Impact of inhomogeneities on the waveform

Perturbed waveform

$$\frac{df_S}{dt_S} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_S^{11/3}$$

- The observed **frequency** is **redshifted** $f_O = \dfrac{f_S}{1+z}$

- **Time intervals** are redshifted $dt_O = (1+z)dt_S$

- These relations are valid in any universe

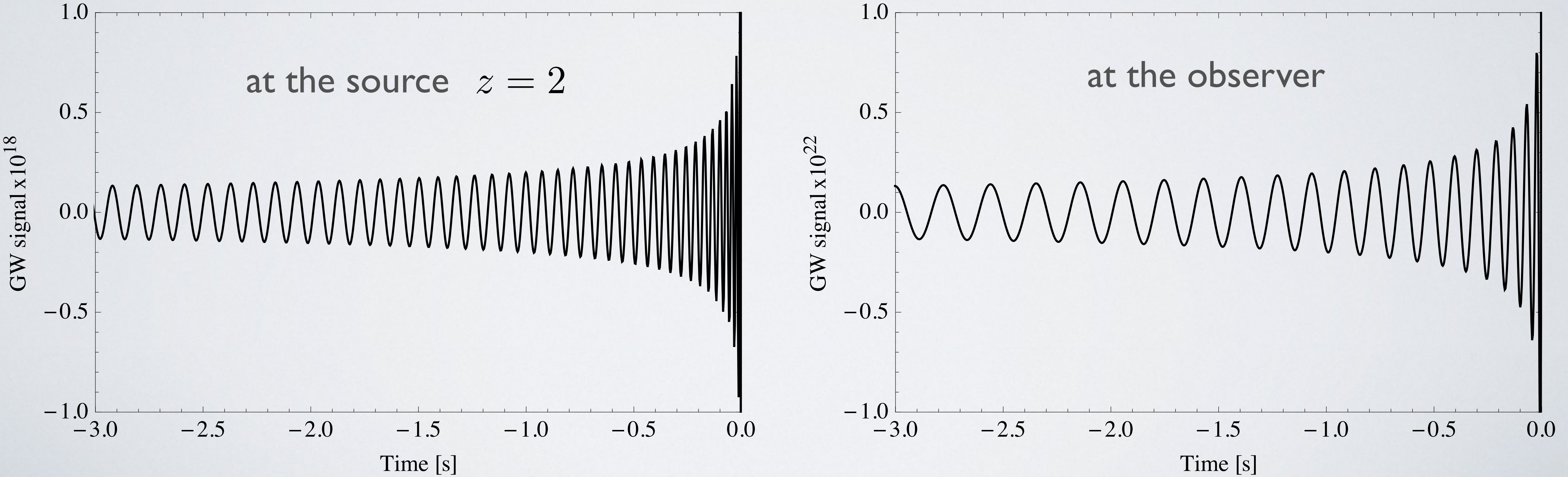
$$(1+z)\frac{d\left[f_O(1+z)\right]}{dt_O} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_O^{11/3}(1+z)^{11/3}$$

Perturbed waveform

Constant redshift $$\phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c}\right)^{5/8} + \phi_c$$

$$\mathcal{M}_c = (1+z)M_c = \frac{a_O}{a_S}\left[1 + \mathbf{v}_S\cdot\mathbf{n} - \mathbf{v}_O\cdot\mathbf{n} + \psi_O - \psi_S - \int_{t_S}^{t_O} dt(\dot\phi + \dot\psi)\right] M_c$$

The perturbations are **degenerated** with a change in the chirp mass



Perturbed waveform

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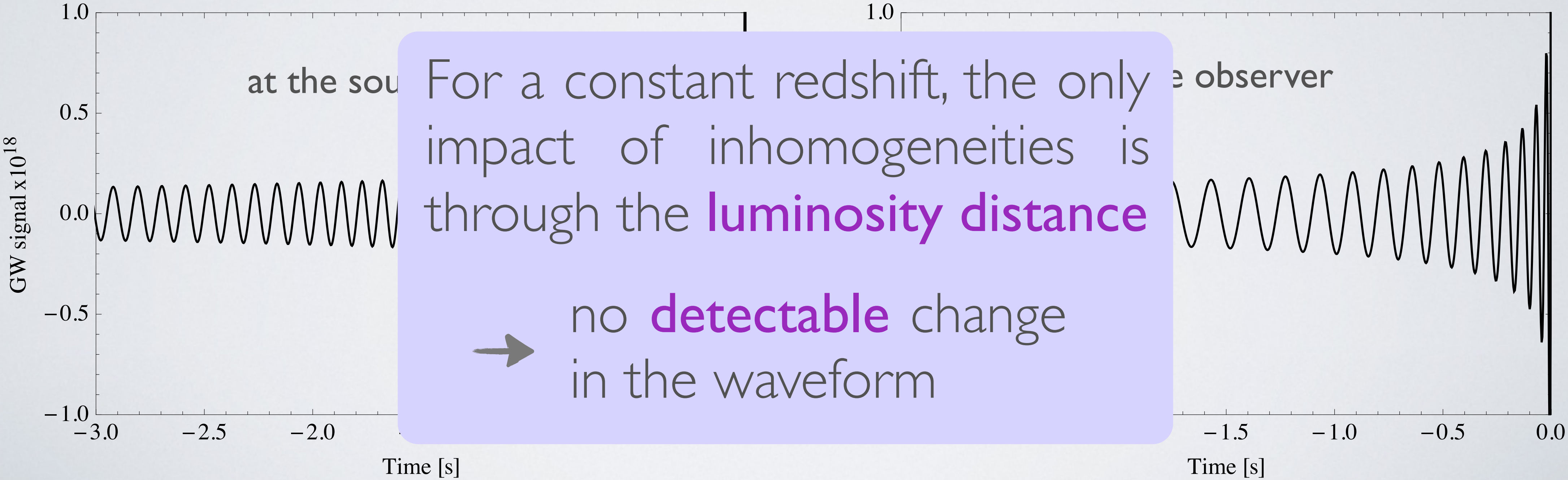
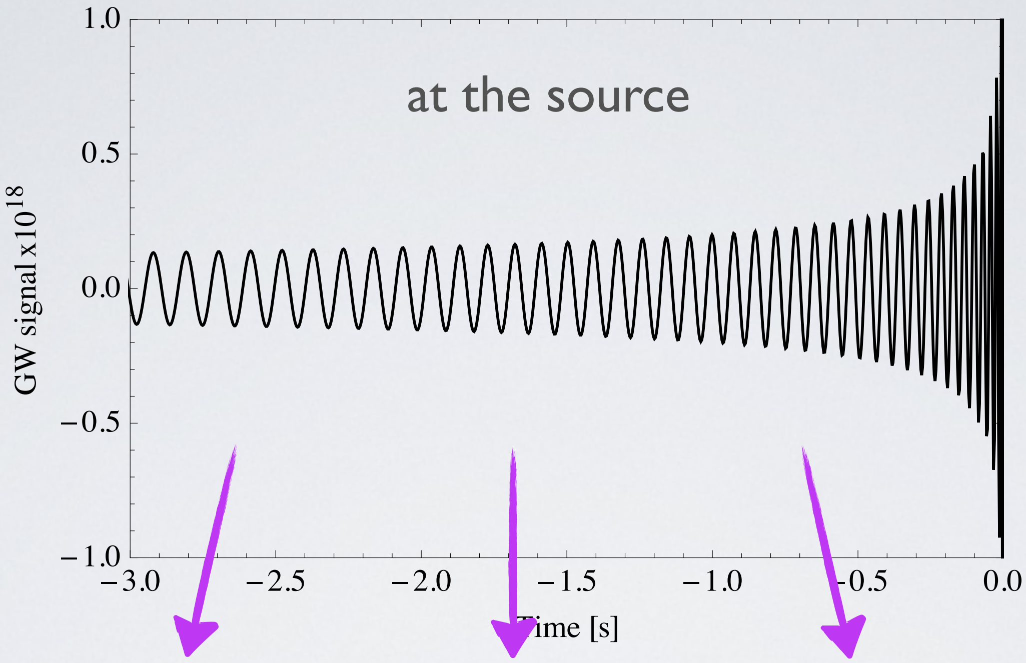


For a constant redshift, the only impact of inhomogeneities is through the **luminosity distance**

→ no **detectable** change in the waveform

Evolving redshift



Different stretches at different times

→ **distortion** in the signal

Evolving redshift

$$(1+z)\frac{d\left[f_O(1+z)\right]}{dt_O} = \frac{96}{5}\pi^{8/3}(GM_c)^{5/3} f_O^{11/3}(1+z)^{11/3}$$

We **expand** the redshift around the time when we start observing

$$z(t'_O) \simeq z(t_O) + \frac{dz}{dt_O}(t_O - t'_O)$$

$$\Phi_O(\tau_O) = -2\left(\frac{\tau_O}{5G\mathcal{M}_c(z)}\right)^{5/8}\left(1 - \frac{5}{8}Y(z)\tau_O\right) + \Phi_c$$

$$Y(z) = \frac{1}{2}\left(H_0 - \frac{H_S \cdot a_S}{a_O}\right)$$

Seto et al. (2001), Nishizawa et al. (2012)

$$+\frac{1}{2}\left(\frac{\dot{\mathbf{v}}_S\cdot\mathbf{n}}{1+z} - \dot{\mathbf{v}}_O\cdot\mathbf{n} + \frac{\dot{\Phi}_S}{1+z} - \dot{\Phi}_O\right)$$

CB, Caprini, Tamanini and Sturani (2016)

Comparison of background and perturbations

acceleration → binary inside the galaxy
acceleration → galaxy inside the cluster
acceleration → cluster with respect to CMB frame

- Circular motion around the galaxy centre $$\dot{v}_S = \frac{v_S^2}{r}$$

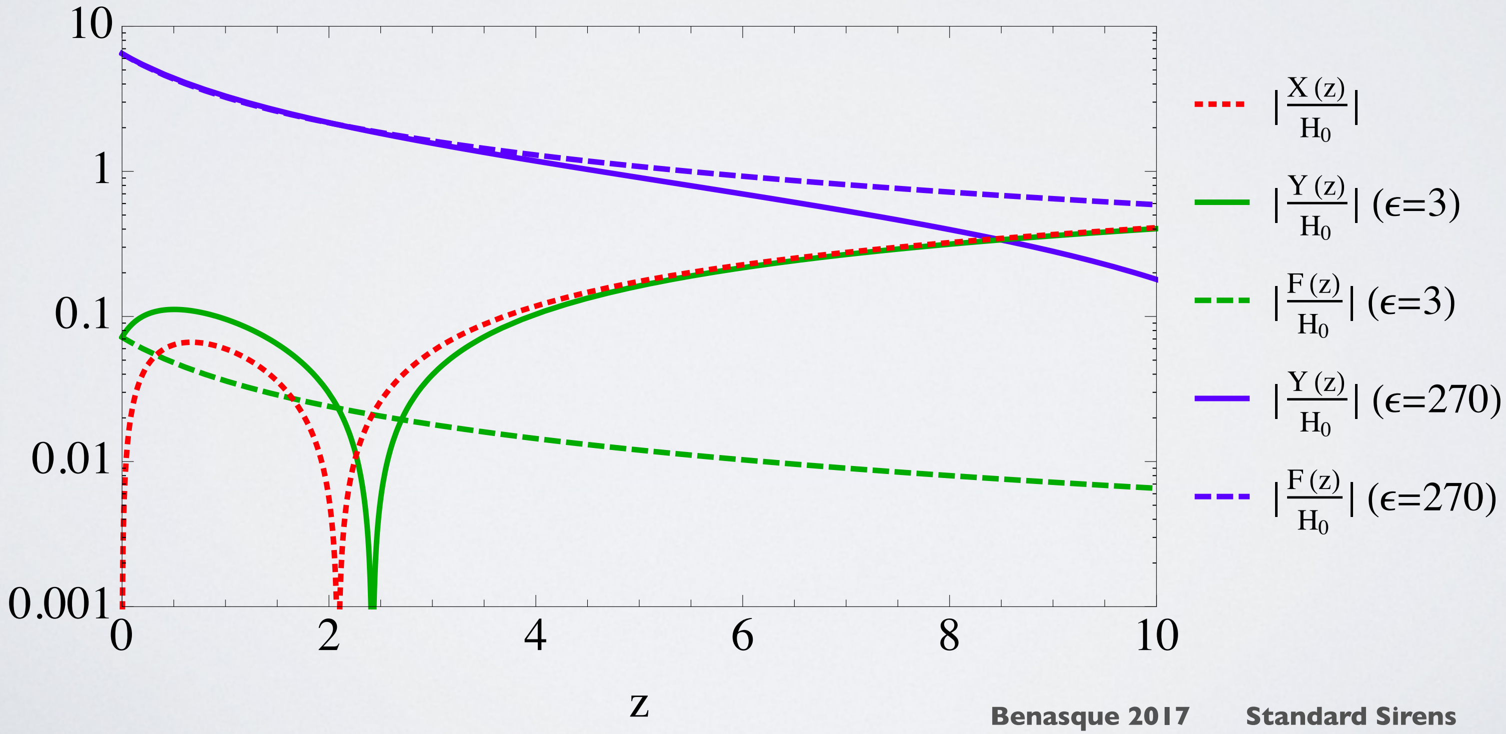
- Viralised motion inside the cluster $$\dot{v}_S = \frac{3}{10}\frac{v_S^2}{r}$$

- Large-scale linear velocity flow $$\dot{v}_S \sim H_0 v_S \sim 10^{-3} H_0$$

Comparison of background and perturbations

- The acceleration depends on the **distance** to the centre and on the **velocity**.
- We introduce a parameter ϵ to quantify the effect

$\epsilon \quad 0 \rightarrow 350 \quad 10^4$ Inayoshi et al (2017)



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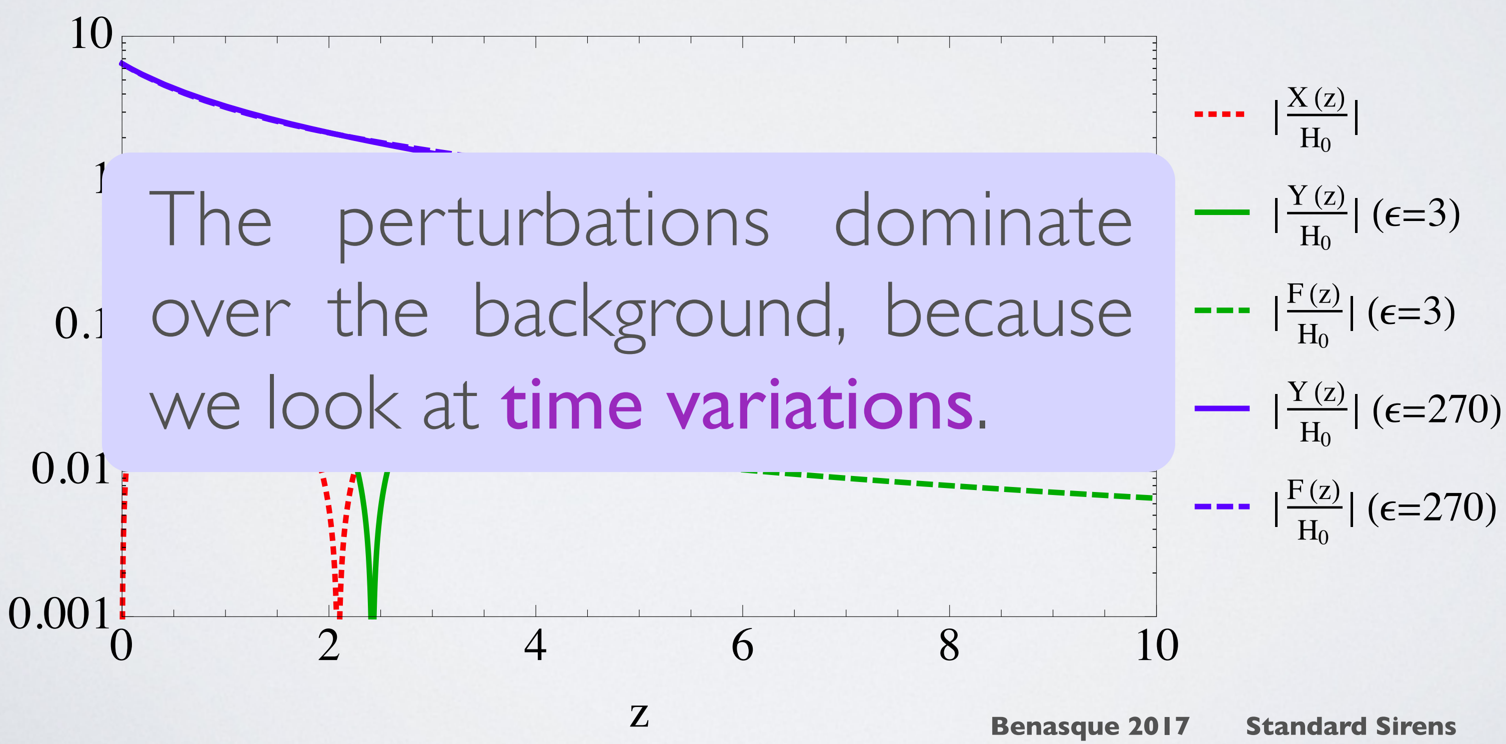
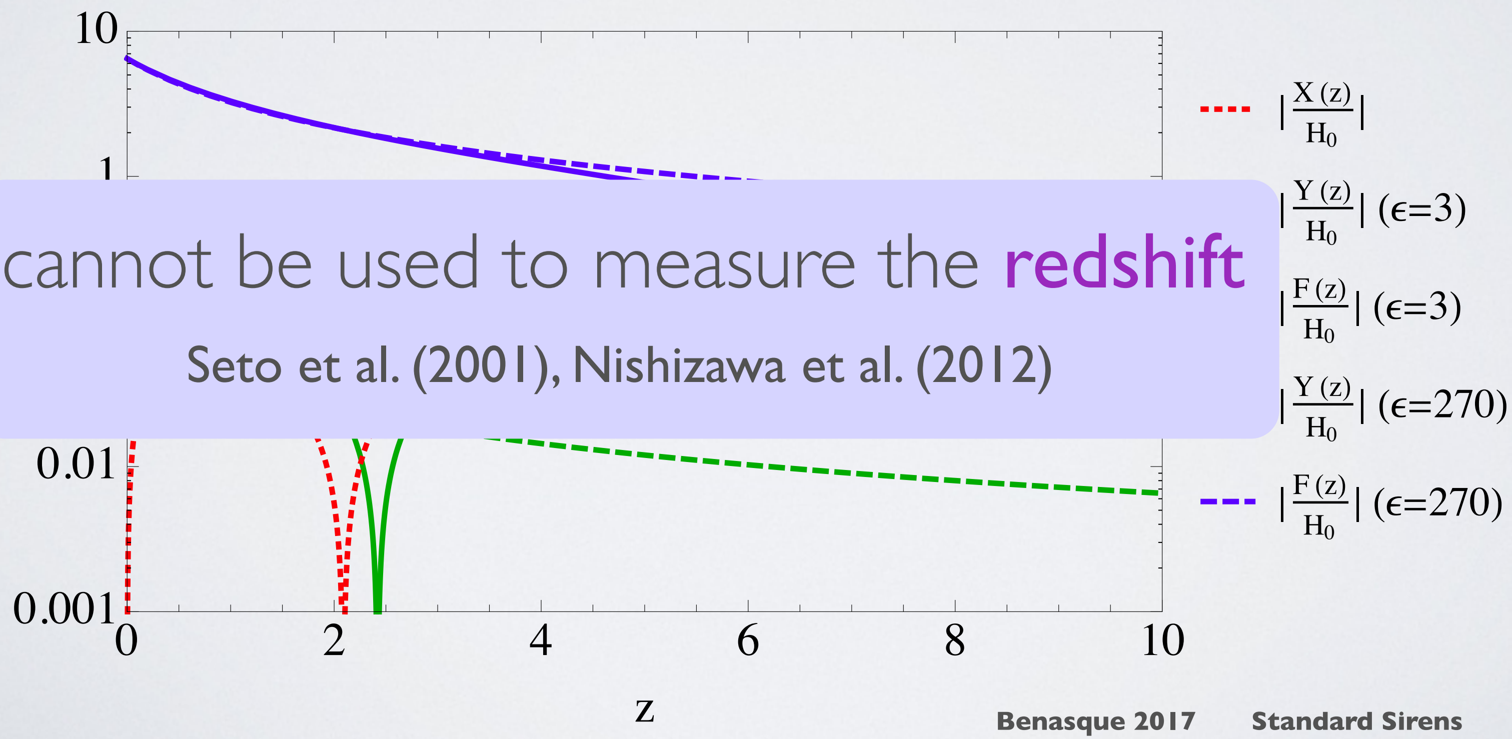


The perturbations dominate over the background, because we look at **time variations**.

Comparison of background and perturbations

- The acceleration depends on the distance to the centre and on the velocity.
- We introduce a parameter ϵ to quantify the effect

$\epsilon \quad 0 \rightarrow 350 \quad 10^4$ Inayoshi et al (2017)



cannot be used to measure the **redshift**

Seto et al. (2001), Nishizawa et al. (2012)

Shift in the phase

- By how much the **phase** has been **shifted** by the effect after a time Δt

$$\Delta\Phi \simeq 0.1\, h\, \frac{Y(z)}{H_0}\left(\frac{50M_\odot}{\mathcal{M}_c(z)}\right)^{\frac{5}{3}}\left(\frac{10^{-3}\mathrm{Hz}}{f_O}\right)^{\frac{5}{3}}\frac{\Delta t}{\mathrm{year}}$$

- Larger effect for **small frequency**: more important far from coalescence (formally a -4PN effect)

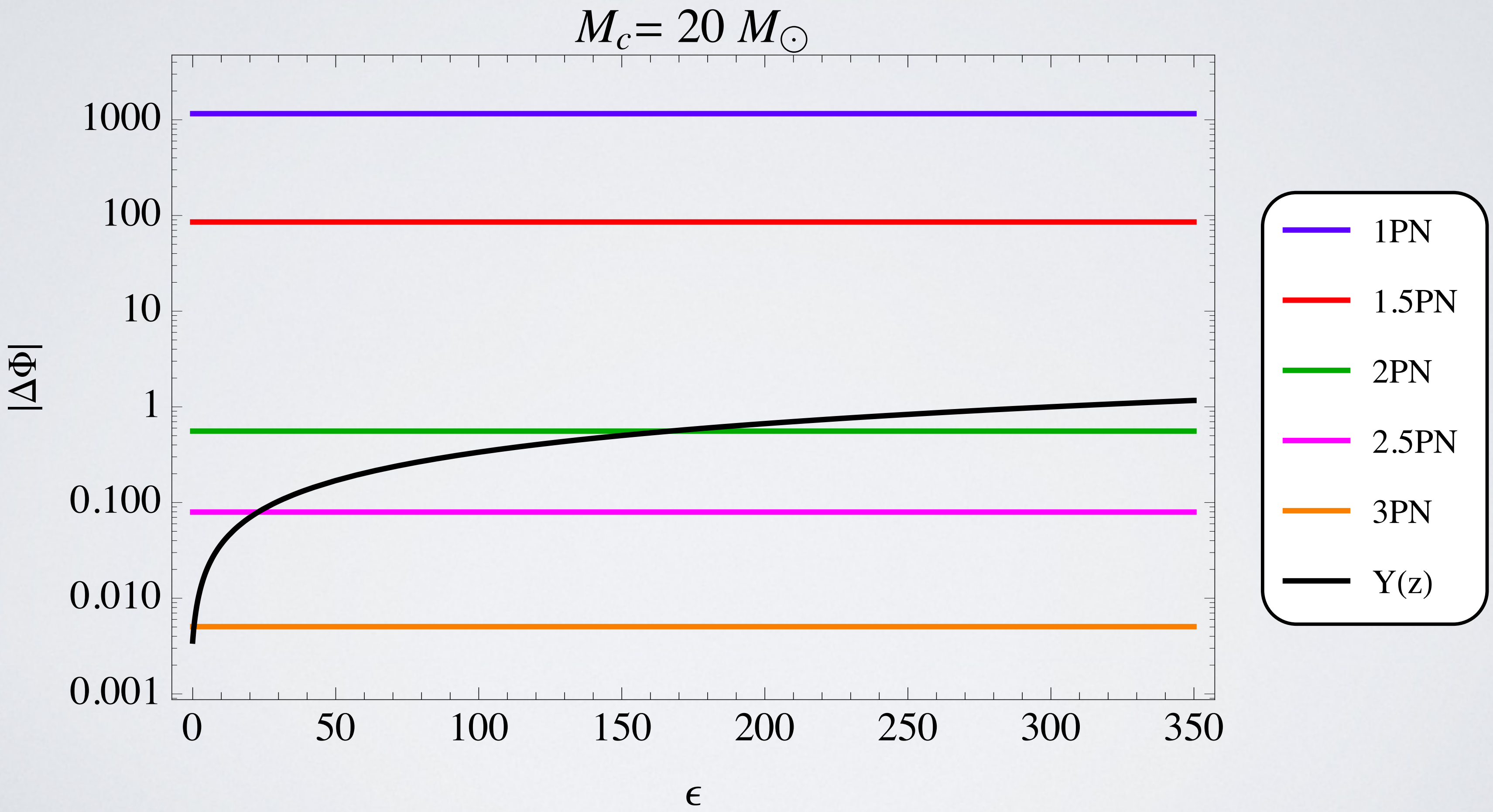
  → not observable by LIGO, but relevant for **LISA**

- Larger effect for **small masses** (more cycles)

- The **longer** we observe, the better.

Shift in the phase

We compare the **shift** in the phase induced by the effect after 5 years of observation with the shift due to **PN effects**



Mismatch

- Do we **miss detections** if we do not include the effect in the template?

- We calculate the **fitting factor** using a Monte Carlo code

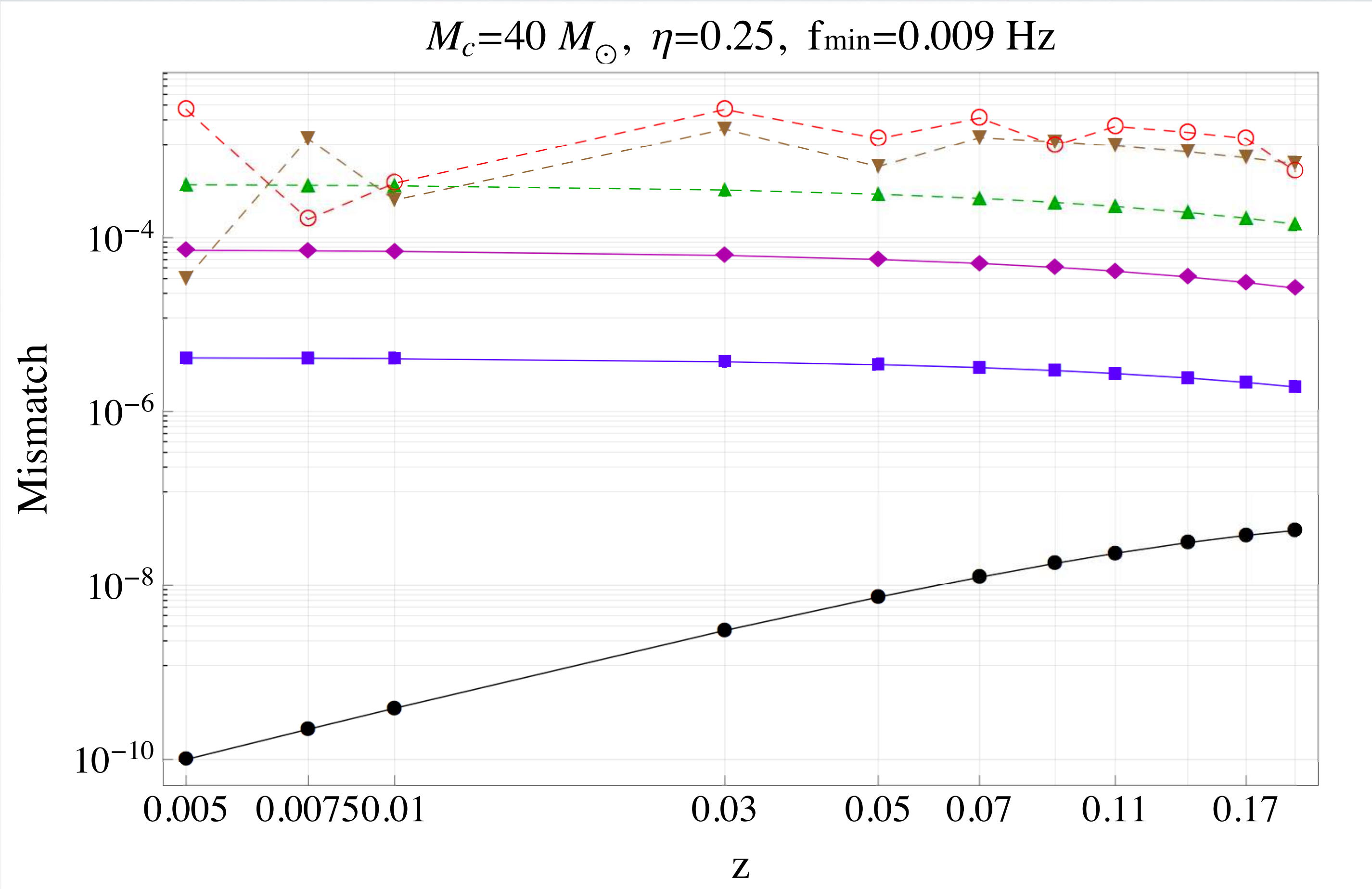
template   signal

$$FF \equiv \underset{\Delta t_c, \Delta\Phi_c, \Delta M_c, \Delta\eta}{\mathrm{Max}} \frac{\langle h_1 | h_2 \rangle}{||h_1||\,||h_2||}$$

- The mismatch is defined as $m \equiv 1 - FF$

Mismatch

ϵ



350

270

120

50

12

0

Loss of detection $\sim 3m$ less than one per mill

Bias in parameters

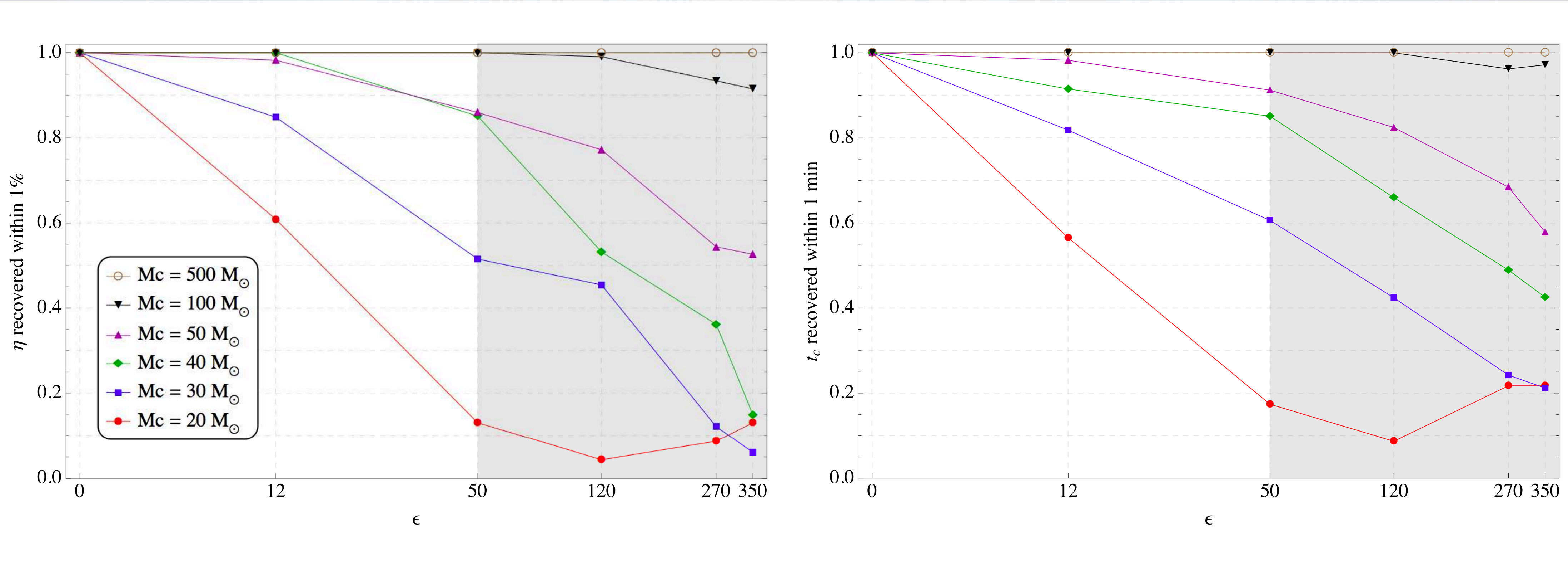
Having a wrong template can affect the determination of the parameters of the binary.

template signal

$$FF \equiv \underset{\Delta t_c, \Delta\Phi_c, \Delta M_c, \Delta\eta}{\mathrm{Max}} \frac{\langle h_1 | h_2 \rangle}{||h_1||\,||h_2||}$$

Bias in the parameters

- symmetric mass ratio $\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$
- time at coalescence t_c



up to a few percent

up to several days

Bias in the parameters

- symmetric mass ratio $\eta = \dfrac{m_1 m_2}{(m_1 + m_2)^2}$
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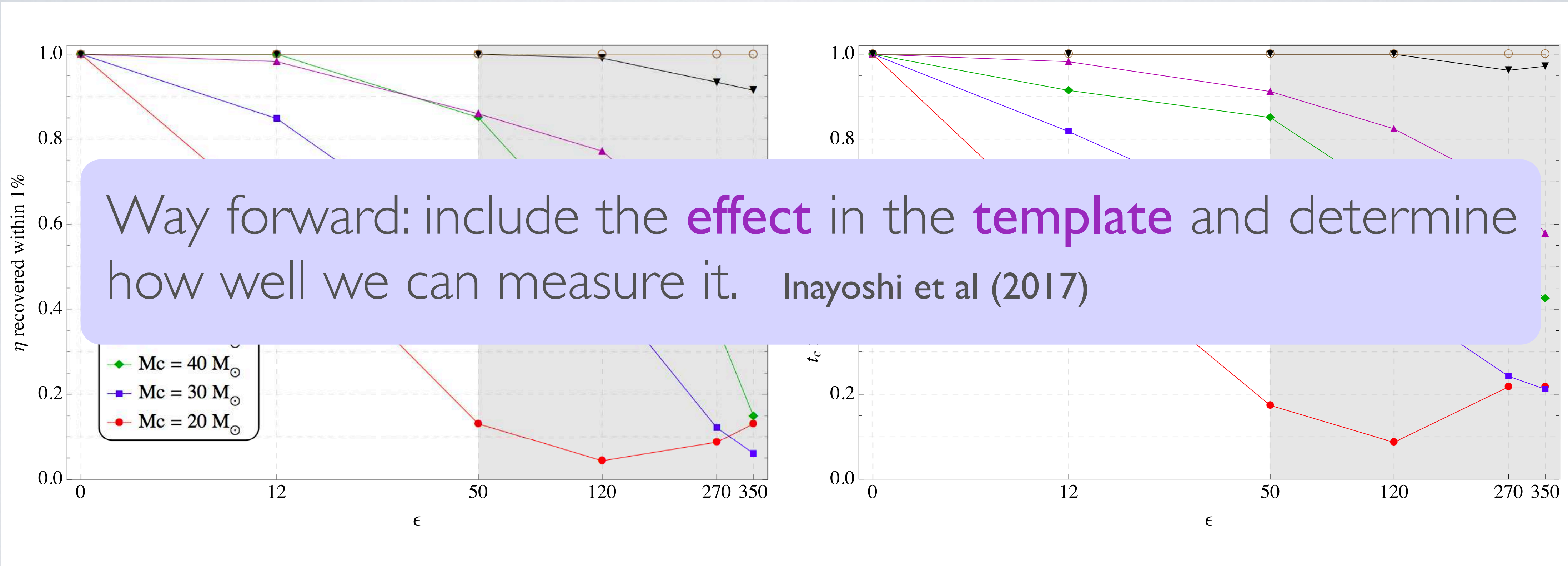

Way forward: include the **effect** in the **template** and determine how well we can measure it. Inayoshi et al (2017)

up to a few percent

up to several days

Conclusion

- Coalescing binaries give a measurement of the **luminosity distance** → standard sirens.

- Assuming a homogeneous and isotropic universe, we can constrain **dark energy**.

- Accounting for **inhomogeneities**, we have **corrections**
  - in the luminosity distance
  - in the waveform

- We can treat those as an additional **noise**, or as a new **signal**.