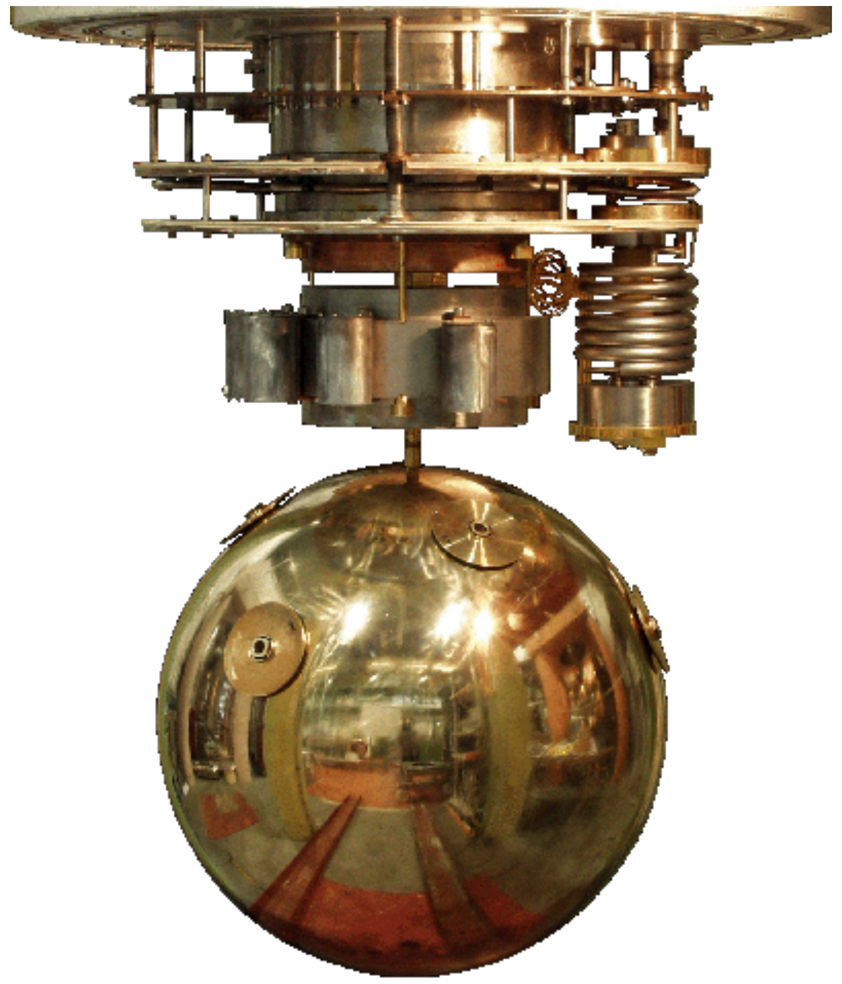
Gravitational Wave Observatories I: History, Status & Future Basic Theory

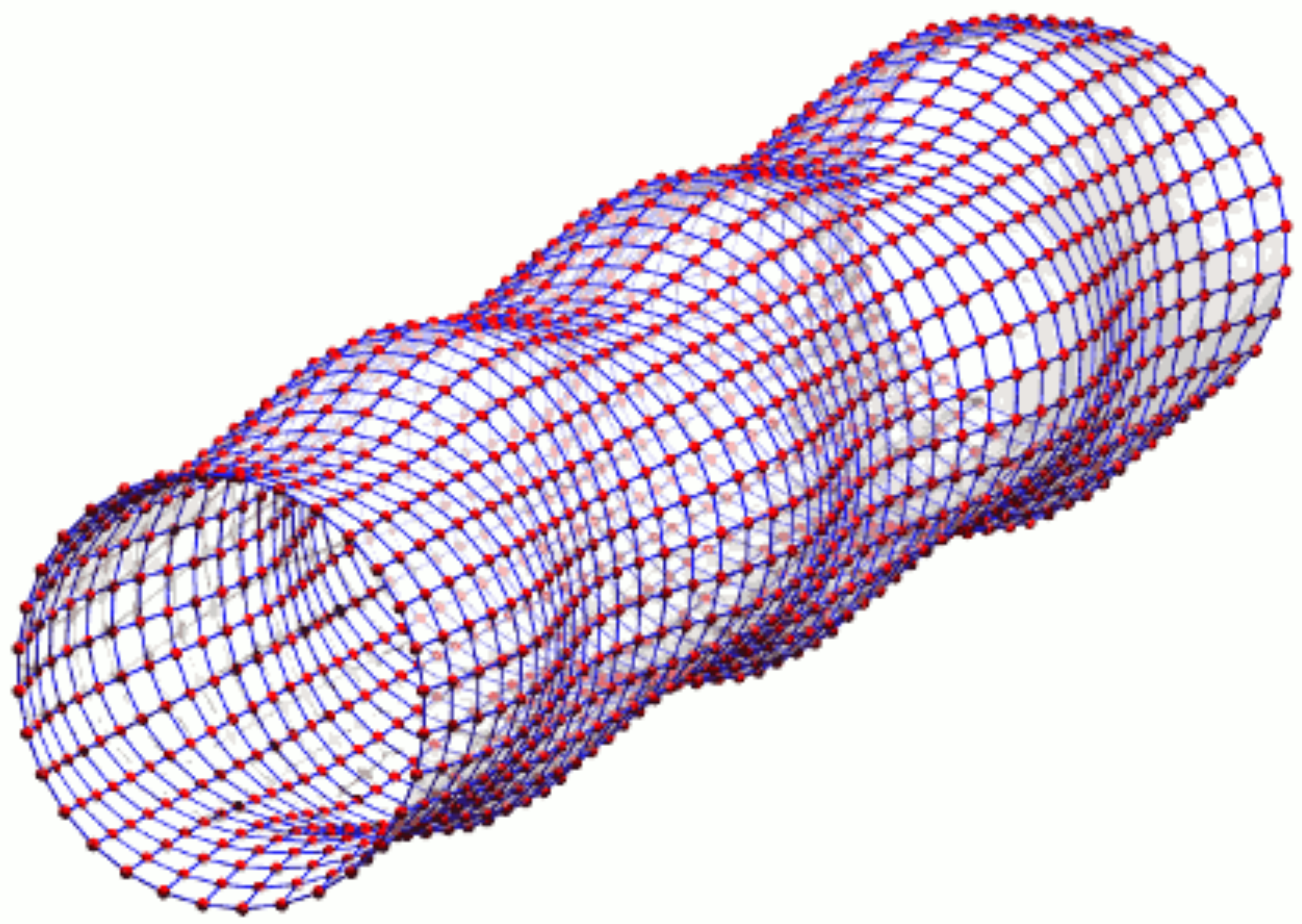
Neil J. Cornish

Gravitational Wave Detection

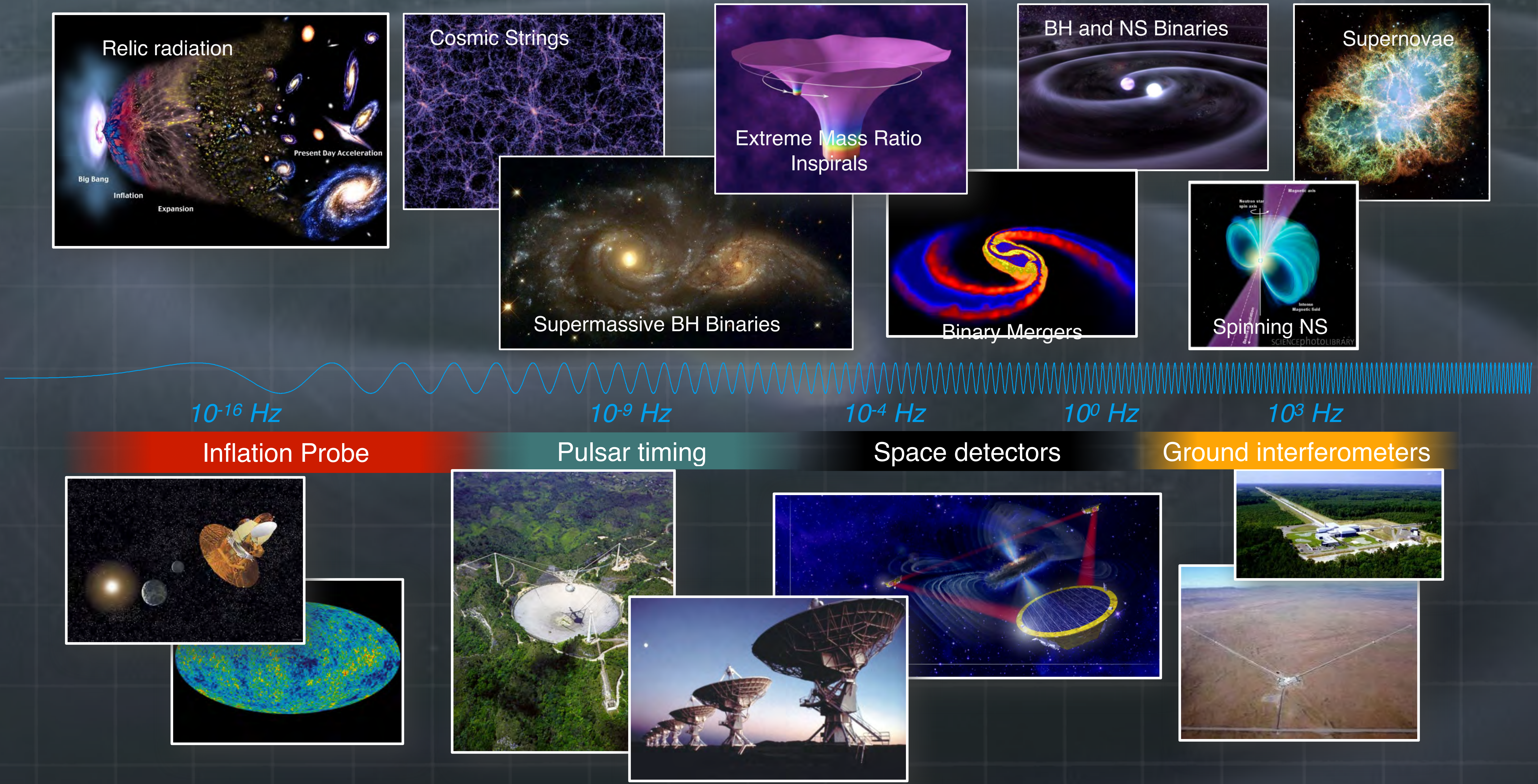
Mechanical/Acoustic

Time of flight





Gravitational Wave Astronomy



Early History

(Saving PTA history, status and future for Friday)

PHYSICAL REVIEW VOLUME 117, NUMBER 1 JANUARY 1, 1960

Detection and Generation of Gravitational Waves*

J. WEBER
University of Maryland, College Park, Maryland
(Received February 9, 1959; revised manuscript received July 20, 1959)

First description of using a mechanical (acoustic) detector

SOVIET PHYSICS JETP VOLUME 16, NUMBER 2 FEBRUARY, 1963

ON THE DETECTION OF LOW FREQUENCY GRAVITATIONAL WAVES

M. E. GERTSENSHTEĬN and V. I. PUSTOVOĬT

First description of using a Michelson interferometer

November 1971 / Vol. 10, No. 11 / APPLIED OPTICS 2495

Photon-Noise-Limited Laser Transducer for Gravitational Antenna

G. E. Moss, L. R. Miller, and R. L. Forward

First experimental tests of a laser interferometer, with input from Chapman and Weiss

Rai Weiss, 1972 design for what became LIGO

QUARTERLY PROGRESS REPORT

No. 105

APRIL 15, 1972

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
RESEARCH LABORATORY OF ELECTRONICS
CAMBRIDGE, MASSACHUSETTS 02139

B. ELECTROMAGNETICALLY COUPLED BROADBAND GRAVITATIONAL ANTENNA

1. Introduction

The prediction of gravitational radiation that travels at the speed of light has been an essential part of every gravitational theory since the discovery of special relativity. In 1918, Einstein,[1] using a weak-field approximation in his very successful geometrical theory of gravity (the general theory of relativity), indicated the form that gravitational waves would take in this theory and demonstrated that systems with time-variant mass quadrupole moments would lose energy by gravitational radiation. It was evident to Einstein that since gravitational radiation is extremely weak, the most likely measurable radiation would come from astronomical sources. For many years the subject of gravitational radiation remained the province of a few dedicated theorists; however, the recent discovery of the pulsars and the pioneering and controversial experiments of Weber[2,3] at the University of Maryland have engendered a new interest in the field.

Weber has reported coincident excitations in two gravitational antennas separated 1000 km. These antennas are high-Q resonant bars tuned to 1.6 kHz. He attributes these excitations to pulses of gravitational radiation emitted by broadband sources concentrated near the center of our galaxy. If Weber's interpretation of these events is correct, there is an enormous flux of gravitational radiation incident on the Earth.

Several research groups throughout the world are attempting to confirm these results with resonant structure gravitational antennas similar to those of Weber. A broadband antenna of the type proposed in this report would give independent confirmation of the existence of these events, as well as furnish new information about the pulse shapes.

The discovery of the pulsars may have uncovered sources of gravitational radiation which have extremely well-known frequencies and angular positions. The fastest known pulsar is NP 0532, in the Crab Nebula, which rotates at 30.2 Hz. The gravitational flux incident on the Earth from NP 0532 at multiples of 30.2 Hz can be 10^{-6} erg/cm^2/s at most. This is much smaller than the intensity of the events measured by Weber. The detection of pulsar signals, however, can be benefited by use of correlation techniques and long integration times.

The proposed antenna design can serve as a pulsar antenna and offers some distinct advantages over high-Q acoustically coupled structures.

2. Description of a Gravitational Wave in the General Theory of Relativity

In his paper on gravitational waves (1918), Einstein showed by a perturbation argument that a weak gravitational plane wave has an irreducible metric tensor in an

GRAVITATIONAL-WAVE ASTRONOMY[1,2]

WILLIAM H. PRESS[3] AND KIP S. THORNE
California Institute of Technology, Pasadena, California

1. INTRODUCTION

The "windows" of observational astronomy have become broader. They now include, along with photons from many decades of the electromagnetic spectrum, extraterrestrial "artifacts" of other sorts: cosmic rays, meteorites, particles from the solar wind, samples of the lunar surface, and neutrinos. With gravitational-wave astronomy, we are on the threshold—or just beyond the threshold—of adding another window; it is a particularly important window because it will allow us to observe phenomena that cannot be studied adequately by other means: gravitational collapse, the interiors of supernovae, black holes, short-period binaries, and perhaps new details of pulsar structure. There is the further possibility that gravitational-wave astronomy will reveal entirely new phenomena—or familiar phenomena in unfamiliar guise—in trying to explain the observations of Joseph Weber.

The future of gravitational-wave astronomy looks bright whether or not

Early claim of detection

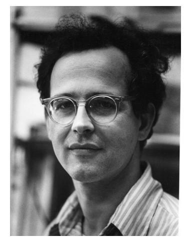
VOLUME 22, NUMBER 24 PHYSICAL REVIEW LETTERS 16 JUNE 1969

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

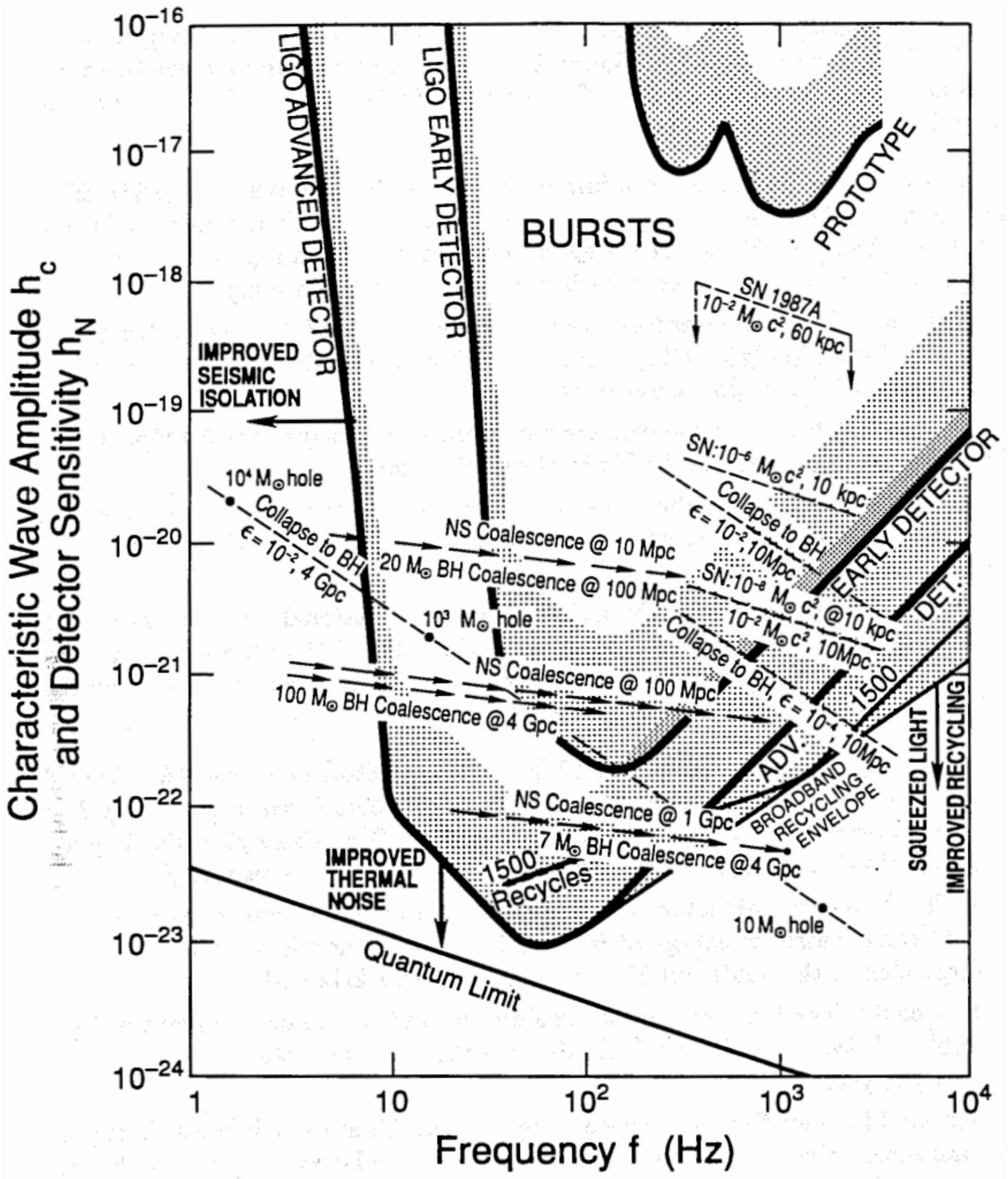
J. Weber
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

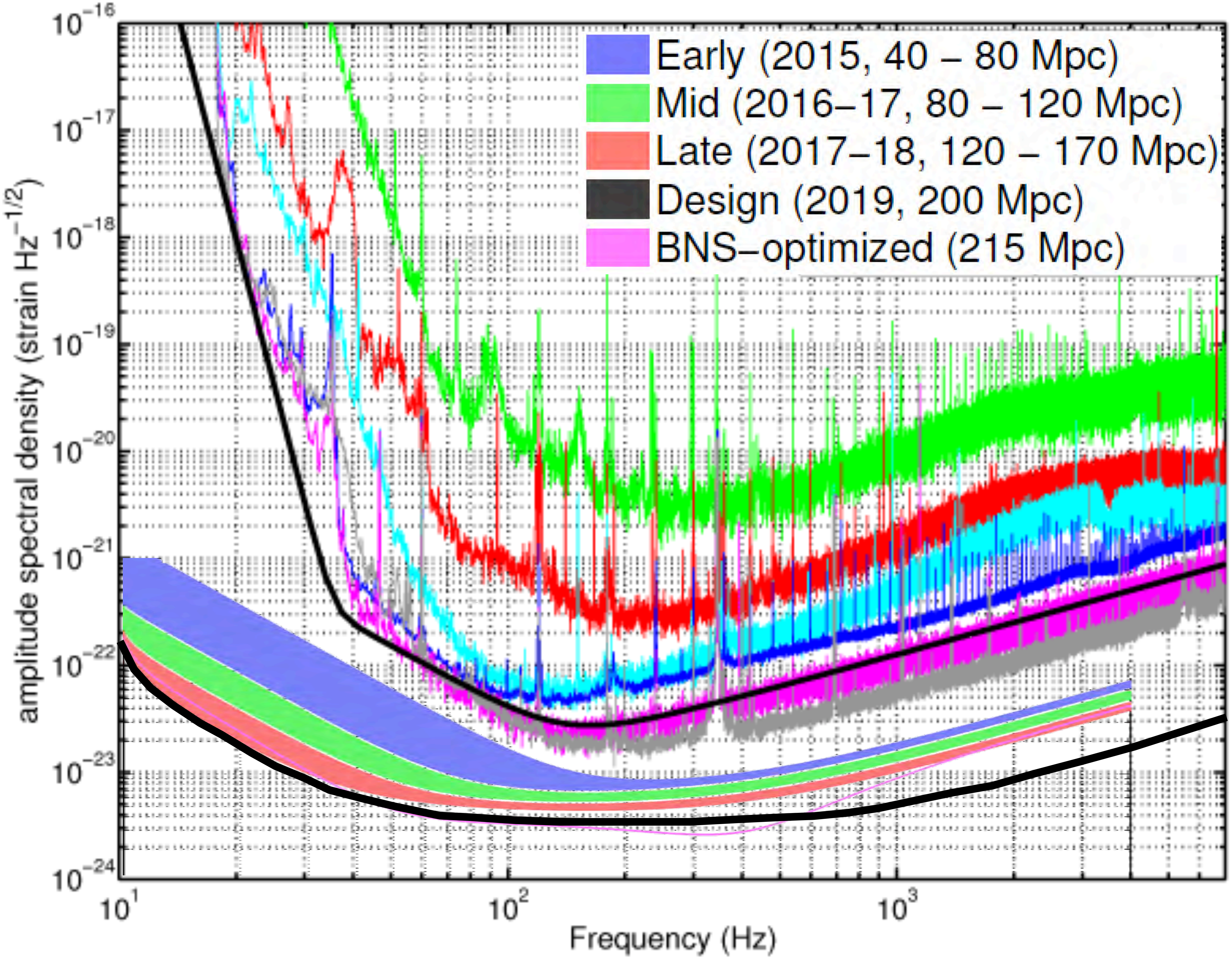
LIGO Timeline

- Conceived in the early 70's, Chapman, Forward, Weiss
- 1984, Caltech and MIT form LIGO collaboration, jointly lead by Drever, Weiss and Thorne   
- 1989 proposal to the National Science Foundation
- 1991 construction approved
- 1998 facility construction complete
- 2002 first observing run for the first generation detectors
- 2015 first observing run for the second generation detectors

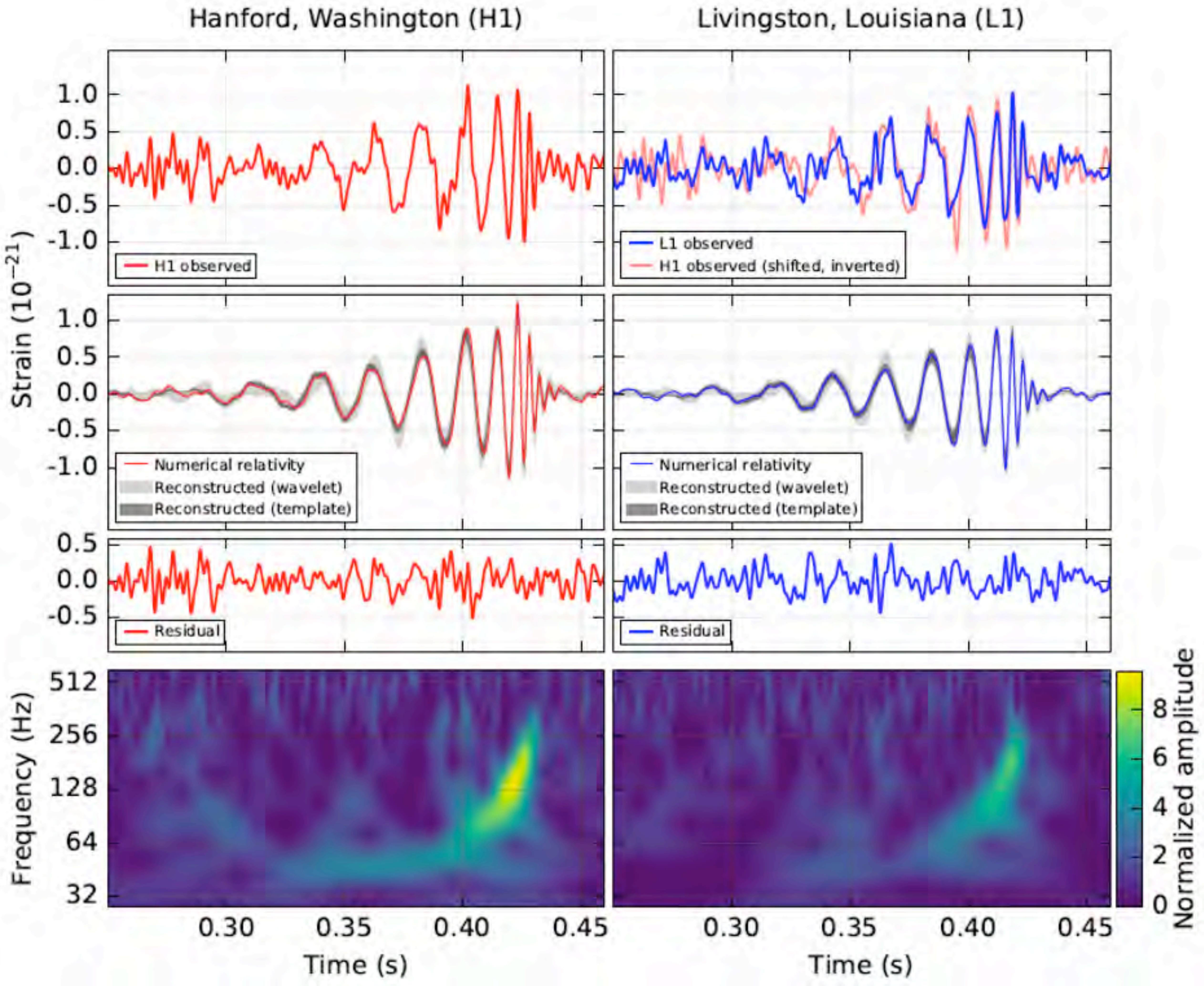
R. E. Vogt, R. W. P. Drever, K. S. Thorne, F. J. Raab and R. Weiss (Caltech & MIT), "Construction, operation, and supporting research and development of a Laser Interferometer Gravitational-wave Observatory", proposal to NSF, 1989



LIGO sensitivity over time



GW150914: At last a signal!



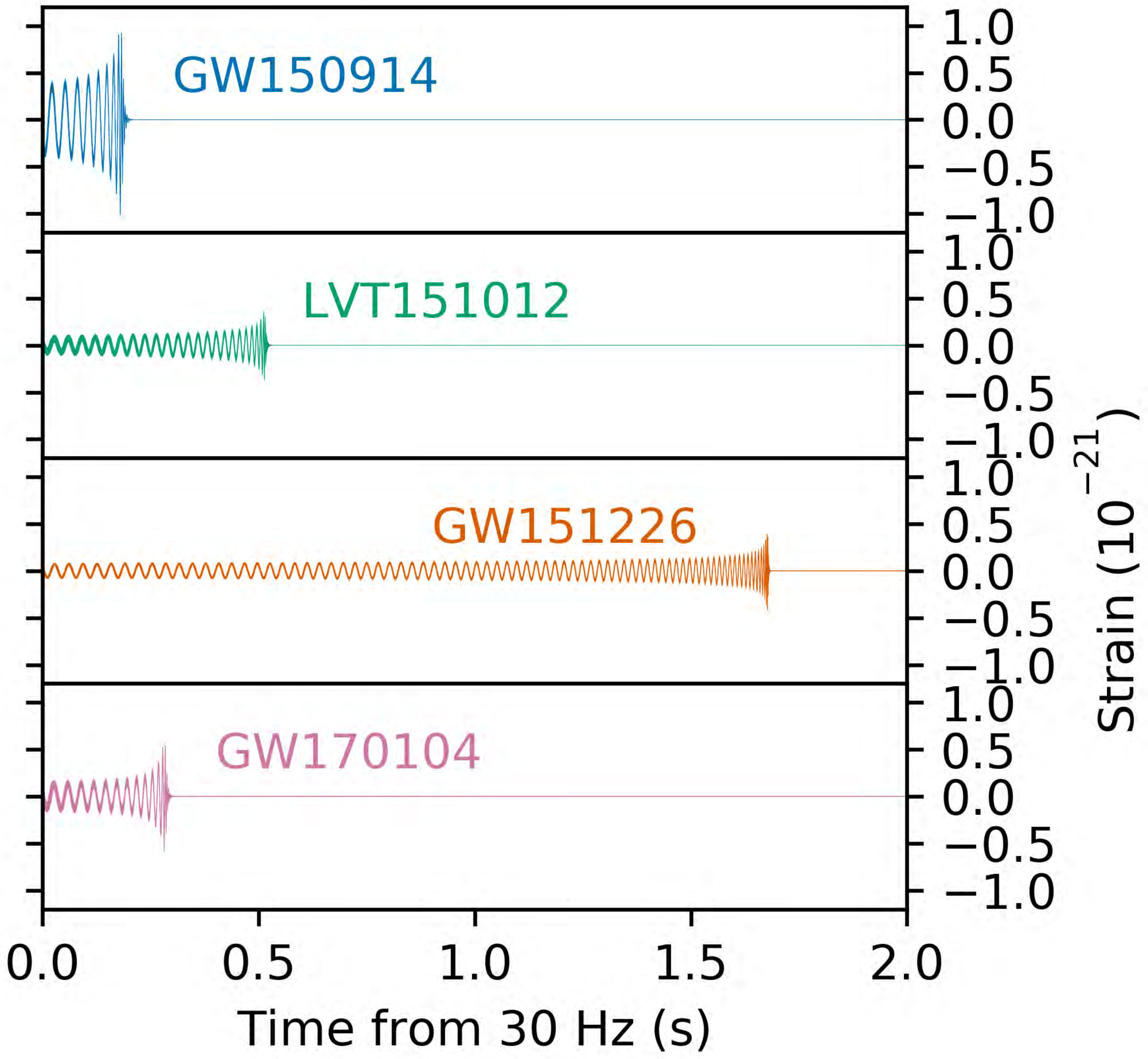
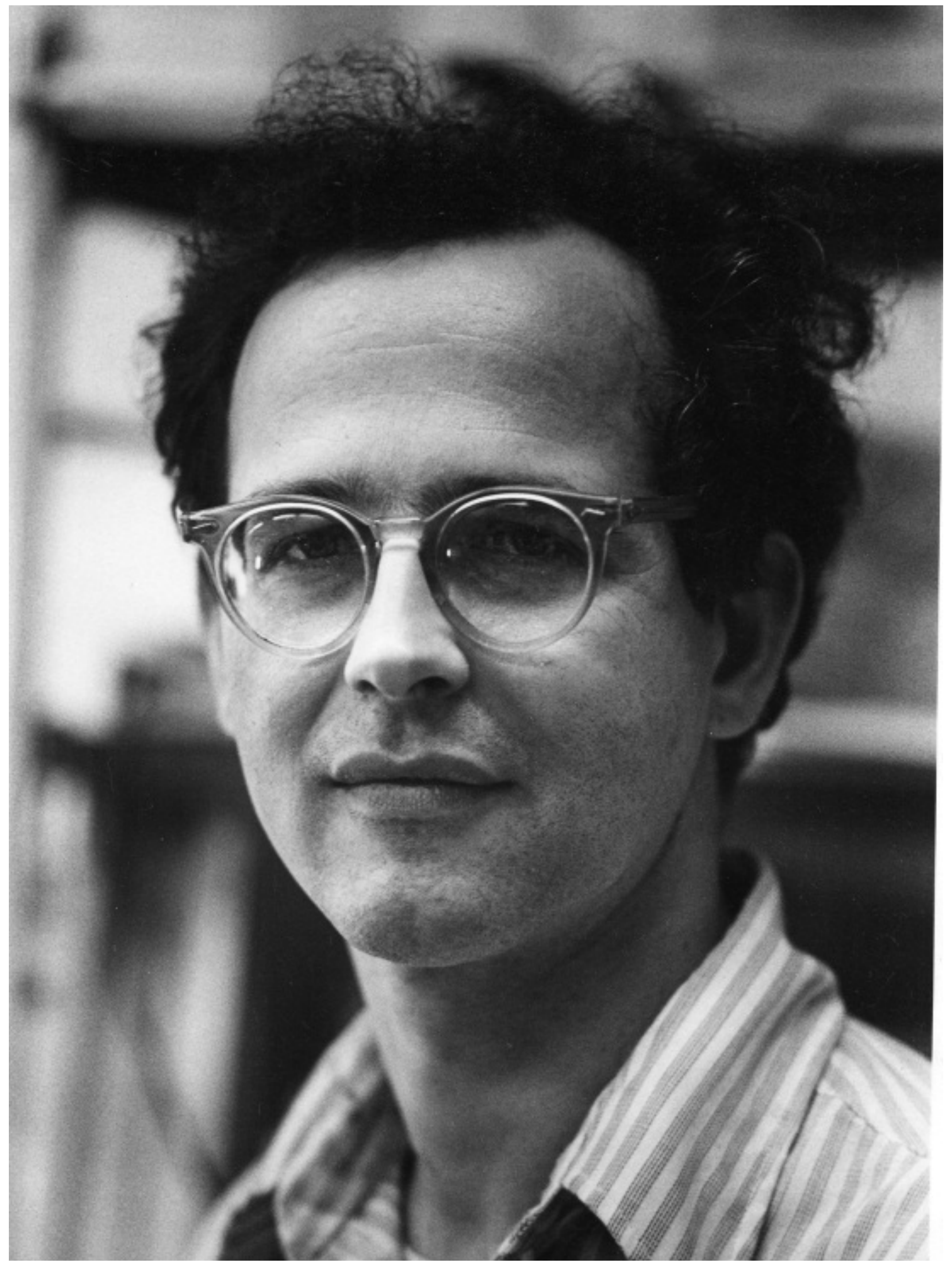


FIG. 1. The gravitational-wave event GW150914 observed by the LIGO Hanford (H1, left column panels) and Livingston (L1, right column panels) detectors. Times are shown relative to September 14, 2015 at 09:50:45 UTC. For visualization, all time series

GW150914: A story 40+ years in the making

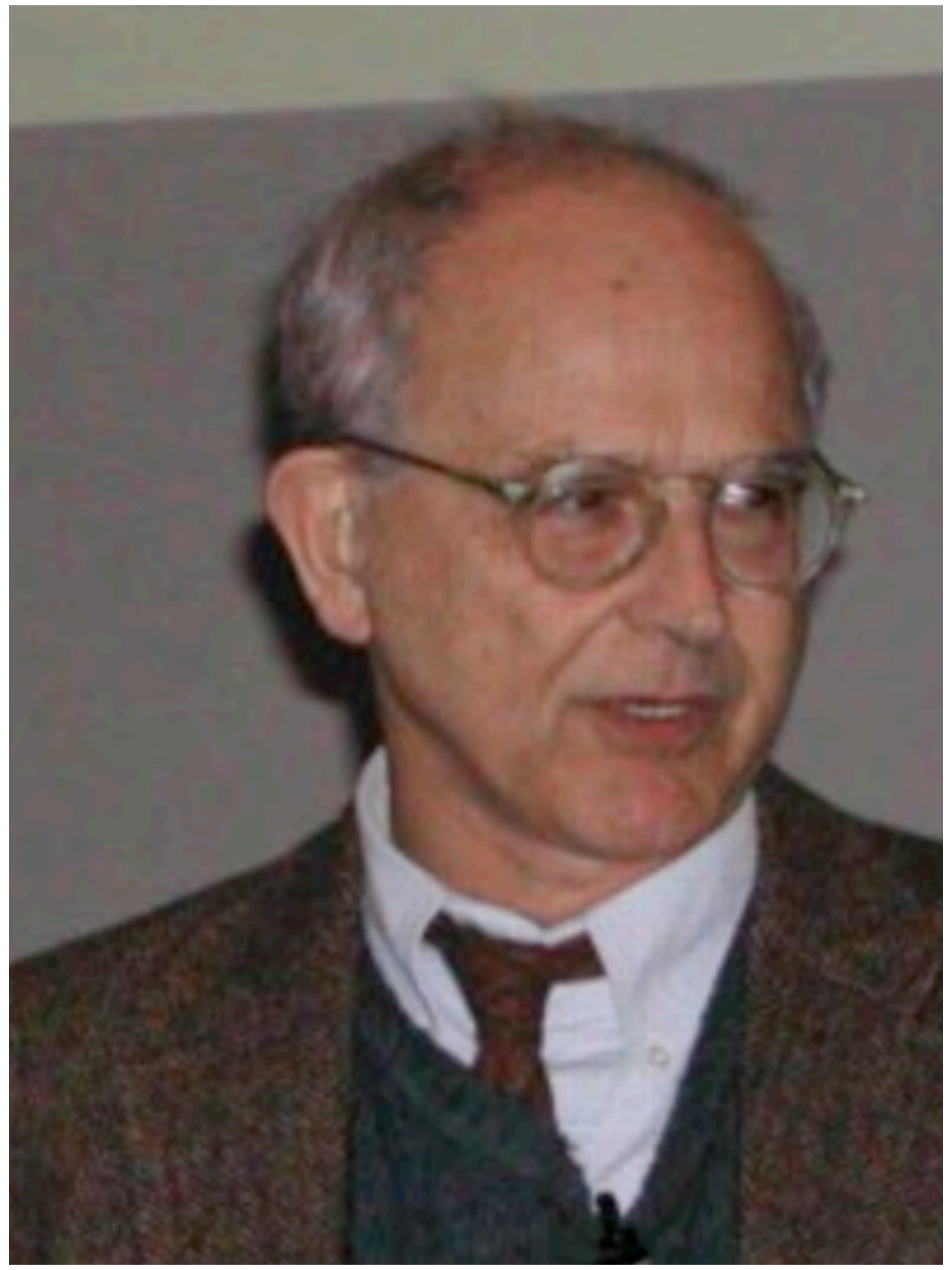


Rai Weiss

Kip Thorne

Ron Drever

GW150914: A story 40+ years in the making



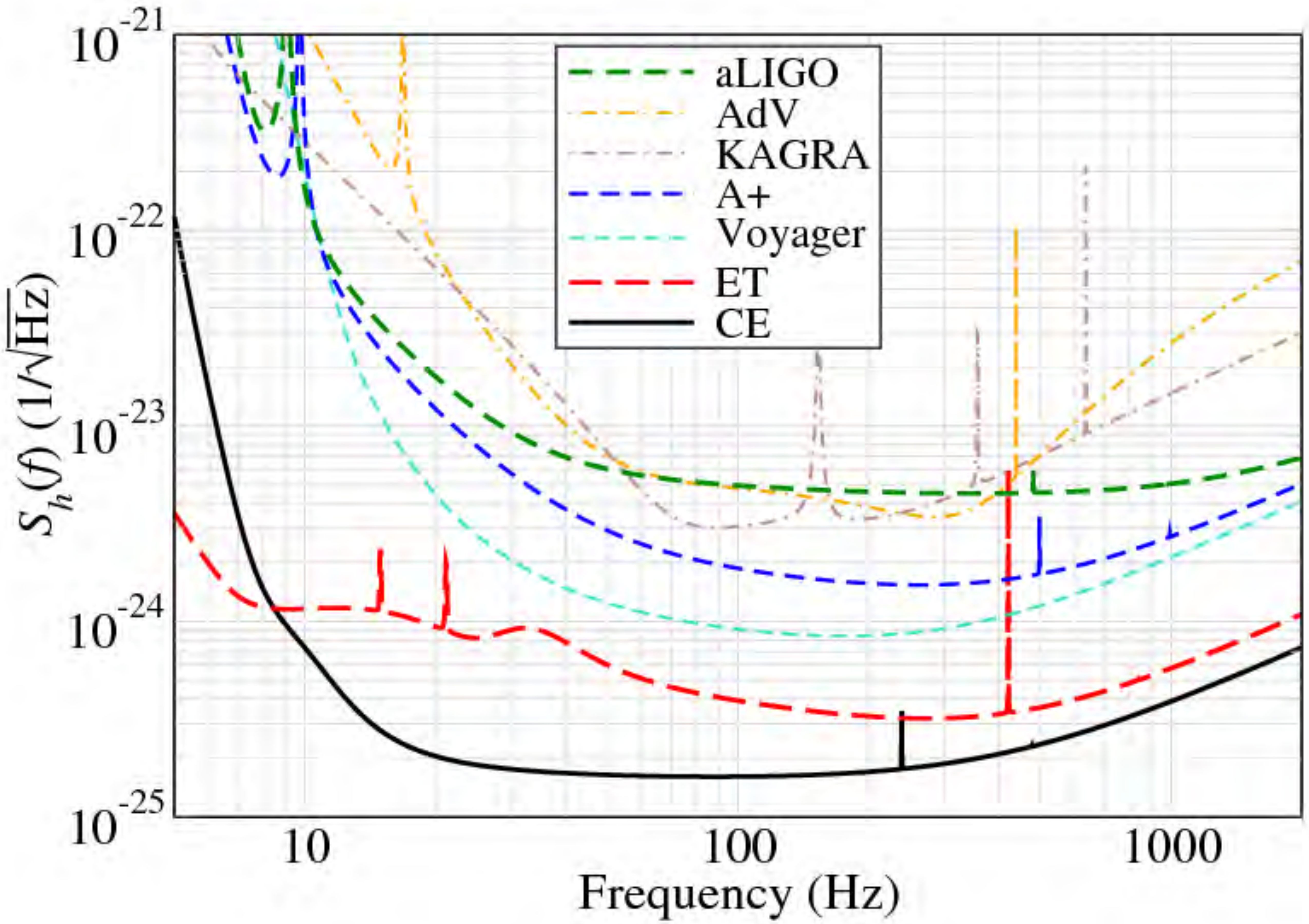
Rai Weiss

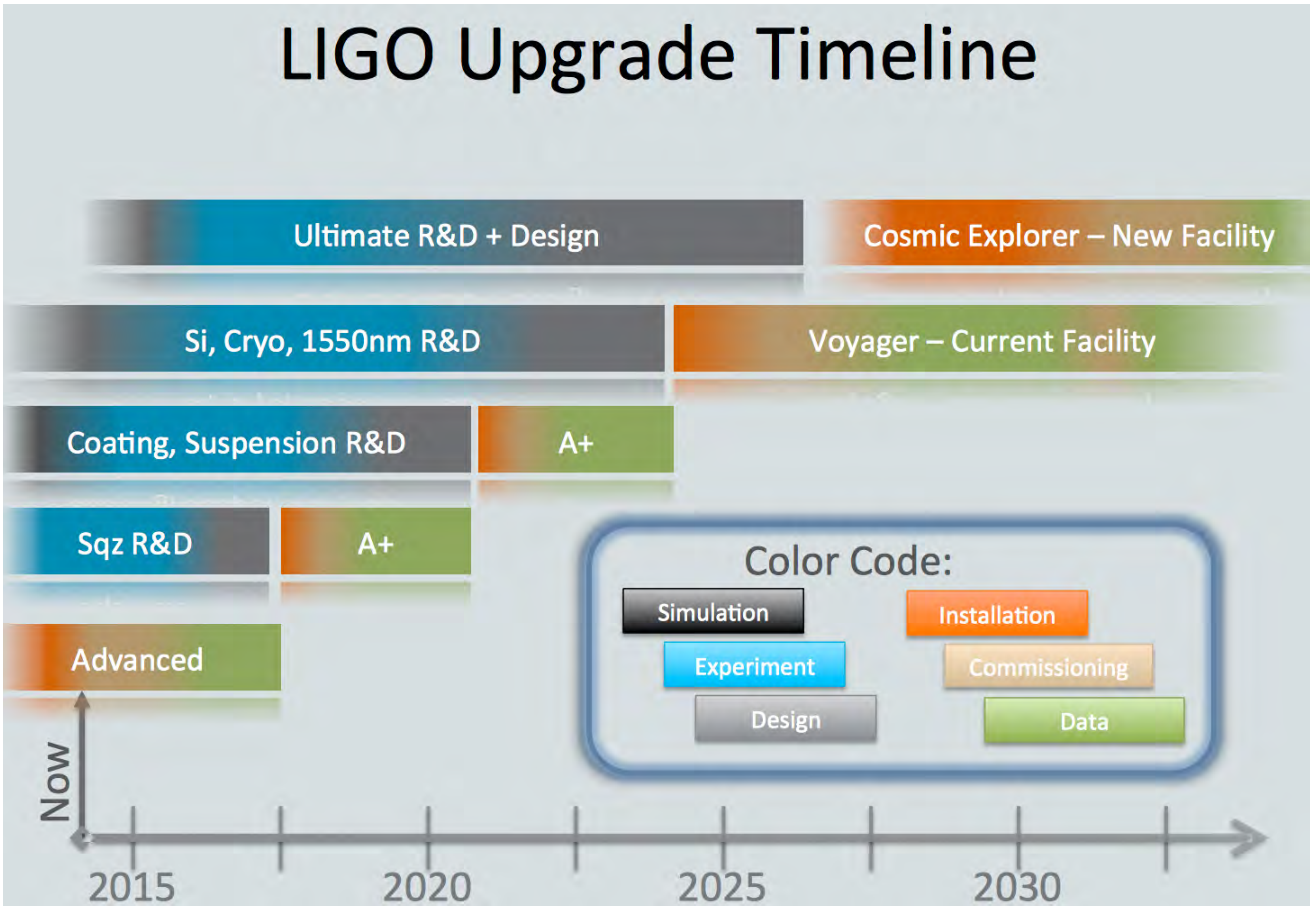
Kip Thorne

Ron Drever


GEO
VIRGO
LIGO H1
LIGO L1
KAGRA
2017+
LIGO-India
2023+
Next steps - a worldwide network

3rd and 4th generation ground-based instruments





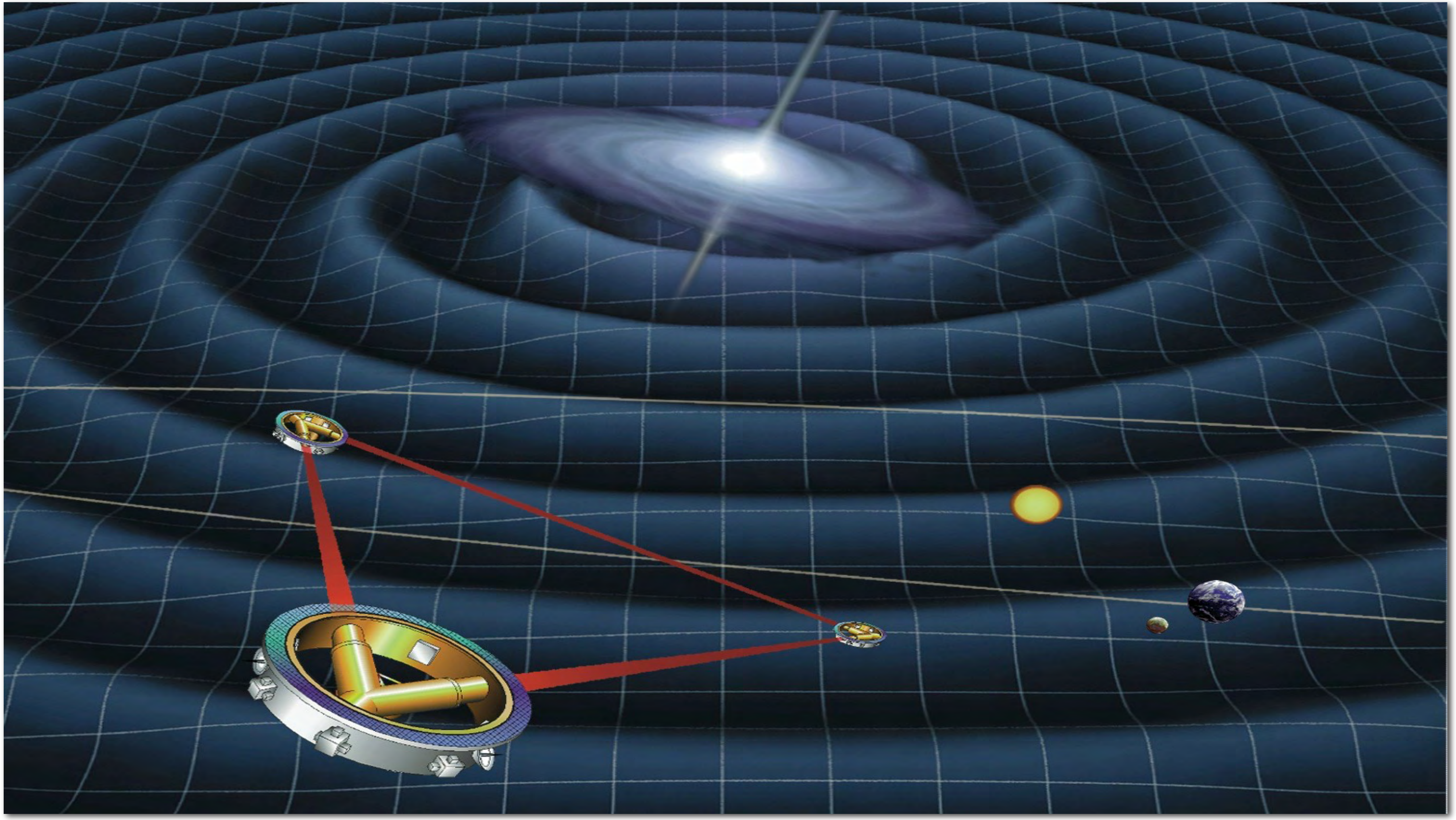
A+: aLIGO upgrade, freq. dep. squeezing, heavier mirrors, more powerful lasers

Voyager: aLIGO upgrade, same facility, cryogenic, more powerful lasers

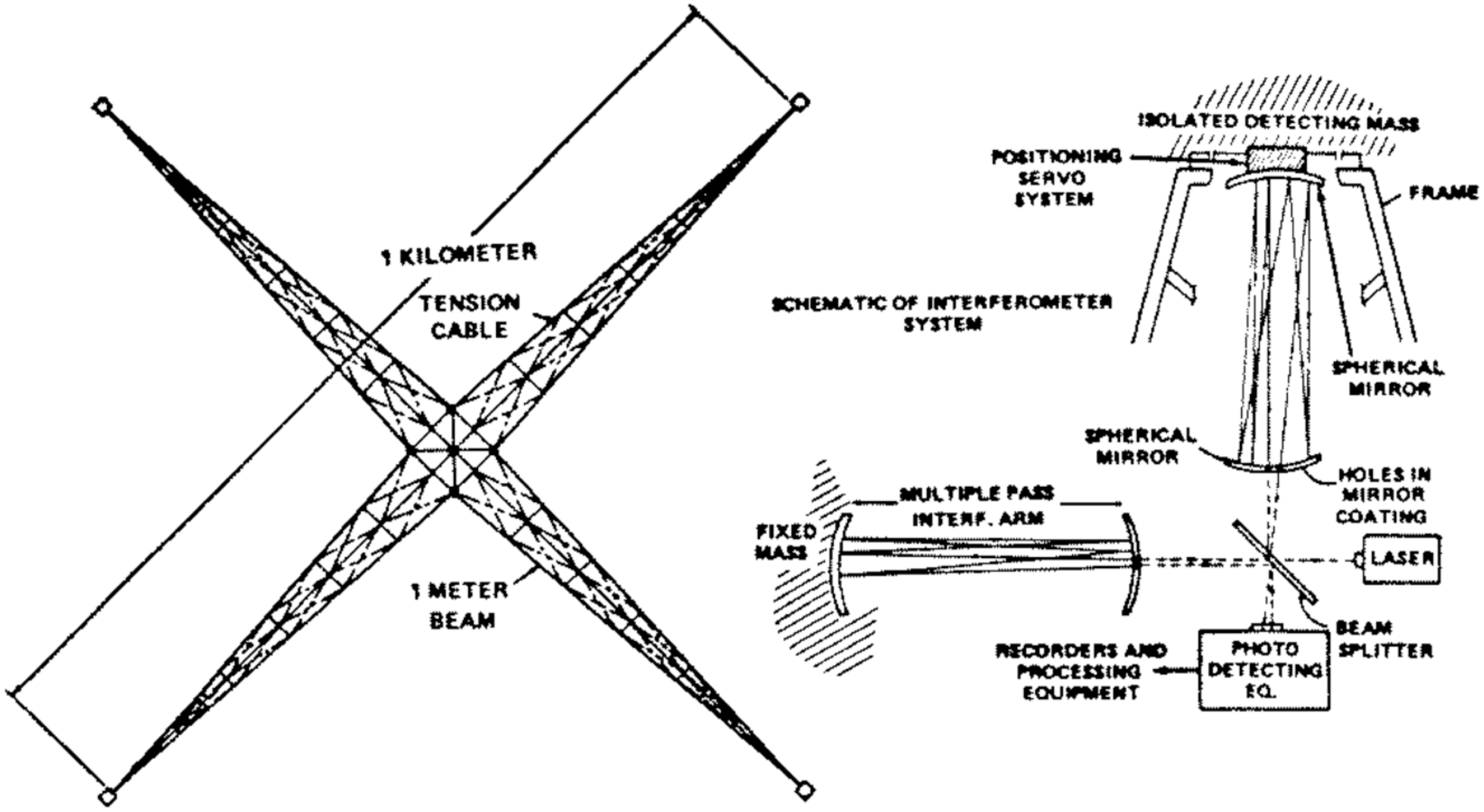
Einstein Telescope: Underground, 10 km, triangular, cryogenic

Cosmic Explorer: New facility, 40 km arms, squeezing etc

Space Interferometers



Gravitational Wave Interferometer: 1974



"LIGO in space"

1978 Design - 16.5 t, $49.5M. Shuttle Launched. To be built in space. Aluminum extruding machine.

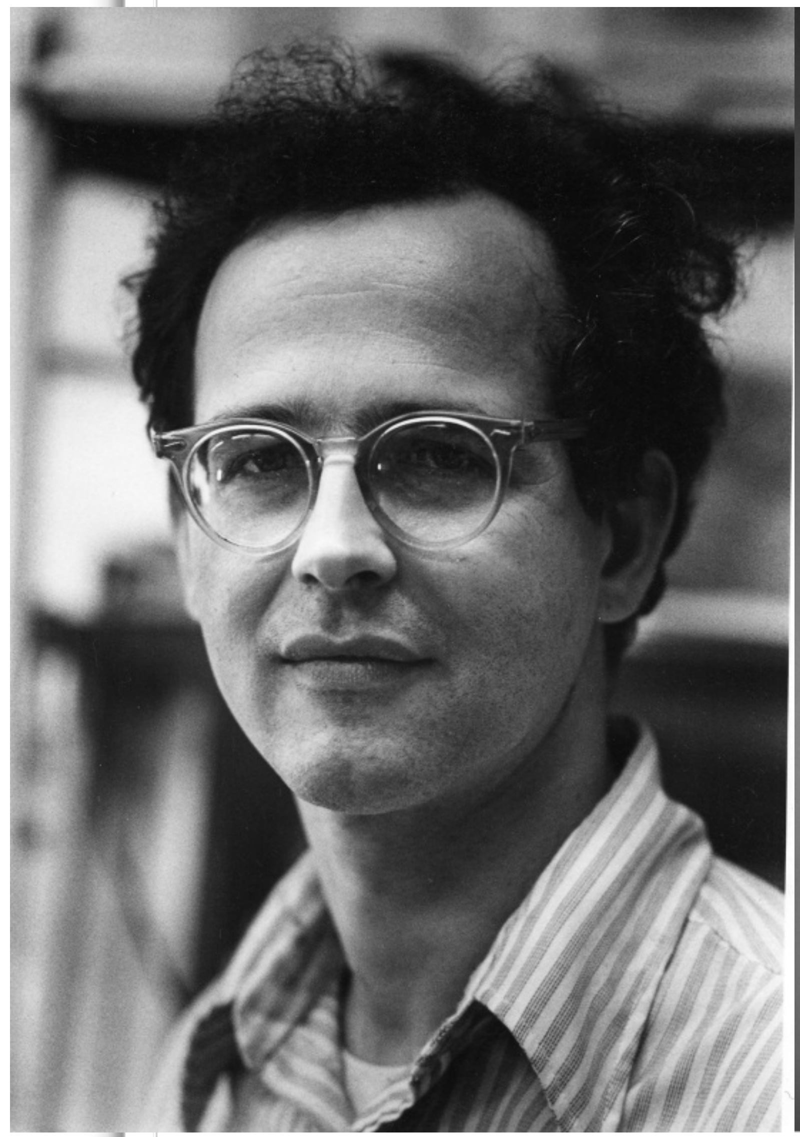
The Weiss Report: 1975

MANAGEMENT AND OPERATIONS

WORKING GROUP FOR SHUTTLE ASTRONOMY

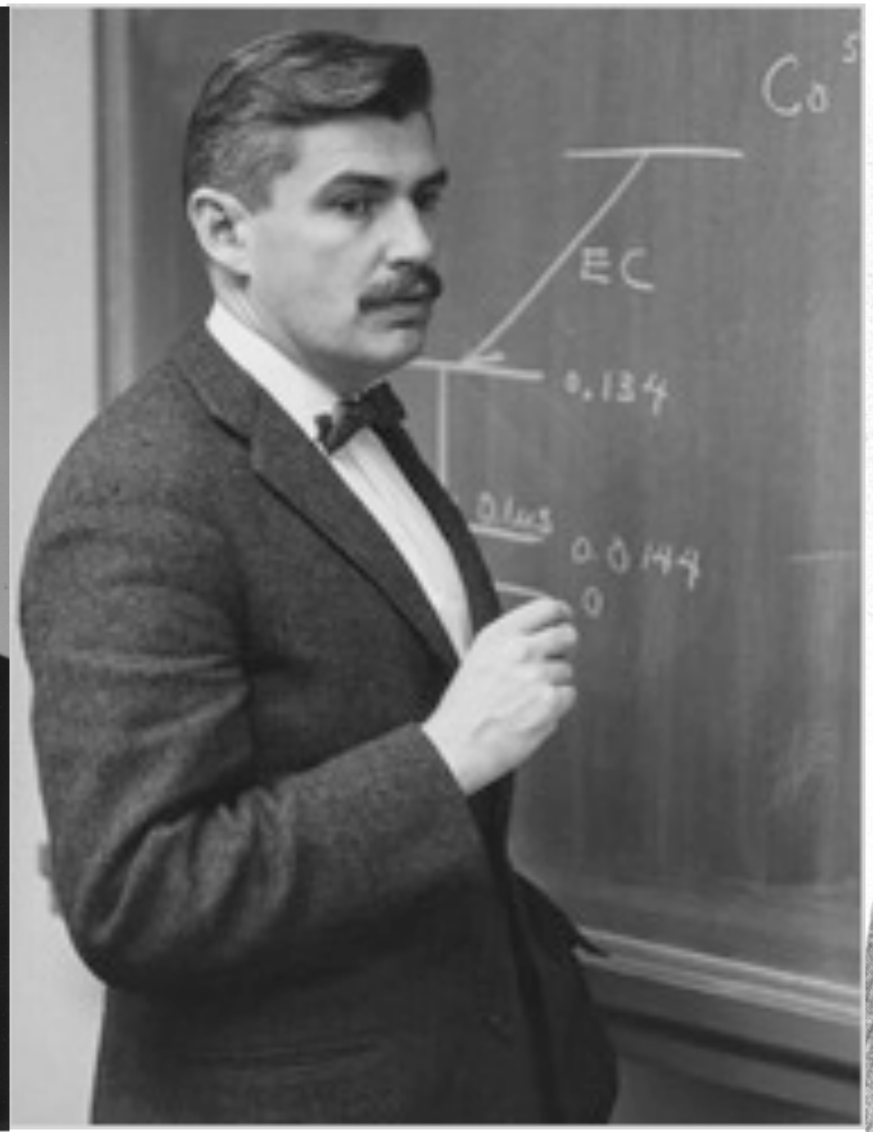
REPORT OF THE SUB-PANEL ON RELATIVITY AND GRAVITATION

Weiss



Bender

Pound



Misner

The Weiss Report: 1975

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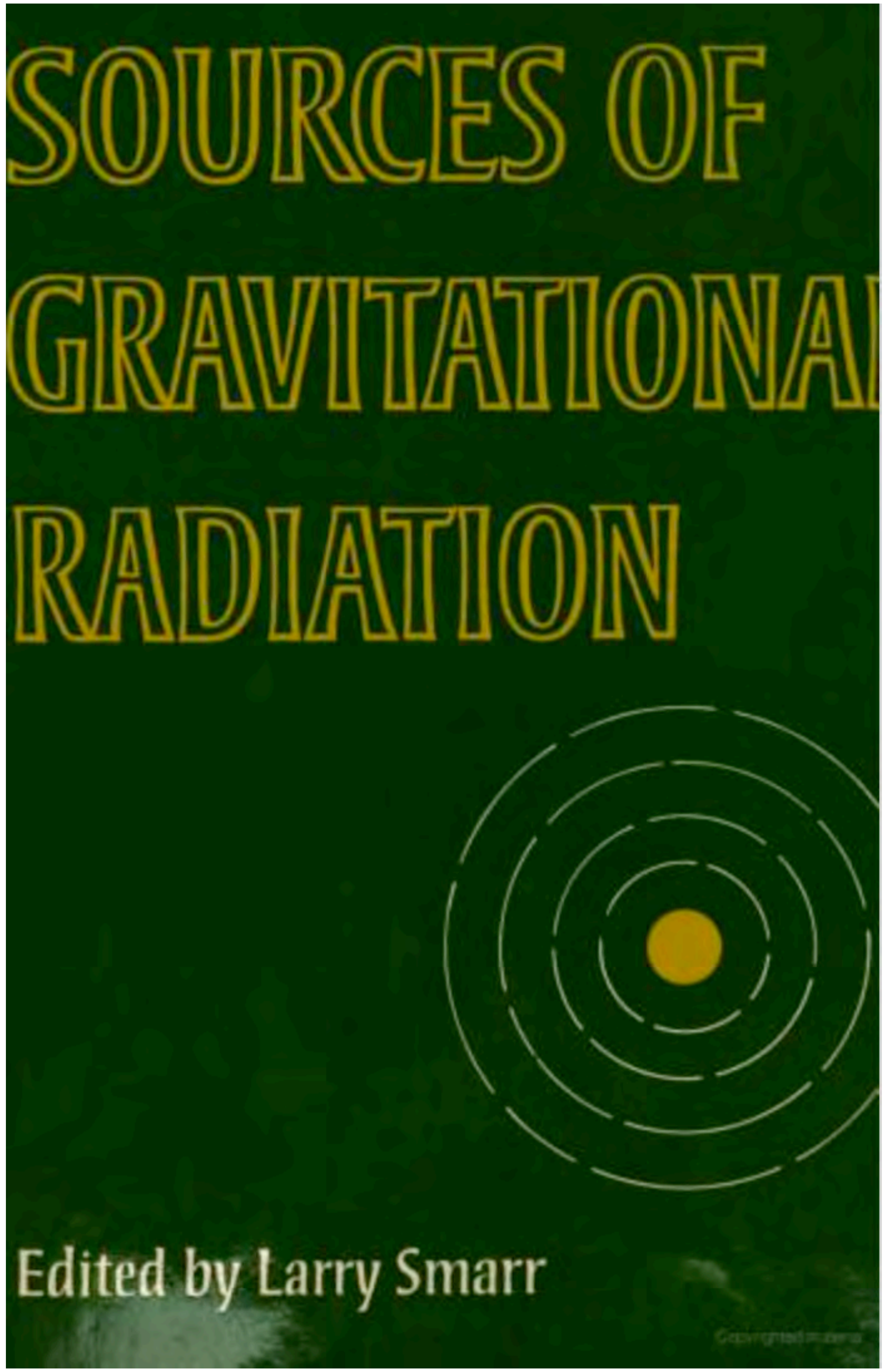
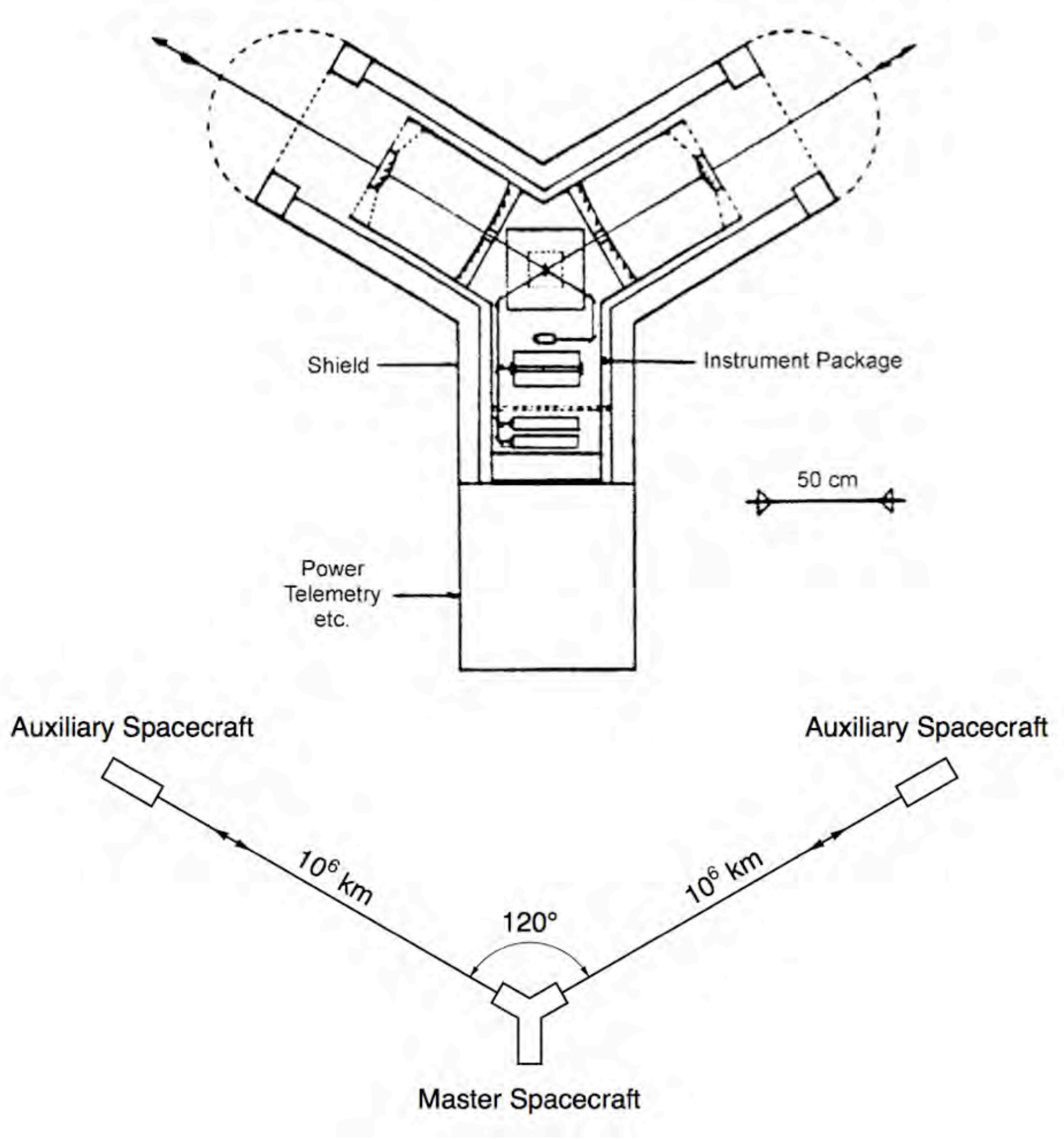
Battelle Workshop, Seattle July 24-August 4, 1978

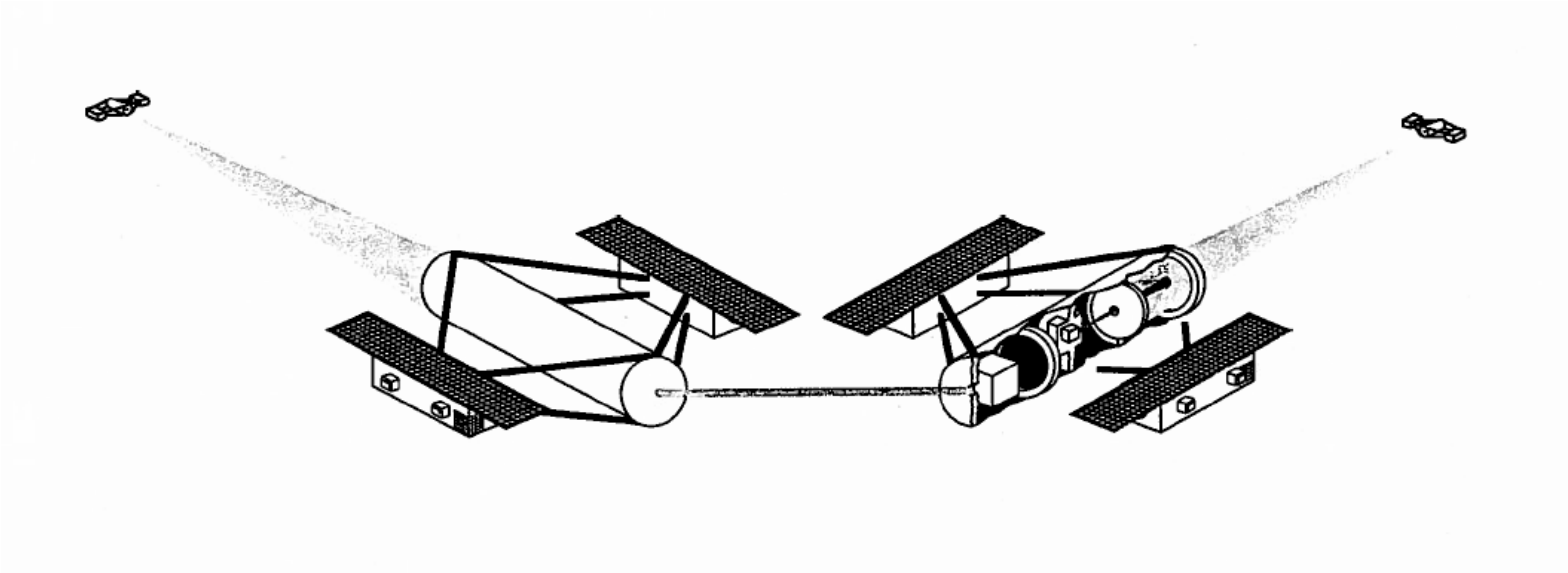


Laser Antenna for Gravitational-radiation Observation in Space (LAGOS): 1981



Faller & Bender 1981
Faller, Bender, Hall, Hils & Vincent 1985

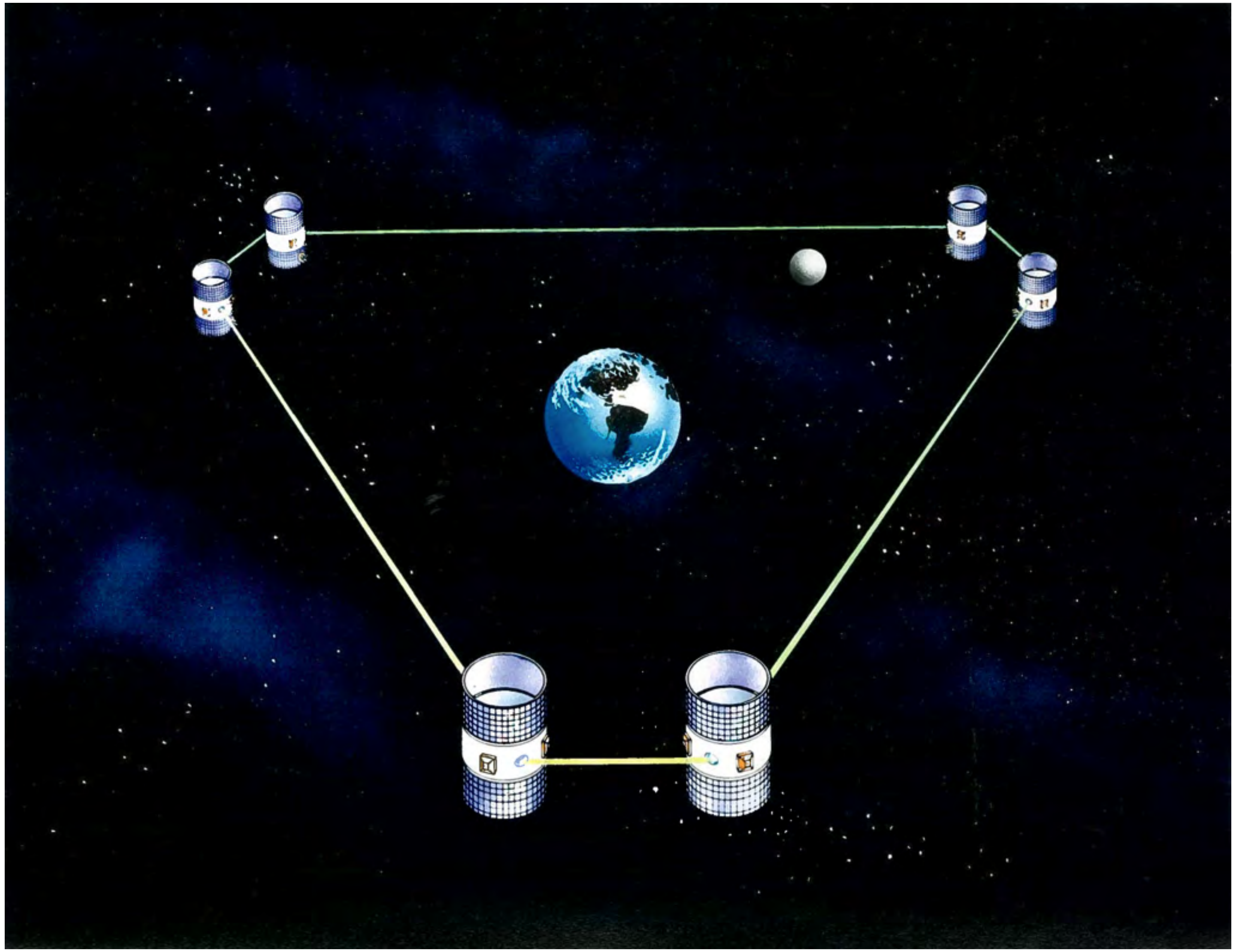
Laser Interferometer Space Antenna (LISA): 1993



ESA M3 candidate May1993

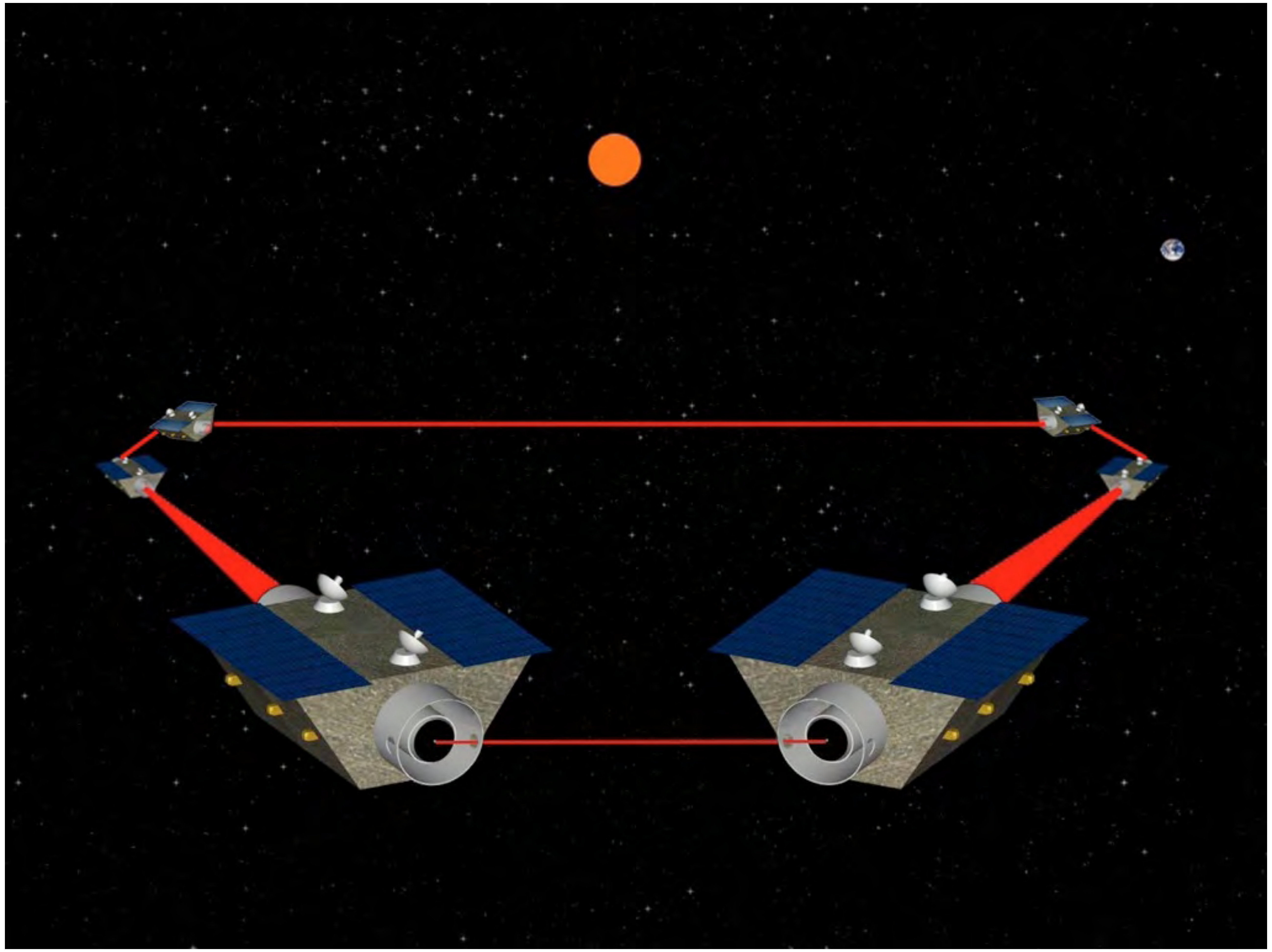
(Jim Hough came up with the LISA acronym in 1992)

Spaceborne Astronomical Gravitational-wave Interferometer To Test Aspects of Relativity and Investigate Unknown Sources (SAGITTARIUS): 1993



ESA M3 candidate (Hellings) 1993

Laser Interferometer Space Antenna for Gravity (LISAG): 1993



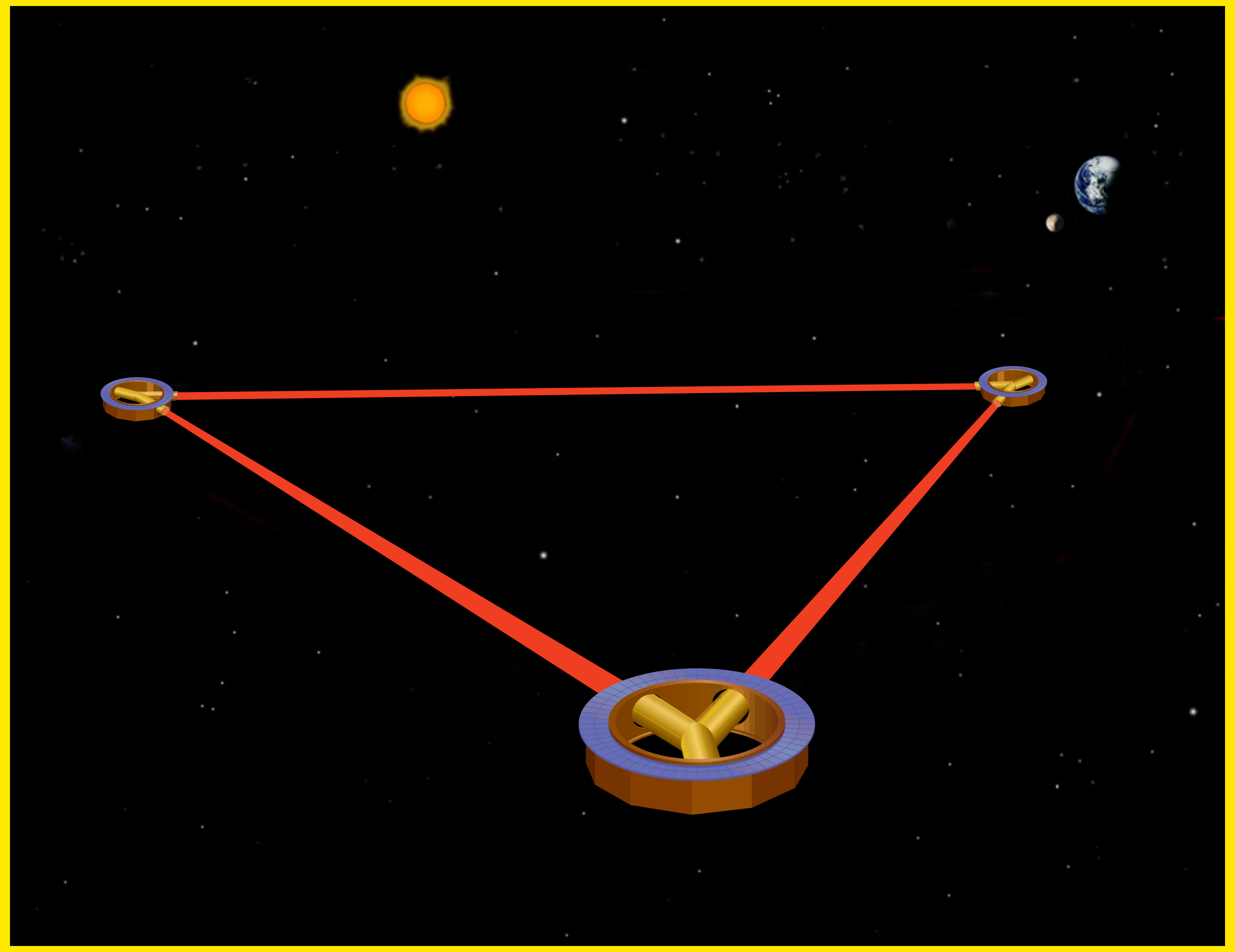
ESA Cornerstone candidate December 1993

Laser Interferometer Space Antenna (LISA)

LISA

Laser Interferometer Space Antenna
for the detection and observation of gravitational waves

An international project in the field of
Fundamental Physics in Space



Pre-Phase A Report

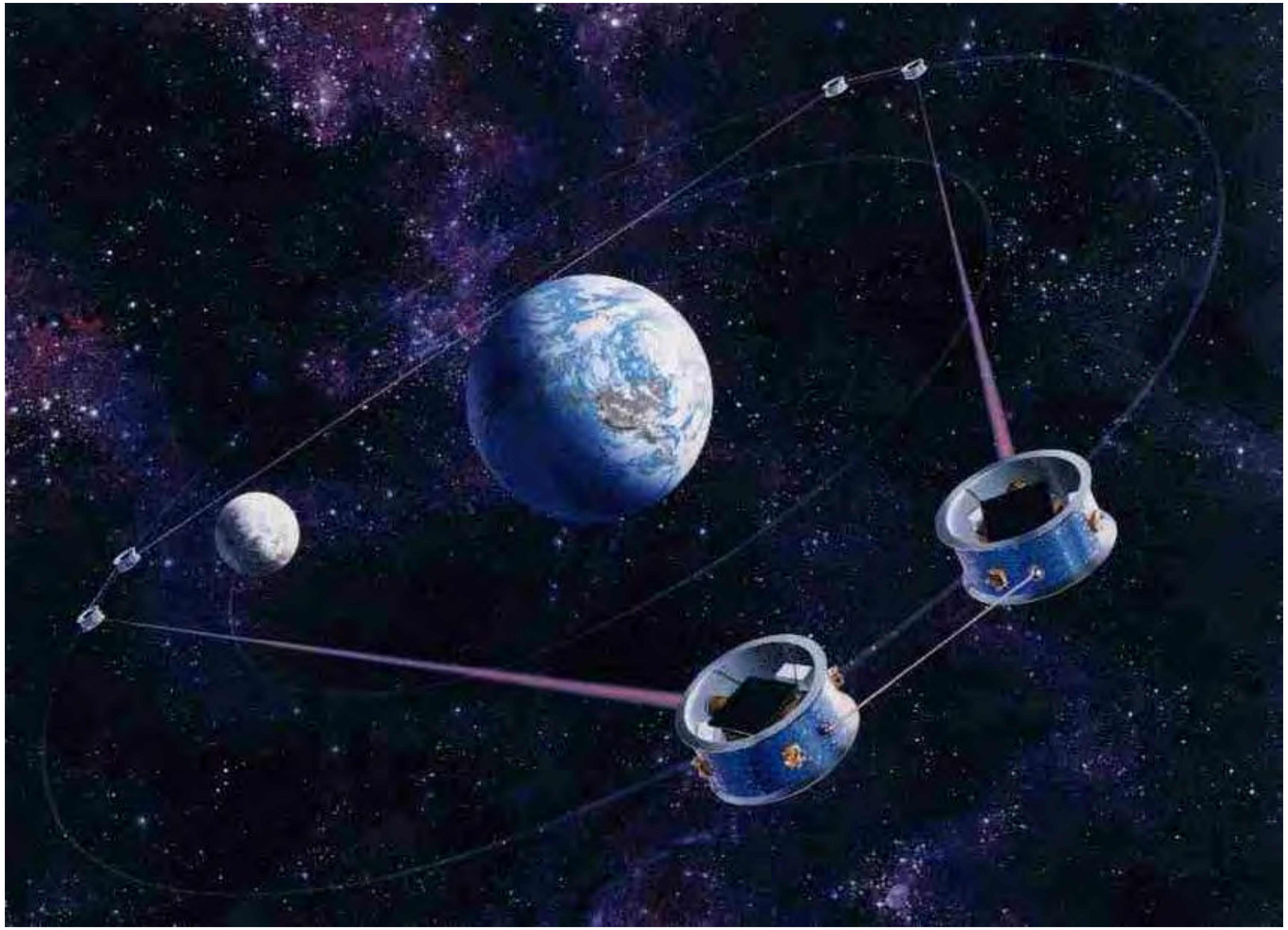
Second Edition

July 1998

MPQ 233 July 1998

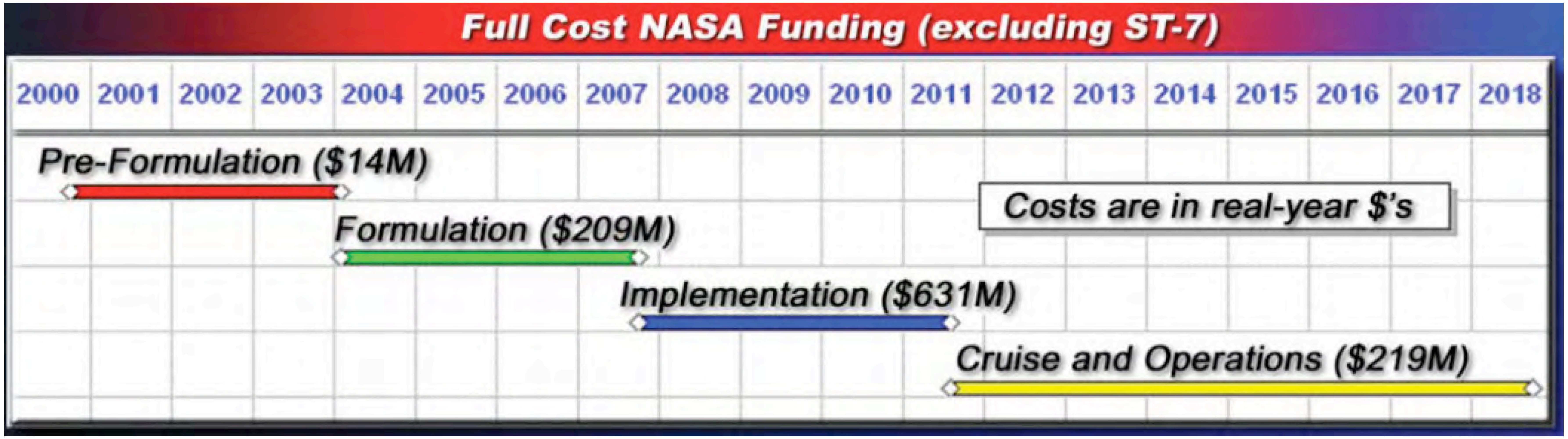
NASA/ESA joint study1996, Yellow Book 1998

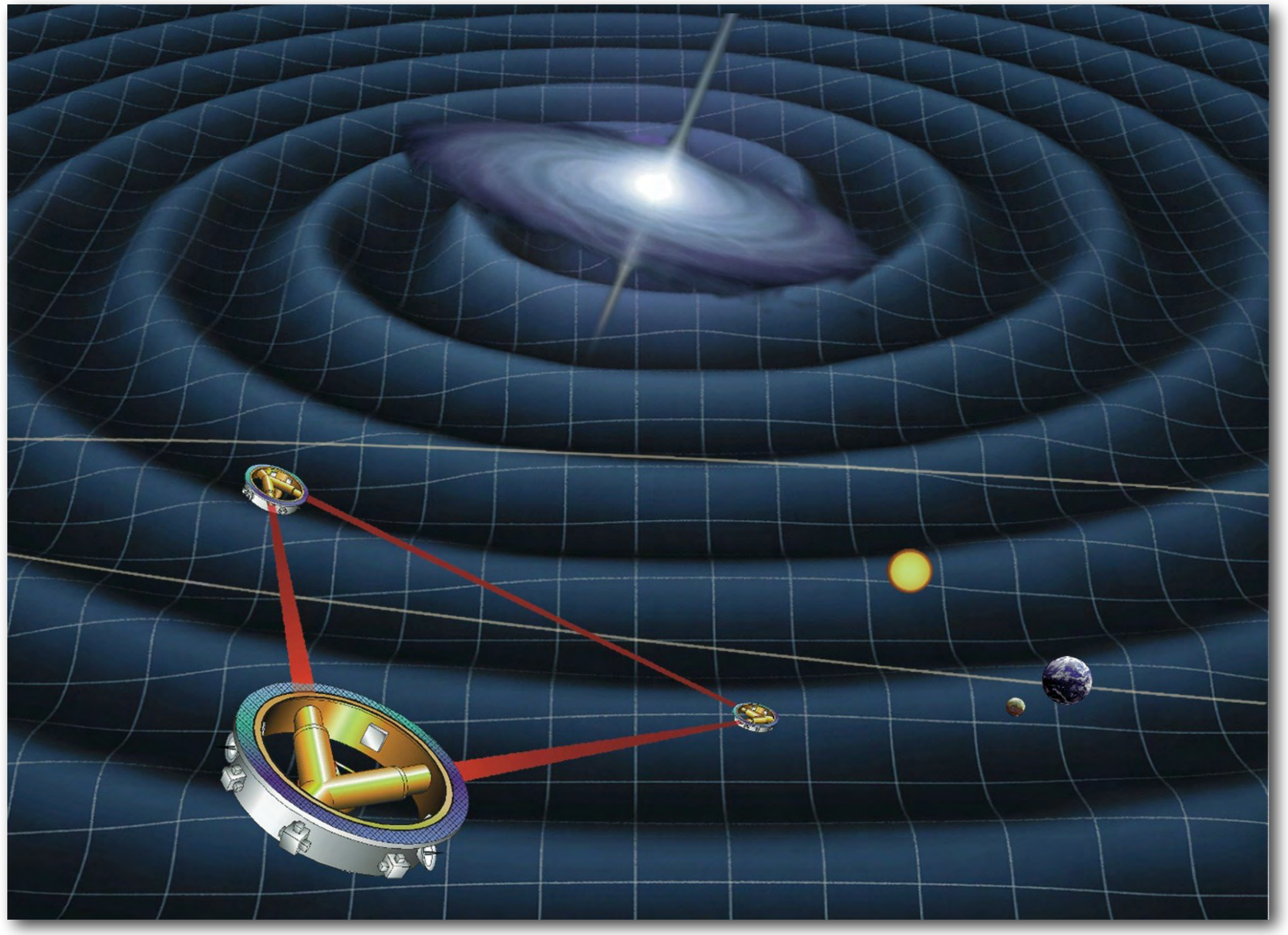
Orbiting Medium Explorer for Gravitational-wave Astrophysics (OMEGA):1998

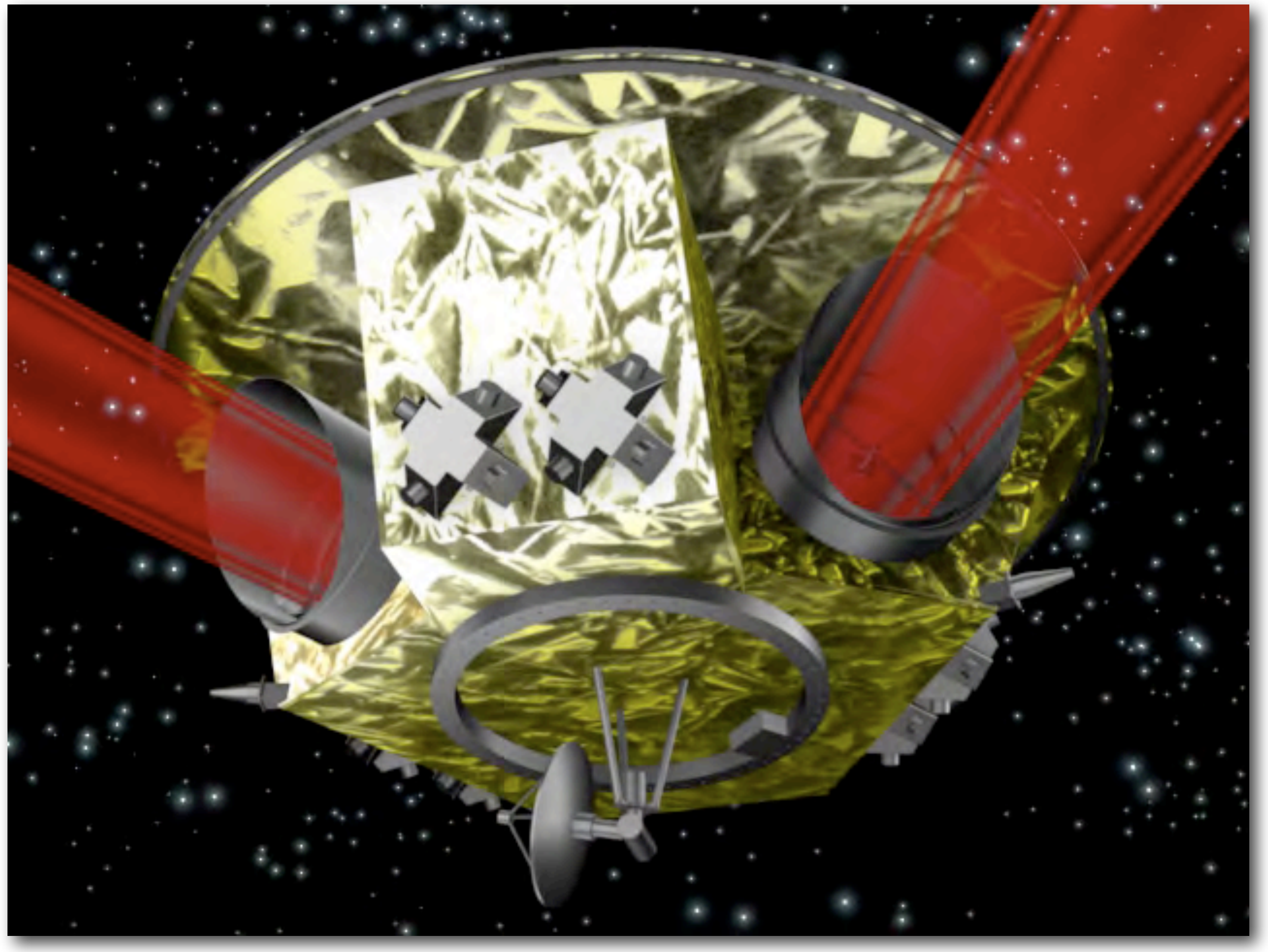


1998 NASA MIDEX proposal (Hellings et al)

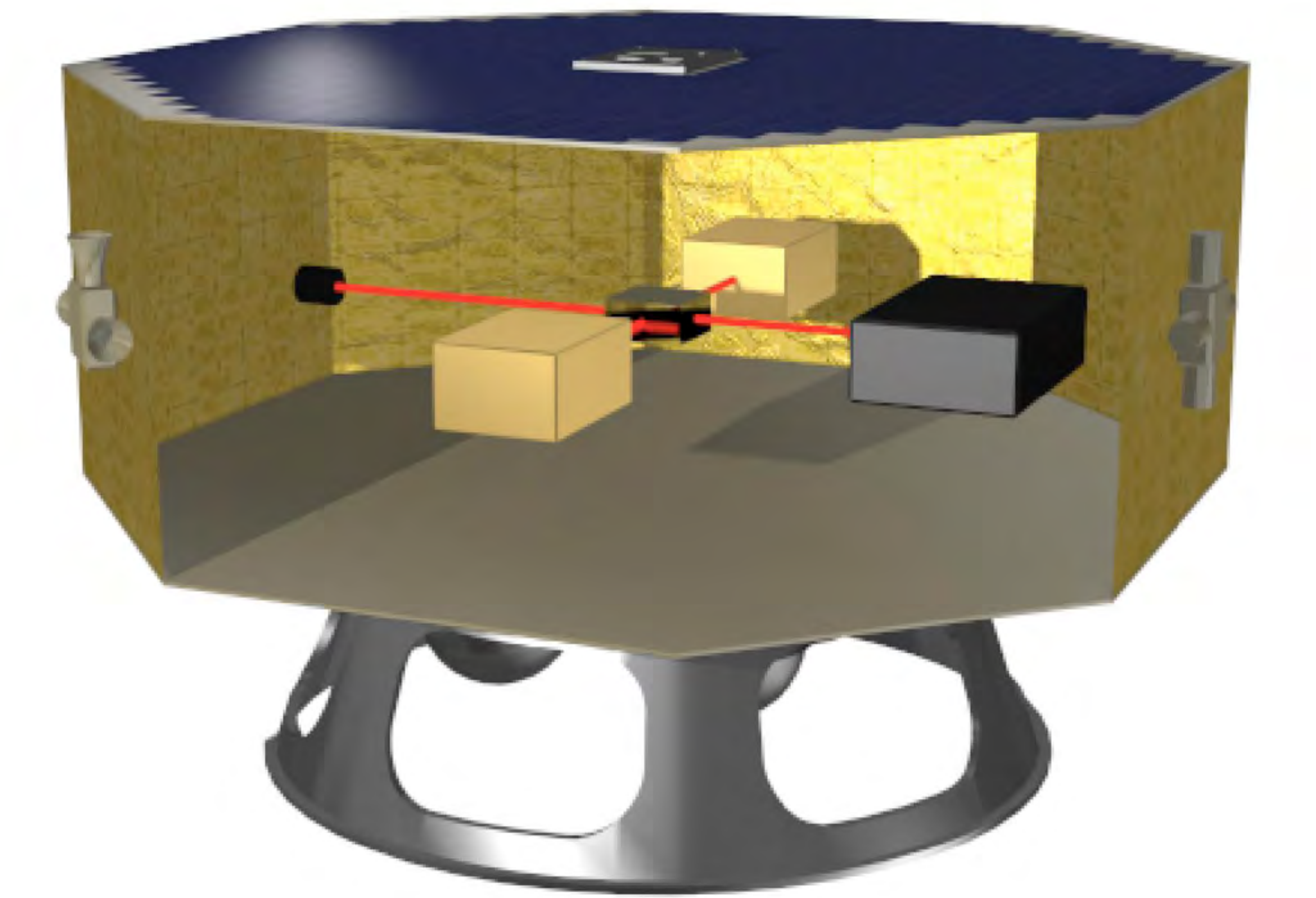
ESA/NASA LISA mission: official start 2001





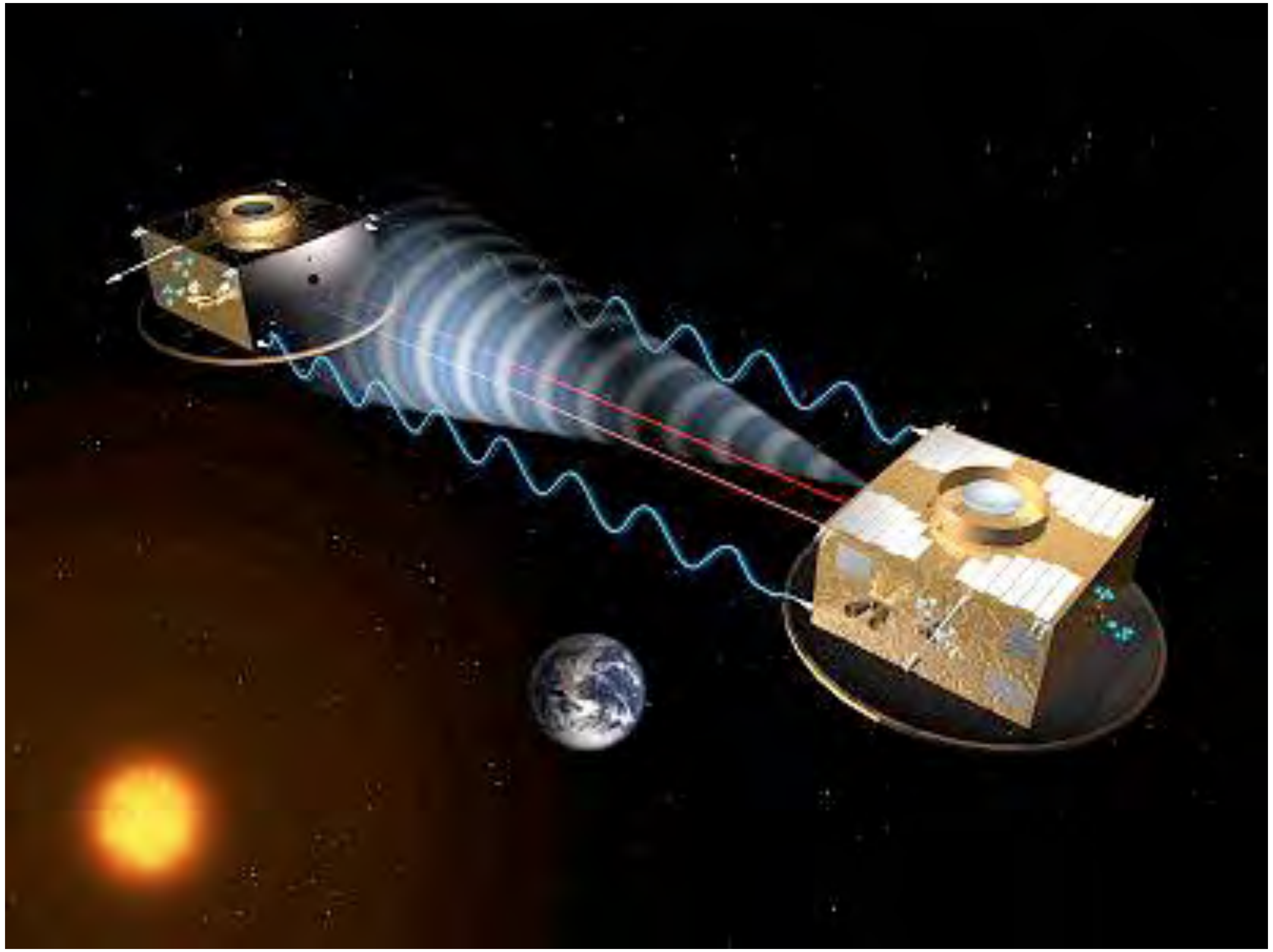


Pathfinder: European LIsa TEchnology (ELITE): 1998



For launch in 2002

Small Missions for Advanced Research in Technology-2 (SMART-2): 2000



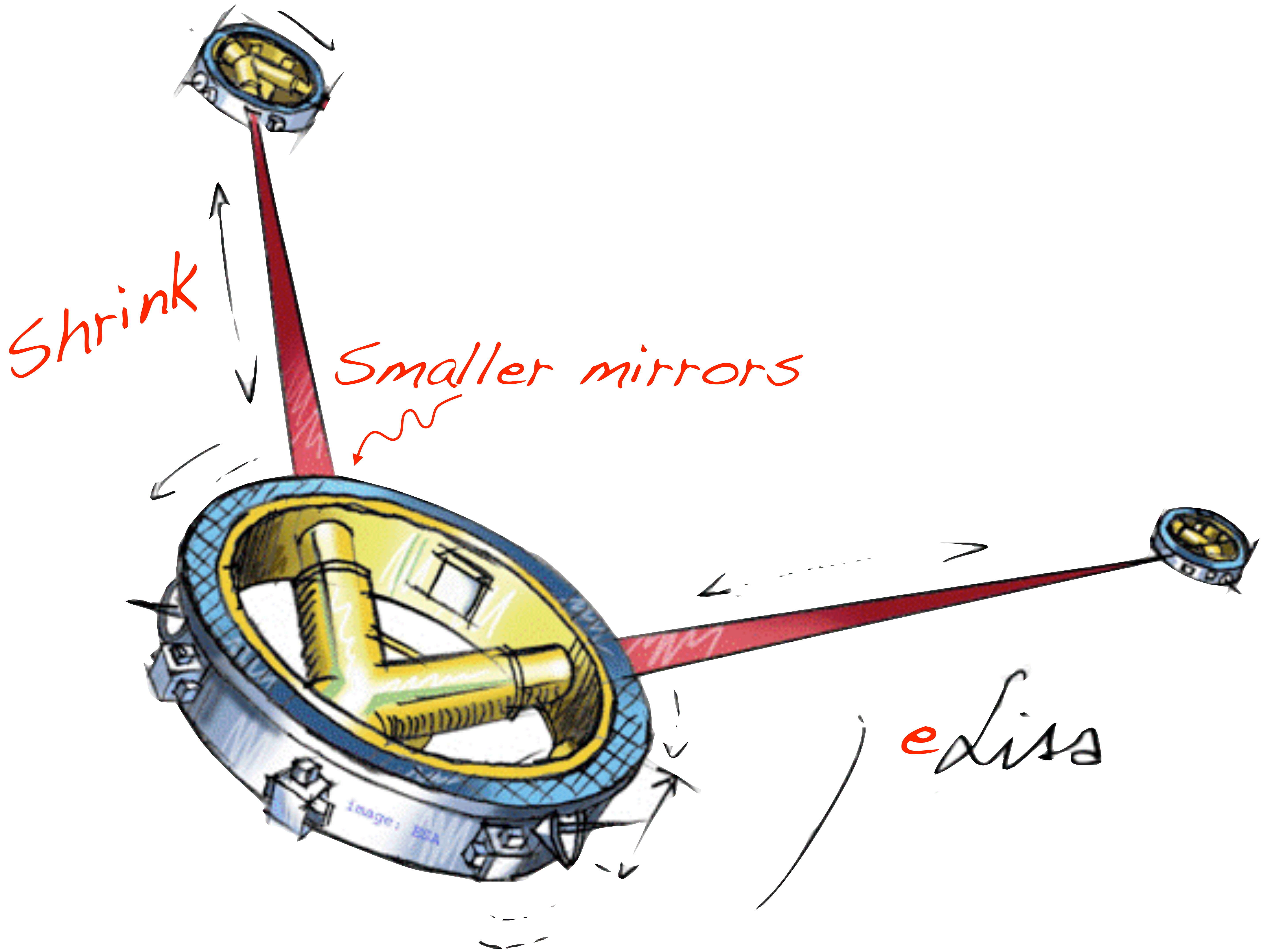
LISA/Darwin Pathfinder. For Launch in 2006

LISA Pathfinder - Space Technology Mission 7: Approved 2002

SMART-2 Descoped to single spacecraft. For launch in ~~2006~~ 2015

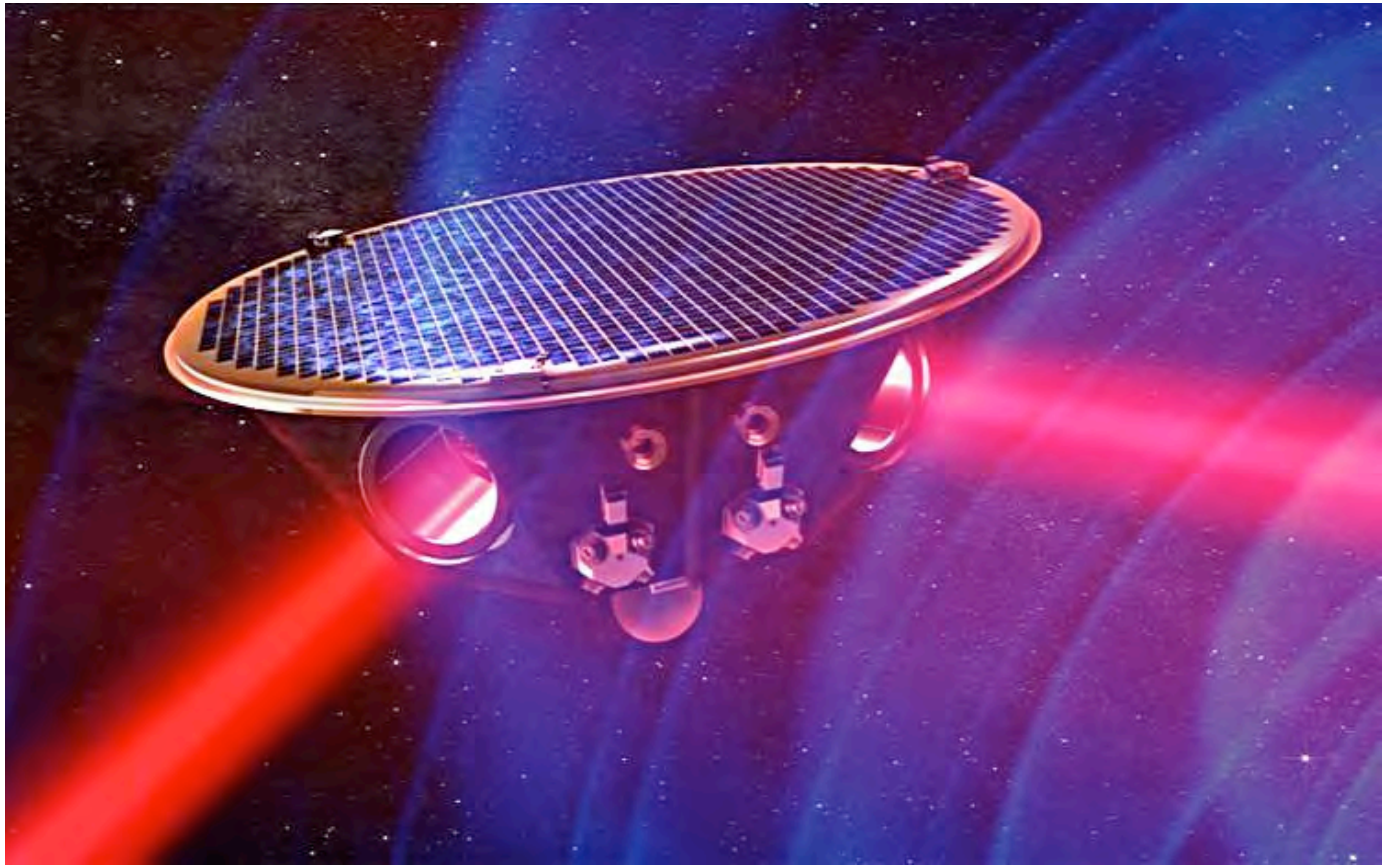
March 2011, The Divorce

eLISA - Descoped LISA proposed for ESA-lead mission (2011)

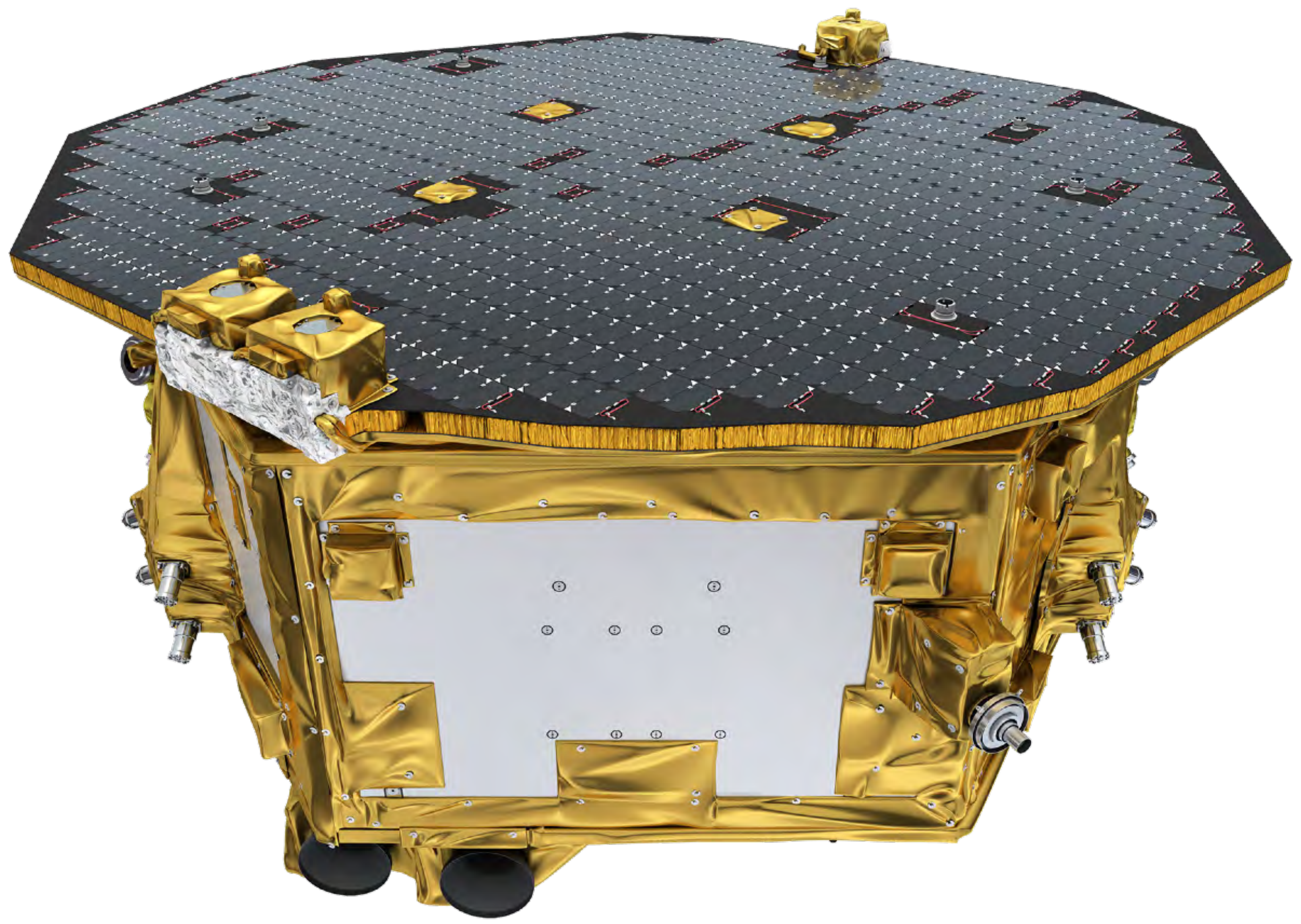


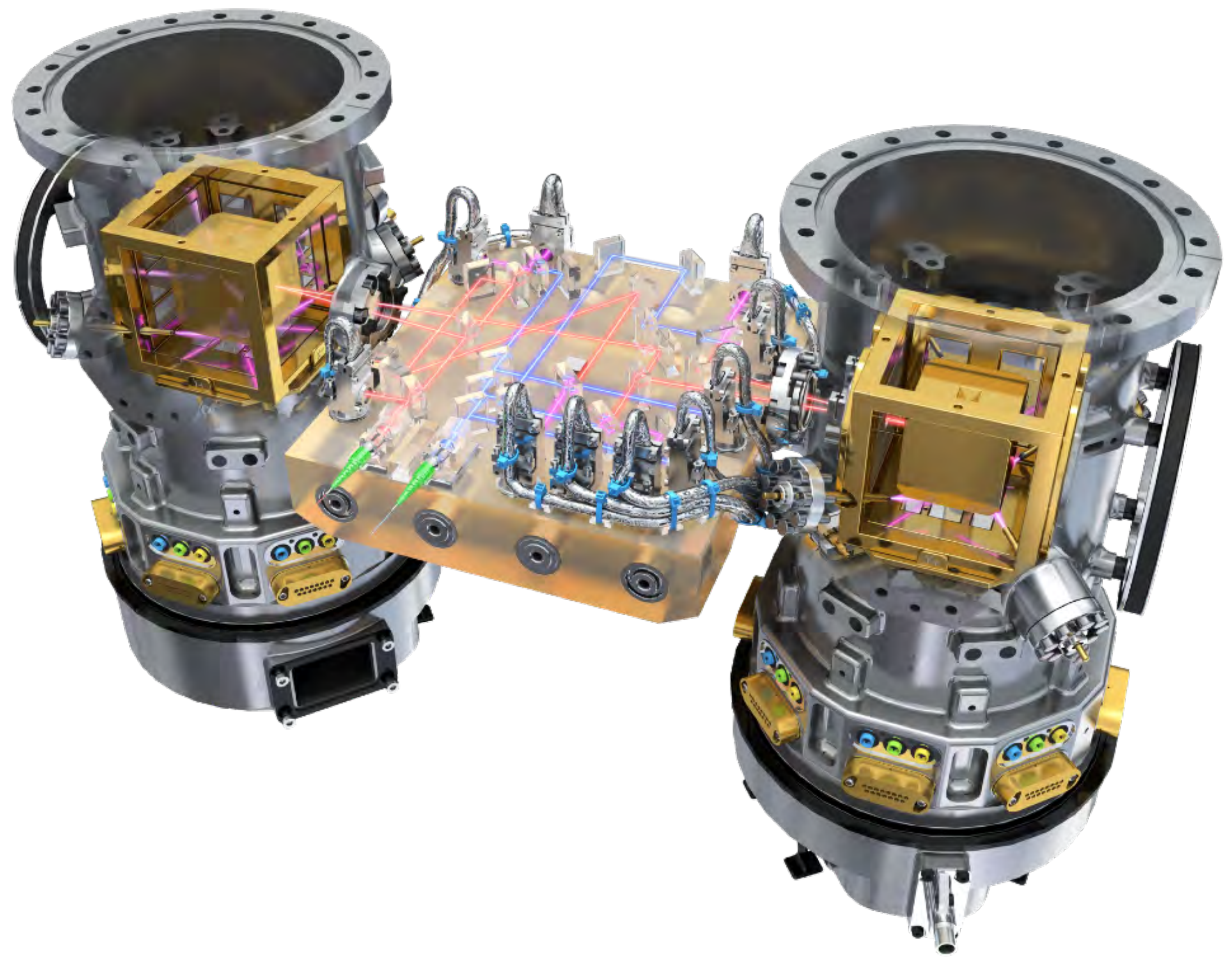
Cosmic Visions L1 Candidate

The Gravitational Universe selected as L3 science theme (2013)

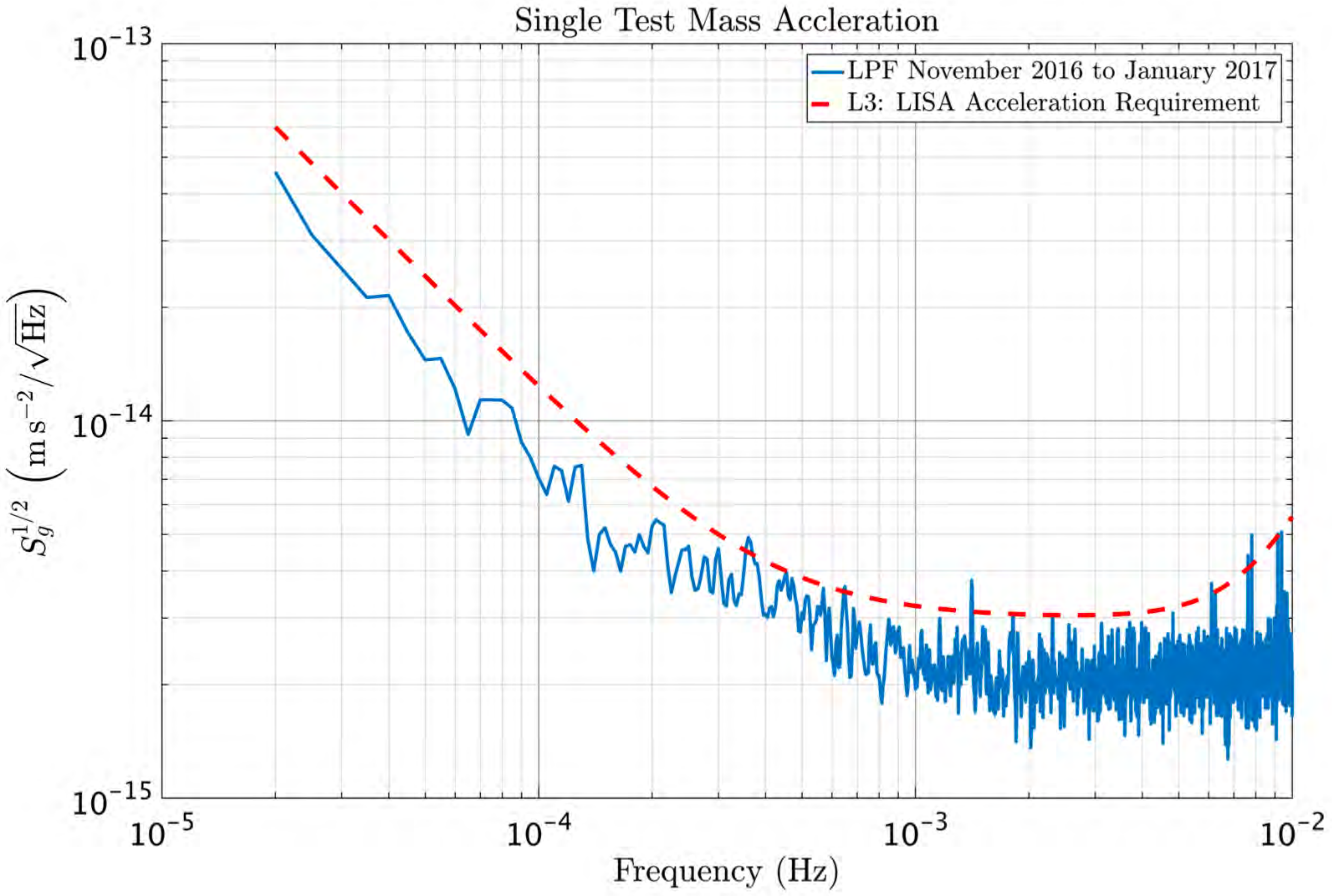


eLISA as candidate mission concept: Launch in 2034

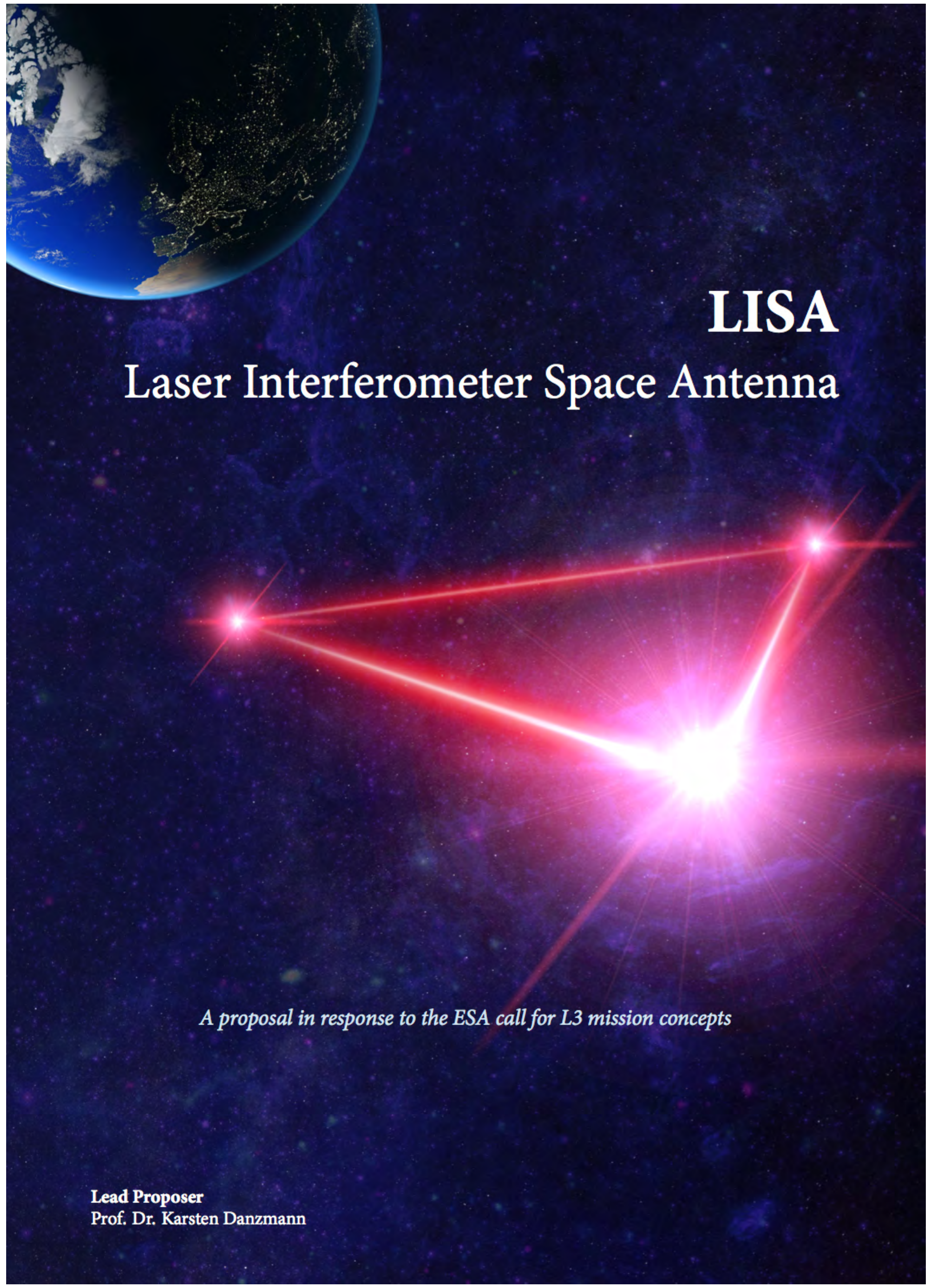


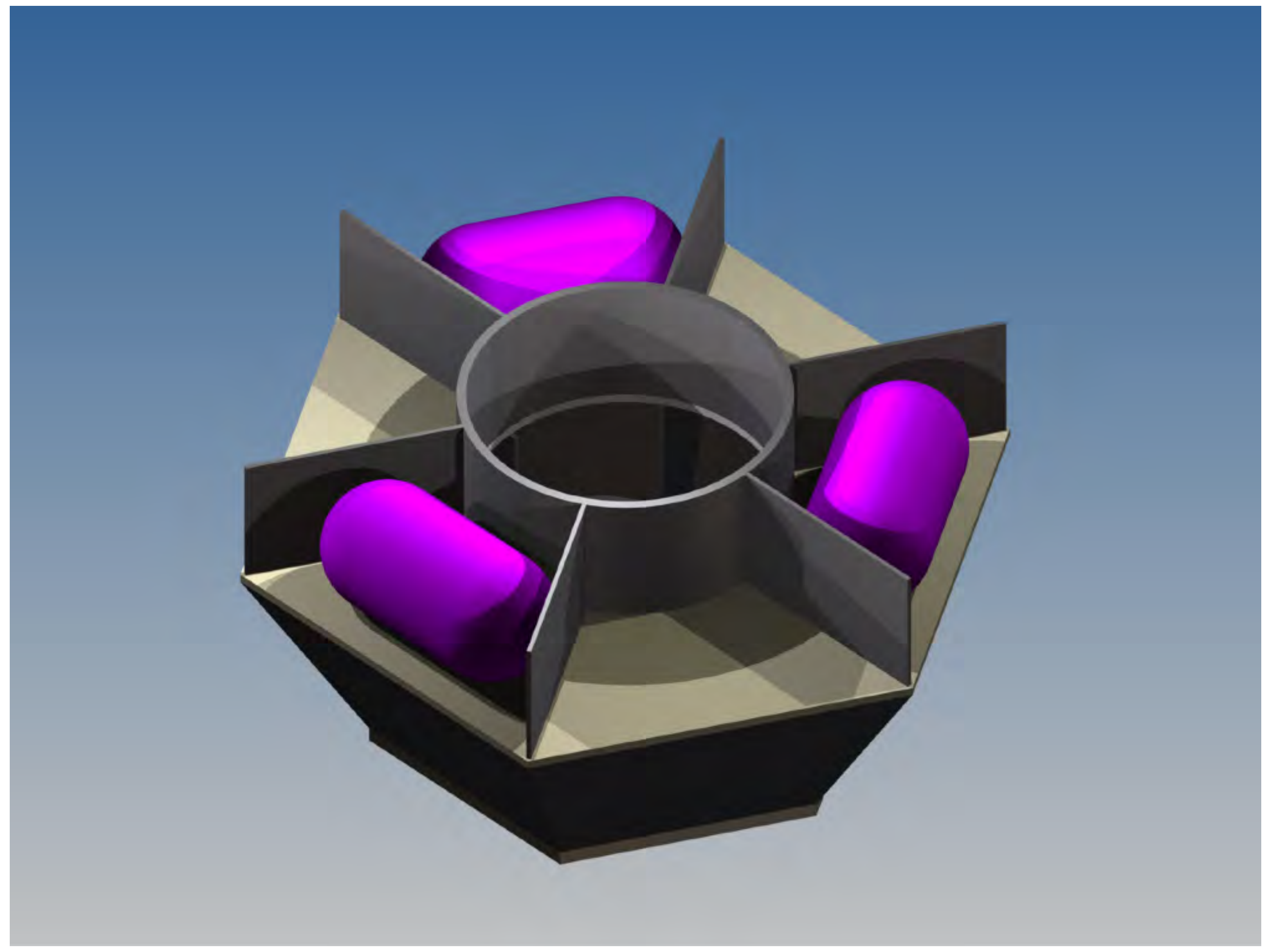


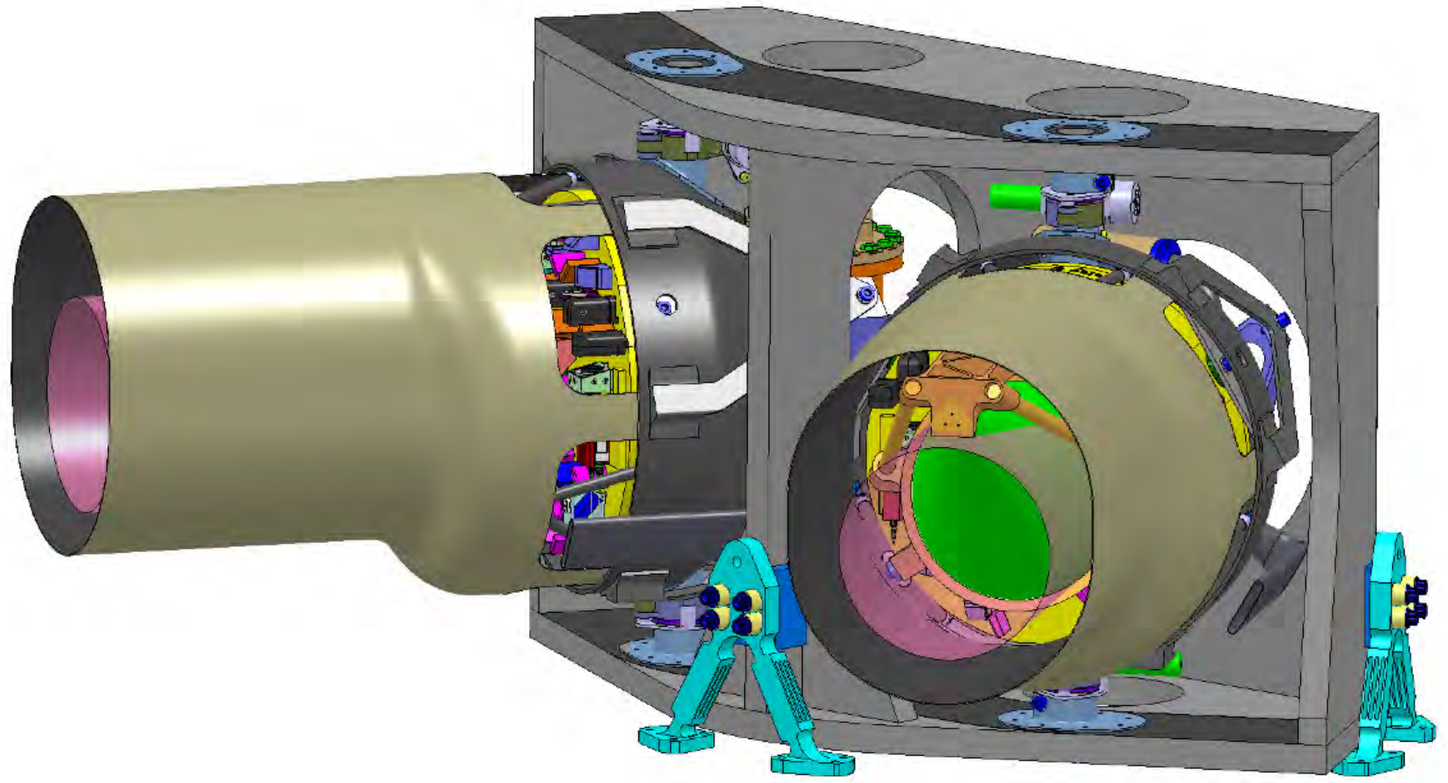
Near perfect free-fall demonstrated by the LISA Pathfinder mission in 2016

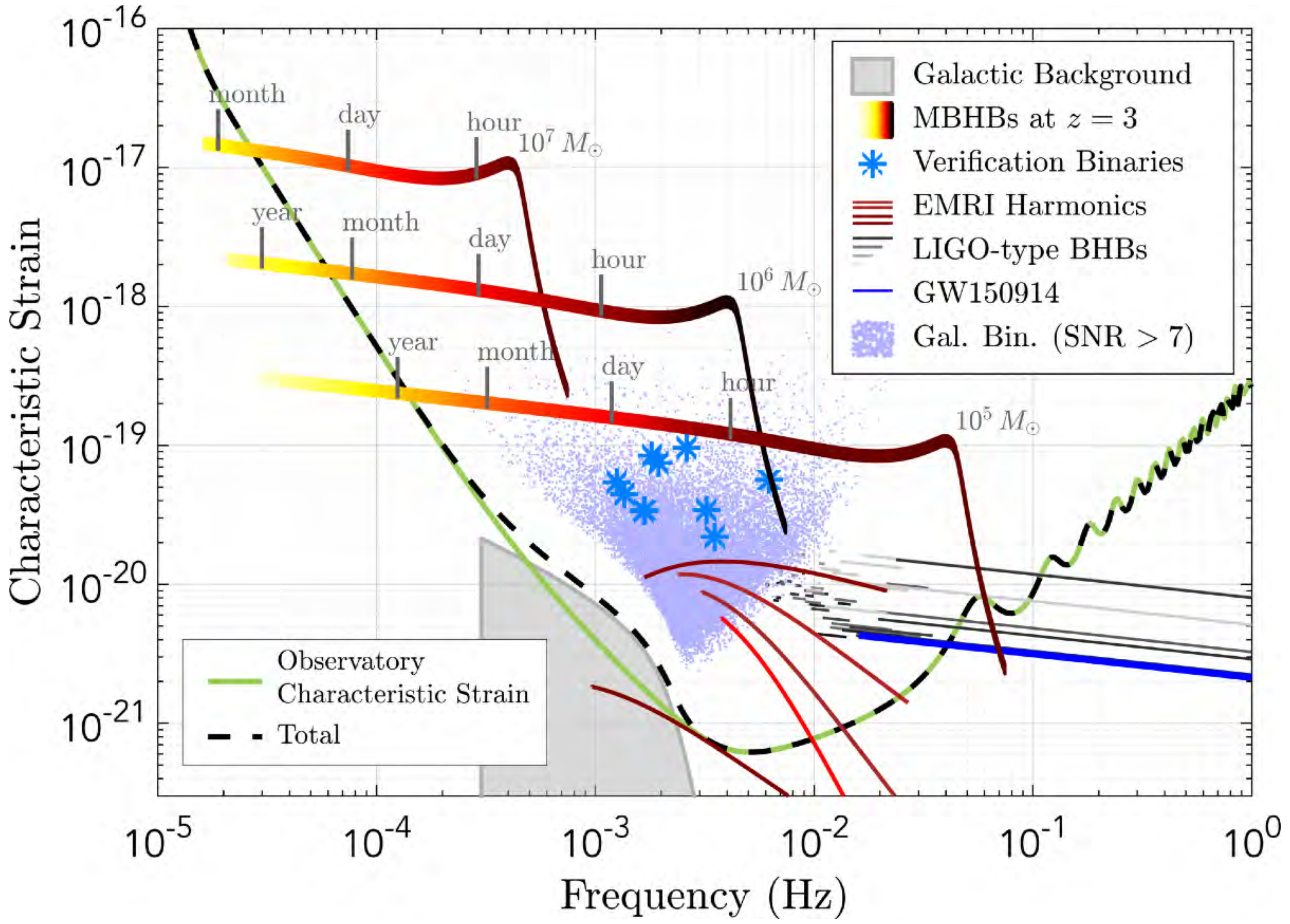


January 2017, LISA mission proposed for ESA L3 science theme

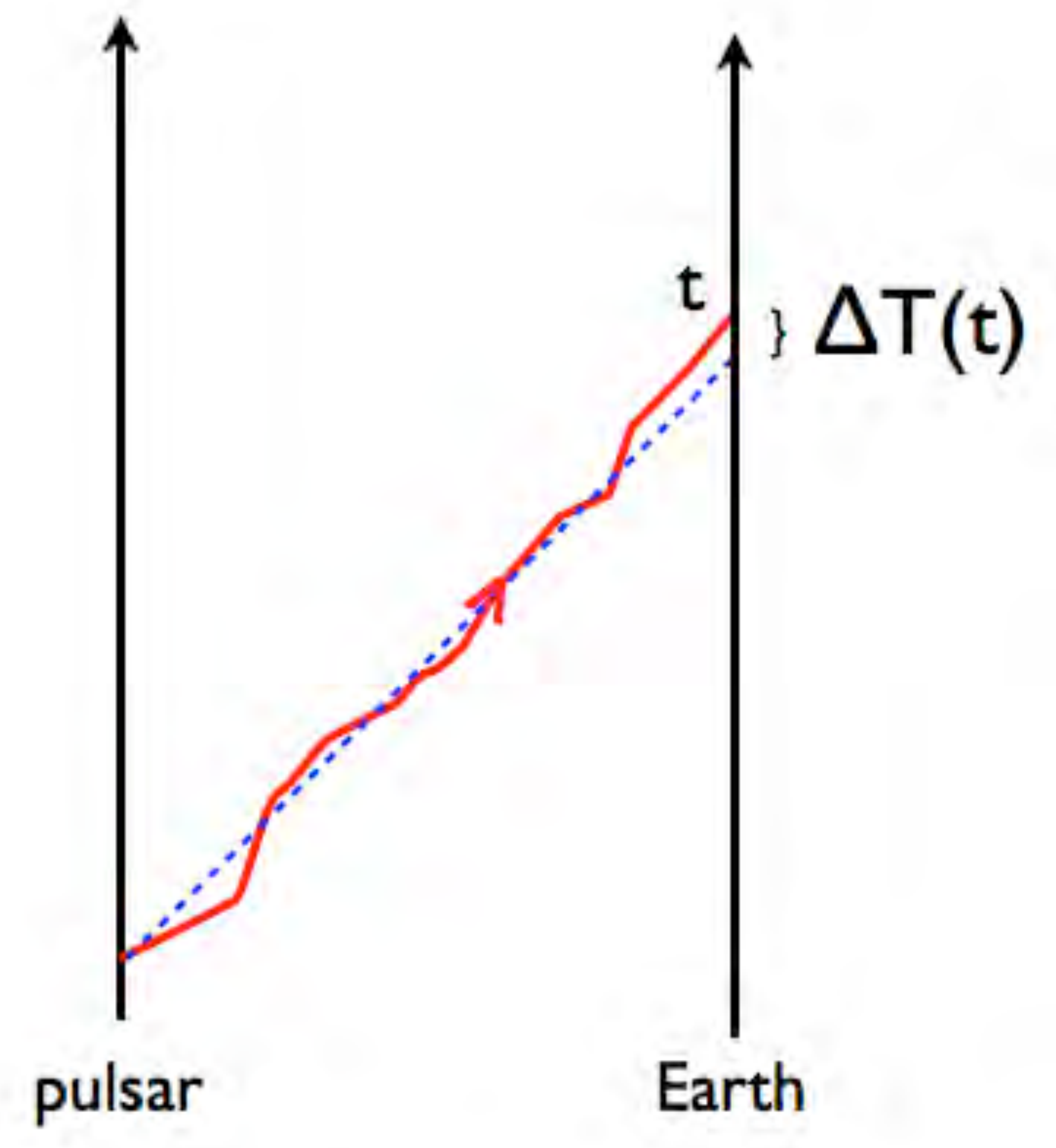




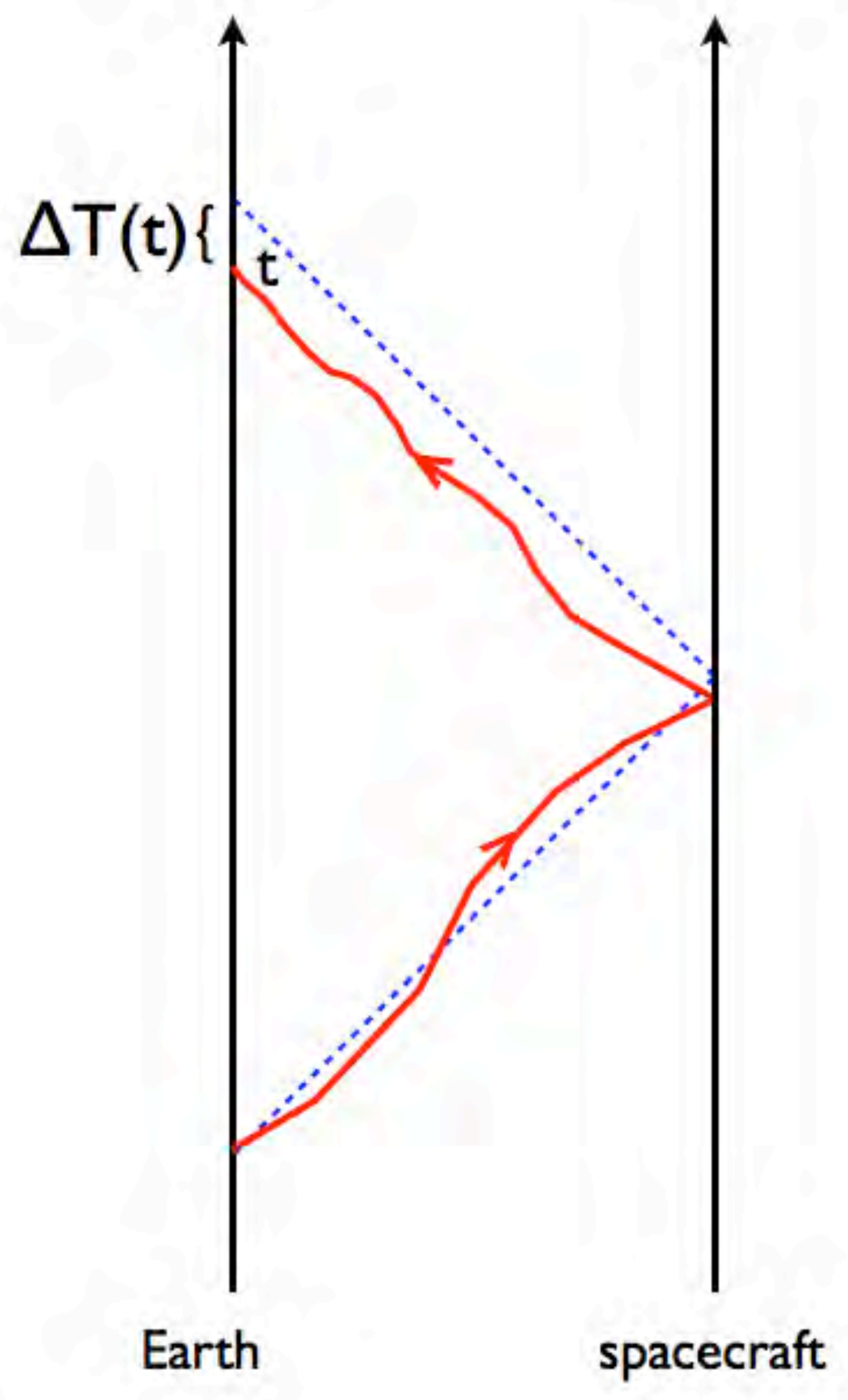




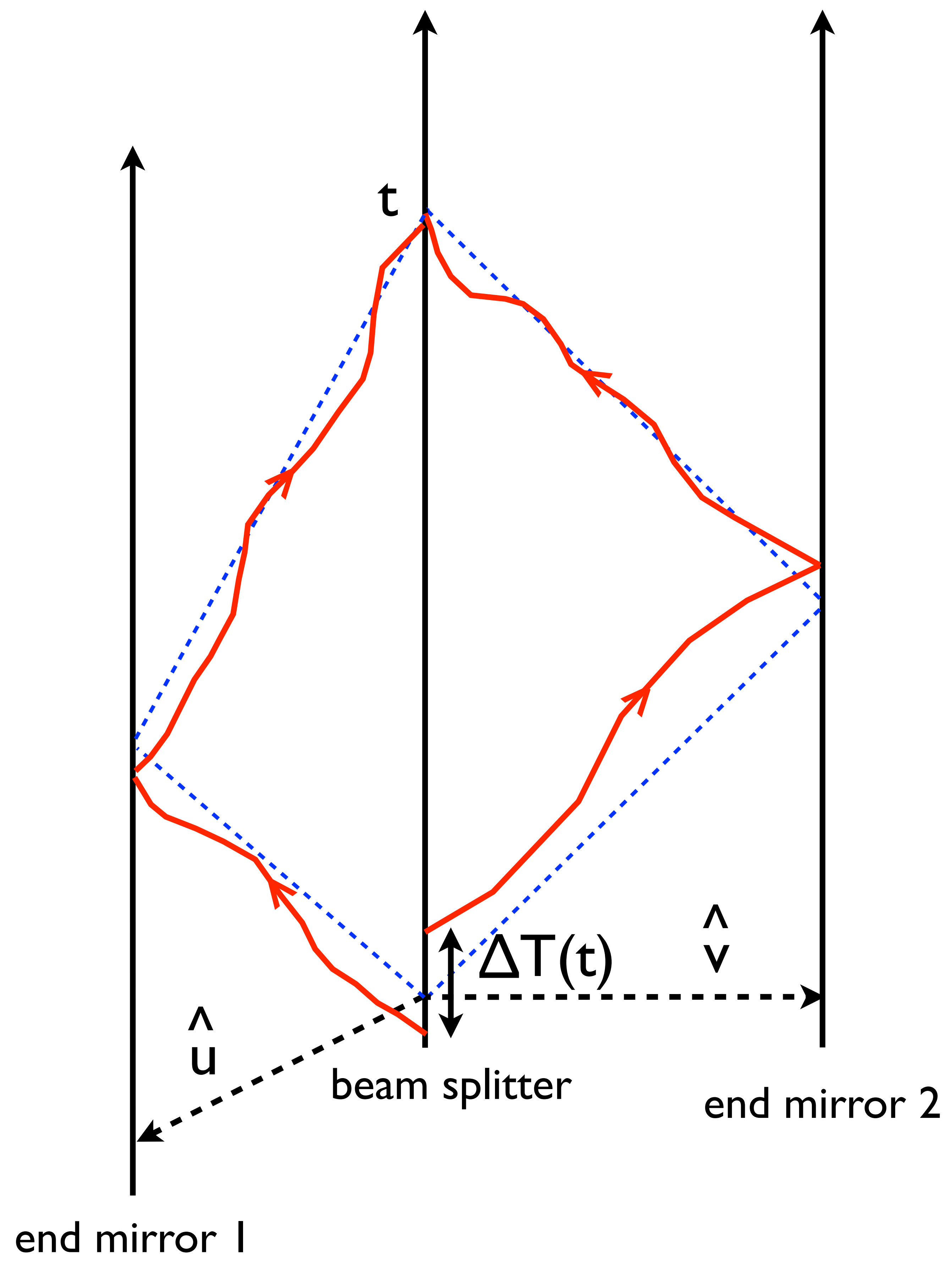
Time of flight detectors



Pulsar Timing



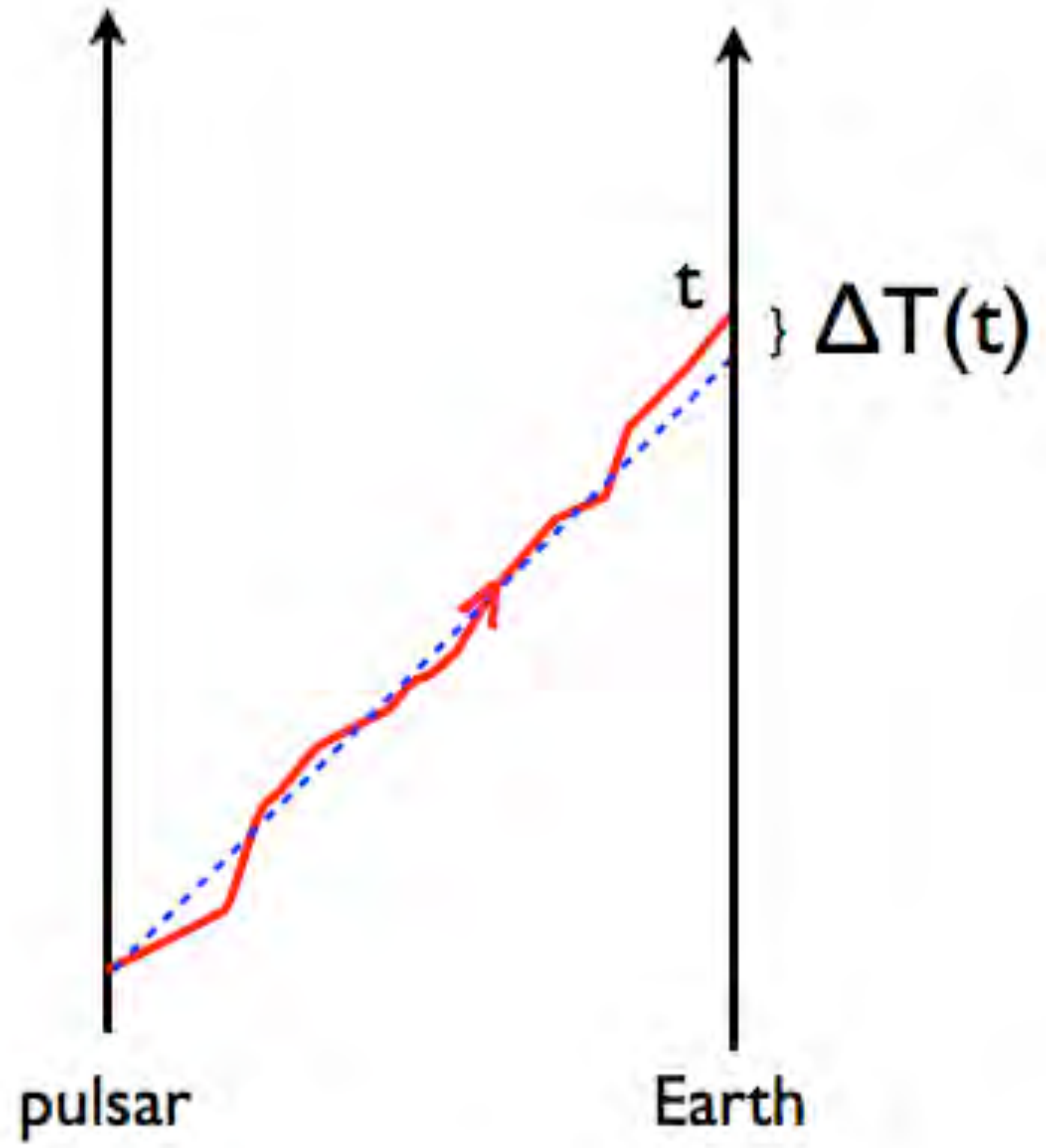
Spacecraft tracking



Laser Interferometers

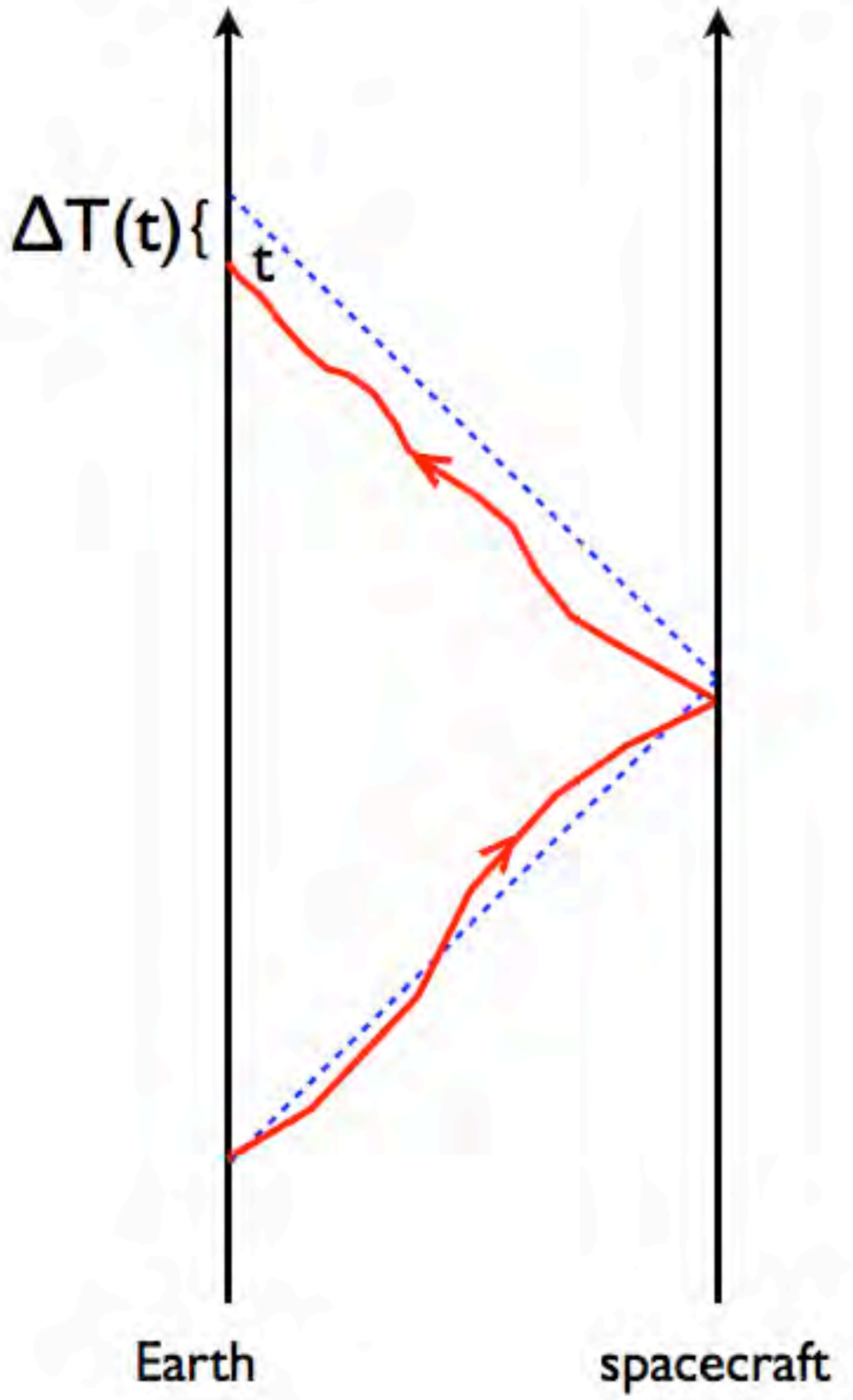
Time of flight detectors

$$\Delta T(t)$$



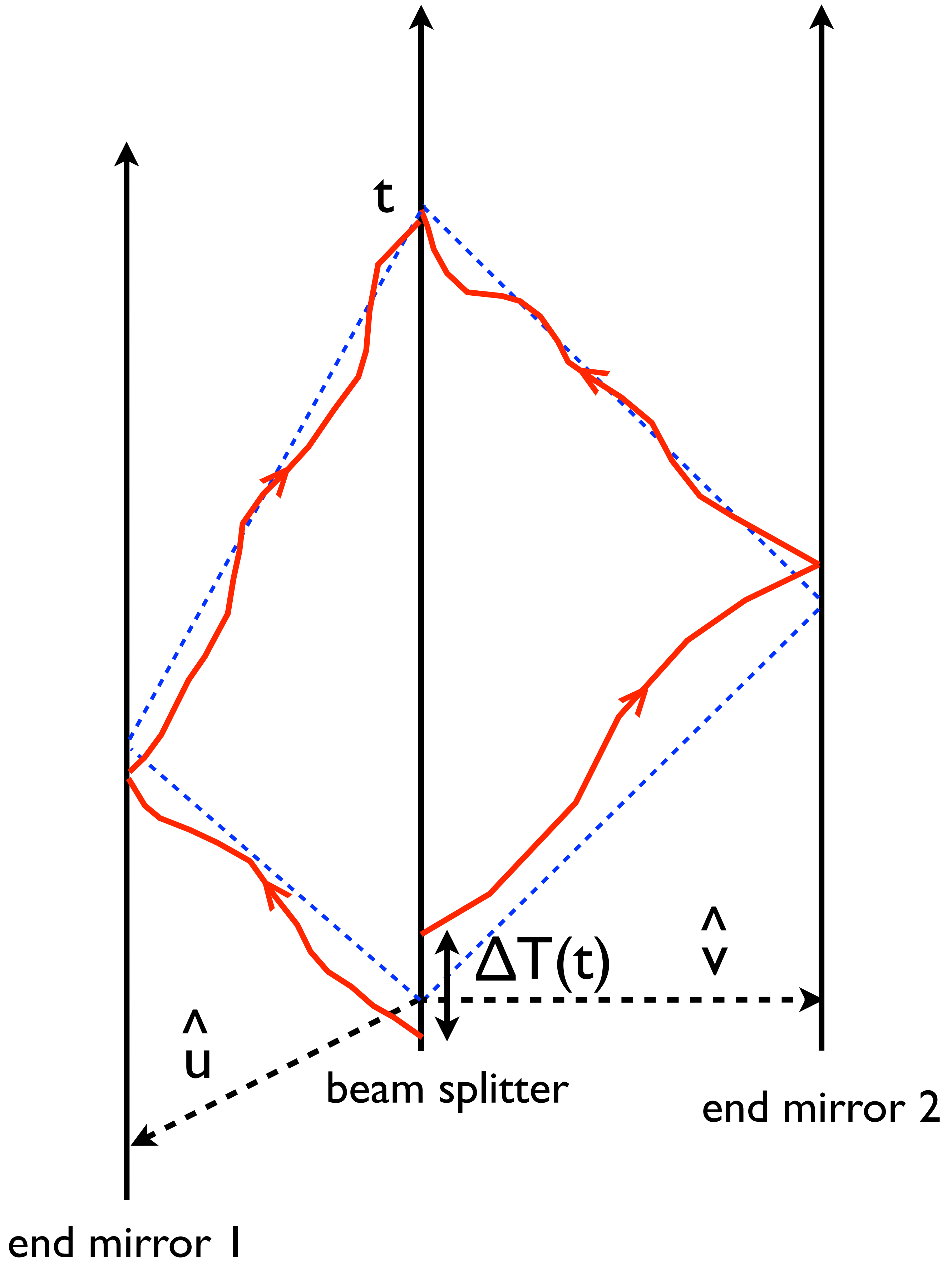
Pulsar Timing

$$\frac{\Delta\nu(t)}{\nu_0} = \frac{d\Delta T(t)}{dt}$$



Spacecraft tracking

$$\Phi(t) = 2\pi\nu_0\Delta T(t)$$

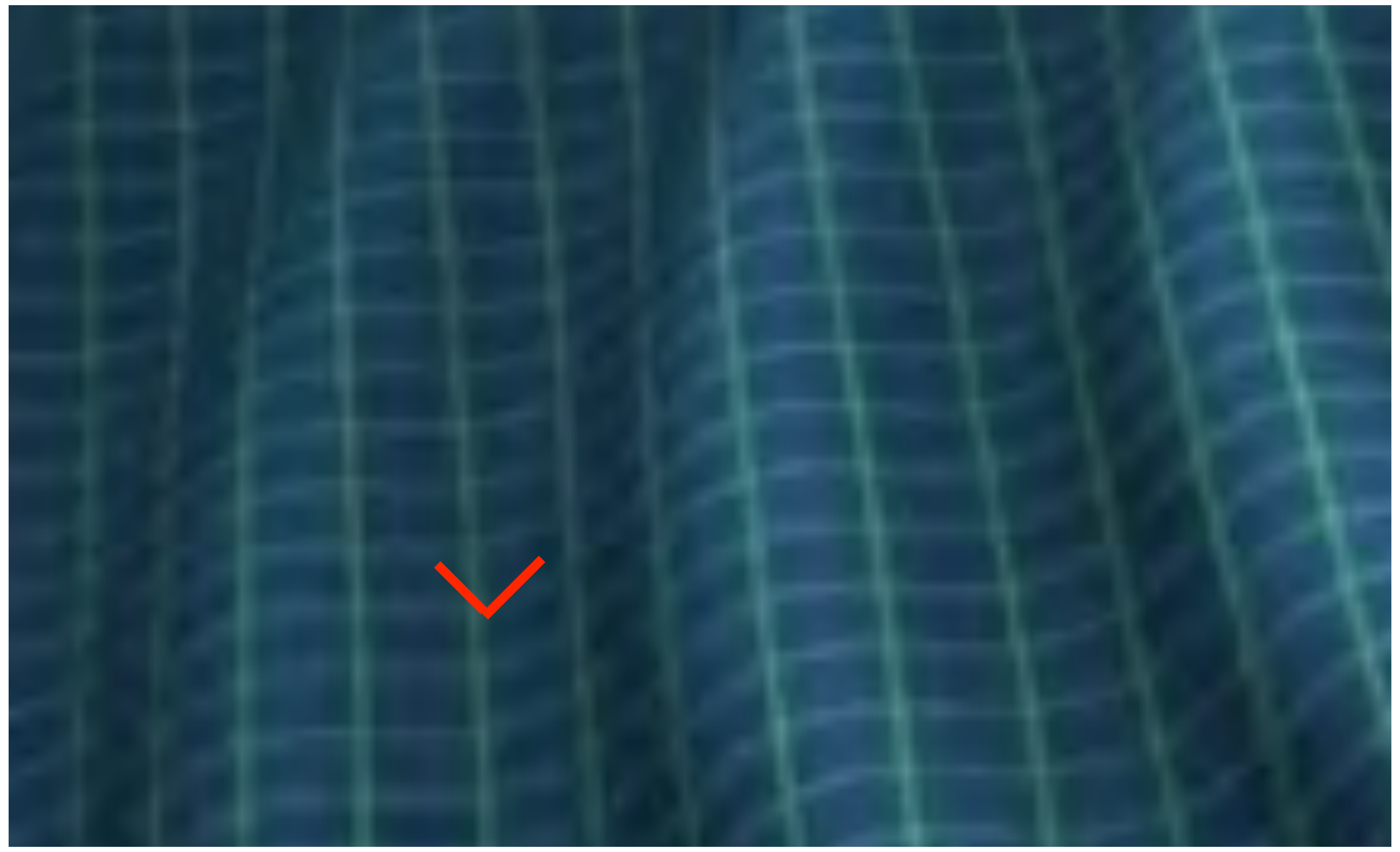


Laser Interferometers

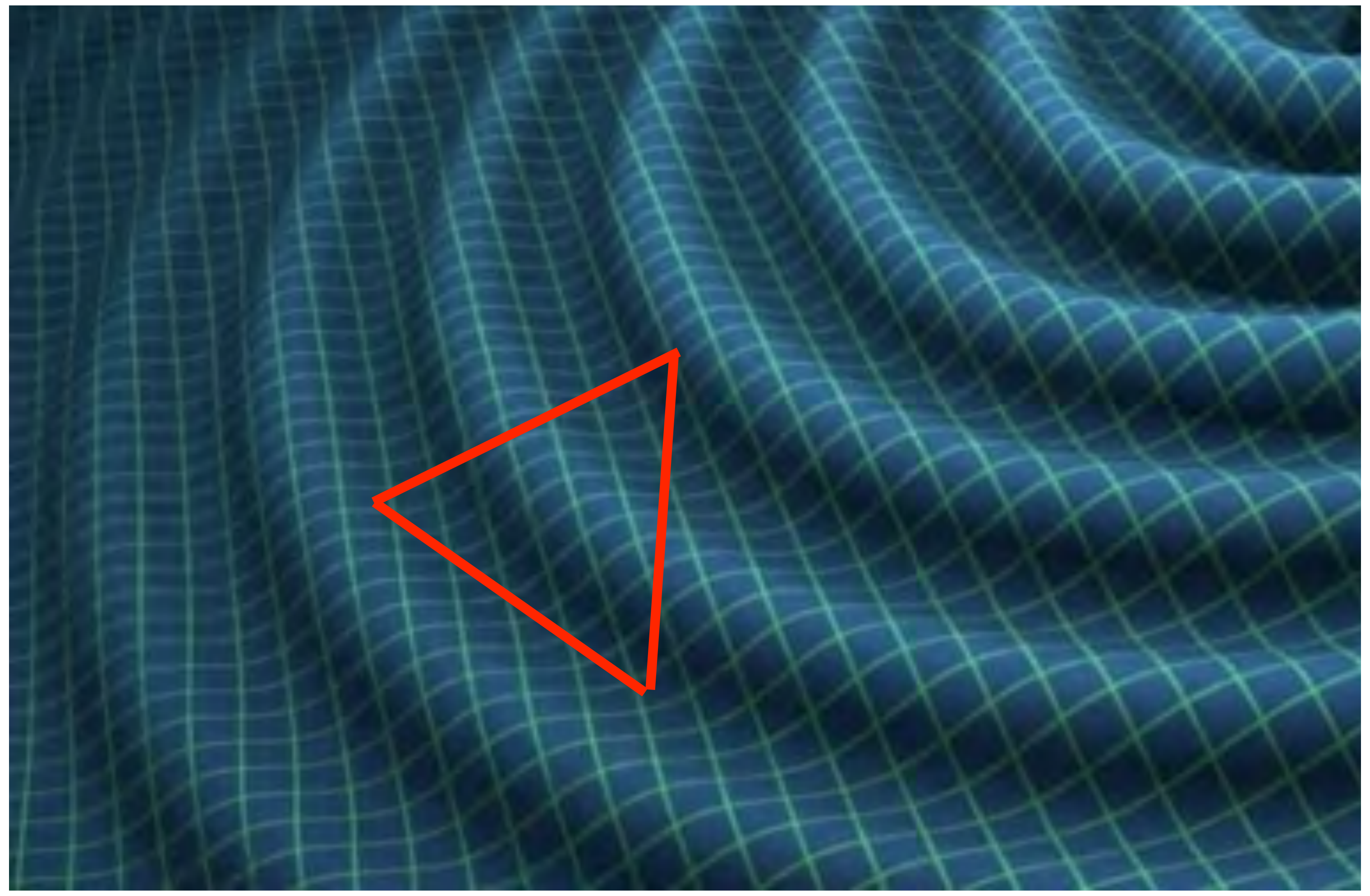
The Long and the Short of it

Beam detector	L (km)	f_* (Hz)	f (Hz)	f/f_*	Relation
Ground-based interferometer	~ 1	$\sim 10^5$	10 to 10^4	10^{-4} to 10^{-1}	$f \ll f_*$
Space-based interferometer	$\sim 10^6$	$\sim 10^{-1}$	10^{-4} to 10^{-1}	10^{-3} to 1	$f \lesssim f_*$
Spacecraft Doppler tracking	$\sim 10^9$	$\sim 10^{-4}$	10^{-6} to 10^{-3}	10^{-2} to 10	$f \sim f_*$
Pulsar timing	$\sim 10^{17}$	$\sim 10^{-12}$	10^{-9} to 10^{-7}	10^3 to 10^5	$f \gg f_*$

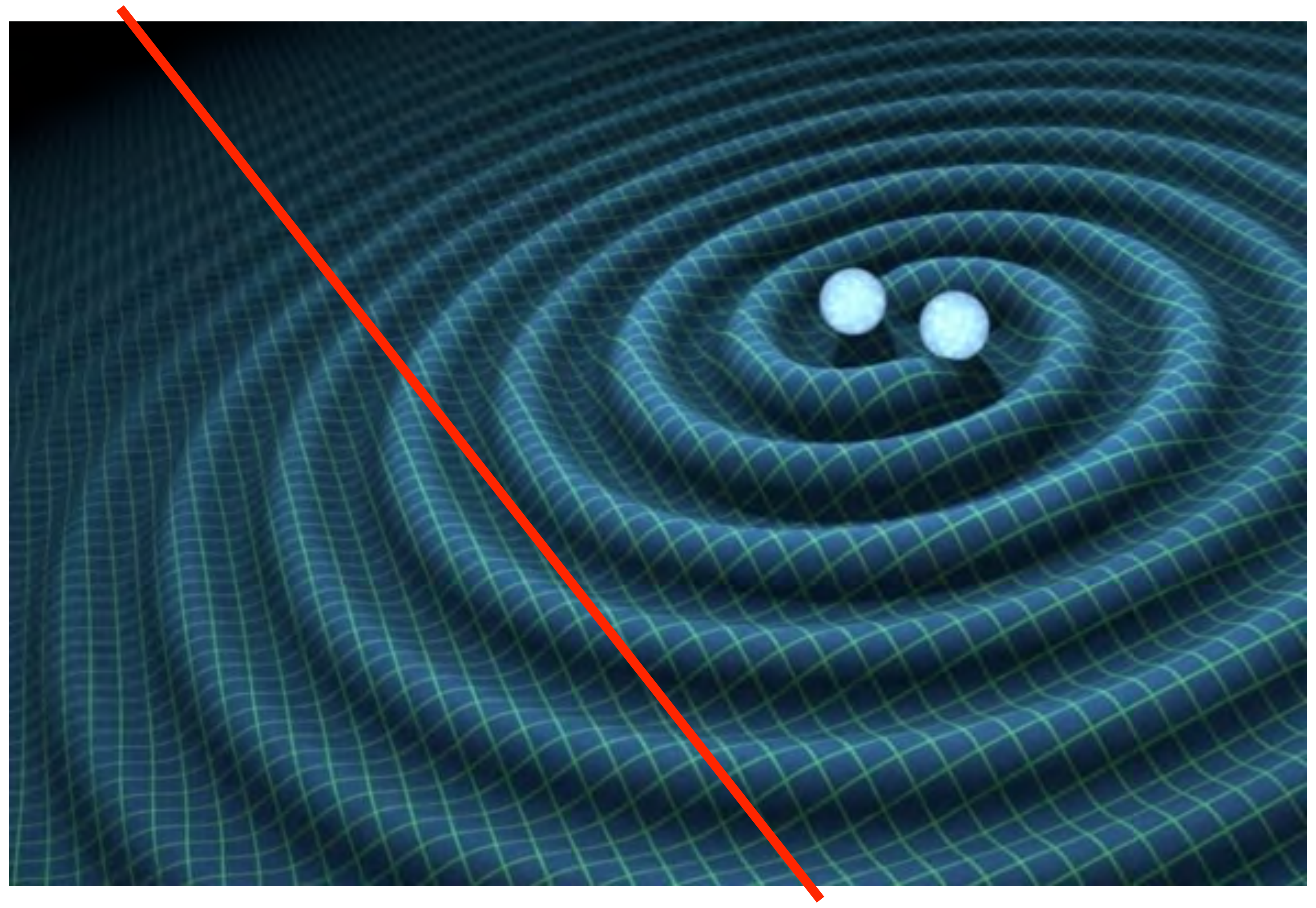
$$f_* = \frac{c}{L}$$



LIGO



LISA



PTA

Review: Gravitational Wave Theory

The Transverse-Traceless gauge is well suited for computing the GW response of time-of-flight detectors

Line element for a plane wave propagating in the +z direction in a TT coordinate system

$$ds^2 = -dt^2 + dx^2(1 + h_+) + dy^2(1 - h_+) + 2h_\times dy dy + dz^2$$

$h_+, h_\times$ are functions of $u = t - z$

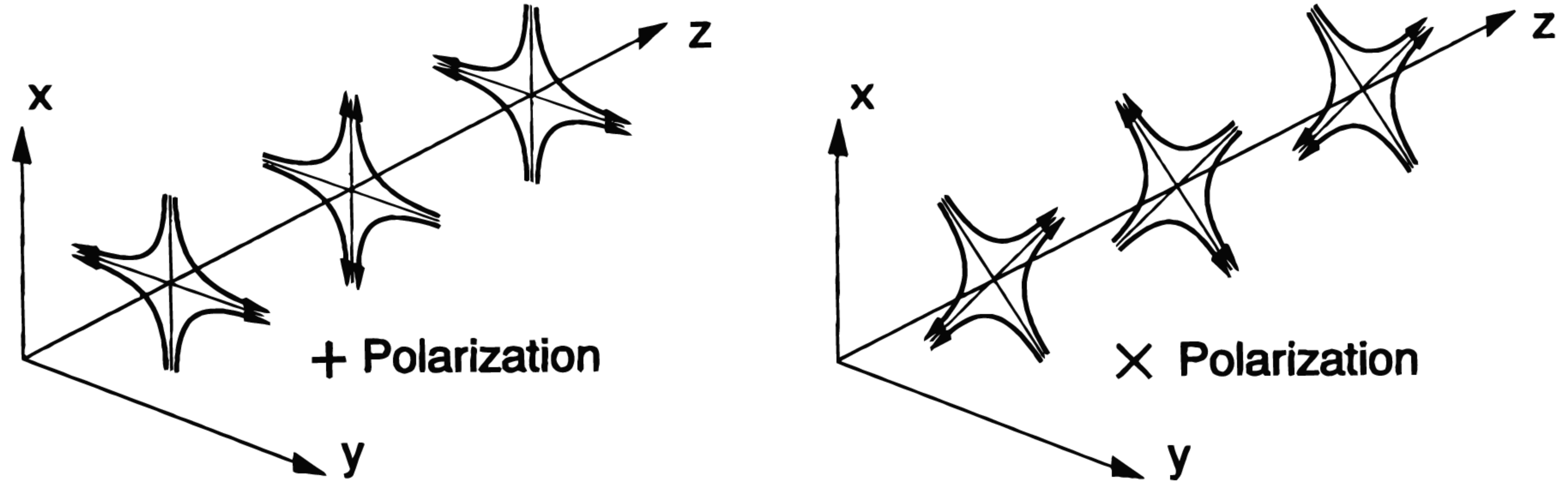
Any wave can be formed from a superposition of plane waves, so results derived using this metric are fully general. Results for other propagation directions and polarization frames can be found by a rotation

Review: Gravitational Wave Theory

$$ds^2 = -dt^2 + dx^2(1 + h_+) + dy^2(1 - h_+) + 2h_\times dydy + dz^2$$

The metric describes flat space with small curvature ripples (time dependent tidal field)

$$R_{txtx} = R_{tyty} = -\frac{1}{2}\ddot{h}_+ \qquad R_{txty} = R_{tytx} = -\frac{1}{2}\ddot{h}_\times$$



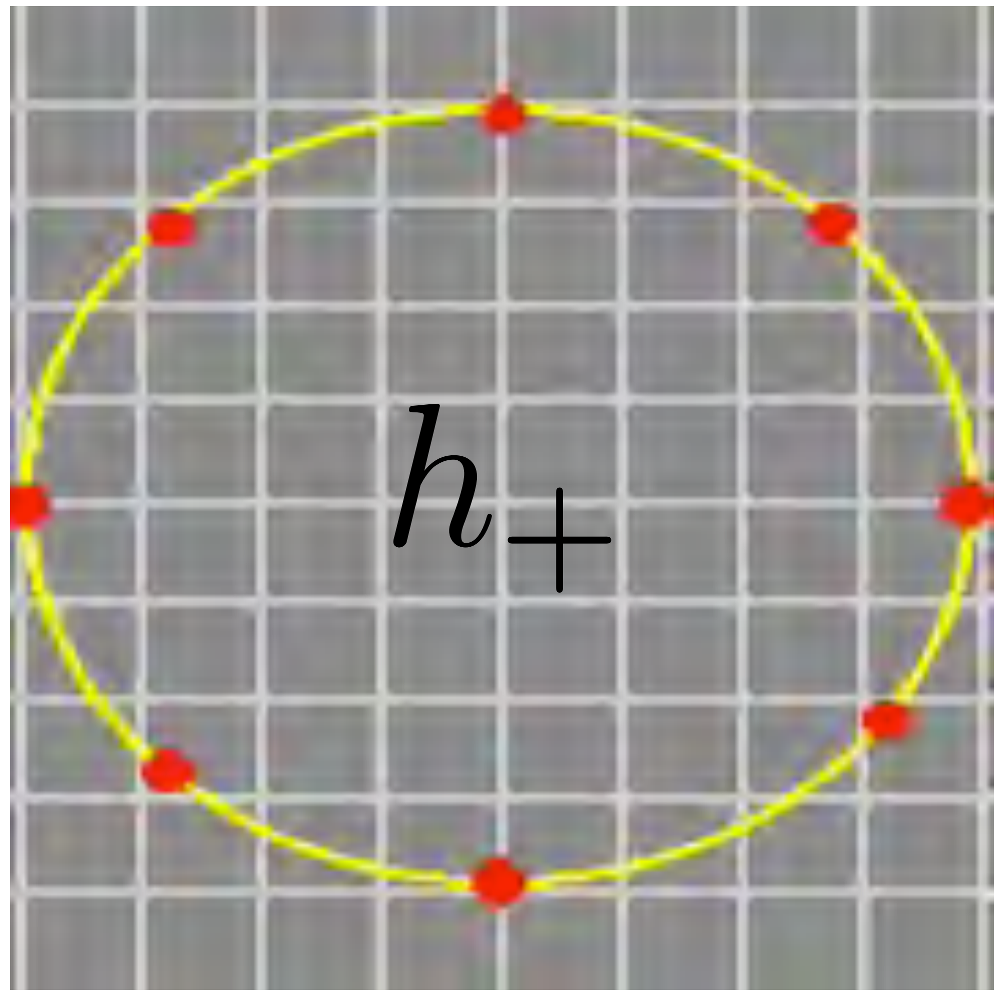
Review: Gravitational Wave Theory

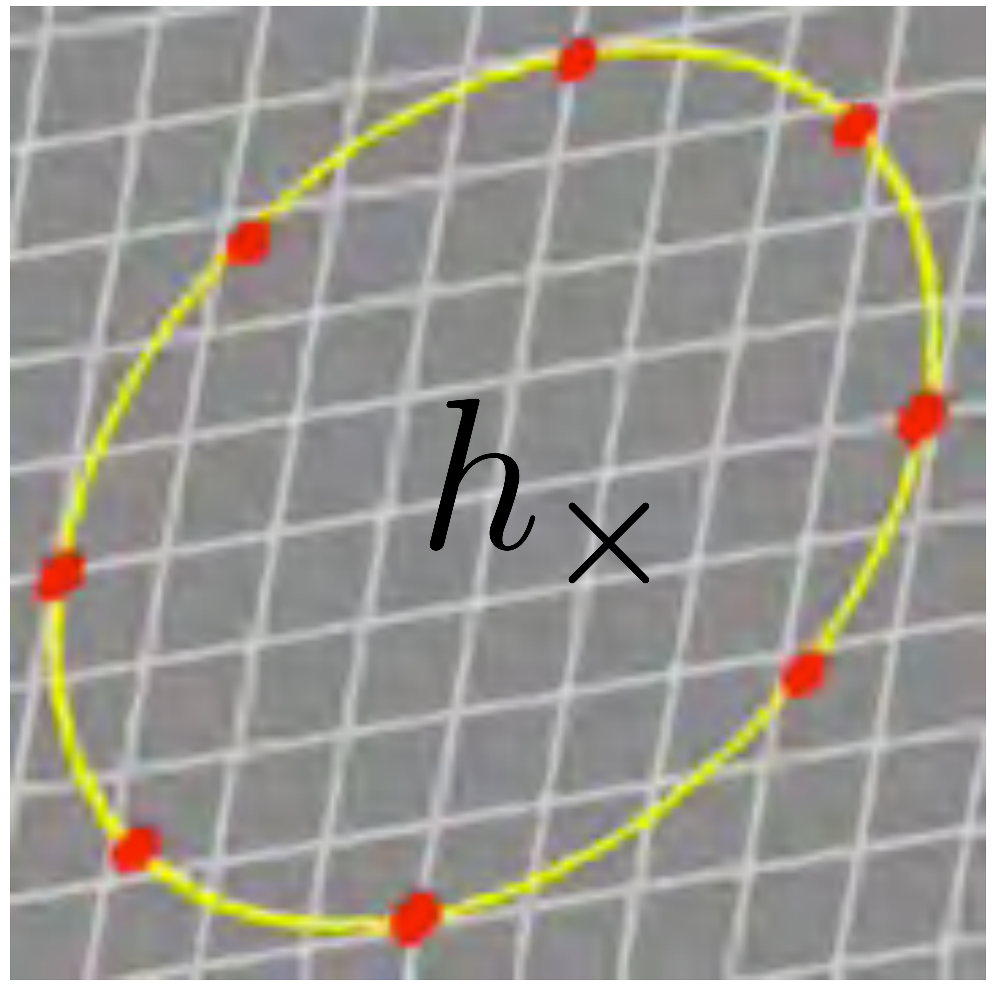
$$ds^2 = -dt^2 + dx^2(1+h_+) + dy^2(1-h_+) + 2h_\times dydy + dz^2$$

The tidal effects can be seen in the acceleration of the separation between two nearby geodesics

Writing the separation vector as $\boldsymbol{\xi} \to \xi(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$

The geodesic deviation equation yields $\ddot{\zeta} = -\frac{1}{\xi} R_{titj}\xi^i\xi^j = \frac{\xi}{2}\sin^2\theta\left(\ddot{h}_+\cos(2\phi) + \ddot{h}_\times\sin(2\phi)\right)$





Animation of $\zeta(t)$ for $\theta = \frac{\pi}{2}$

Review: Gravitational Wave Theory

$$ds^2 = -dt^2 + dx^2(1 + h_+) + dy^2(1 - h_+) + 2h_\times dydy + dz^2$$

The tidal stretching picture is only valid for geodesics separated by much less than the wavelength of the gravitational wave. Applies to LIGO, but not pulsar timing or LISA.

If the detector is much smaller than the GW wavelength, then it is convenient to work in a locally inertial coordinate system (RNC):

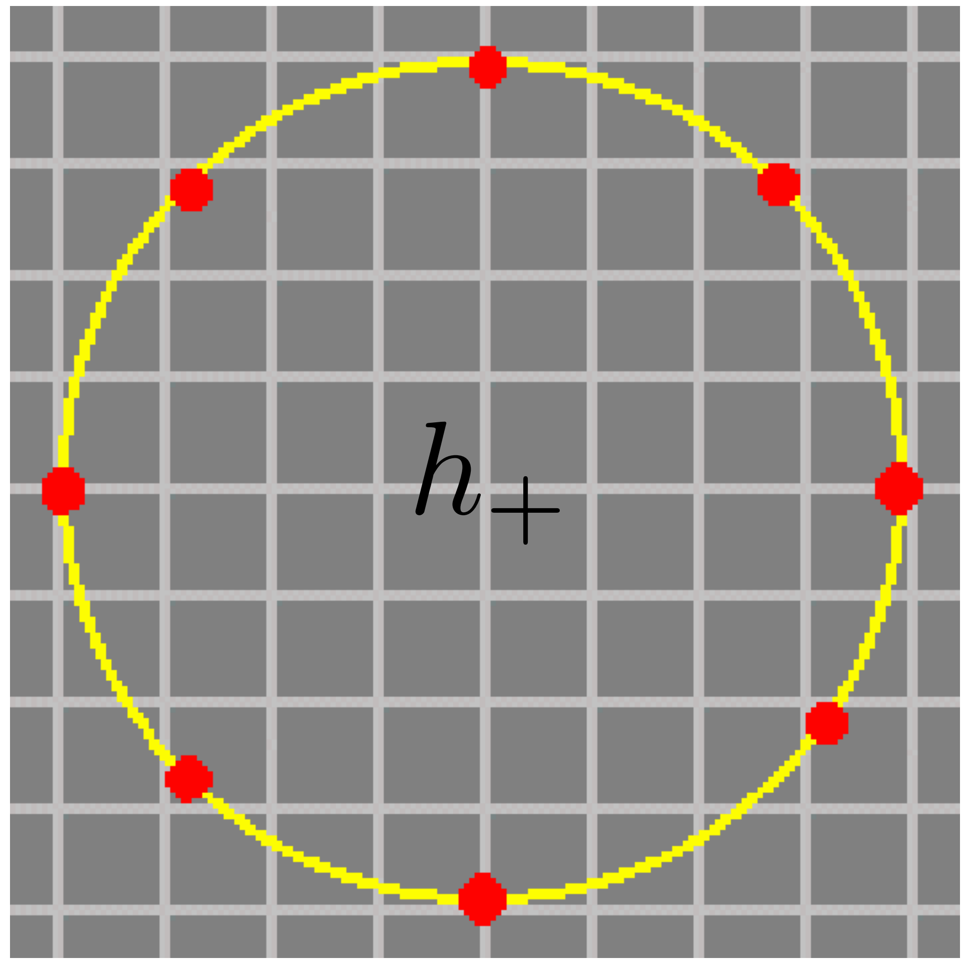
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + \frac{1}{2}(dt - dz)^2 \left(\ddot{h}_+(x^2 - y^2) - 2\ddot{h}_\times\, xy\right)$$

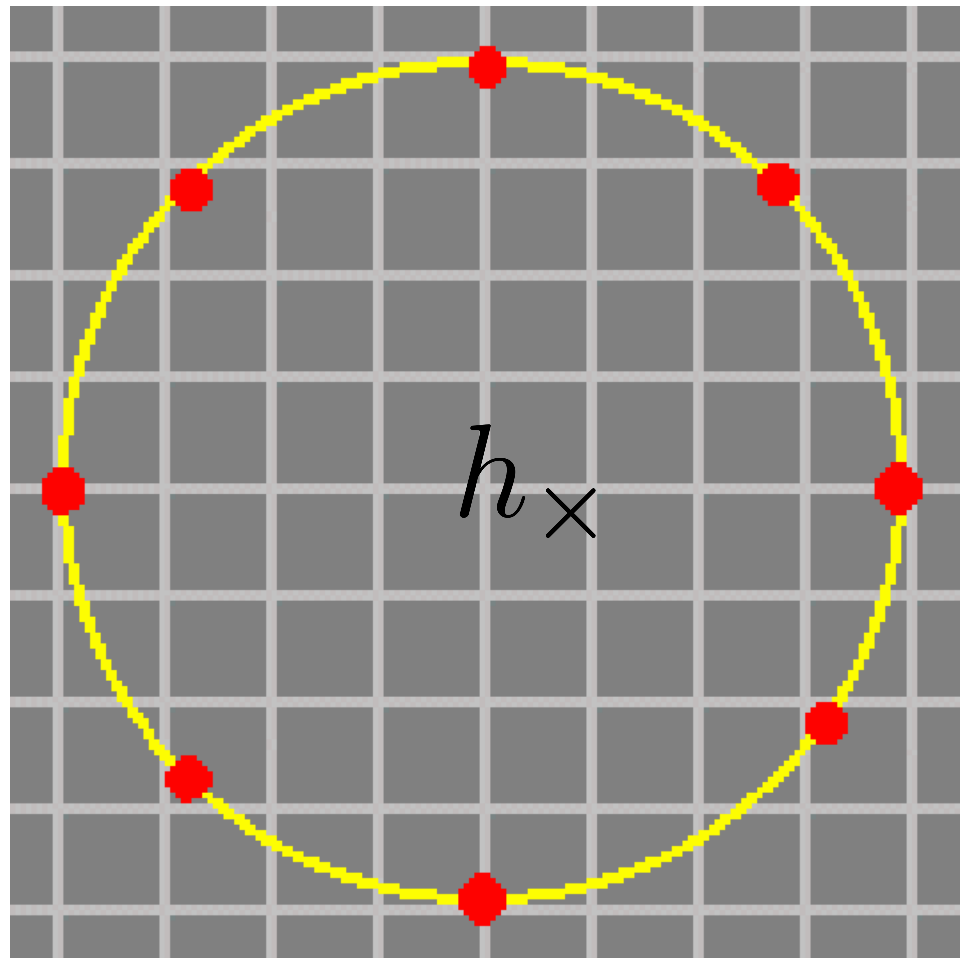
Maggiore refers to this as the proper detector frame in his textbook (where he also includes the local gravitational acceleration and Coriolis forces)

Review: Gravitational Wave Theory

$$ds^2 = -dt^2 + dx^2(1 + h_+) + dy^2(1 - h_+) + 2h_\times dydy + dz^2$$

A defining feature of the TT gauge is that the coordinate acceleration vanishes $\ddot{x}^i = 0$





Animation of a GW directed perpendicular to a ring of test particles in the TT gauge
(this animation even works in PDF and printed format!)

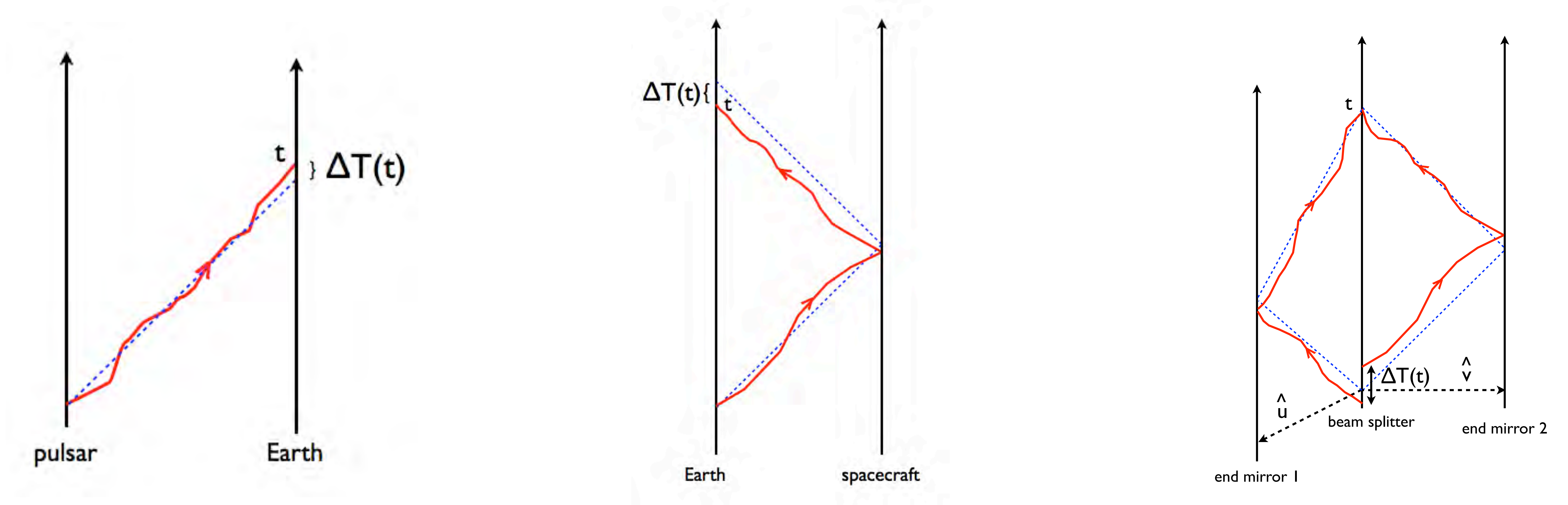
Time of flight computed in TT gauge

$$ds^2 = -dt^2 + dx^2(1 + h_+(u)) + dy^2(1 - h_+(u)) + 2h_\times(u)dydy + dz^2$$

$$= -dvdu + dx^2(1 + h_+(u)) + dy^2(1 - h_+(u)) + 2h_\times(u)dydy$$

where $u = t - z, \quad v = t + z$

All time-of-flight detectors require us to compute the time it takes a photon to travel from one event to another in the spacetime perturbed by a GW. Some require multiple trips



Time of flight computed in TT gauge

$$ds^2 = -dudv^2 + dx^2(1 + h_+(u)) + dy^2(1 - h_+(u)) + 2h_\times(u)dydy + dz^2$$

Have to solve for null geodesics in this metric. We could integrate the geodesic equations, but the spacetime has lots of symmetry, and hence conserved quantities. No integration needed!

Killing vectors $\vec{\partial}_x,\ \vec{\partial}_y,\ \vec{\partial}_v$ Photon worldline $x^\alpha(\lambda)$ Photon 4-velocity $S^\alpha = \frac{dx^\alpha}{d\lambda}$

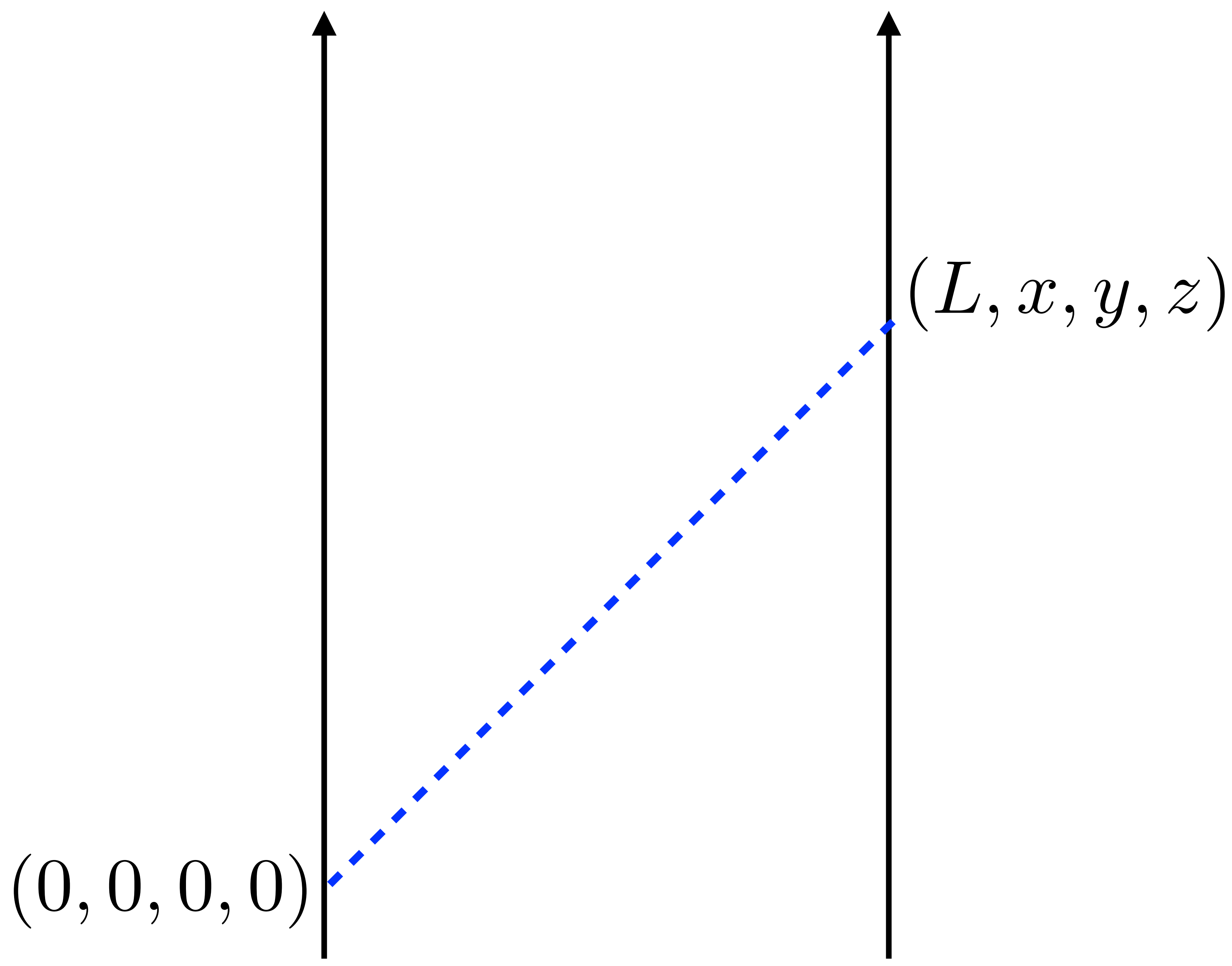
Killing vectors yield three constants of motion $S_x(\lambda) = \alpha_x, \quad S_y(\lambda) = \alpha_y, \quad S_v(\lambda) = \alpha_z$

and we have the fourth condition $S_\alpha S^\alpha = 0$

[Derivation follows N. J. Cornish, Phys.Rev.D80:087101,2009, arXiv:0910.4372]

Time of flight computed in TT gauge

$$ds^2 = -dudv^2 + dx^2(1 + h_+(u)) + dy^2(1 - h_+(u)) + 2h_\times(u)dydy + dz^2$$



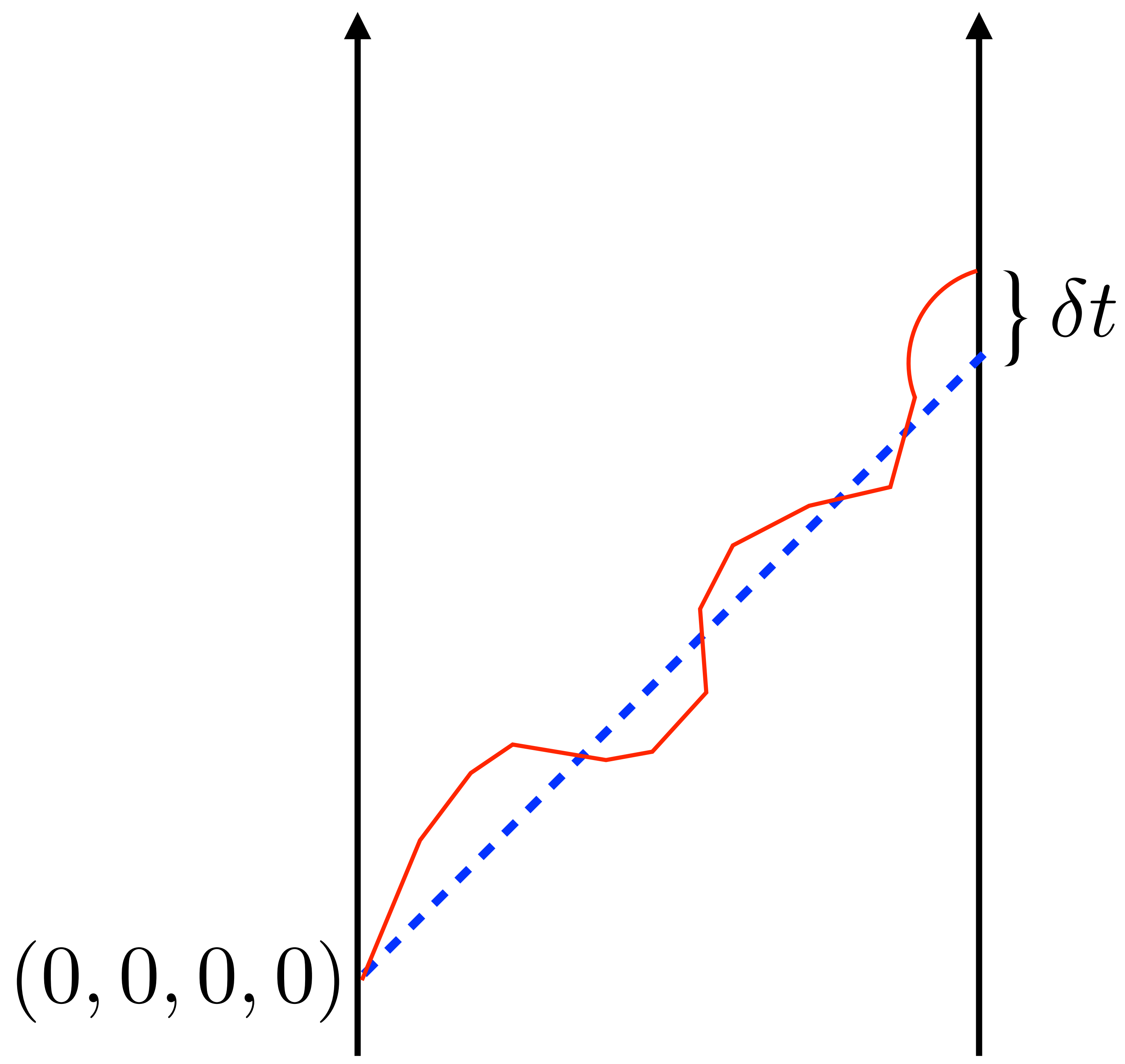
Path from $(0,0,0,0)$ to (L,x,y,z) in unperturbed spacetime has

$$\alpha_x = \frac{x}{\lambda_2 - \lambda_1}, \quad \alpha_y = \frac{y}{\lambda_2 - \lambda_1}, \quad \alpha_z = -\frac{L - z}{2(\lambda_2 - \lambda_1)}$$

$$t = L = \sqrt{x^2 + y^2 + z^2}$$

Time of flight computed in TT gauge

$$ds^2 = -dudv^2 + dx^2(1+h_+(u)) + dy^2(1-h_+(u)) + 2h_\times(u)dydy + dz^2$$



When GWs are present we have to change our aim:

$$\delta\alpha_x = \frac{1}{(L-z)(\lambda_2-\lambda_1)}(xH_{xx}+yH_{xy})$$

$$\delta\alpha_y = \frac{1}{(L-z)(\lambda_2-\lambda_1)}(yH_{yy}+xH_{xy})$$

$$\delta\alpha_z = -\frac{\delta t}{2(\lambda_2-\lambda_1)}$$

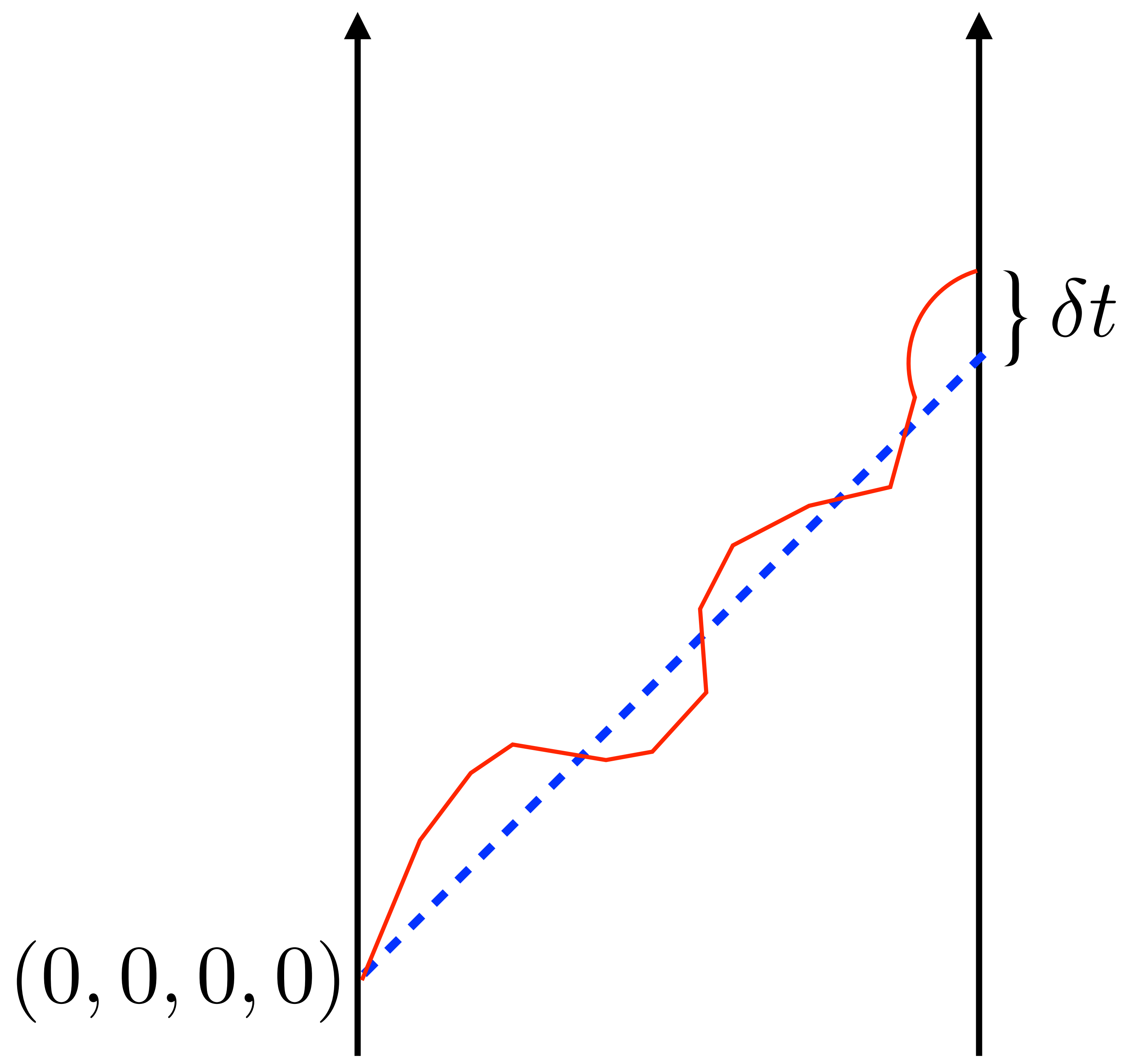
$$H_{ij} = \int_0^{L-z} h_{ij}(u)du$$

$$\delta t = \frac{1}{2L(L-z)}(x^2H_{xx}+y^2H_{yy}+2xyH_{xy})$$

$$(h_{xx} = -h_{yy} = h_+, \quad h_{xy} = h_\times)$$

Time of flight computed in TT gauge

$$\delta t = \frac{1}{2L(L-z)}(x^2 H_{xx} + y^2 H_{yy} + 2xyH_{xy})$$



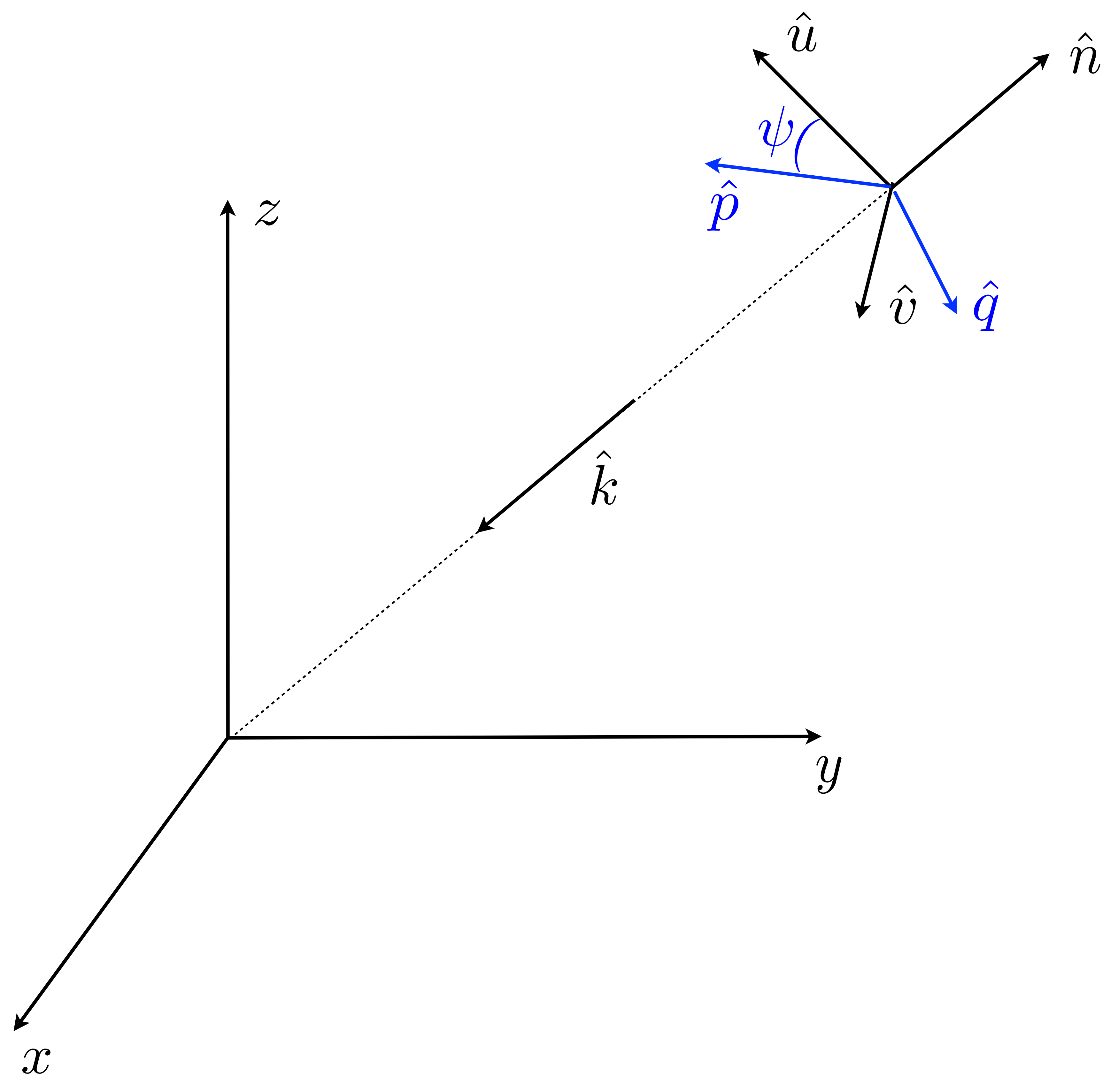
Coordinate independent version:

$$\Delta\tau_{12} = \frac{(\hat{a}\otimes\hat{a}) : \mathbf{H}[u_1, u_2]}{2(1-\hat{k}\cdot\hat{a})} \qquad (u = k_\alpha x^\alpha)$$

Here $\hat{a}$ is a unit vector along the detector arm and $\hat{k}$ is the GW propagation direction

$$\mathbf{H}[u_1, u_2] = \int_{u_1}^{u_2} \mathbf{h}(u)\, du \qquad \mathbf{h} = h_+(u)\, \epsilon^+ + h_\times(u)\, \epsilon^\times$$

General coordinate system



$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{u} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{v} = \sin\phi\,\hat{x} - \cos\phi\,\hat{y}$$

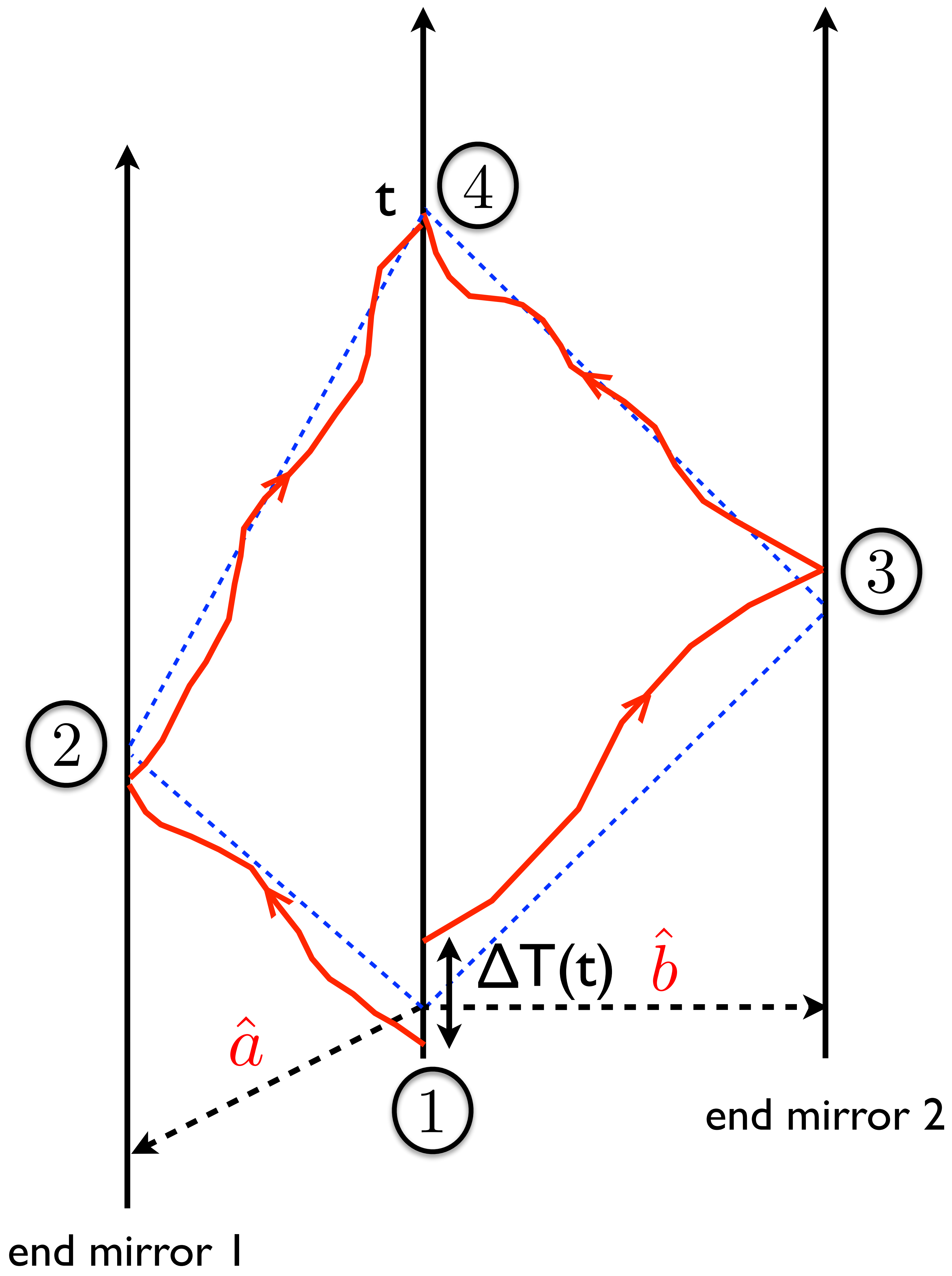
$$\mathbf{e}^{+} = \hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}$$
$$\mathbf{e}^{\times} = \hat{u}\otimes\hat{v} + \hat{v}\otimes\hat{u}$$

$$\mathbf{h} = h_{+}\boldsymbol{\epsilon}^{+} + h_{\times}\boldsymbol{\epsilon}^{\times}$$

$$\begin{aligned}\epsilon^{+} &= \hat{p}\otimes\hat{p} - \hat{q}\otimes\hat{q} \\ &= \cos 2\psi\,\mathbf{e}^{+} - \sin 2\psi\,\mathbf{e}^{\times}\end{aligned}$$

$$\begin{aligned}\epsilon^{\times} &= \hat{p}\otimes\hat{q} + \hat{q}\otimes\hat{p} \\ &= \sin 2\psi\,\mathbf{e}^{+} + \cos 2\psi\,\mathbf{e}^{\times}\end{aligned}$$

Example: Laser interferometer in the long wavelength limit



$$\Delta T(t) = \Delta\tau_{12} + \Delta\tau_{24} - \Delta\tau_{13} - \Delta\tau_{34}$$

$$h(t) \equiv \frac{\Delta T(t)}{2L} \approx \underbrace{\frac{1}{2}\left[\hat{a}\otimes\hat{a} - \hat{b}\otimes\hat{b}\right]}_{\text{Detector tensor}} : \mathbf{h}(t)$$

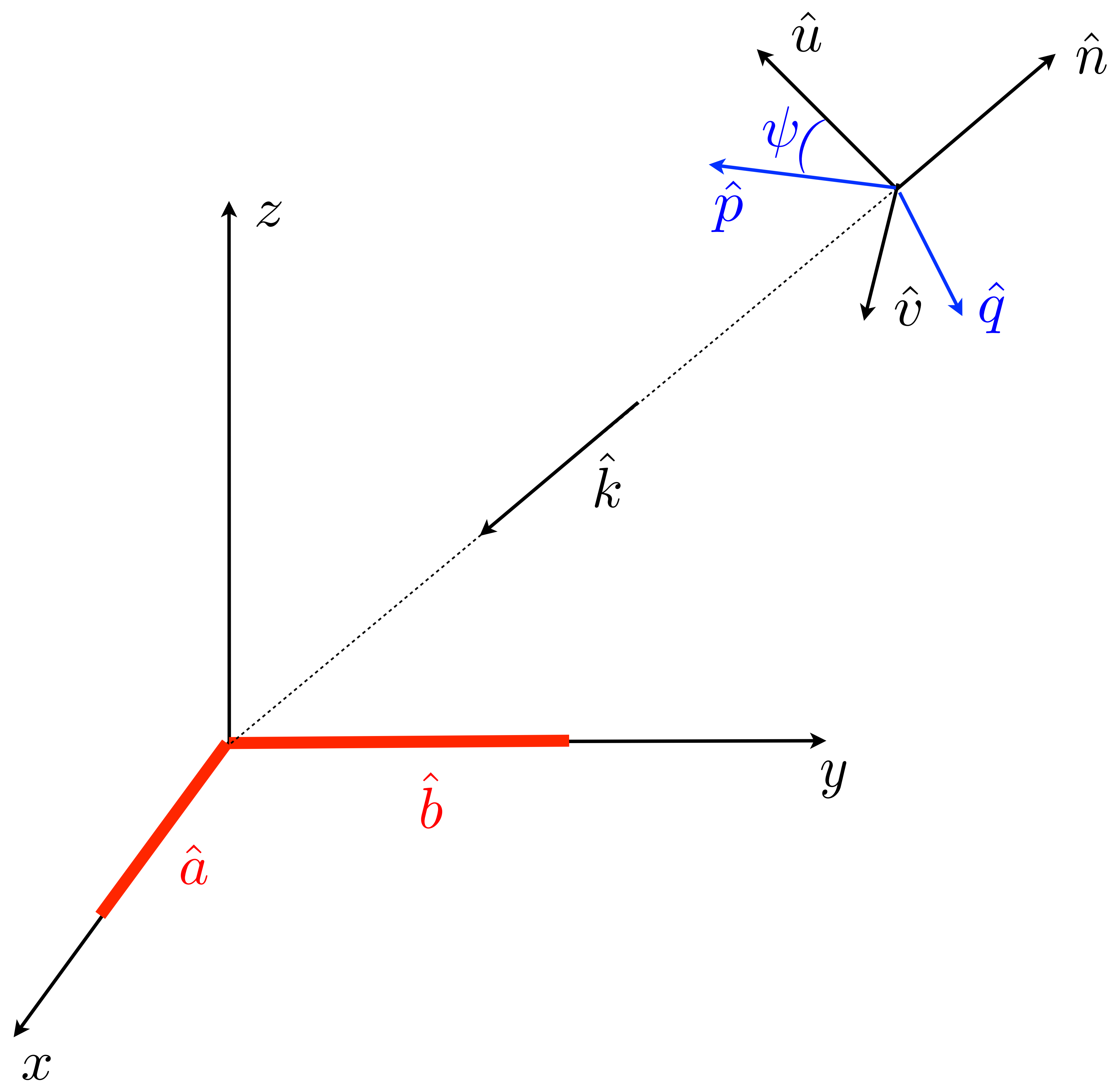
$$\mathbf{h}(t) = h_{+}(t)\boldsymbol{\epsilon}^{+} + h_{\times}(t)\boldsymbol{\epsilon}^{\times}$$

Polarization tensors

Antenna Pattern Functions

$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{u} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{v} = \sin\phi\,\hat{x} - \cos\phi\,\hat{y}$$

$$\mathbf{e}^{+} = \hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}$$
$$\mathbf{e}^{\times} = \hat{u}\otimes\hat{v} + \hat{v}\otimes\hat{u}$$



$$(\hat{a}\otimes\hat{a}) : \mathbf{e}^{+} \quad = \quad \cos^2\theta\cos^2\phi - \sin^2\phi$$

$$(\hat{a}\otimes\hat{a}) : \mathbf{e}^{\times} \quad = \quad \cos\theta\sin 2\phi$$

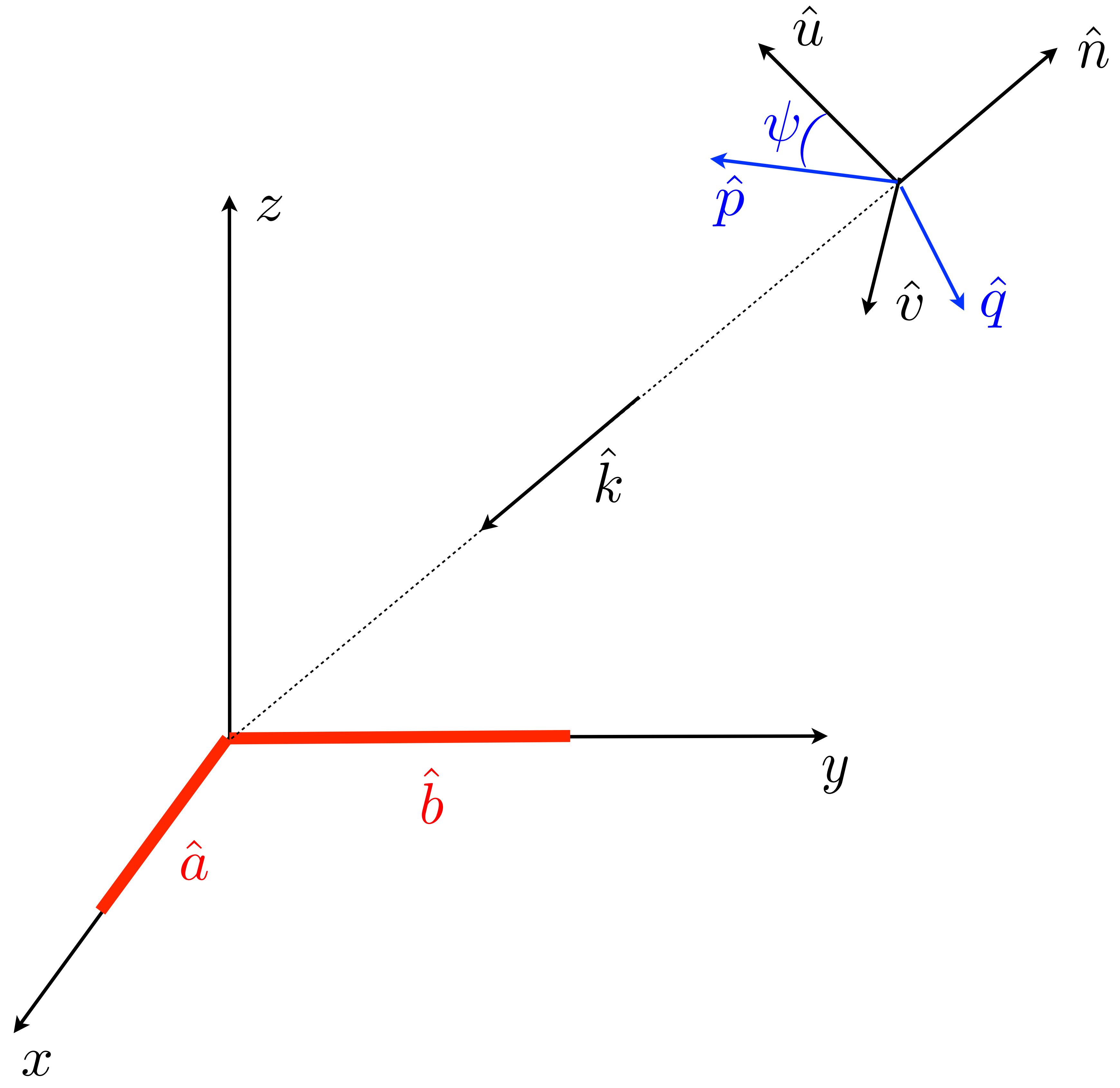
$$(\hat{b}\otimes\hat{b}) : \mathbf{e}^{+} \quad = \quad \cos^2\theta\sin^2\phi - \cos^2\phi$$

$$(\hat{b}\otimes\hat{b}) : \mathbf{e}^{\times} \quad = \quad -\cos\theta\sin 2\phi$$

Antenna Pattern Functions

$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{u} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{v} = \sin\phi\,\hat{x} - \cos\phi\,\hat{y}$$

$$\mathbf{e}^{+} = \hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}$$
$$\mathbf{e}^{\times} = \hat{u}\otimes\hat{v} + \hat{v}\otimes\hat{u}$$



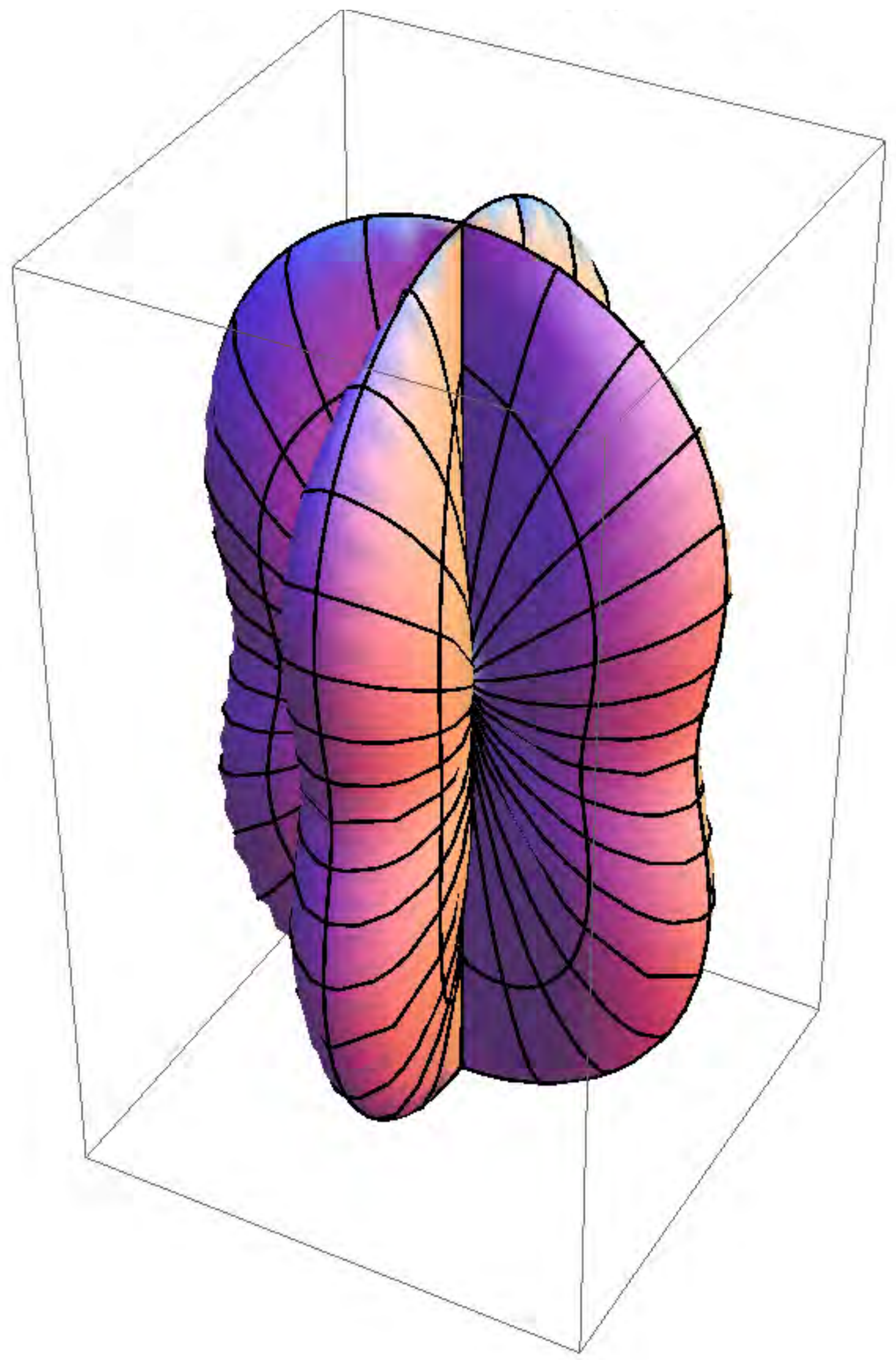
$$h = F^{+}h_{+} + F^{\times}h_{\times}$$

$$\begin{aligned} F^{+} &= \frac{1}{2}(\hat{a}\otimes\hat{a} - \hat{b}\otimes\hat{b}) : \epsilon^{+} \\ &= \frac{1}{2}(1+\cos^2\theta)\cos(2\phi)\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi \end{aligned}$$

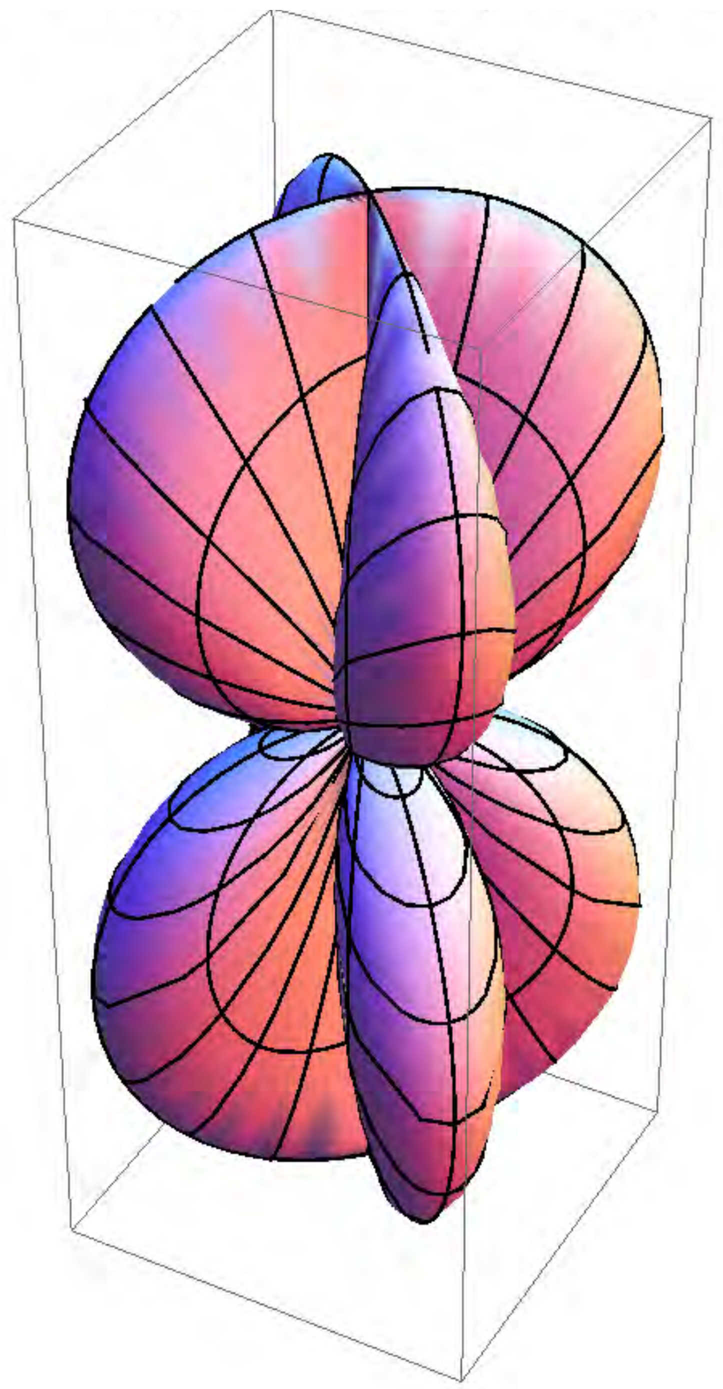
$$\begin{aligned} F^{\times} &= \frac{1}{2}(\hat{a}\otimes\hat{a} - \hat{b}\otimes\hat{b}) : \epsilon^{\times} \\ &= \frac{1}{2}(1+\cos^2\theta)\cos(2\phi)\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi \end{aligned}$$

Antenna Pattern Functions

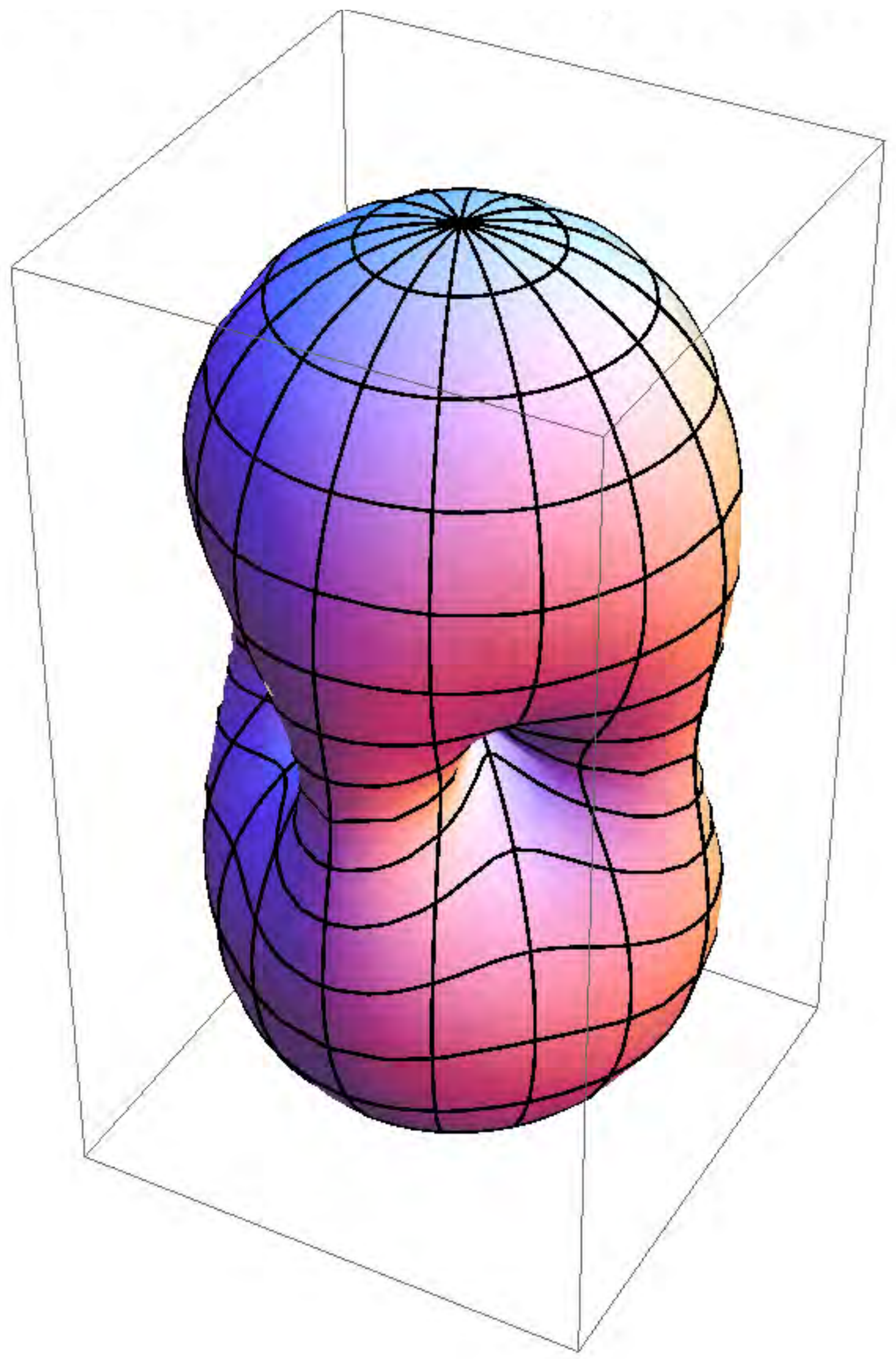
F_+



$F_\times$

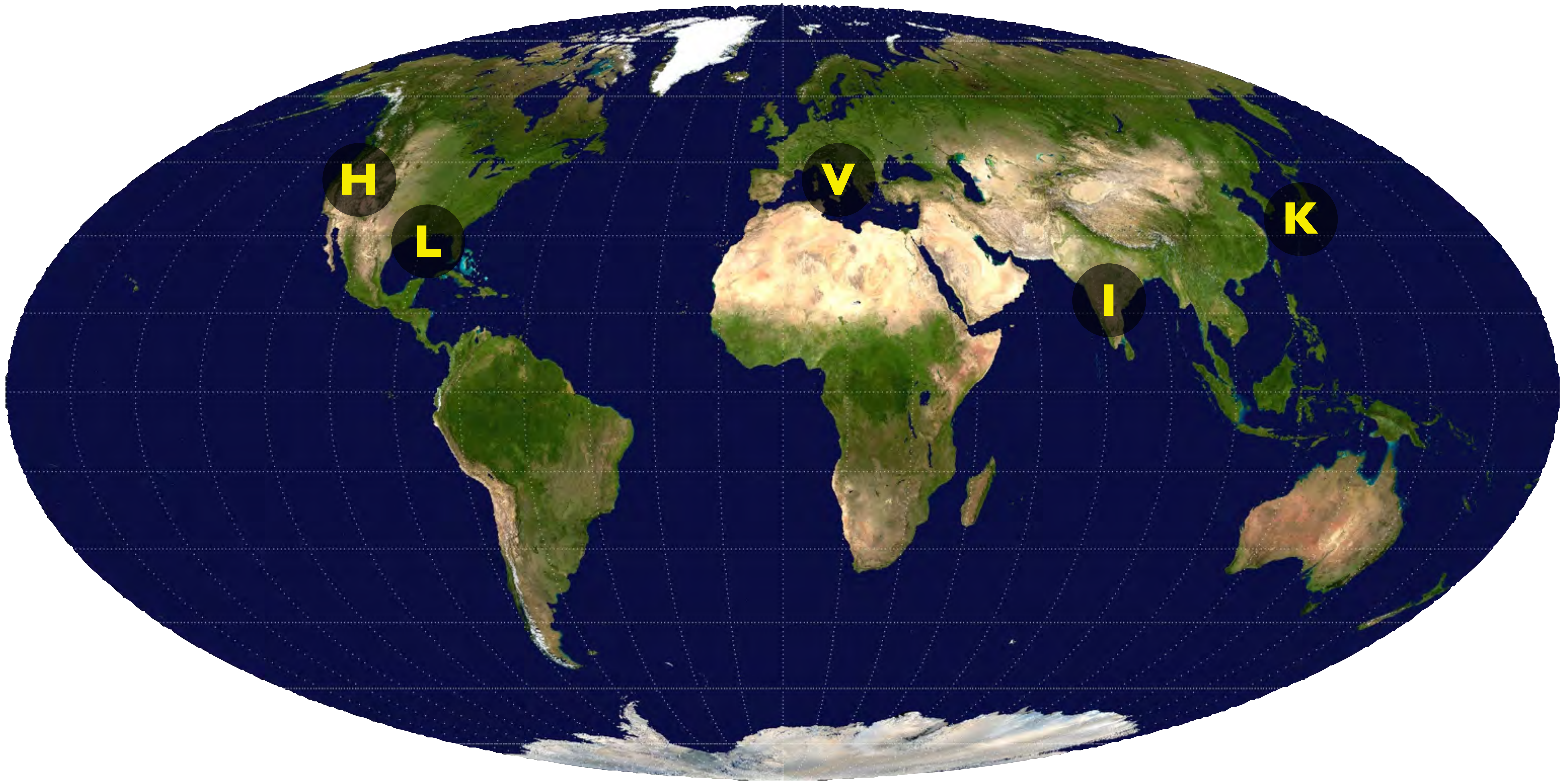


$F = \sqrt{F_+^2 + F_\times^2}$

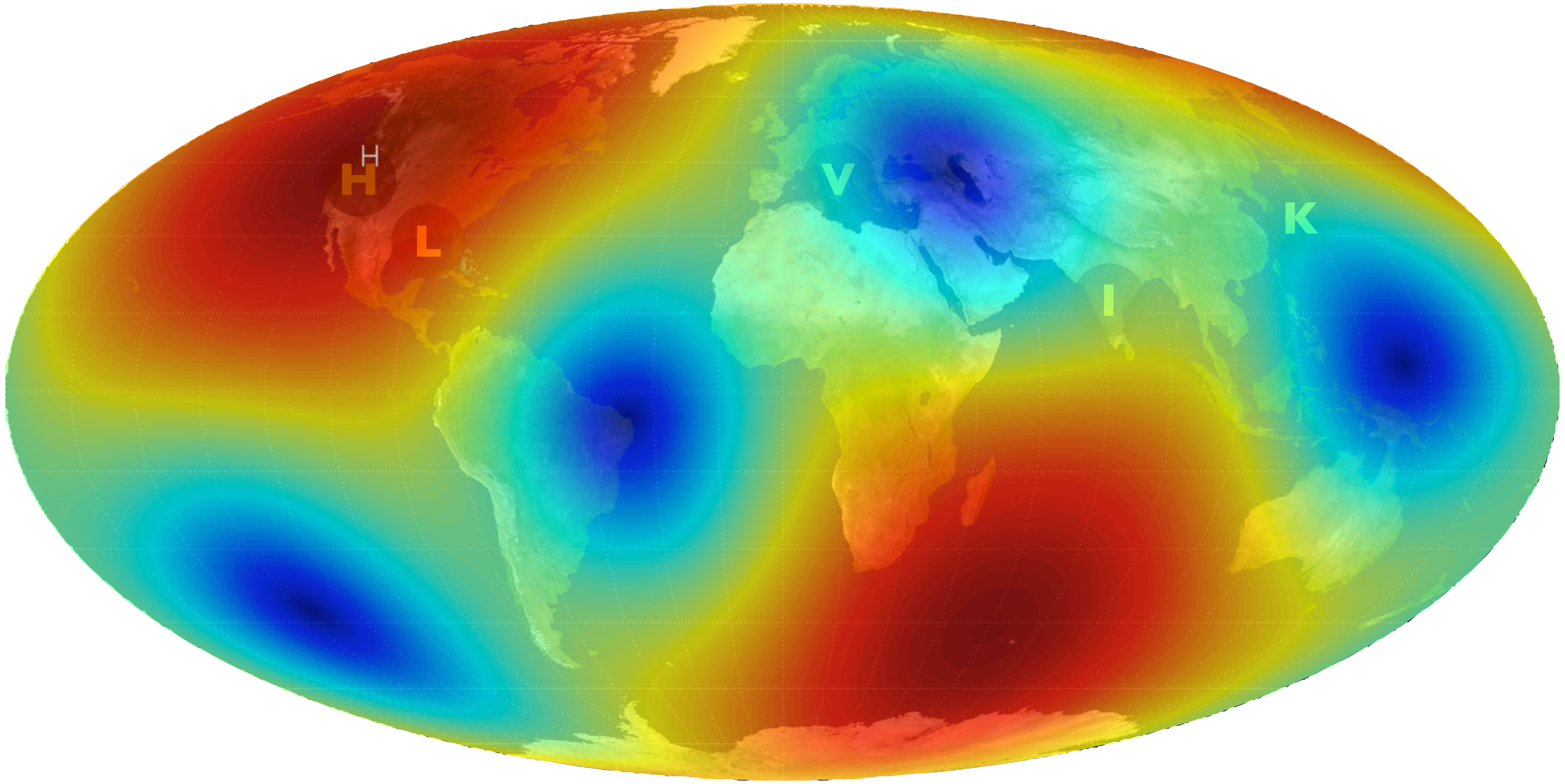


Polarization averaged

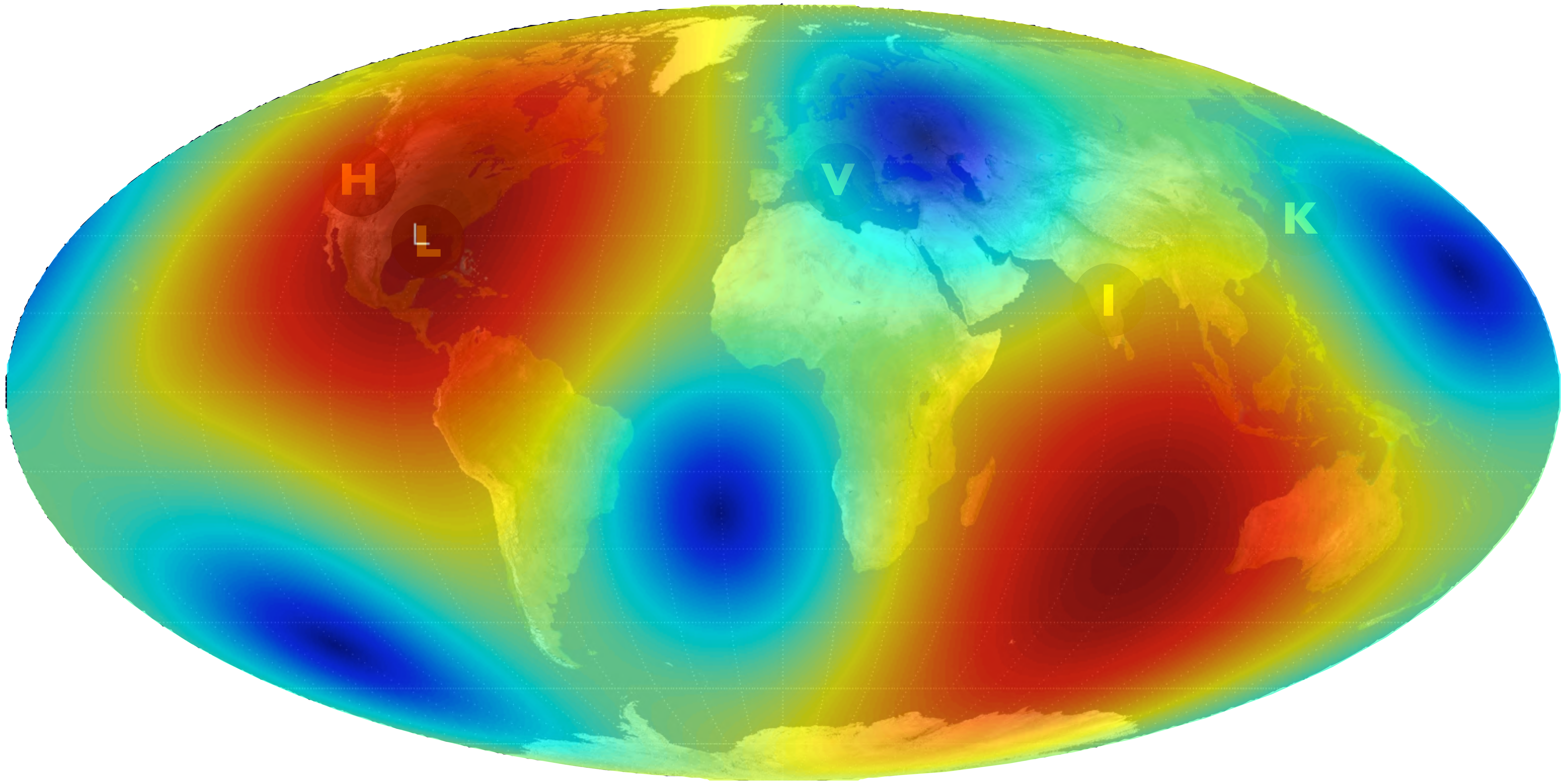
Terrestrial Network



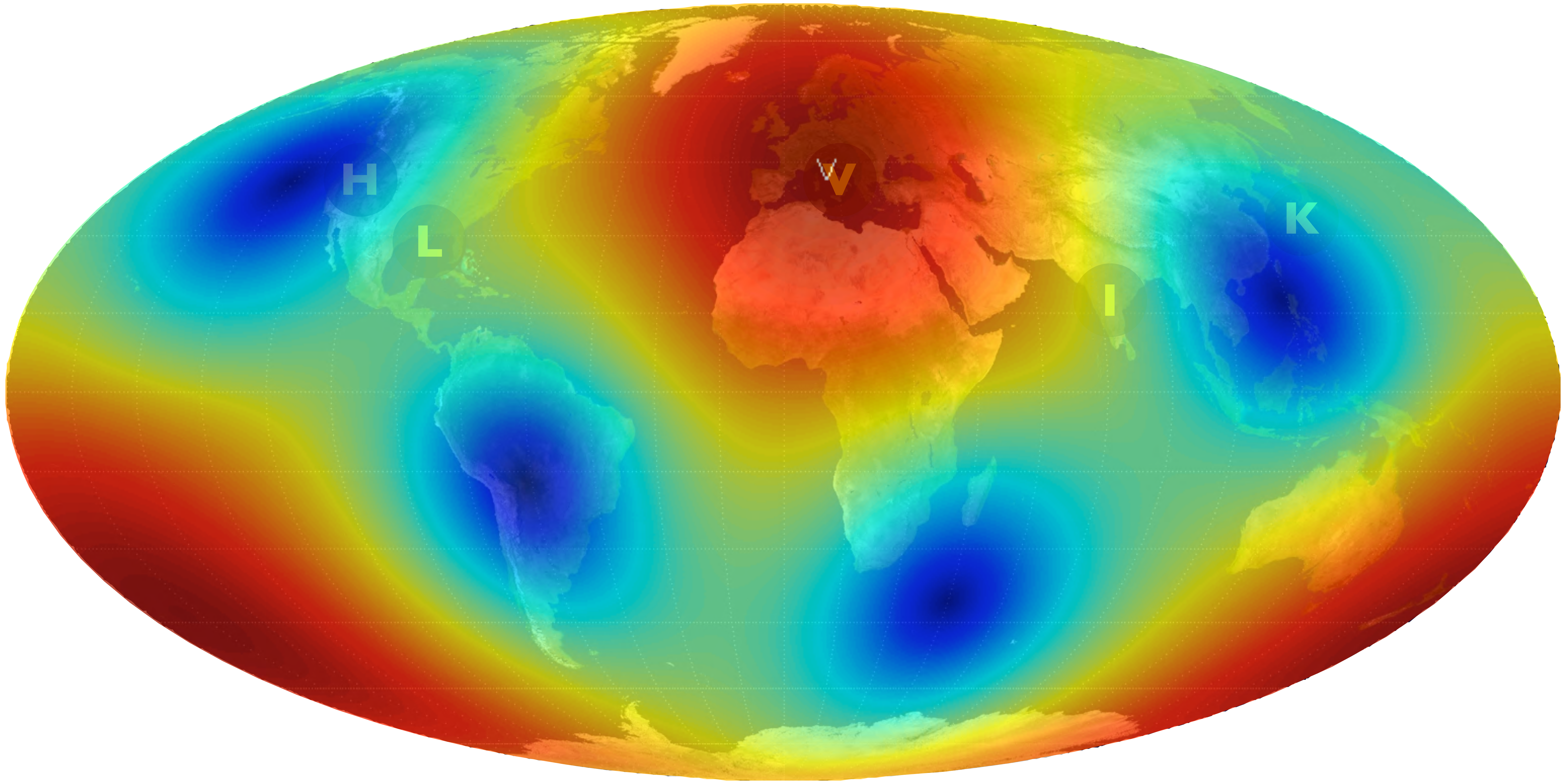
Terrestrial Network



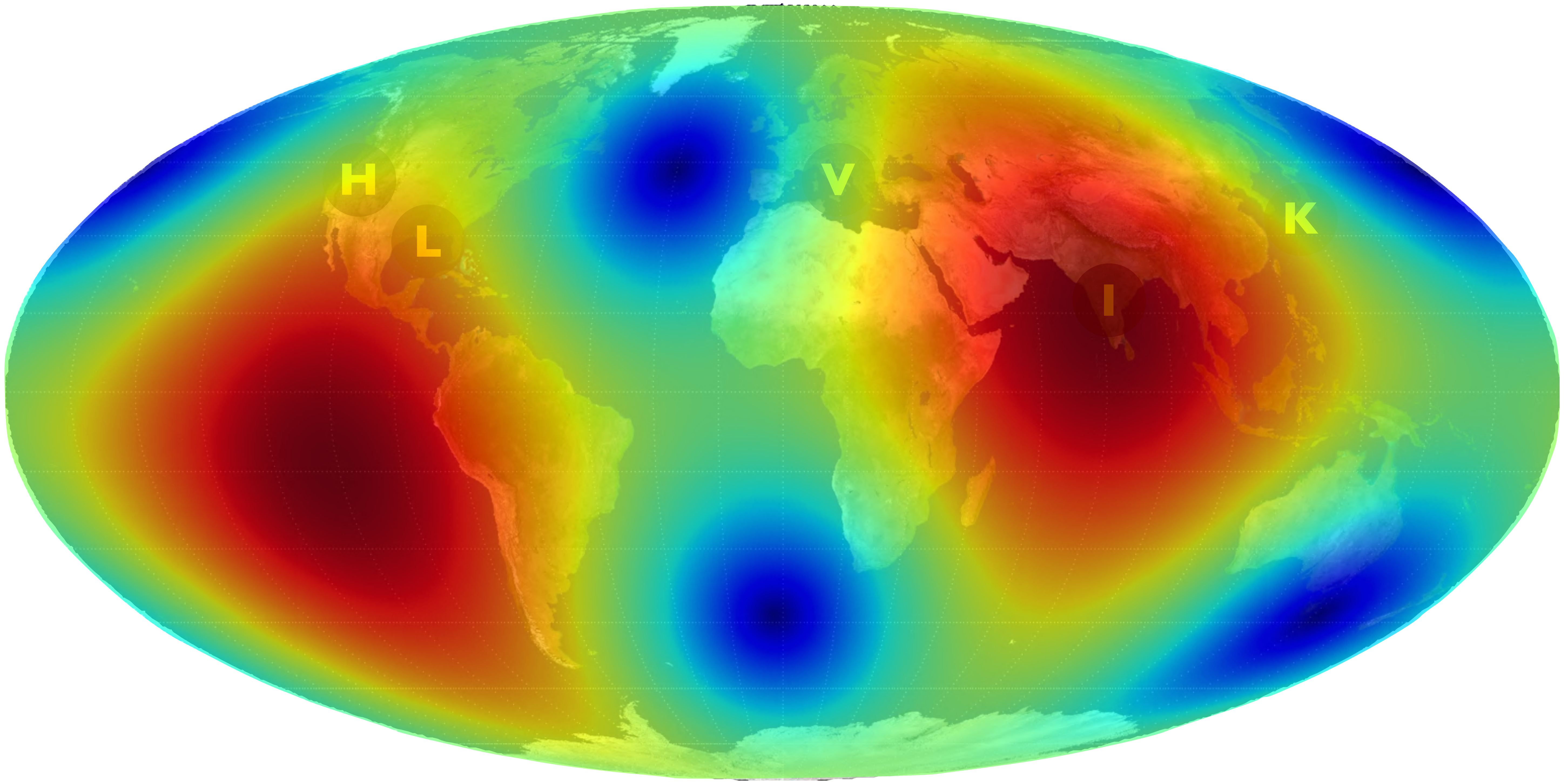
Terrestrial Network



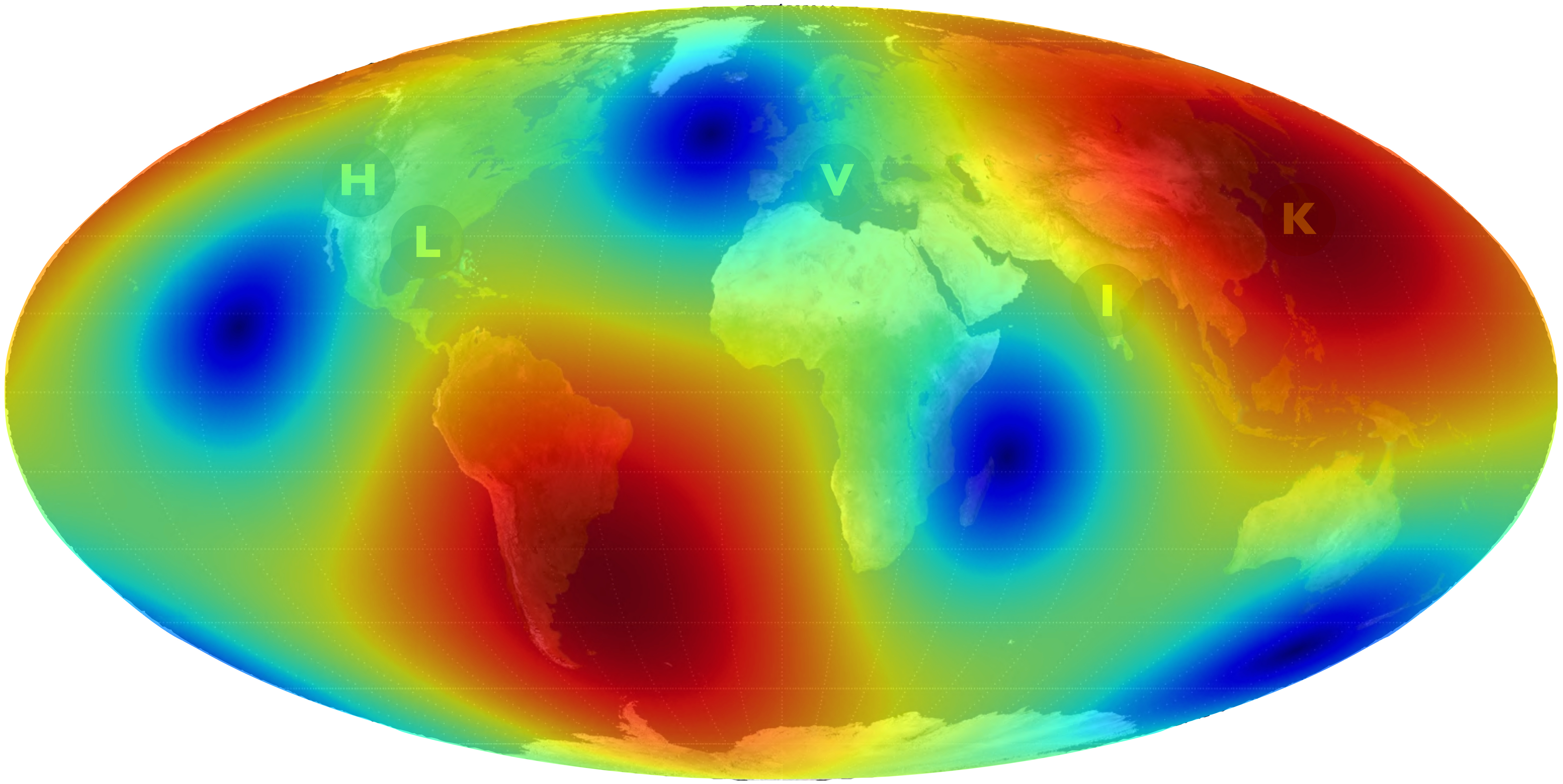
Terrestrial Network



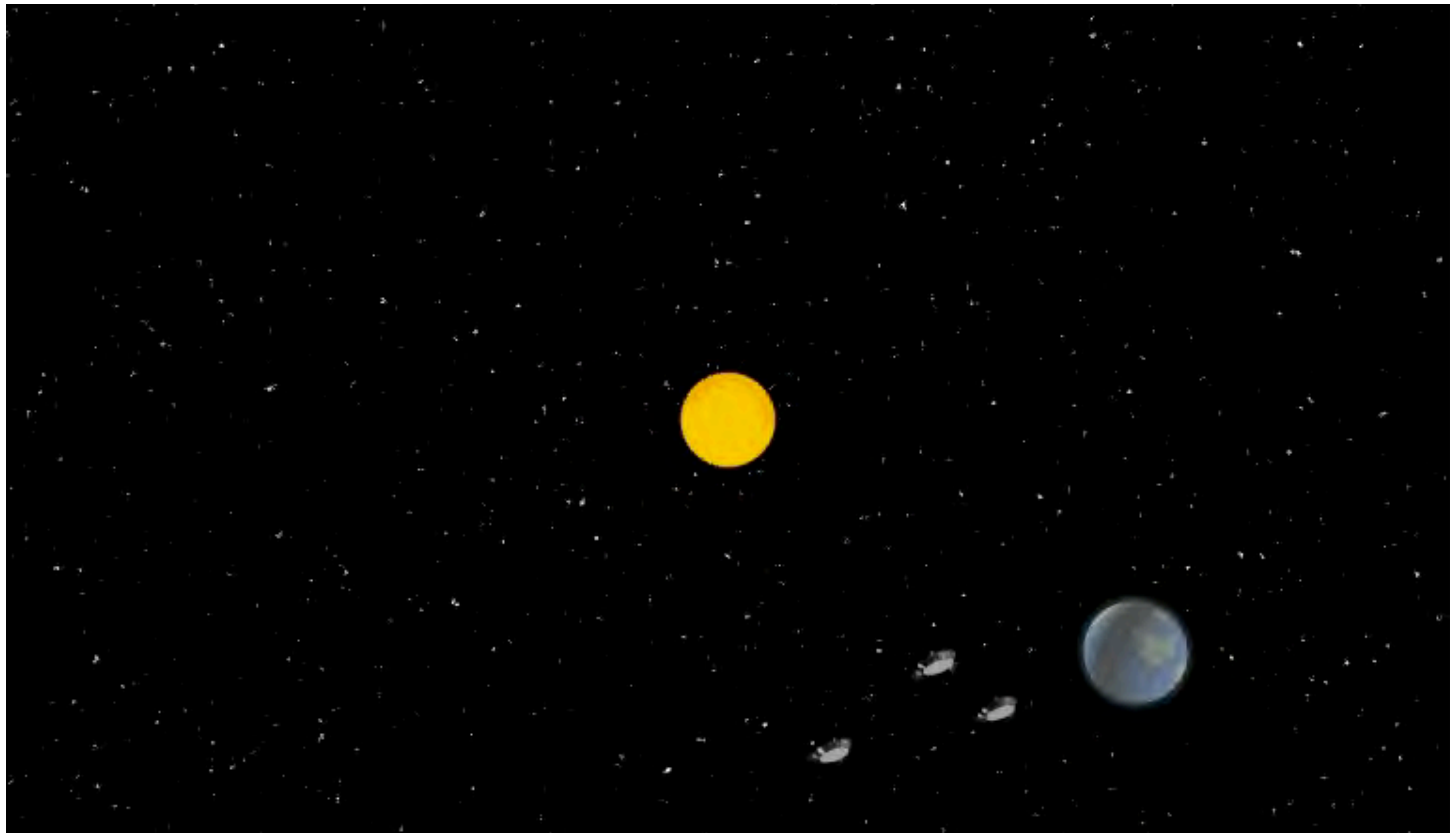
Terrestrial Network

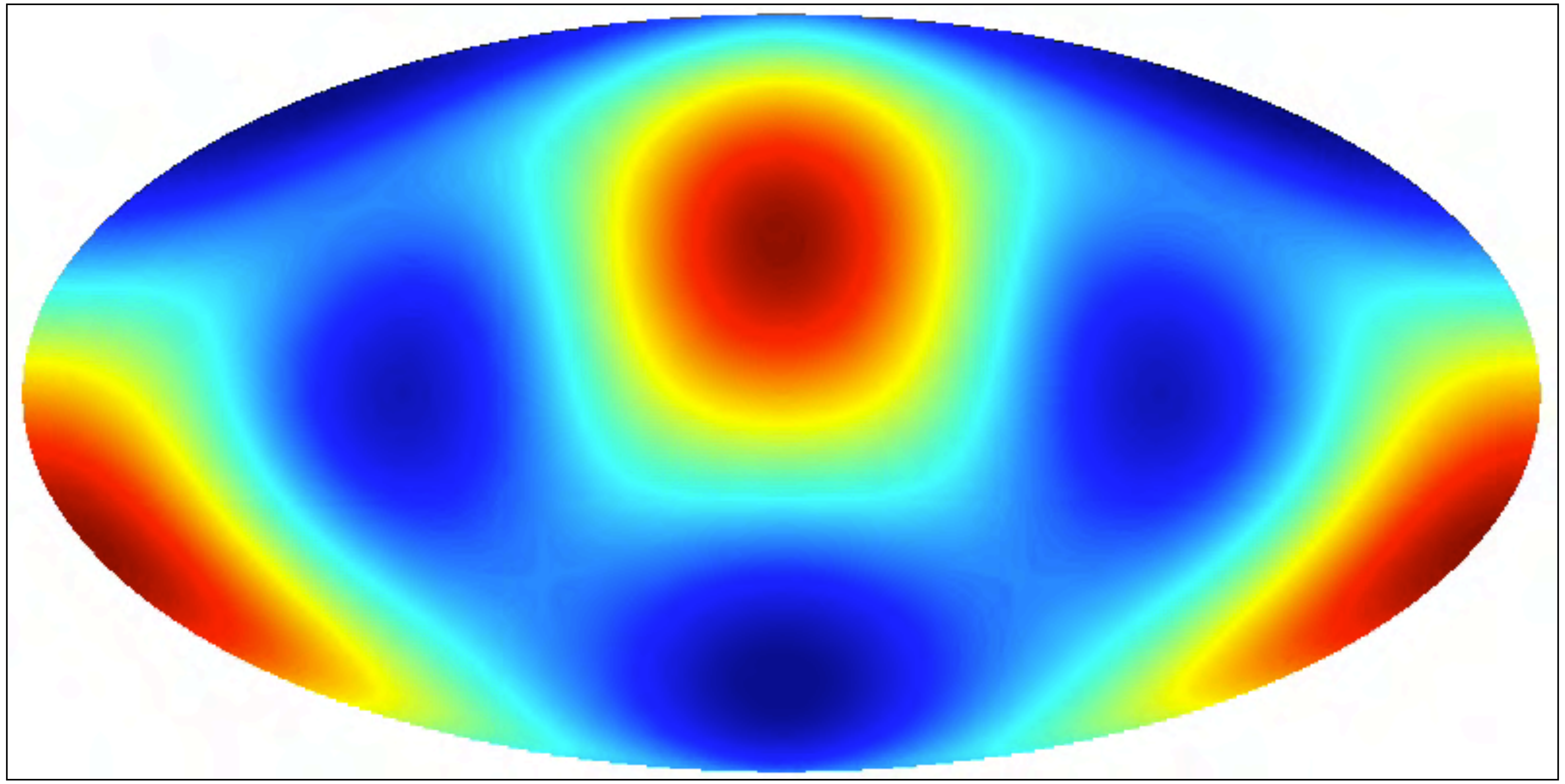


Terrestrial Network



Laser Interferometer Space Antenna





Low frequency response

Beyond the low frequency approximation

$$\Delta\tau_{12} = \frac{(\hat{a}\otimes\hat{a}):\mathbf{H}[u_1,u_2]}{2(1-\hat{k}\cdot\hat{a})} \qquad \mathbf{H}[u_1,u_2] = \int_{u_1}^{u_2}\mathbf{h}(u)\,du \qquad (u = k_\alpha x^\alpha)$$

Example: $\mathbf{h}(u) = A\cos(\omega(t-\hat{k}\cdot\mathbf{x}))\,\boldsymbol{\epsilon}^+$

$$\Delta\tau_{12} = \underbrace{\frac{L}{2}((\hat{a}\otimes\hat{a}):\boldsymbol{\epsilon}^+)}_{\text{Long wavelength one-arm antenna pattern}}\,\underbrace{\mathrm{sinc}\left[\frac{\omega L}{2}(1-\hat{k}\cdot\hat{a})\right]}_{\text{Finite arm-length correction to antenna pattern}}\cos\left[\underbrace{\omega(t+\frac{L}{2}-\frac{\hat{k}\cdot(\mathbf{x_1}+\mathbf{x_2})}{2}}_{\text{Phase of the wave at midpoint of arm}}\right]$$

Long wavelength one-arm antenna pattern

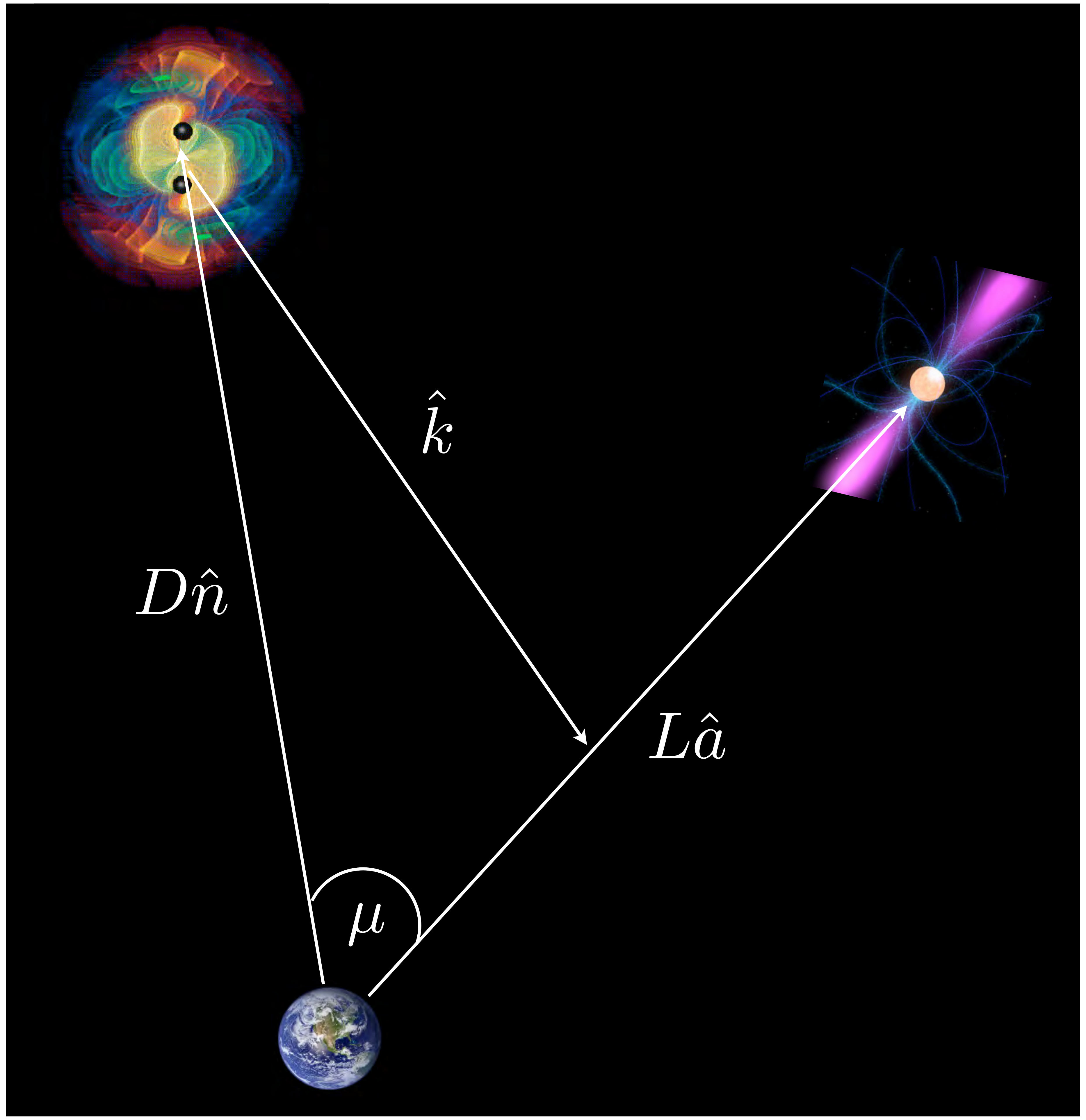
Finite arm-length correction to antenna pattern

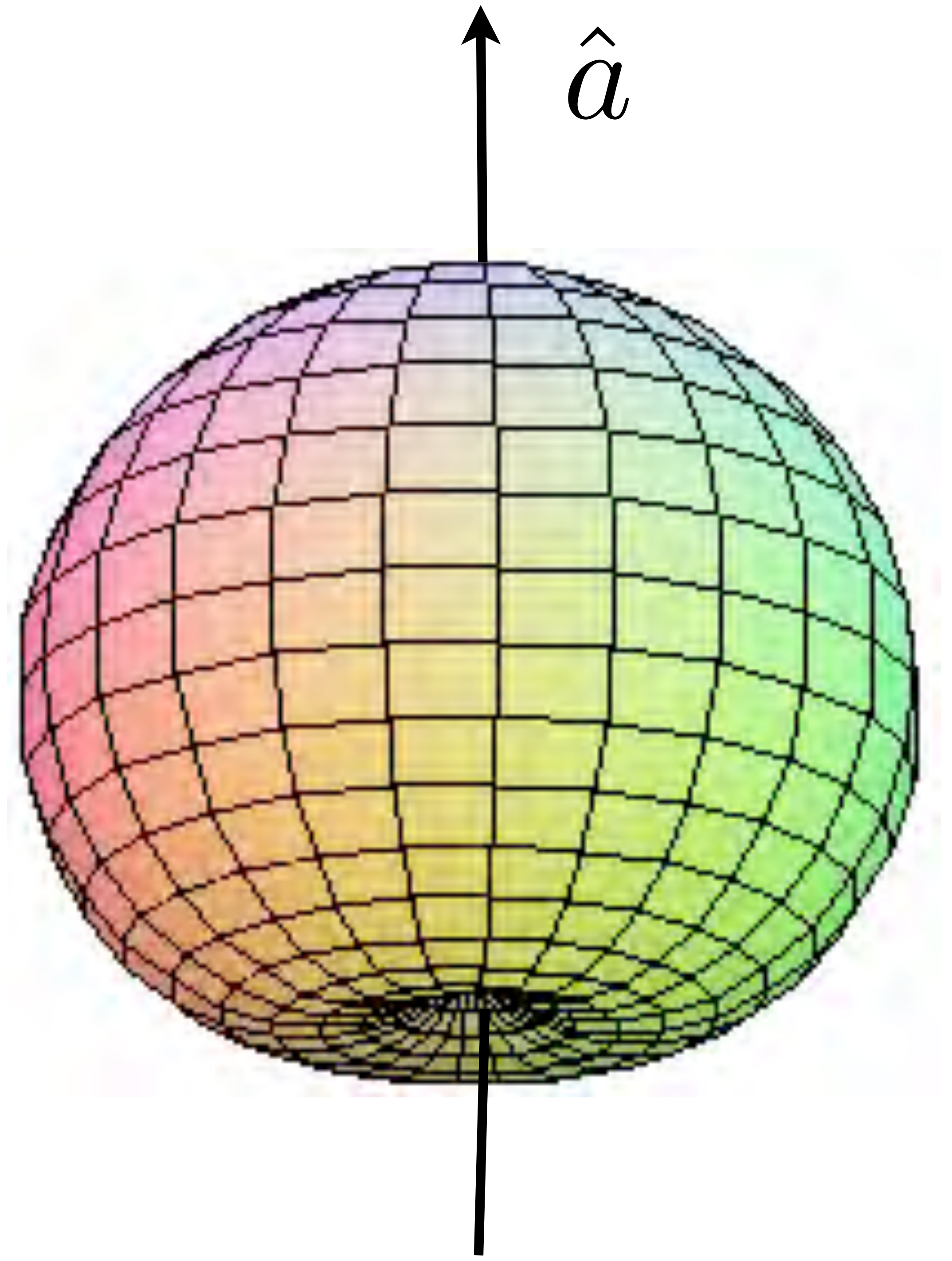
Phase of the wave at midpoint of arm

Pulsar Timing

$$\tau_{\rm GW}(t) = -\frac{L}{2}\int_{-1}^{0} (\hat{a}\otimes\hat{a}) : \mathbf{h}(t+L\xi, -\hat{a}\xi L)\, d\xi$$

$$\hat{k} = -\hat{n} - \xi\frac{L}{D}(\hat{a} - \hat{n}\cos\mu)$$





Short wavelength limit

$$(\hat{a}\otimes\hat{a}) : \mathbf{H} = (1+\cos\mu)(H_{+}\cos 2\psi + H_{\times}\sin 2\Psi)$$

(Ignoring *L*/*D* amplitude corrections)