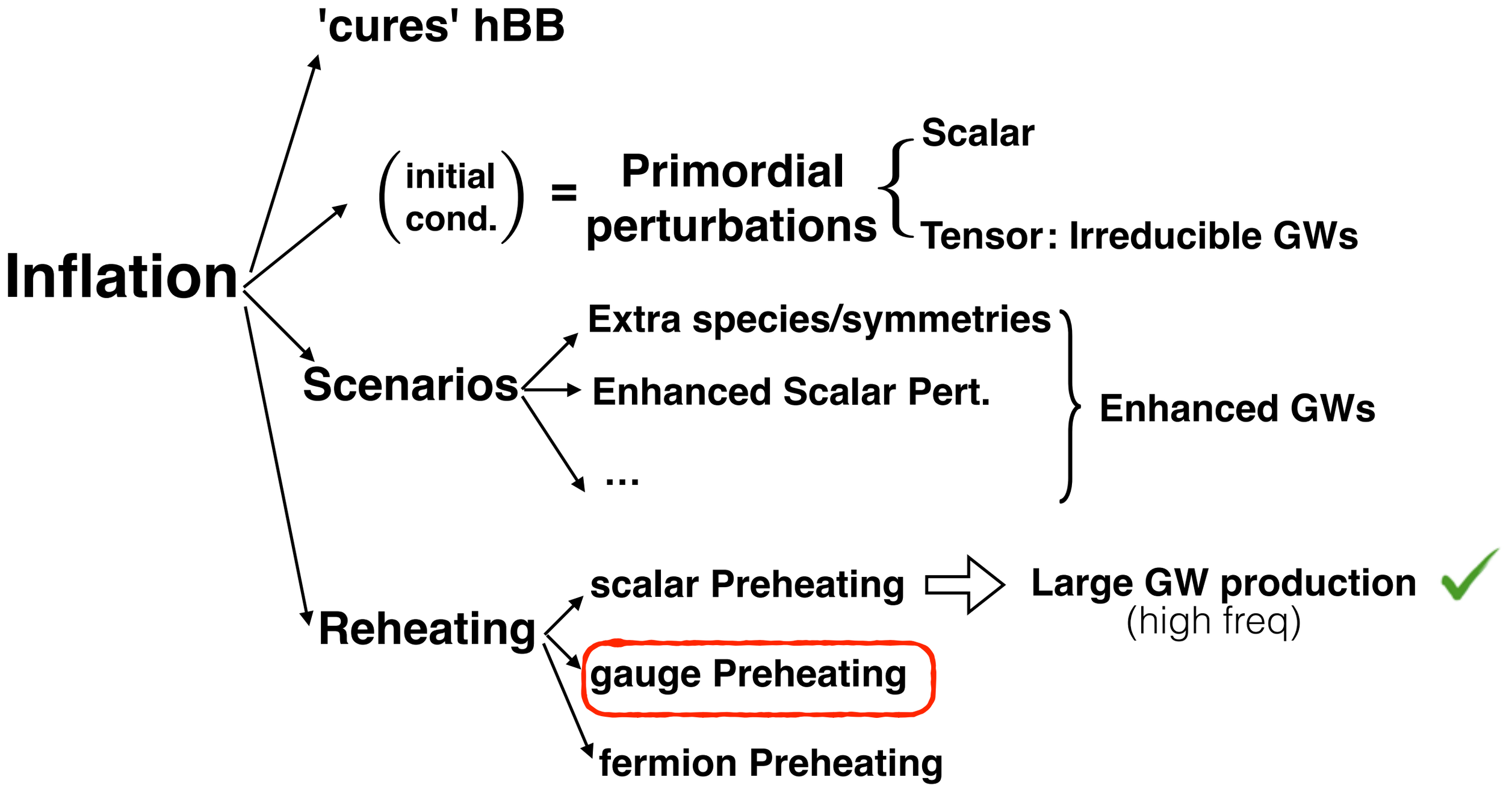

GAUGE (P)REHEATING

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The Abelian-Higgs+Inflaton model

$$L = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a} + Tr[(D_\mu \Phi)^{+} D^{\mu} \Phi] + \frac{1}{2}(\partial_\mu \chi)^2 - V(\Phi, \chi)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi, \chi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 + \frac{g^2}{2}\phi^2\chi^2 + \frac{1}{2}m^2\chi^2$$

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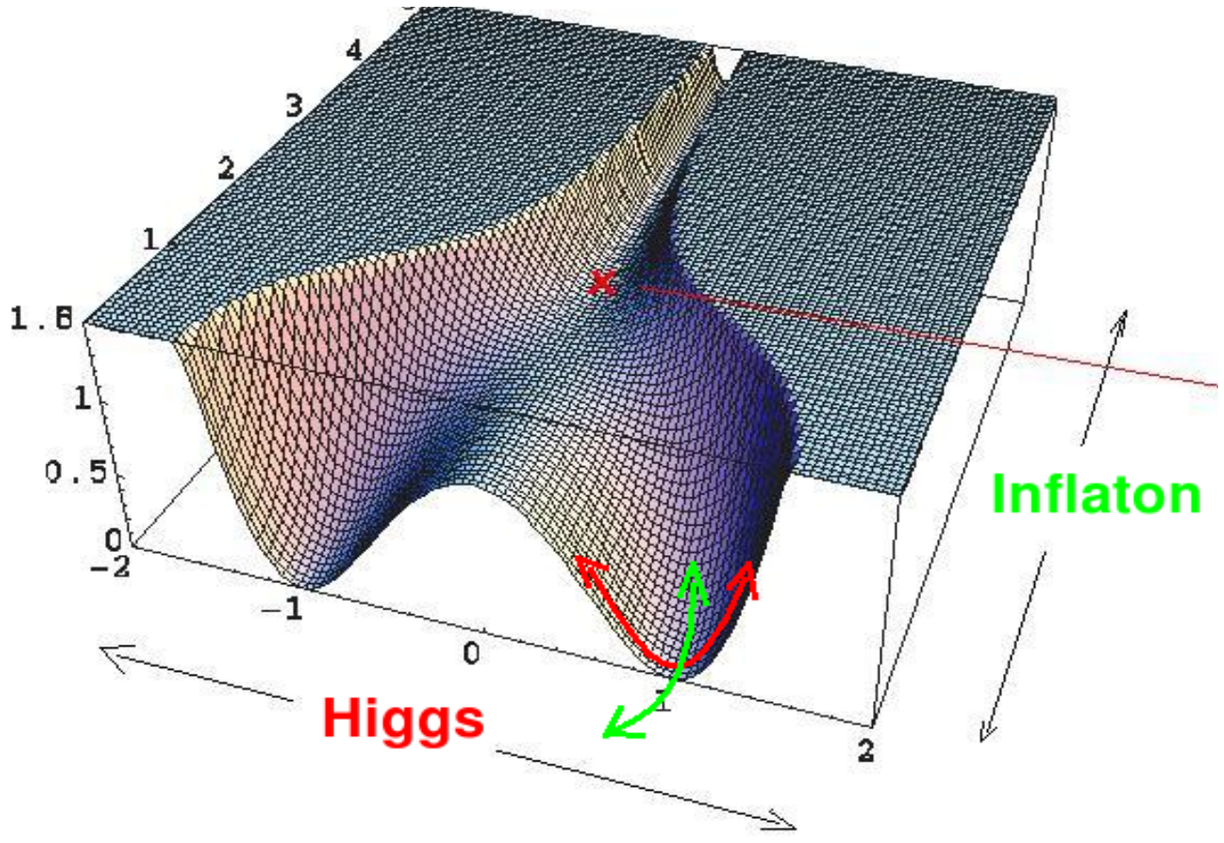

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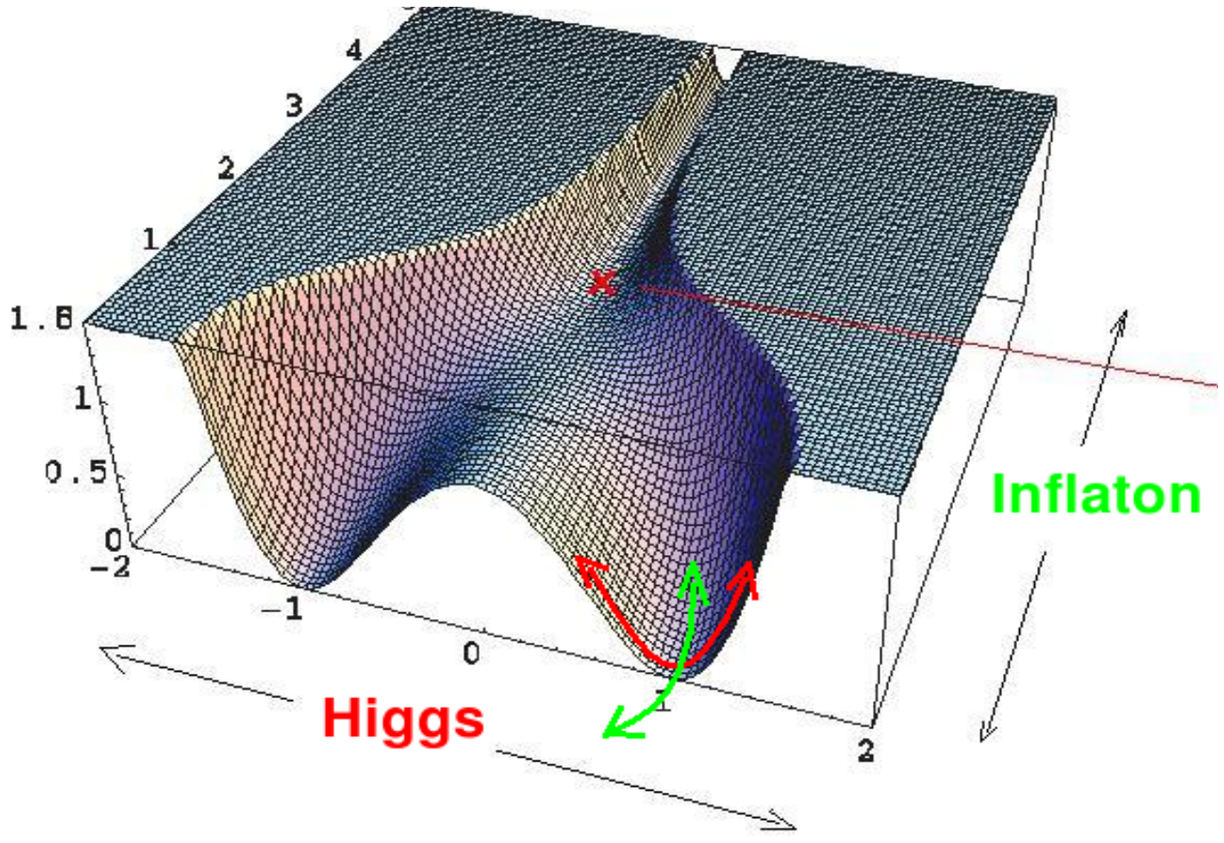

… but now there are gauge field(s) !

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EOM: { Minkowski, Temporal Gauge ($A_0 = 0$)

$$\ddot{\varphi} - D_i D_i \varphi + V_{,\varphi^*} = 0 \quad \rightarrow \text{SCALARS eom}$$

$$\ddot{A}_i - \partial_j \partial_j A_i + \partial_i \partial_j A_j = 2e^2 \,\mathrm{Im}\,[\varphi^* D_i \varphi] \quad \rightarrow \text{VECTORS eom}$$

$$\partial_i \dot{A}_i = 2e^2 \,\mathrm{Im}\,[\varphi^* \dot{\varphi}] \ . \quad \rightarrow \text{GAUSS law}$$

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→ SCALARS eom

→ VECTORS eom

→ GAUSS law

GW EOM

$$\ddot{h}_{ij} - \partial_k \partial_k h_{ij} = 16\pi G \, \Pi^{\mathrm{TT}}_{ij}$$

$$\Pi^{\mathrm{TT}}_{ij} = \left[\partial_i \chi \partial_j \chi + 2\,\mathrm{Re}\left[D_i \varphi \left(D_j \varphi\right)^*\right] - B_i B_j - E_i E_j\right]^{\mathrm{TT}}$$

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COVARIANT

MAGNETIC

ELECTRIC

GAUGE (P)REHEATING

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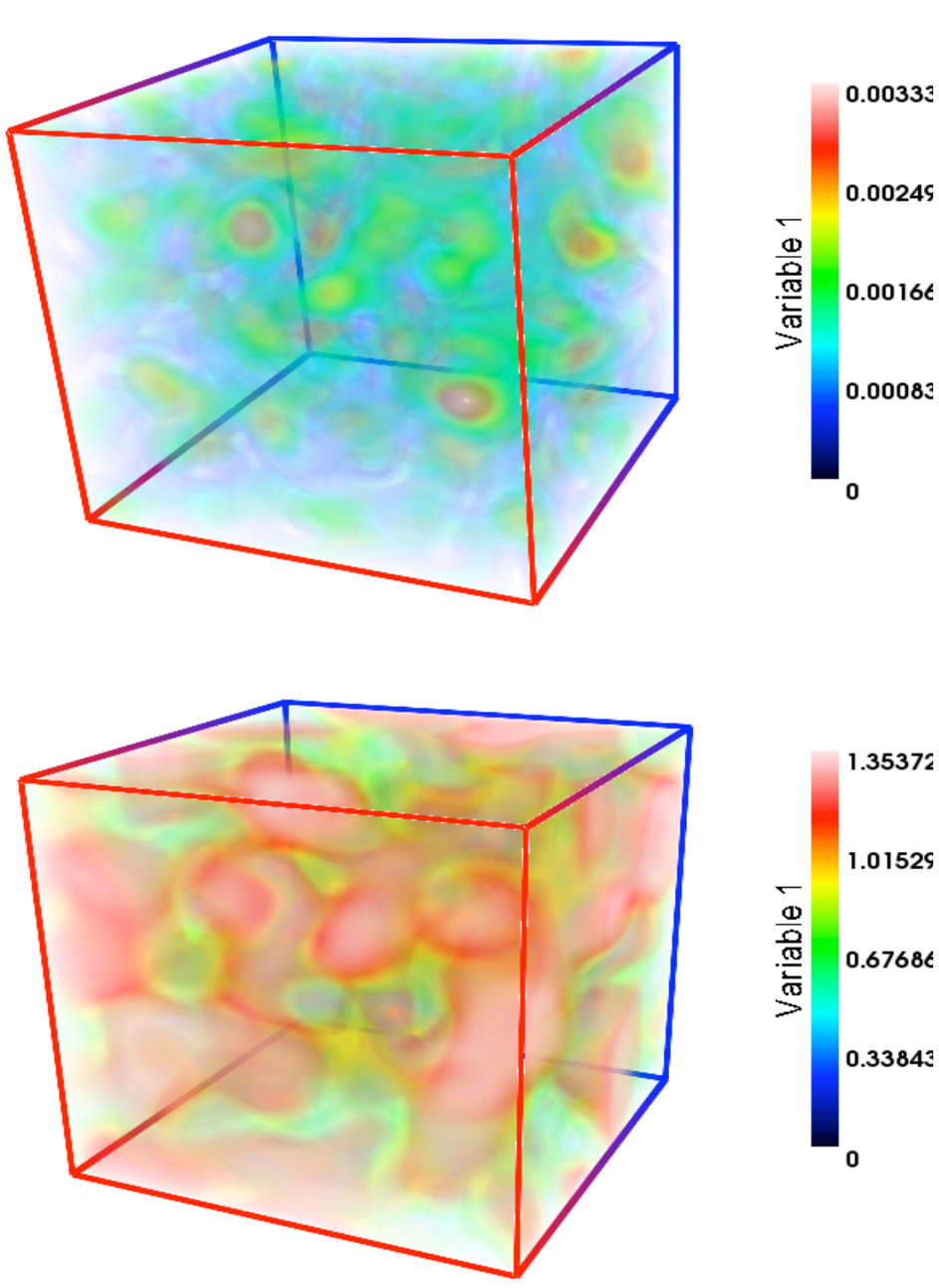
TECHNICAL NOTE : { LATTICE GAUGE TECHNIQUES

~ $O(dx^2)$, ~ $O(dt^2)$

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

DYNAMICS OF THE HIGGS: mt $= 5.5 \rightarrow$ mt $= 23$

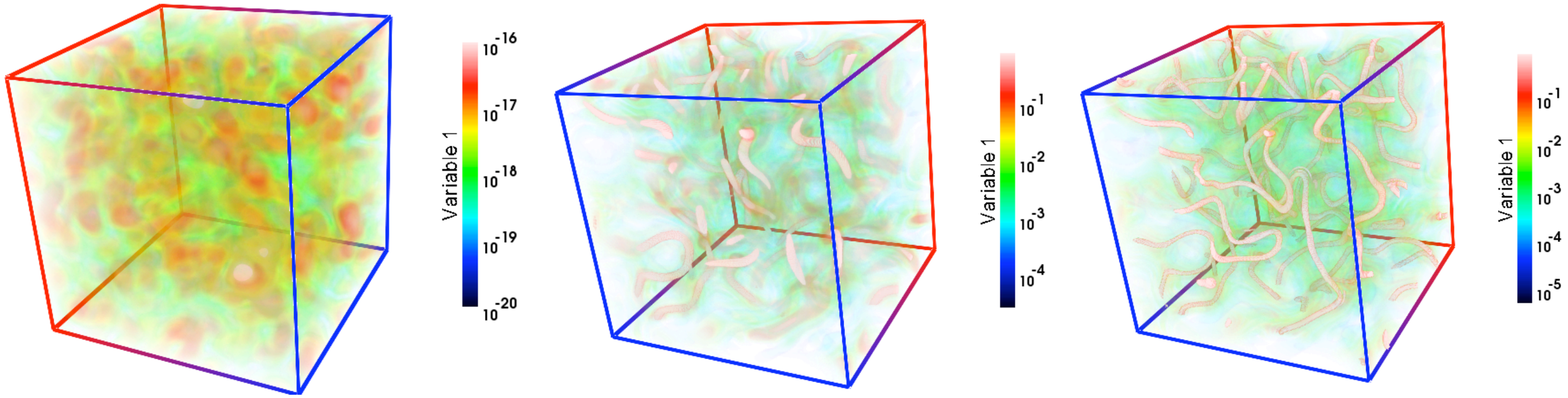


Dufaux et al 2010

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

DYNAMICS OF THE MAGNETIC FIELD: $mt = 5.5 \rightarrow mt = 17$



Dufaux et al 2010

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What's going on !?

Cosmic Strings are formed

Dufaux et al 2010

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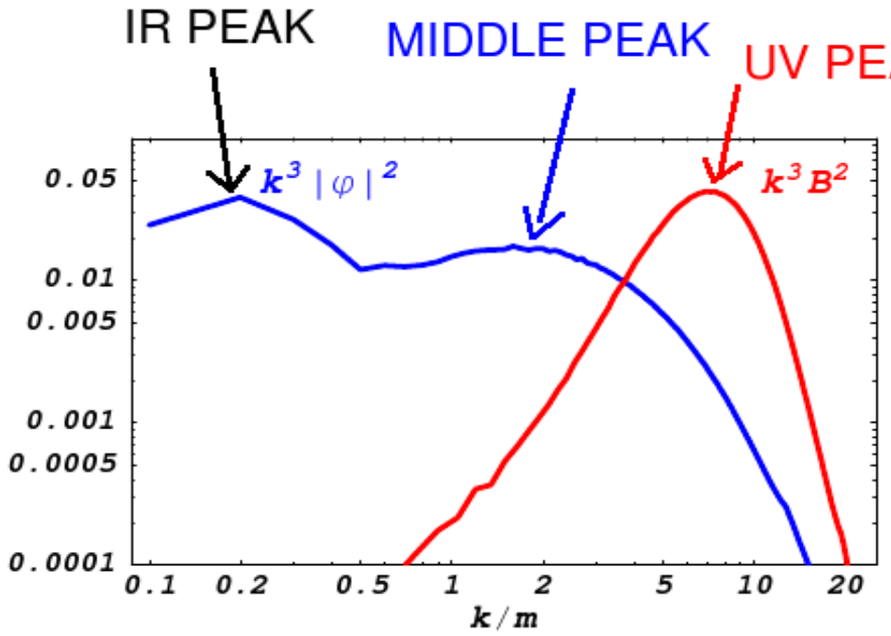
(Topological Defects ⟶ 5th Lecture)

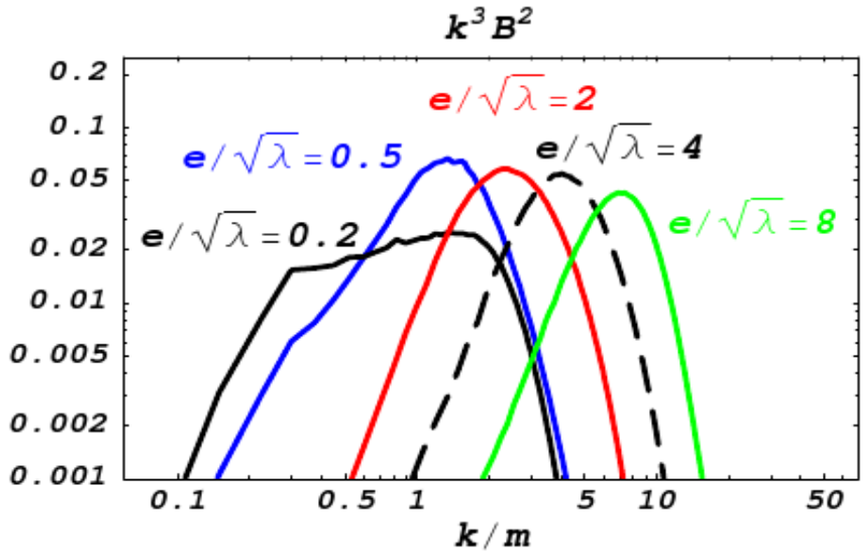
Dufaux et al 2010

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

SCALARS AND VECTORS' SPECTRA:





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The Abelian-Higgs+Inflaton model

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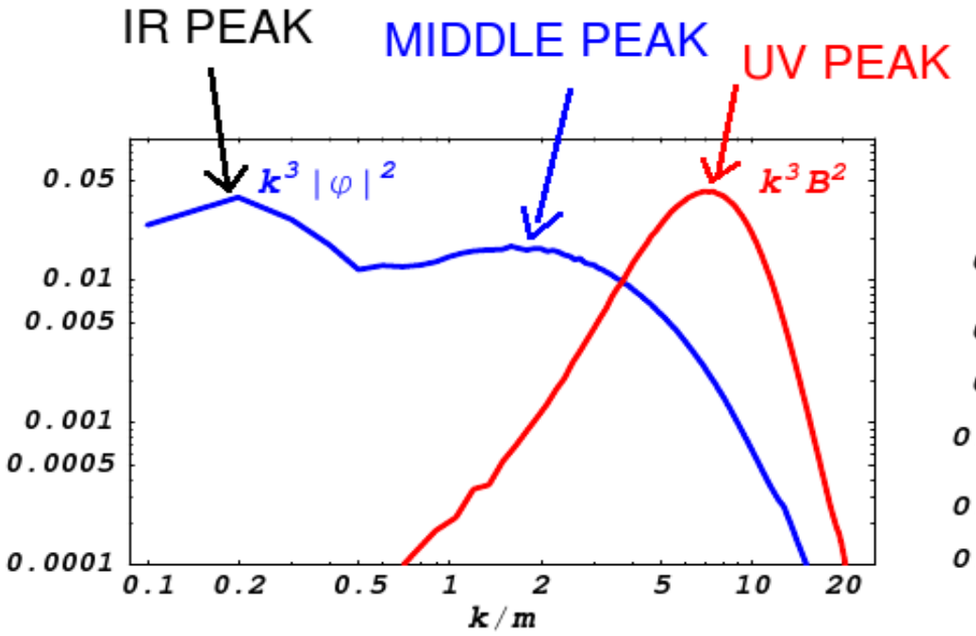

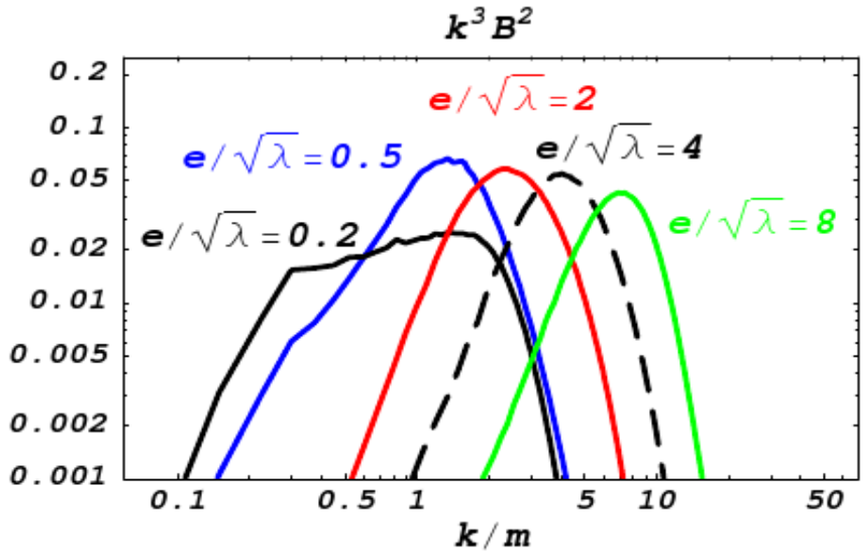

PARAMETERS ABELIAN-HIGGS Model: $m \equiv \sqrt{\lambda}v,\ \lambda/g^2,\ e/\sqrt{\lambda}\ ,\ V_c$

MIDDLE PEAK: { Higgs mass / (Inflaton Velocity)$^{1/3}$ } --> Tachyonic Scale, Bubbles' Size

IR PEAK : Inflaton Velocity, Higgs+Inflaton Couplings (Dufaux et al 2009)

UV PEAK: Vector mass / Higgs Mass

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

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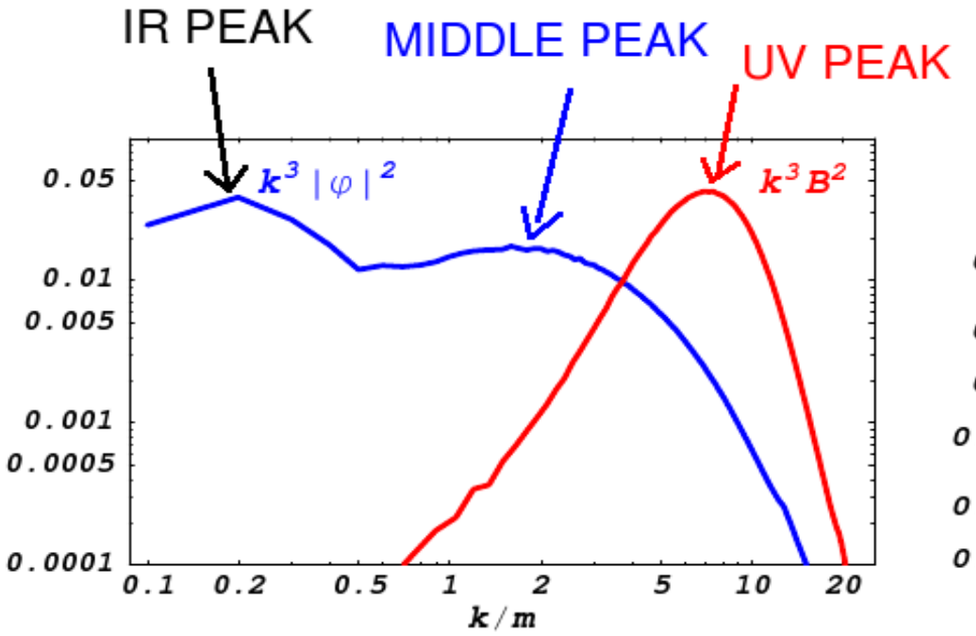

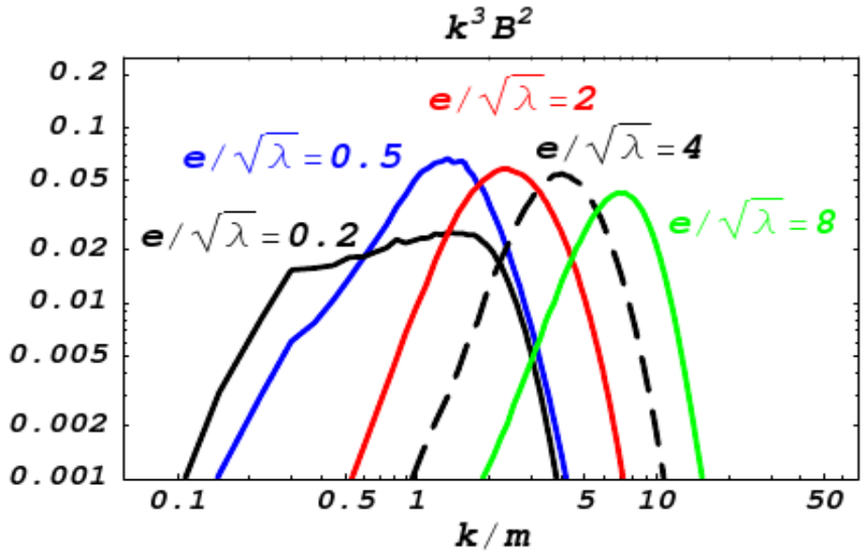

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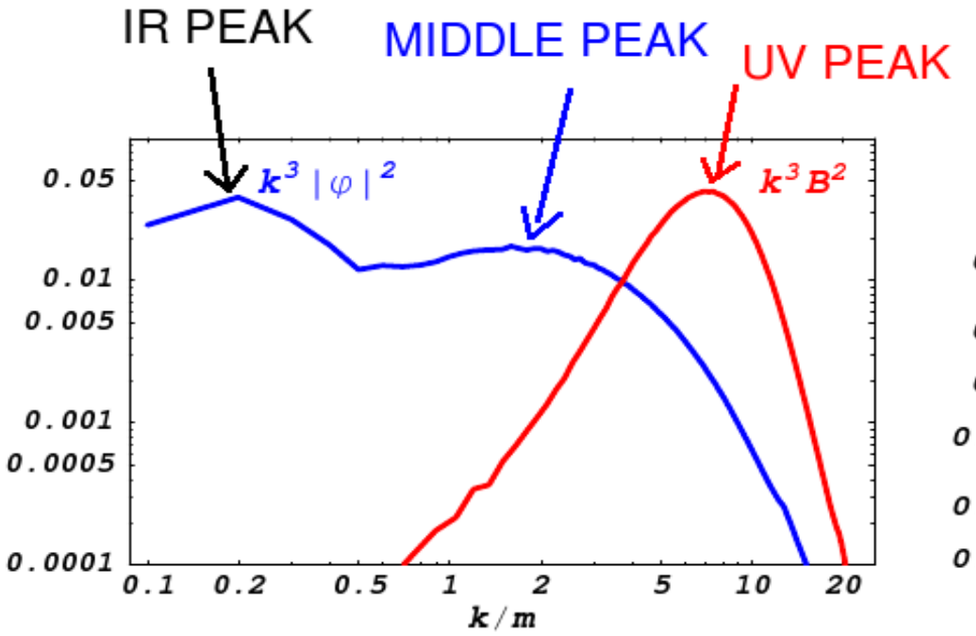

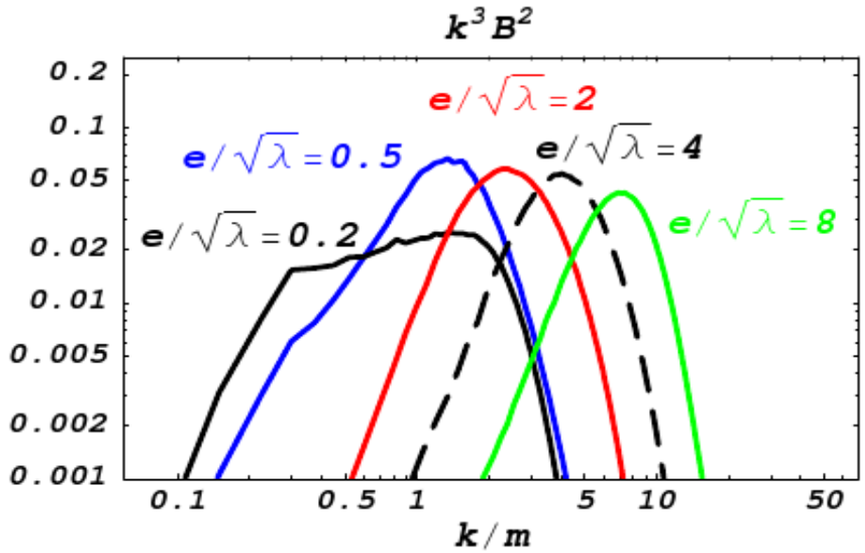

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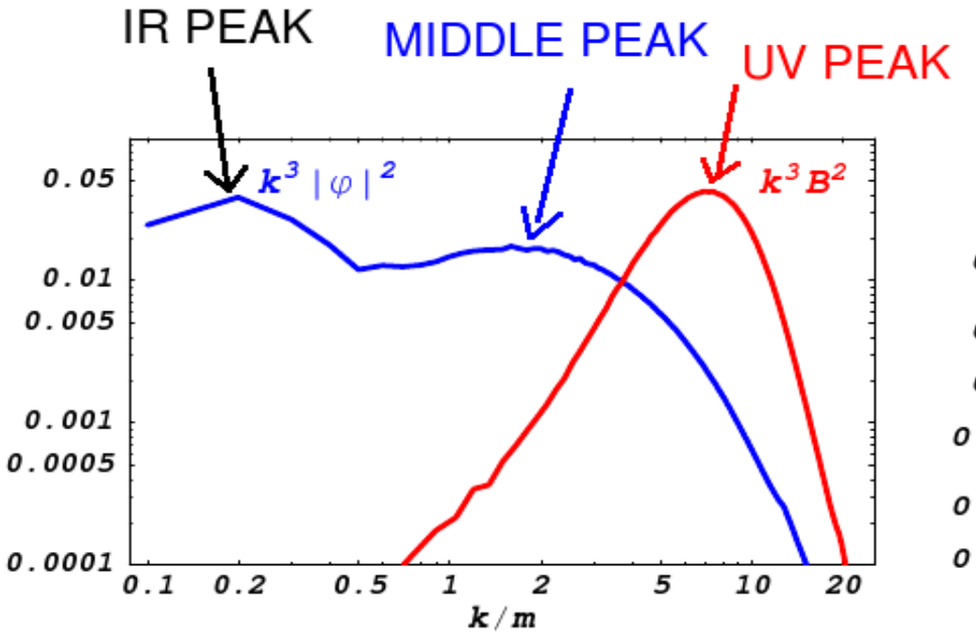

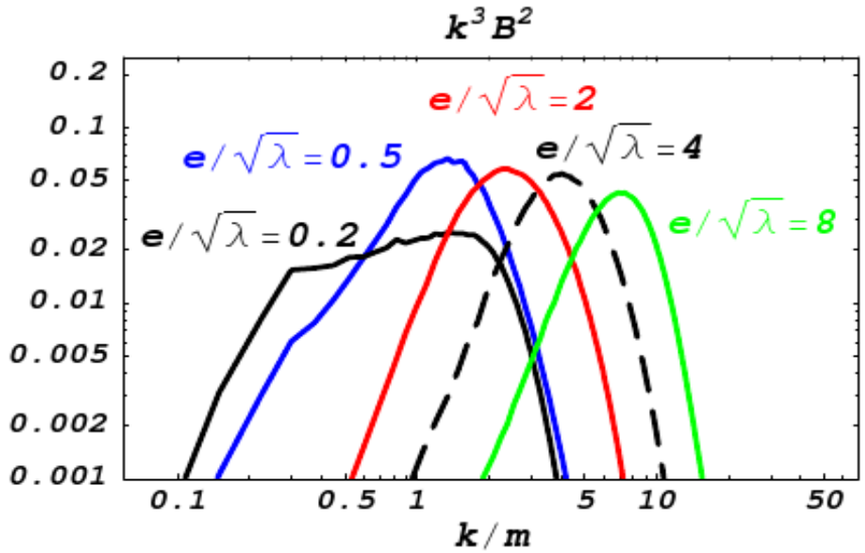

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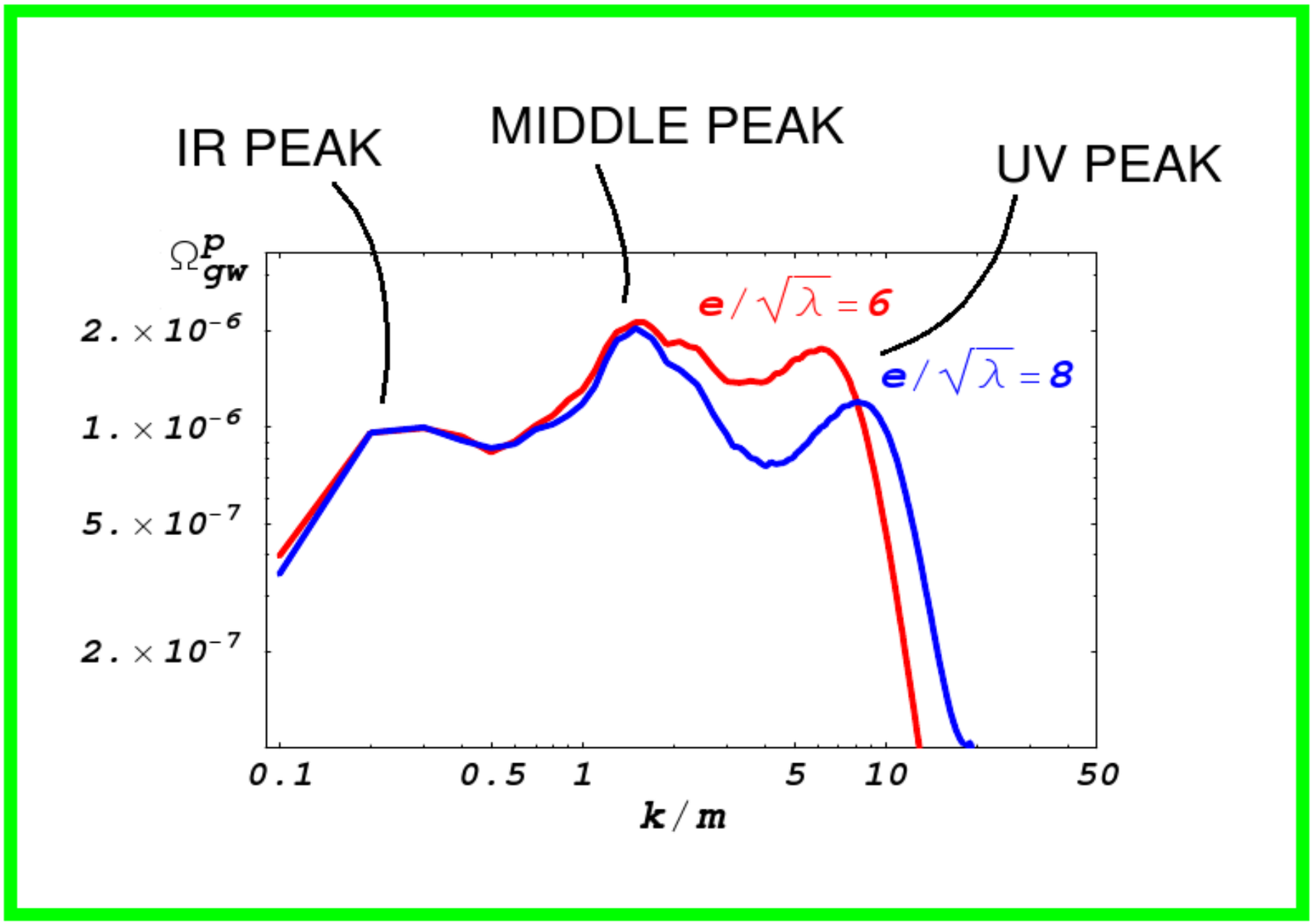
GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GRAVITATIONAL WAVES SPECTRA:

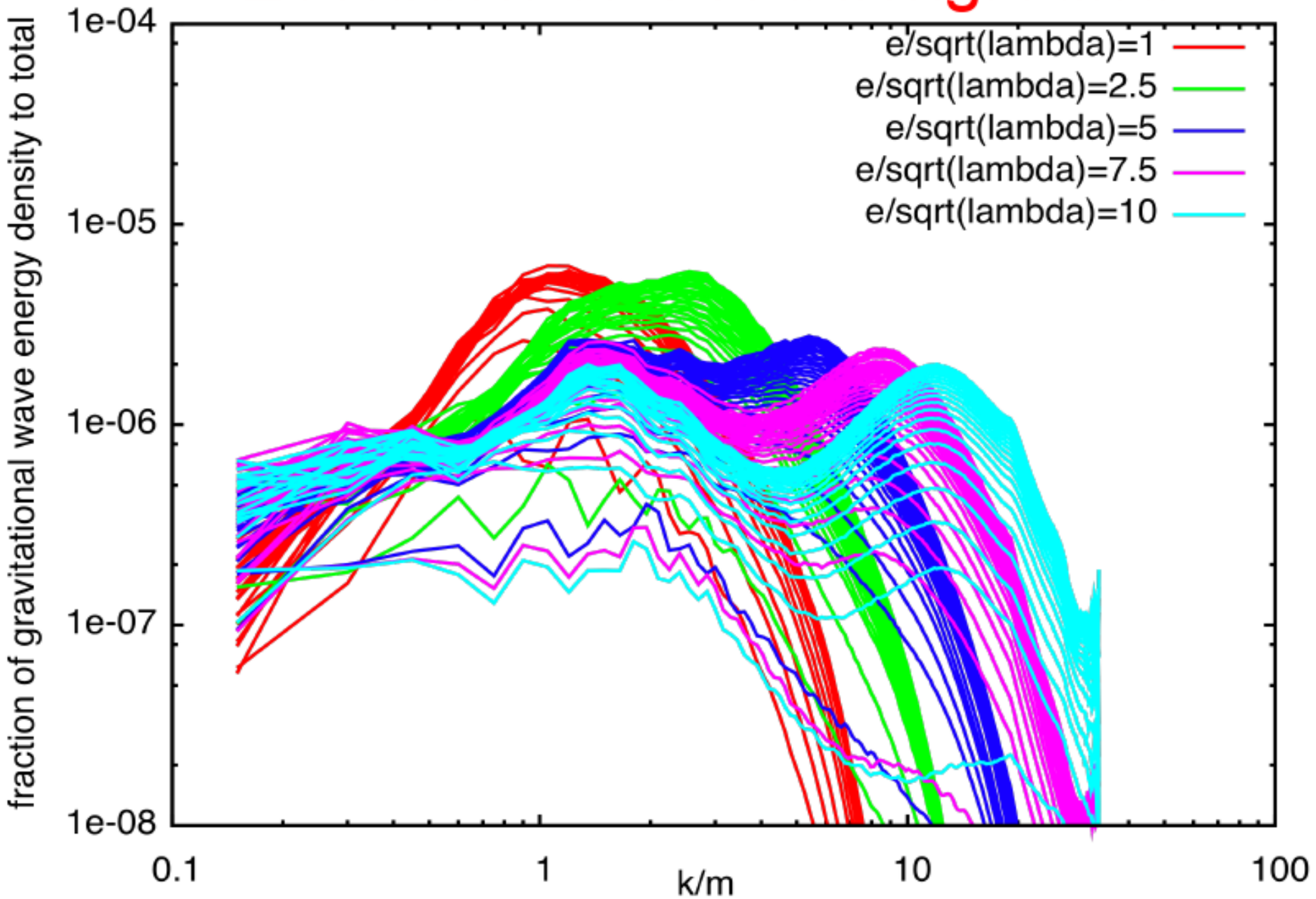


GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GRAVITATIONAL WAVES SPECTRA:

Evolution of GW background



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

GW SPECTRA: ANALYTICS:

IR and Middle Peaks' Amplitude: $\mathsf{F}(g, \lambda, V_c)$

UV peak Amplitude: Lattice Simulations

$$f_1 \lesssim f\,(g, \lambda, V_c) \quad \text{(IR peak)}$$

$$f_2 \approx \lambda^{1/4}\, 10^{11}\,\text{Hz} \quad \text{(Middle peak)}$$

$$f_3 \approx \frac{e}{\sqrt{\lambda}}\, \lambda^{1/4}\, 10^{11}\,\text{Hz} \quad \text{(UV peak)}$$

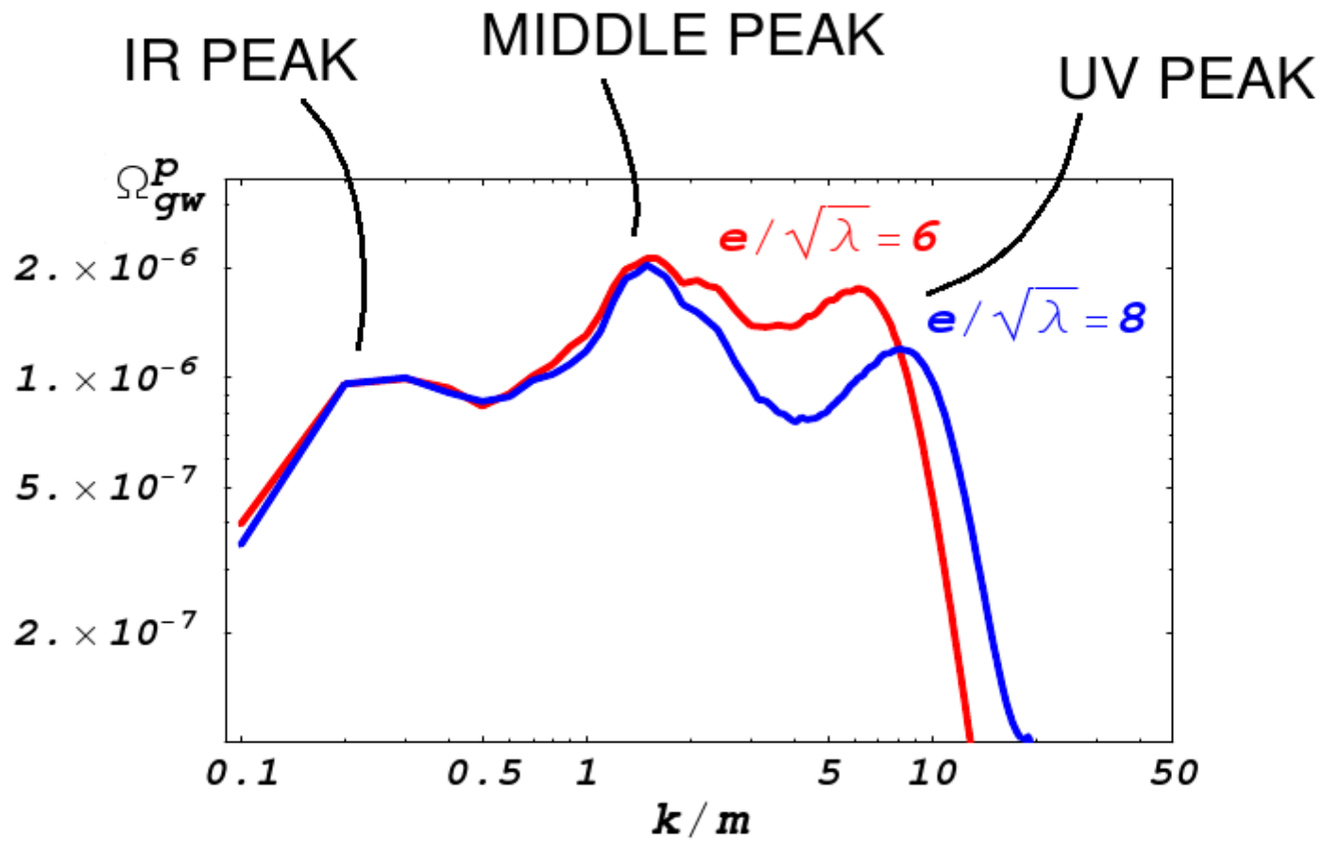
RED-SHIFTED FREQUENCIES

$f\,(g, \lambda, V_c)$ ⟶ Dufaux et al '09

GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks ! (particle physics spectroscopy)



GAUGE (P)REHEATING

The Abelian-Higgs+Inflaton model

Several Peaks ! (particle physics spectroscopy)

$$\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,,$$

Large amplitude(s) !

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The Abelian-Higgs+Inflaton model

Several Peaks ! (particle physics spectroscopy)

$$\Omega_{\rm GW}^{(o)} \sim 10^{-11}\,, \quad @ \quad f_o \sim 10^8 - 10^9 \ {\rm Hz}$$

Large amplitude(s) ! ... but at high Frequency !

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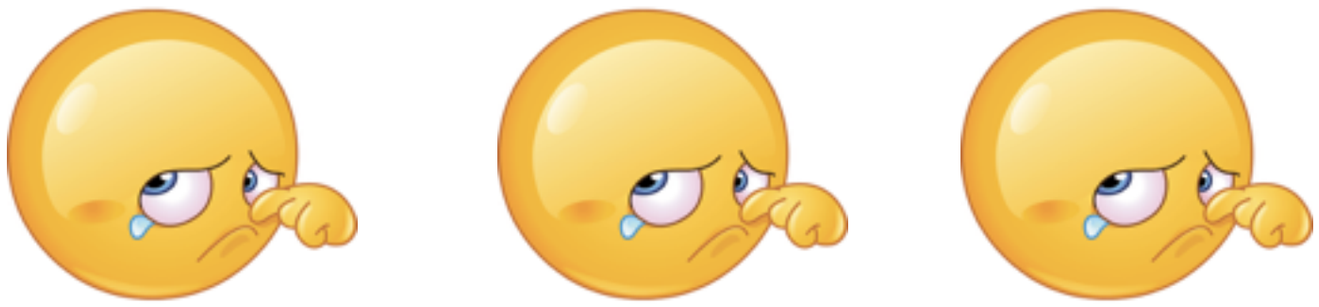
Large amplitude(s) ! ... but at high Frequency !

Very unfortunate... no high frequency detectors !



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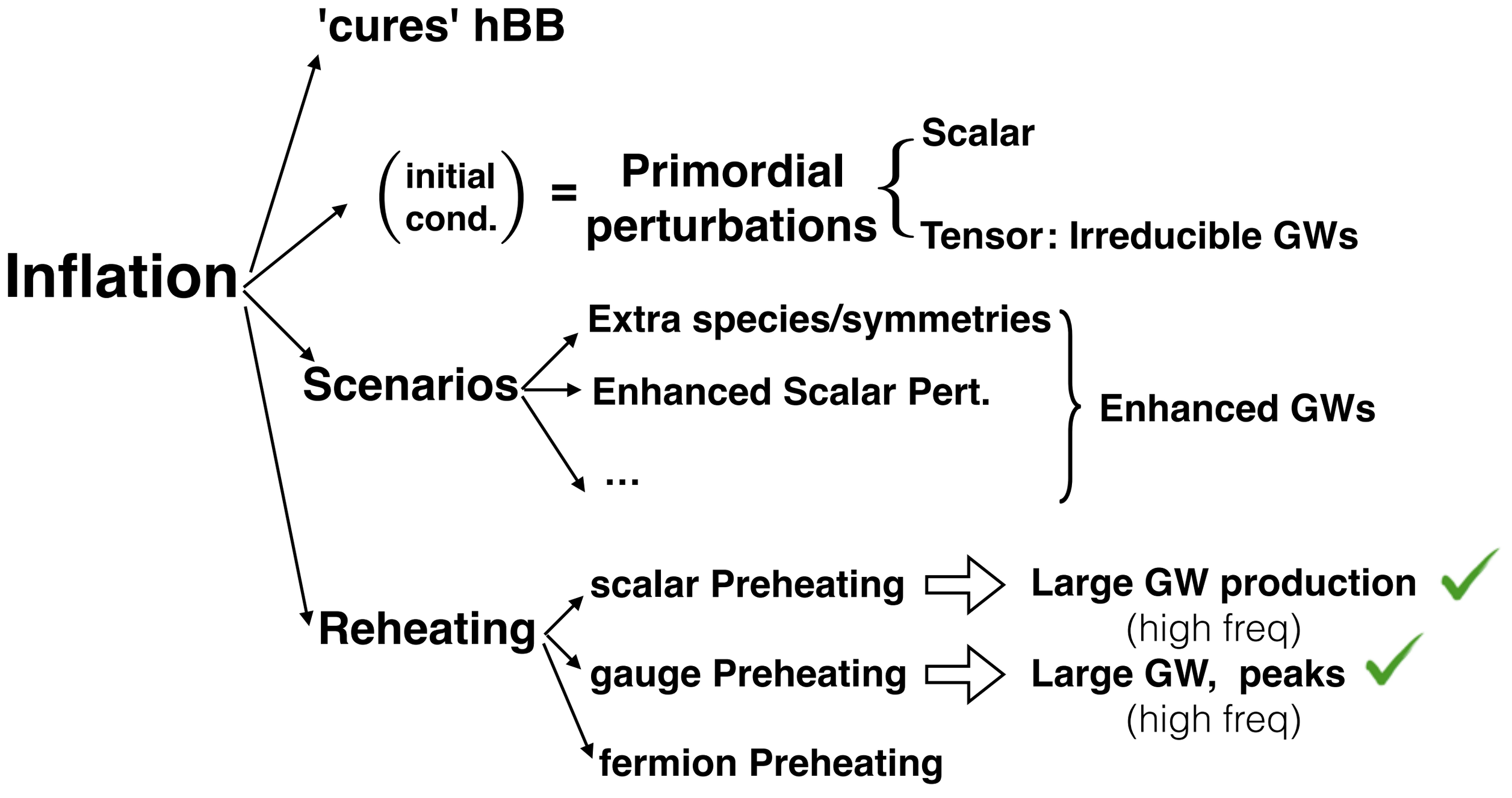

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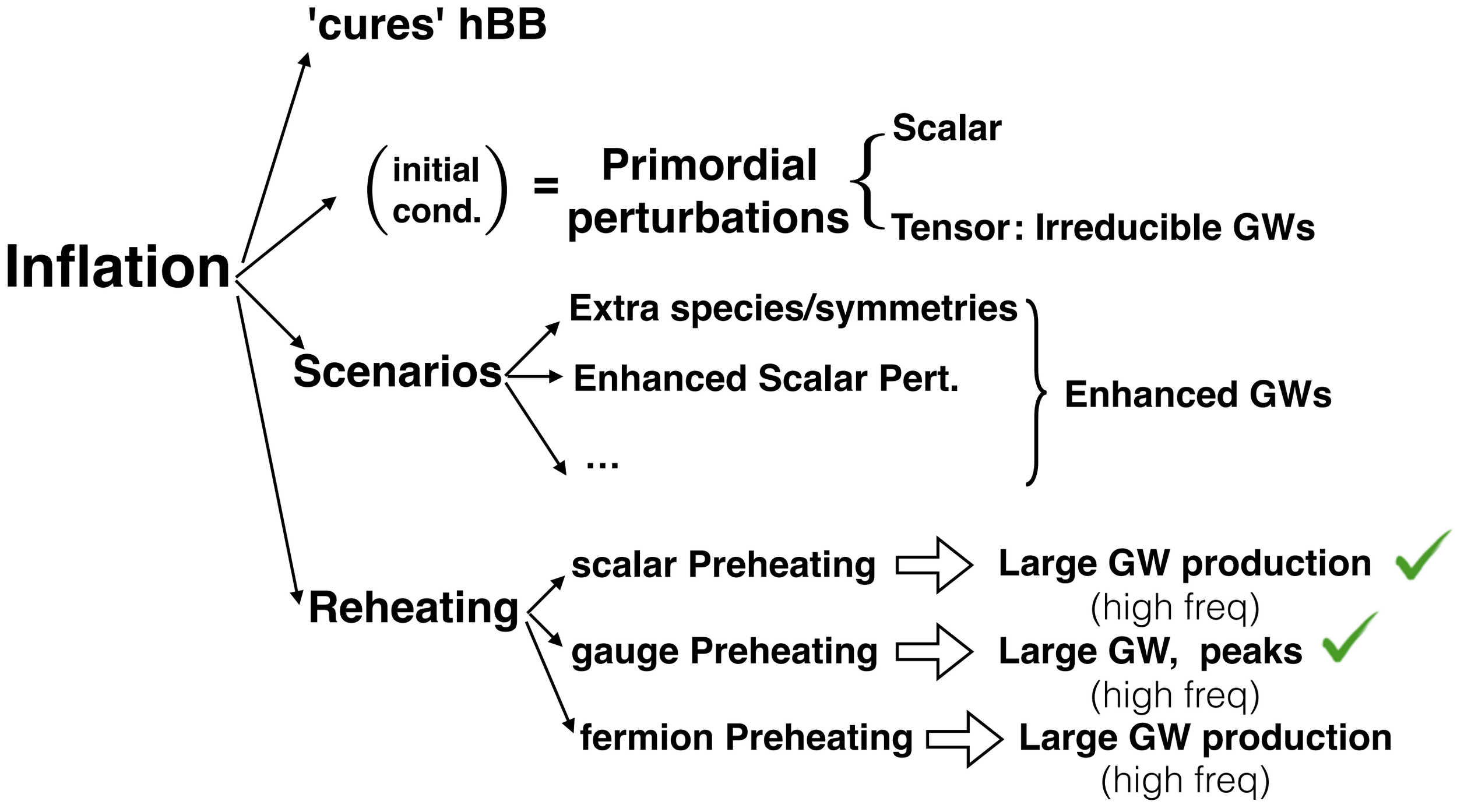

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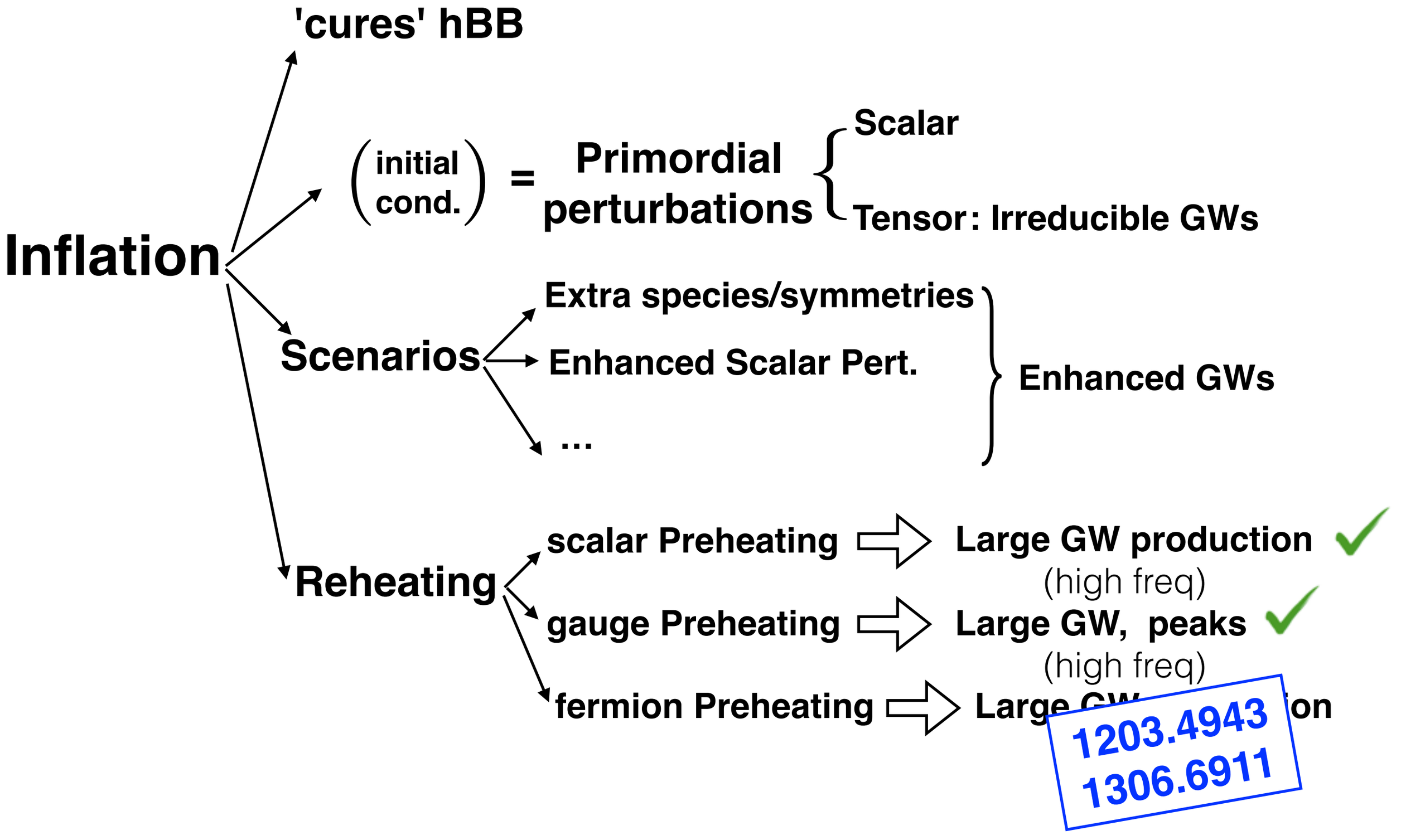

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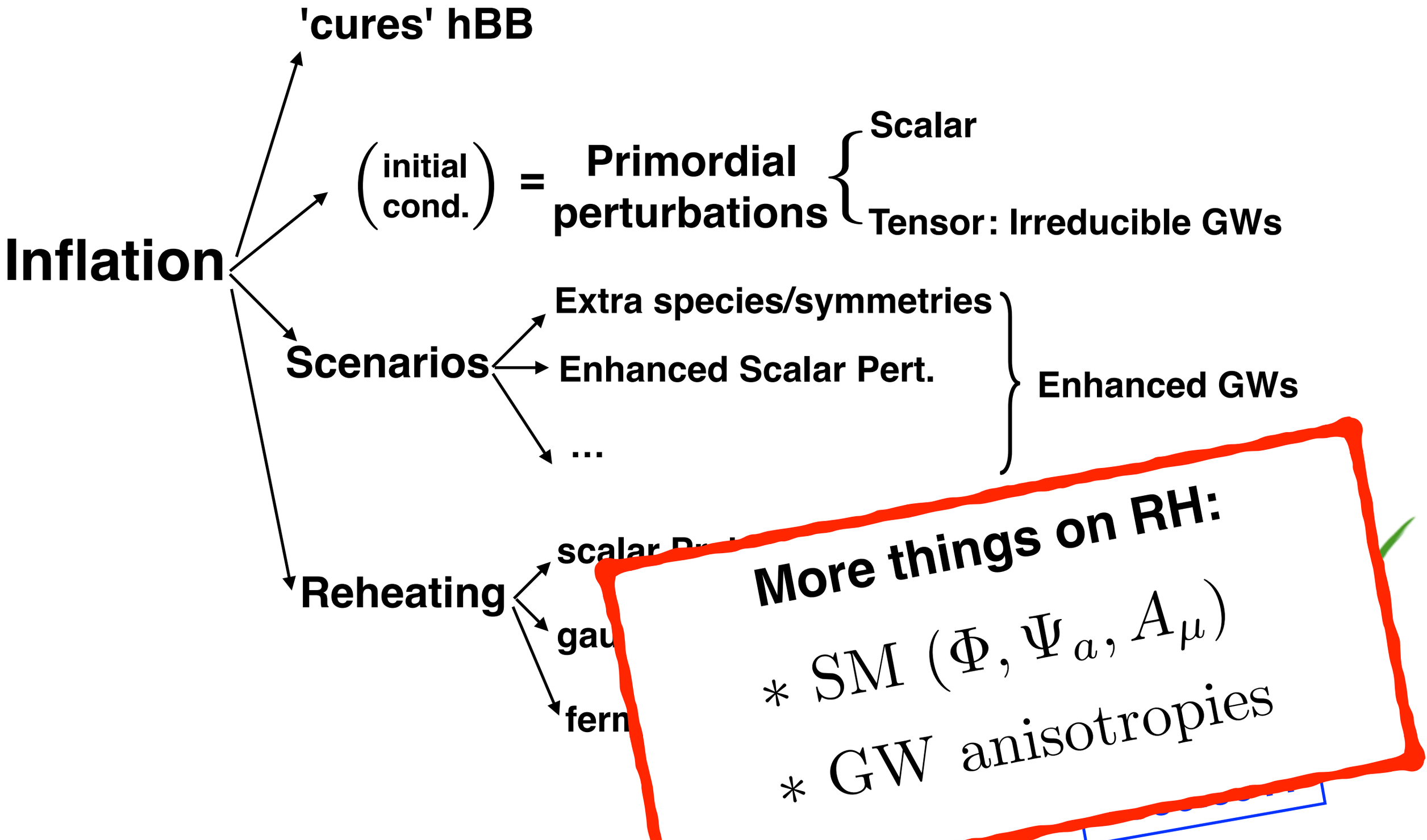

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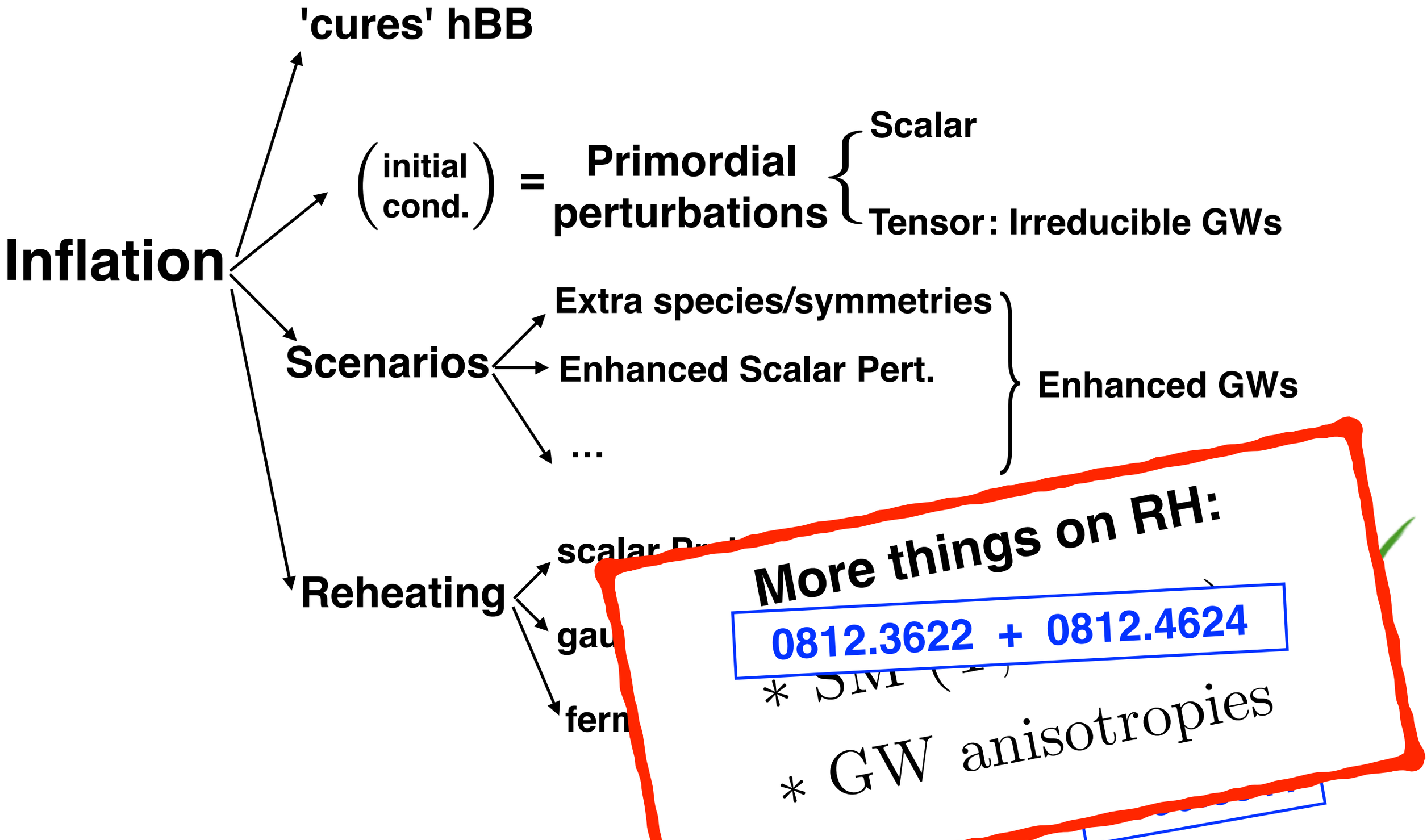

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