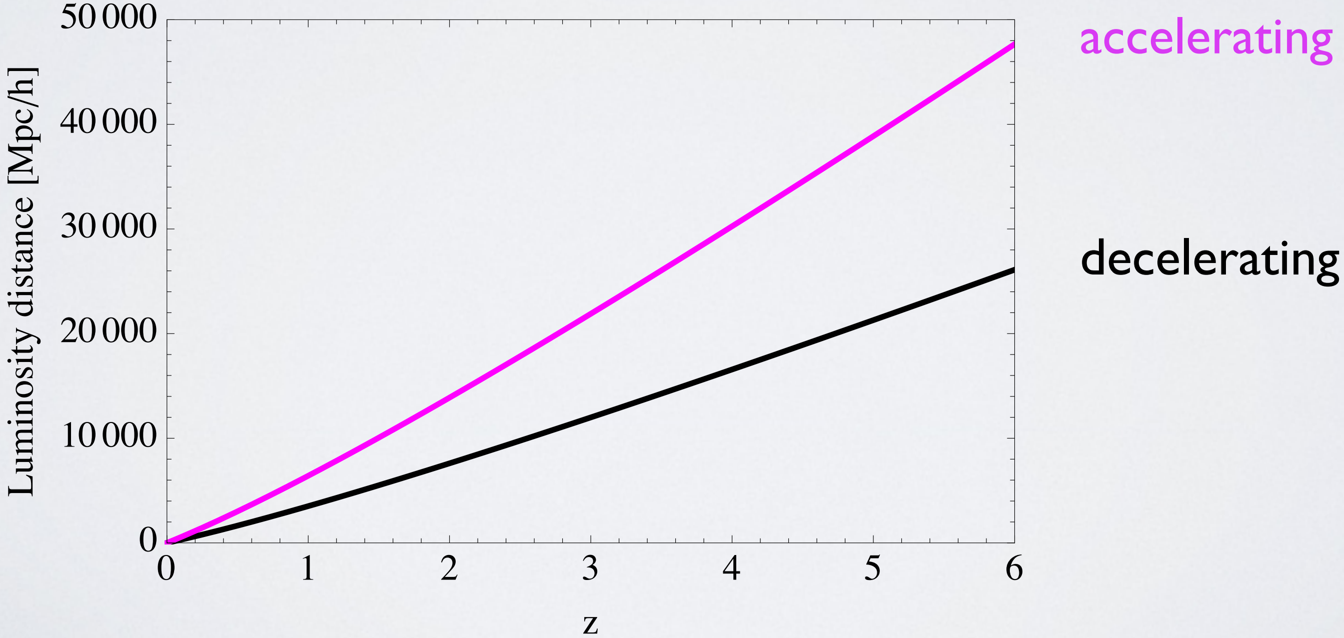
Standard Sirens

Standard Sirens

- We look at GWs emitted by **coalescing binaries**

- From the waveform we measure the **luminosity distance**.

- If we have in addition a measurement of the redshift, we have a point of the curve $d_L(z)$.



Outline

- Derivation of the **waveform** for inspiraling binaries.
- **Propagation** of GWs in our universe, assuming a homogeneous and isotropic metric.

  → dependence in the **luminosity distance**

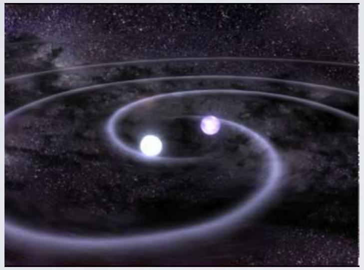
- What can we learn from the luminosity distance?
- Expression of $d_L(z)$ in a FLRW metric.
- **Forecasts** on dark energy from LISA.

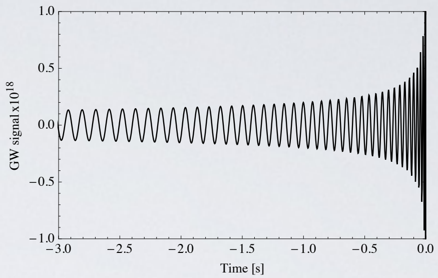
Outline

- Propagation of GWs in an **inhomogeneous** universe.
- Two relevant effects
  - Inhomogeneities change the **luminosity distance**
  - Inhomogeneities change the **waveform**
- What is this impact of inhomogeneities on GWs **detection** and **interpretation**?

M. Maggiore Gravitational Waves

Inspiraling binaries





See lectures by Luc Blanchet

GWs are defined as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

We study the propagation in a **flat space-time**, i.e. $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$

$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$ obeys a **wave equation** $\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$

Inspiraling binaries

- We calculate the solution outside the source.

- We use the TT (transverse-traceless) gauge

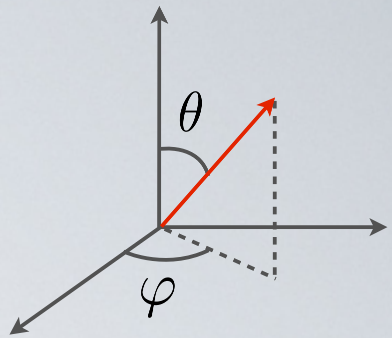
  → $h_{\mu\nu}$ described by two **polarisations** h_+ and $h_\times$

- We **expand** at lowest order in v/c

- GWs are generated by variations of the **quadrupole** moment of the source

$$M^{ij} = \int d^3x\, T^{00}(t, \mathbf{x}) x^i x^j$$

Quadrupole formula



$$h_+(t,\theta,\varphi) = \frac{G}{r}\Big[\ddot{M}_{11}(\cos^2\varphi - \sin^2\varphi\cos^2\theta) + \ddot{M}_{22}(\sin^2\varphi - \cos^2\varphi\cos^2\theta)$$
$$- \ddot{M}_{33}\sin^2\theta - \ddot{M}_{12}\sin 2\varphi(1+\cos^2\theta) + \ddot{M}_{13}\sin\varphi\sin 2\theta$$
$$+ \ddot{M}_{23}\cos\varphi\sin 2\theta\Big]$$

$$h_\times(t,\theta,\varphi) = \frac{G}{r}\Big[(\ddot{M}_{11} - \ddot{M}_{22})\sin 2\varphi\cos\theta + 2\ddot{M}_{12}\cos 2\varphi\cos\theta$$
$$- 2\ddot{M}_{13}\cos\varphi\sin\theta + 2\ddot{M}_{23}\sin\varphi\sin\theta\Big]$$

Quadrupole of a binary system

$$\rho(t,\mathbf{x}) = m_1\delta(\mathbf{x}-\mathbf{x}_1(t)) + m_2\delta(\mathbf{x}-\mathbf{x}_2(t))$$

$$M^{ij}(t) = \int d^3x\, x^i x^j \rho(t,\mathbf{x}) = m_1 x_1^i x_1^j + m_2 x_2^i x_2^j$$

We can simplify the derivation by going to the reference frame of the **center of mass**

$$M^{ij}(t) = \mu x_0^i(t) x_0^j(t)$$

With $\mu = \frac{m_1 m_2}{m_1+m_2}$ and $\mathbf{x}_0(t) = \mathbf{x}_1 - \mathbf{x}_2$

If we know the **trajectory** $\mathbf{x}_0(t)$ we know $\ddot{M}^{ij}$

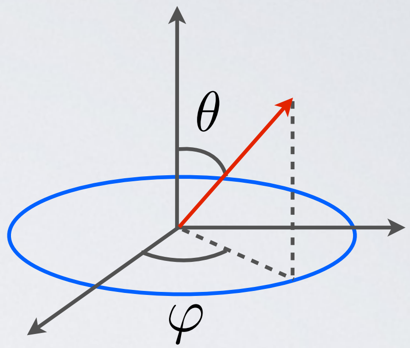
Step 1: circular orbit

We assume that the relative coordinate $\mathbf{x}_0(t)$ describes a **circle**

The trajectory is fixed → **no backreaction** from GW emission

$$x_0(t) = R\cos(\omega_B t + \pi/2)$$

$$y_0(t) = R\sin(\omega_B t + \pi/2)$$



chirp mass $M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

$$f_{\rm GW} = \frac{\omega_{\rm GW}}{2\pi} = \frac{2\omega_B}{2\pi}$$

$$h_+(t,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}(\pi f_{\rm GW})^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(2\pi f_{\rm GW} t_{\rm ret} + 2\varphi\right)$$

$$h_\times(t,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}(\pi f_{\rm GW})^{2/3}\cos\theta\sin\left(2\pi f_{\rm GW} t_{\rm ret} + 2\varphi\right)$$

Step 2: inspiral orbit

- Emitting GWs **costs energy** → the orbit decreases

$$\frac{dE_{\rm orbit}}{dt} = -P$$

- We assume a **quasi-circular** motion $|\dot{R}| \ll R \cdot \omega_B$ → $\dot{\omega}_B \ll \omega_B^2$

$$E_{\rm orbit} = E_{\rm kin} + E_{\rm pot} = -\frac{G m_1 m_2}{2R}$$

- Energy decreases → R decreases → ω_B increases

→ more loss of energy → **coalescence**

Step 2: inspiral orbit

$$E_{\rm orbit} = -\left(\frac{G^2 M_c^5\, \omega_{\rm GW}^2}{32}\right)^{1/3} \quad \rightarrow \quad \frac{dE_{\rm orbit}}{dt} = -\left(\frac{G^2 M_c^5}{32}\right)^{1/3} \frac{2\,\dot\omega_{\rm GW}}{3\,\omega_{\rm GW}^{1/3}}$$

- We compare this with the **power** carried away by **GWs**

- Energy-momentum tensor $t_{\mu\nu} = \frac{1}{32\pi G}\langle \partial_\mu h_{\alpha\beta}\partial_\nu h^{\alpha\beta}\rangle$ ↘ average over many periods

$$\rho = t^{00} = \frac{1}{16\pi G}\langle \dot h_+^2 + \dot h_\times^2\rangle$$

- The power is the **energy** carried away **per** unit of **time**

$$P = \frac{32}{5G}\left(\frac{G M_c\, \omega_{\rm GW}}{2}\right)^{10/3}$$

Evolution of the frequency

$$\frac{dE_{\rm orbit}}{dt} = -P \quad \rightarrow \quad \dot{f}_{\rm GW} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_{\rm GW}^{11/3}$$

Solution $$f_{\rm GW} = \frac{1}{\pi}(GM_c)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8} \qquad \tau = t - t_c$$

What about the **waveform**?

$$x_0(t) = R(t)\cos\left(\int_{t_0}^{t} dt'\omega_S(t')\right) \qquad y_0(t) = R(t)\sin\left(\int_{t_0}^{t} dt'\omega_S(t')\right)$$

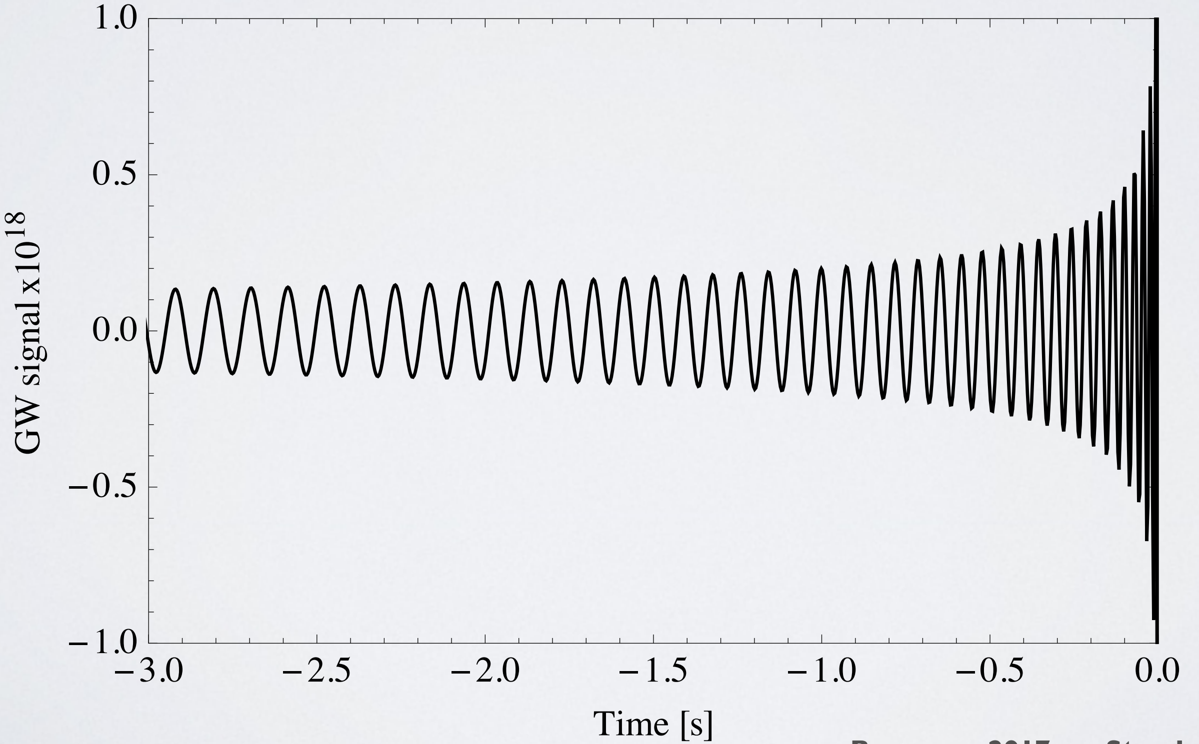
We define the **phase** $$\phi(t) \equiv \int_{t_0}^{t} dt'\omega_{\rm GW}(t') = \int_{t_0}^{t} dt' 2\omega_S(t')$$

Wave-form

$$h_+(t,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}\big(\pi f_{\rm GW}(\tau)\big)^{2/3}\frac{1+\cos^2\theta}{2}\cos\big(\phi(\tau)\big)$$

$$h_\times(\tau,\theta,\varphi) = \frac{4}{r}(GM_c)^{5/3}\big(\pi f_{\rm GW}(\tau)\big)^{2/3}\cos\theta\sin\big(\phi(\tau)\big)$$

$$\phi(\tau) = -2\left(\frac{\tau}{5GM_c}\right)^{5/8} + \phi_c$$



Expanding universe

Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

We use our solution to describe the GW **close** to the **source**

$$r \to r_{\rm phys} = a_S\, r$$ → comoving coordinate

$$h_+(\tau_S) = \frac{4}{a_S r}(GM_c)^{5/3}\big(\pi f_S(\tau_S)\big)^{2/3}\frac{1+\cos^2\theta}{2}\cos\big(\phi_S(\tau_S)\big)$$

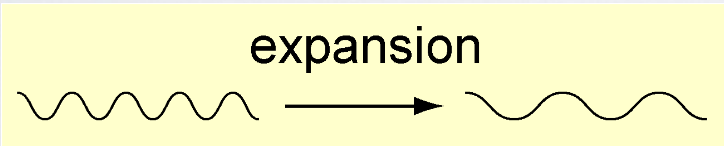
$$h_\times(\tau_S) = \frac{4}{a_S r}(GM_c)^{5/3}\big(\pi f_S(\tau_S)\big)^{2/3}\cos\theta\sin\big(\phi_S(\tau)\big)$$

From the source to the observer

- Propagation in an **expanding** universe

$$\Box \bar{h}_{\mu\nu} = 0 \qquad\qquad \frac{1}{r} \rightarrow \frac{1}{ar}$$

- The wavelength is **stretched** by the expansion



$$f_O = \frac{f_S}{1+z}$$

Redshift: $1 + z = \dfrac{a_O}{a_S}$

From the source to the observer

- **Time intervals** are affected by the expansion $dt_O = (1+z)dt_S$

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3}f_S^{11/3}$$

$$(1+z)\frac{d\big[f_O(1+z)\big]}{dt_O} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3}f_O^{11/3}(1+z)^{11/3}$$

- If the redshift is **constant** during the time of observation

$$\frac{df_O}{dt_O} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c(z))^{5/3}f_O^{11/3} \qquad \mathcal{M}_c = (1+z)M_c$$

$$f_O(\tau_O) = \frac{1}{\pi}(G\mathcal{M}_c)^{-5/8}\left(\frac{5}{256\,\tau_O}\right)^{3/8}$$

From the source to the observer

Phase at the observer $\phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8} + \phi_c$

→ The phase is **constant** along null geodesics $\phi_O(\tau_O) = \phi_S(\tau_S)$

$$k^\mu = \partial^\mu \phi \quad \text{and} \quad k^\mu k_\mu = 0 \quad \rightarrow \quad k^\mu \partial_\mu \phi = 0$$

$$h_+(\tau_O) = \frac{4}{a_O r(1+z)}(G\mathcal{M}_c)^{5/3}\big(\pi f_O(\tau_O)\big)^{2/3}\frac{1+\cos^2\theta}{2}\cos\big(\phi_O(\tau_O)\big)$$

$$h_\times(\tau_O) = \frac{4}{a_O r(1+z)}(G\mathcal{M}_c)^{5/3}\big(\pi f_O(\tau_O)\big)^{2/3}\cos\theta\sin\big(\phi_O(\tau_O)\big)$$

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luminosity distance

$$h_+(\tau_O) = \frac{4}{a_O r(1+z)}(G\mathcal{M}_c)^{5/3}\left(\pi f_O(\tau_O)\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(\phi_O(\tau_O)\right)$$

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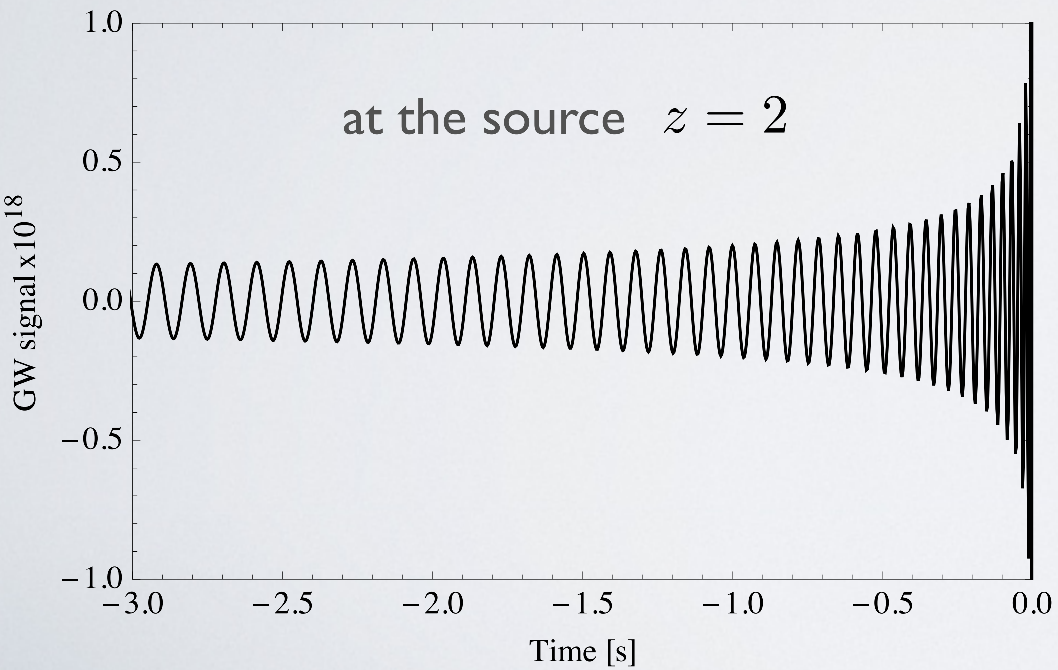
$$h_+(\tau_O) = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}\big(\pi f_O(\tau_O)\big)^{2/3}\frac{1+\cos^2\theta}{2}\cos\big(\phi_O(\tau_O)\big)$$

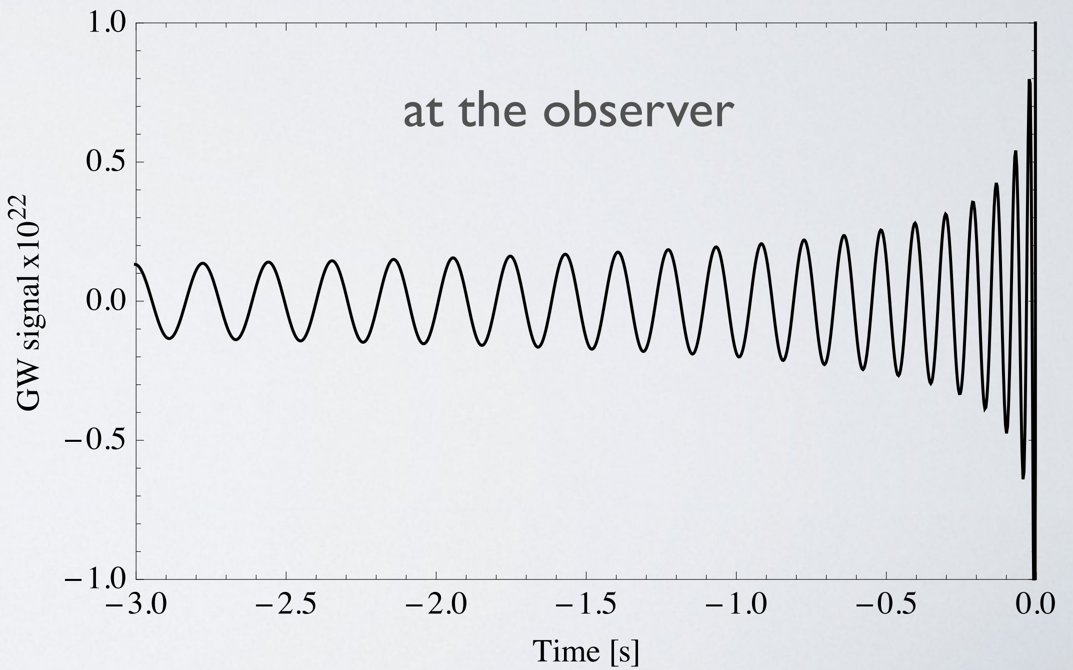
$$h_\times(\tau_O) = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}\big(\pi f_O(\tau_O)\big)^{2/3}\cos\theta\sin\big(\phi_O(\tau_O)\big)$$

Impact of the expansion

- The **amplitude** is diluted with the distance $d_L = \sqrt{\dfrac{L}{4\pi F}}$

- The **frequency** is redshifted.



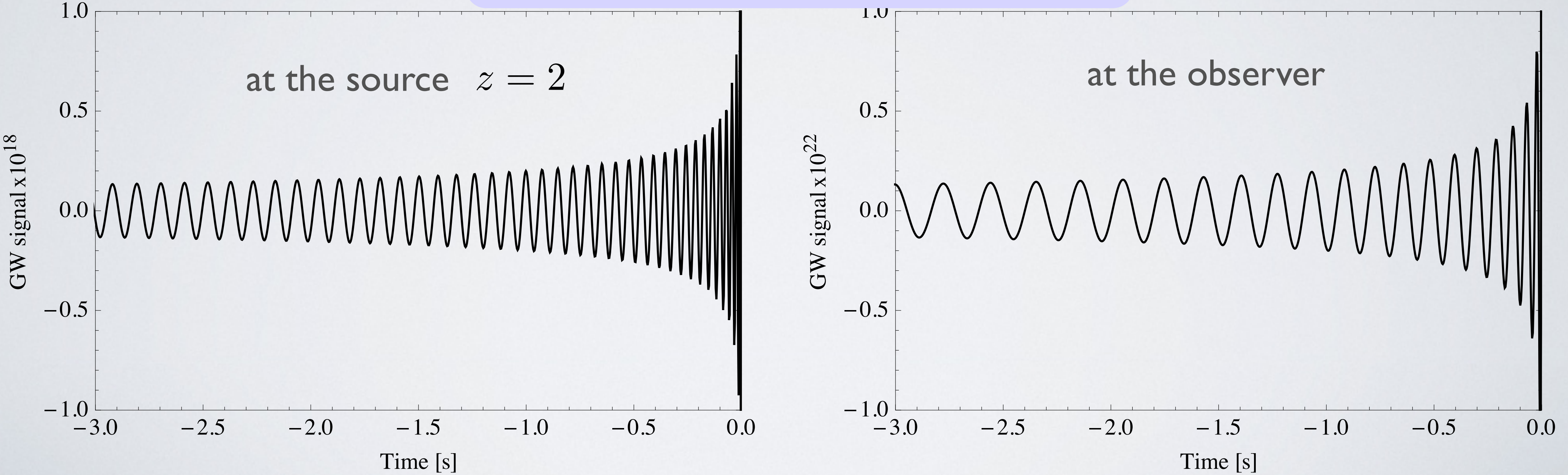


Impact of the expansion

- The **amplitude** is diluted with the distance $d_L = \sqrt{\frac{L}{4\pi F}}$

- The **frequenc**

the change in the waveform is **degenerated** with a change in the **chirp mass**



Information

- What can we **learn** if we measure h_+ and $h_\times$?

- We measure $$\frac{df_O}{dt_O} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c(z))^{5/3} f_O^{11/3}$$

→ measurement of the redshifted **chirp mass** $\mathcal{M}_c$

- Ratio of the amplitude $$\frac{A_+}{A_\times} = \frac{1+\cos^2\theta}{2\cos\theta}$$

→ measurement of the **orientation** of the binary

$$h_+(\tau_O) = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}\left(\pi f_O(\tau_O)\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(\phi_O(\tau_O)\right)$$

Information

- What can we **learn** if we measure h_+ and $h_\times$?

- We measure $$\frac{df_O}{dt_O} = \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c(z))^{5/3} f_O^{11/3}$$

→ measurement of the redshifted **chirp mass** $\mathcal{M}_c$

- Ratio of the amplitude $$\frac{A_+}{A_\times} = \frac{1+\cos^2\theta}{2\cos\theta}$$

We can measure directly the **luminosity distance**

$$h_+(\tau_O) = \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}\left(\pi f_O(\tau_O)\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos\left(\phi_O(\tau_O)\right)$$

Standard sirens

- Why the name: standard sirens?

- In (bad) analogy with supernovae

- Supernovae type Ia emit the same energy when exploding

  we know their luminosity

  from the flux we infer $$d_L = \sqrt{\frac{L}{4\pi F}}$$

- Standard candles $=$ objects that emit same luminosity

Standard sirens

- By analogy we say: with GWs we can measure directly the luminosity distance → **standard sirens**

- Not very good analogy because binary systems **do not** all emit the **same energy**: h_+ and $h_\times$ depend on the system.

- We have enough information from the two polarisations and the waveform to **measure** the **distance**.

- **Advantage**: we do not rely on similarity between objects.

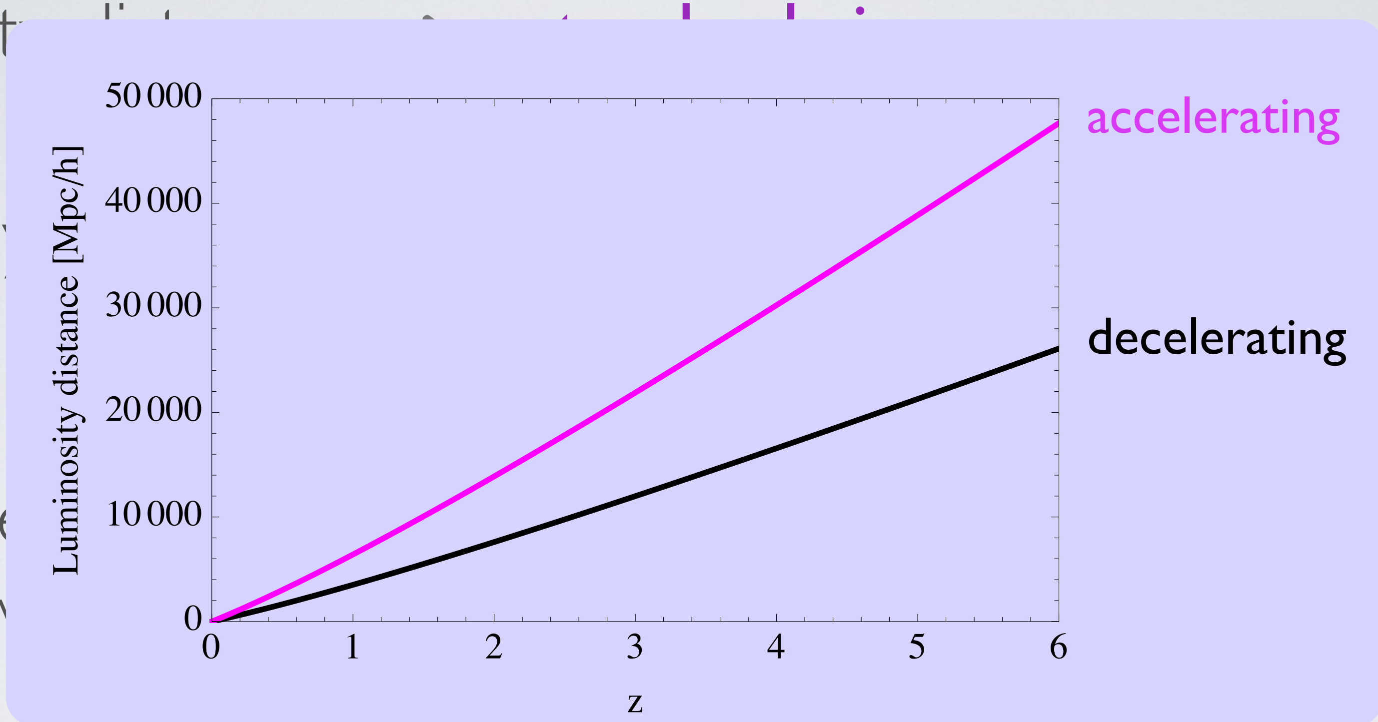
- **Problem**: we do not have a measurement of the redshift.

Standard sirens

- By analogy we say: with GWs we can measure directly the luminosit

- Not ver **not** all emit the ystem.

- We have risations and the



- **Advantage**: we do not rely on similarity between objects.

- **Problem**: we do not have a measurement of the redshift.

Electro-magnetic counterpart

- To do **cosmology** we need $d_L(z)$

- With multiple detectors we can **locate** the position of the source (with some uncertainty).

- We can try and see if the system **emits** something in the **optical.**

  See lecture by Alexis Coleiro

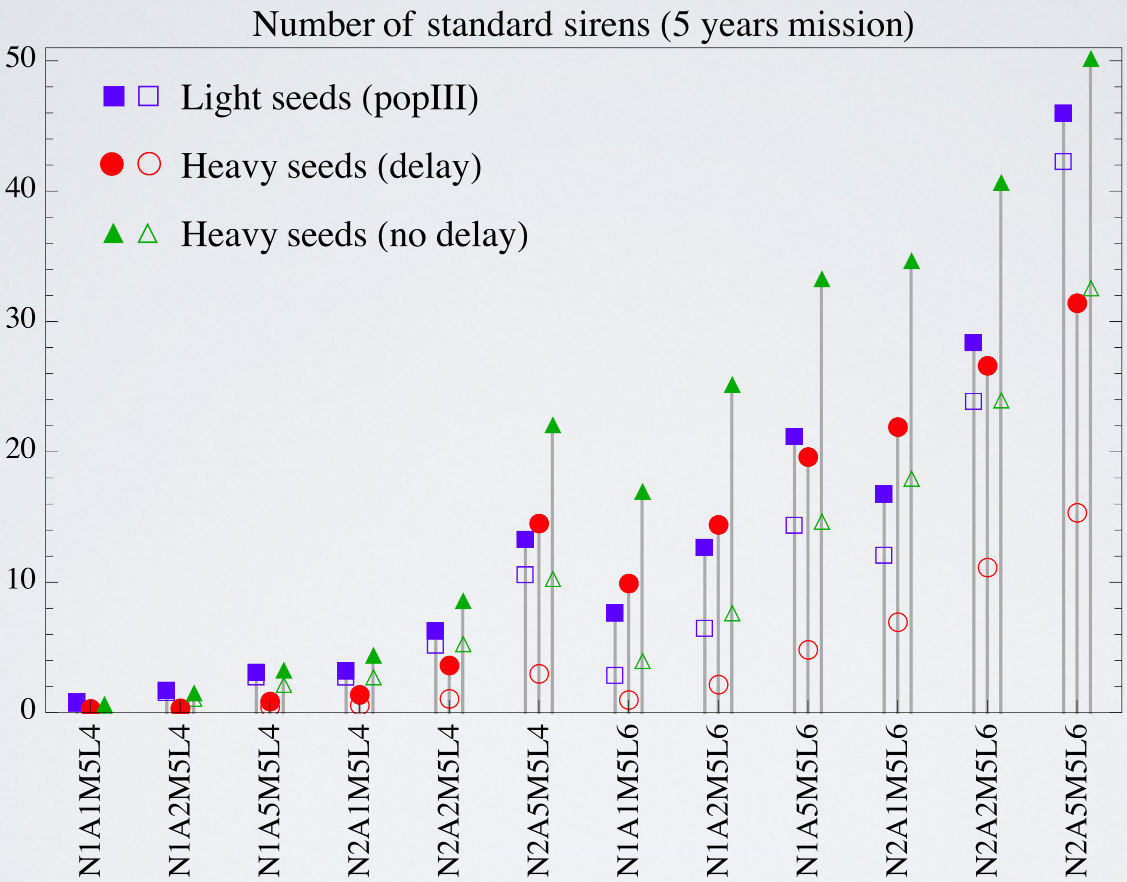
- We can **use cosmology** to infer the redshift from the distance, use this to determine in which galaxy the binary is and then measure the redshift of the galaxy.

What can we learn from the luminosity distance?

Black Board

Number of standard sirens with electro-magnetic counterpart

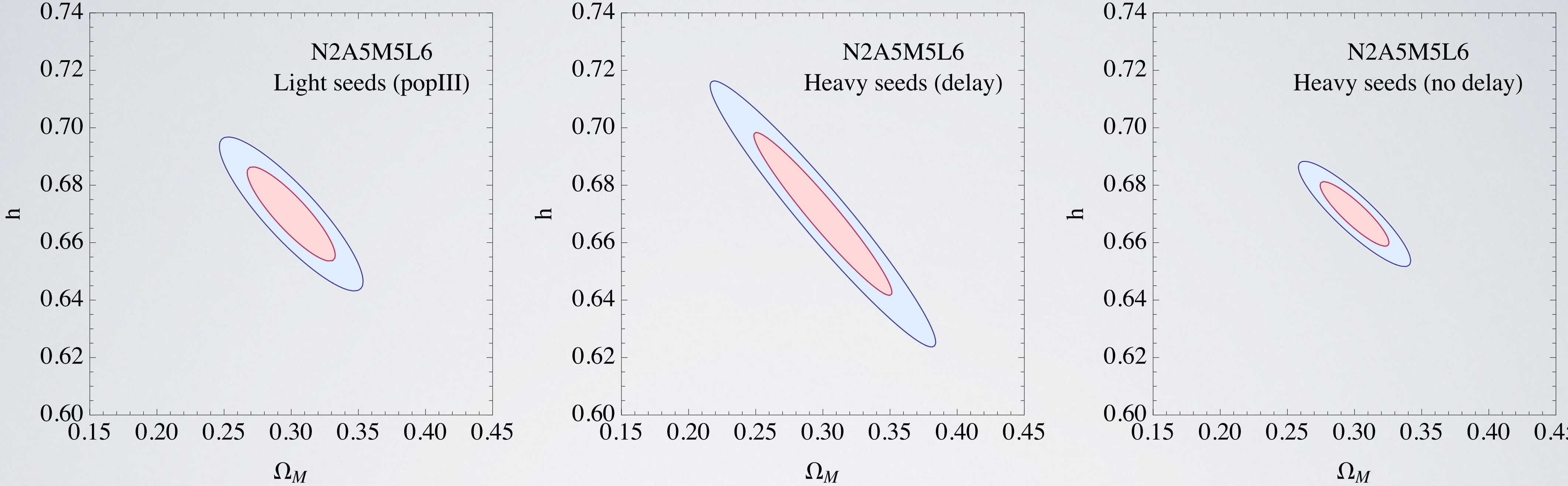
See lectures by Alberto Sesana



Tamanini et al., arXiv:1601.07112

Constraints on the amount of dark energy

Tamanini et al., arXiv:1601.07112



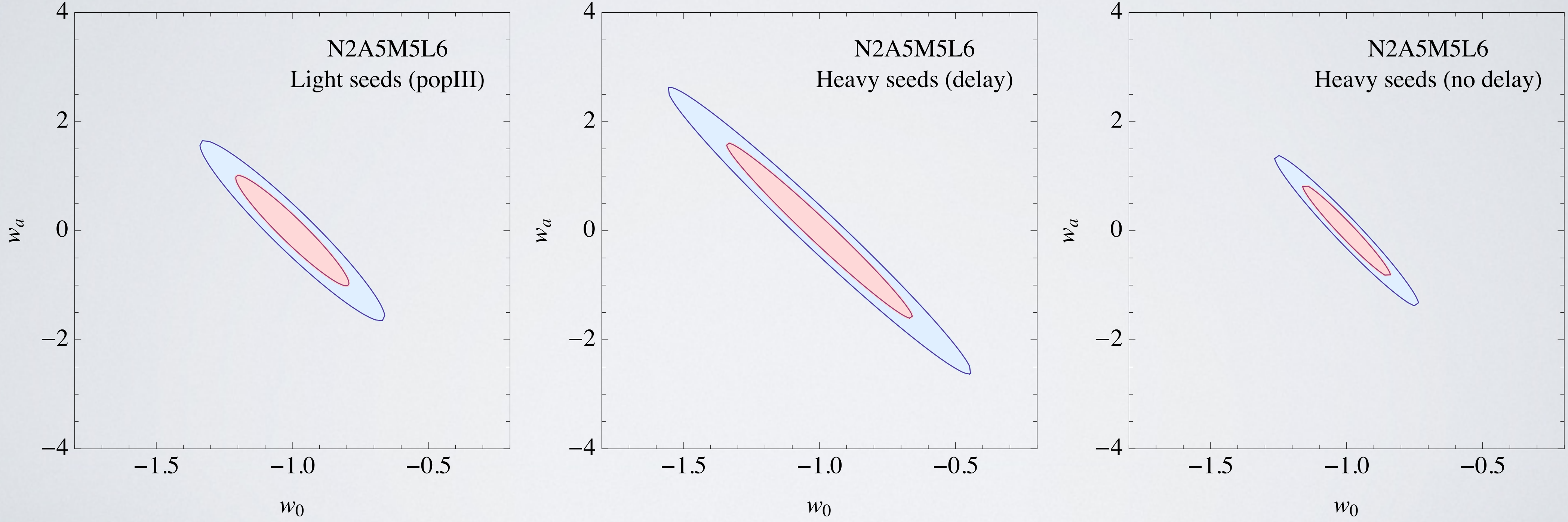
$w = -1$ $\qquad$ $\Omega_m + \Omega_\Lambda = 1$

Measurement of the **Hubble constant** $1\,\%$

Constraints on the equation of state

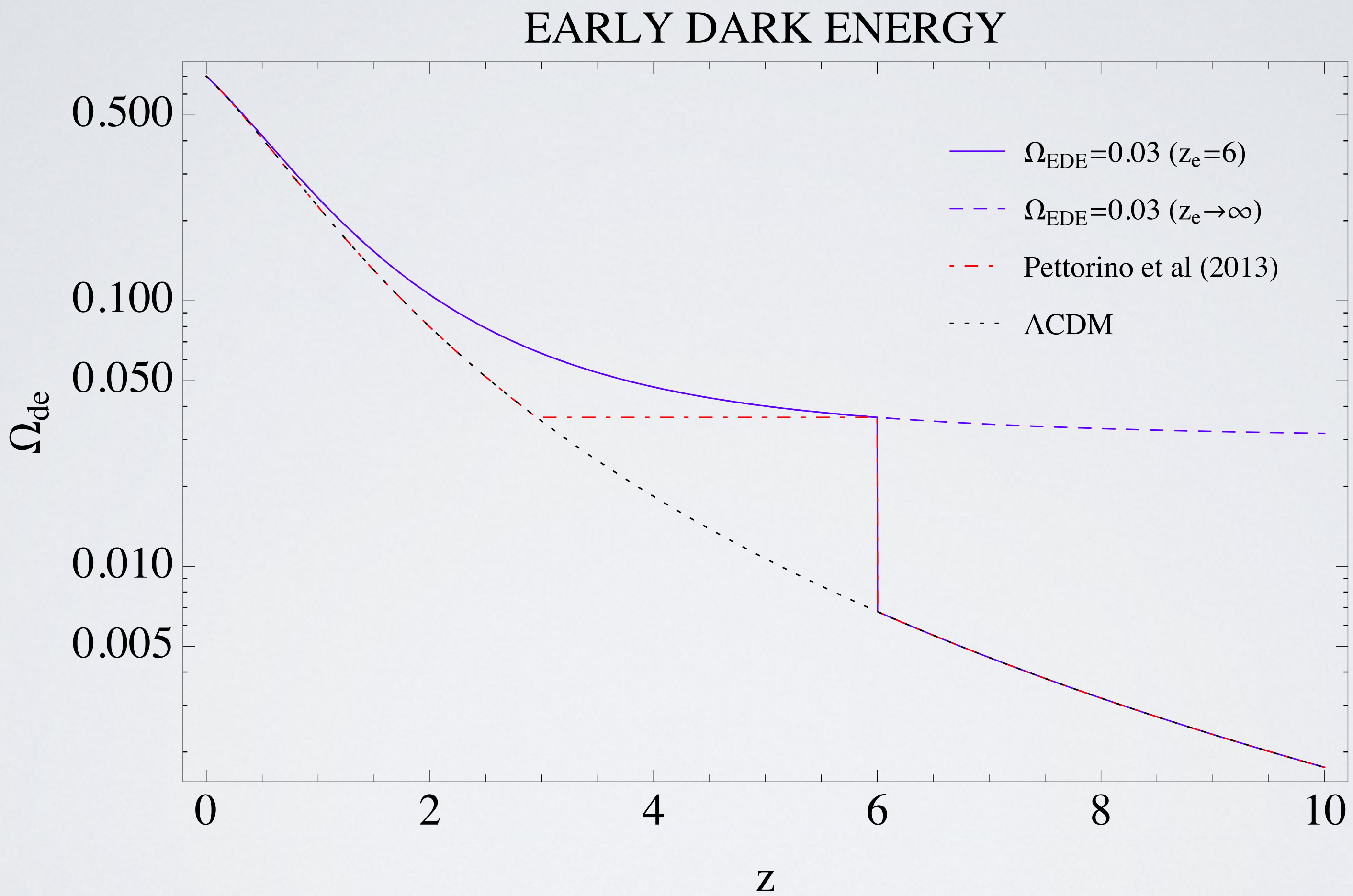
$$w = w_0 + w_a \frac{z}{1+z}$$

Tamanini et al., arXiv:1601.07112



Caprini and Tamanini, arXiv:1607.08755

Early dark energy



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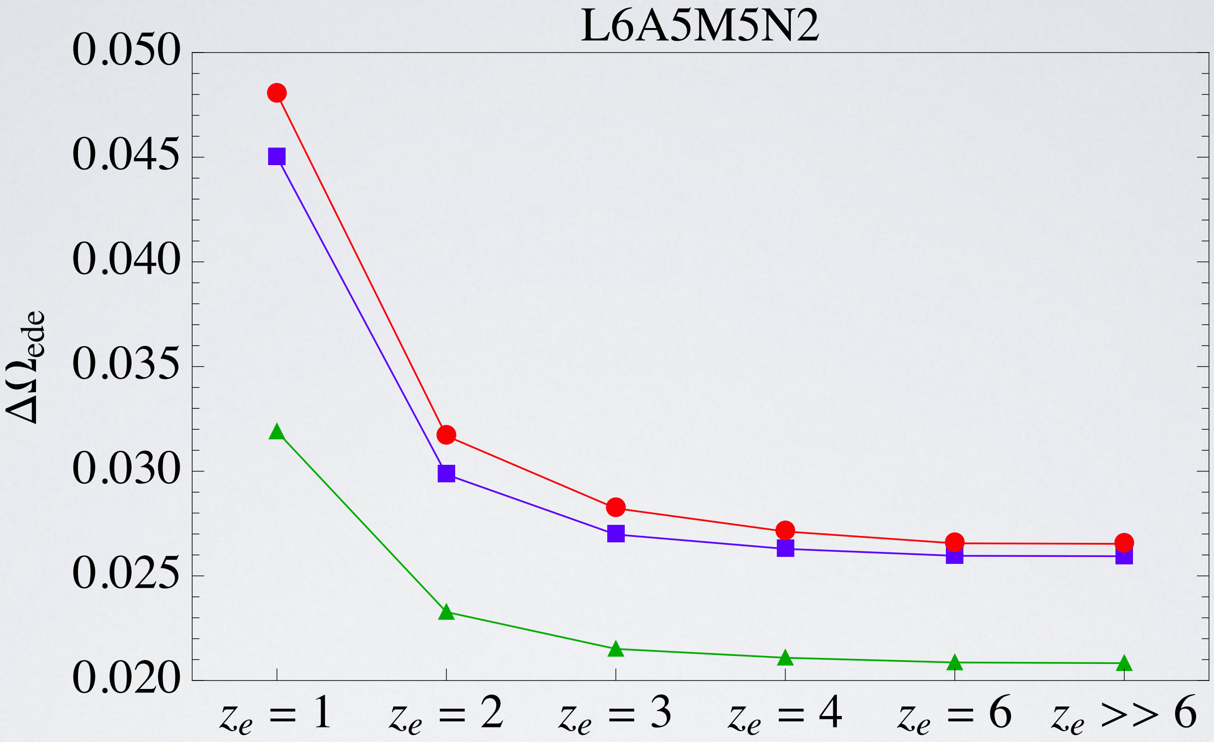
Early dark energy



For $z_e < 10$ standard sirens give better constraints than the CMB.