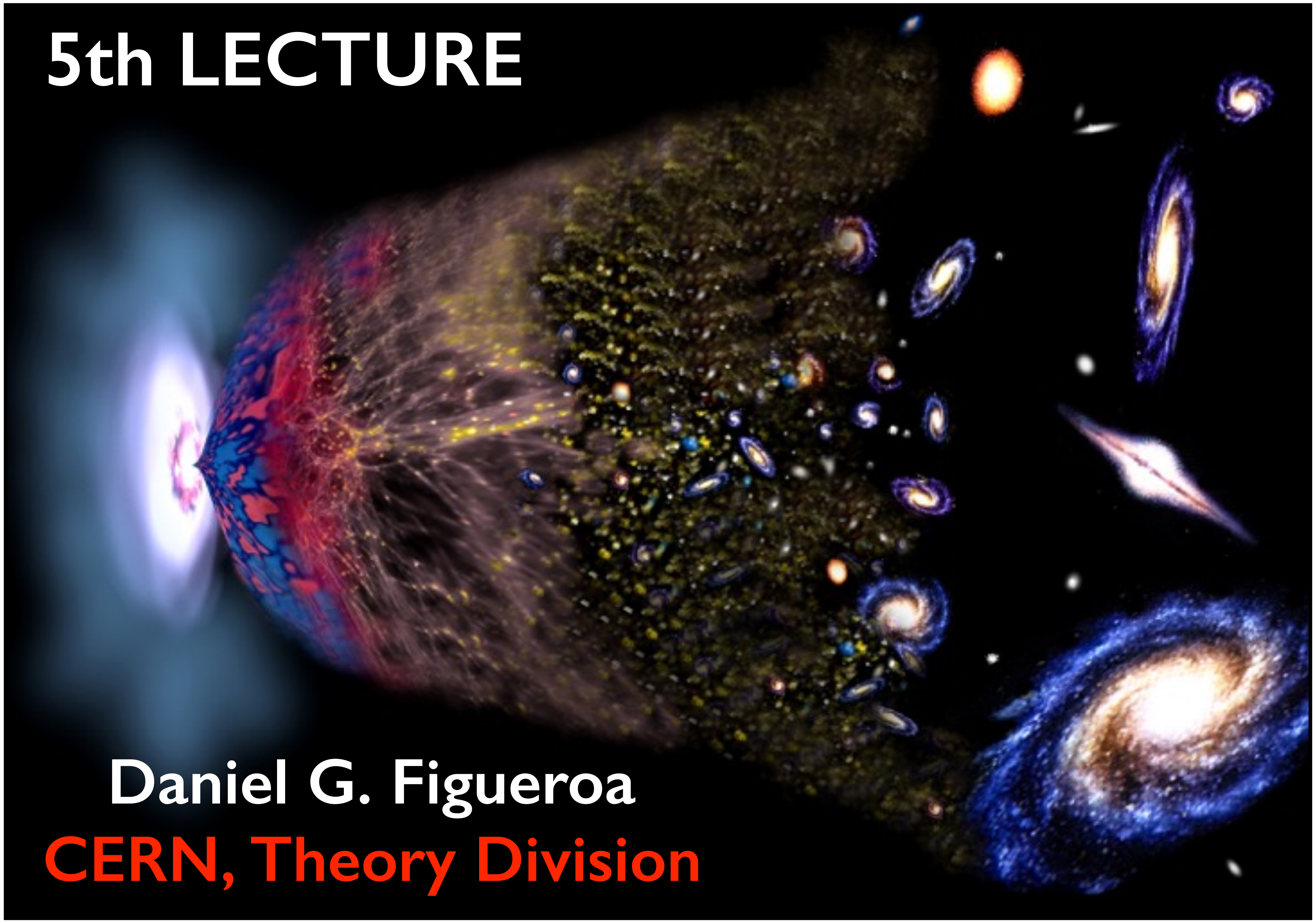
GRAVITATIONAL WAVES
PROBE OF THE EARLY UNIVERSE



School on Gravitational Waves for Cosmology and Astrophysics, Benasque, May 28 - June 10, 2017

EARLY UNIVERSE

Inflation

'cures' hBB

(initial cond.) = Primordial perturbations

- Scalar: CMB ΔT anisotropies (eventually Galaxies)
- Tensor: Irreducible GWs (CMB B-mode Pol.)

Scenarios

- Extra species/symmetries
- Enhanced Scalar Pert.
- ...

Enhanced GWs

EARLY UNIVERSE

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Enhanced GWs

Reheating

- scalar Preheating
- gauge Preheating
- fermion Preheating

Large GW (high freq)

EARLY UNIVERSE

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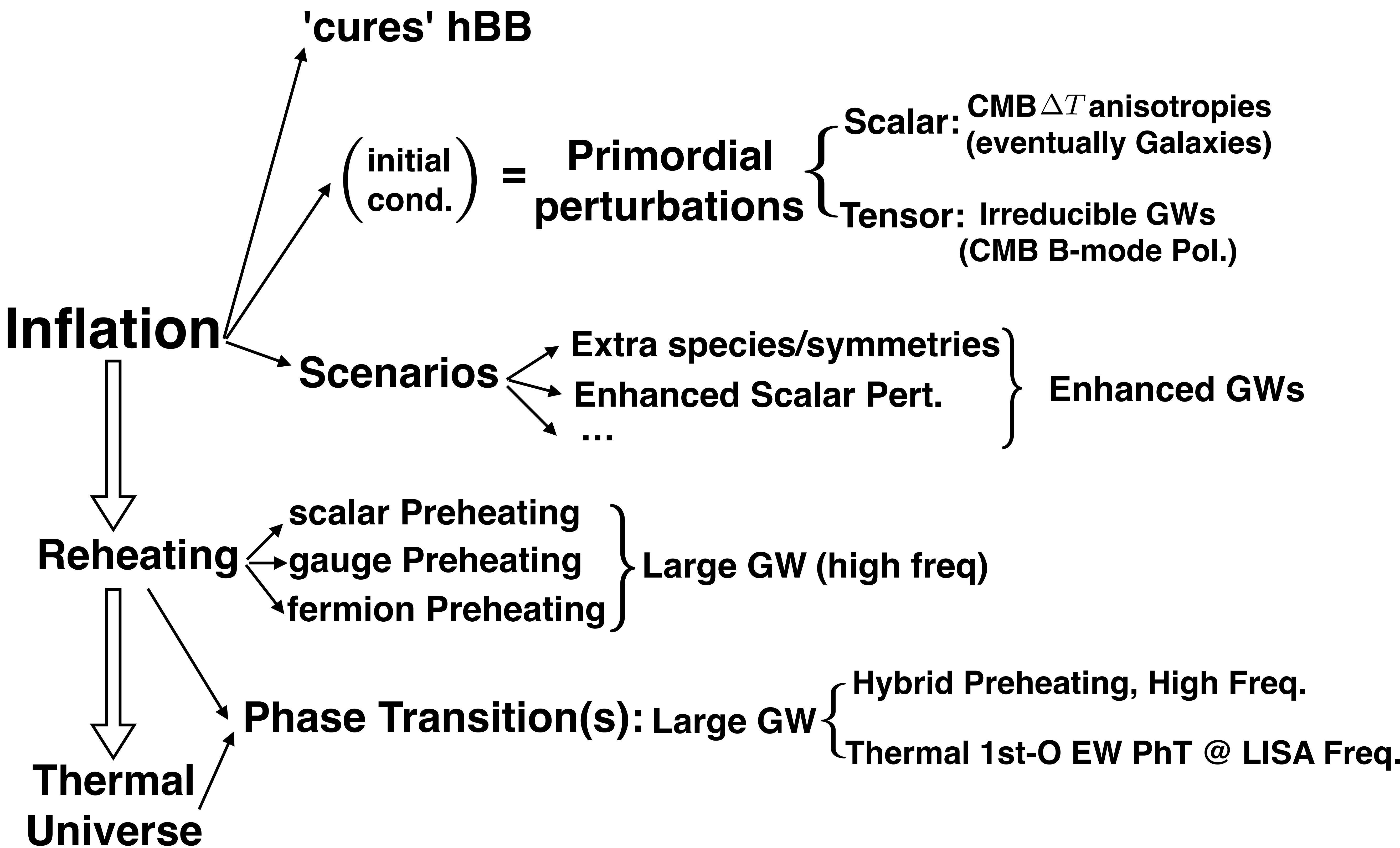
Large GW (high freq)

Thermal Universe

Phase Transition(s): Large GW

Thermal 1st-O EW PhT @ LISA Freq.

EARLY UNIVERSE



EARLY UNIVERSE

Inflation

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Scenarios

Extra species/symmetries

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...

Enhanced GWs

Reheating

scalar Preheating

gauge Preheating

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Large GW (high freq)

Thermal Universe

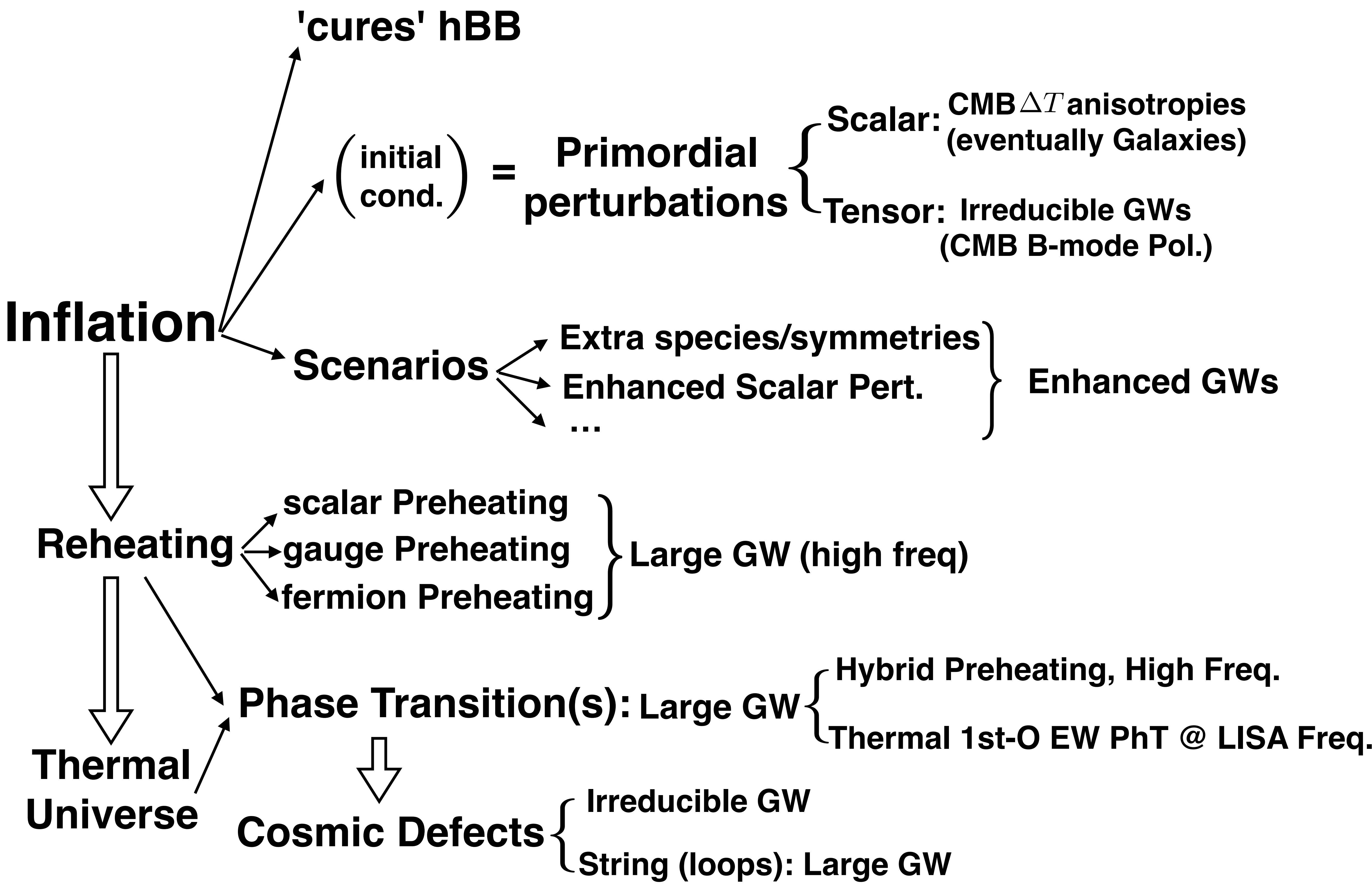
Phase Transition(s): Large GW

Hybrid Preheating, High Freq.

Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects

EARLY UNIVERSE



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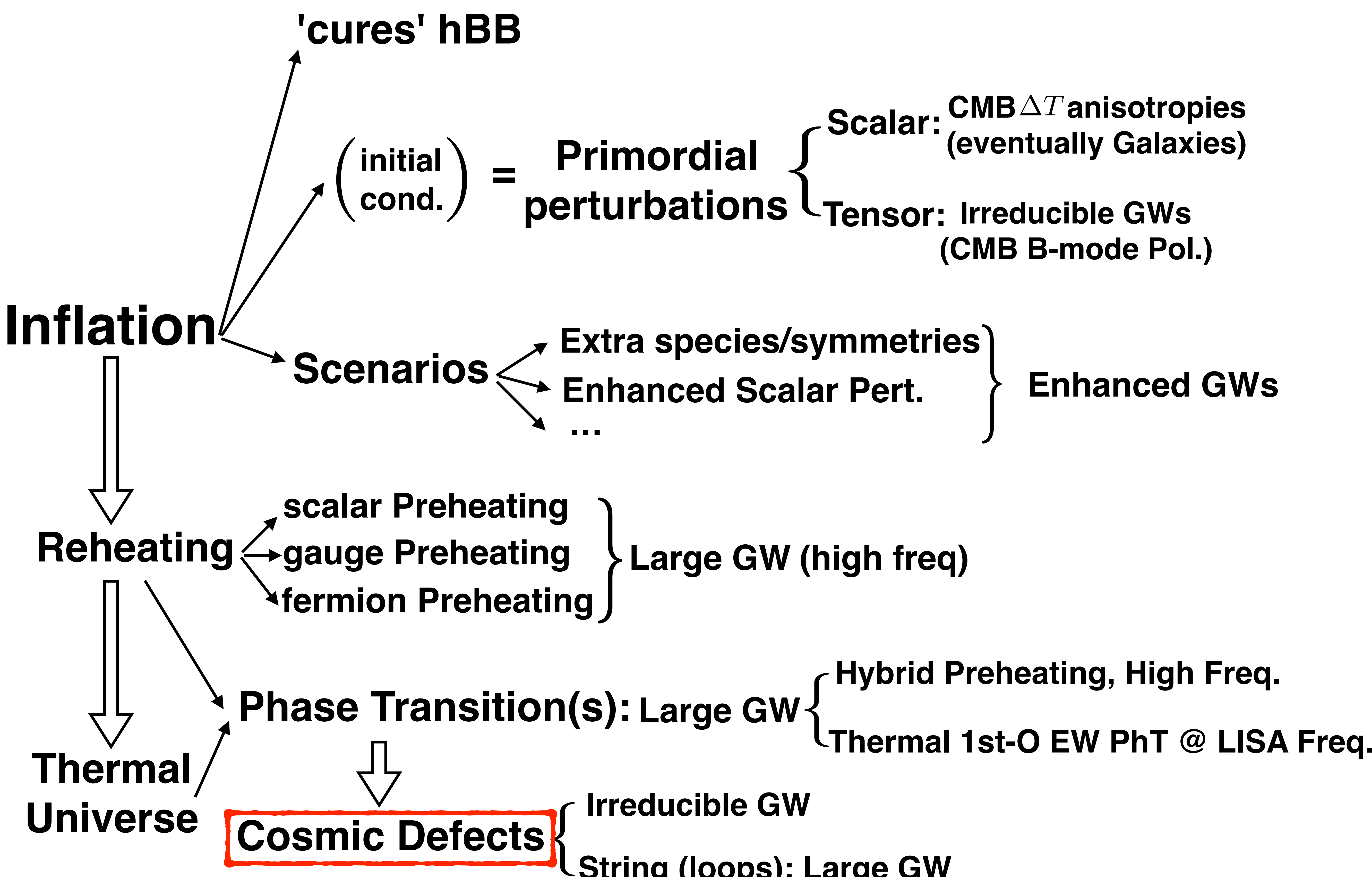


Introduction to Cosmic Defects

Topology of cosmic domains and strings

T W B Kibble
Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, UK

Received 11 March 1976

Abstract. The possible domain structures which can arise in the universe in a spontaneously broken gauge theory are studied. It is shown that the formation of domain walls, strings or monopoles depends on the homotopy groups of the manifold of degenerate vacua. The subsequent evolution of these structures is investigated. It is argued that while theories generating domain walls can probably be eliminated (because of their unacceptable gravitational effects), a cosmic network of strings may well have been formed and may have had important cosmological effects.

Kibble pioneered the study of topological defect generation in the early universe.

Introduction to Cosmic Defects

Kibble'76

…ve a specific example in mind. Let us consider an N-component real scalar field ϕ with a Lagrangian invariant under the orthogonal group $\mathrm{O}(N)$, and coupled in the usual way to $\frac{1}{2}N(N-1)$ vector fields represented by an antisymmetric matrix B_μ. We can take

$$L=\tfrac{1}{2}(D_\mu\phi)^2-\tfrac{1}{8}g^2(\phi^2-\eta^2)^2+\tfrac{1}{8}\mathrm{Tr}(B_{\mu\nu}B^{\mu\nu}) \tag{1}$$

with

$$D_\mu\phi=\partial_\mu\phi-eB_\mu\phi$$

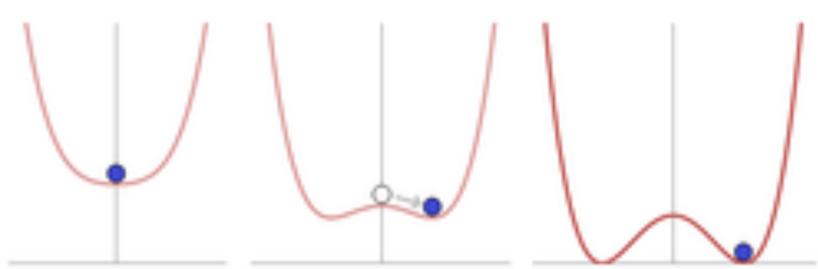
$$B_{\mu\nu}=\partial_\nu B_\mu-\partial_\mu B_\nu+e[B_\mu, B_\nu].$$

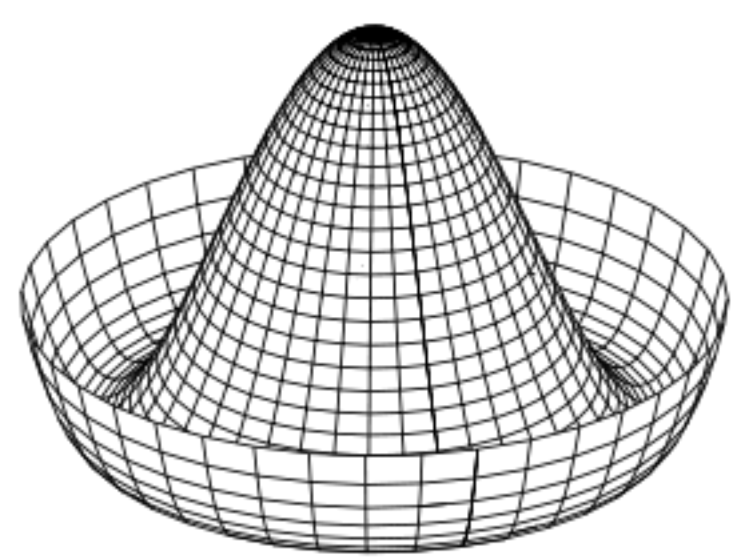
The coupling constants g and e are not necessarily related, but we shall assume that they are of a similar order of magnitude (and both small).

At zero temperature the $\mathrm{O}(N)$ symmetry here is spontaneously broken to $\mathrm{O}(N-1)$, with ϕ acquiring a vacuum expectation of order η. In the tree approximation,

$$\langle\phi\rangle^2=\eta^2 \tag{2}$$

so that the manifold of degenerate vacua is an $(N-1)$ sphere S^{N-1}.





Introduction to Cosmic Defects

Kibble'76

Higgs True V.E.V.

$$V(\phi) = \tfrac{1}{8}g^2(\phi^2 - \eta^2)^2 + \tfrac{1}{48}[(N+2)g^2 + 6(N-1)e^2]T^2\phi^2, \qquad (3)$$

as in the Landau–Ginsberg theory of superconductivity. (See for example Schrieffer 1964.) The minimum occurs at $\phi = 0$ and so the symmetry is unbroken for T larger than the transition temperature

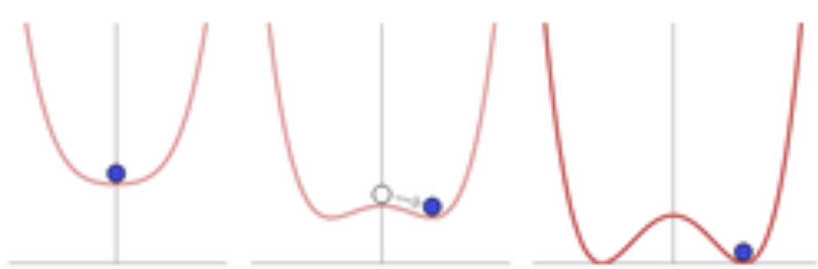
$$T_c = \eta\left(\frac{N+2}{12} + \frac{N-1}{2}\frac{e^2}{g^2}\right)^{-1/2}. \qquad (4)$$

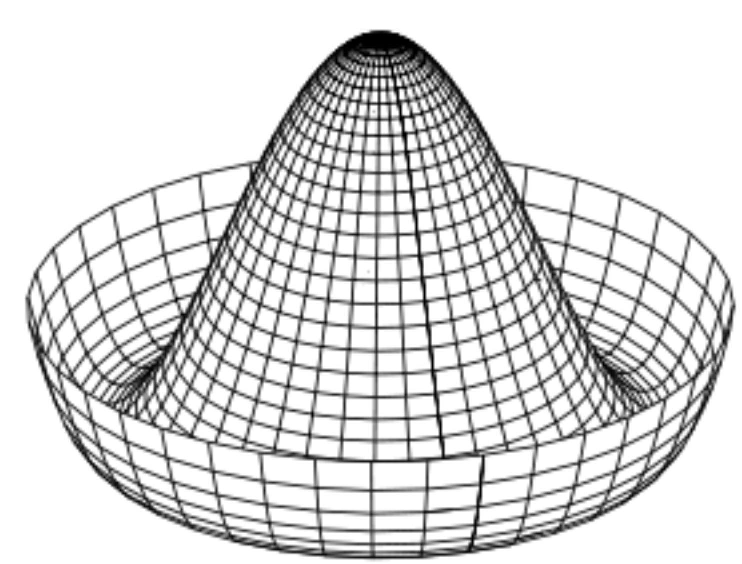
Critical Temperature

This is the normal phase. Below T_c, we have an ordered phase: ϕ acquires a vacuum expectation value, which plays the role of the order parameter, and whose magnitude is determined by

$$\langle\phi\rangle^2 = \eta^2[1 - (T^2/T_c^2)]. \qquad (5)$$

Higgs VEV vs Temp $(T \leq T_c)$

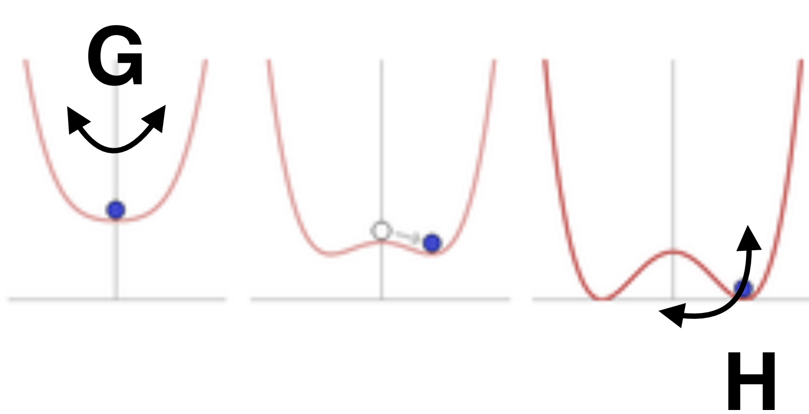


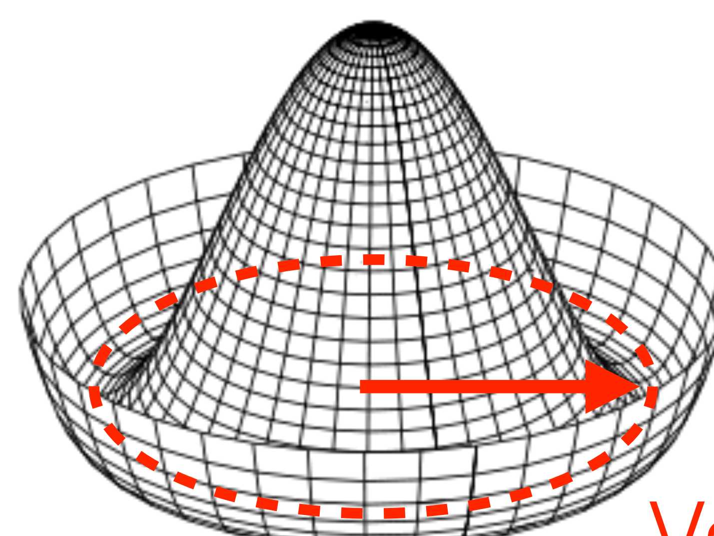


Introduction to Cosmic Defects

Kibble'76

us recall the more general situation. In a model with symmetry group G, the vacuum expectation value $\langle\phi\rangle$ will be restricted to lie on some orbit of G. If H is the isotropy subgroup of G at one point $\langle\phi\rangle$, i.e. the subgroup of transformations leaving $\langle\phi\rangle$ unaltered, then the orbit may be identified with the coset space $M = G/H$. Physically H is the subgroup of unbroken symmetries, and M is the manifold of degenerate vacua. As we shall see, the topological properties of M (specifically its homotopy groups) largely determine the geometry of possible domain structures.



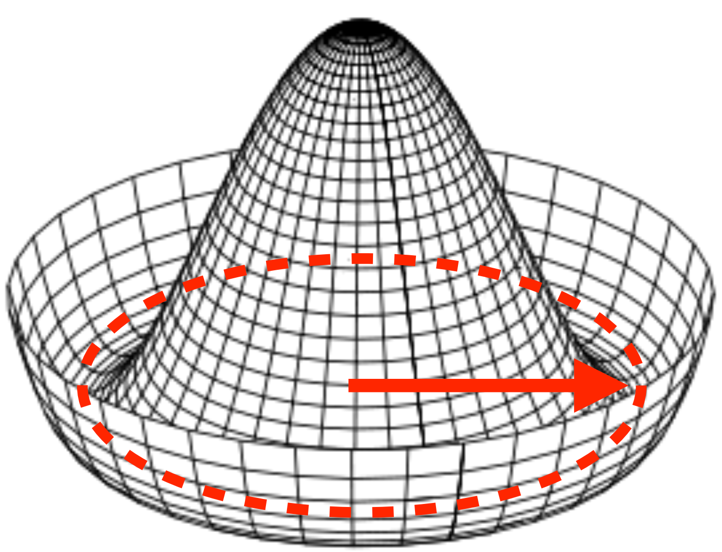


Introduction to Cosmic Defects

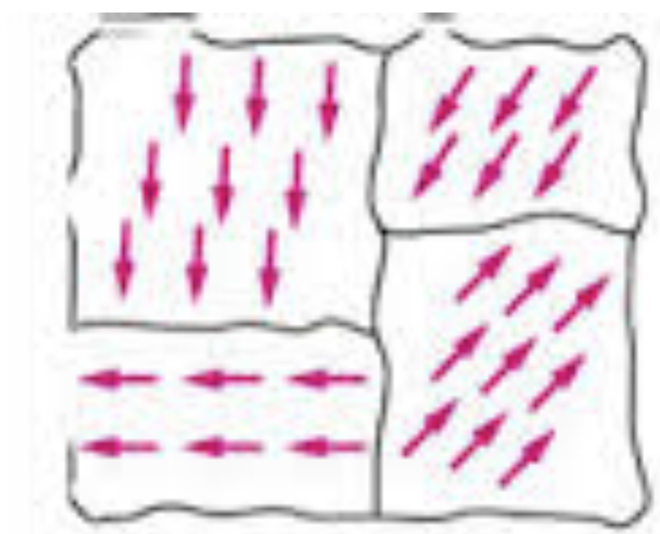
3. Formation of protodomains

For T near T_c there will be large fluctuations in ϕ. Once T has fallen well below T_c, we may expect ϕ to have settled down with a non-zero expectation value corresponding to some point on M. No point is preferred over any other. As in an isotropic ferromagnet cooled below its Curie point the choice will be determined by whatever small fields happen to be present, arising from random fluctuations. Moreover this choice will be made independently in different regions of space, provided they are far enough apart. (What is far enough we shall discuss shortly.) Thus we can anticipate the formation of an initial domain structure with the expectation value of ϕ, the order parameter, varying from region to region in a more or less random way. Of course for energetic reasons a constant or slowly varying $\langle\phi\rangle$ is preferred and so much of this initially chaotic variation will quickly die away. The interesting question is whether any residue remains—in particular whether normal regions can be 'trapped' like flux tubes in a superconductor.

Kibble'76



M = G/H

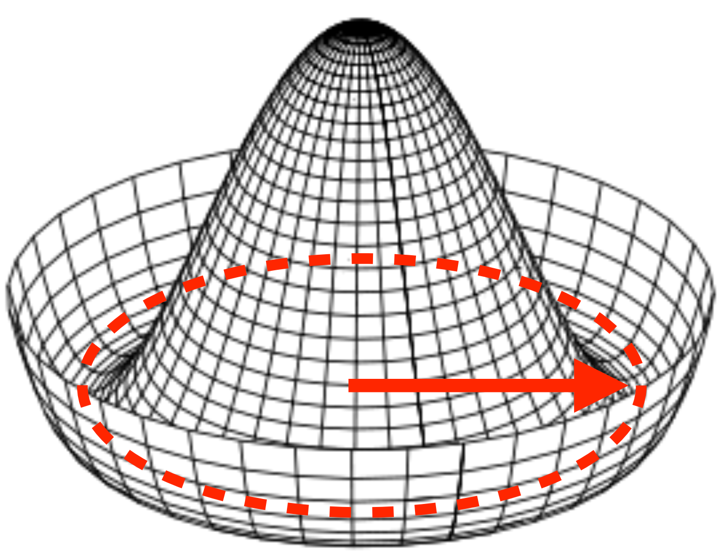


Introduction to Cosmic Defects

6. Conclusions and discussion

On this basis we showed that a domain structure can be expected to arise. The topological character of this structure depends on the homotopy groups $\pi_k(M)$ of the manifold M of degenerate vacua. Domain walls can form if $\pi_0(M)$ is nontrivial, i.e. if M is non-connected. If it has n connected components we find an n-phase emulsion. The formation of cosmic strings requires that $\pi_1(M)$ be nontrivial, i.e. that M is not formed of simply connected components. Finally, 'monopoles' can form if $\pi_2(M)$ is nontrivial.

Kibble'76



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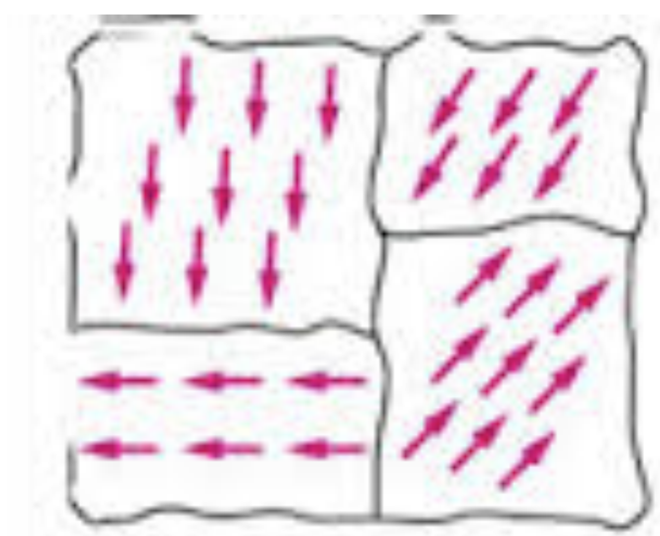

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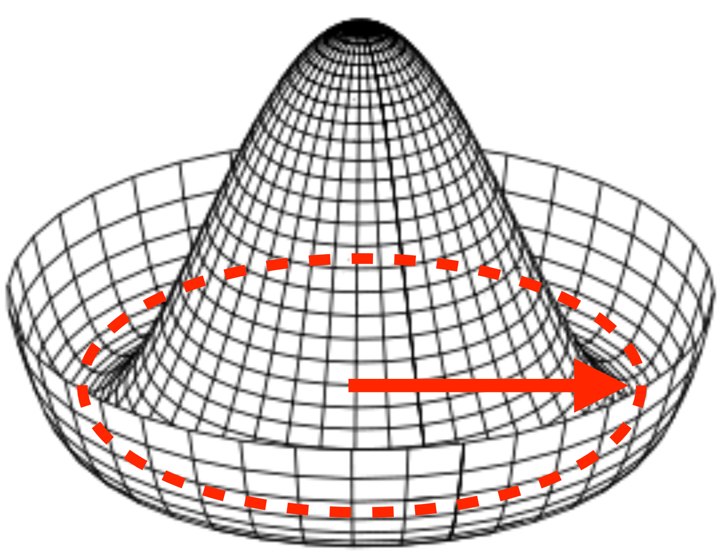
homotopy groups

Kibble'76



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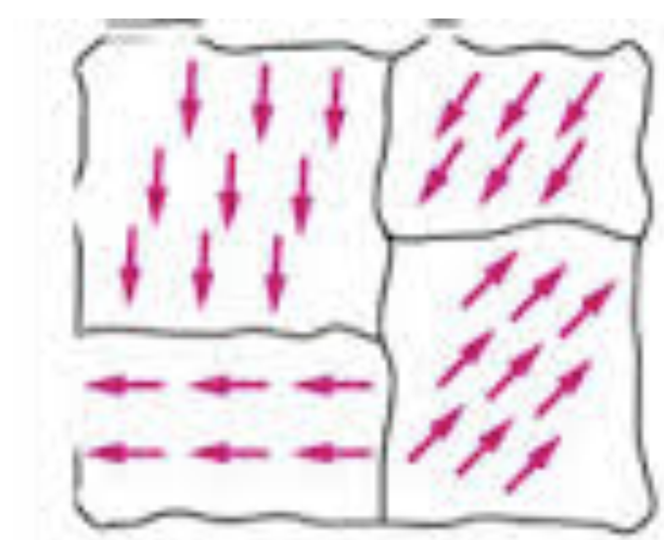
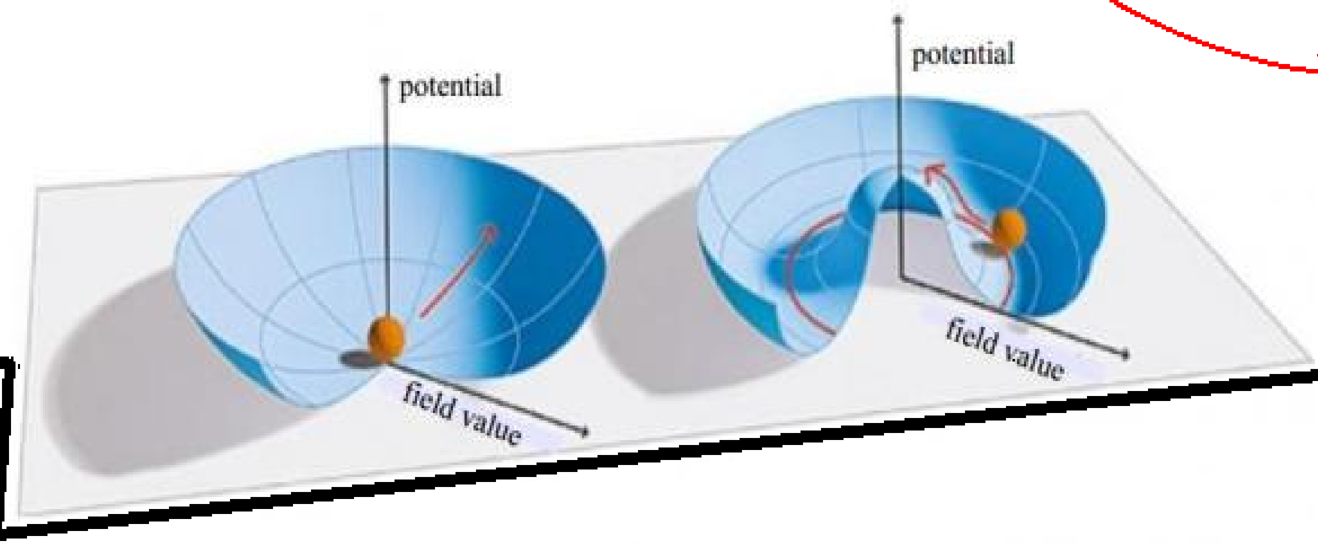


Introduction to Cosmic Defects

$$V = \frac{\lambda}{4}\left(|\Phi|^2 - v^2\right)^2 + V_{\rm int}(\Phi, \chi, T)$$

(1$^{\rm st}$ Order, 2$^{\rm nd}$ Order, Cross-Over)

$$V_{\rm int} \sim \begin{cases} g_{\rm T}^2|\Phi|^2T^2 & (\text{ THERMAL }) \\ g^2|\Phi|^2\chi^2 & (\text{ FIELD INT.}) \end{cases}$$



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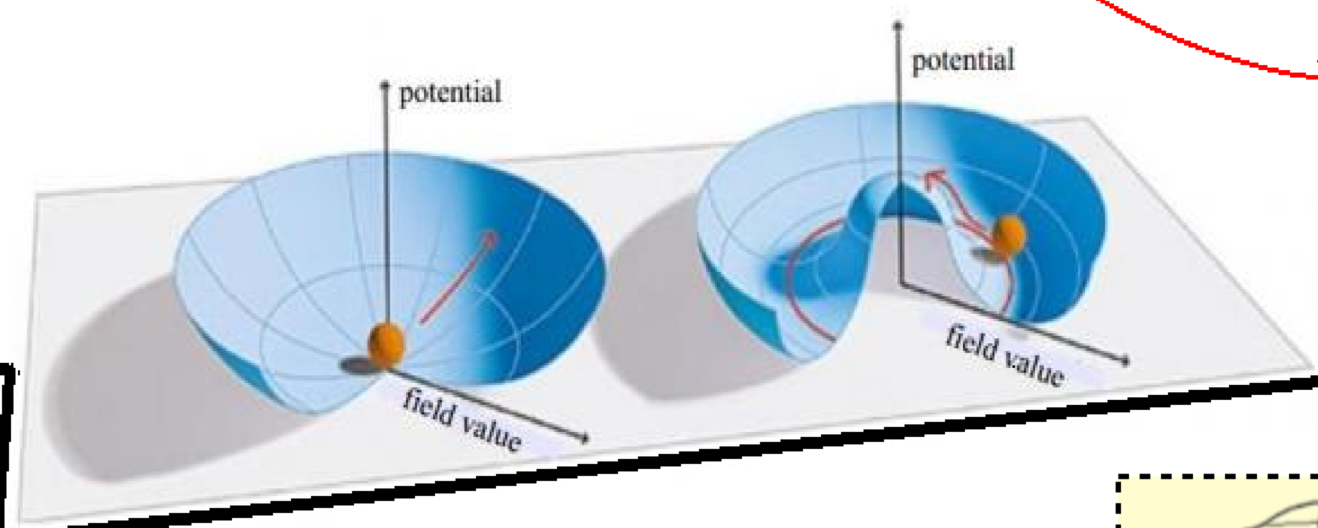

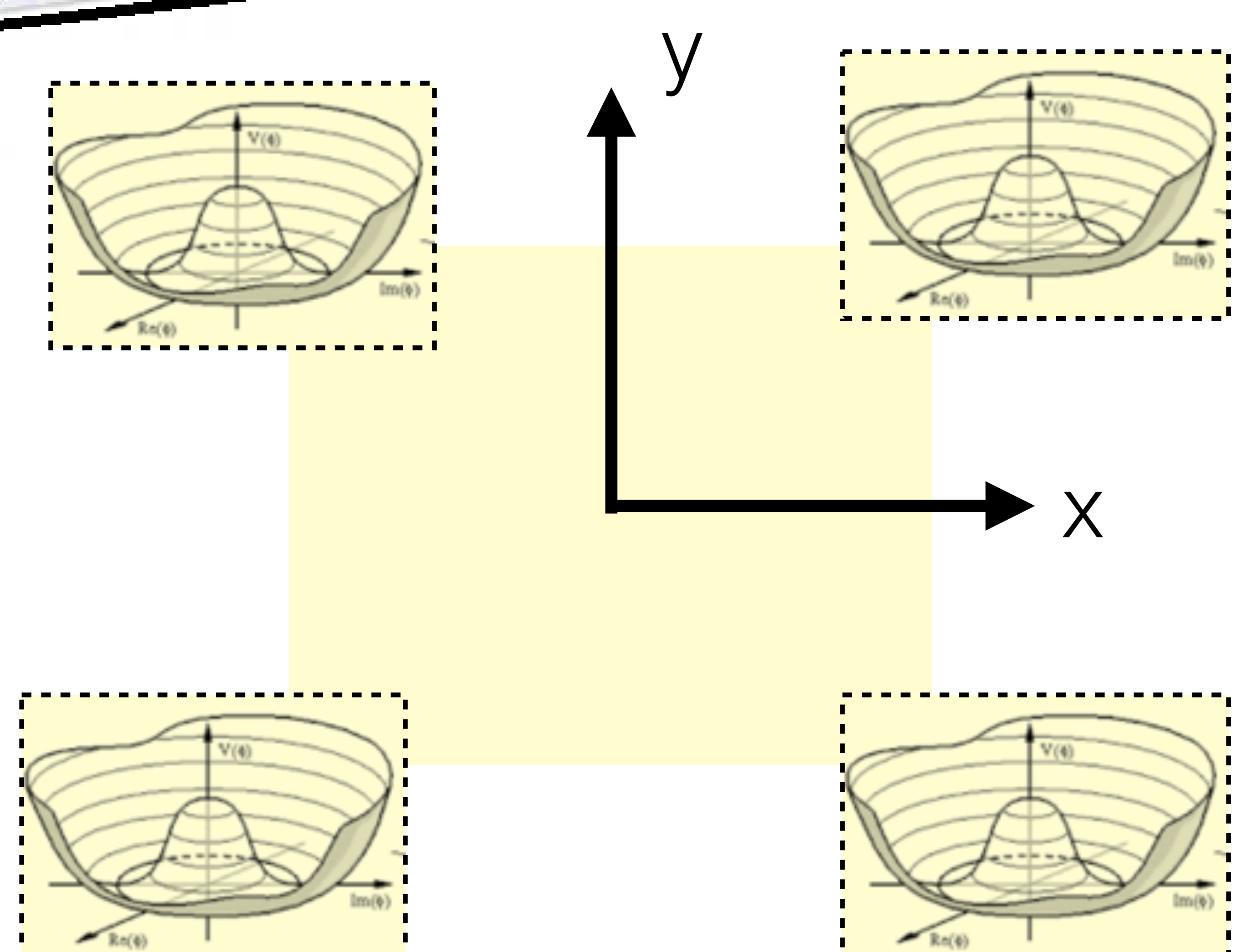

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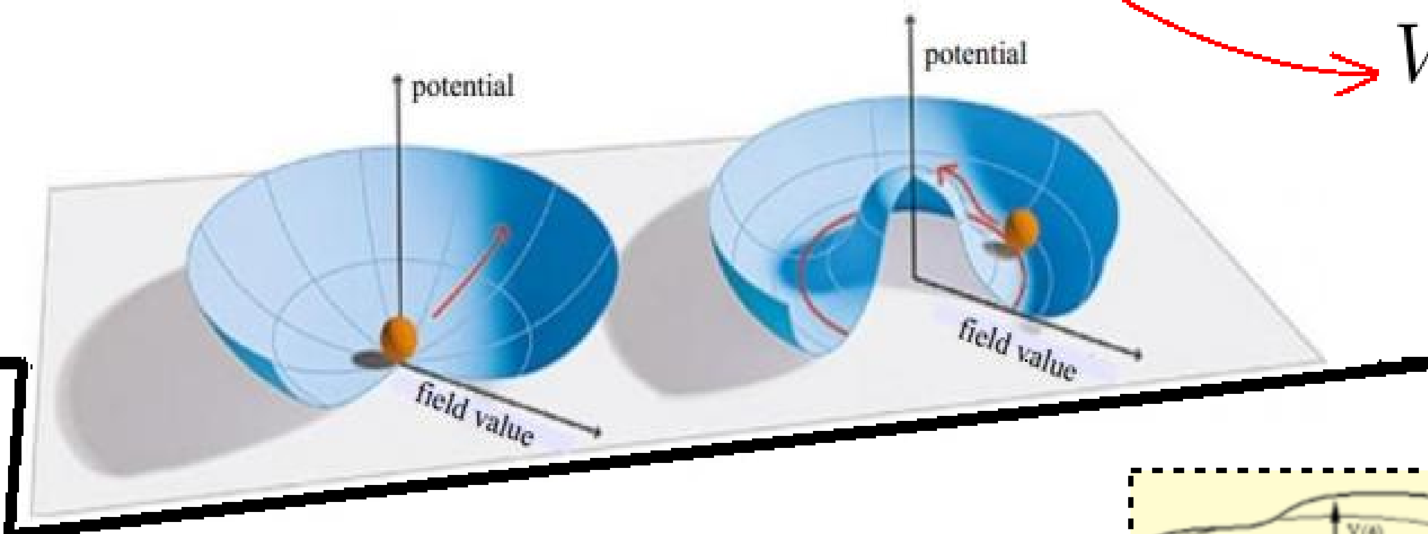

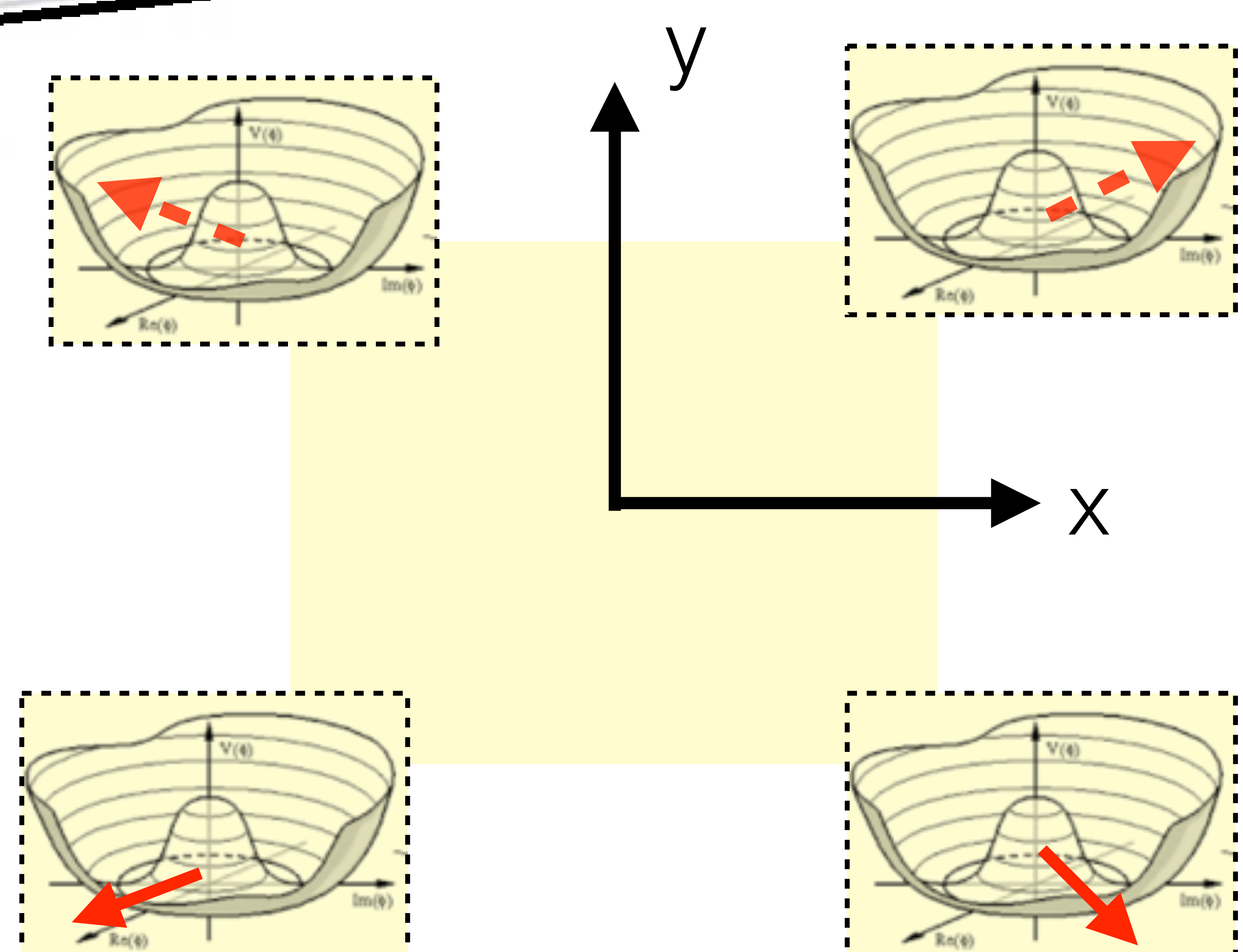

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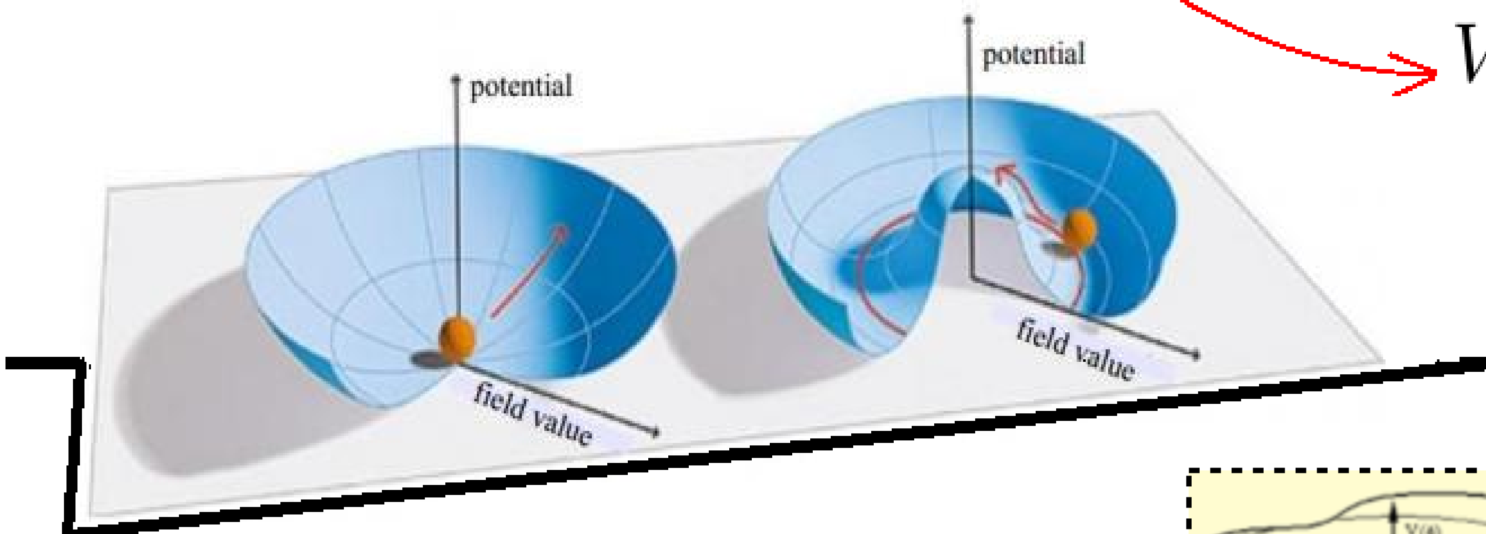

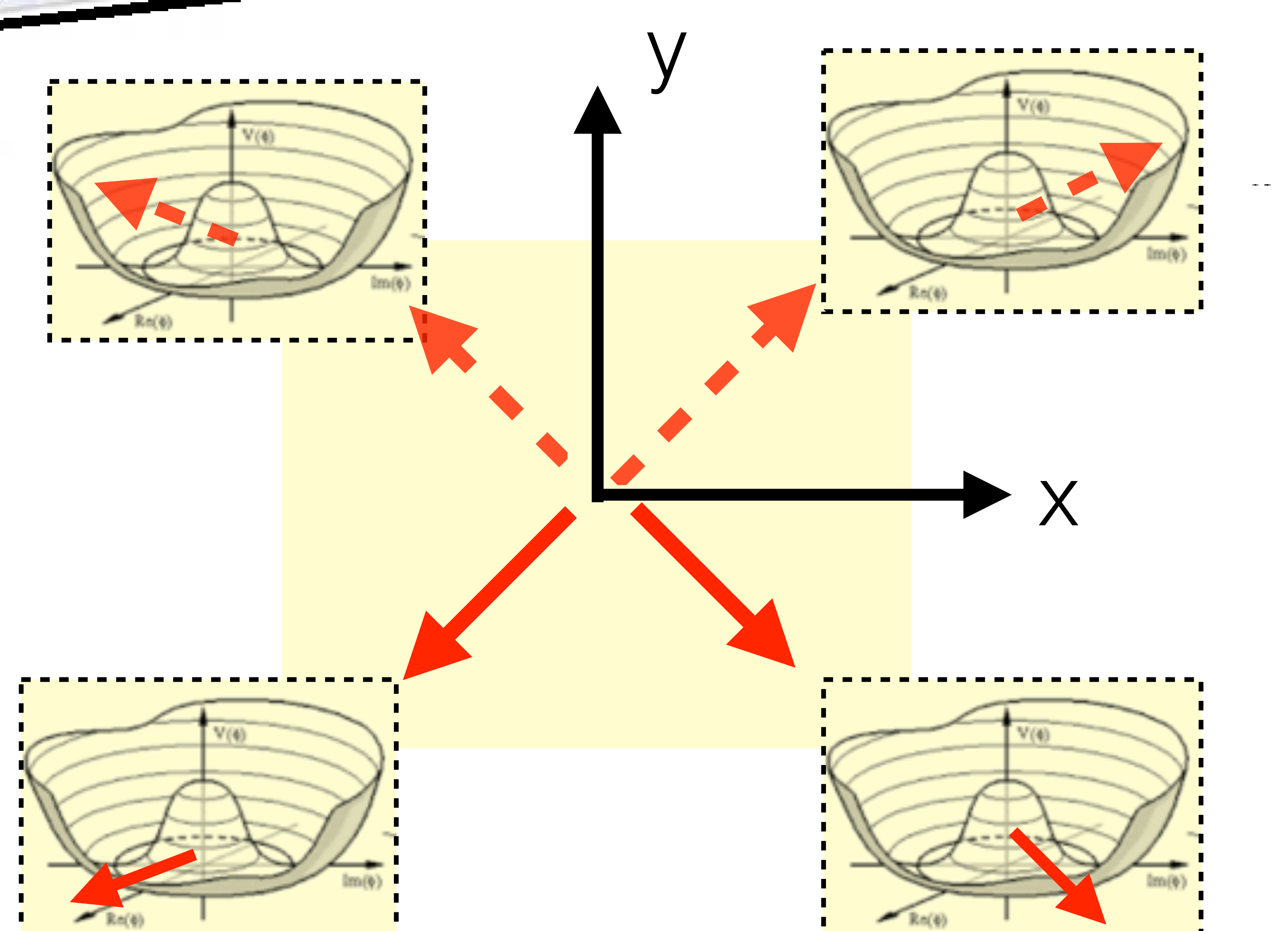

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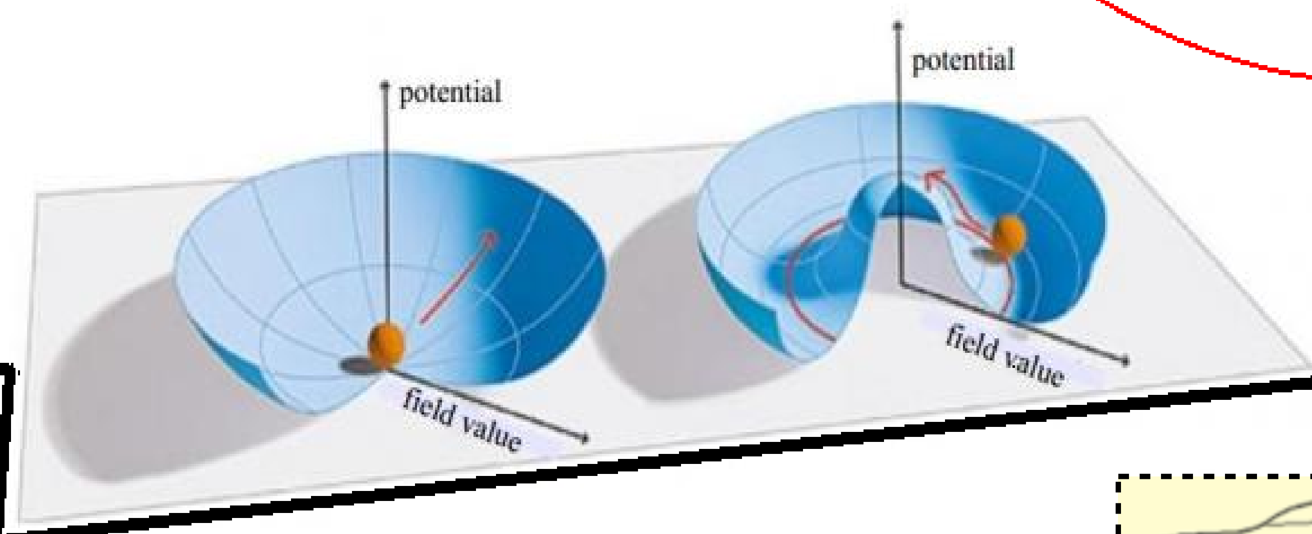

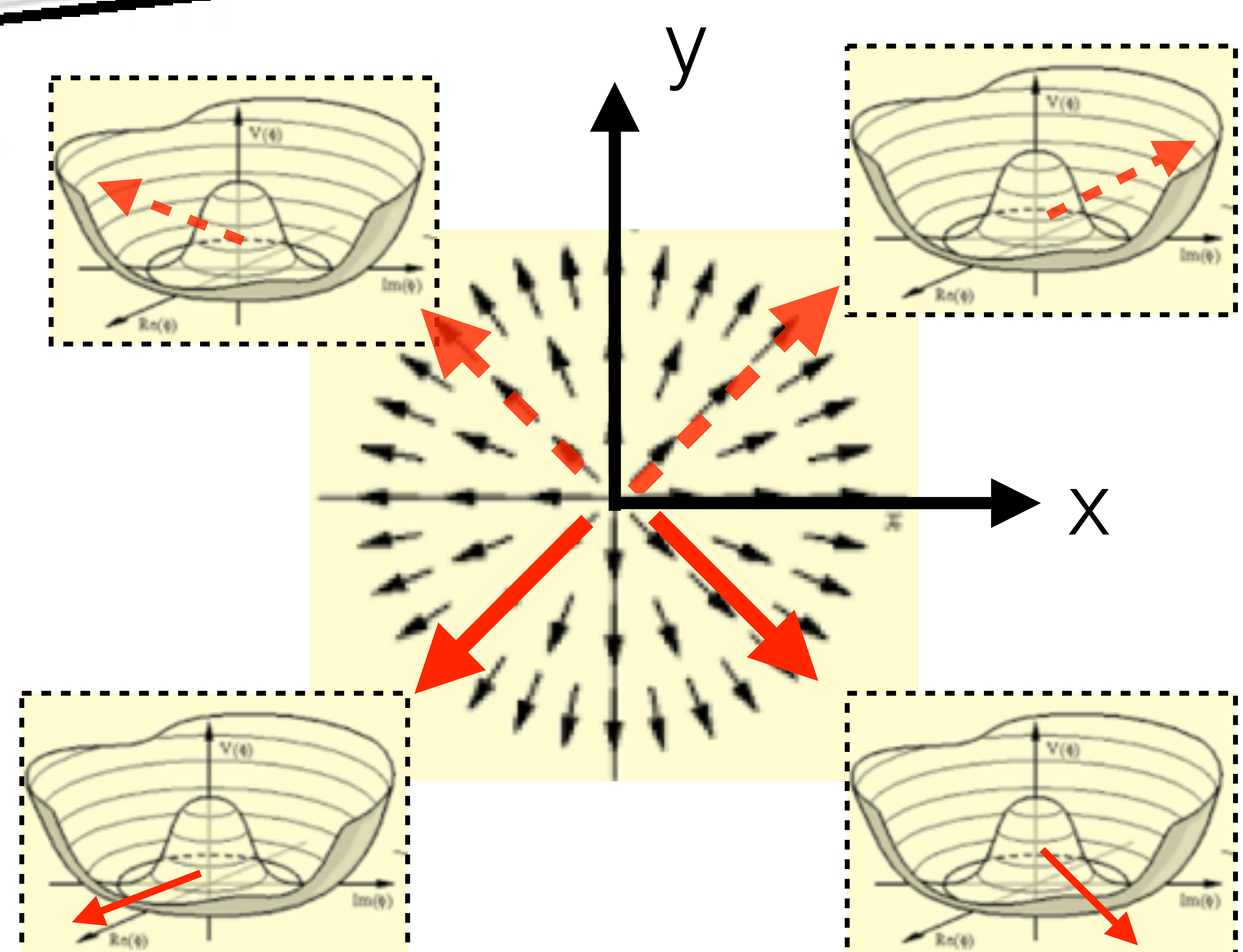

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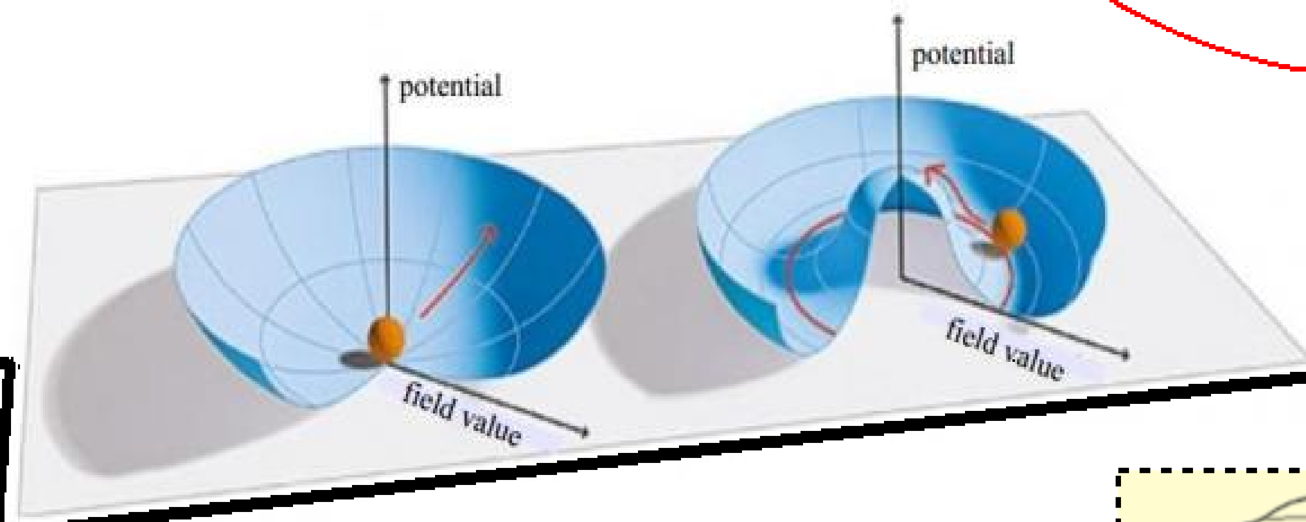

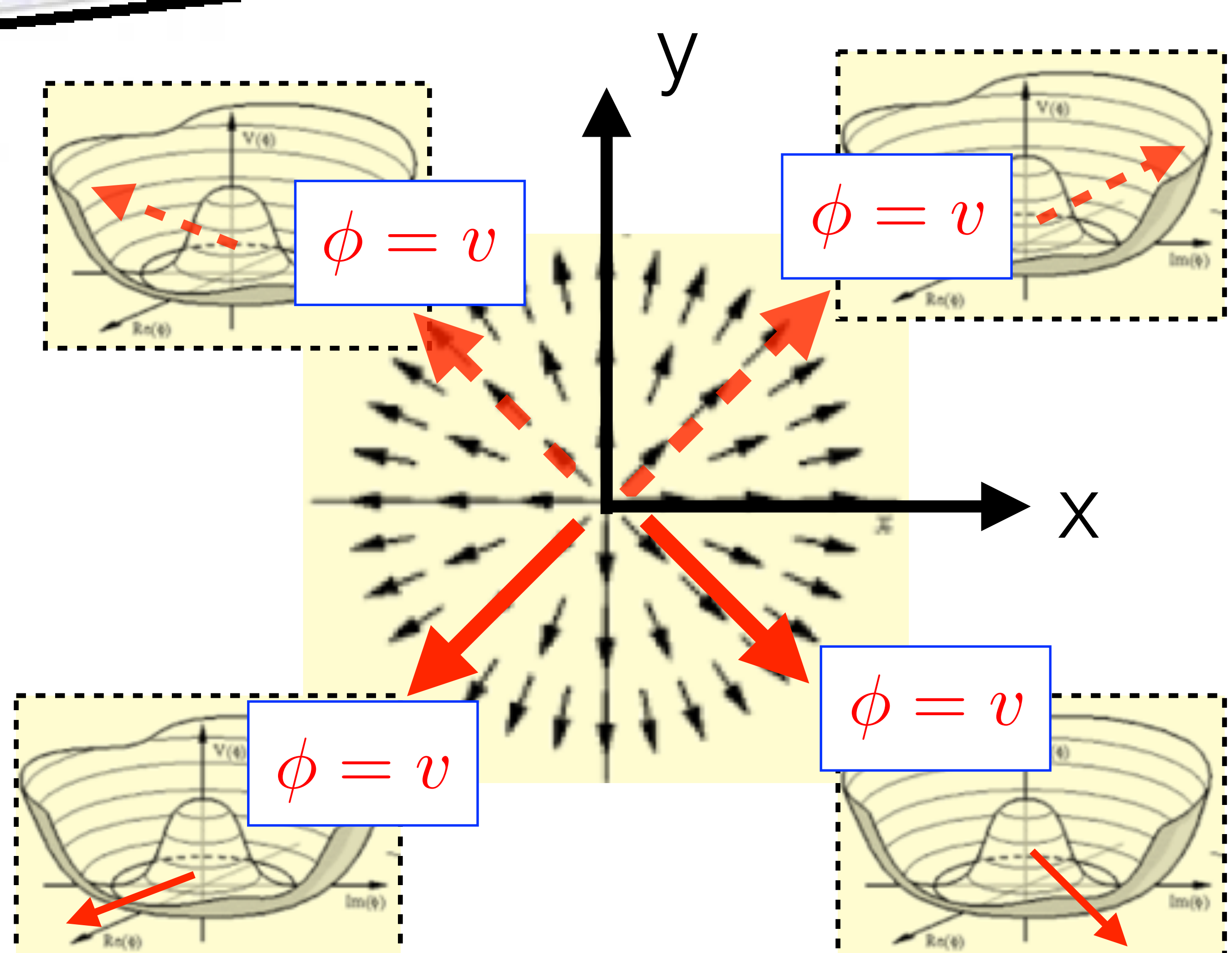

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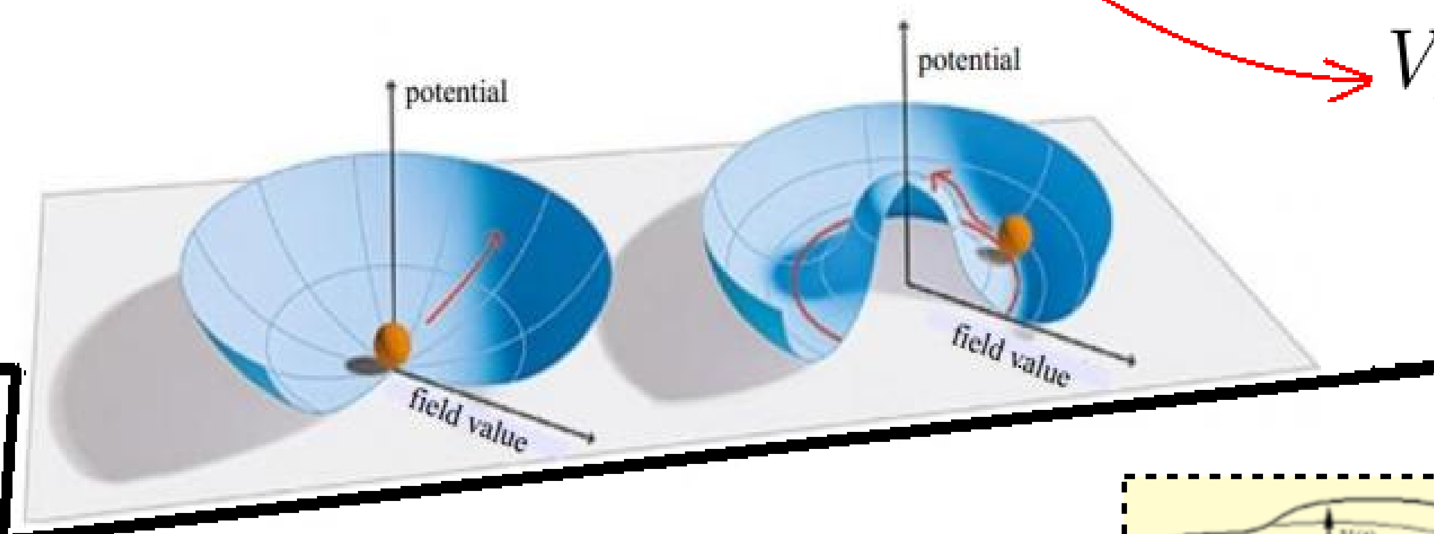
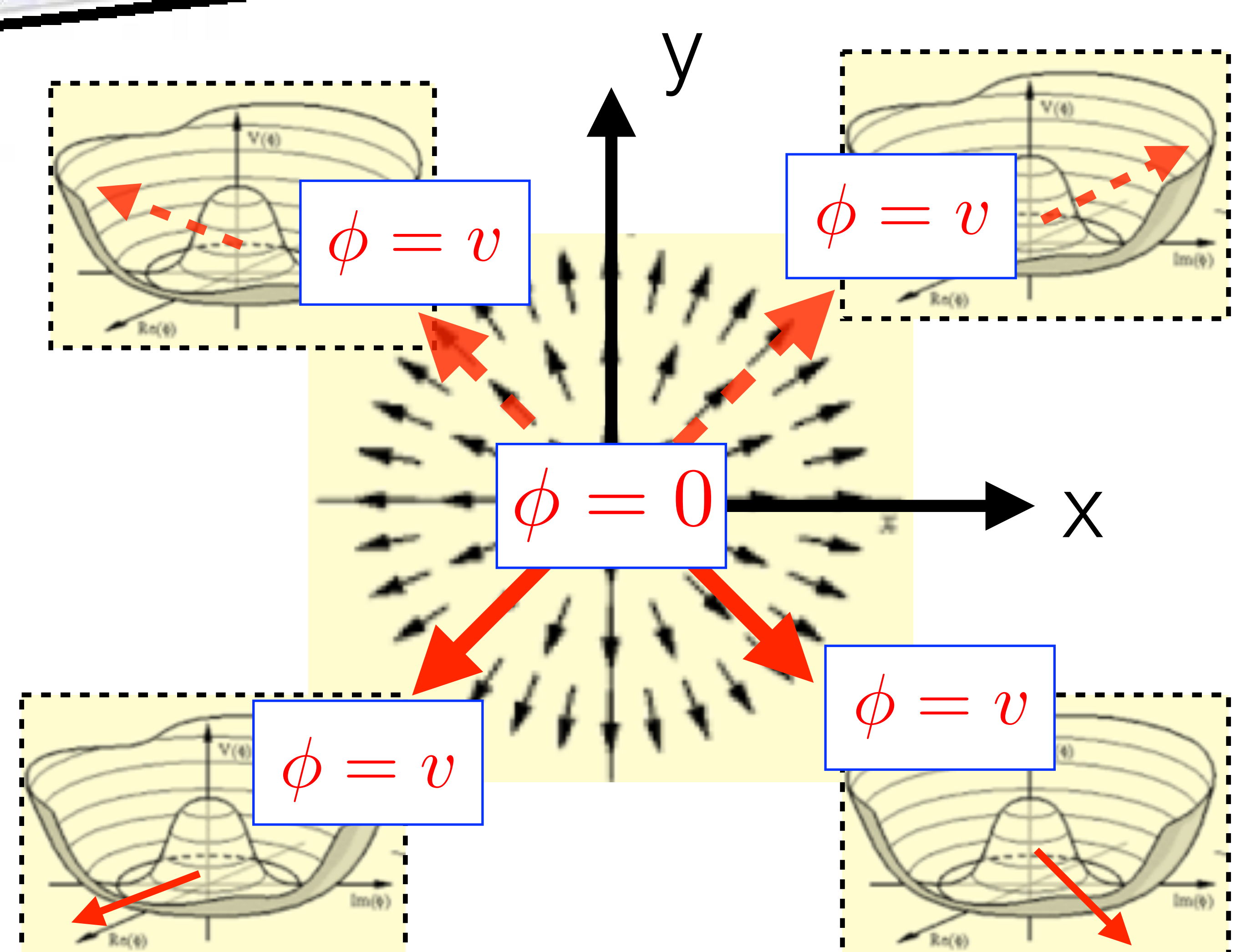
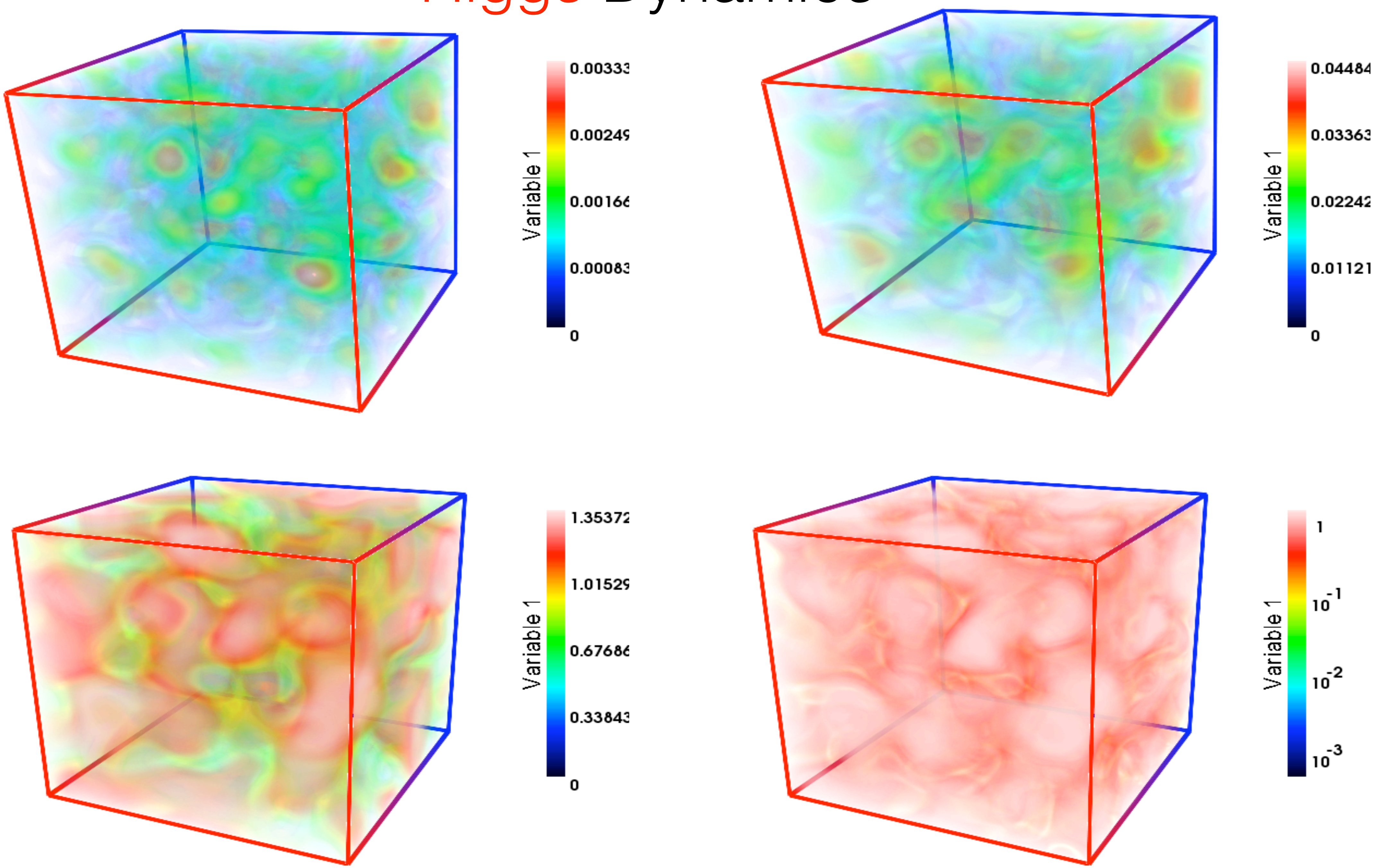




Introduction to Cosmic Defects

U(1) Breaking (after Hybrid Inflation)

Higgs Dynamics

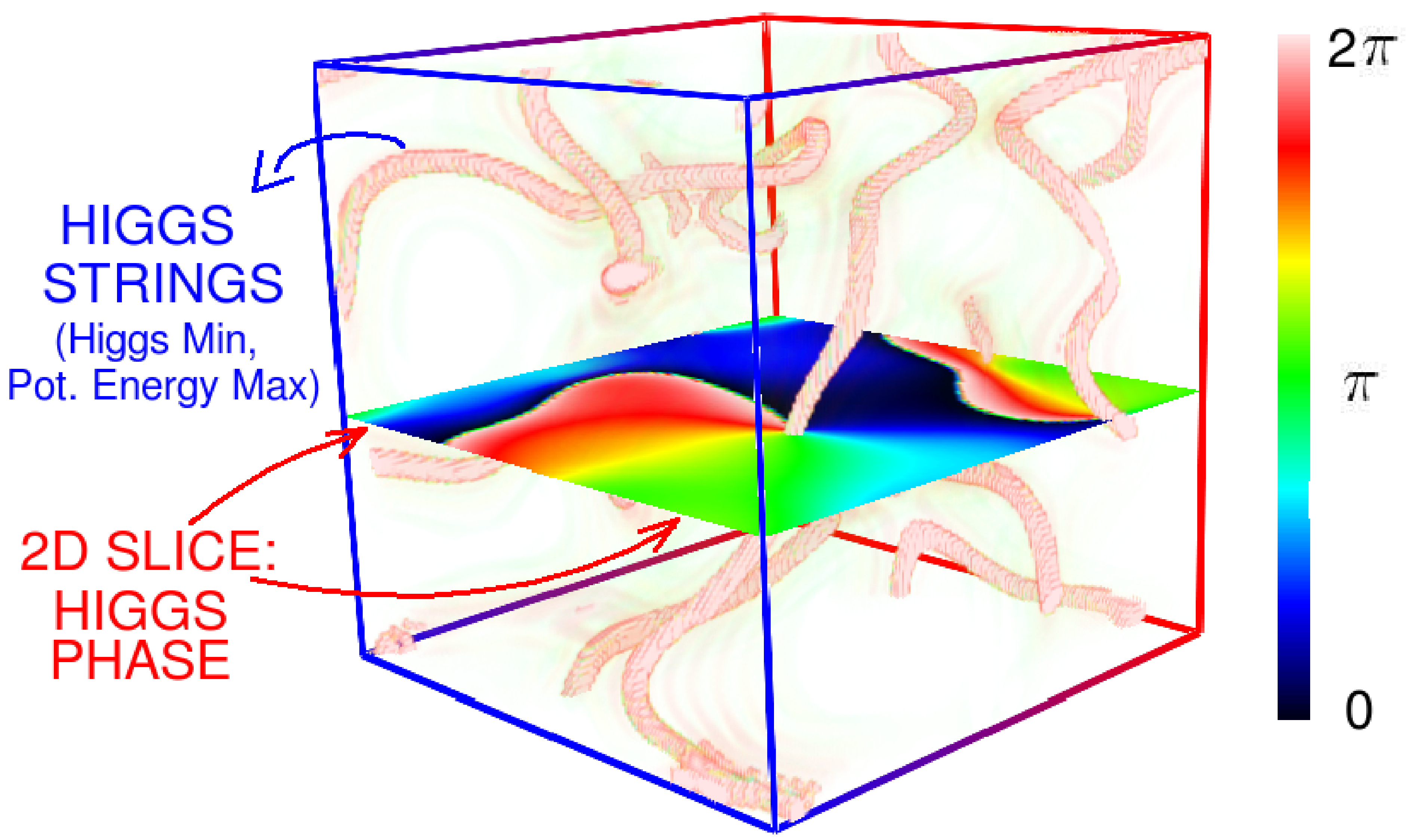


Dufaux et al PRD 2010

Introduction to Cosmic Defects

U(1) Breaking (after Hybrid Inflation)

SNAPSHOT OF THE HIGGS $(\mathrm{mt} = 17)$



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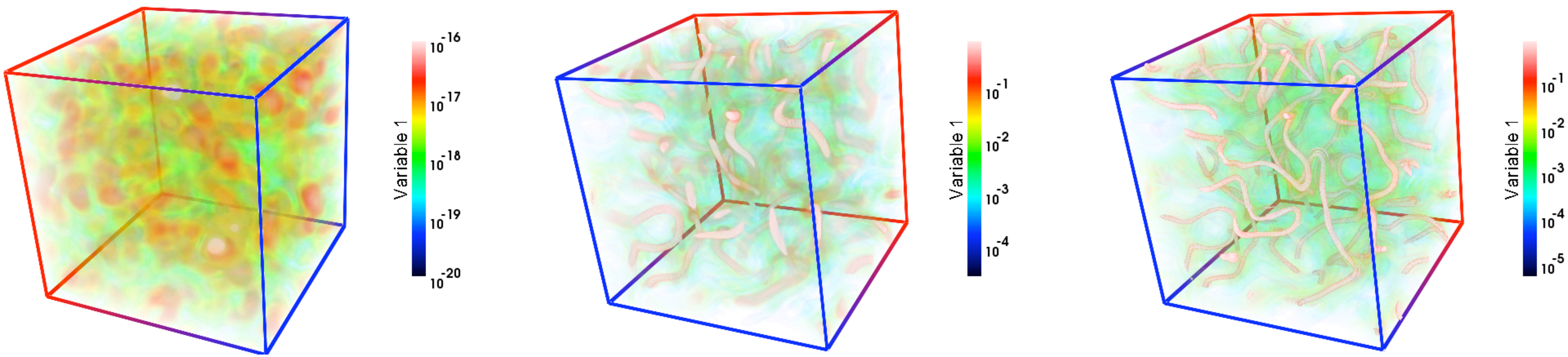
Introduction to Cosmic Defects

U(1) Breaking (after Hybrid Inflation)

MAGNETIC FIELD DYNAMICS



Magnetic Field energy density

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Introduction to Cosmic Defects

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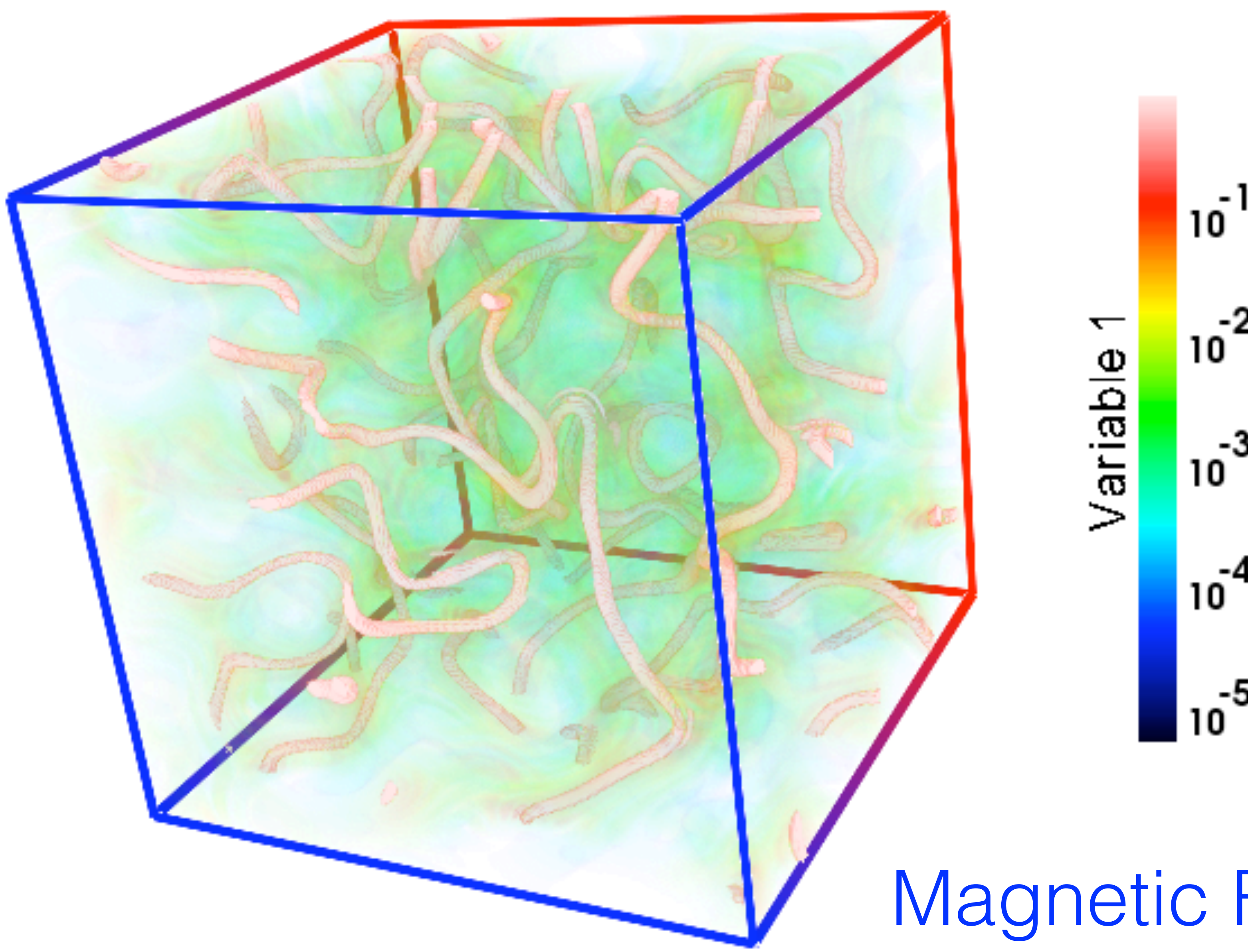

Magnetic Field
energy density

Dufaux et al PRD 2010

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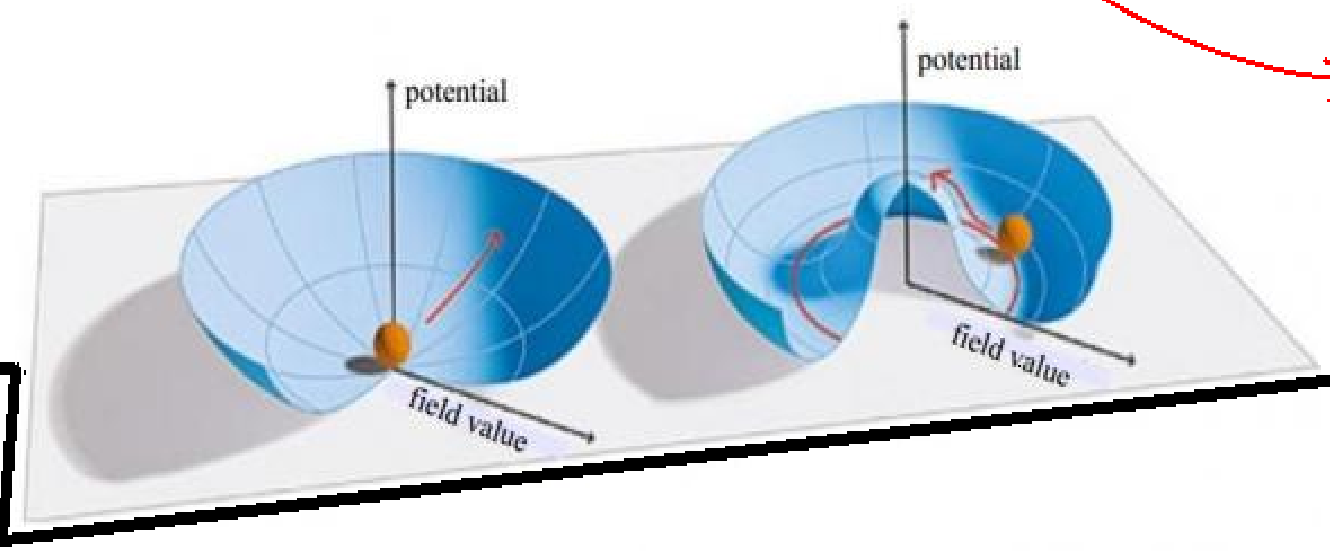

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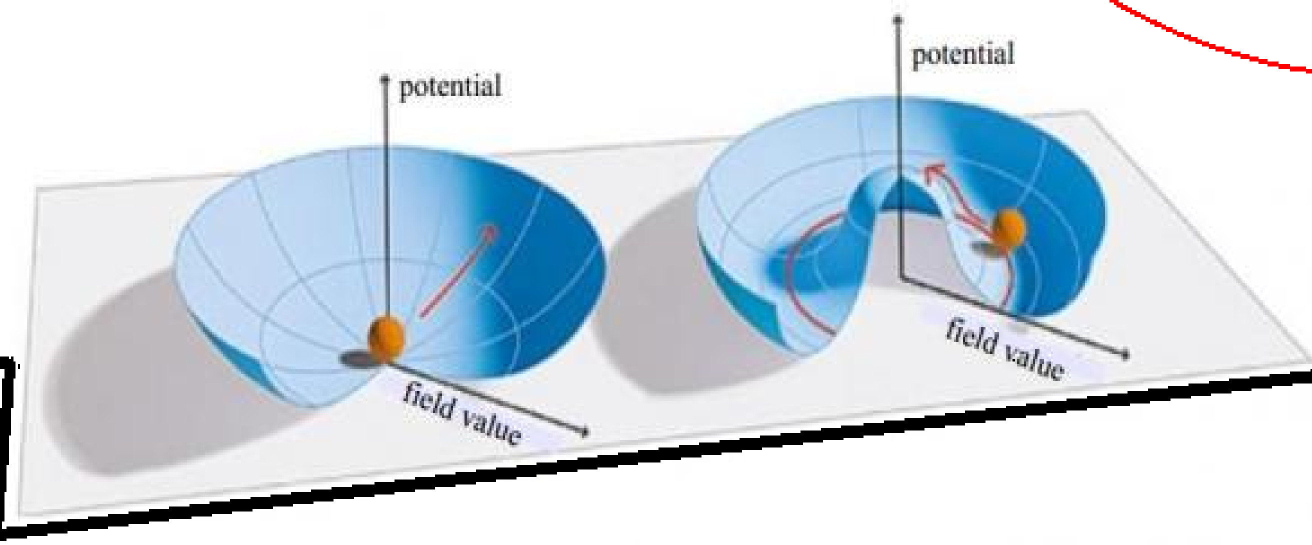
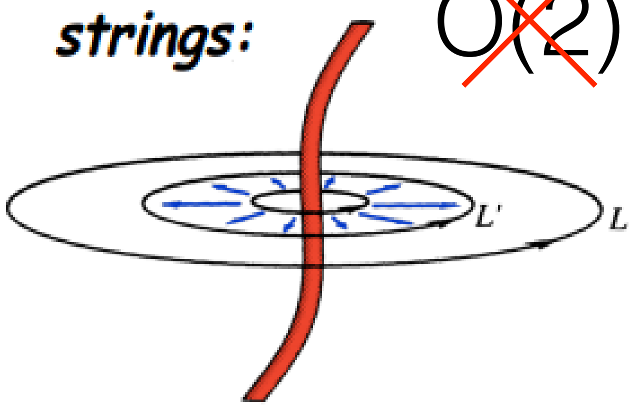
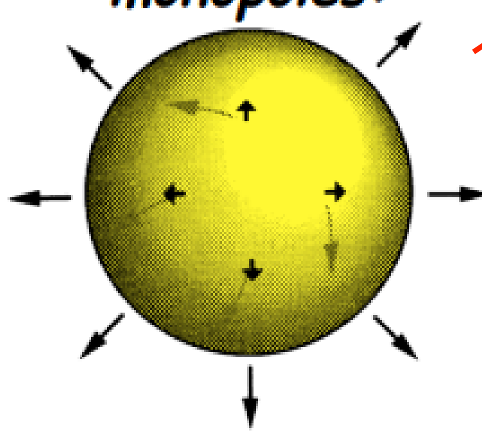


ZOOLOGY:

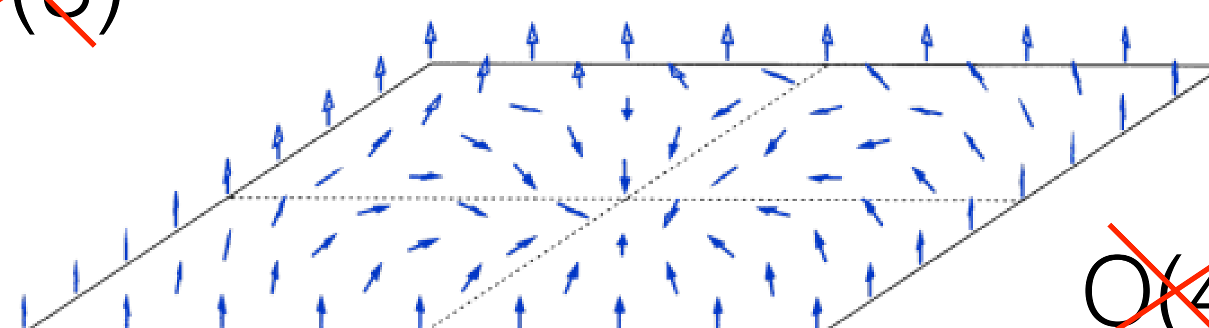
strings: ~~O(2)~~



monopoles: ~~O(3)~~



texture: (non-topological)



~~O(4)~~

MICRO-PHYSICS → COSMIC DEFECTS

(M = G/H)

Introduction to Cosmic Defects

$$\text{DEFECTS: Aftermath of PhT} \rightarrow \begin{cases} \begin{cases} \text{Domain Walls} \\ \text{Cosmic Strings} \\ \text{Cosmic Monopoles} \end{cases} \\ \text{Non} - \text{Topological} \end{cases}$$

Introduction to Cosmic Defects

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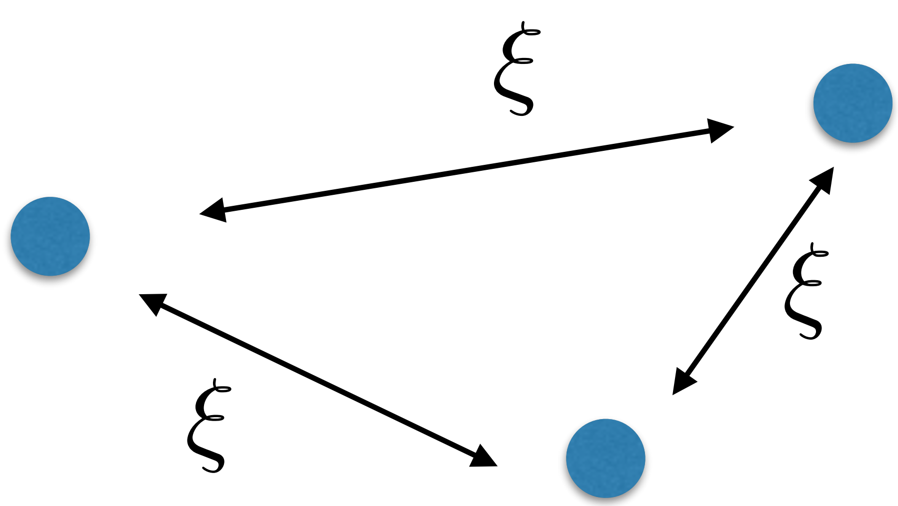
CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$



Introduction to Cosmic Defects

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CAUSALITY & MICROPHYSICS $\Rightarrow$ Corr. Length: $\xi(t) = \lambda(t)\, H^{-1}(t)$

(Kibble' 76)

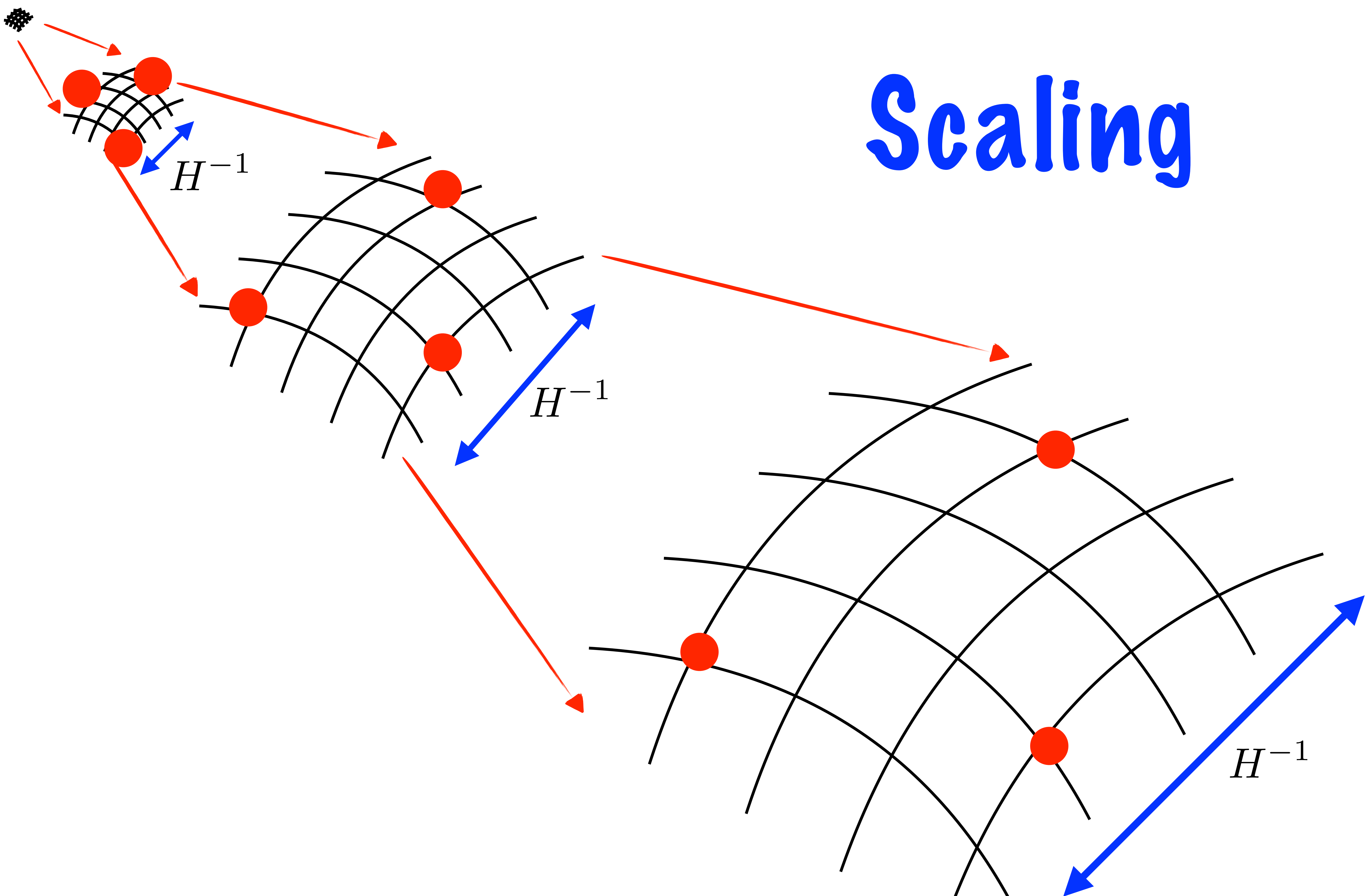
SCALING: $\lambda(t) = \text{const.} \quad \rightarrow \quad \lambda \sim 1 \quad \Longrightarrow \quad k/\mathcal{H} = kt$

comoving momentum

conformal time

Introduction to Cosmic Defects

Scaling



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  - ...

  Enhanced GWs

Reheating

- scalar Preheating
- gauge Preheating
- fermion Preheating

  Large GW (high freq)

Thermal Universe

Phase Transition(s): Large GW
- Hybrid Preheating, High Freq.
- Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects ✓
- Irreducible GW
- String (loops): Large GW

EARLY UNIVERSE

Inflation

- 'cures' hBB
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Thermal Universe

Cosmic Defects ...: Large GW

Let's first review GW **Stochastic Backgrounds**

Stochastic GW backgrounds

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \longrightarrow \text{Volume Average}$$

Stochastic GW backgrounds

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)}\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle$$

→ ensemble average

→ Inflation: QM

→ Other: Random

Stochastic GW backgrounds

$$\begin{aligned}\rho_{\rm GW}(t) &= \frac{1}{32\pi G a^2(t)}\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle \\ &= \frac{1}{32\pi G a^2(t)}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}\; e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\left\langle \dot{h}_{ij}\left(\mathbf{k},t\right)\dot{h}^*_{ij}\left(\mathbf{k}',t\right)\right\rangle\end{aligned}$$

Stochastic GW backgrounds

$$\rho_{\mathrm{GW}}(t) = \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle$$

$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3} \, e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')} \left\langle \dot{h}_{ij}(\mathbf{k},t)\, \dot{h}^*_{ij}(\mathbf{k}',t)\right\rangle$$

$$\boxed{\left\langle \dot{h}_{ij}(\mathbf{k},t)\, \dot{h}^*_{ij}(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

Stochastic GW backgrounds

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)} \int \frac{dk}{k}\; k^3\, \mathcal{P}_{\dot h}(k,t) \;\; = \int \frac{d\rho_{\rm GW}}{d\log k}\, d\log k$$

$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

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Define: $\bar{h}_{ij}\,(\mathbf{x},t) \;=\; a(t)h_{ij}\,(\mathbf{x},t)$

Stochastic GW backgrounds

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EOM: $\ddot{\bar{h}}_{ij}\left(\mathbf{x},t\right) - \left(\nabla^2 + \frac{\ddot{a}(t)}{a(t)}\right)\bar{h}_{ij}\left(\mathbf{x},t\right) = 16\pi G\, a(t)\Pi^{\rm TT}_{ij}\left(\mathbf{x},t\right)$

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Green Function

$$\boxed{\dot h_{ij}(\mathbf{k},t) = \frac{16\pi G}{k a(t)}\int_{t_I}^{t} dt' a(t')\, \mathcal{G}(k(t-t'))\, \Pi^{\rm TT}_{ij}(\mathbf{k},t')}$$

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$$\boxed{\left\langle \Pi^{\rm TT}_{ij}(\mathbf{k},t)\,\Pi^{\rm TT}_{ij}(\mathbf{k}',t')\right\rangle \equiv (2\pi)^3\, \Pi^2(k,t,t')\,\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

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$$\begin{aligned}\langle \mathcal{G}(\mathbf{k},t,t')\mathcal{G}(\mathbf{k},t,t'')\rangle_{T_k} &\equiv \frac{1}{T_k}\int_t^{t+T_k} d\tilde t\; \mathcal{G}(\mathbf{k},\tilde t,t')\mathcal{G}(\mathbf{k},\tilde t,t'') \\ &\underset{kt\,\gg\,1}{=} \frac{1}{2}(k^2+\mathcal{H}^2(t))\cos[k(t'-t'')]\end{aligned}$$

Stochastic GW backgrounds

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$$\boxed{@\,k \gg \mathcal{H}}$$

EARLY UNIVERSE

Inflation → 'cures' hBB

Inflation → (initial cond.) = Primordial perturbations:
- Scalar: CMB ΔT anisotropies (eventually Galaxies)
- Tensor: Irreducible GWs (CMB B-mode Pol.)

Inflation → Scenarios:
- Extra species/symmetries
- Enhanced Scalar Pert.
- ...

} Enhanced GWs

⇓

Reheating:
- scalar Preheating
- gauge Preheating
- fermion Preheating

} Large GW (high freq)

⇓

Thermal Universe

Reheating / Thermal Universe → Phase Transition(s): Large GW
- Hybrid Preheating, High Freq.
- Thermal 1st-O EW PhT @ LISA Freq.

⇓

Cosmic Defects ✓

Let's first review GW **Stochastic Backgrounds** ✓

...(s): Large GW

EARLY UNIVERSE

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Cosmic Defects

- $\boxed{\text{Irreducible GW}}$
- String (loops): Large GW

Irreducible GW emission from a Defect Network

1) Theorem: GW from Evolution of Defect Networks

2) Analytical Calculations: Large-N

3) O(N) Lattice Simulations

4) Full Spectrum

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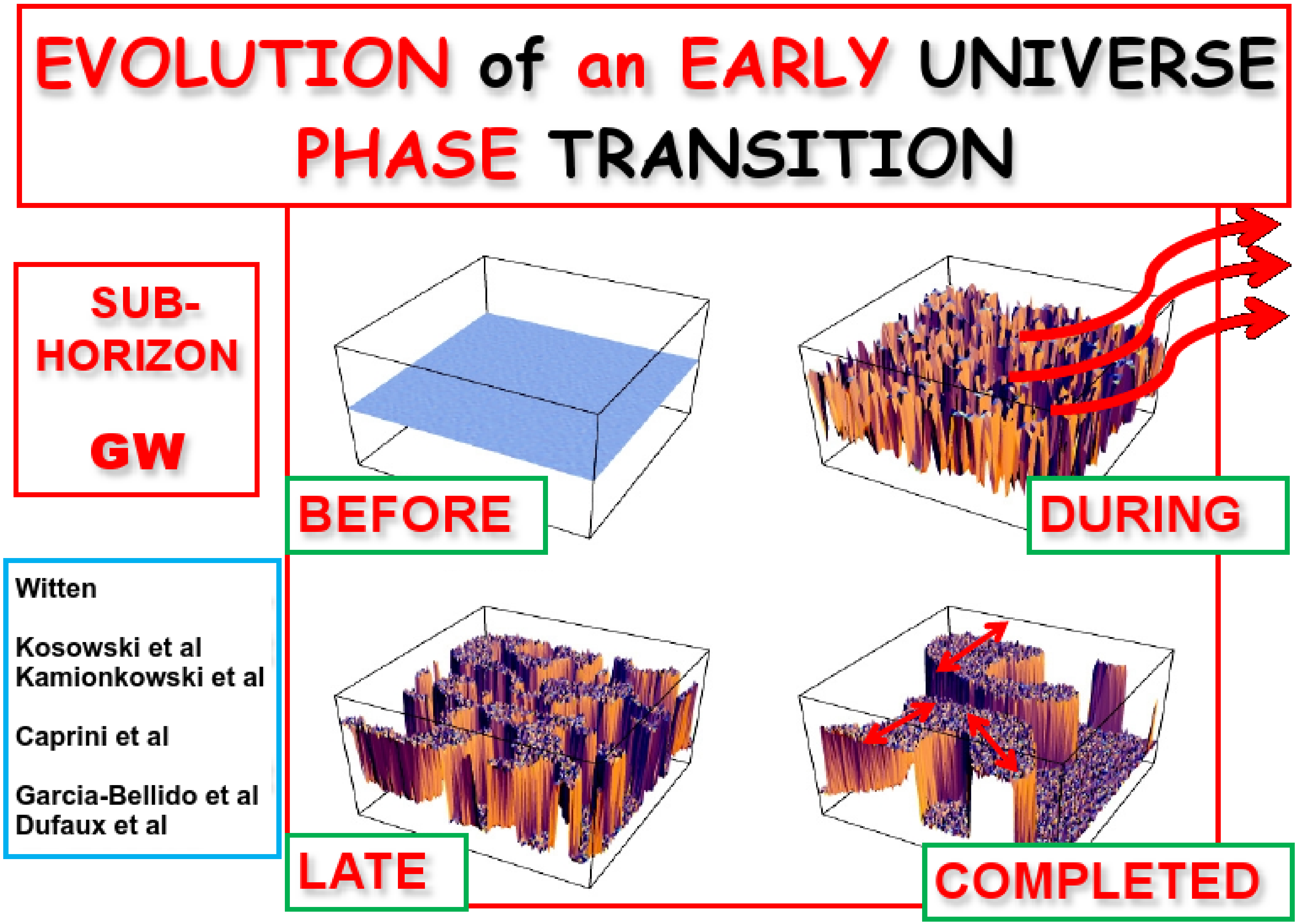
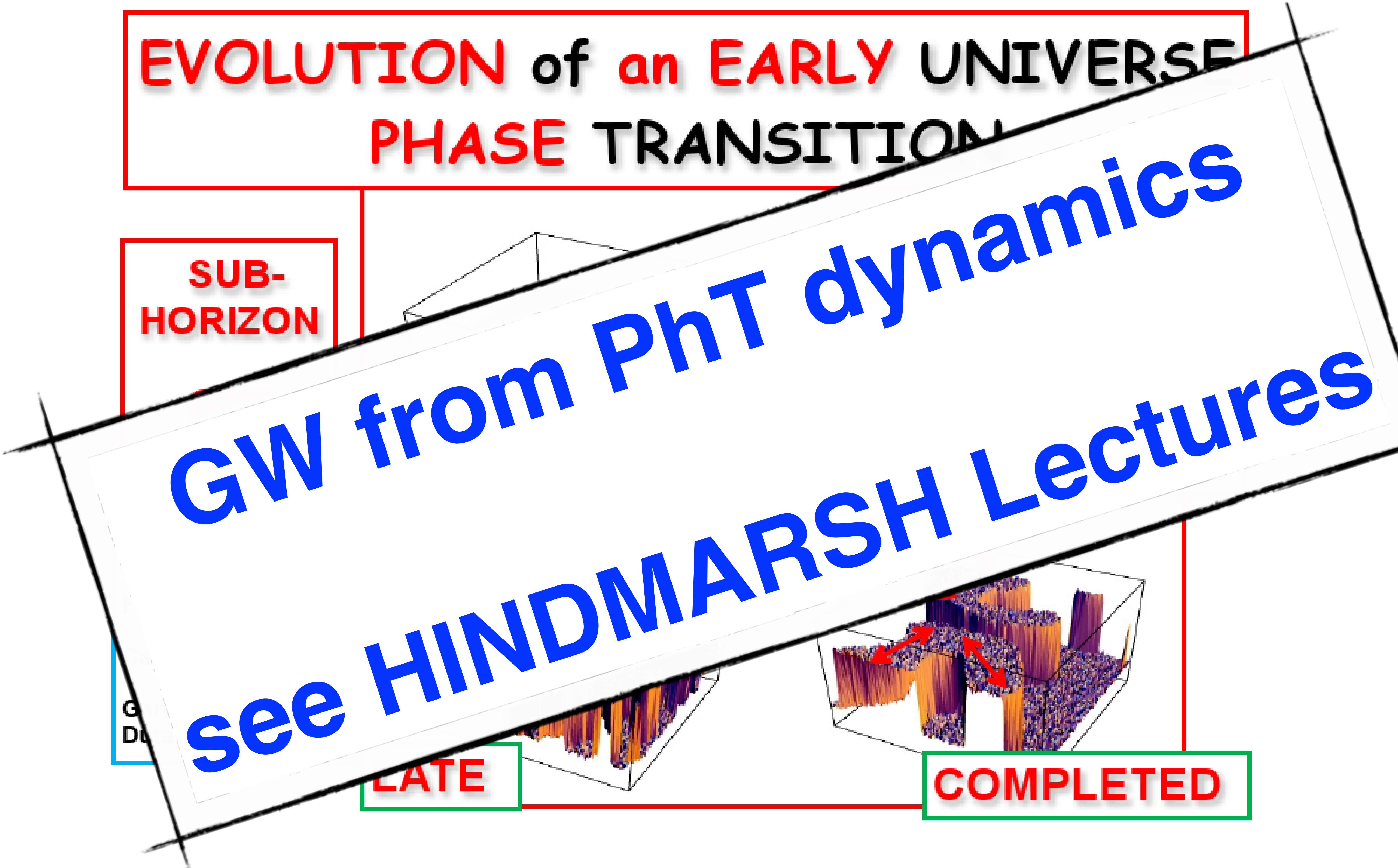


1) Theorem: GW from Evolution of Defect Networks



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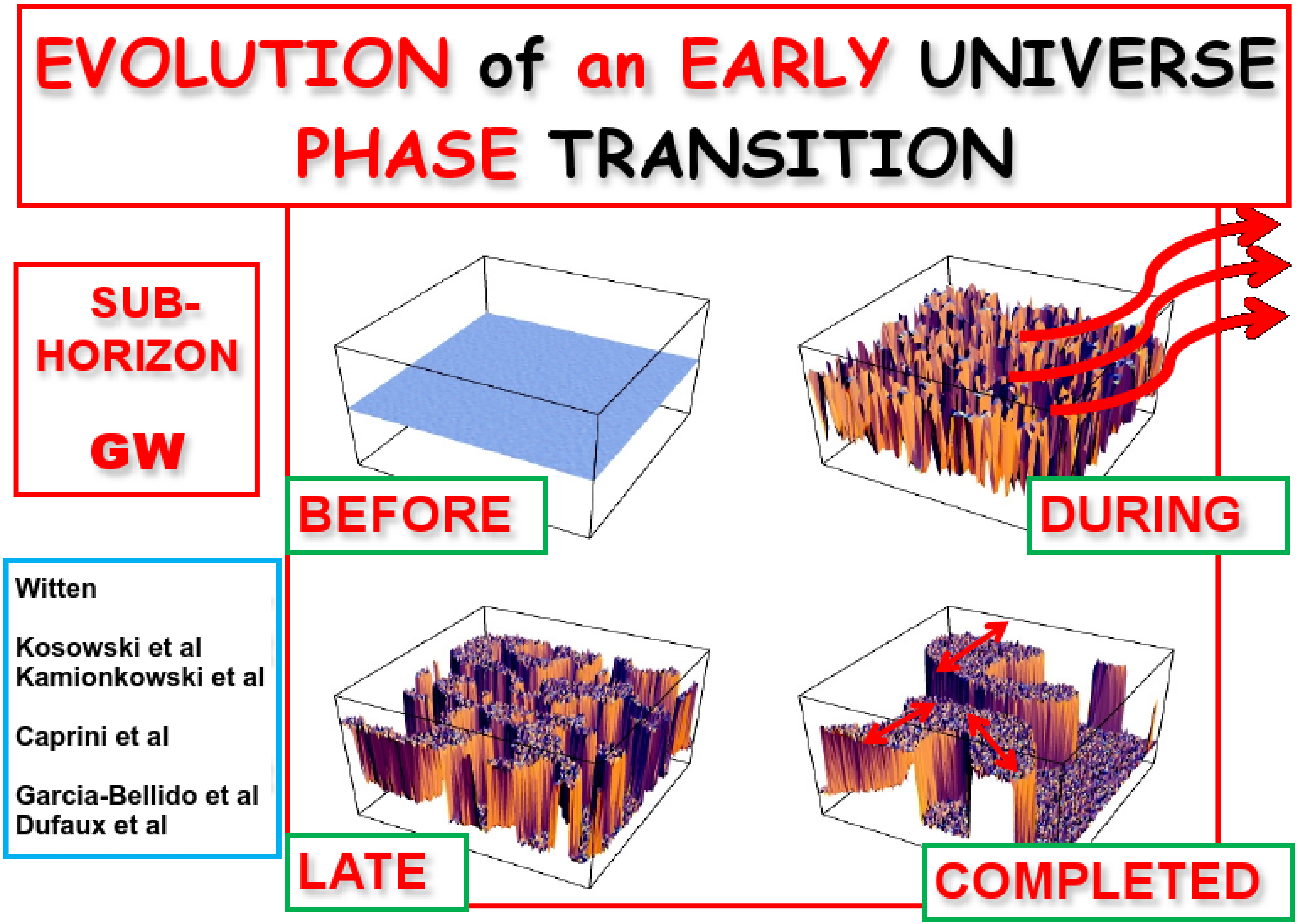

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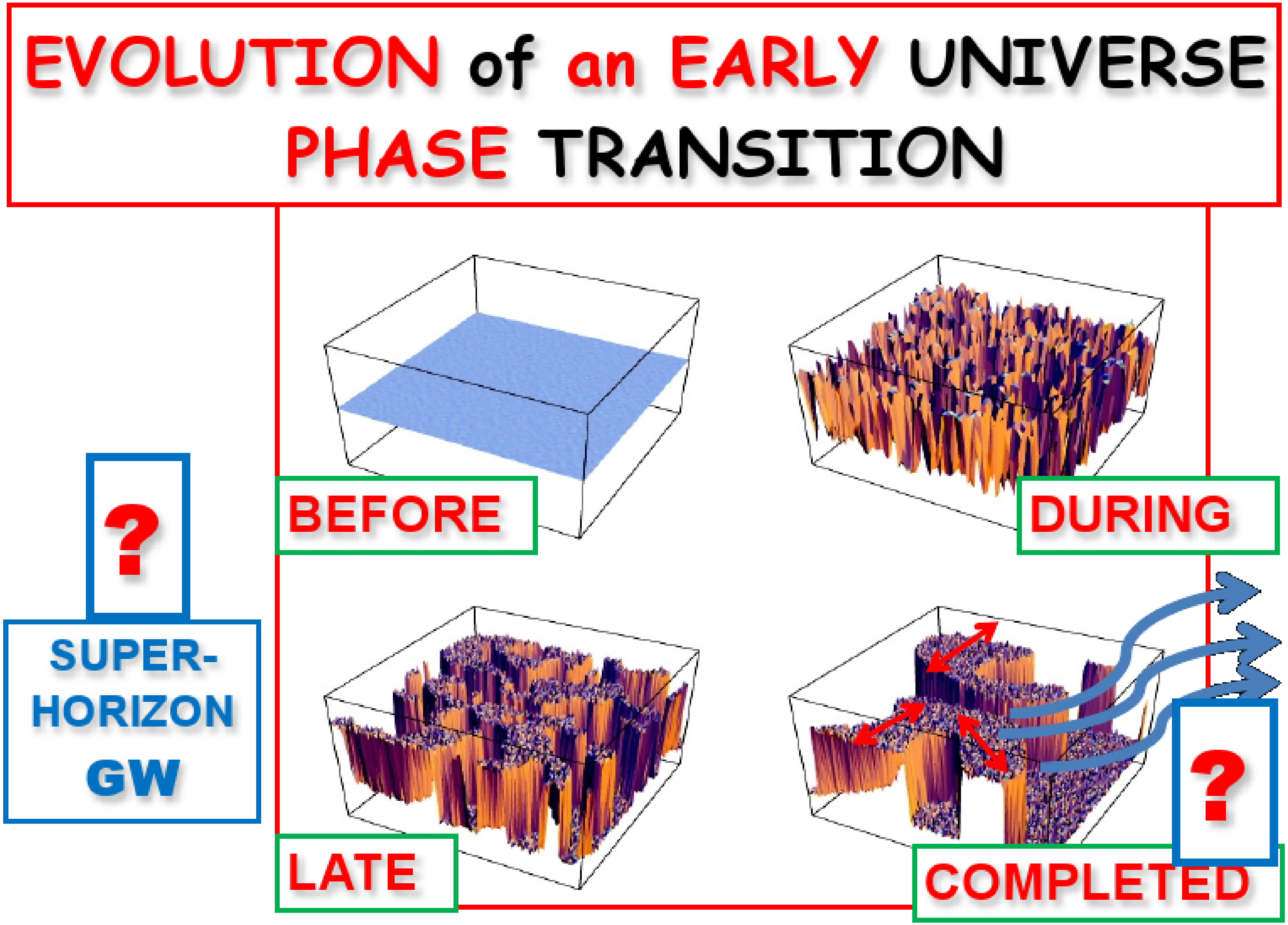

1) Theorem: GW from Evolution of Defect Networks

DEFECTS: GW Source $\rightarrow \{T_{ij}\}^{\rm TT} \propto \{\partial_i\phi\partial_j\phi, E_iE_j, B_iB_j\}^{\rm TT}$

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UTC: $\langle T_{ij}^{\rm TT}(\mathbf{k},t)T_{ij}^{\rm TT}(\mathbf{k}',t')\rangle = (2\pi)^3\,\Pi^2(k,t_1,t_2)\;\delta^3(\mathbf{k}-\mathbf{k}')$

(Unequal Time Correlator)

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(Unequal Time Correlator)

Stochastic Background of Gravitational Waves

GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2a^4(t)}\int dt_1dt_2\;a(t_1)a(t_2)\;\cos(k(t_1-t_2))\;\Pi^2(k,t_1,t_2)$$

Comoving Conformal

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SCALING

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Rad. Dom

SCALING

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GW spectrum: $(x_i \equiv kt_i)$ Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{\rm V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)}\left[\int dx_1dx_2\ \sqrt{x_1x_2}\ \cos(x_1-x_2)\ U(x_1,x_2)\right]$$

Rad. Dom **SCALING**

1) Theorem: GW from Evolution of Defect Networks

GW spectrum: $(x_i \equiv k t_i)$ R.D. and SCALING

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$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{\rm V}{M_p}\right)^4 \frac{M_p^2}{a^4(t)}\, F_U} \qquad F_U \sim \text{Const. (Dimensionless)}$$

DGF, Hindmarsh, Urrestilla, PRL 2013

1) Theorem: GW from Evolution of Defect Networks

GW today:

VEV

Scaling @ RD

$$\boxed{\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{32}{3}\left(\frac{\rm V}{M_p}\right)^4 \Omega_{\rm rad}^{(o)}\, F_U\,, \quad \text{(SCALE INV.!)}}$$

Defect type

$$F_U \equiv \int_0^x dx_1 dx_2 \sqrt{x_1 x_2}\, cos(x_1 - x_2) U(x_1, x_2)$$

DGF, Hindmarsh, Urrestilla, PRL 2013

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$\forall$ PhT (1st, 2nd, ...), $\forall$ Defects (top. or non-top.)

DGF, Hindmarsh, Urrestilla, PRL 2013

Irreducible GW emission from a Defect Network

✓ 1) Theorem: GW from Evolution of Defect Networks

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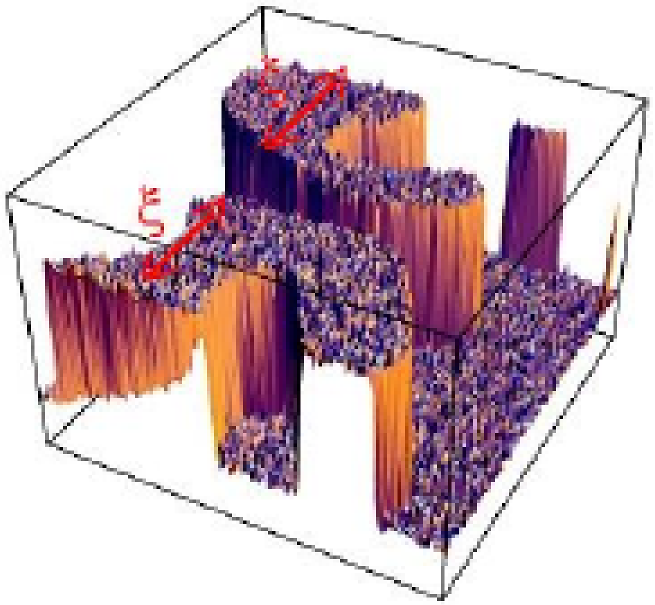
GLOBAL PHASE TRANSITION

$$O(N) \rightarrow O(N\text{-}1): \left[\begin{array}{ll} \sum_a \phi_a^2 = v^2 & \text{(CONSTRAINT)} \\ \Box \phi_a + V'(\phi_a) = 0 & \text{(EOM)} \end{array}\right] \Rightarrow \boxed{\Box \phi_a + (\partial_\mu \phi_b \cdot \partial^\mu \phi_b)\, \phi_a = 0}$$



UNIVERSE EXPANDING
(CAUSAL HORIZON)

FIELD SELF-ORDERS
(ξ↑↑ , ξ < 1/H)

2) Analytical Calculations: Large-N

GLOBAL PHASE TRANSITION

$$\mathbf{O(N)} \rightarrow \mathbf{O(N\text{-}1)}: \begin{cases} \sum_a \phi_a^2 = v^2 & \textbf{(CONSTRAINT)} \\ \Box \phi_a + V'(\phi_a) = 0 & \textbf{(EOM)} \end{cases} \Rightarrow \boxed{\Box \phi_a + (\partial_\mu \phi_b \cdot \partial^\mu \phi_b)\, \phi_a = 0}$$

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$$\beta_a \equiv \phi_a / v \quad \Rightarrow \quad \sum_a \eta^{\mu\nu} \partial_\mu \beta^a \partial_\nu \beta^a = N \langle \eta^{\mu\nu} \partial_\mu \beta^1 \partial_\nu \beta^1 \rangle = \bar{T}(\eta) = T_o \eta^{-2}$$

(N >> 1)

(Turok & Spergel '91)

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$$\bar{T}(\eta) = T_o \eta^{-2}$$

(Turok & Spergel '91)

LARGE-N LIMIT:
(N >> 1)

$$\boxed{\phi_a(\mathbf{k},\eta) = (k\eta)^{\frac{1}{2}-\gamma} C_1(\mathbf{k})\, J_{\gamma+1}(k\eta)} \quad (a = \eta^\gamma)$$

$(k\eta_* < 1$, Super-Horizon Scales$)$

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$$\left\langle \beta^a(\mathbf{k},\eta)\beta^{*b}(\mathbf{k}',\eta') \right\rangle = A\left(\frac{\eta\eta'}{\eta_*^2}\right)^{3/2} \frac{J_\nu(k\eta)J_\nu(k'\eta')}{(k\eta)^\nu(k'\eta')^\nu} \left\langle \beta^a(\mathbf{k},\eta_*)\beta^{*b}(\mathbf{k}',\eta_*) \right\rangle$$

$$= (2\pi)^3 6\pi^2 A(\eta\eta')^{3/2} \frac{J_\nu(k\eta)J_\nu(k\eta')}{(k\eta)^\nu(k\eta')^\nu} \frac{\delta_{ab}}{N} \delta(\mathbf{k}-\mathbf{k}')$$

$$\boxed{(\beta_a \equiv \phi_a/v)}$$

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$$\Pi^2(k,\eta,\eta') = \int\frac{d^3q}{(2\pi)^3}\,q^4\left[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2\right]^2 \boxed{\mathcal{P}^{ab}_\phi(|\mathbf{q}|,\eta,\eta')\mathcal{P}^{ab}_\phi(|\mathbf{k}-\mathbf{q}|,\eta,\eta')}$$

2) Analytical Calculations: Large-N

$\phi_a(\mathbf{k},\eta) \rightarrow T_{\mu\nu}(\phi_a) \rightarrow \Pi^{TT}_{\mu\nu}(\phi_a) \rightarrow \Box h_{\mu\nu} = 16\pi G\,\Pi^{TT}_{\mu\nu}$

FIELD FLUCTUATIONS	STRESS tensor	ANISOTROPIC (TT) STRESS tensor	GW EQUATIONS (TT metric perturb.)

$$\rho_{GW} = \frac{<\dot{h}_{\mu\nu}\dot{h}^{\mu\nu}>}{16\pi G} = \int \frac{d\rho_{GW}(k,\eta)}{d\log k} d\log k \Rightarrow \boxed{\Omega_{GW}(k,\eta) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}(k,\eta)}{d\log k}}$$

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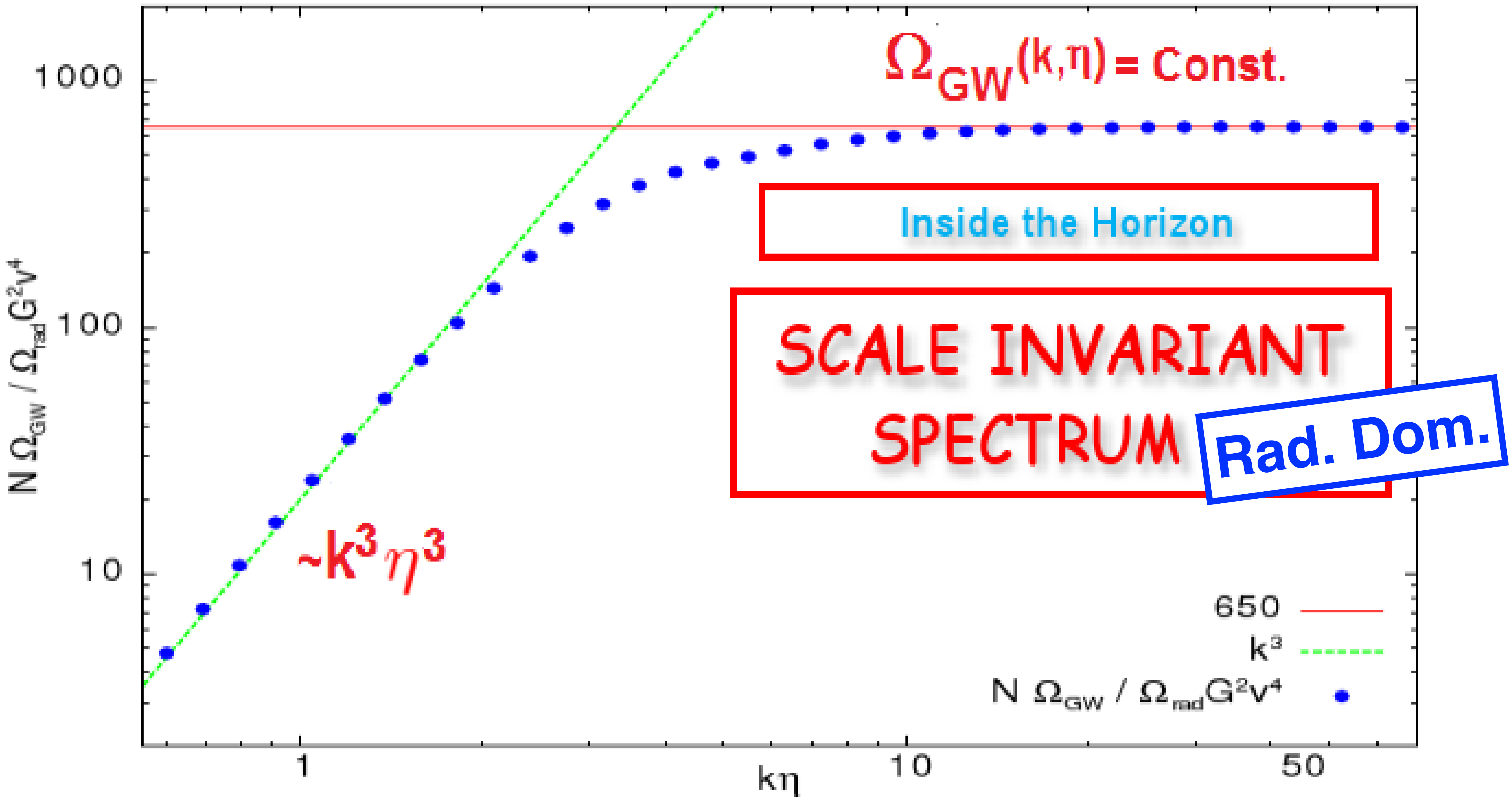

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SCALE INVARIANT SPECTRUM

(FREQ. INDEPENDENT)

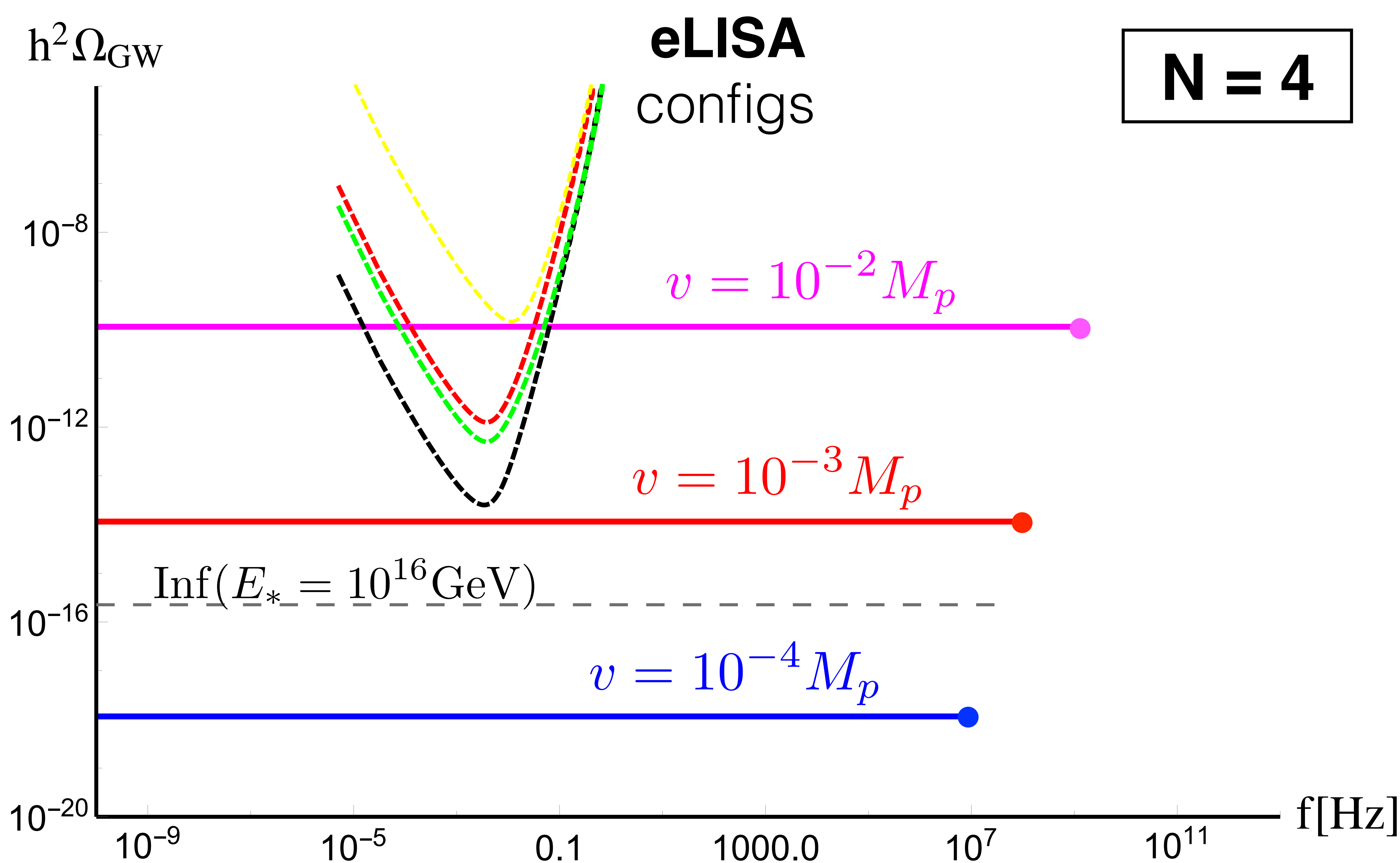
Rad. Dom.

$$\boxed{\Omega_{GW}(k,\eta_0) \simeq \frac{651}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4}$$

$$\boxed{\mathcal{R} \equiv \frac{\Omega_{GW}(k,\eta_0)}{\Omega_{GW}^{(\rm inf)}(k,\eta_0)} \simeq \frac{\mathcal{O}(10^2)}{\rm N}}$$

Jones-Smith et al 2008, Fenu et al 2008

2) Analytical Calculations: Large-N



Irreducible GW emission from a Defect Network

✓ 1) Theorem: GW from Evolution of Defect Networks

✓ 2) Analytical Calculations: Large-N

3) O(N) Lattice Simulations

4) Full Spectrum

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GW today: $\forall$ PhT (1st, 2nd, ...), $\forall$ Defects (top. or non-top.)

$$\Omega_{GW}^{(o)} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\rho_{\rm GW}}{d\log k}\right)_o = \frac{32}{3}\left(\frac{\rm V}{M_p}\right)^4 \Omega_{\rm rad}^{(o)}\, F_U\,, \quad \text{(SCALE INV.!)}$$

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VEV

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Defect type $\longrightarrow$ $F_U^{(2)}, F_U^{(3)}, F_U^{(4)}, F_U^{(\rm N>4)}$

Strings, Monopoles, Textures, …

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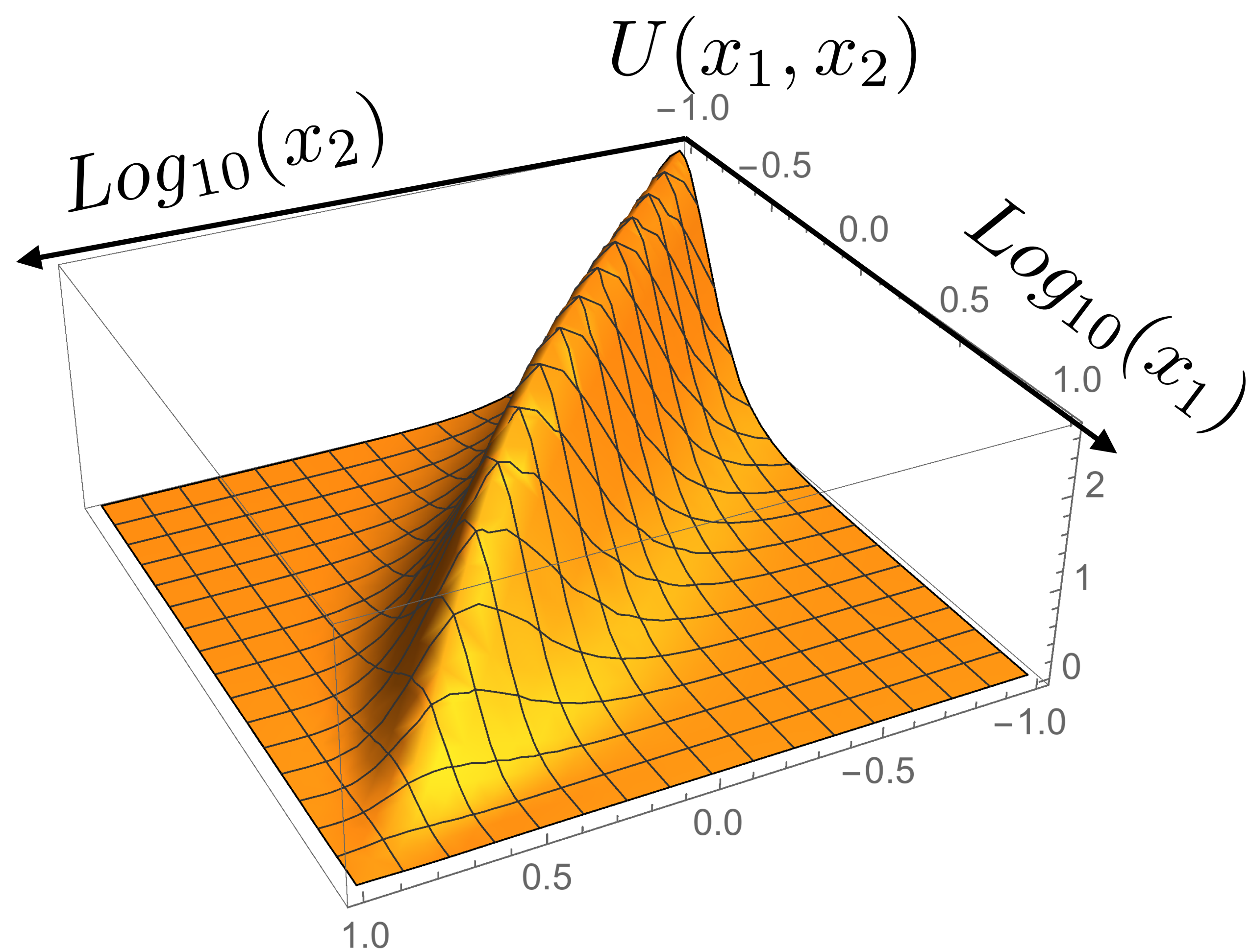
LATTICE SIMULATIONS.!
GLOBAL SYM. BREAKING
$1024^3 \rightarrow U(x_1, x_2) \rightarrow F_U$

(N = 2, 3, 4, 8, 12, 20)

Hindmarsh & Urrestilla



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$$\boxed{\frac{\Omega_{GW}^{Sim(N)}}{\Omega_{GW}^{(\rm Analytics)}} = \left\{\begin{array}{ll} 1.3, & (\rm N = 12) \\ 1.8, & (\rm N = 8) \\ 3.9, & (\rm N = 4) \\ 7.3, & (\rm N = 3) \\ 130, & (\rm N = 2) \end{array}\right\}}$$

DGF, Hindmarsh, Urrestilla, PRL 2013

$$\Omega_{\rm GW}^{\rm num} = \Omega_{\rm GW}^{\rm th}\left(a_0 + \frac{a_1}{N} + \frac{a_2}{N^2} + \cdots\right),$$

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$$\Omega_{\rm GW}^{\rm num} = \Omega_{\rm GW}^{\rm th}\left(a_0 + \cancel{\frac{a_1}{N}} + \frac{a_2}{N^2} + \cdots\right),$$

$$a_0 \simeq 1.1,\; a_2 \simeq 45$$

$$(N \geq 3)$$

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4) General Features

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Scaling @ RD

$$F_U \equiv \int_0^x dx_1 dx_2 \sqrt{x_1 x_2} cos(x_1 - x_2) U(x_1, x_2)$$

What about Matt-Dom (MD) modes?

4) General Features

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GW spectrum: Expansion UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \frac{k^3}{M_p^2 a^4(t)}\int dt_1 dt_2\; a(t_1)a(t_2)\;\cos(k(t_1 - t_2))\,\frac{{\rm V}^4}{\sqrt{t_1 t_2}}\,U(kt_1, kt_2)$$

SCALING

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Matt. Dom

SCALING

4) General Features

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What about Matt-Dom (MD) modes?

$$(x_i \equiv k t_i)$$

GW spectrum:

Expansion

UTC

$$\frac{d\rho_{\rm GW}}{d\log k}(k,t) \propto \left(\frac{v}{M_p}\right)^4 \frac{M_p}{k^2 a^4(t)}\left[\int_{x_{\rm eq}}^{x} dx_1 dx_2\;\; x_1^{\frac{3}{2}} x_2^{\frac{3}{2}}\;\; \cos(x_1 - x_2)\;\; U(x_1, x_2)\right]$$

Matt. Dom

SCALING

Scale-Dependence !

4) General Features

Total GW Spectrum

$$\boxed{h^2\Omega_{\rm GW}^{(\rm o)} = h^2\Omega_{\rm rad}^{(\rm o)}\left(\frac{V}{M_p}\right)^4\left[F_U^{(\rm R)} + F_U^{(\rm M)}\left(\frac{k_{\rm eq}}{k}\right)^2\right]}$$

$$F_U^{(\rm R)} \equiv \frac{32}{3}\int_0^x dx_1 dx_2\, (x_1x_2)^{1/2}\cos(x_1 - x_2)\, U_{\rm RD}(x_1, x_2)$$

RD

Scaling

$$F_U^{(\rm M)} \equiv \frac{32}{3}\frac{(\sqrt{2}-1)^2}{2}\int_{x_{\rm eq}}^x dx_1 dx_2\, (x_1x_2)^{3/2}\cos(x_1 - x_2)\, U_{\rm MD}(x_1, x_2)$$

MD

Scaling

4) General Features

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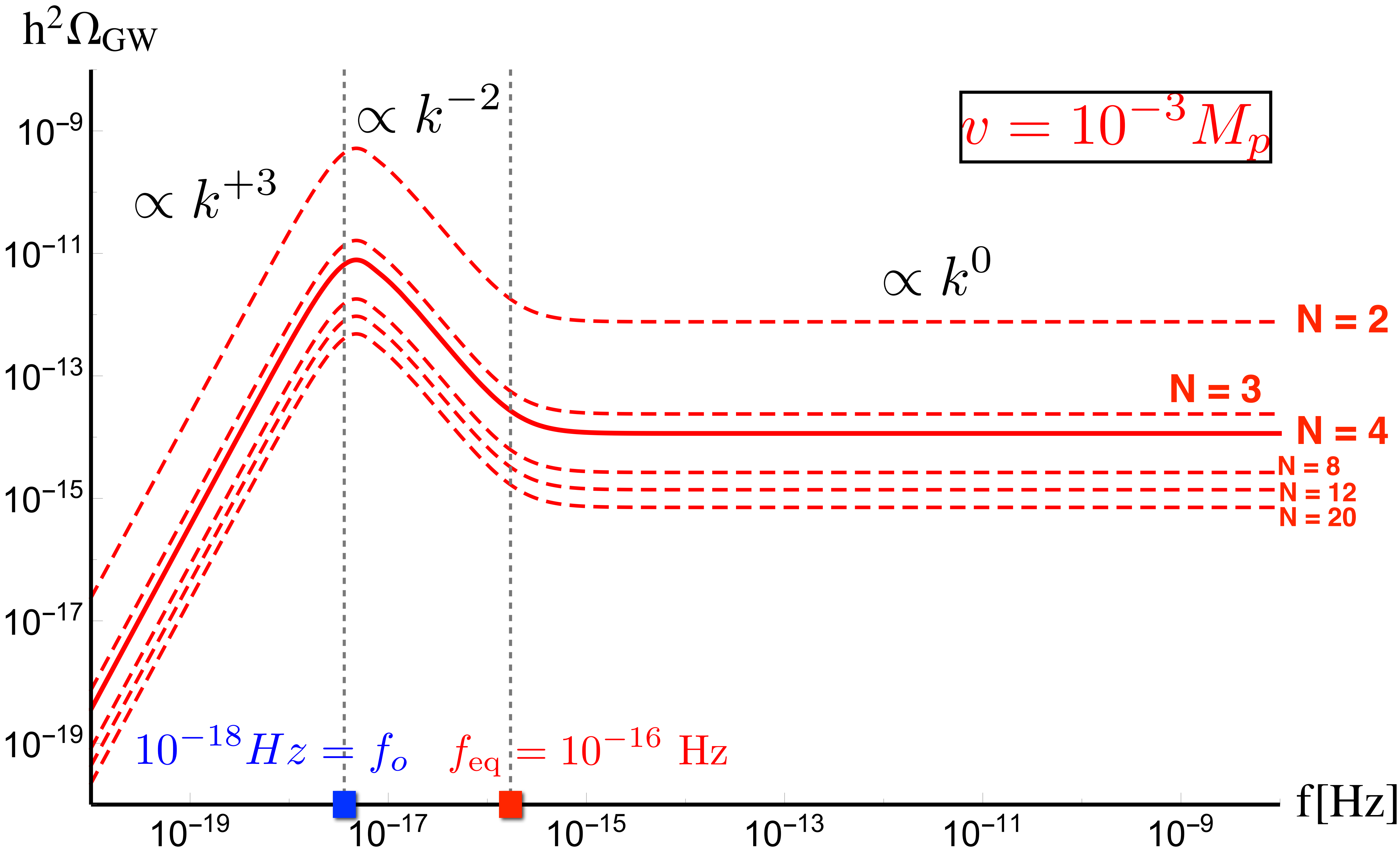

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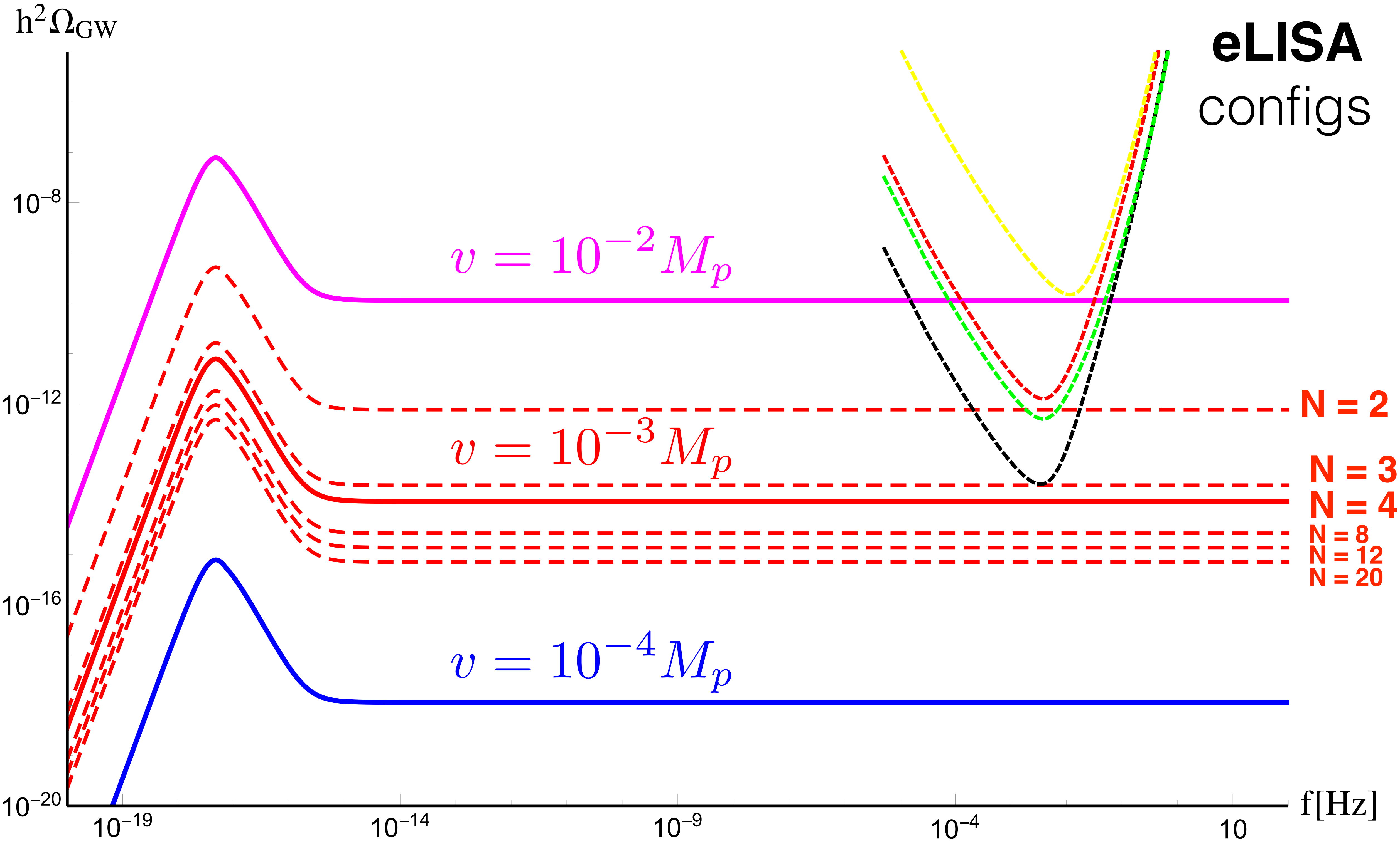

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$\forall$ PhT (1st, 2nd, ...), $\forall$ Defects (top. or non-top.)

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$$f < f_o \sim 10^{-18}{\rm Hz}\,, \quad f_o < f < f_{\rm eq} \sim 10^{-15}{\rm Hz}\,, \quad f_{\rm eq} < f < f_* \lesssim 10^8{\rm Hz}$$

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Large-N analytical, Global Lattice Sim's, AH Lattice Sim's

Irreducible Background!

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What about Abelian-Higgs Strings?

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$$F_U^{(\mathrm{AH})} \equiv \frac{32}{3}\int_0^x dx_1 dx_2\, a(x_1)\, a(x_2)\, \cos(x_1 - x_2)\, U_{\mathrm{AH}}(x_1, x_2)$$

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AH-team: Hindmarsh, Lizarraga Kunz & Urrestilla

AH

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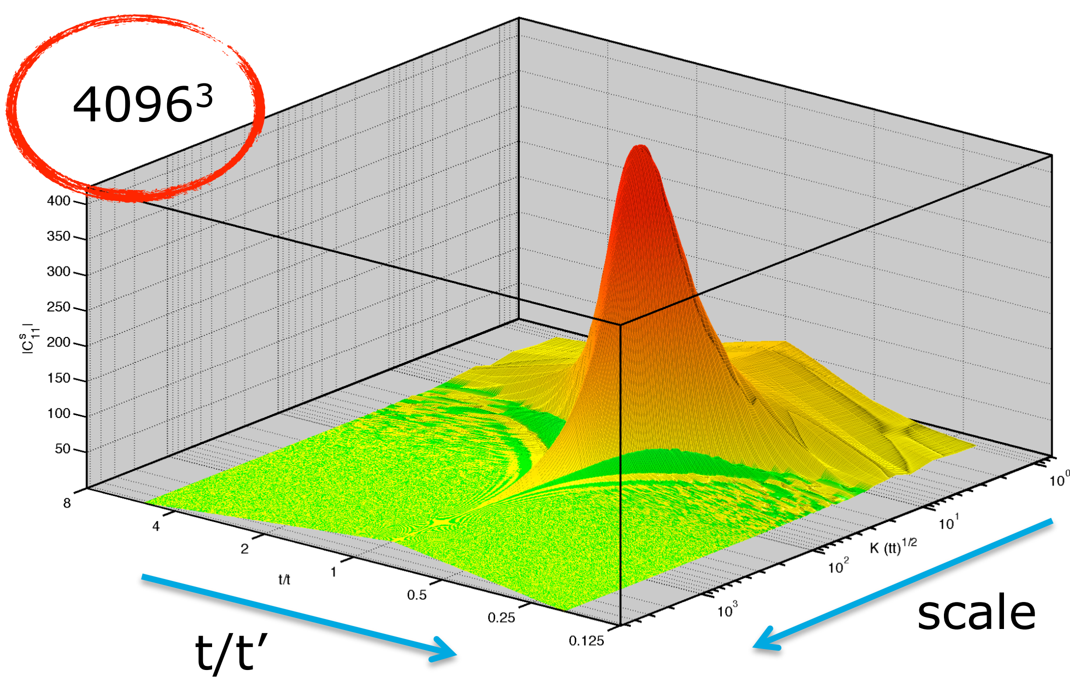

$$\left[\; a(x) \equiv a_{\mathrm{eq}} \left(\left[(\sqrt{2} - 1)(x/x_{\mathrm{eq}}) + 1 \right] - 1 \right) \;\right]$$

(figs. by Joanes Lizarraga)

What about Abelian-Higgs Strings?

$$F_U^{(\mathrm{AH})} \equiv \frac{32}{3}\int_0^x dx_1 dx_2\, a(x_1)\, a(x_2)\, \cos(x_1 - x_2)\, U_{\mathrm{AH}}(x_1, x_2)$$

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Kunz & Urrestilla

Work in progress !

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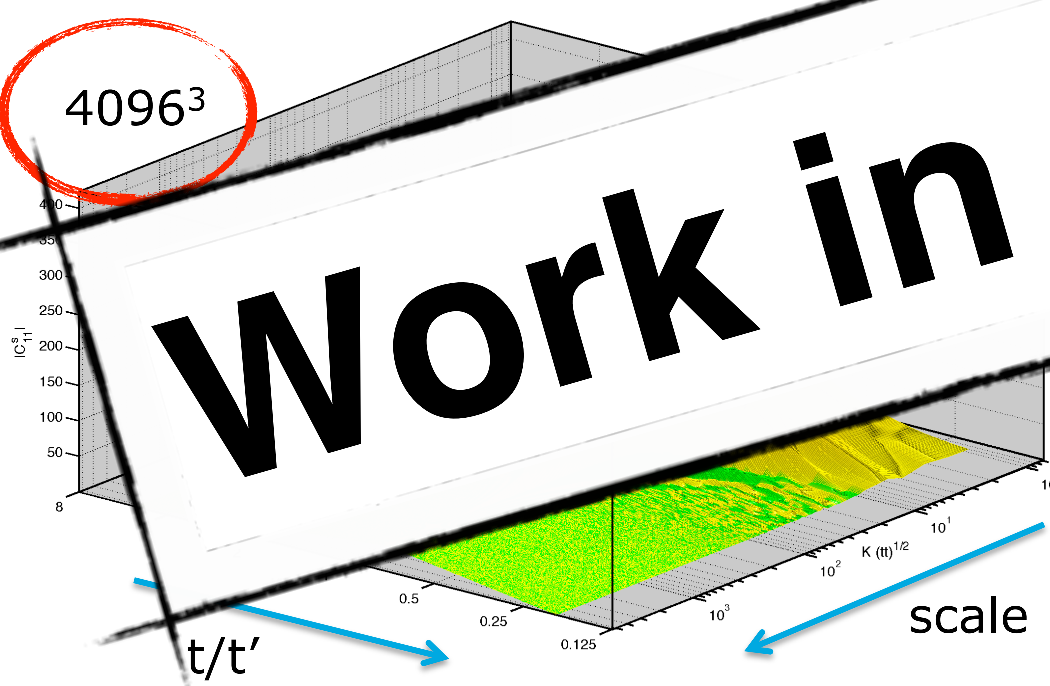

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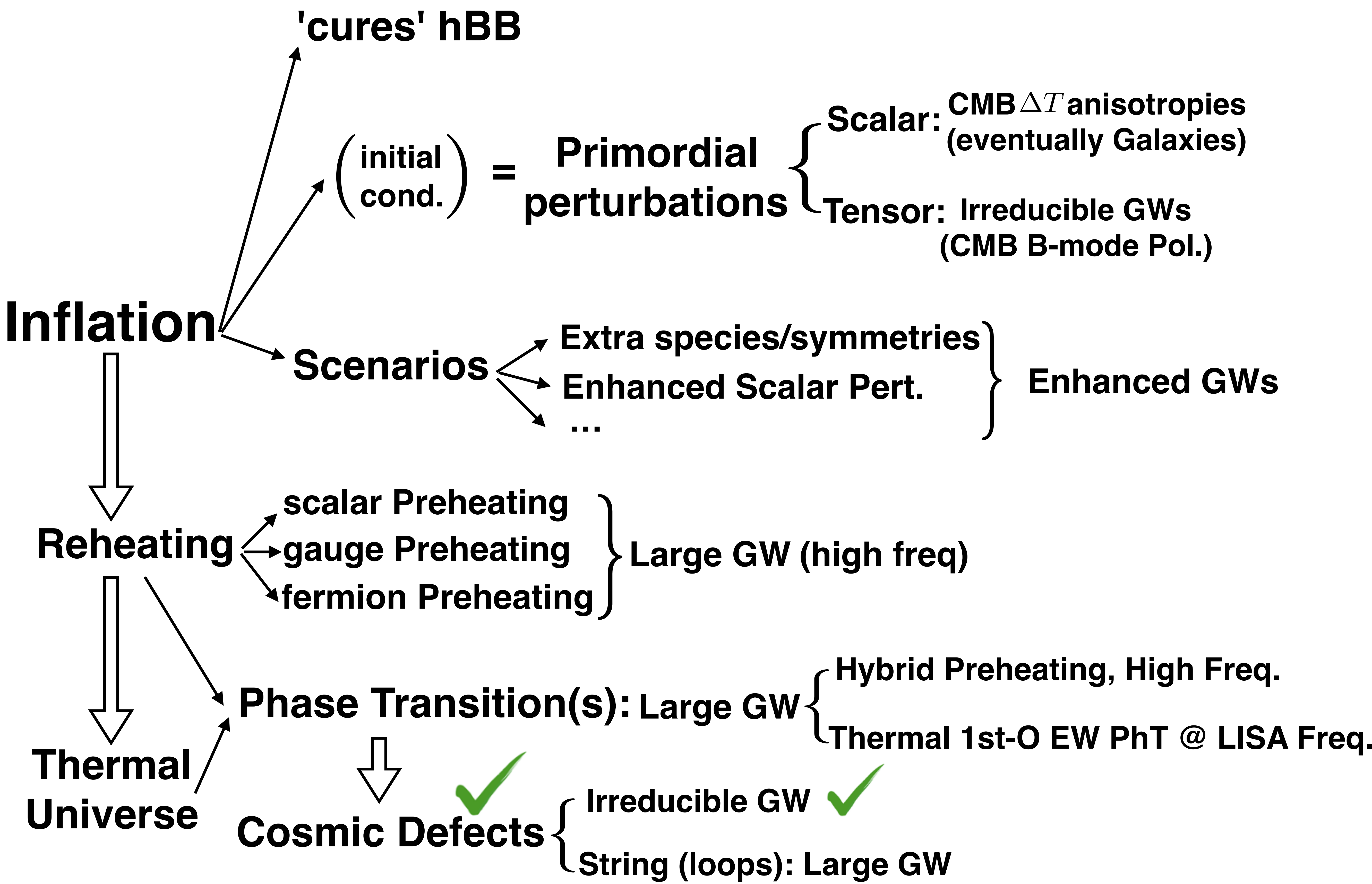
EARLY UNIVERSE



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- 'cures' hBB
- (initial cond.) = Primordial perturbations
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- Scenarios
  - Extra species/symmetries
  - Enhanced Scalar Pert.
  - ...
  - } Enhanced GWs

Reheating

- scalar Preheating
- gauge Preheating
- fermion Preheating
- } Large GW (high freq)

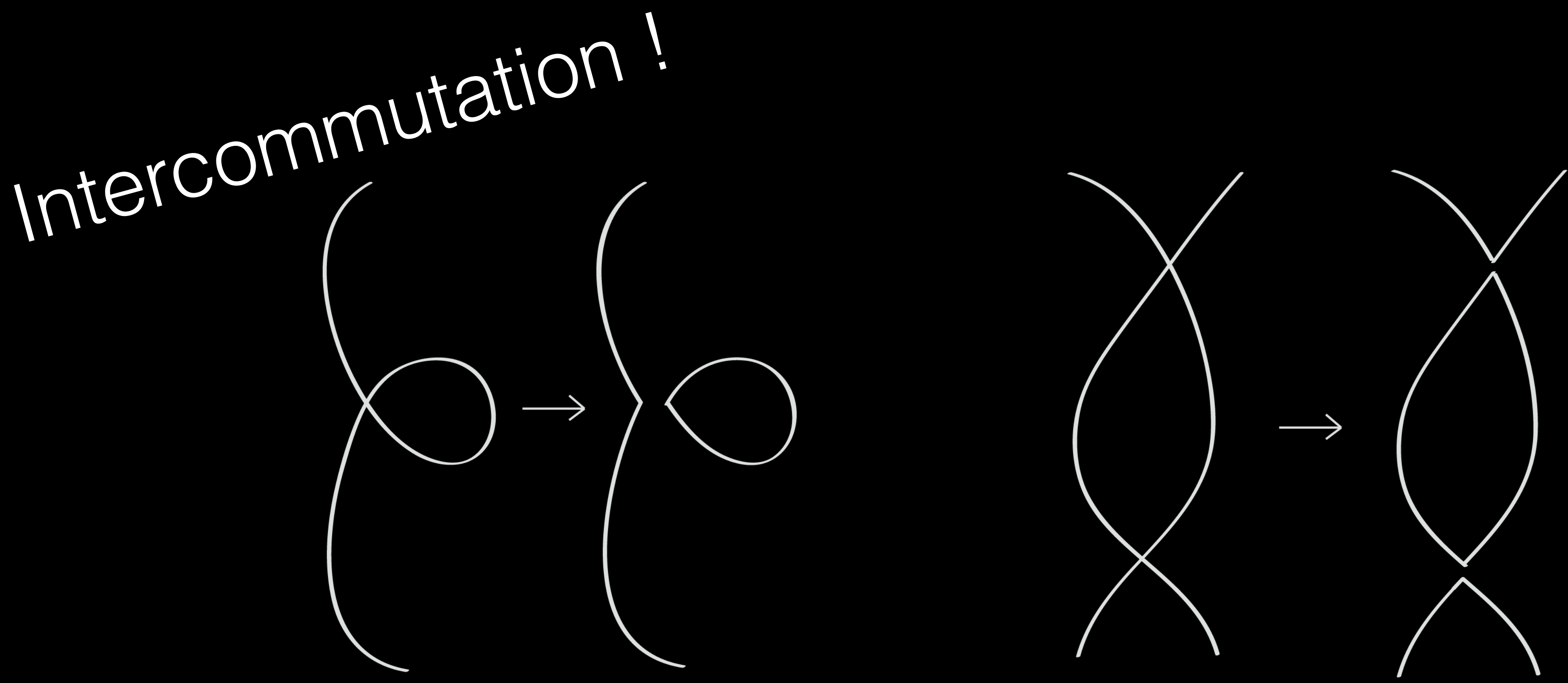
Thermal Universe

Phase Transition(s): Large GW
- Hybrid Preheating, High Freq.
- Thermal 1st-O EW PhT @ LISA Freq.

Cosmic Defects
- Irreducible GW
- $\boxed{\text{String (loops): Large GW}}$

Cosmic Strings Network: Loop configurations

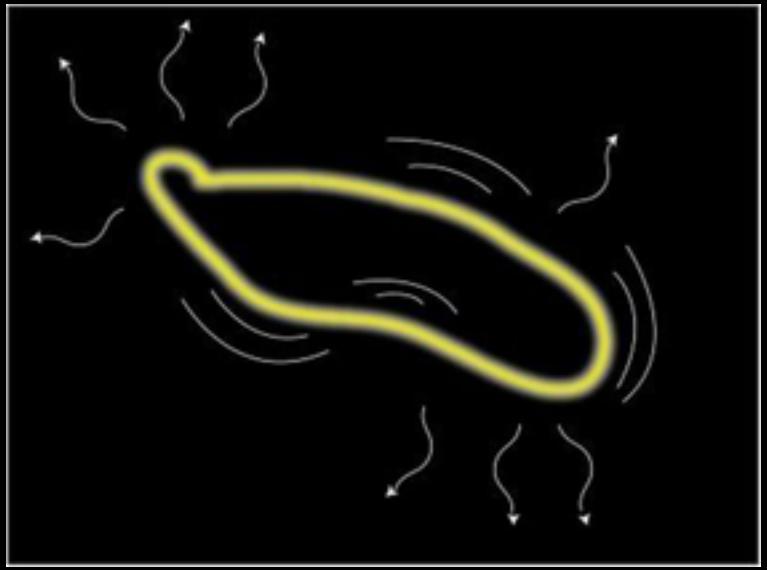
A cosmic string network formed by:
1) ‘Infinite' long cosmic strings
2) (subhorizon)Cosmic string loops



- Cosmic strings: $p = 1$
- Cosmic superstrings: $p \in [10^{-3}, 1]$

Cosmic Strings Network: Loop configurations

Loops once formed, decay by radiation emission



$\longrightarrow$

* GW emission
* Boson emission
* UHCR
* …

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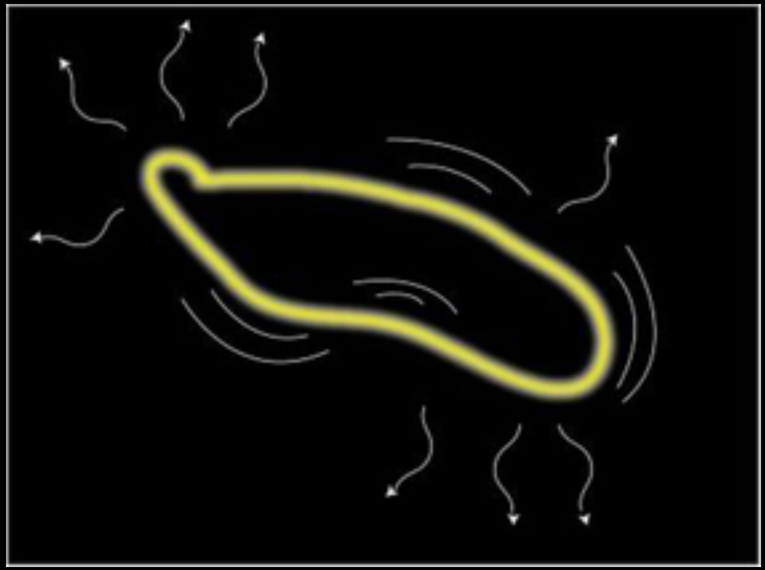

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Extra emission of GWs !

(Vilenkin '81)

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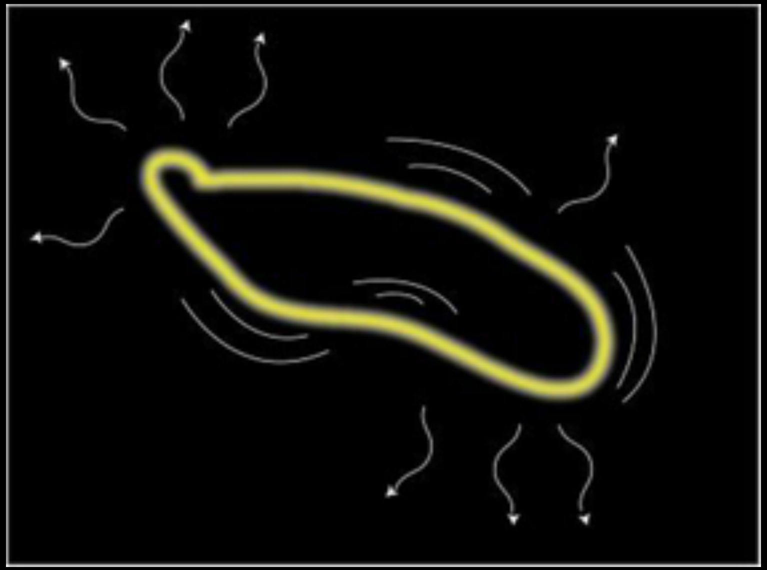

$\longrightarrow$ **Extra emission of GWs !**

* Boson emission
* UHCR
* …

(Vilenkin '81)

Assuming GW emission dominates …

Given a loop number density $n(\ell, t)$

$$\Omega_{\rm gw}(f) = \frac{2G\mu^2 c^3}{\rho_{\rm crit} a^5(t_0) f} \sum_{j=1}^{\infty} j P_j \int_{t_{\rm f}}^{t_0} a^5(t') n_j(f, t') dt'$$

Cosmic Strings Network: Loop configurations

Cosmic string loop (length l) oscillates under tension μ

Cosmic Strings Network: Loop configurations

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➡ **emits GWs in a series of harmonic modes**

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Extra emission of GWs ! **(Vilenkin '81)**

$$\frac{d\rho^{(\mathrm{o})}}{df} \equiv \Gamma G\mu^2 \int_{t_*}^{t_o} dt \left(\frac{a(t)}{a_o}\right)^3 \int_0^{\alpha/H(t)} dl\, l n(l,t)\, \mathcal{P}((a_o/a(t)) f l)$$

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expansion history

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GW power emission

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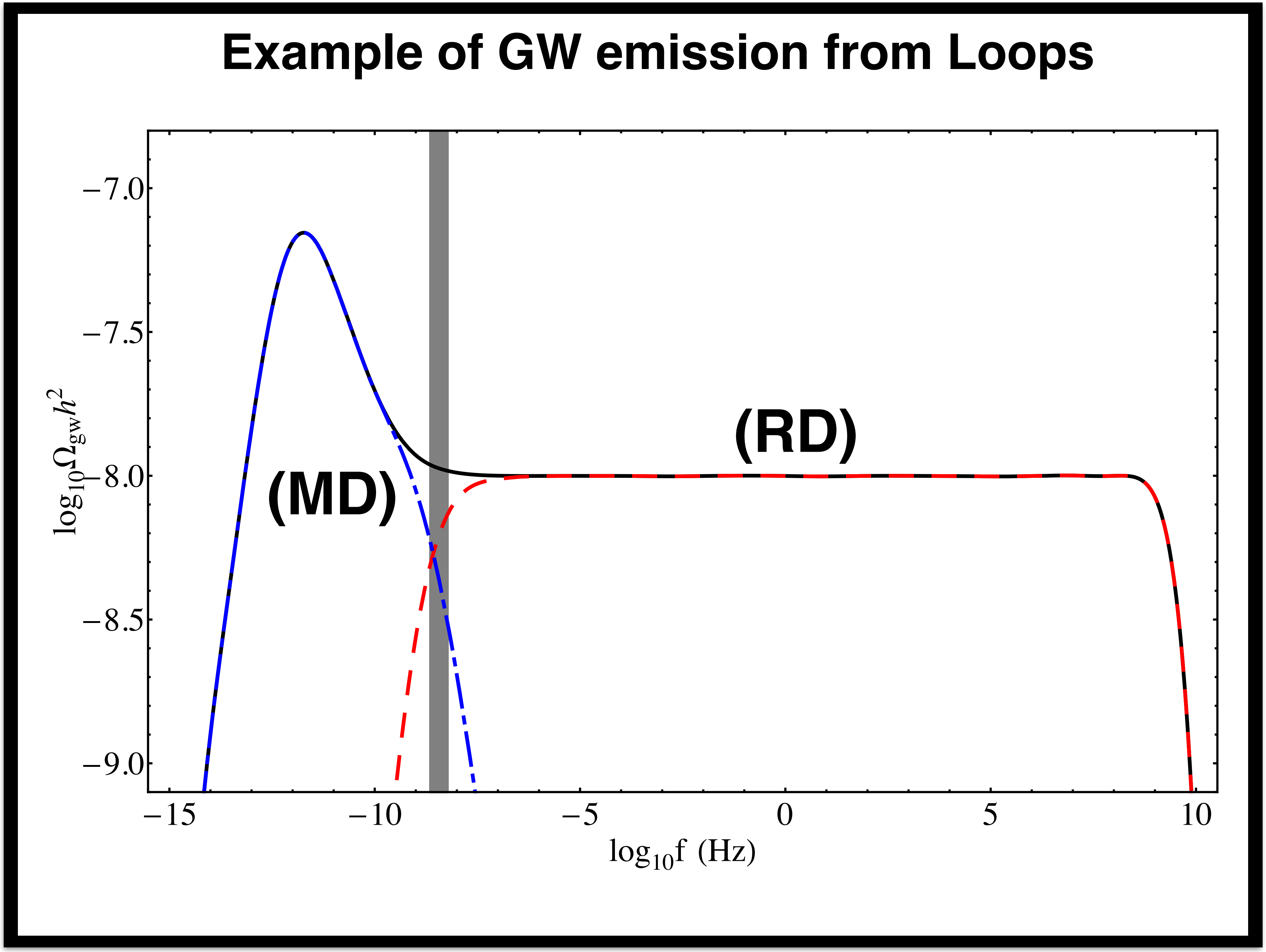
expansion history | **length** | **number density** | **GW power emission**

$\propto 1/(fl)^{q+1}$

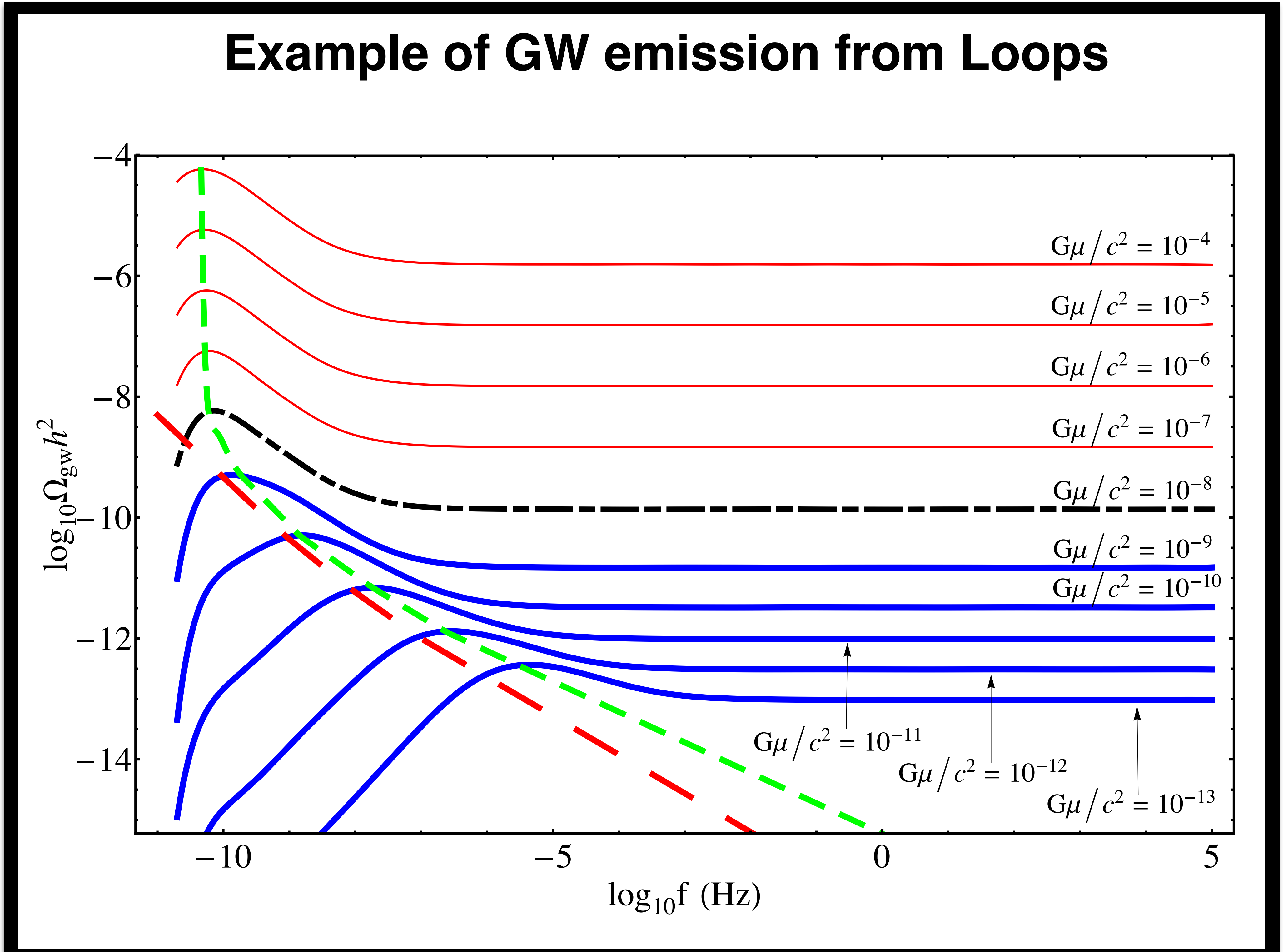
features (kinks,cusps,...)

Cosmic Strings Network: Loop configurations



Sanidas et al 2012

Cosmic Strings Network: Loop configurations



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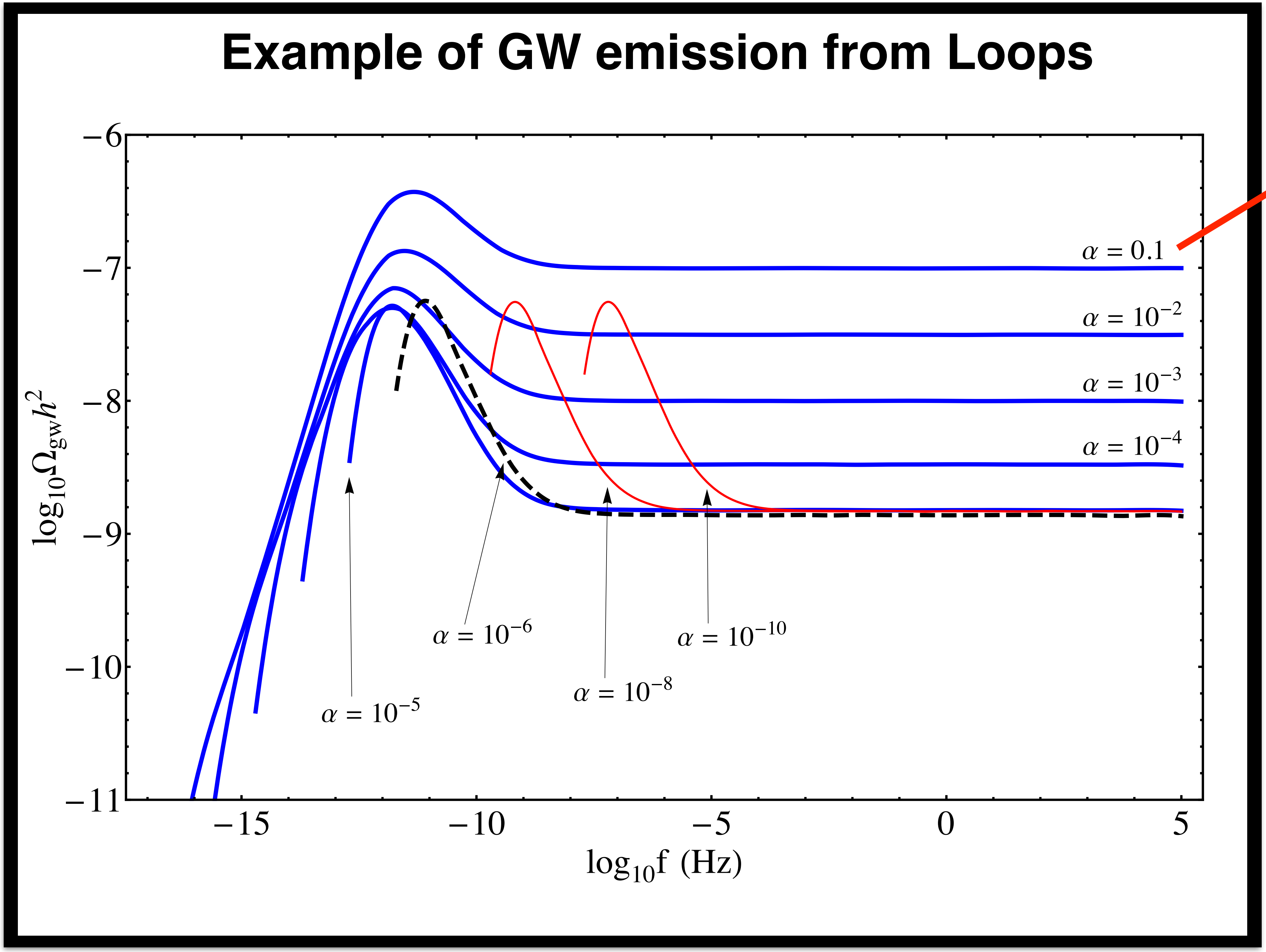

α

loop size

(relative to horizon)

Sanidas et al 2012

Cosmic Strings Network: Loop configurations

GW from string loops $\neq$ **GW from "Infinite"-Strings**
(particular emission) **(irreducible emission)**

Cosmic Strings Network: Loop configurations

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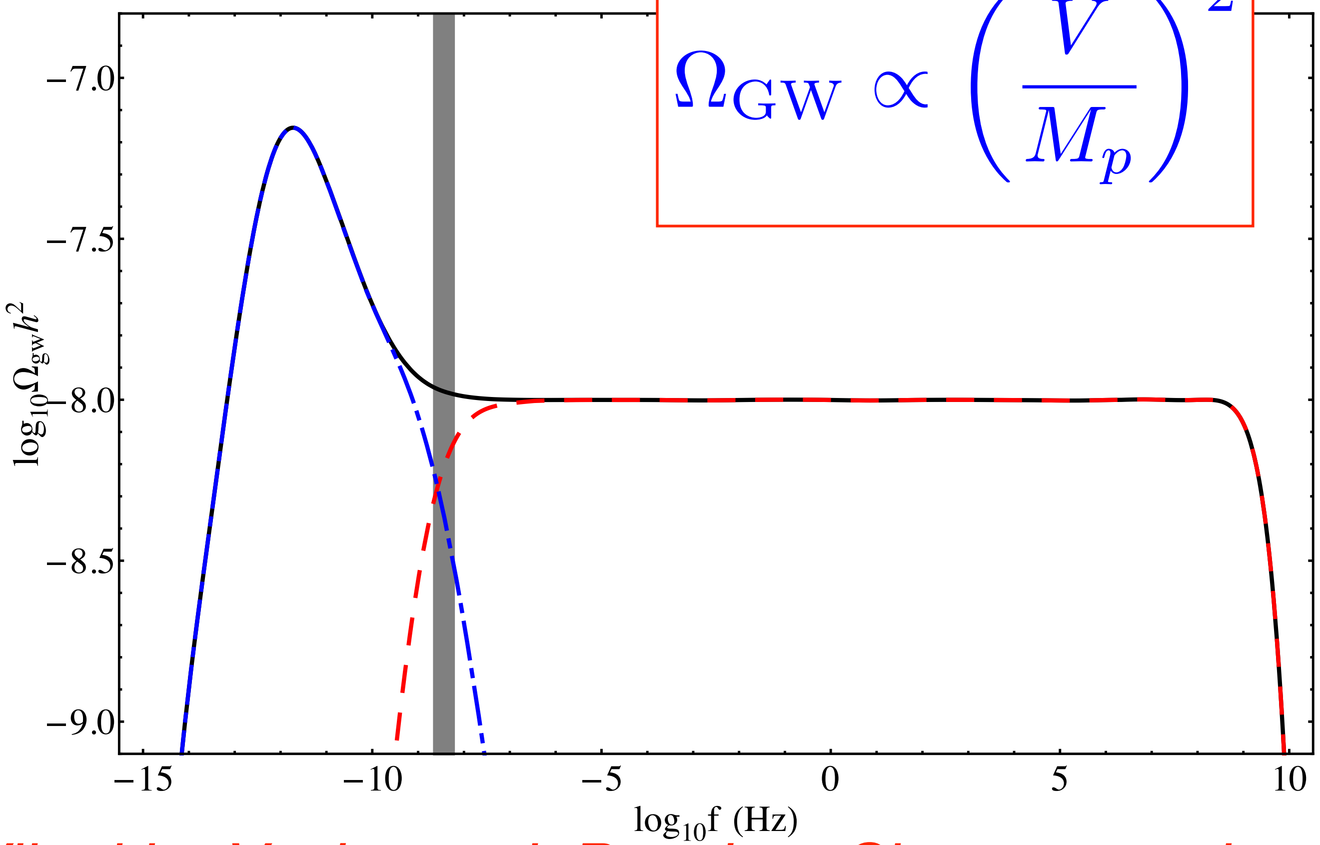
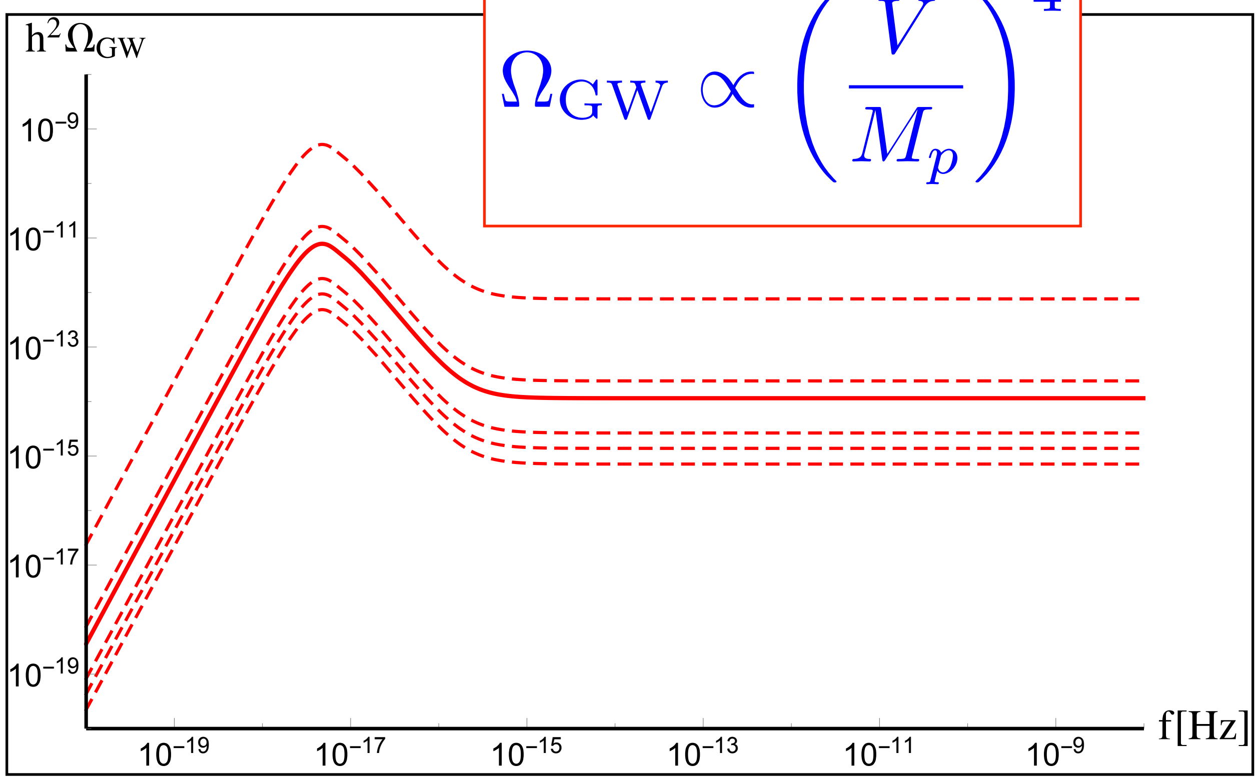
$$\Omega_{\rm GW} \propto \left(\frac{V}{M_p}\right)^2$$



Vilenkin, Vachaspati, Bouchet, Siemens et al, Sanidas et al, Blanco-Pillado et al, … 1981 - 2016

$$\Omega_{\rm GW} \propto \left(\frac{V}{M_p}\right)^4$$



DGF, Hindmarsh, Urrestilla, work in progress 2013-2016

Cosmic Strings Network: Loop configurations

GW from string loops $\neq$ GW from "Infinite"-Strings
(particular emission) (irreducible emission)

Ask Drew, Helfer, Pazouli, ...

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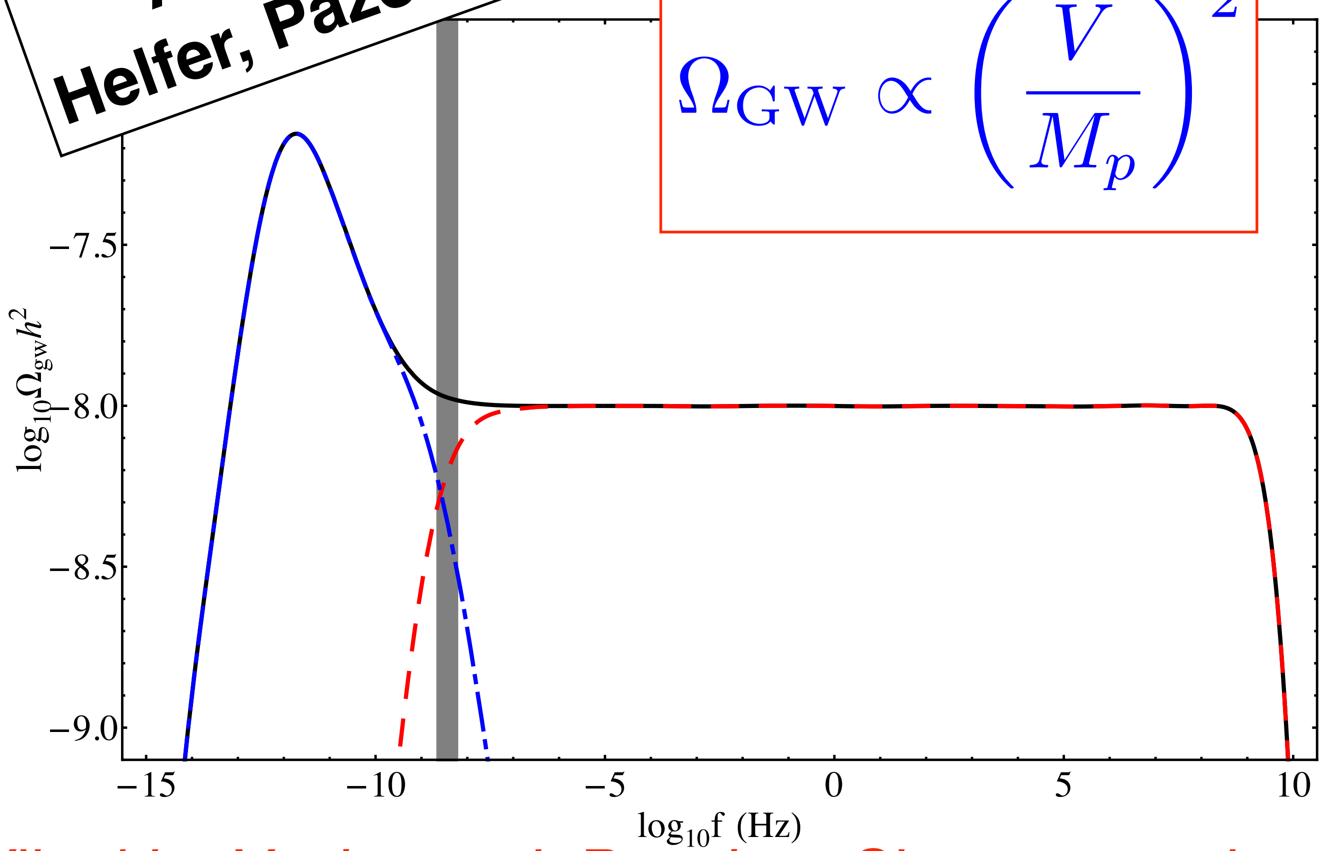

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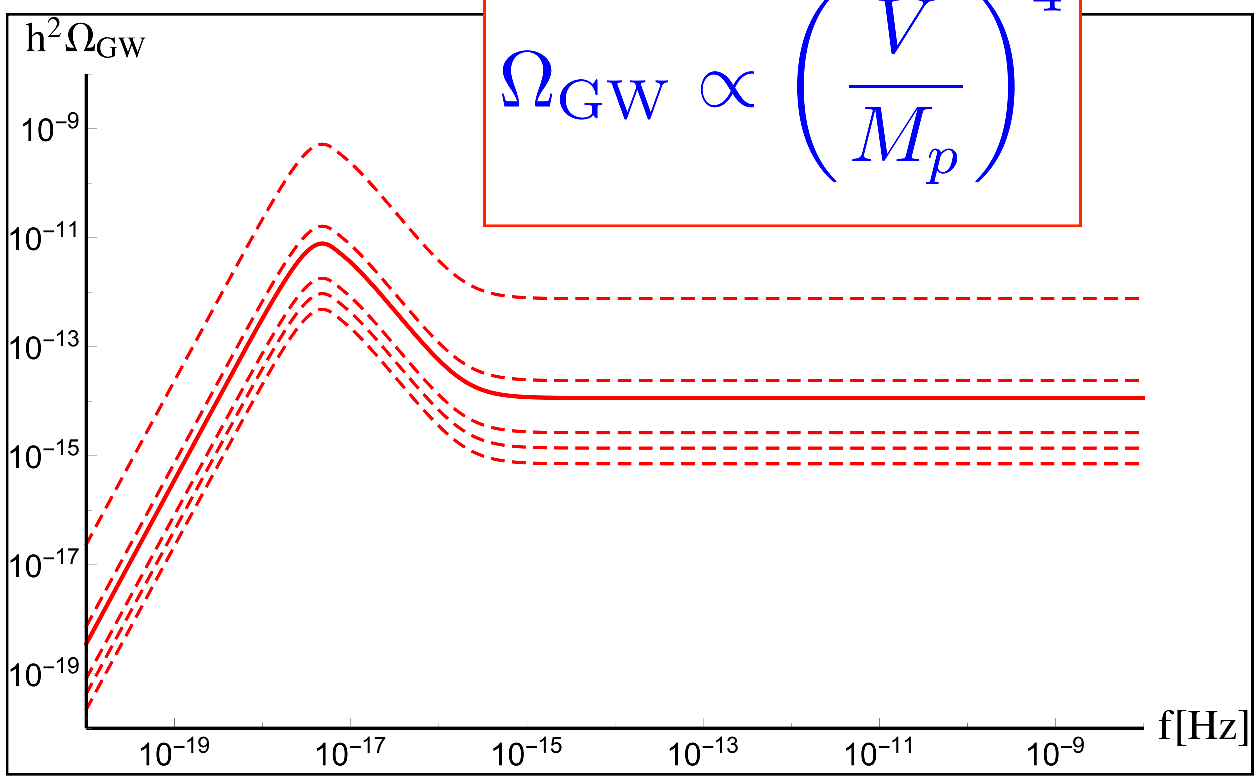

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Cosmic Strings Network: Loop configurations

Extra emission of GWs ! (Vilenkin '81)

Results for 6 links, SNR=20

LISA Prospects

- A1M2
  Conservative limit: $G\mu/c^2 < 4.4 \times 10^{-10}$
  Large loops: $G\mu/c^2 < 1.5 \times 10^{-16}$
- A2M2
  Conservative limit: $G\mu/c^2 < 1.1 \times 10^{-10}$
  Large loops: $G\mu/c^2 < 2.1 \times 10^{-17}$
- A2M5
  Conservative limit: $G\mu/c^2 < 7.0 \times 10^{-11}$
  Large loops: $G\mu/c^2 < 1.3 \times 10^{-17}$
- A5M5
  Conservative limit: $G\mu/c^2 < 1.4 \times 10^{-11}$
  Large loops: $G\mu/c^2 < 4.4 \times 10^{-18}$ $\rightarrow$ $\boxed{v \lesssim 10^{10} GeV}$

(From Sanidas et al,LISA GW cosmology 3rd encounter)

EARLY UNIVERSE in GWs

'cures' hBB

Inflation

$\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

Scalar: CMB ΔT anisotropies (eventually Galaxies)

Tensor: Irreducible GWs (CMB B-mode Pol.)

Scenarios

Extra species/symmetries

Enhanced Scalar Pert.

...

Enhanced GWs

Reheating

scalar Preheating

gauge Preheating

fermion Preheating

Large GW (high freq)

Thermal Universe

Phase Transition(s): Large GW

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Irreducible GW

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(initial) Primordial

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GRAVITATIONAL WAVES AS A PROBE OF THE EARLY UNIVERSE

(very brief) SUMMARY

0) Cosmology

Early Universe

1) GWs from Inflation

2) GWs from Preheating

3) GWs from Phase Transitions

4) GWs from Cosmic Defects

GRAVITATIONAL WAVES AS A PROBE OF THE EARLY UNIVERSE

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0) Cosmology → cool !… but we already knew this !

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High amplitude, unlike detection

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[*At LISA if EWPT is strong 1st order] [**By LISA/PTA, If large loops present]

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… GW definition

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GRAVITATIONAL WAVES AS A PROBE OF THE EARLY UNIVERSE

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… **GW definition**

complicated, but robust

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Coming Soon …

*** Caprini & Figueroa, REVIEW on GWs from Early Universe**
(expected Sept 2017**)**

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ALL about
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*** Prague Particle Cosmology school (1 week): Sep 4 - 8**

http://www-ucjf.troja.mff.cuni.cz/~iss2017/ISS2017_main.html

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Inflation	Antonio Riotto
Hot early Universe	Rocky Kolb
Neutrinos in cosmology	Sergio Pastor
Dark matter	Thomas Schwetz-Mangold
Gravitational waves	Daniel Figueroa

Coming Now …

770 slides afterwards…

Coming Now …

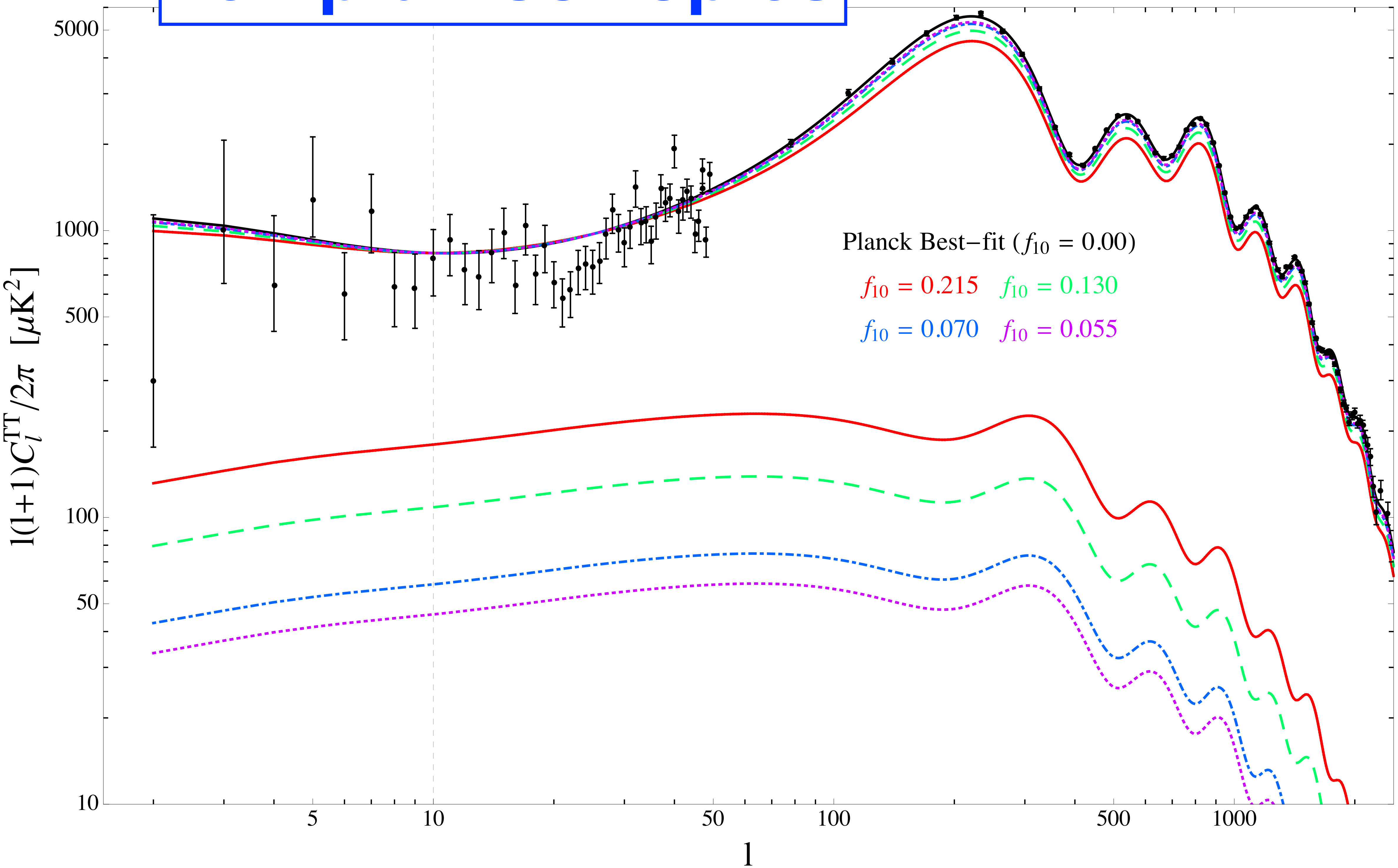
770 slides afterwards…

I thank you for your attention !

CMB (Back) SLIDES

Cosmic Microwave Background

Temp-anisotropies



Durrer et al, JCAP 2014

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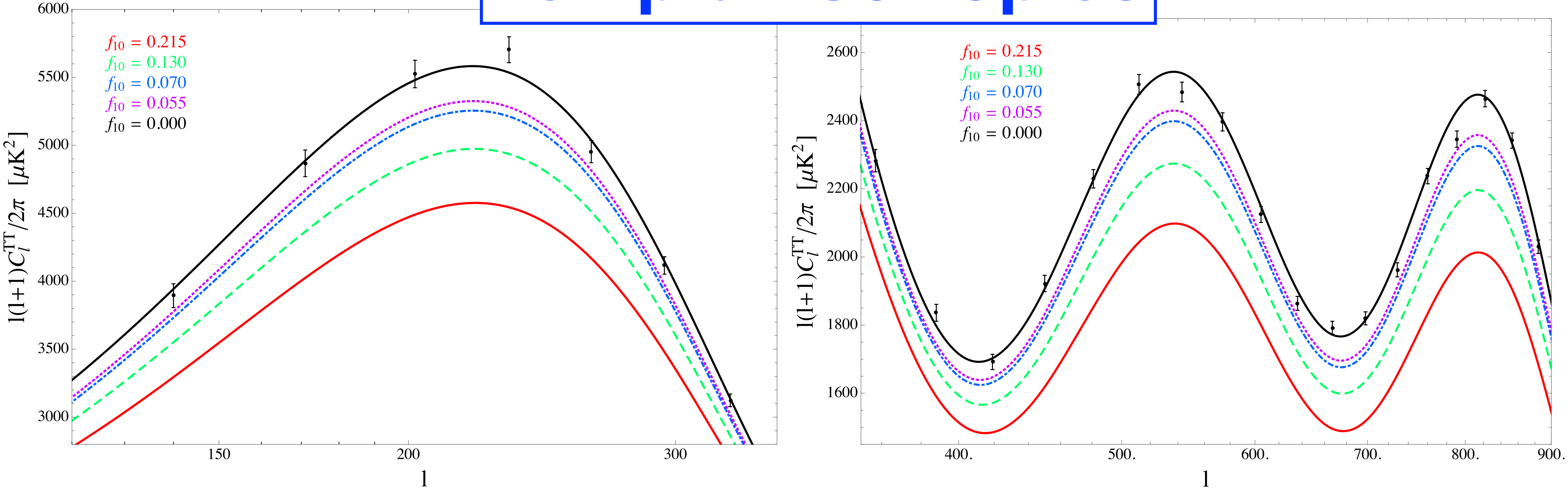

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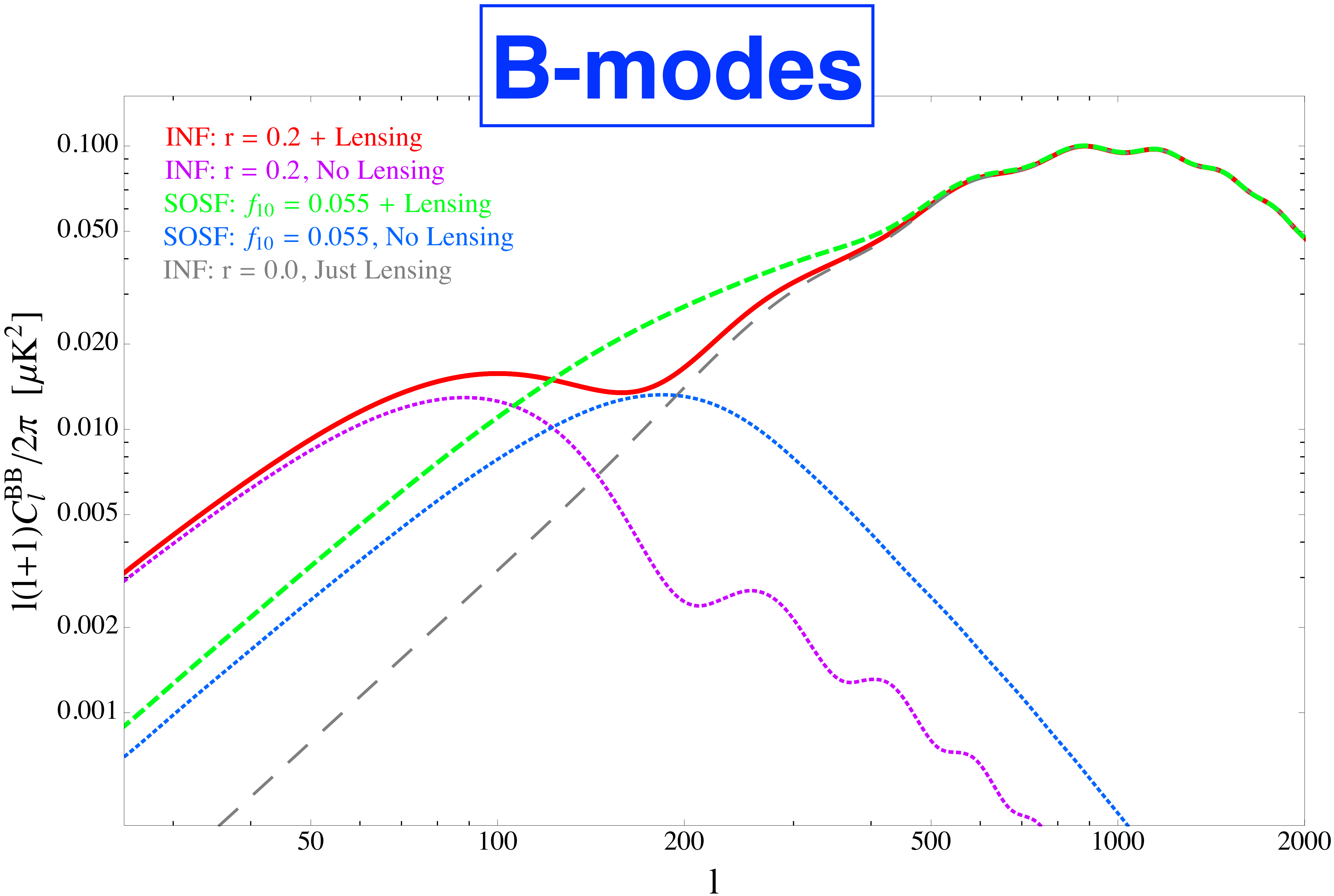

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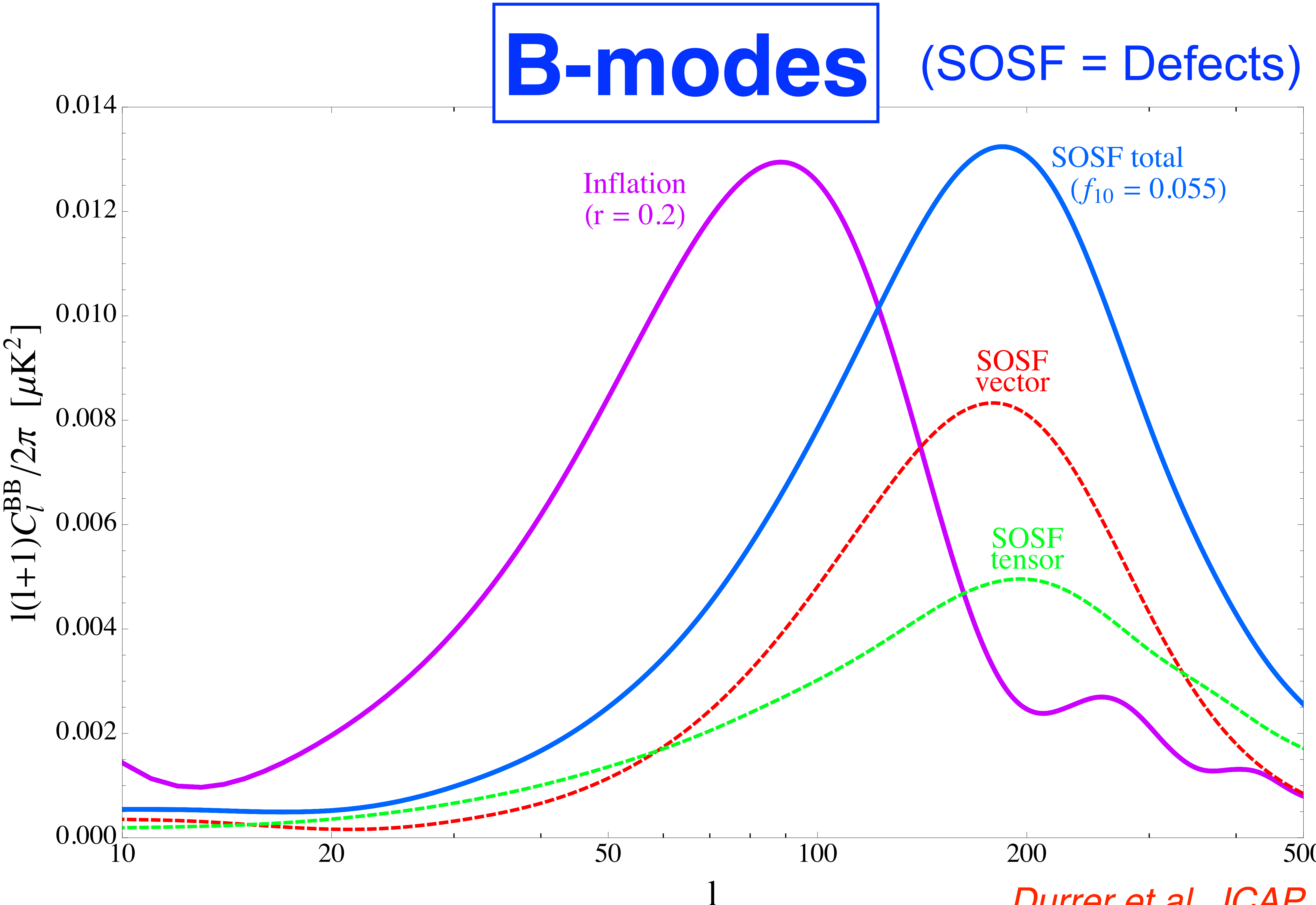

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