Propagation of gravitational waves in a inhomogeneous universe

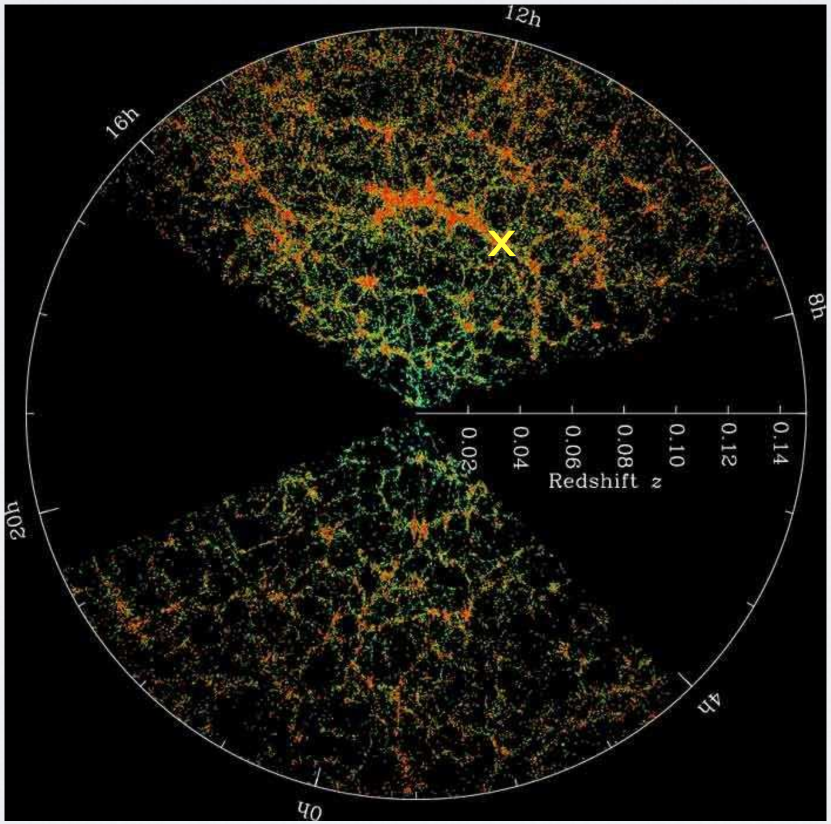
Perturbed universe

- Until now we have assumed a FLRW universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

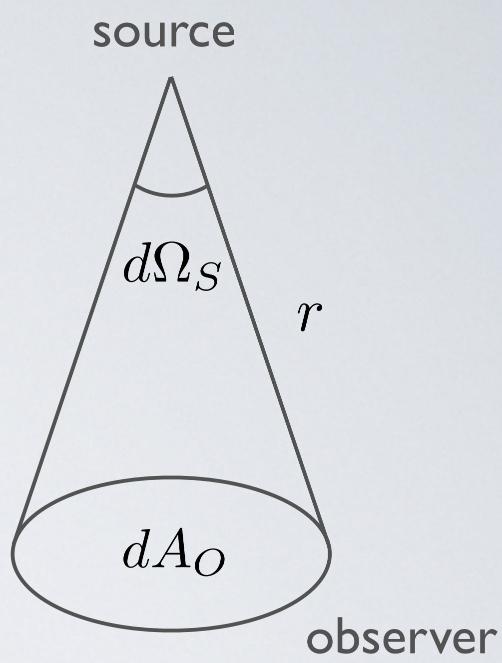
- In reality matter is **inhomogeneously** distributed

Credit: M. Blanton, SDSS



Perturbations to the luminosity distance

$$d_L = \sqrt{\frac{L}{4\pi F}}$$ depends on:



- **Energy** of GW $$\frac{E_S}{E_O} = 1 + z$$
- **Time** interval $$\frac{dt_O}{dt_S} = 1 + z$$
- The relation between **surface** and **angle** $$\frac{dA_O}{d\Omega_S}$$

$$\rightarrow \quad d_L = (1+z)\sqrt{\frac{dA_O}{d\Omega_S}}$$

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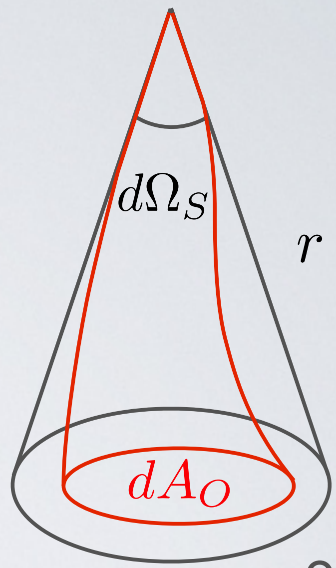

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$$\rightarrow \quad d_L = (1+z)\sqrt{\frac{dA_O}{d\Omega_S}}$$

↙ perturbed ↘ perturbed

Redshift perturbations

Surface perturbations

Black Board

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

$$d_L(z_S,\mathbf{n}) = \chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S}d\chi\frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi)$$
$$+\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\mathbf{v}_S\cdot\mathbf{n}-\Phi_S-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\Psi_S$$
$$+\frac{1}{\chi_S}\int_0^{\chi_S}d\chi(\Phi+\Psi)-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\int_0^{\chi_S}d\chi(\Phi'+\Psi')\Bigg\}$$

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

gravitational lensing

$$d_L(z_S,\mathbf{n}) = \chi_S(1+z_S)\Bigg\{1 - \int_0^{\chi_S} d\chi \frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi)$$
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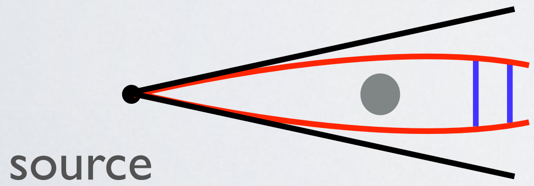

larger flux → smaller distance

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

$$d_L(z_S,\mathbf{n}) = \chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S} d\chi \frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi)$$

Doppler

$$+\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\mathbf{v}_S\cdot\mathbf{n} - \Phi_S - \left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\Psi_S$$

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larger energy → smaller distance

Luminosity distance perturbations

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Doppler

more expansion → larger distance

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

$$
\begin{aligned}
d_L(z_S,\mathbf{n}) = &\chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S} d\chi \frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi)\\
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\end{aligned}
$$

gravitational redshift

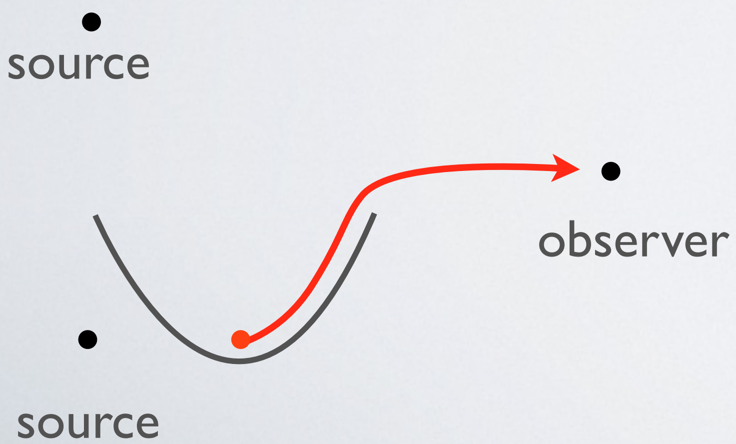
smaller energy → larger distance

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

$$d_L(z_S,\mathbf{n}) = \chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S}d\chi\frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi) + \left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\mathbf{v}_S\cdot\mathbf{n}-\Phi_S-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\Psi_S + \frac{1}{\chi_S}\int_0^{\chi_S}d\chi(\Phi+\Psi)-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\int_0^{\chi_S}d\chi(\Phi'+\Psi')\Bigg\}$$

gravitational redshift



less expansion → smaller distance

Luminosity distance perturbations

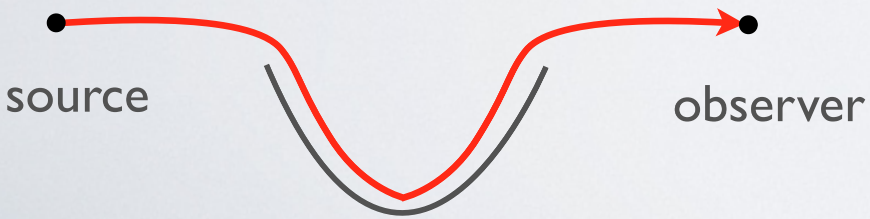
CB, Durrer and Gasparini (2005)

$$d_L(z_S,\mathbf{n}) = \chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S} d\chi \frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi)$$

$$+\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\mathbf{v}_S\cdot\mathbf{n}-\Phi_S-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\Psi_S$$

Shapiro time-delay

$$+\frac{1}{\chi_S}\int_0^{\chi_S} d\chi(\Phi+\Psi)-\left(1-\frac{1}{\mathcal{H}_S\chi_S}\right)\int_0^{\chi_S} d\chi(\Phi'+\Psi')\Bigg\}$$



decrease velocity → smaller distance

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

$$
\begin{aligned}
d_L(z_S,\mathbf{n}) =& \chi_S(1+z_S)\Bigg\{1-\int_0^{\chi_S} d\chi\frac{(\chi_S-\chi)}{2\chi_S\chi}\Delta_\Omega(\Phi+\Psi) \\
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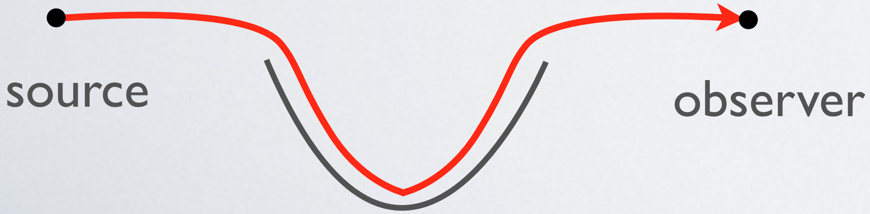
\end{aligned}
$$

Integrated Sachs-Wolfe



increase energy → smaller distance

Luminosity distance perturbations

CB, Durrer and Gasparini (2005)

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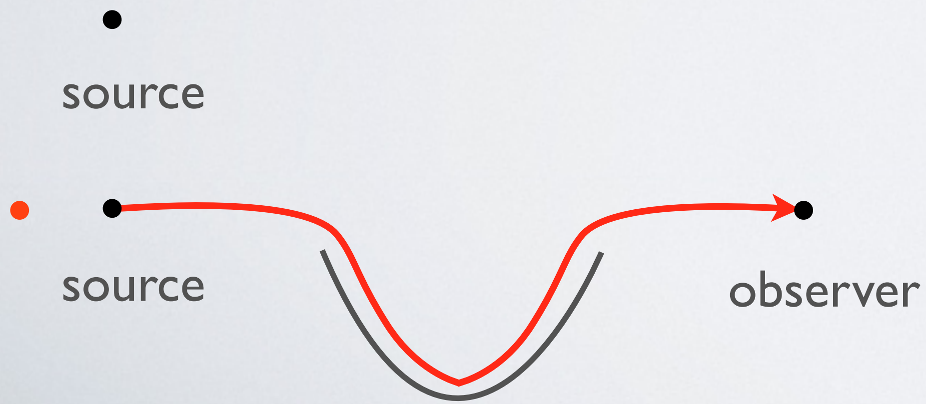


more expansion → larger distance

Importance of the perturbations

- Φ, Ψ and $\mathbf{v}_S$ are **statistical** variables

  → we cannot calculate $d_L(z_S, \mathbf{n})$ for **individual** sources.

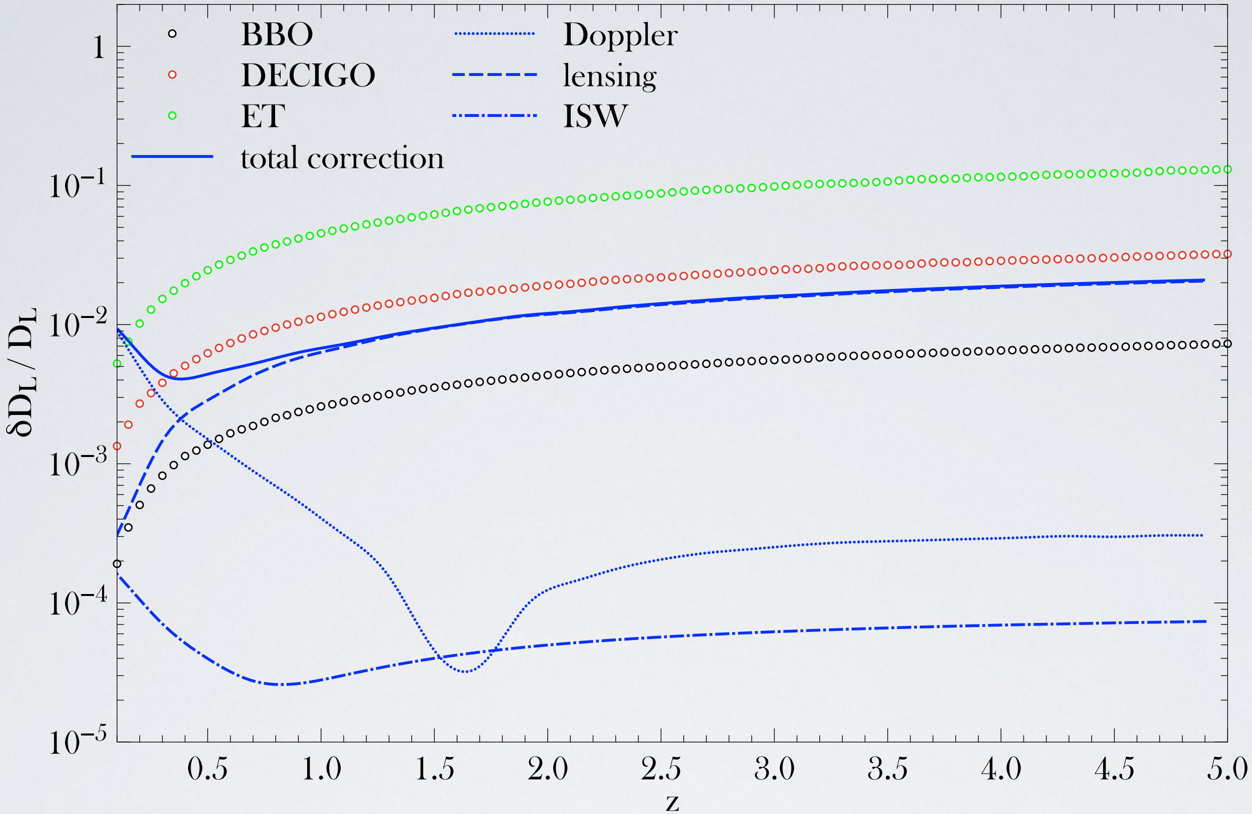
- What is **in average** the impact of the perturbations on d_L ?

  By construction $\langle \Psi \rangle = 0$ → $\langle d_L(z_S, \mathbf{n}) \rangle = (1 + z_S)\chi_S$

- The **variance** $\sigma_{d_L} = \sqrt{\left\langle \Big(d_L(z_S, \mathbf{n}) - \bar{d}_L(z_S) \Big)^2 \right\rangle}$

  tells us how far for the mean one measurement can be.

Importance of the perturbations



Bertacca et al., 1702.01750

Holz and Hughes, 2005

- With **non-linearities**, the lensing generates $5-10\%$ corrections.
- This has to be accounted for as **uncertainties** on the measure of the distance. There are proposals to delense the signal.

Angular power spectrum

$$d_L(z_S, \mathbf{n}) = \sum_{\ell m} a_{\ell m}(z_S) Y_{\ell m}(\mathbf{n})$$

$$C_\ell(z_S, z_{S'}) = \langle a_{\ell m}(z_S) a^*_{\ell m}(z_{S'}) \rangle$$

- The angular power spectrum tells us the amplitude of the perturbations on a **scale** $\theta \sim \frac{\pi}{\ell}$

- **Maximum** ℓ given by precision on the **localisation** of the source

10 degrees $\ell \sim 20$

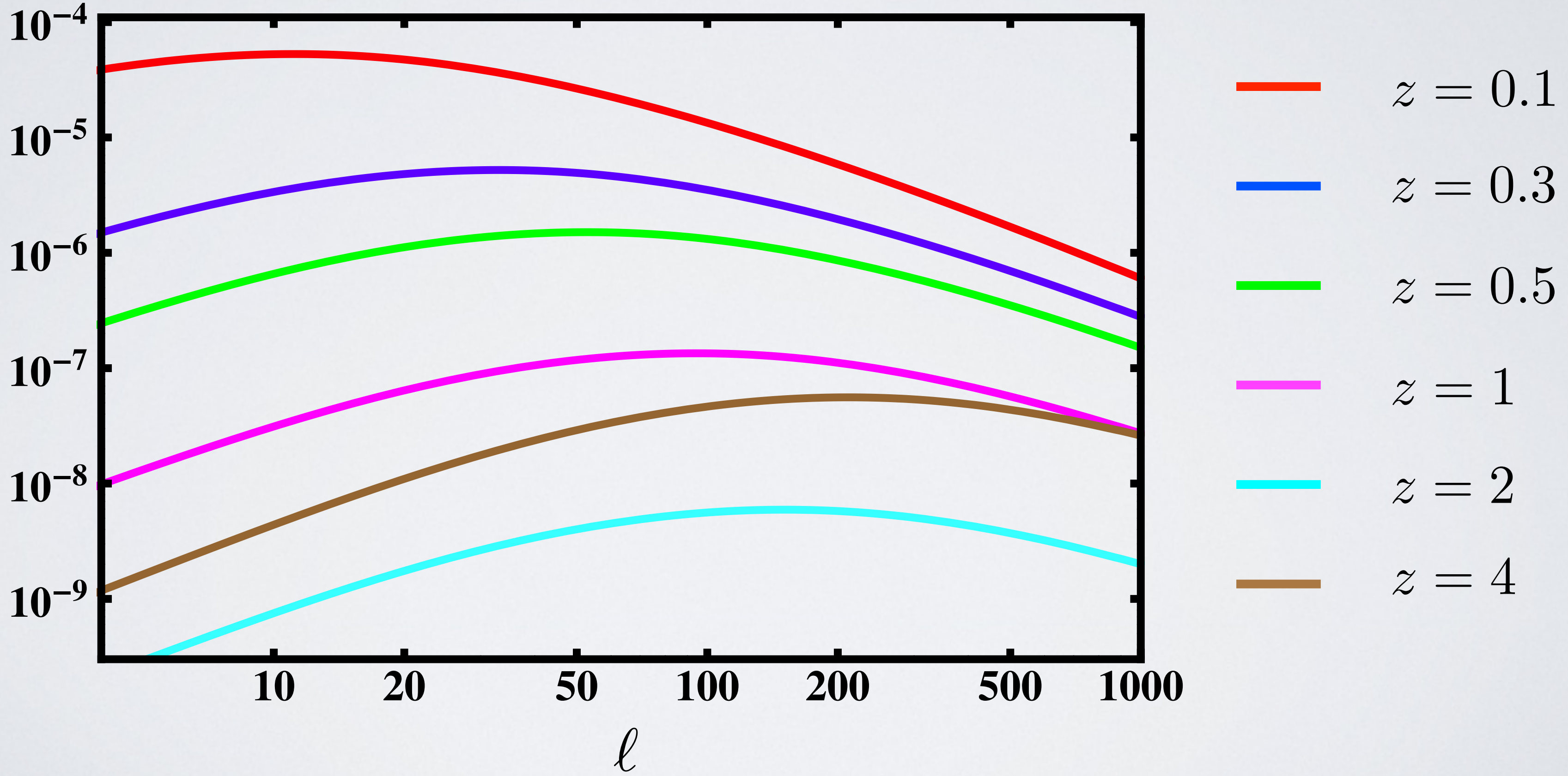
1 degree $\ell \sim 200$

Peculiar velocities on the luminosity distance

$$\frac{\delta d_L}{d_L} = \left(1 - \frac{1}{\mathcal{H}_S \chi_S}\right) \mathbf{v}_S \cdot \mathbf{n}$$

$$\frac{\ell(\ell+1)}{2\pi} C_\ell^{(d_L)}$$

CB, Durrer and Gasparini (2005)



Lensing on the luminosity distance

$$\frac{\delta d_L}{d_L} = -\int_0^{\chi_S} d\chi \, \frac{(\chi_S - \chi)}{2\chi_S \chi} \Delta_\Omega(\Phi + \Psi)$$

CB, Durrer and Gasparini (2005)