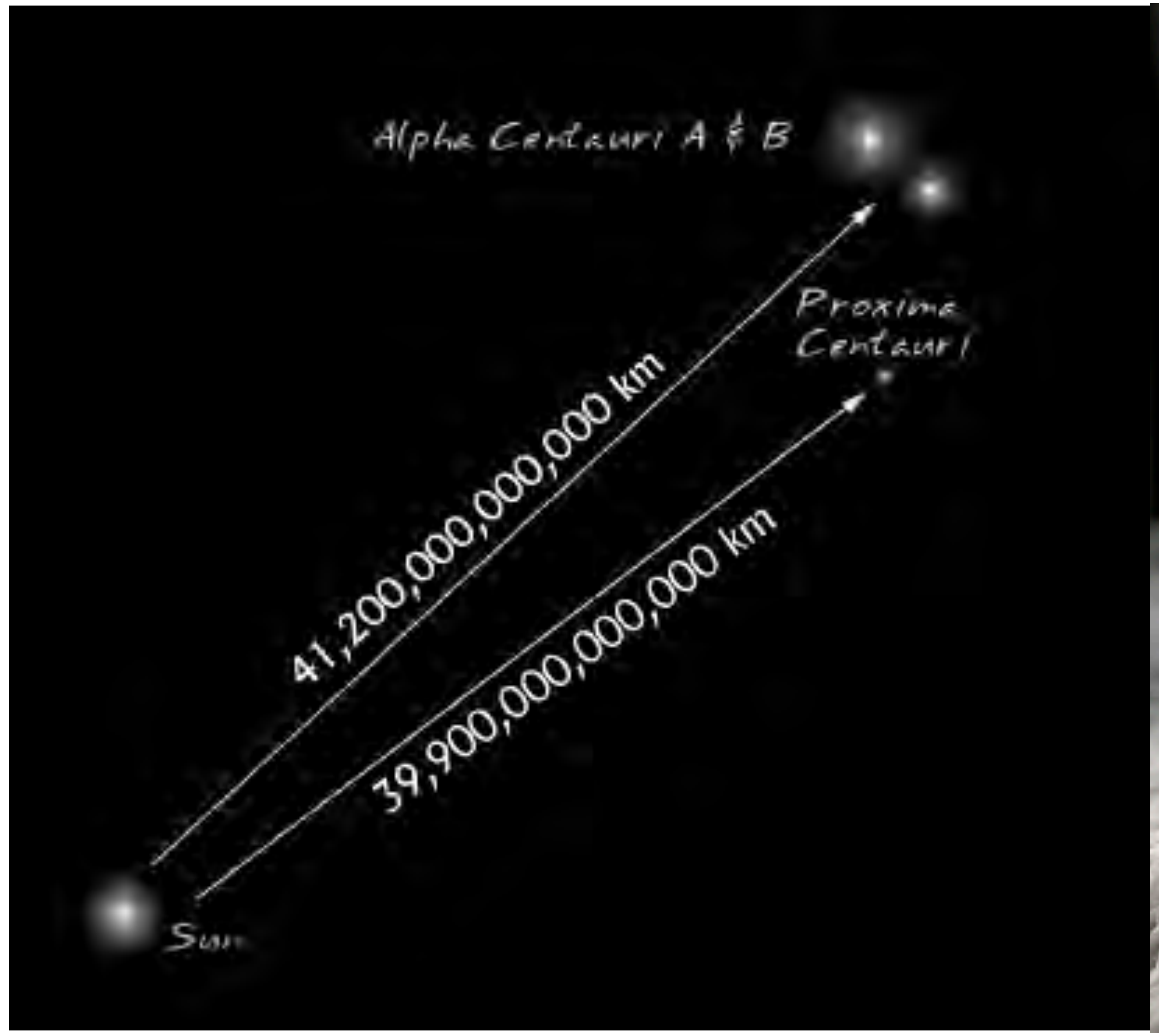
Gravitational Wave Observatories III: Ground Based Interferometers

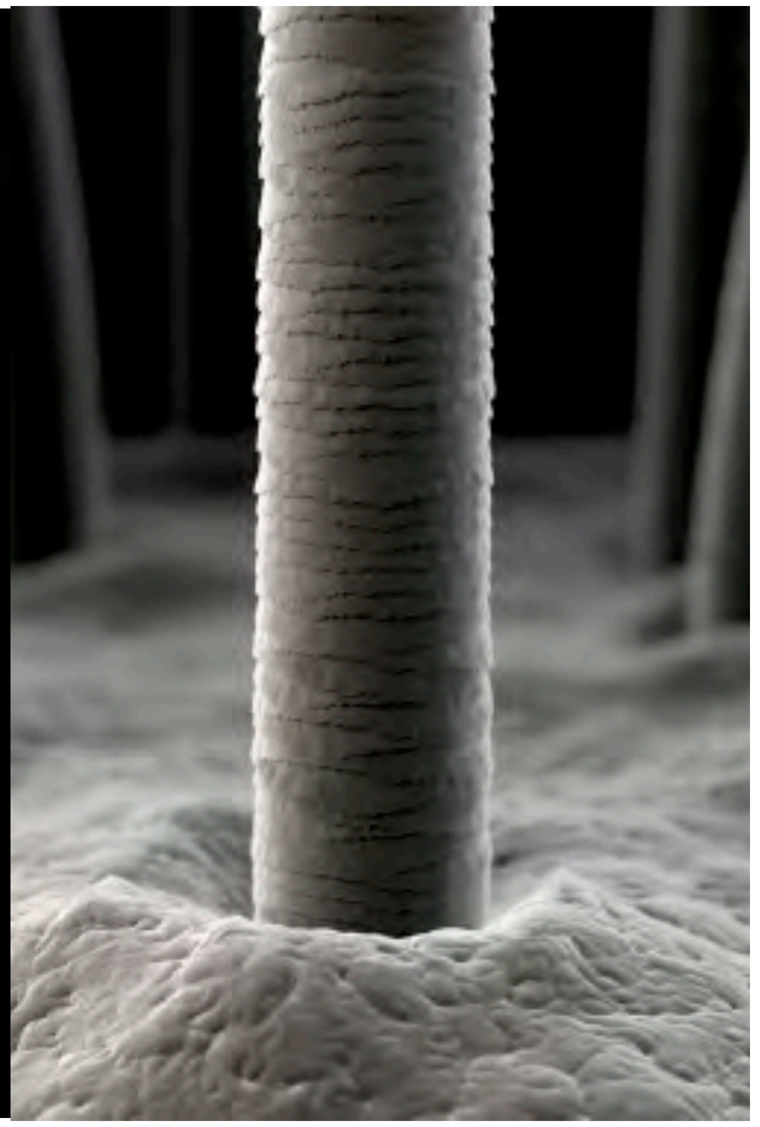
Neil J. Cornish

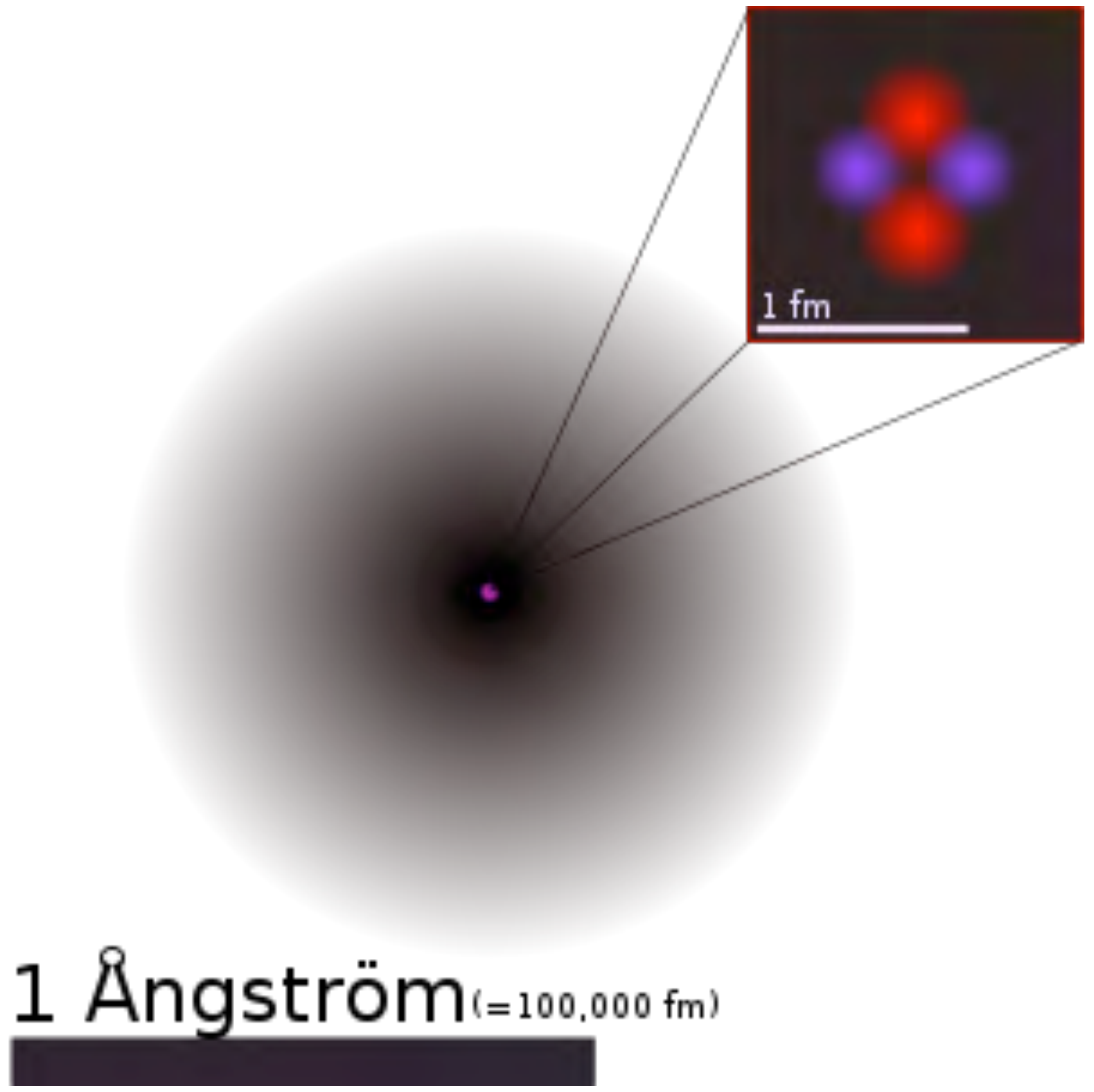
Outline

- Interferometer design
- Sources of noise
- LIGO data analysis, theory and search results
- Astrophysical rates

Gravitational Wave Telescopes

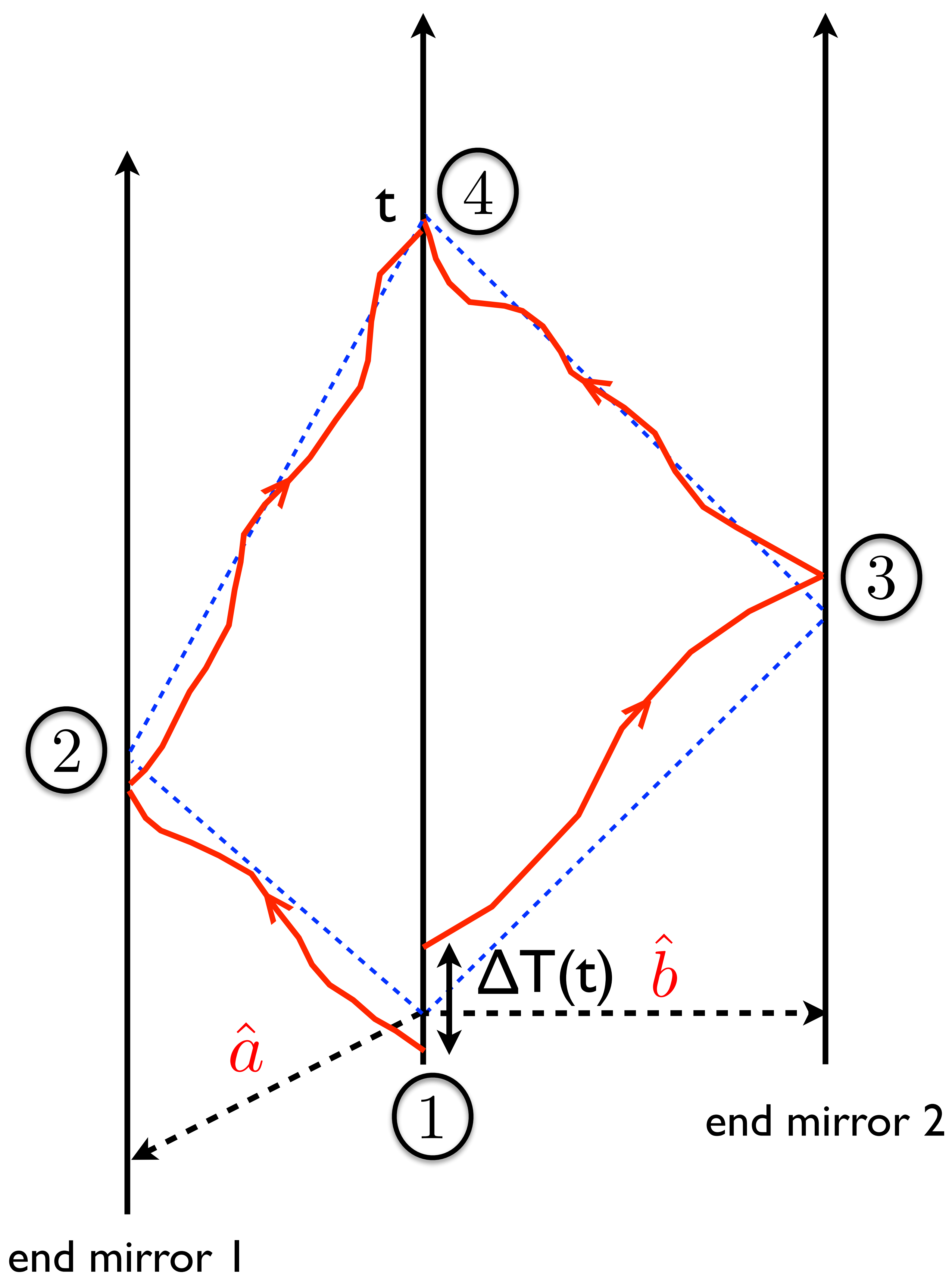






Interferometer Design

$$\Phi(t) = 2\pi\nu_0\Delta T(t)$$



Consideration 1 $\Delta T \propto h\,L \quad \text{for} \quad f_{\rm gw} < \frac{c}{L}$

Want L as large as possible up to $L \sim \frac{c}{10^3\,\text{Hz}} = 300\,\text{km}$

Expensive!

Solution: folded arms

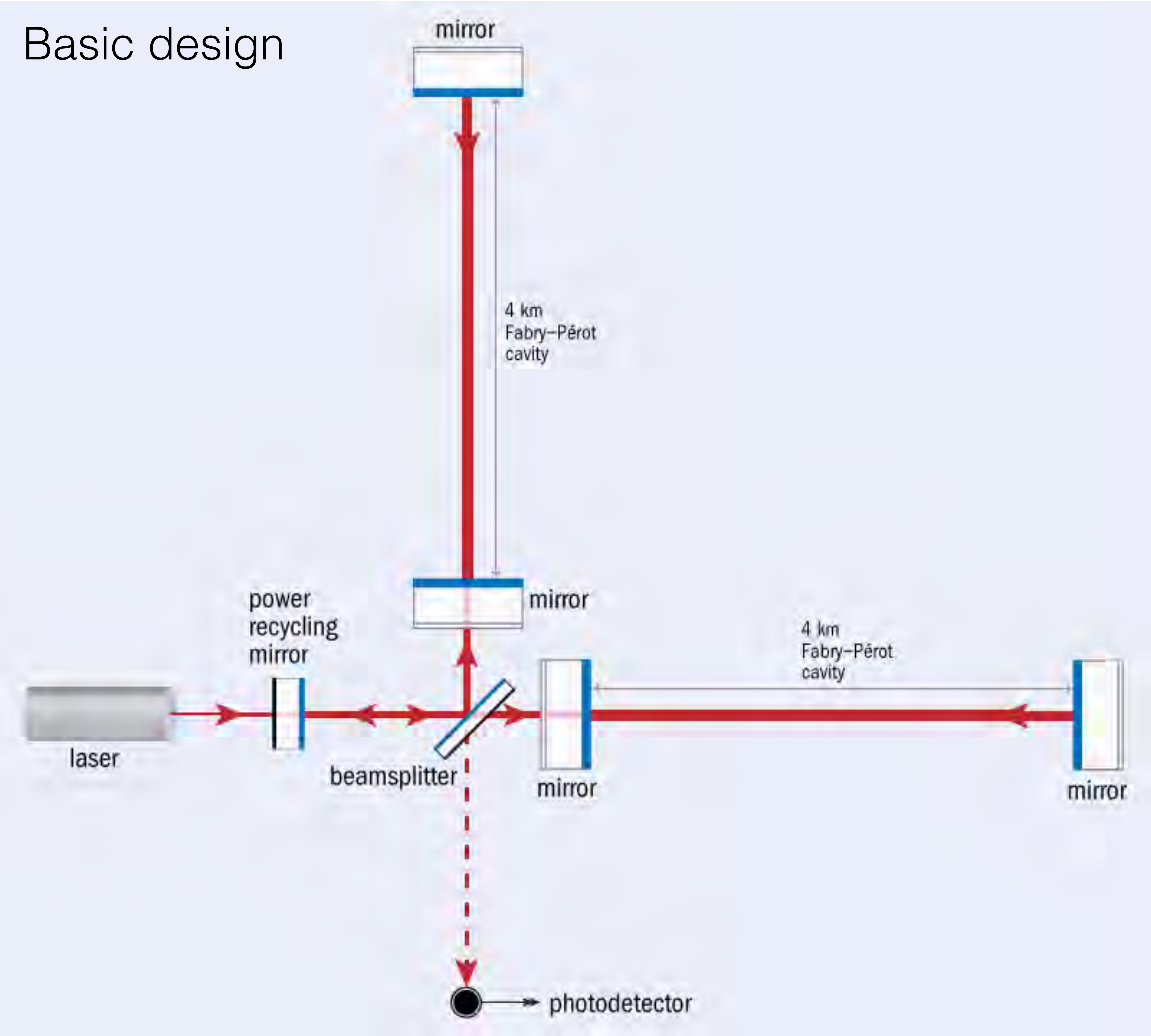
Consideration 2 $\Phi \propto \nu_0\Delta T$

Fluctuations in Laser frequency can masquerade as GW signal

Partial Solution: Highly stable lasers

Solution: Michelson topology - cancel laser frequency noise

Interferometer Design



The 4 km Fabry-Perot cavities effectively fold the arms

Can calculate the response using basic E&M, electric field transmission and reflection coefficients at each mirror. (See Maggiore's text)

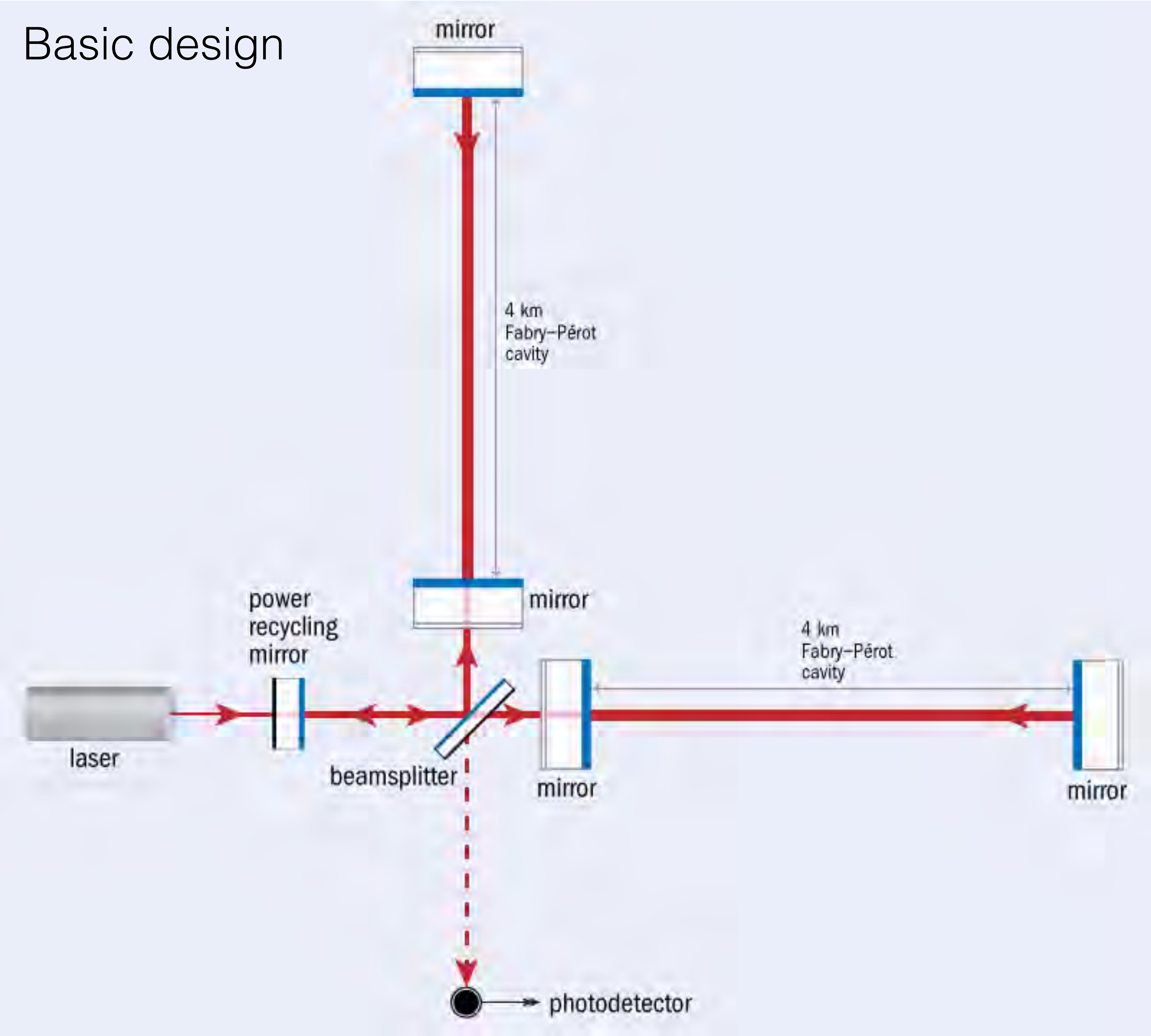
In the long wavelength limit, phase shift is

$$|\Delta\Phi_{\mathrm{FP}}| = \left(\frac{\nu_0 h L}{c}\right)\left[\frac{8\mathcal{F}}{\sqrt{1+(f_{\mathrm{gw}}/f_p)^2}}\right]$$

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$ Cavity Finesse

$$f_p = \frac{1}{4\pi\tau_s} \approx \frac{c}{4\,\mathcal{F}L}$$ Pole frequency

Single Fabry-Perot Cavity



$$|\Delta\Phi_{\mathrm{FP}}| = \left(\frac{\nu_0 h L}{c}\right)\left[\frac{8\mathcal{F}}{\sqrt{1+(f_{\mathrm{gw}}/f_p)^2}}\right]$$

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$ Cavity Finesse

$$f_p = \frac{1}{4\pi\tau_s} \approx \frac{c}{4\,\mathcal{F}L}$$ Pole frequency

Reflectivities are very close to unity

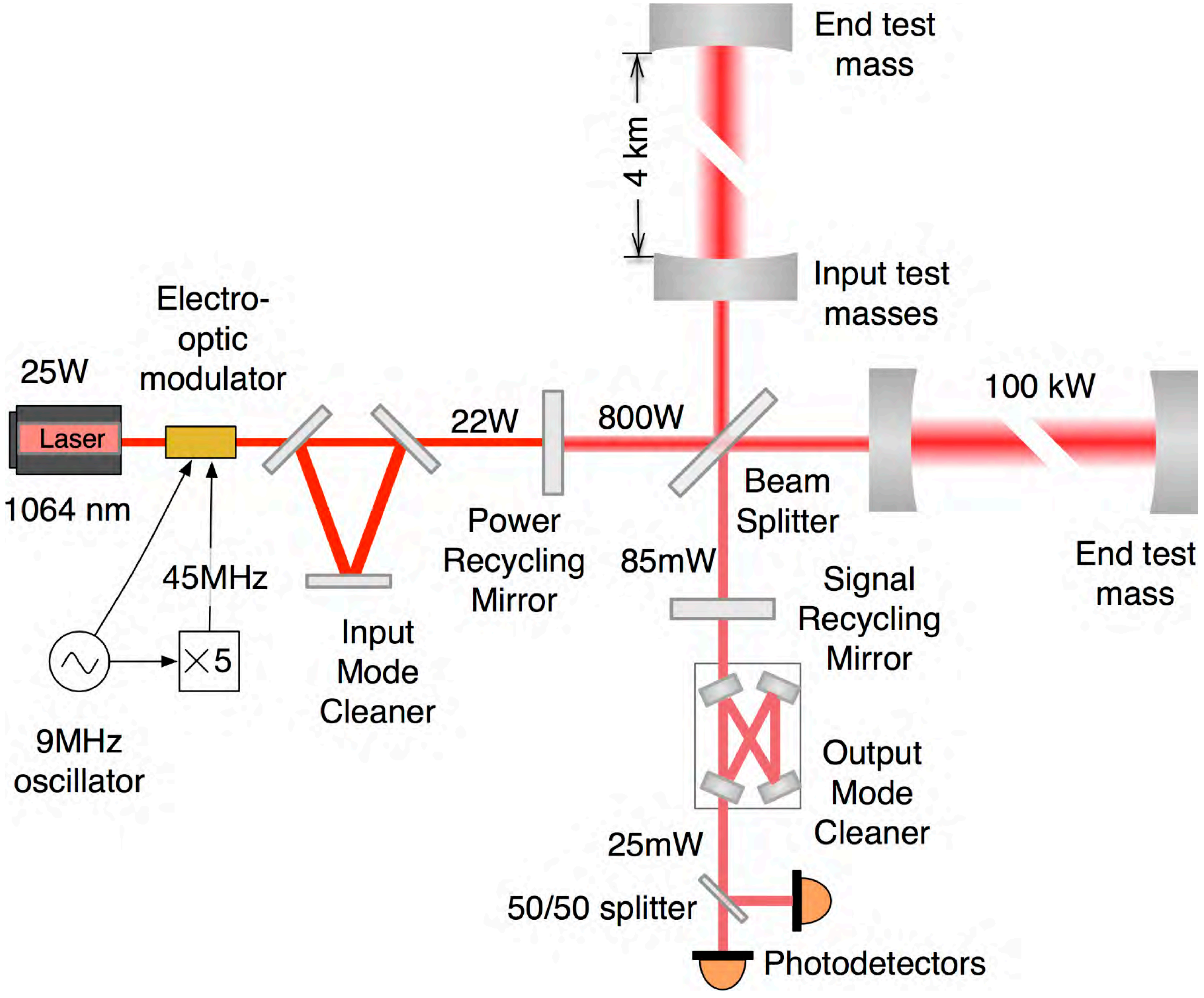
e.g. advanced LIGO $\mathcal{F} = 450$ $f_p = 42\,\mathrm{Hz}$

Note that increasing the Finesse improves the low frequency sensitivity (good), but lowers the pole frequency (bad)

Advanced LIGO gets around this problem by using signal recycling

Michelson Interferometer with coupled Fabry-Perot Cavities

The actual LIGO design is much more complicated. FP cavities are coupled by a power recycling and signal recycling mirrors



There is a common mode and a differential mode

$$L_+ = \frac{L_1 + L_2}{2} \qquad L_- = \frac{L_1 - L_2}{2}$$

Differential mode contains the GW signal

$$f_- = \frac{2}{4\pi L_+} \ln\left(\frac{1 - r_i r_s}{r_e r_i - r_e r_s (t_i^2 + r_i^2)}\right)$$

$$i = \text{input mirror}$$
$$e = \text{end mirror}$$
$$s = \text{signal recycling mirror}$$

Advanced LIGO $\quad f_- = 350 \text{ Hz}$

Transfer function $\quad \dfrac{1}{\sqrt{1 + (f_{gw}/f_-)^2}}$

The gain from folding and recycling is now a complicated combination of terms. Comes out at a factor of ~1,100

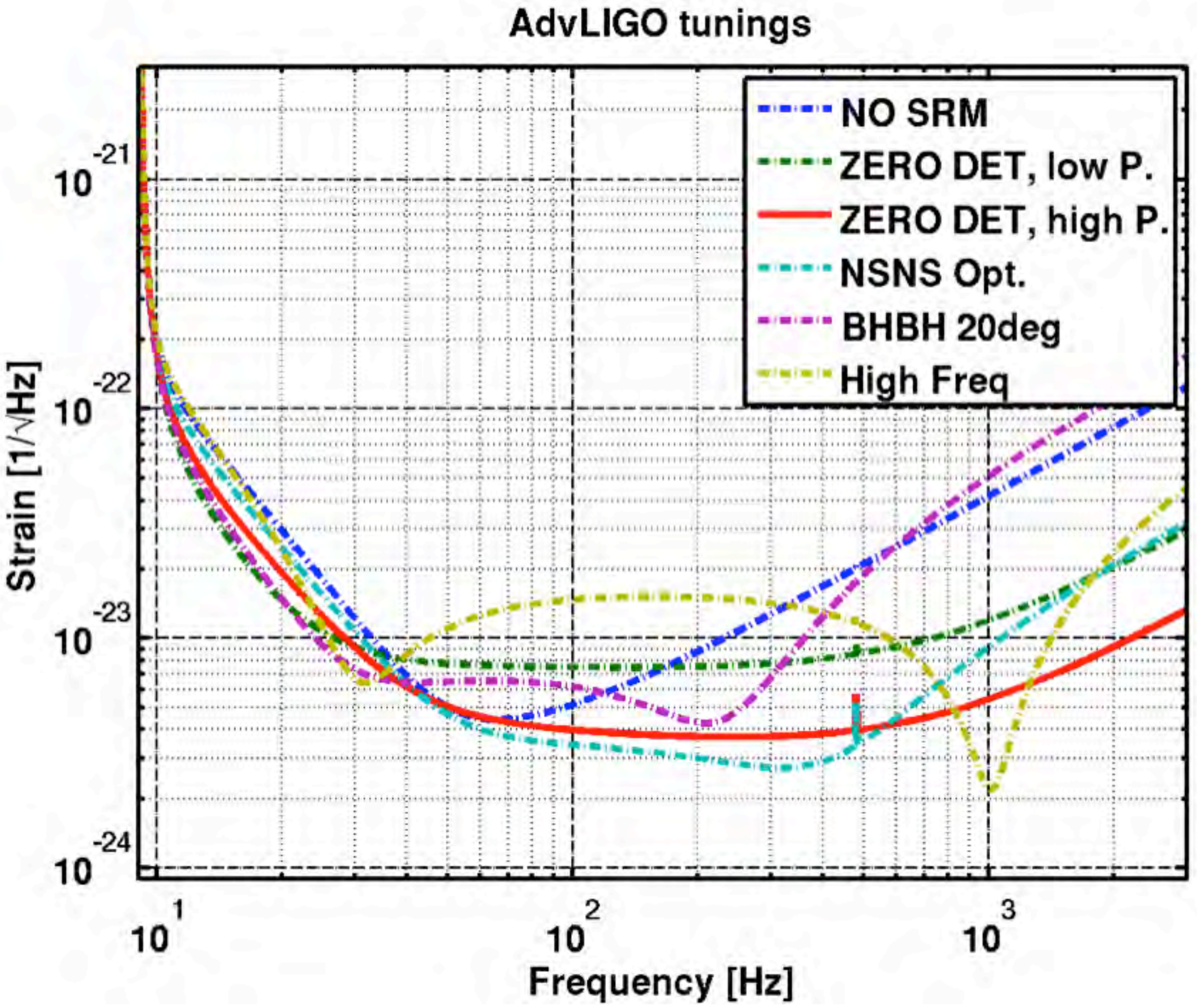
Signal recycling and response shaping

The cavity transfer function for the Michelson-Fabry-Perot topology with signal recycling allows us to shape the response and target particular signals by changing the distance to the signal recycling mirror

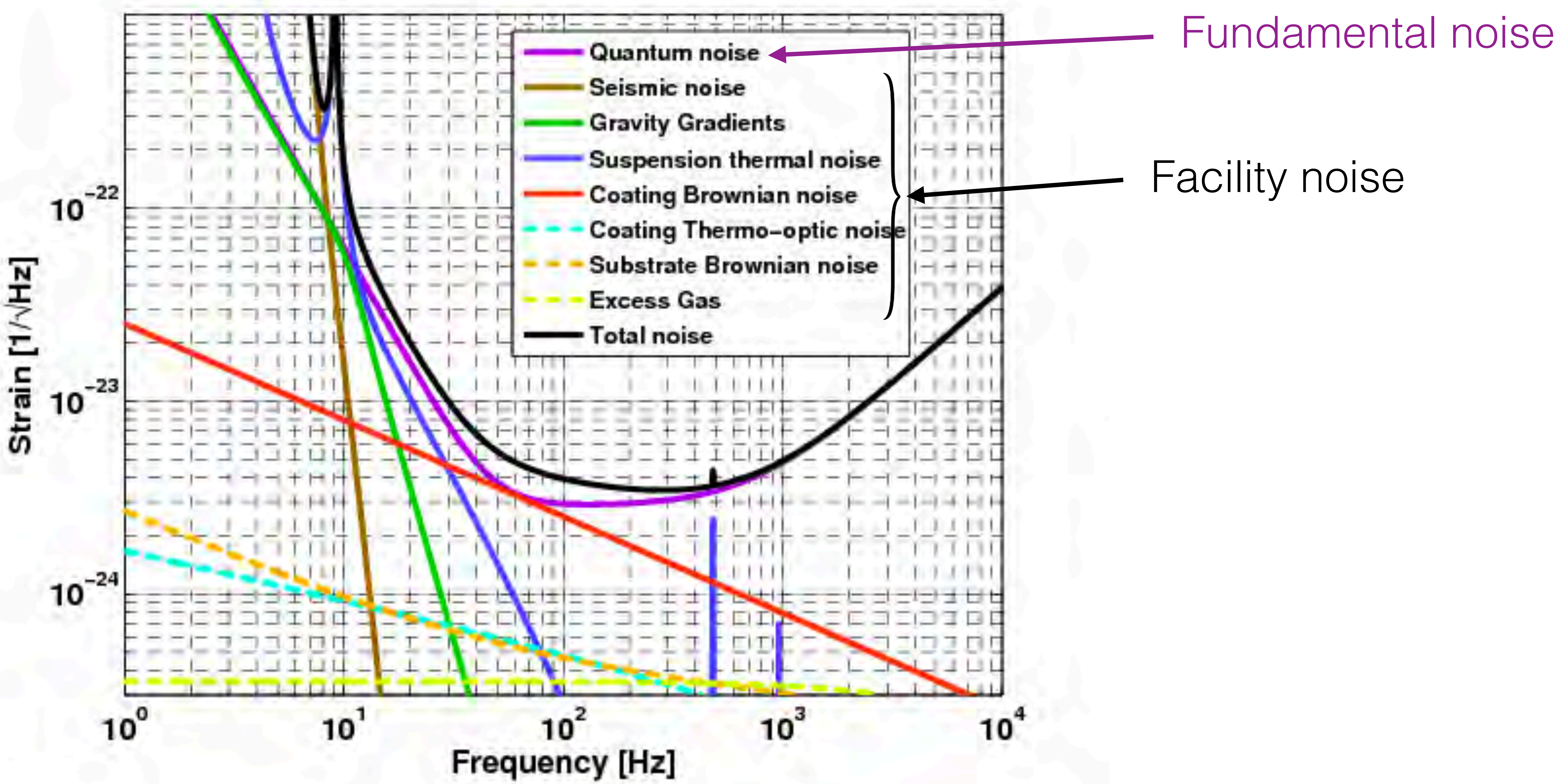
$$C(f) = \frac{t_s e^{-i(2\pi f \ell_s/c + \phi_s)}}{1 - r_s \left(\frac{r_i - e^{-4\pi i f L/c}}{1 - r_i e^{-4\pi i f L/c}} \right) e^{-2i(2\pi f \ell_s/c + \phi_s)}}$$

$$\phi_s = 2\pi \nu_0 \, \ell_s / c$$

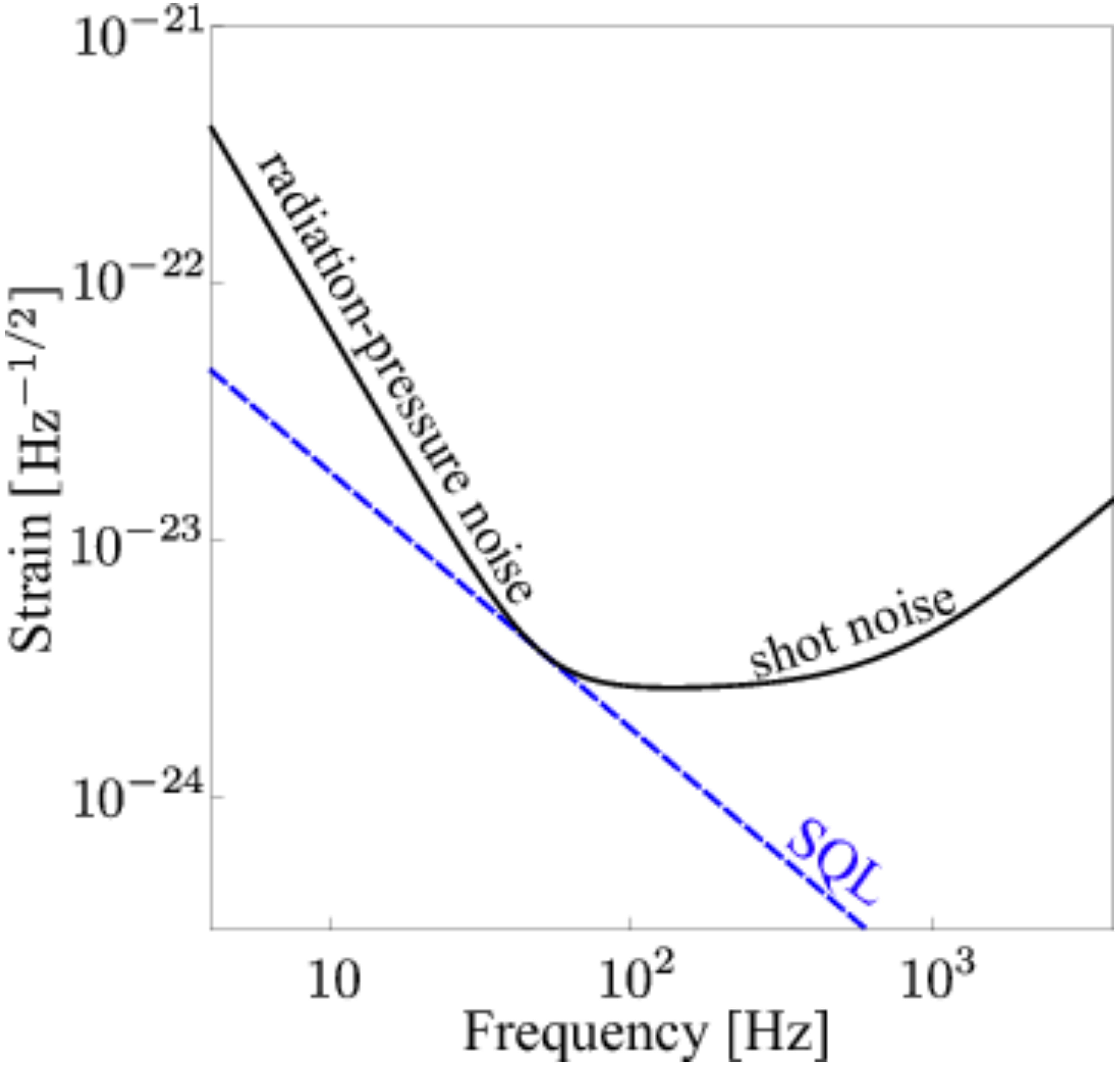
The pole frequency quoted on the last slide was for the zero detuned configuration



Sources of Noise



Fundamental (quantum) noise

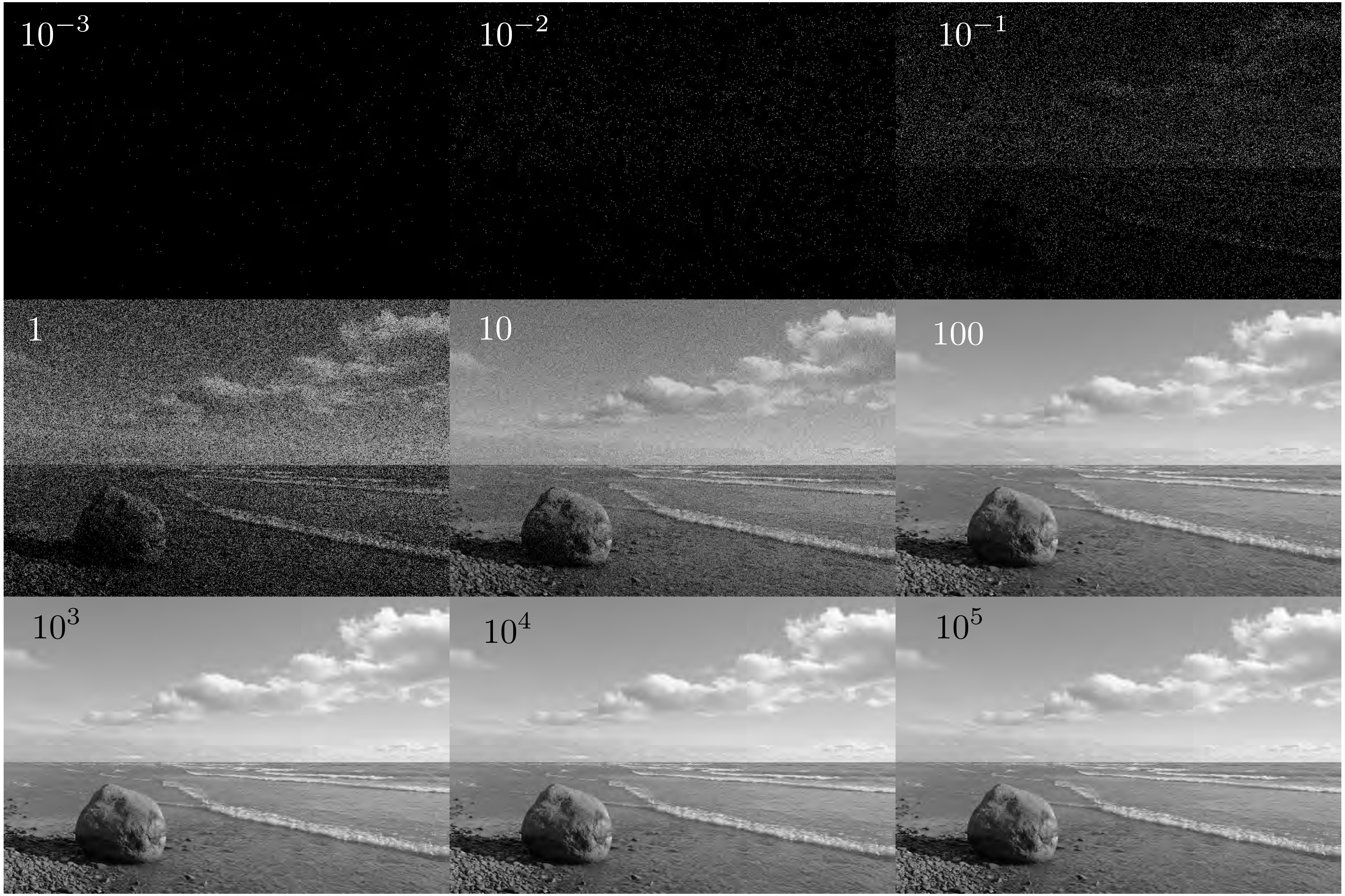


$$S_{\text{shot}}^{1/2} \propto \frac{1}{\sqrt{I_0}}$$

$$S_{\text{rp}}^{1/2} \propto \sqrt{I_0}$$

$$\text{SQL}\Delta \quad x \Delta p \geq \hbar$$

Shot noise: Digital camera, photons per pixel



$$S_{\text{shot}}^{1/2} \propto \frac{1}{\sqrt{I_0}}$$

Radiation pressure noise

The test masses are essentially free (inertial) in the horizontal direction for frequencies above the pendulum frequency of the suspension

$$\ddot{x} = \frac{F_{\text{rad}}}{M}$$

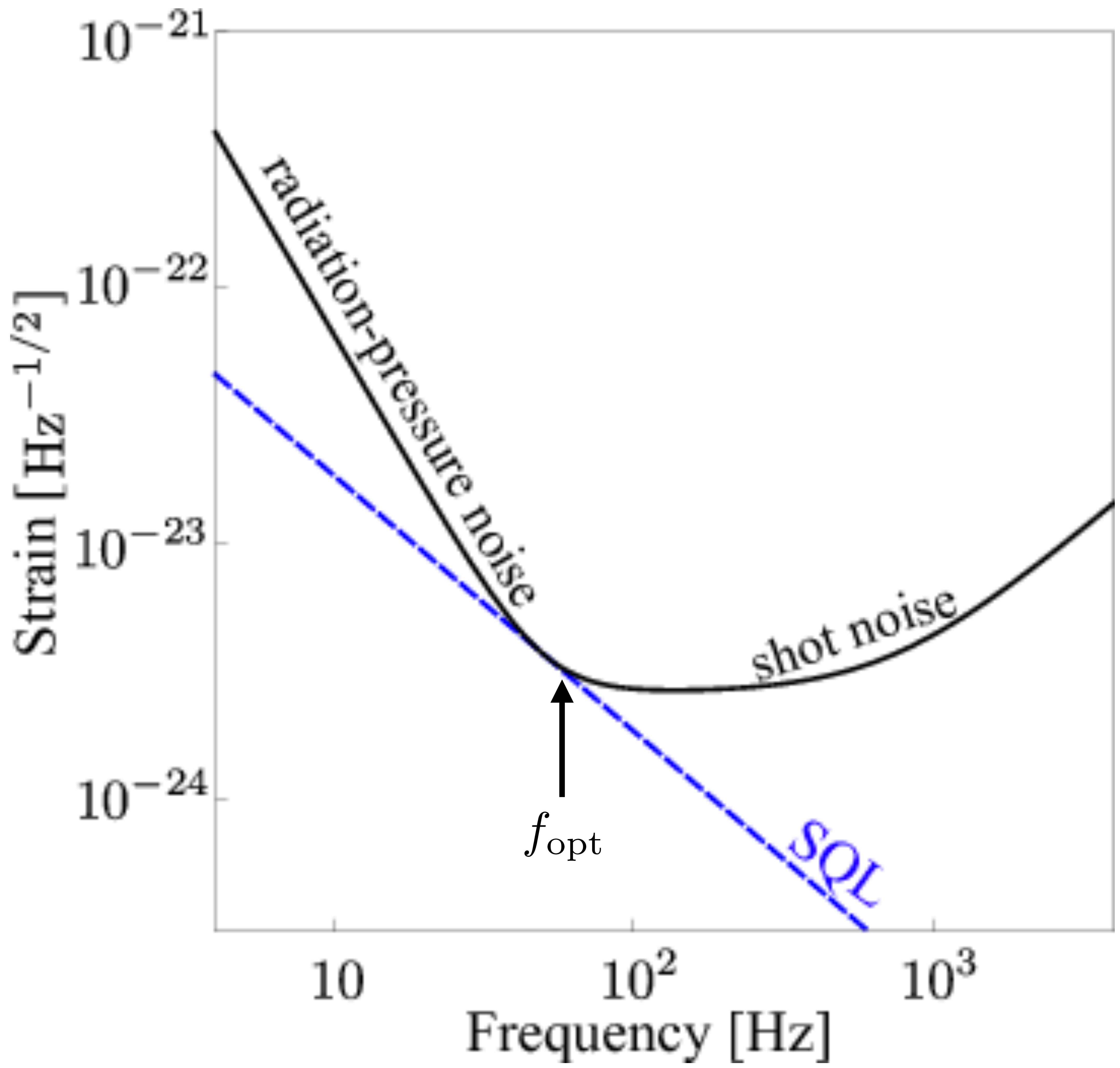
$$\Rightarrow \quad \tilde{x} = -\frac{\tilde{F}_{\text{rad}}}{4\pi^2 f^2 M}$$

$$\mathbb{E}[\tilde{x}] = 0$$

$$\mathbb{E}[\tilde{x}\tilde{x}^*] \propto \frac{I_0}{f^4 M^2}$$

$$\Rightarrow \quad S_{\text{rp}}^{1/2} \propto \frac{\sqrt{I_0}}{f^2 M}$$

Fundamental (quantum) noise



The intensity of the laser light is frequency dependent due to the cavity response

RP Shot

$$S_{\rm Q} = \frac{\hbar}{\pi^2 f^2 M L^2}\left(\frac{I(f)}{\pi^2 f^2 M c} + \frac{\pi^2 f^2 M c}{I(f)}\right)$$

$$I(f) \sim \frac{I_0}{1 + (f/f_-)^2}$$

SQL is reached when the Shot and RP contributions are equal (minimum of $S_{\rm Q}$)

$$S_{\rm SQL} = \frac{2\hbar}{\pi^2 f_{\rm opt}^2 M L^2}$$

Standard Quantum Limit

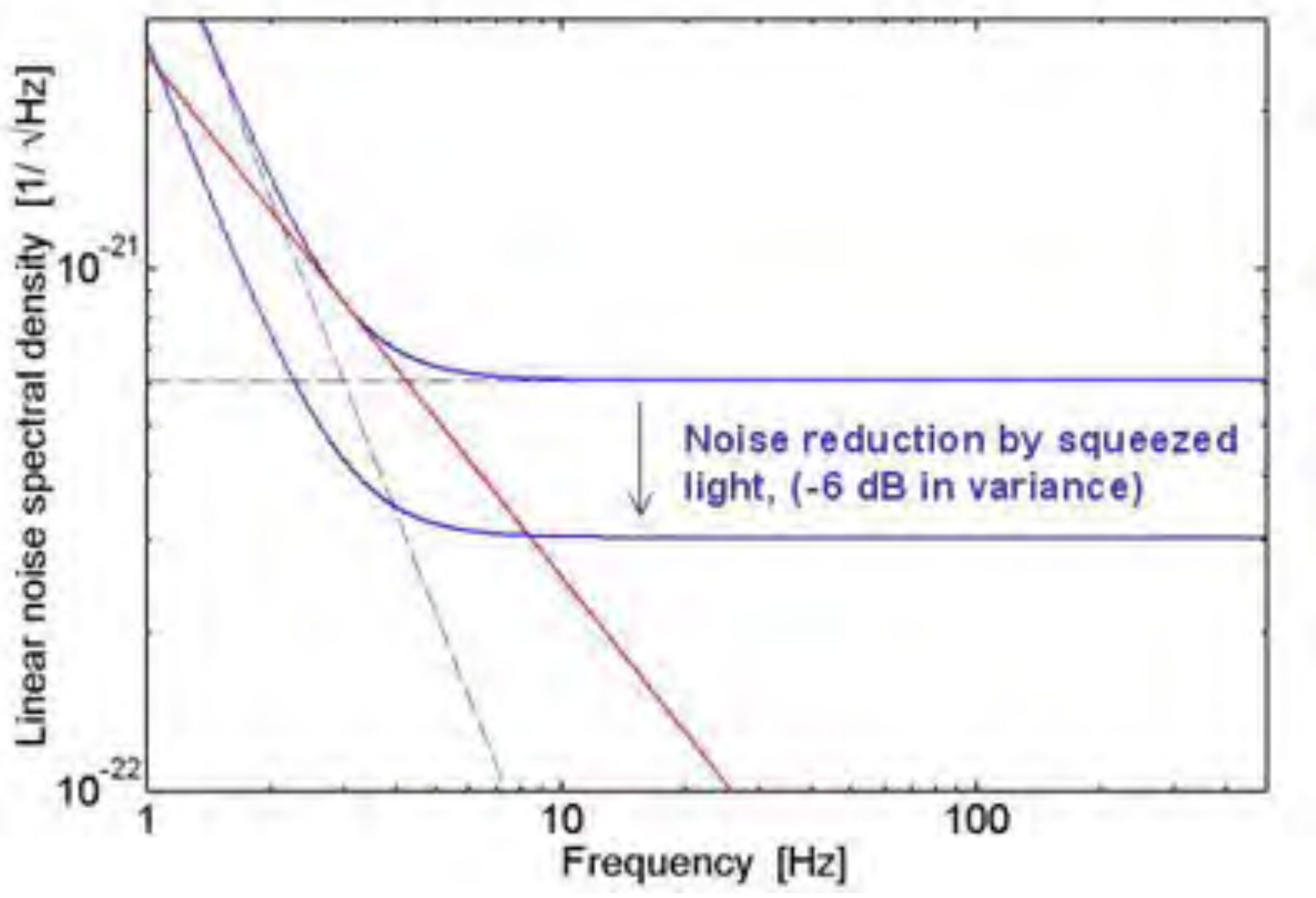
$$\mathrm{SQL}\Delta \quad x\Delta p \geq \hbar$$

$$S_{\mathrm{SQL}} = \frac{2\hbar}{\pi^2 f_{\mathrm{opt}}^2 M L^2}$$

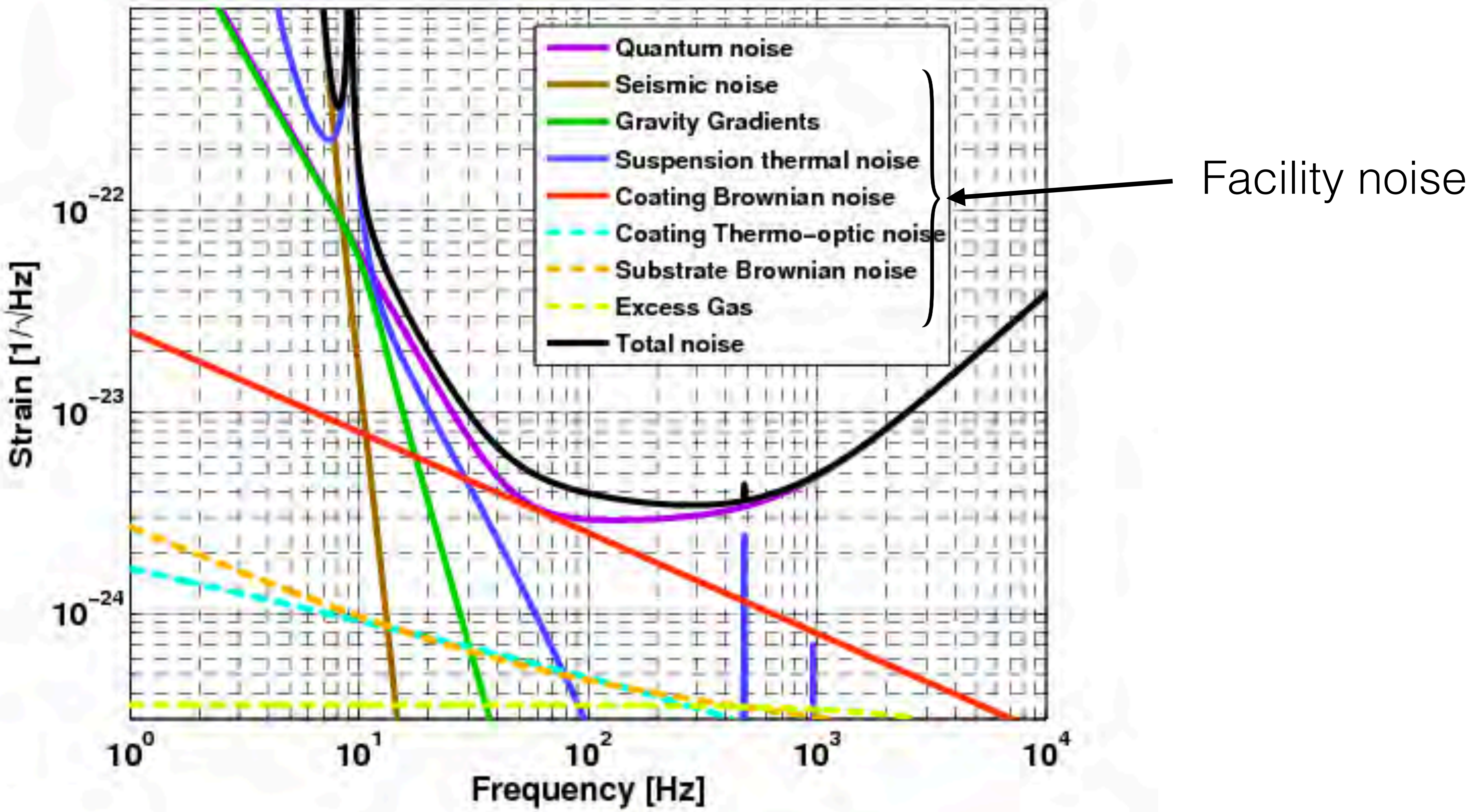
The SQL is not a fundamental limit to GW detector sensitivity

1) Measure momentum change rather than position change (speedmeter)

2) The RP and Shot noise can be made to be correlated. Allows us to reshape uncertainty ellipse using squeezed light (Quantum no-demolition measurement)



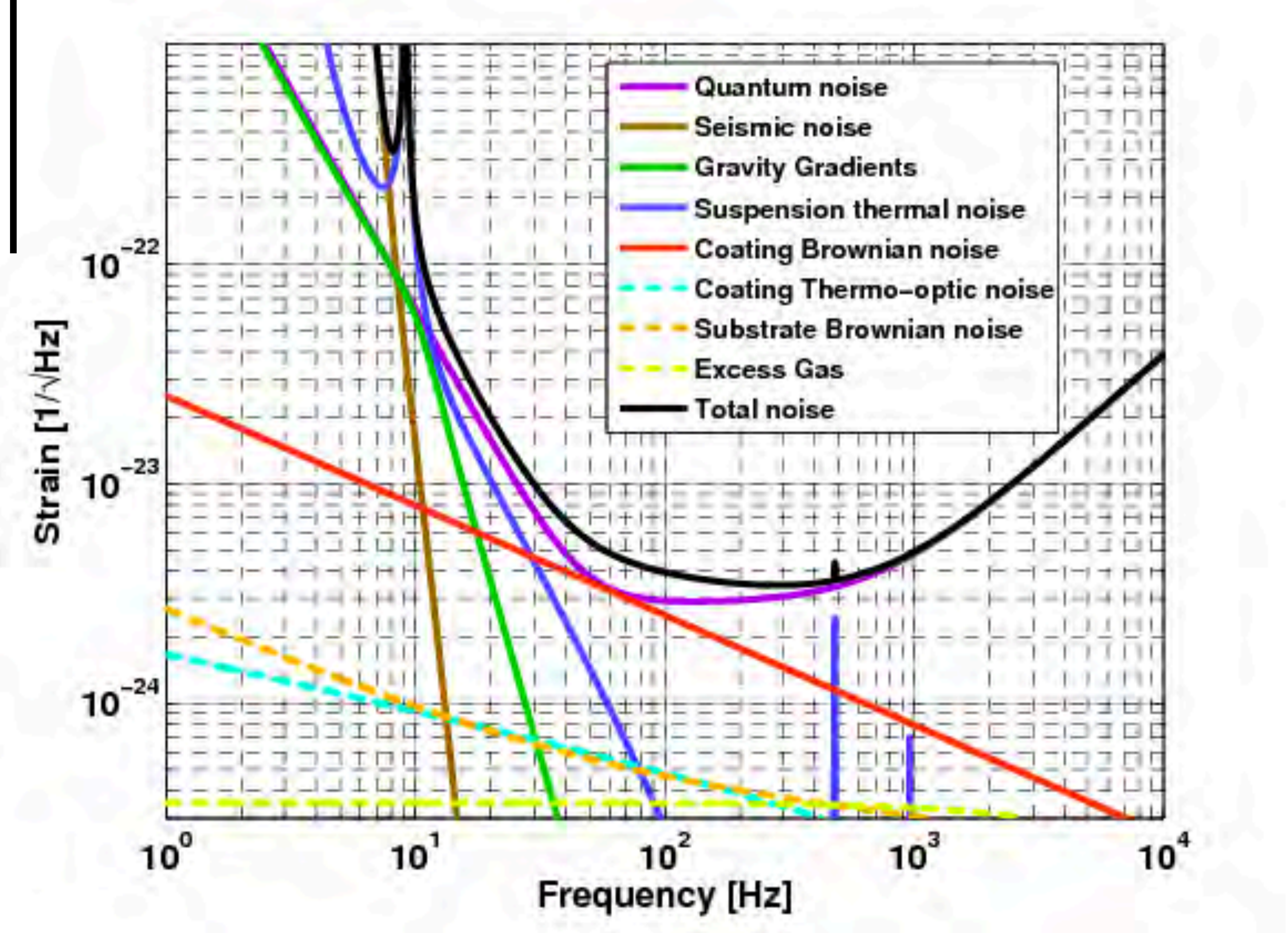
Facility Noise



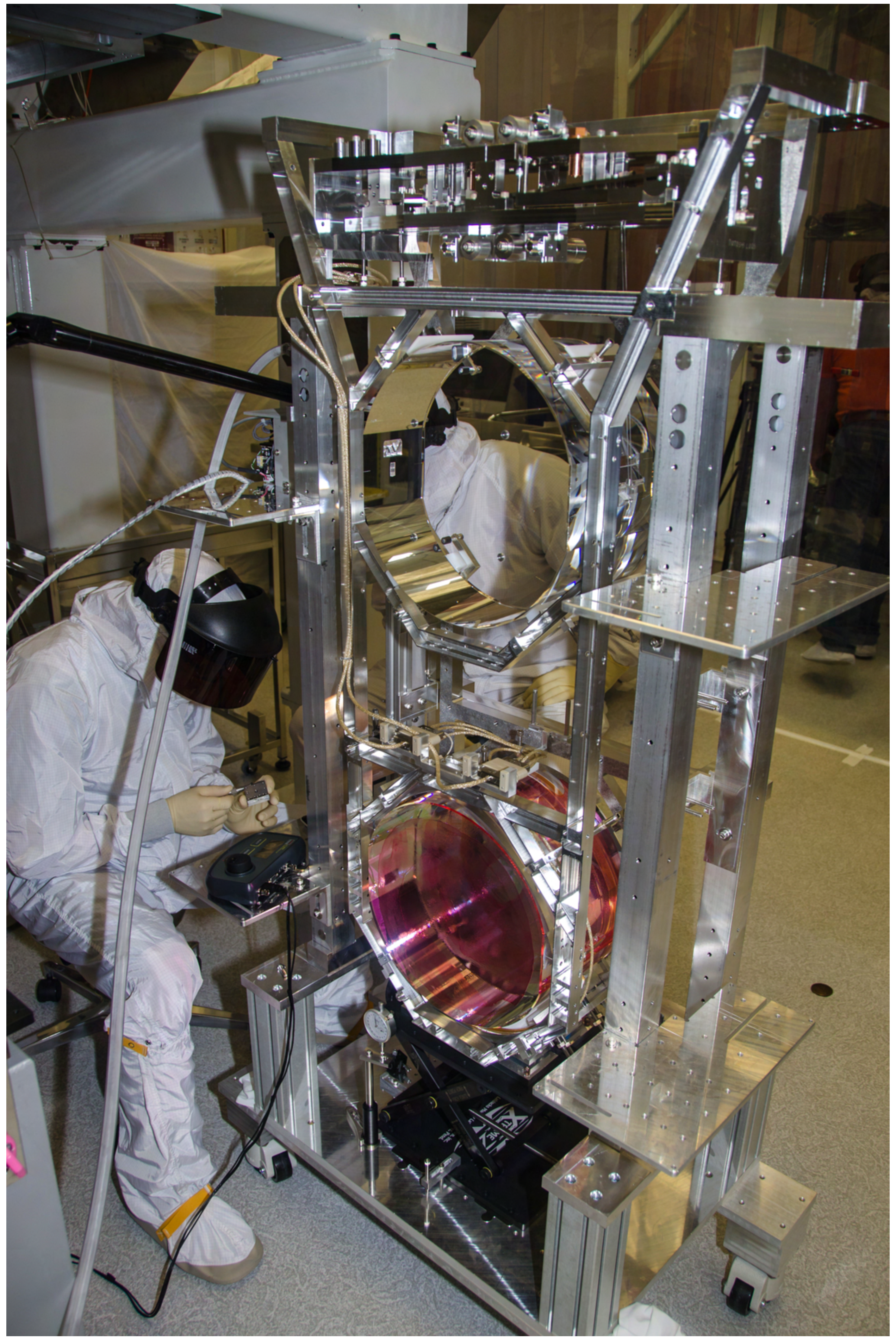
Seismic Noise

10^{10} too large

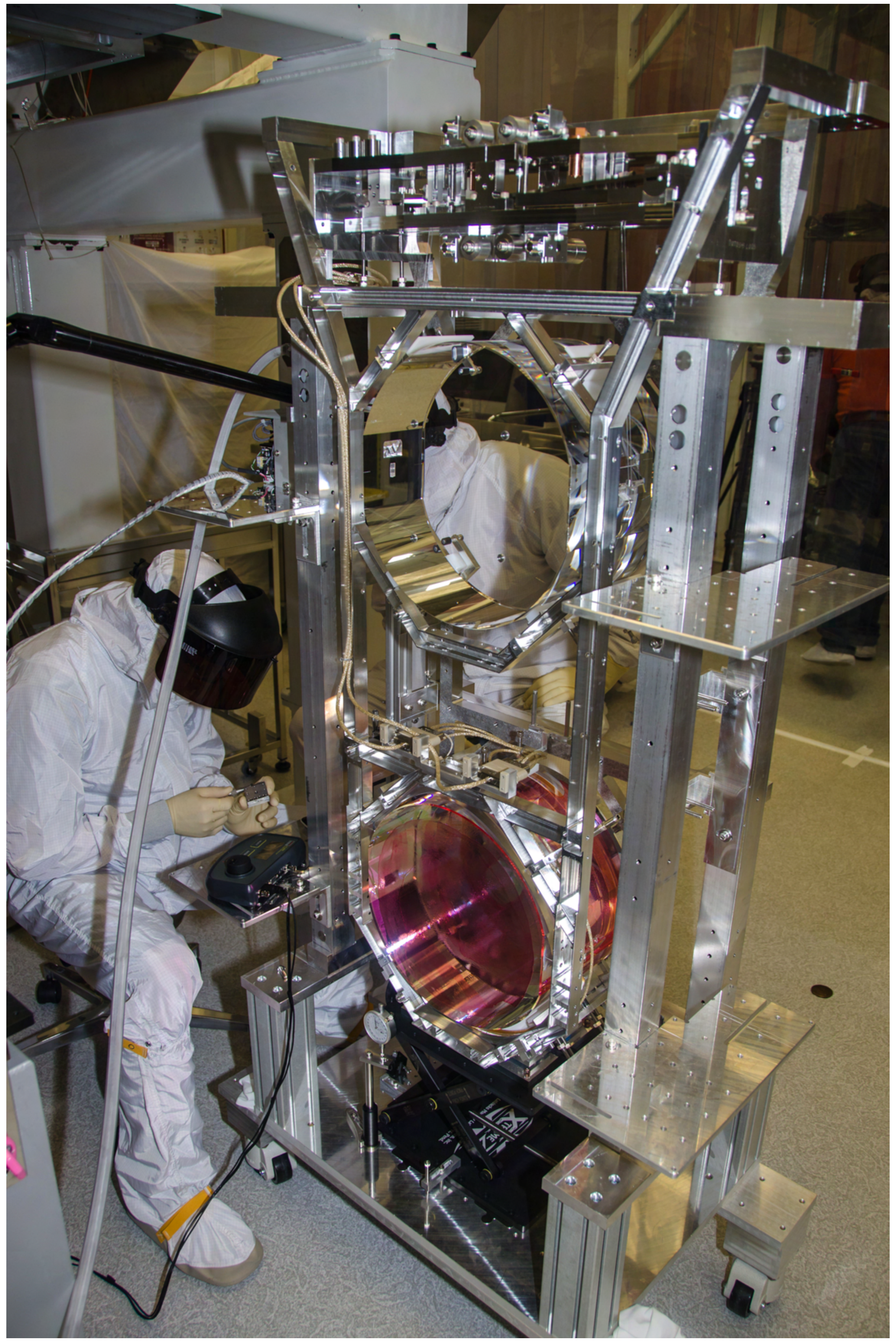
$$S_{\rm seis}^{1/2} \sim 10^{-12} \left(\frac{10\,{\rm Hz}}{f}\right)^2 \,{\rm Hz}^{-1/2}$$



Need one of these



Seismic Noise



$$S_{\mathrm{seis}}^{1/2} \sim 10^{-12} \left(\frac{10\,\mathrm{Hz}}{f} \right)^2 \mathrm{Hz}^{-1/2}$$

Single pendulum suspension

$$S_{\mathrm{seis,filt}}^{1/2} = \frac{1}{|1 - (f/f_{\mathrm{pend}})^2|} S_{\mathrm{seis}}^{1/2}$$

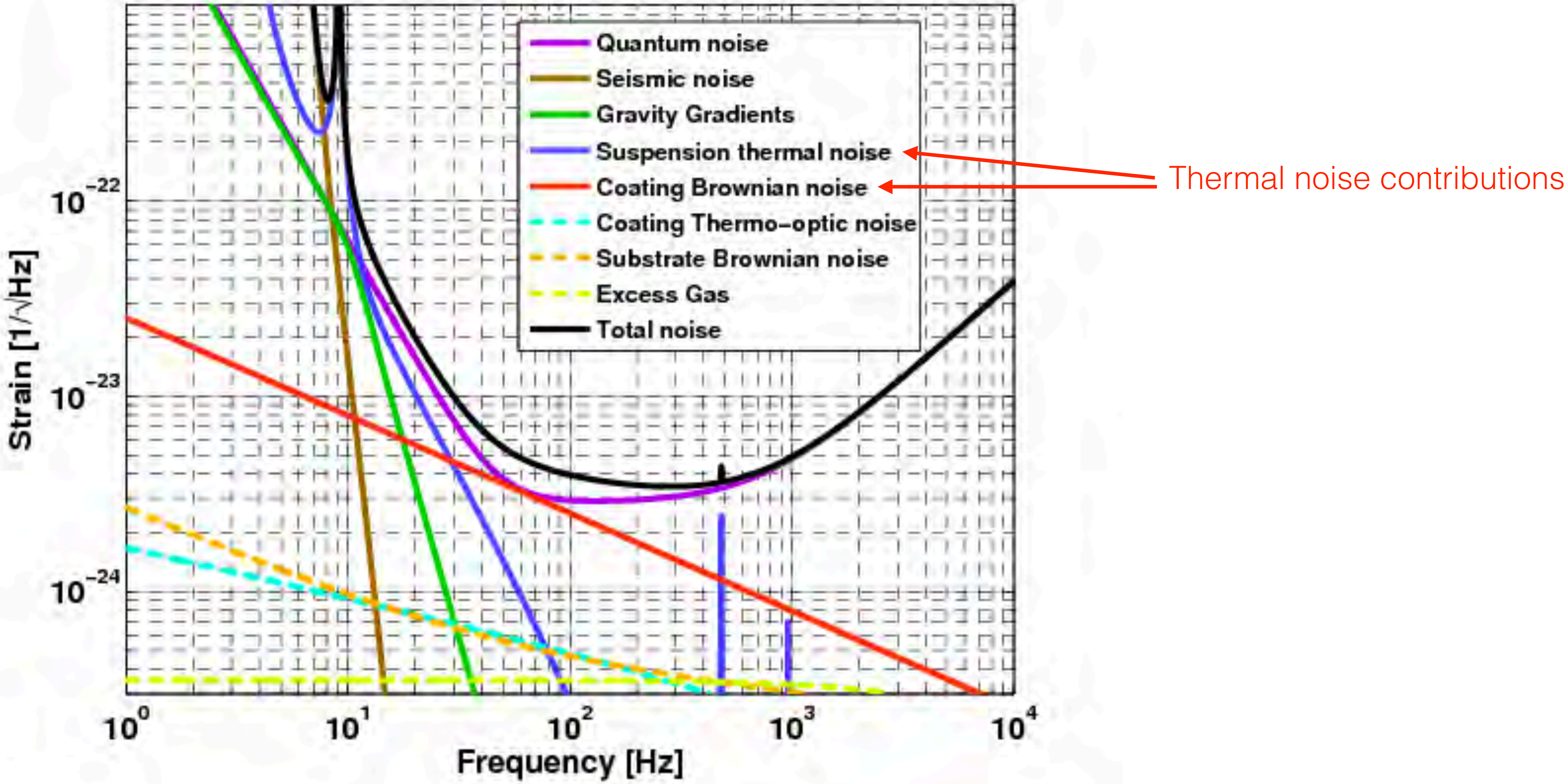
$$f_{\mathrm{pend}} \sim 1\,\mathrm{Hz}$$

$$S_{\mathrm{seis,filt}}^{1/2} \approx 10^{-12} \left(\frac{f_{\mathrm{pend}}}{f} \right)^2 \left(\frac{10\,\mathrm{Hz}}{f} \right)^2 \mathrm{Hz}^{-1/2}$$

aLIGO 5-stage pendulum suspension

$$S_{\mathrm{seis,filt}}^{1/2} \approx 10^{-12} \left(\frac{f_{\mathrm{pend}}}{f} \right)^{10} \left(\frac{10\,\mathrm{Hz}}{f} \right)^2 \mathrm{Hz}^{-1/2}$$

Thermal Noise



Thermal Noise

Fluctuation-dissipation theorem: PSD of fluctuations of a system in equilibrium at temperature *T* is determined by the dissipative terms that return the system to equilibrium

$$S_T(f) = \frac{4k_B T}{(2\pi f)^2}\mathbb{R}[Y(f)]$$

For an *anelastic* spring with loss angle ϕ Hooke's law becomes $F = -k\,x\,(1+i\phi)$

$$S_T(f) = \frac{k_B T}{2\pi^3 M f}\frac{f_{\mathrm{res}}^2\phi(f)}{(f_{\mathrm{res}}^2 - f^2)^2 + f_{\mathrm{res}}^4\phi^2(f)}$$

Mirror Coating Thermal Noise

$$S_T(f) = \frac{k_B T}{2\pi^3 M f} \frac{f_{\rm res}^2 \phi(f)}{(f_{\rm res}^2 - f^2)^2 + f_{\rm res}^4 \phi^2(f)}$$

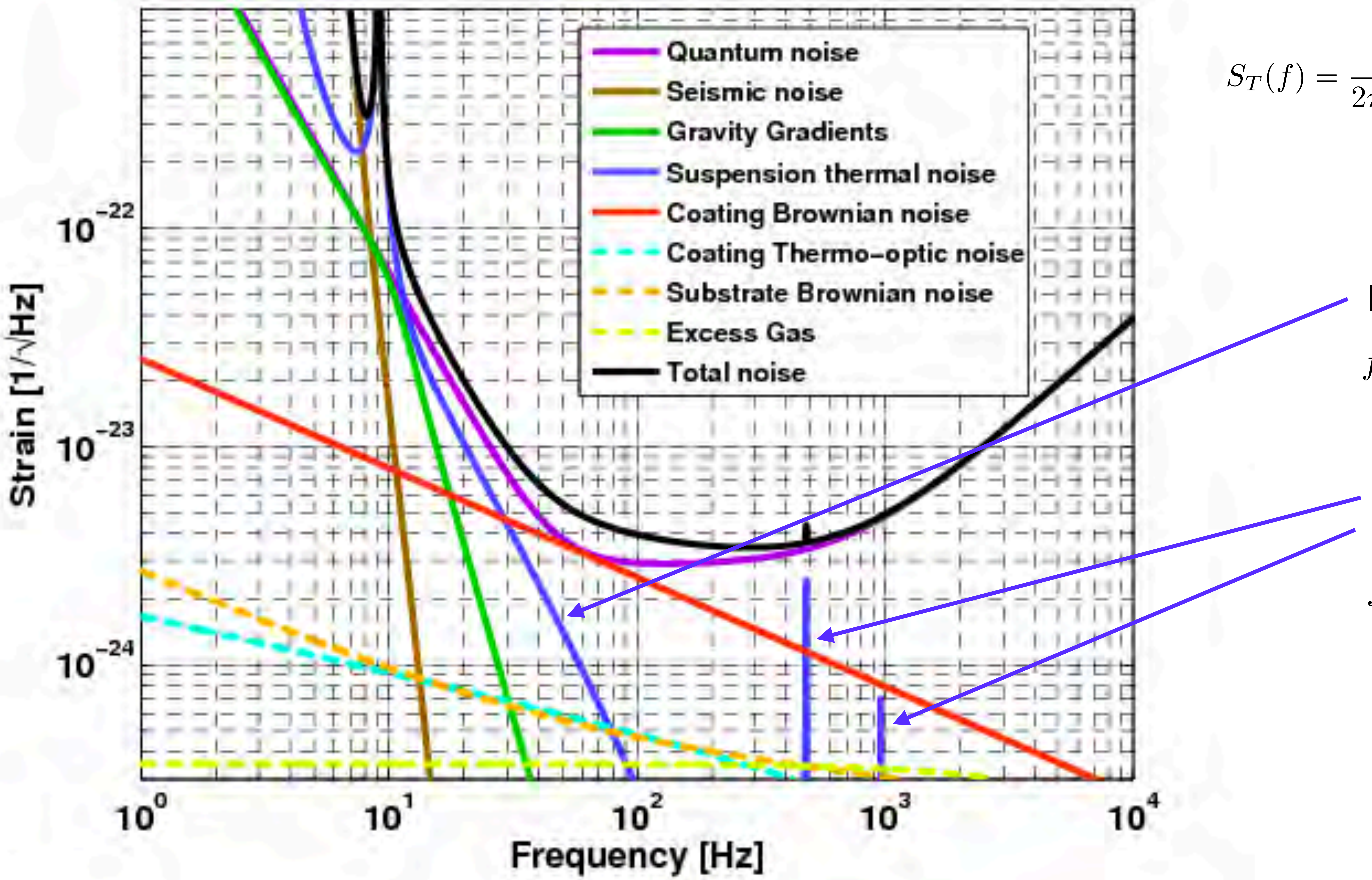
The resonant frequencies for the coatings are very high (tens of kHz), and the loss angle small (millionths)

$$S_{\rm MC} = \left(\frac{2 k_B T \phi}{\pi^3 M L^2 f_{\rm MC}^2}\right) \frac{1}{f}$$

For advanced LIGO

$$S_{\rm MC}^{1/2} = 2.5 \times 10^{-24} \left(\frac{100\,{\rm Hz}}{f}\right)^{1/2} {\rm Hz}^{-1/2}$$

Suspension Thermal Noise



$$S_T(f) = \frac{k_B T}{2\pi^3 M f} \frac{f_{\rm res}^2 \phi(f)}{(f_{\rm res}^2 - f^2)^2 + f_{\rm res}^4 \phi^2(f)}$$

Pendulum mode

$f_{\rm pen} = 9\,{\rm Hz}$

Violin modes

$f_{\rm v} = 500,\ 1000, \ldots\ {\rm Hz}$

Pendulum Suspension Thermal Noise

$$S_T(f) = \frac{k_B T}{2\pi^3 M f} \frac{f_{\rm res}^2 \phi(f)}{(f_{\rm res}^2 - f^2)^2 + f_{\rm res}^4 \phi^2(f)}$$

$$f_{\rm res} = f_{\rm pen} \ll f$$

For advanced LIGO

$$S_{\rm pen}^{1/2} = 3.5 \times 10^{-25} \left(\frac{100\,\mathrm{Hz}}{f} \right)^{5/2} \mathrm{Hz}^{-1/2}$$

Violin mode Thermal Noise

$$S_T(f) = \frac{k_B T}{2\pi^3 M f} \frac{f_{\rm res}^2 \phi(f)}{(f_{\rm res}^2 - f^2)^2 + f_{\rm res}^4 \phi^2(f)}$$

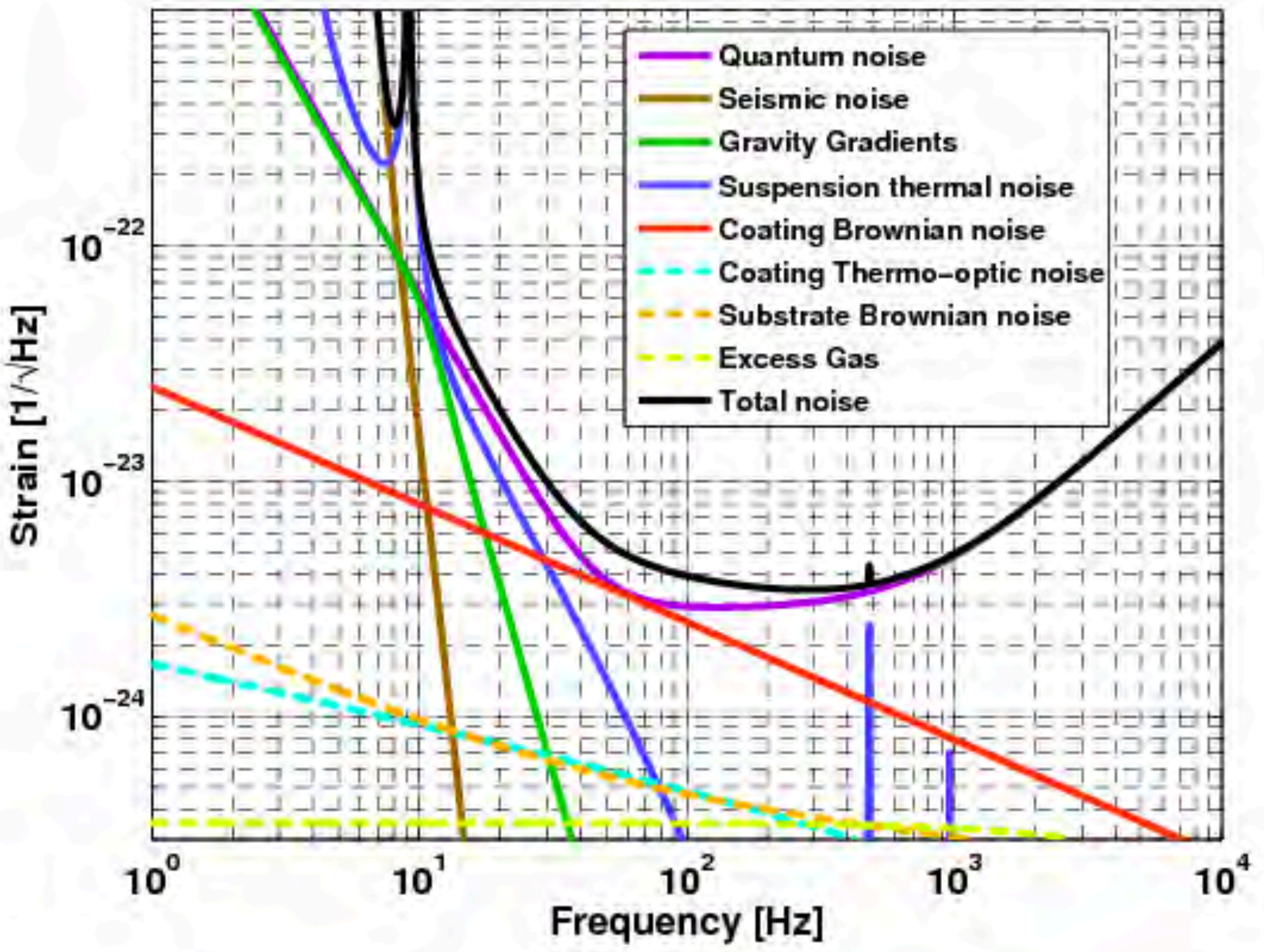
$$f_{\rm res} = f_{\rm v} = 500\,{\rm Hz},\ 1000\,{\rm Hz},\ \ldots$$

For advanced LIGO, first harmonic

$$S_{\rm v}^{1/2} = \frac{3 \times 10^{-24}}{1 + (f_{\rm v}^2 - f^2)^2/\delta f^4}\ {\rm Hz}^{-1/2}$$

$$\delta f = f_{\rm v}\phi^{1/2} \approx 2\,{\rm Hz}$$

Gravity Gradient Noise

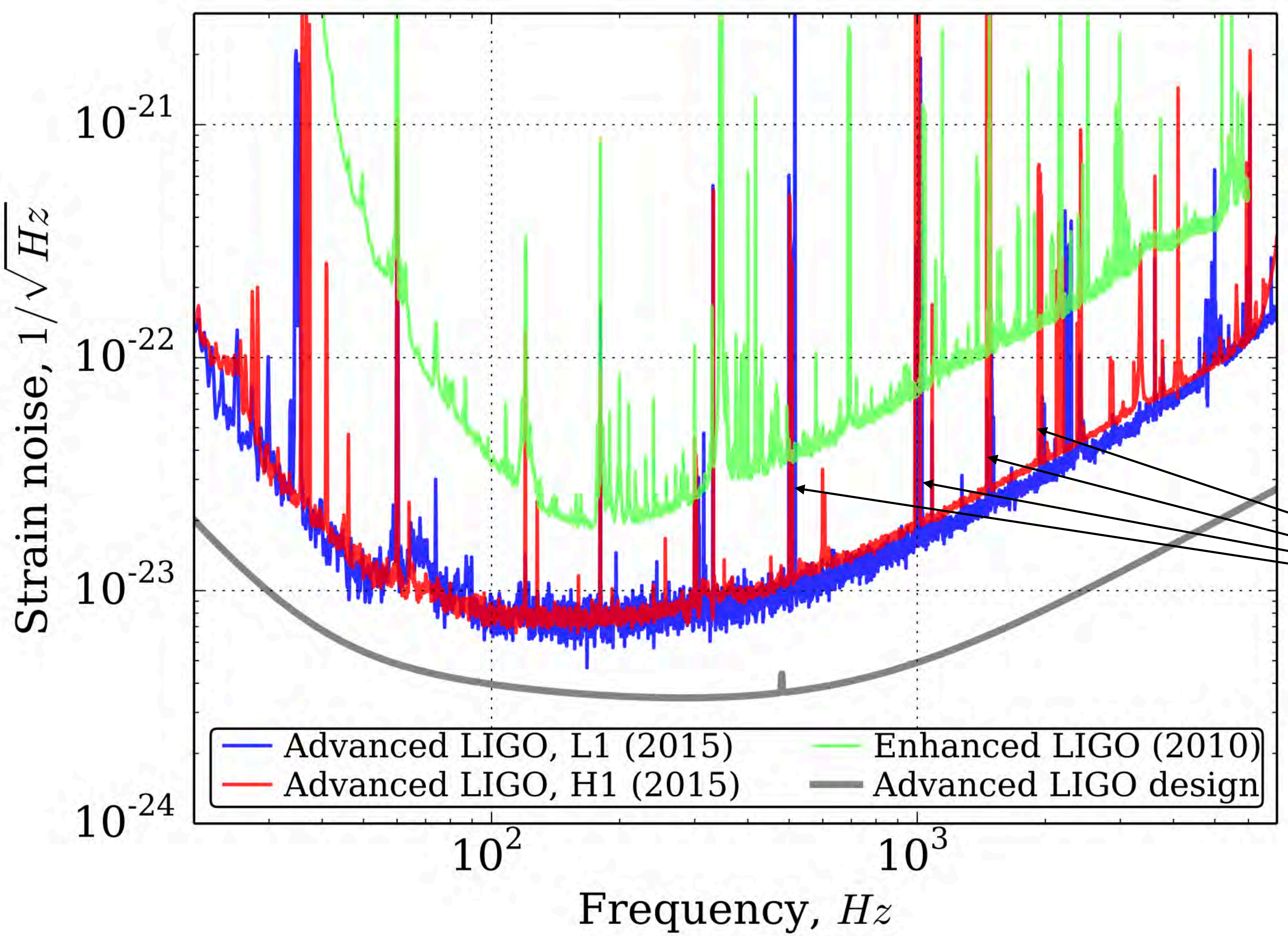


We can't escape Newton

$$\ddot{\mathbf{x}} = G \int \frac{\delta\rho(\mathbf{x}', t)}{|\mathbf{x} - \mathbf{x}'|^3} (\mathbf{x} - \mathbf{x}')\, d^3\mathbf{x}$$

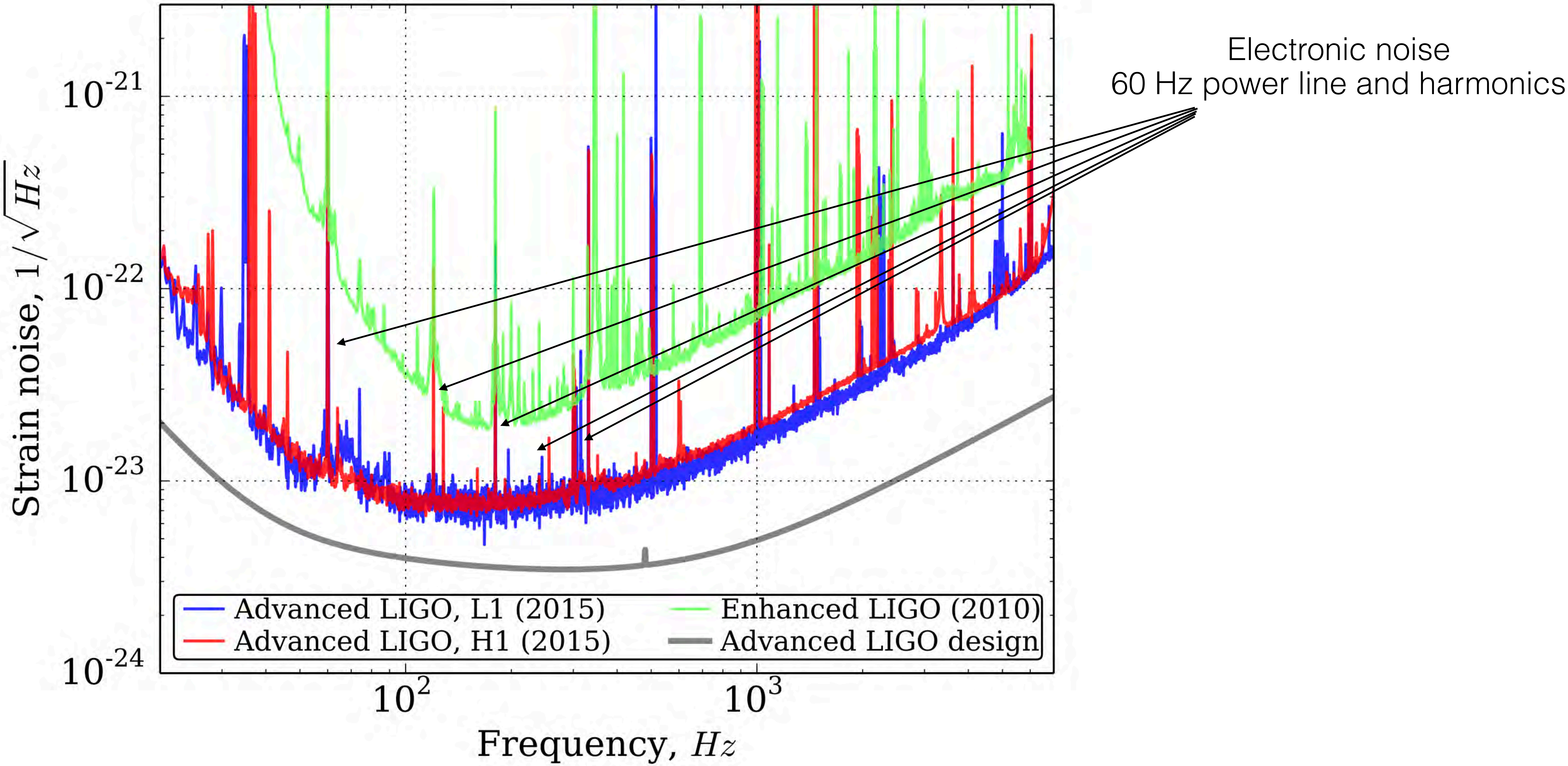
This is why we need LISA!

Real aLIGO noise spectra from O1

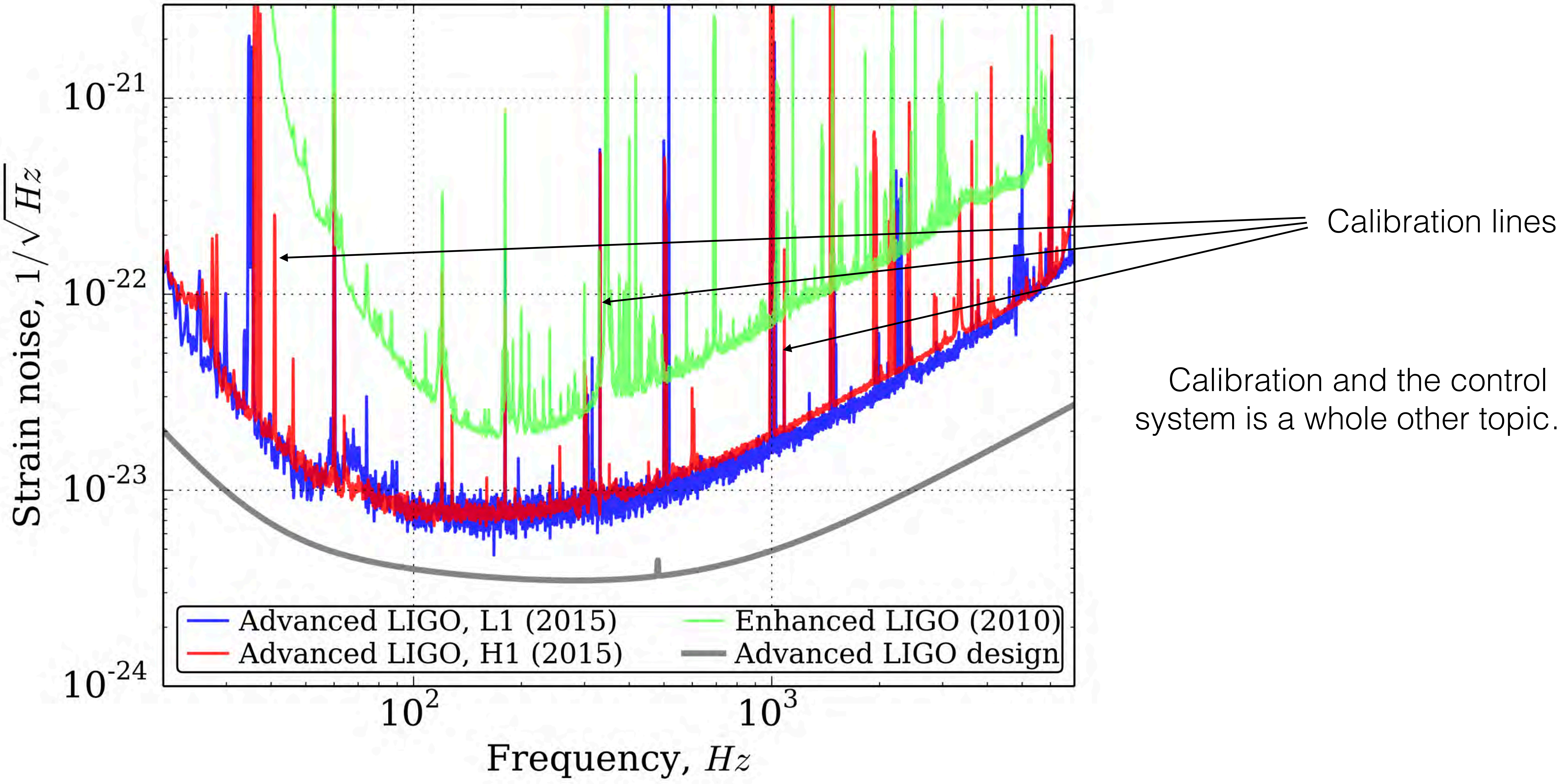


Suspension Thermal noise
Violin modes

Real aLIGO noise spectra from O1



Real aLIGO noise spectra from O1/S6



Calibration and the control system is a whole other topic…

LIGO searches for binary inspired signals

Recall: Likelihood for Stationary Gaussian Noise

$$p(\mathbf{d}|\vec{\lambda}) = \text{const.}\ e^{-\frac{\chi^2(\vec{\lambda})}{2}}$$

$$\chi^2(\vec{\lambda}) = (\mathbf{d} - \mathbf{h}(\vec{\lambda})|\mathbf{d} - \mathbf{h}(\vec{\lambda}))$$

$$(\mathbf{a}|\mathbf{b}) = 2\int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f) + \tilde{a}^*(f)\tilde{b}(f)}{S_n(f)}\, df$$

Suppose that we have two hypotheses:

H_1 : A signal with parameters $\vec{\lambda}$ is present

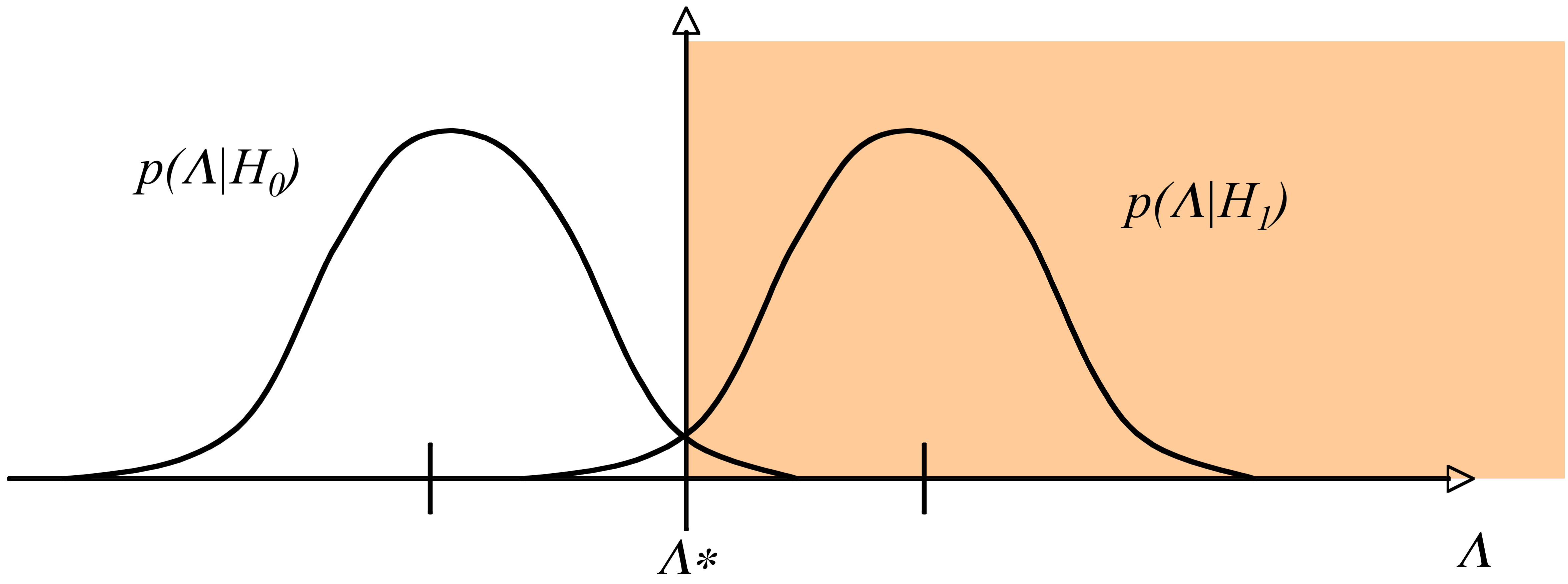
H_0 : No signal is present

Likelihood ratio: $$\Lambda(\vec{\lambda}) = \frac{p(\mathbf{d}|\mathbf{h}(\vec{\lambda}), H_1)}{p(\mathbf{d}, H_0)}$$

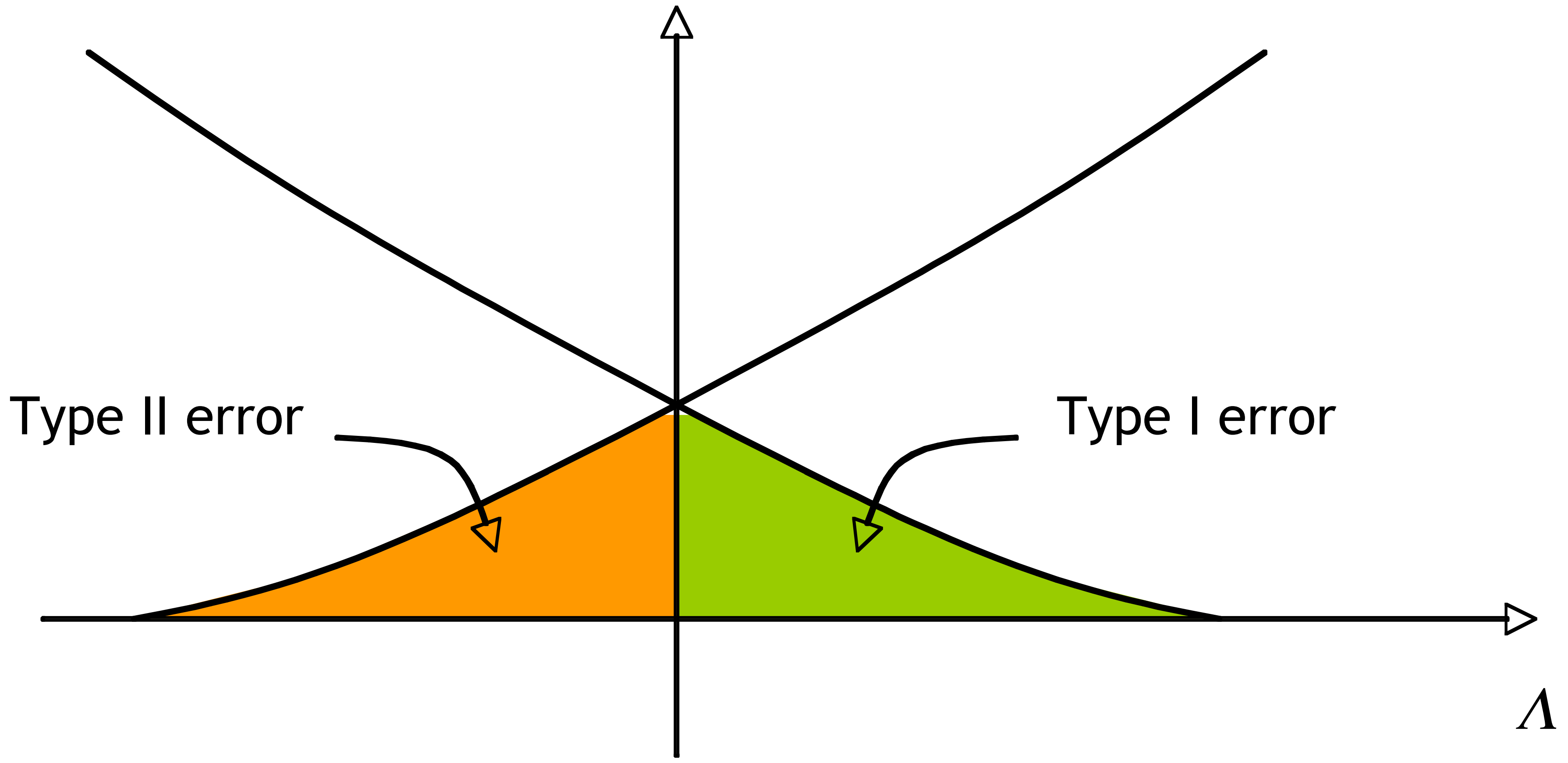
For Gaussian noise: $$\Lambda(\vec{\lambda}) = e^{-(\mathbf{d}|\mathbf{h}) + \frac{1}{2}(\mathbf{h}|\mathbf{h})}$$

Frequentist Hypothesis Testing

Λ – Detection Statistic $\quad H_0$ – Noise Hypothesis $\quad H_1$ – Noise + Signal Hypothesis



Set threshold Λ_* such that $\Lambda > \Lambda_*$ favors hypothesis H_1



Type I error - False Alarm

Type II error - False Dismissal

Neyman-Pearson Theorem

For a fixed false alarm rate, the false dismissal rate is minimized by the likelihood ratio statistic

$$\Lambda(\vec{\lambda}) = \frac{p(\mathbf{d}|\mathbf{h}(\vec{\lambda}), H_1)}{p(\mathbf{d}, H_0)}$$

The likelihood ratio is maximized over the signal parameters.

The rho statistic is often used in place of the likelihood ratio

$$\text{Writing } \mathbf{h} = \rho\hat{h} \text{ where } (\hat{h}|\hat{h}) = 1 \qquad \Lambda(\vec{\lambda}) = e^{\,\rho(\mathbf{d}|\hat{h}) - \frac{1}{2}\rho^2}$$

Maximizing wrt rho

$$\frac{\partial\Lambda(\vec{\lambda})}{\partial\rho} = 0 \qquad \Rightarrow \quad \rho(\vec{\lambda}) = (\mathbf{d}|\hat{h}(\vec{\lambda}))$$

$$\log\Lambda(\vec{\lambda}) = \frac{1}{2}\rho^2(\vec{\lambda})$$

The rho statistic and SNR

The signal-to-noise ratio (SNR) is defined:

$$\begin{aligned}
\mathrm{SNR} &= \frac{\text{Expected value when signal present}}{\text{RMS value when signal absent}} \\
&= \frac{E[\rho]}{\sqrt{E[\rho_0^2] - E[\rho_0]^2}} \\
&= (h|\hat{h}) \\
&= \sqrt{(h|h)}
\end{aligned}$$

In practice, the detector noise is not perfectly Gaussian, and variants of the rho statistic are now used, notably the "new SNR" statistic, introduced by B. Allen Phys.Rev. D71 (2005) 062001

$$\mathrm{SNR}^2 = 4 \int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}\, df$$

Matched Filtering = Maximum Likelihood

Convolve the data with a filter (template) K: $Y = \int dt \int du\, K(t-u) d(u)$

$$\begin{aligned} \text{SNR} &= \frac{E[Y]}{\sqrt{E[Y_0^2] - E[Y_0]^2}} \\ &= \frac{2\int_0^\infty df (\tilde{h}^*(f)\tilde{K}(f) + \tilde{h}(f)\tilde{K}^*(f))}{\sqrt{4\int_0^\infty df |\tilde{K}(f)|^2 S(f)}} \end{aligned}$$

Maximizing the SNR yields $\tilde{K}(f) = \frac{\tilde{h}(f)}{S(f)}$ $\Rightarrow$ $Y = (\mathbf{d}|\mathbf{h}) = \rho\sqrt{(h|h)}$

Frequentist Detection Threshold

For stationary, Gaussian noise the detection statistic ρ is Gaussian distributed.

For the null hypothesis we have
$$p_0(\rho) = \frac{1}{\sqrt{2\pi}}\, e^{-\rho^2/2}$$

For the detection hypothesis we have
$$p_1(\rho) = \frac{1}{\sqrt{2\pi}}\, e^{-(\rho^2 - \mathrm{SNR}^2)/2}$$

Setting a threshold of ρ_* gives the false alarm and false dismissal probabilities
$$P_{\rm FA} = \frac{1}{2}\mathrm{erfc}(\rho_*/\sqrt{2})$$
$$P_{\rm FD} = \frac{1}{2}\mathrm{erfc}((\rho_* - \mathrm{SNR})/\sqrt{2})$$

LIGO/Virgo analyses do not use SNR thresholds, but rather use False Alarm Rate thresholds

$$\mathrm{FAR} = \frac{P_{\rm FA}}{T_{\rm obs}}$$

e.g. FAR = One in million years and an observation time of one year

$$P_{\rm FA} = 10^{-6} \quad \text{aka} \quad 4.9\,\sigma \qquad\qquad \rho_* = 4.8$$

Grid Based Searches

Goal is to lay out a grid in parameter space that is fine enough to catch any signal with some good fraction of the maximum matched filter SNR

The match measures the fractional loss in SNR in recovering a signal with a template and defines a natural metric on parameter space:

$$M(\vec{x},\vec{y}) = \frac{(h(\vec{x})|h(\vec{y}))}{\sqrt{(h(\vec{x})|h(\vec{x}))(h(\vec{y})|h(\vec{y}))}}$$

Taylor expanding $M(\vec{x},\vec{x}+\Delta\vec{x}) = 1 - g_{ij}\Delta x^i \Delta x^j + \dots$

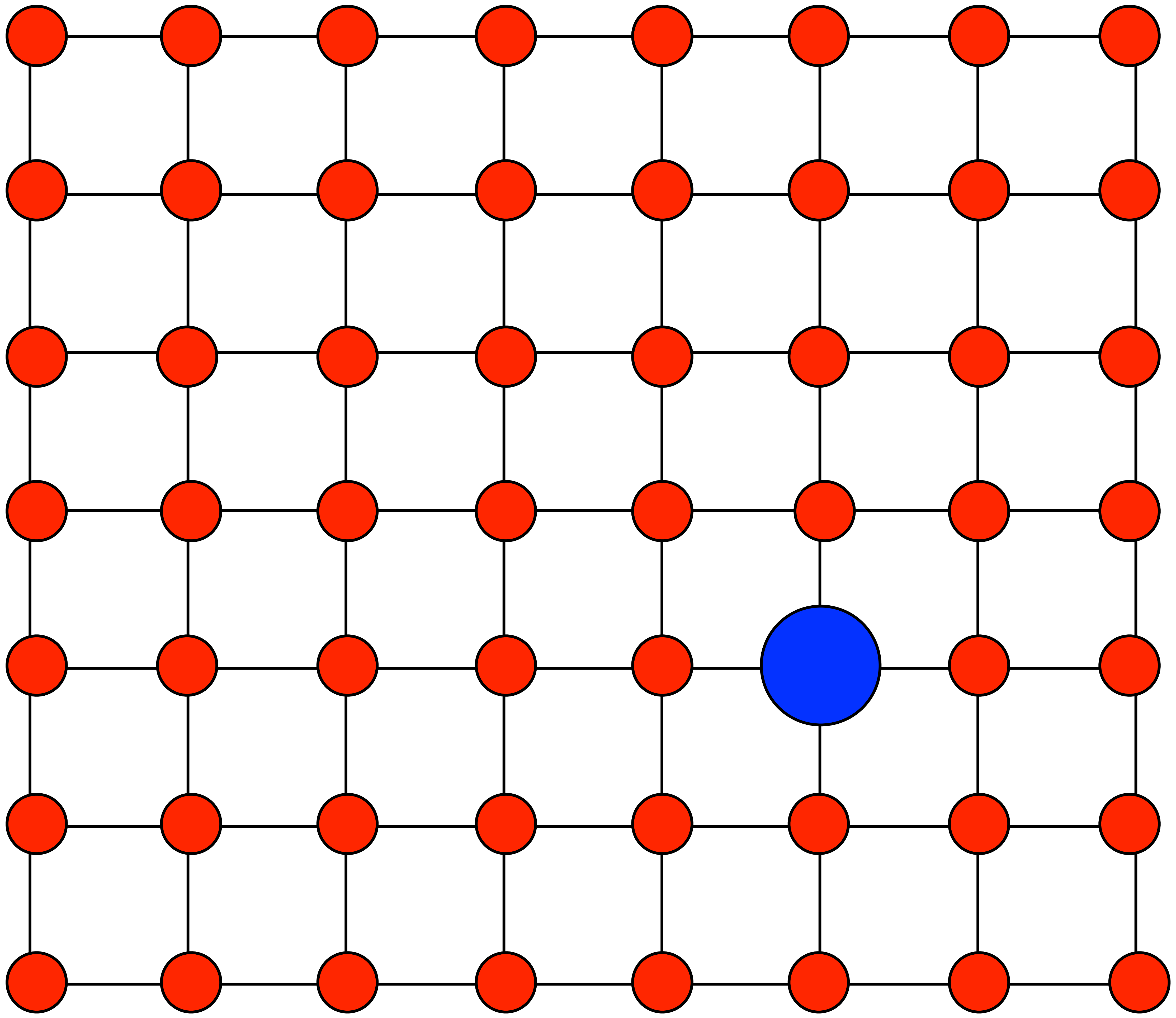
where $g_{ij} = \frac{(h_{,i}|h_{,j})}{(h|h)} - \frac{(h|h_{,i})(h|h_{,j})}{(h|h)^2}$ (Owen Metric)

Number of templates (for a hypercube lattice in D dimensions)

$$N = \frac{V}{\Delta V} = \frac{\int d^D x \sqrt{g}}{(2\sqrt{(1-M_{\rm min})}/D)^D}$$

Cost grows geometrically with D for any lattice

LIGO Style Grid Searches



Typically 2-3 dimensional, 1000's points

Reducing the cost of a search

In most cases it is possible to analytically maximize over 3 or more parameters

Distance:

The unit normalized template $\hat{h}$ defines a reference distance $\bar{D}$

Scaling this template to distance D gives

$$h = \frac{\bar{D}}{D}\,\hat{h}$$

The distance is then estimated from the data as

$$D = \frac{\bar{D}}{(d|\hat{h})}$$

Reducing the cost of a search

Phase Offset:

Generate two templates $h(\phi = 0)$ and $h(\phi = \pi/2)$

Then $$(d|h)_{\max\,\phi} = \sqrt{(d|h(0))^2 + (d|h(\pi/2))^2}$$

Easy to see this in the Fourier domain.

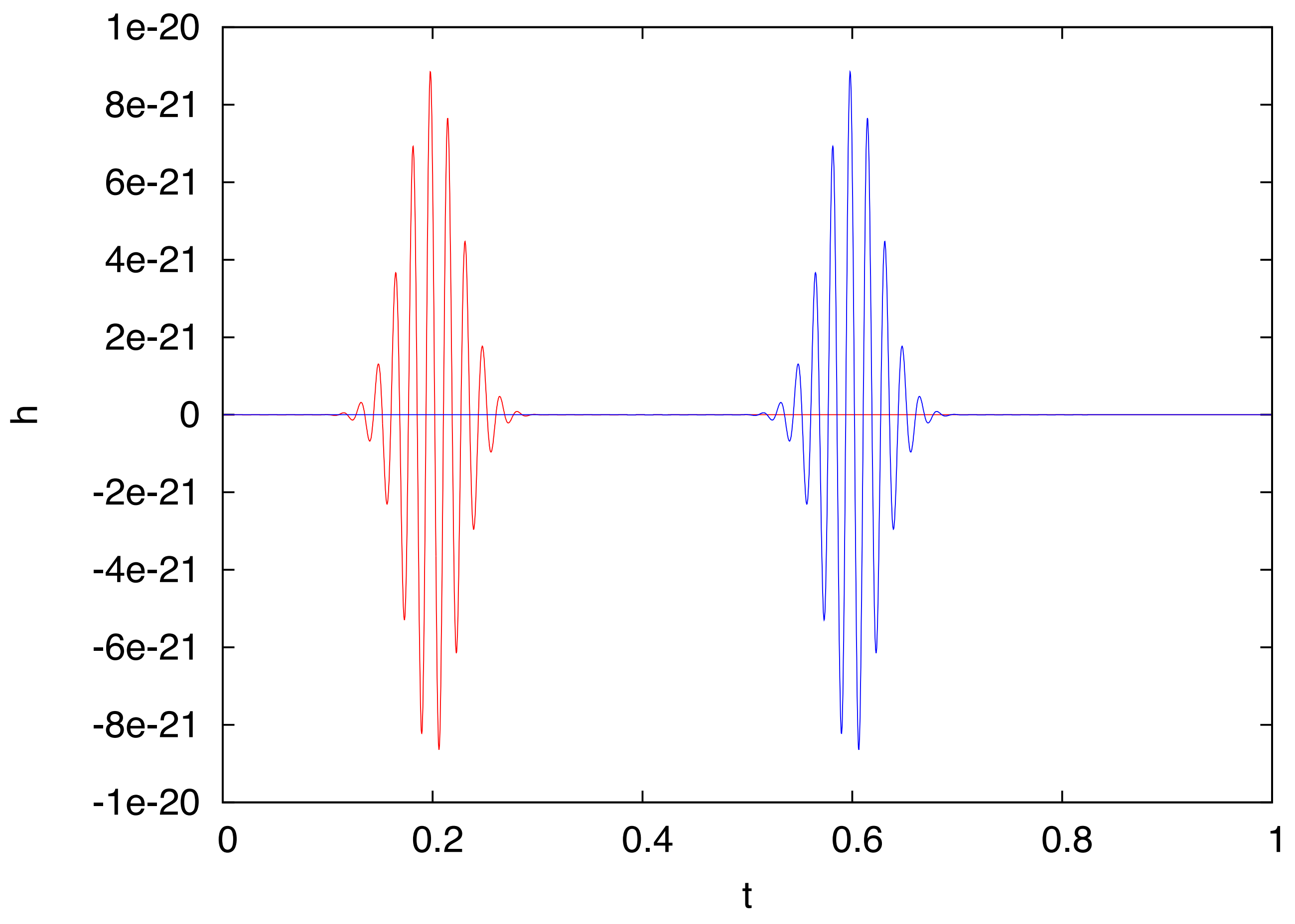
Suppose $\tilde{d} = \tilde{h}_0\, e^{i\phi}$, then

$$(d|h(0)) = (h_0|h_0)\cos\phi$$

$$(d|h(\pi/2)) = (h_0|h_0)\sin\phi$$

Reducing the cost of a search

Time Offset:



Fourier transform treats time as periodic - use this to our advantage

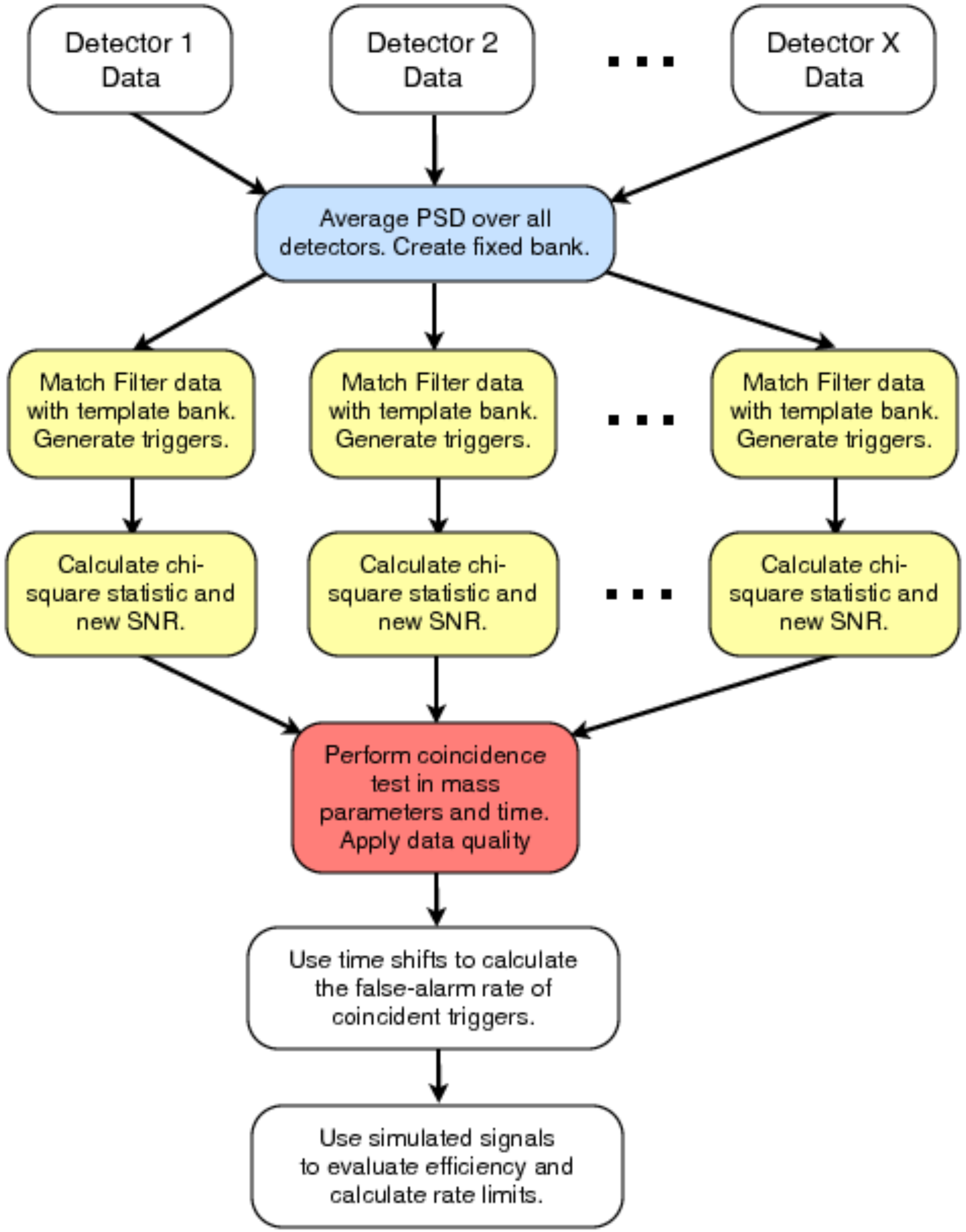
Compute the inverse Fourier transform of the product of the Fourier transforms:

$$(d|h)(\Delta t) = 4 \int \frac{\tilde{d}^*(f)\tilde{h}(f)}{S(f)} \, e^{2\pi i f \Delta t} \, df$$

Then if the template and data differ by a time shift: $d(t) = h(t - t_0)$

$$(d|h)_{\max\, t} = (d|h)(\Delta t = t_0)$$

Workflow for pyCBC search



Template bank constructed

Matched filtering is done per-detector (not coherent)

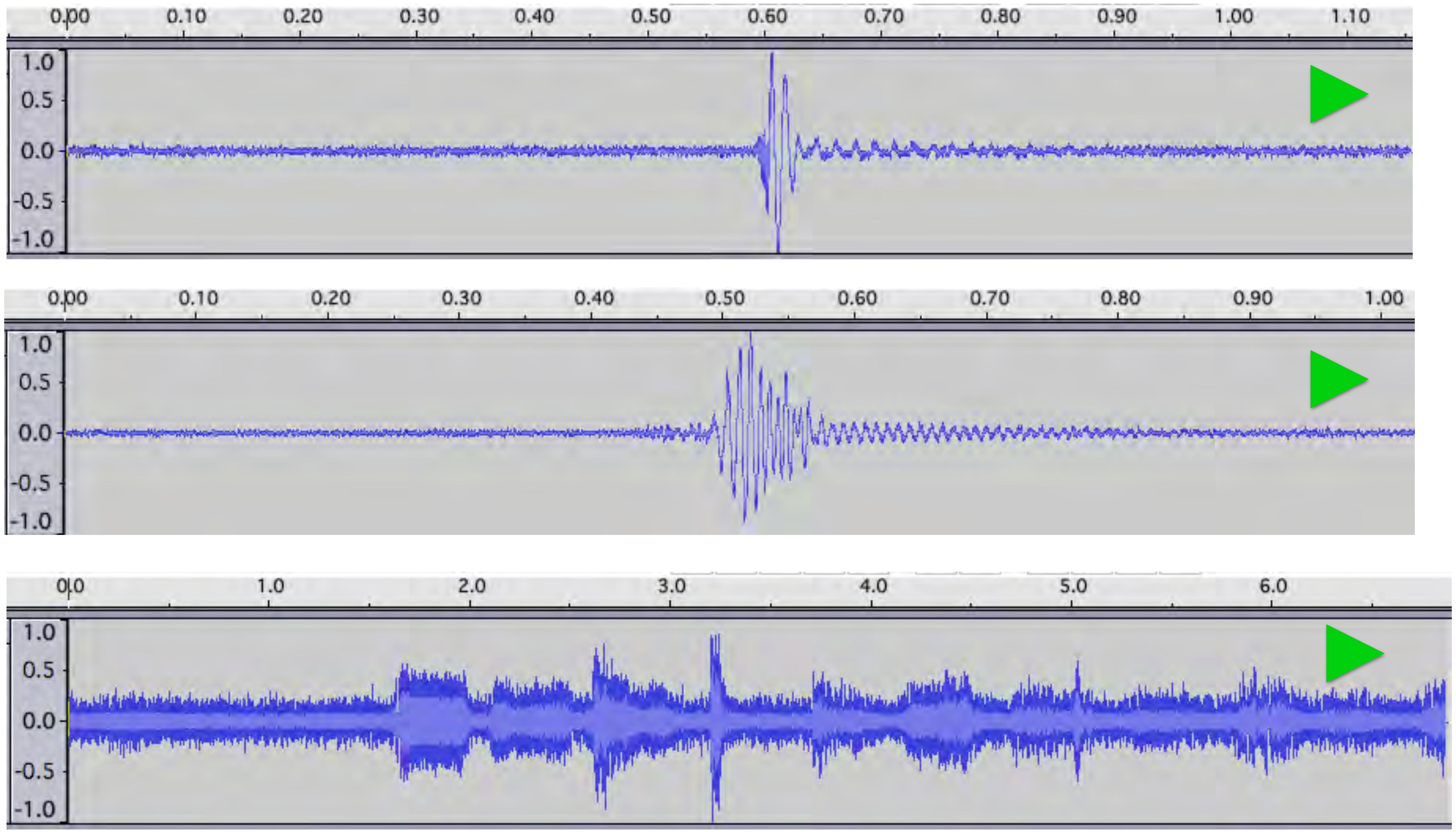
Detection statistic computed ("new SNR")

Coincidence in time/mass enforced
Data quality vetoes applied

Monte Carlo background to compute FAR vs new SNR

Things that go bump in the night

(Bandpass filtered, whitened, time domain)



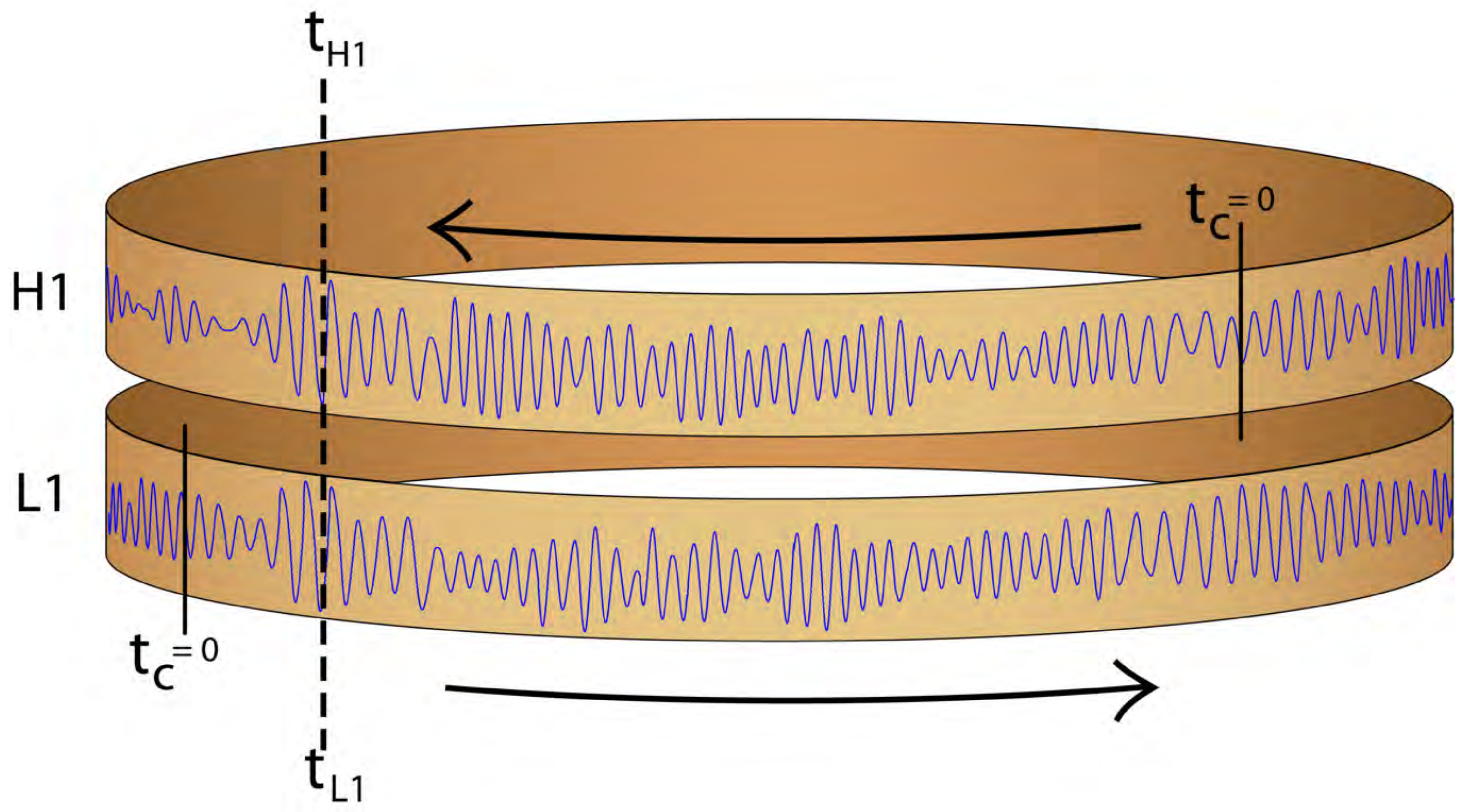
Samples from the Syracuse Audio Study of Glitches

Contending with non-stationary, non-Gaussian noise

Non-stationary: → Adiabatic drifts in the PSD — - work with short data segments

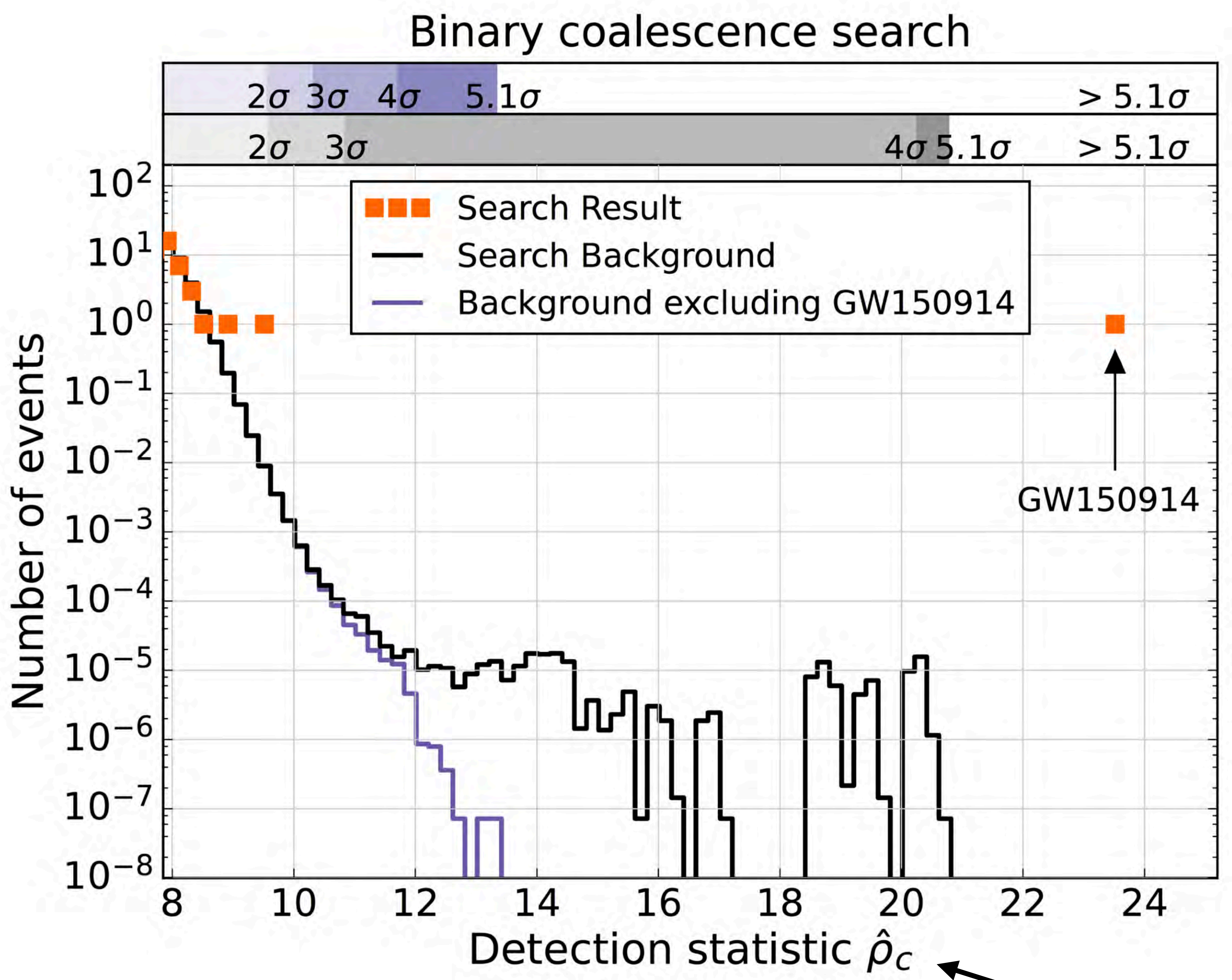
Non-stationary: → Glitches — - vetoes and time-slides

Non-Gaussian: → Glitches



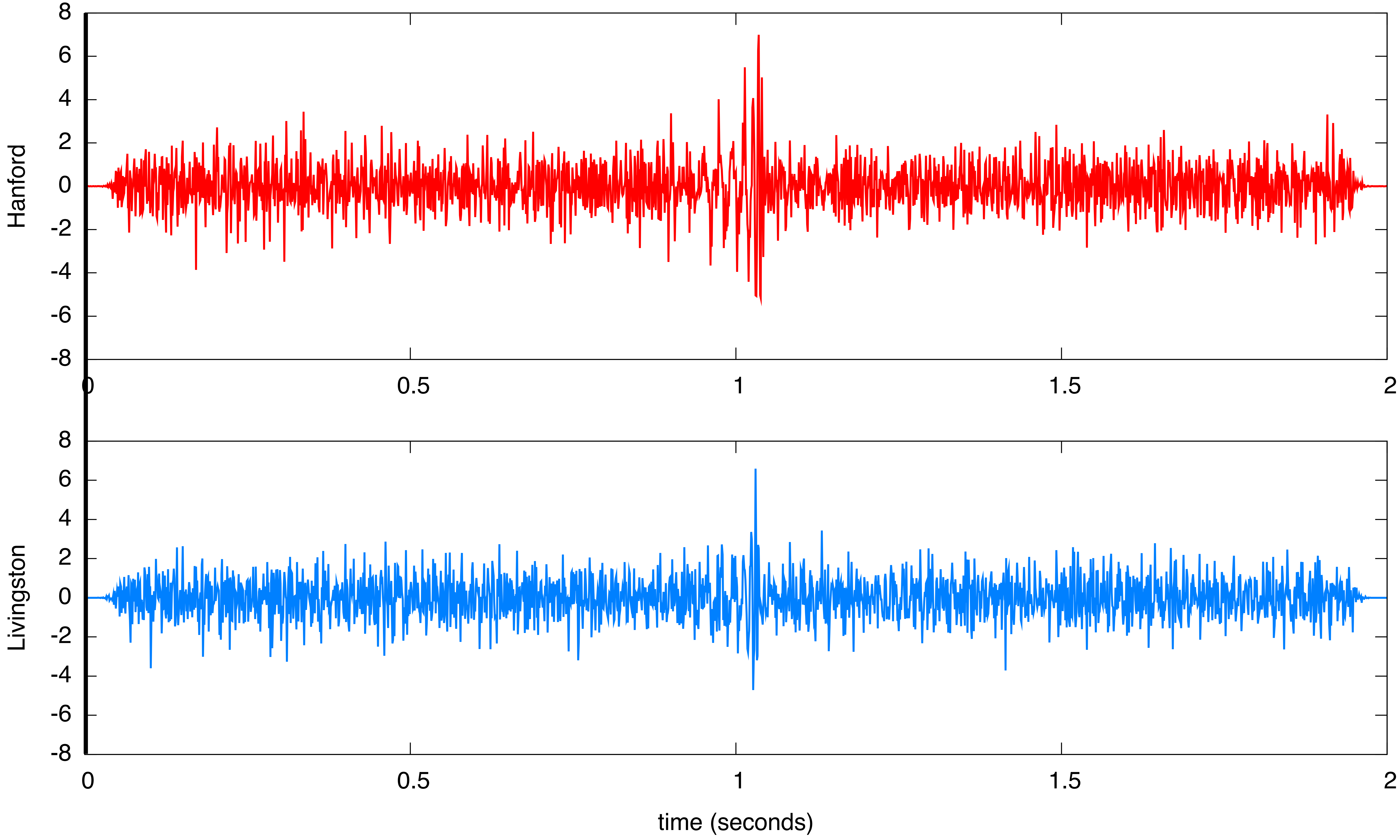
Analysis of 16 days of data from September 2015

Divide by 16 days to get FAR

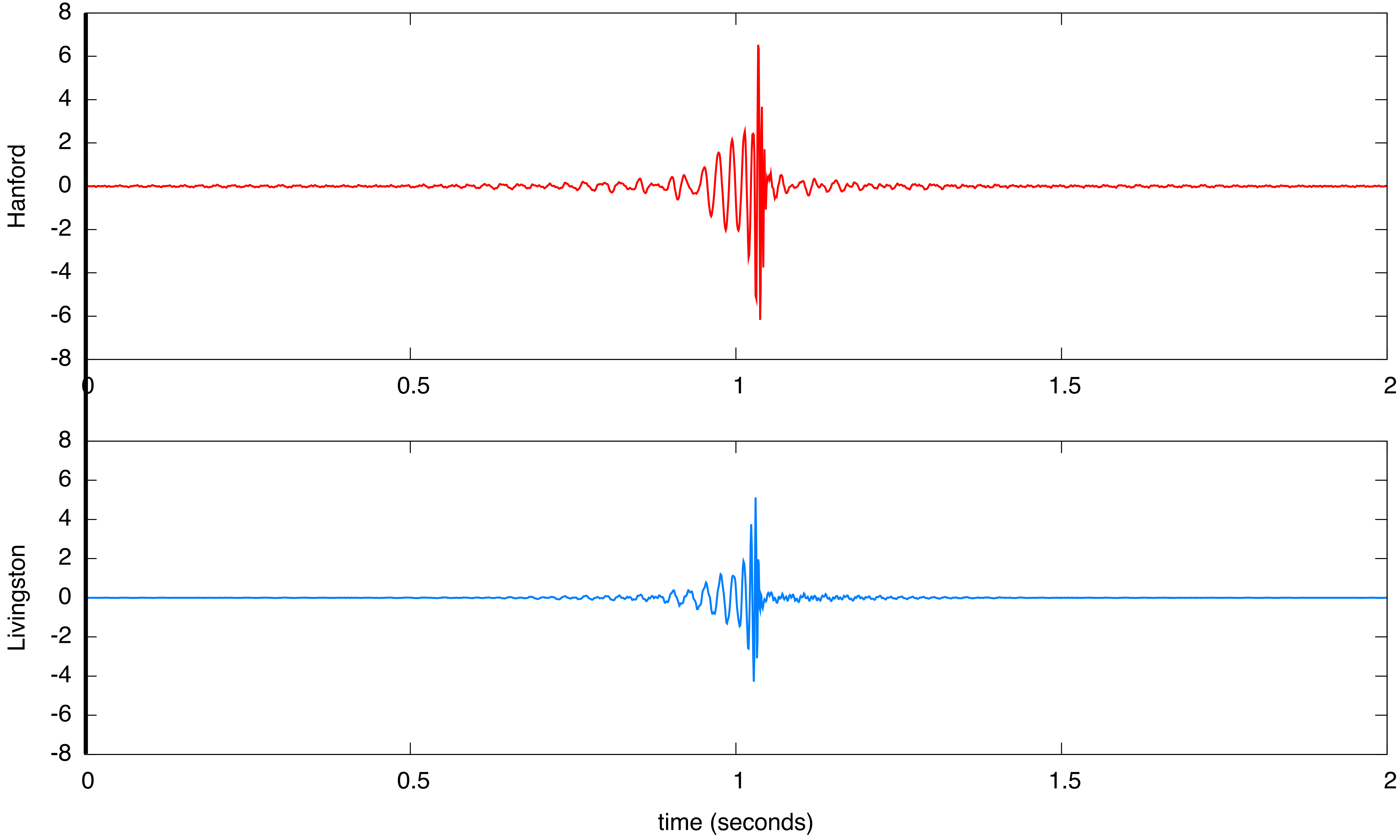


"new SNR"

Early Morning, September 14 2015



BayesWave reconstruction of the signal



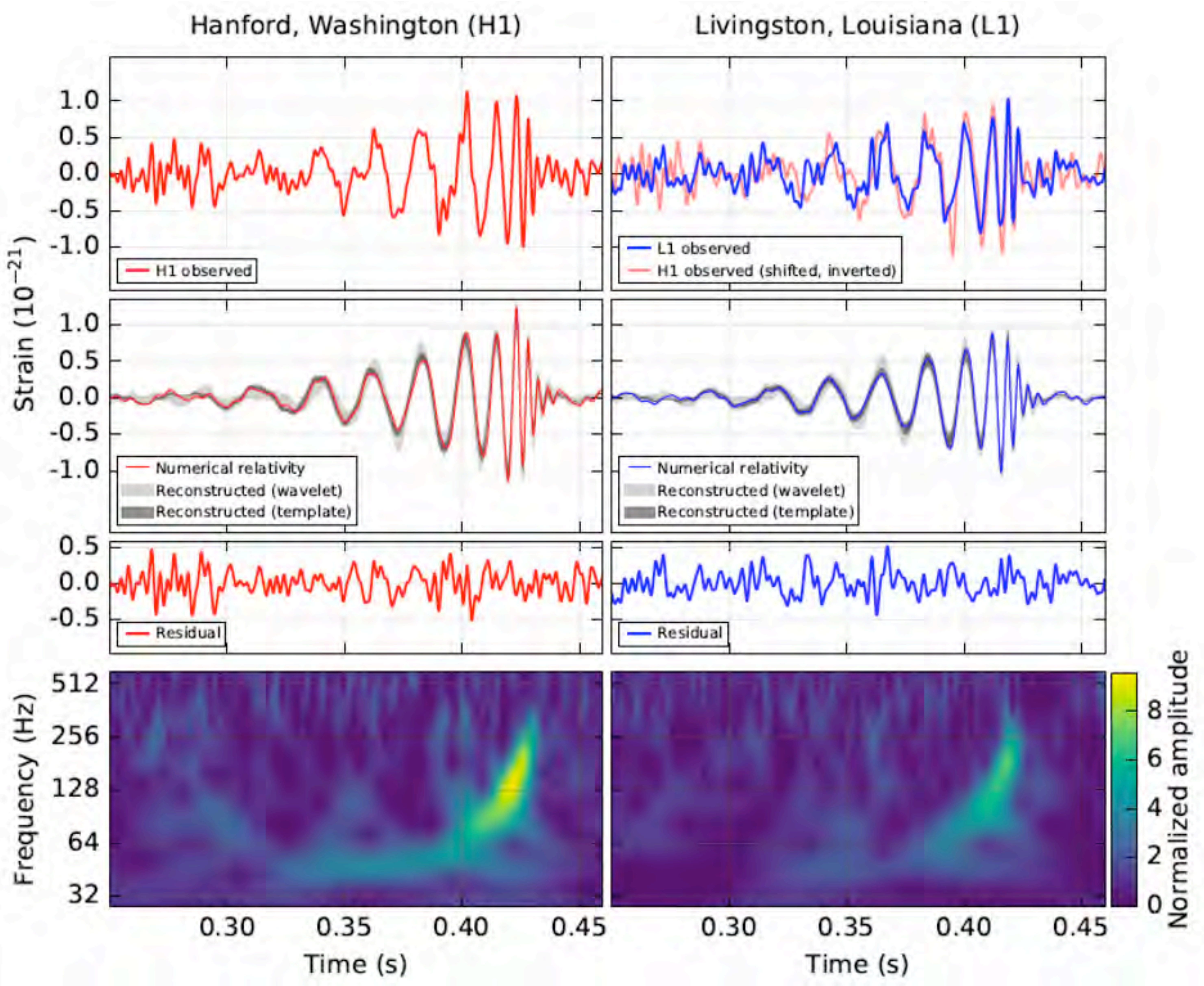
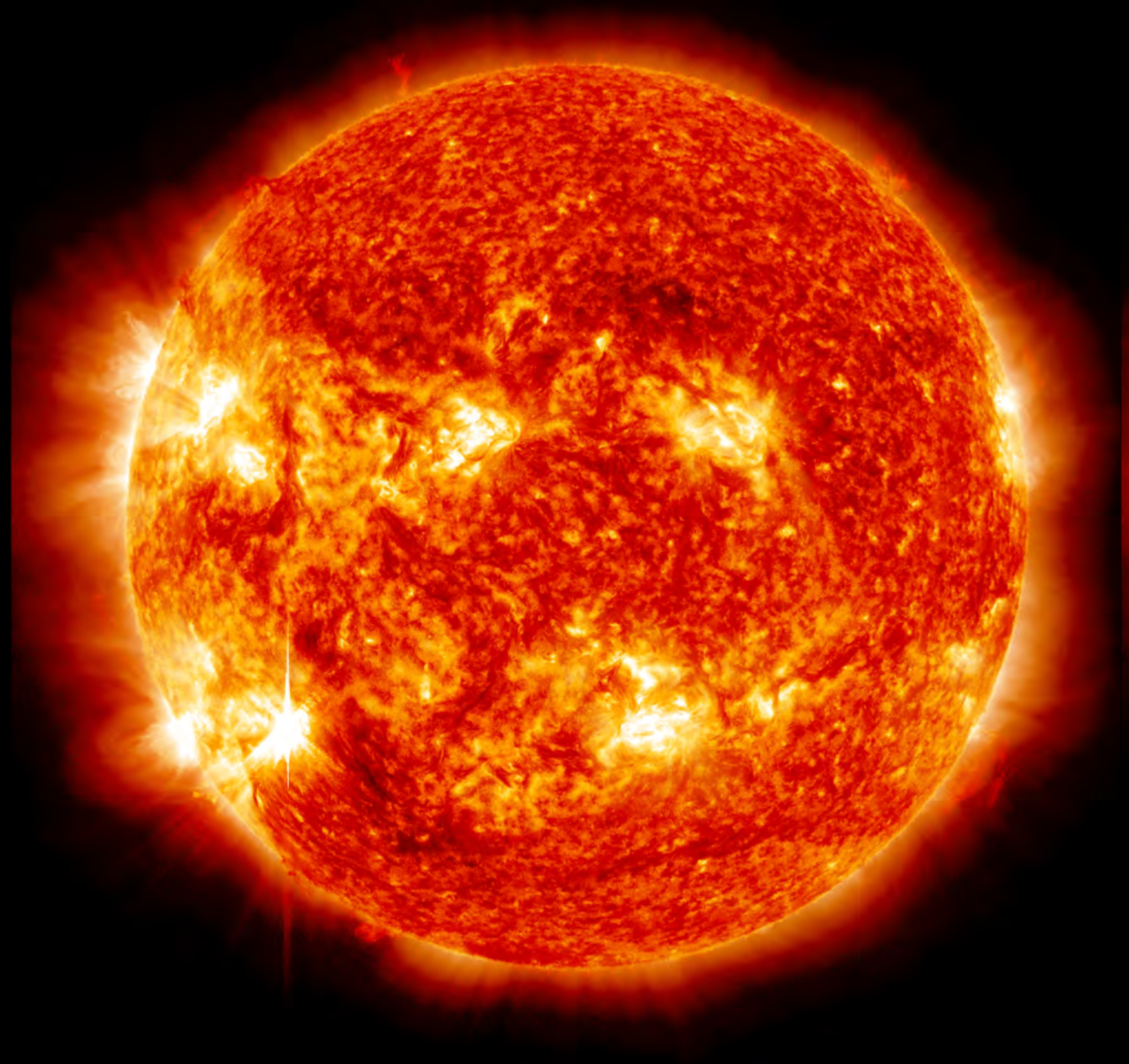
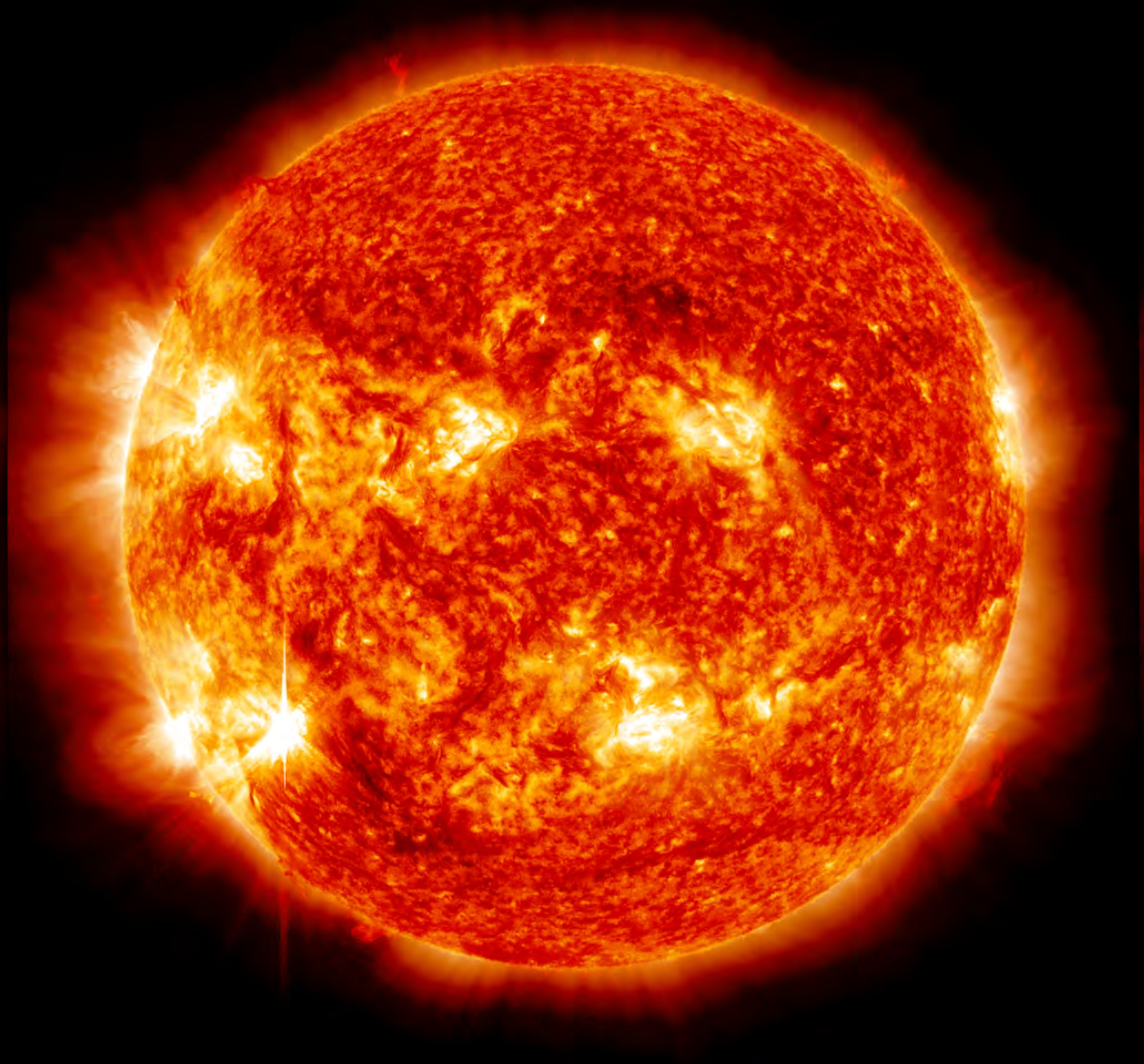
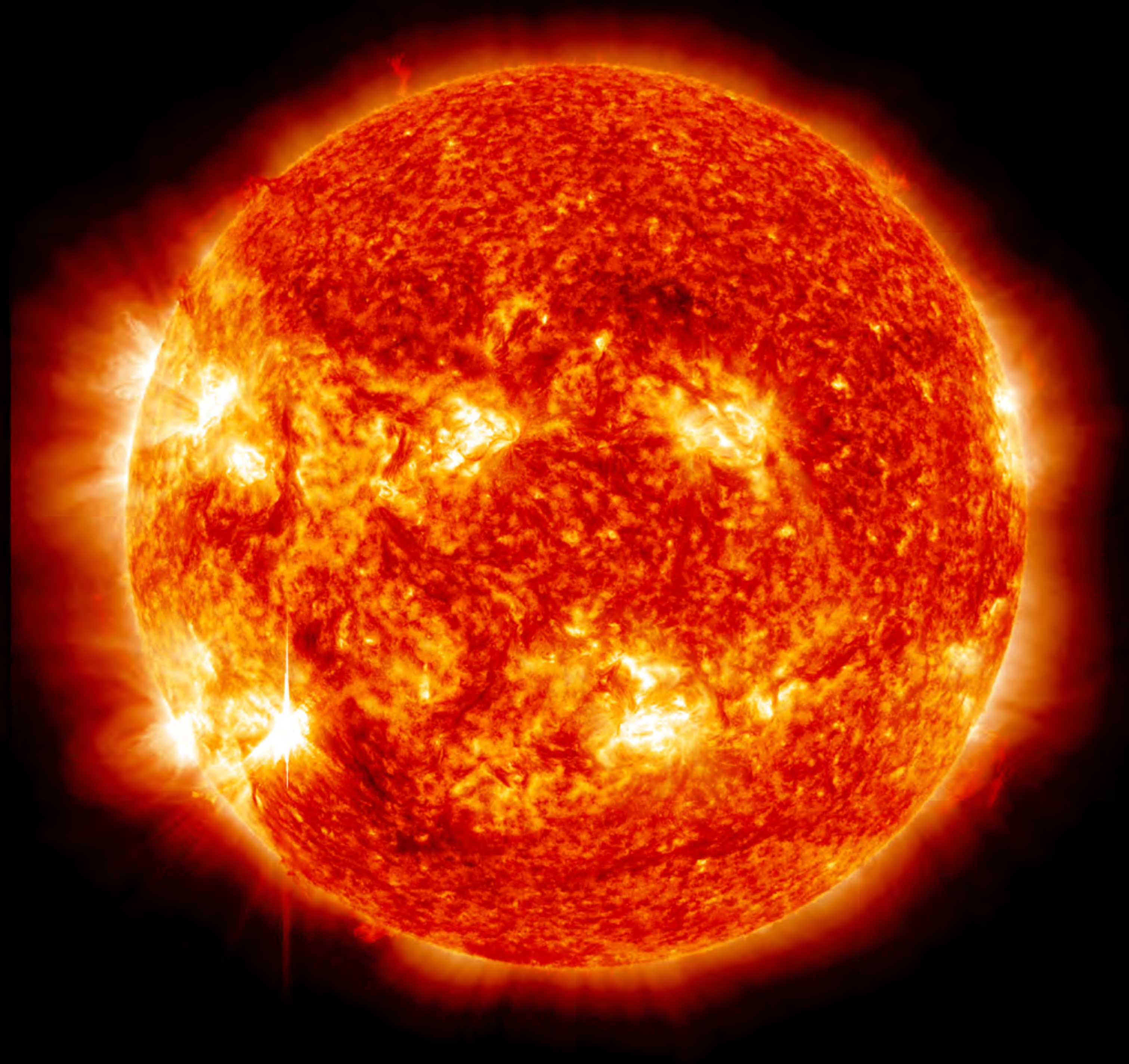
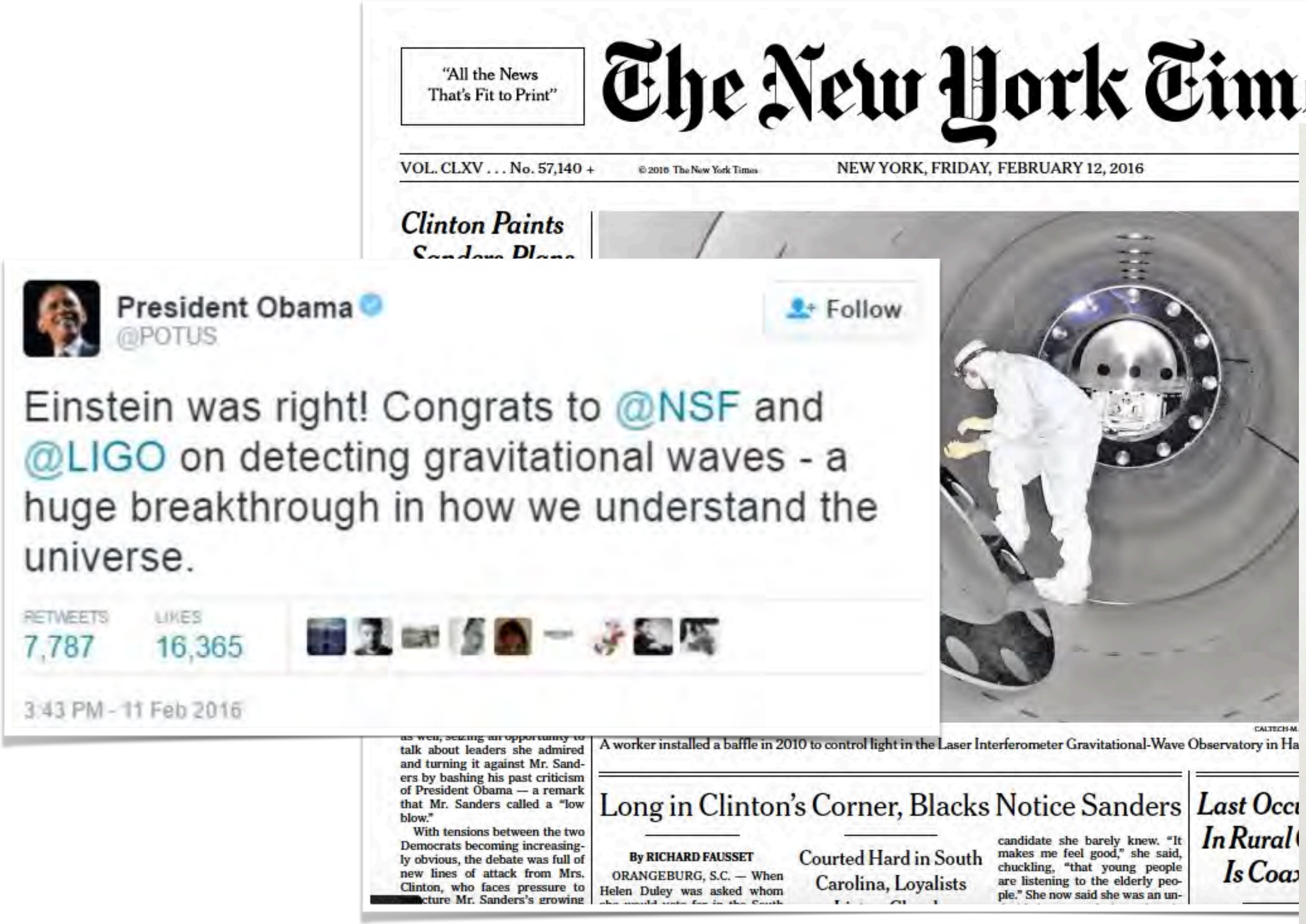


FIG. 1. The gravitational-wave event GW150914 observed by the LIGO Hanford (H1, left column panels) and Livingston (L1, right column panels) detectors. Times are shown relative to September 14, 2015 at 09:50:45 UTC. For visualization, all time series



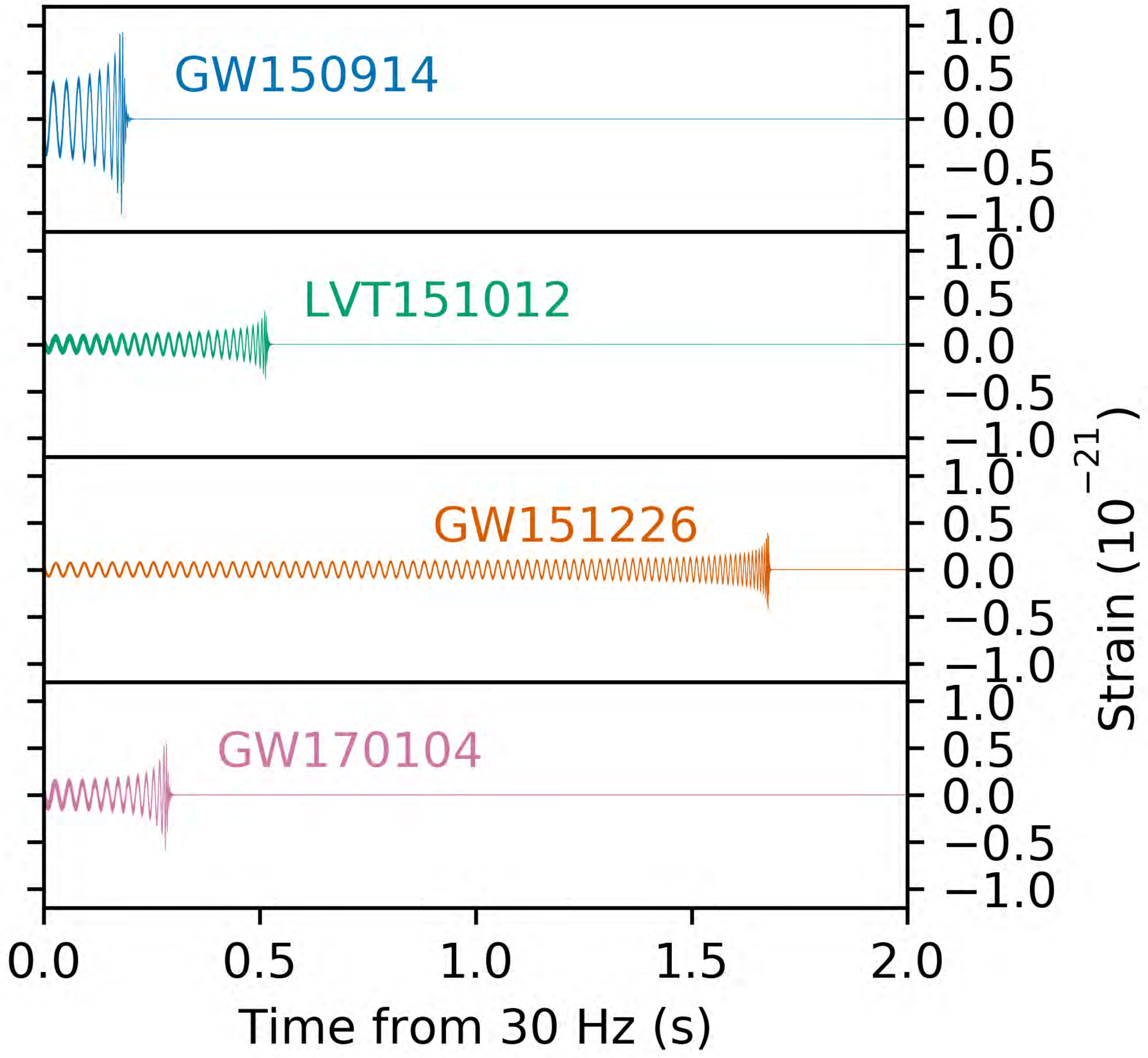


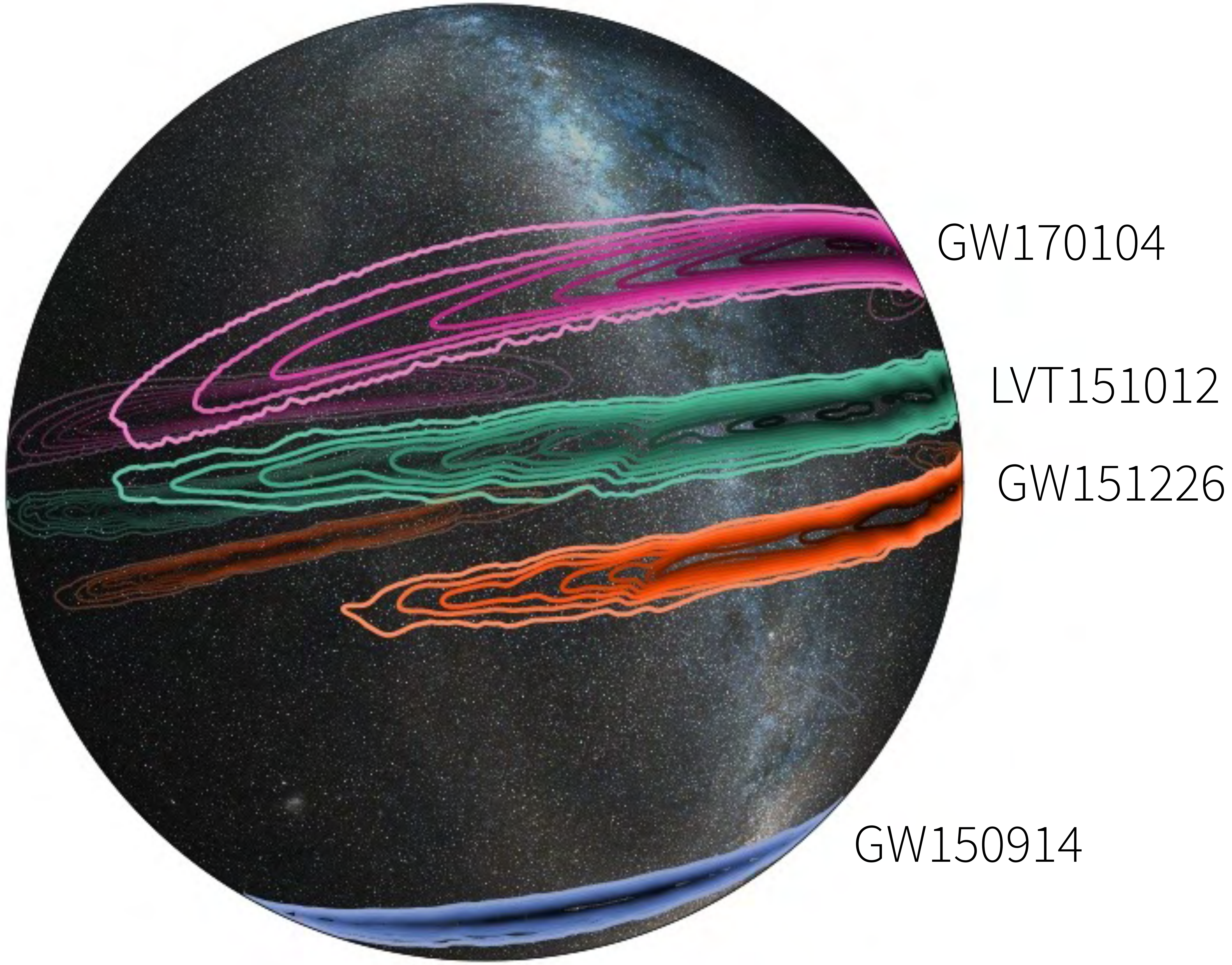



"All the News That's Fit to Print"
The New York Times
Late Edition
Today, some sunshine giving way to times of clouds, cold, high 28. Tonight, a flurry or heavier squall late, low 15. Tomorrow, windy, frig-
VOL. CLXV . . . No. 57,140 +
© 2016 The New York Times
NEW YORK, FRIDAY, FEBRUARY 12, 2016
Clinton Paints
talk about leaders she admired and turning it against Mr. Sanders by bashing his past criticism of President Obama — a remark that Mr. Sanders called a "low blow."
With tensions between the two Democrats becoming increasingly obvious, the debate was full of new lines of attack from Mrs. Clinton, who faces pressure to
A worker installed a baffle in 2010 to control light in the Laser Interferometer Gravitational-Wave Observatory in Ha
Long in Clinton's Corner, Blacks Notice Sanders
By RICHARD FAUSSET
ORANGEBURG, S.C. — When Helen Duley was asked whom
Courted Hard in South Carolina, Loyalists
candidate she barely knew. "It makes me feel good," she said, chuckling, "that young people are listening to the elderly people." She now said she was an un-
Last Occu
In Rural
Is Coax
President Obama
@POTUS
Follow
Einstein was right! Congrats to @NSF and @LIGO on detecting gravitational waves - a huge breakthrough in how we understand the universe.
RETWEETS
7,787
LIKES
16,365
3:43 PM - 11 Feb 2016

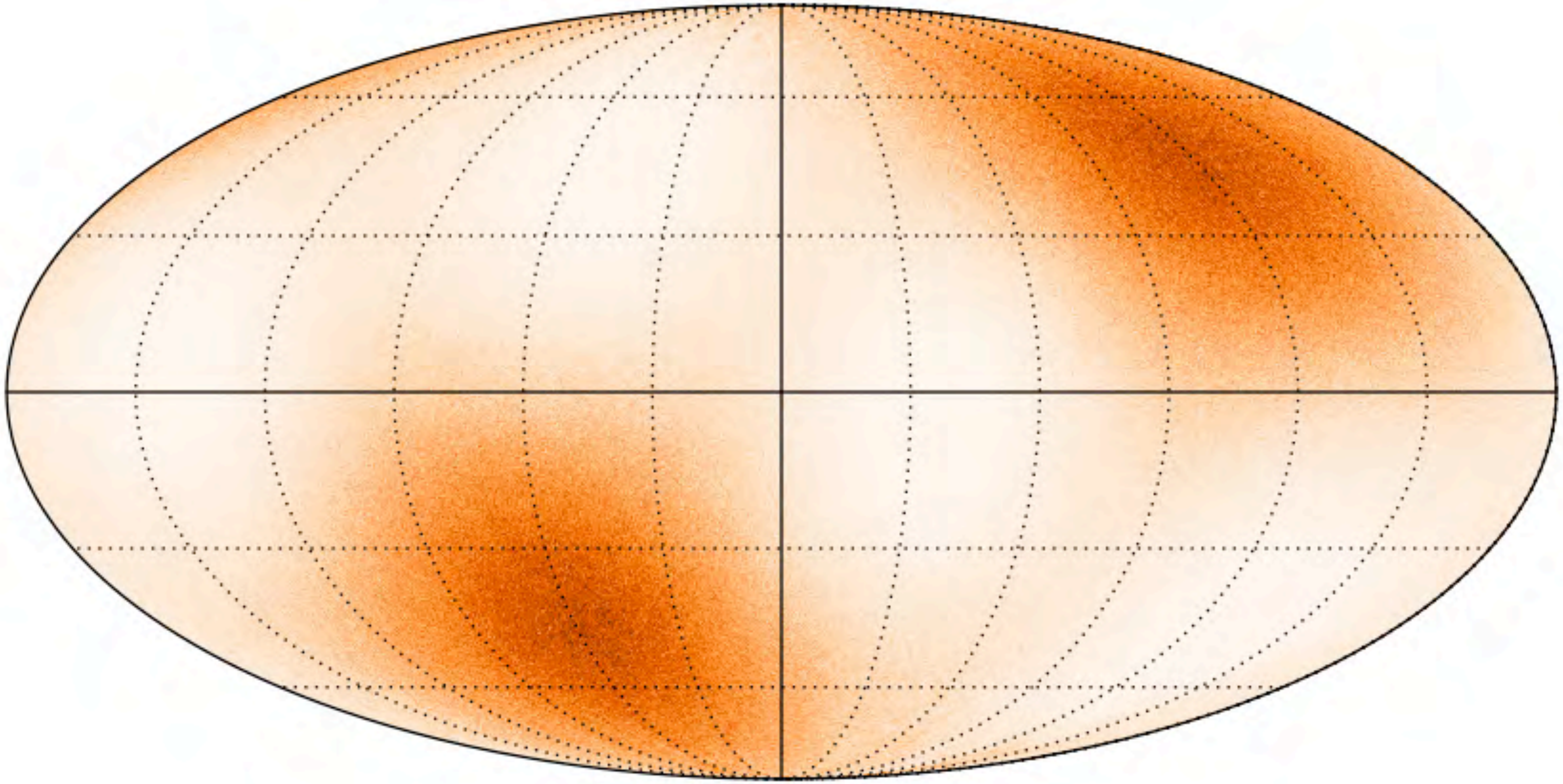

BACKCOUNTRY BEHAVIOR
BOMBS AWAY
BOZEMAN DAILY
CHRONICL
DAILYCHRONICLE.COM
FRIDAY, FEBRUARY 12, 2016
Jury rules against woman in 'wrongful birth' case
Einstein was right
'Now we can hear the universe:' Einstein's gravity waves detected, MSU scientists members of discovery team
With FBI ring tightening, last Oregon occupiers surrender
47 29

LIGO detections to-date



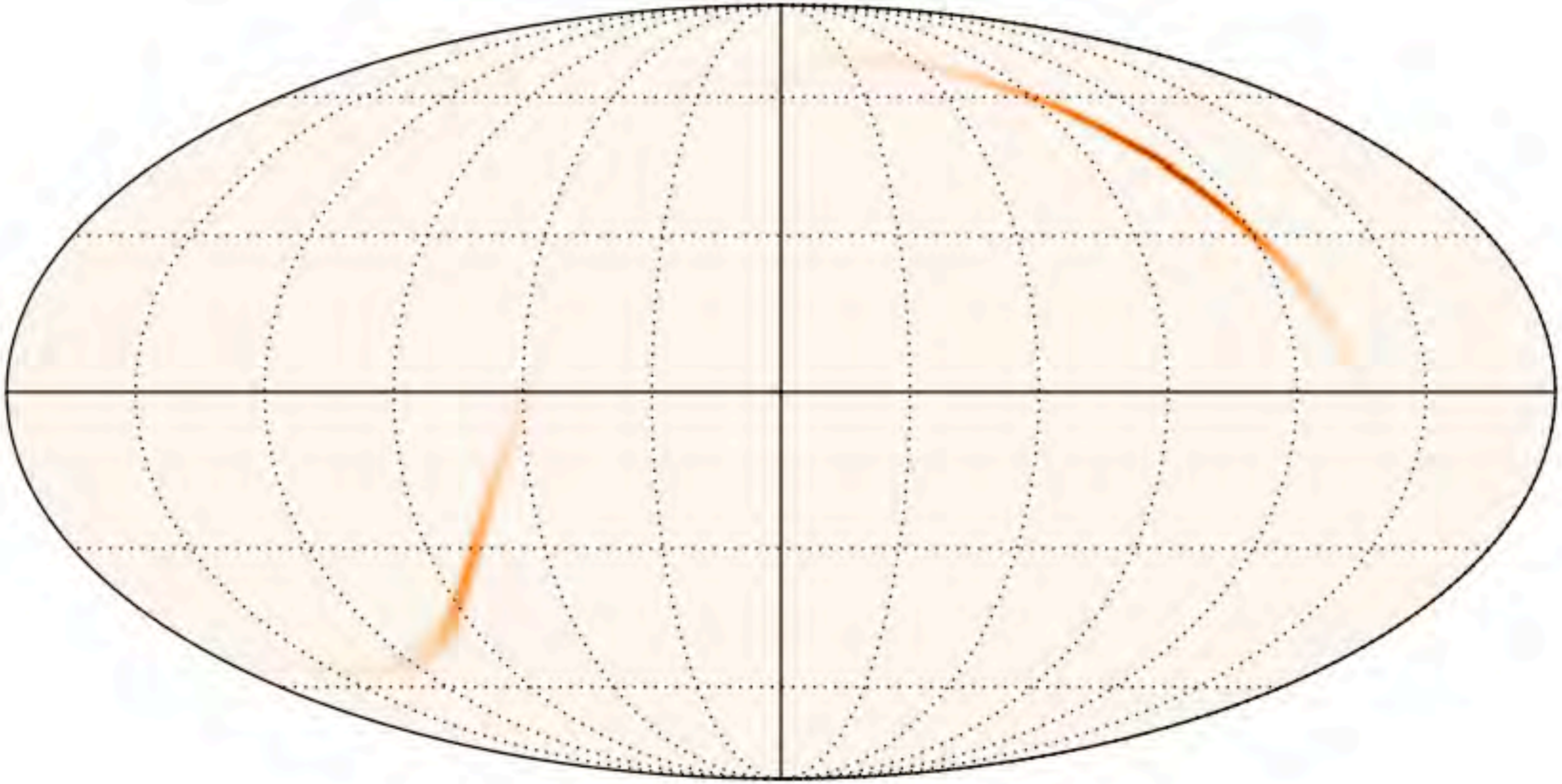


Triangulating the Source



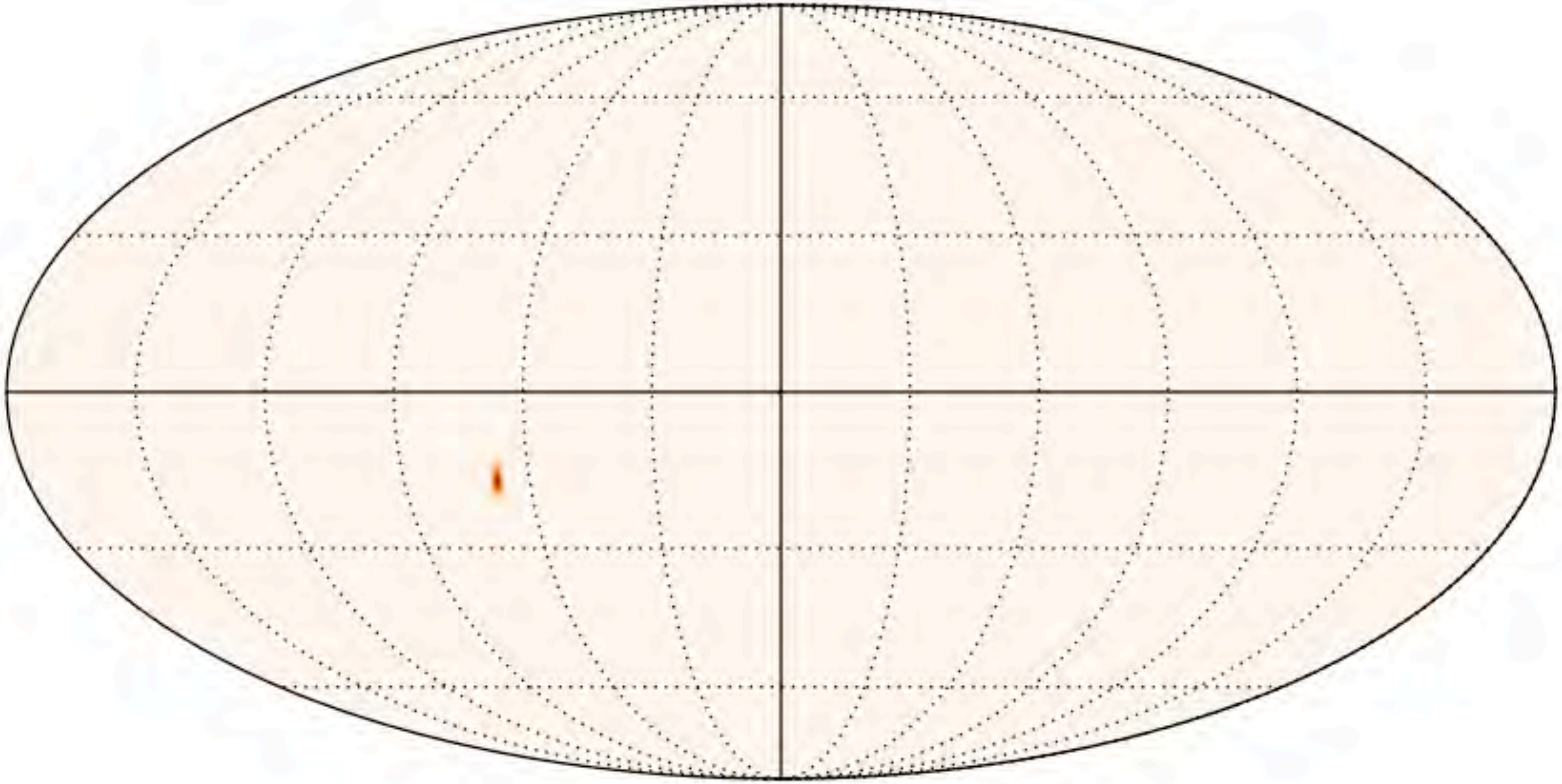
Hanford

Triangulating the Source



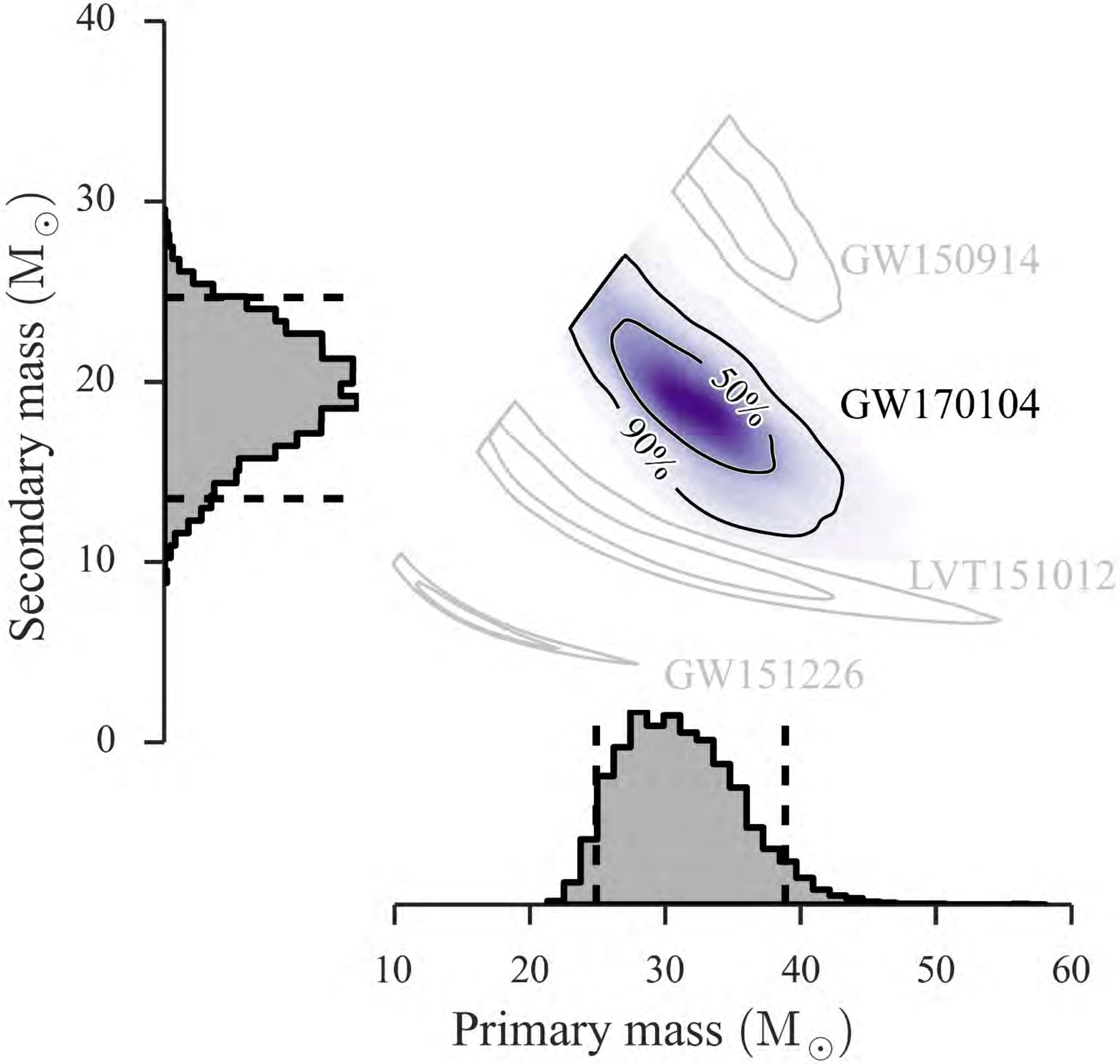
Hanford + Livingston

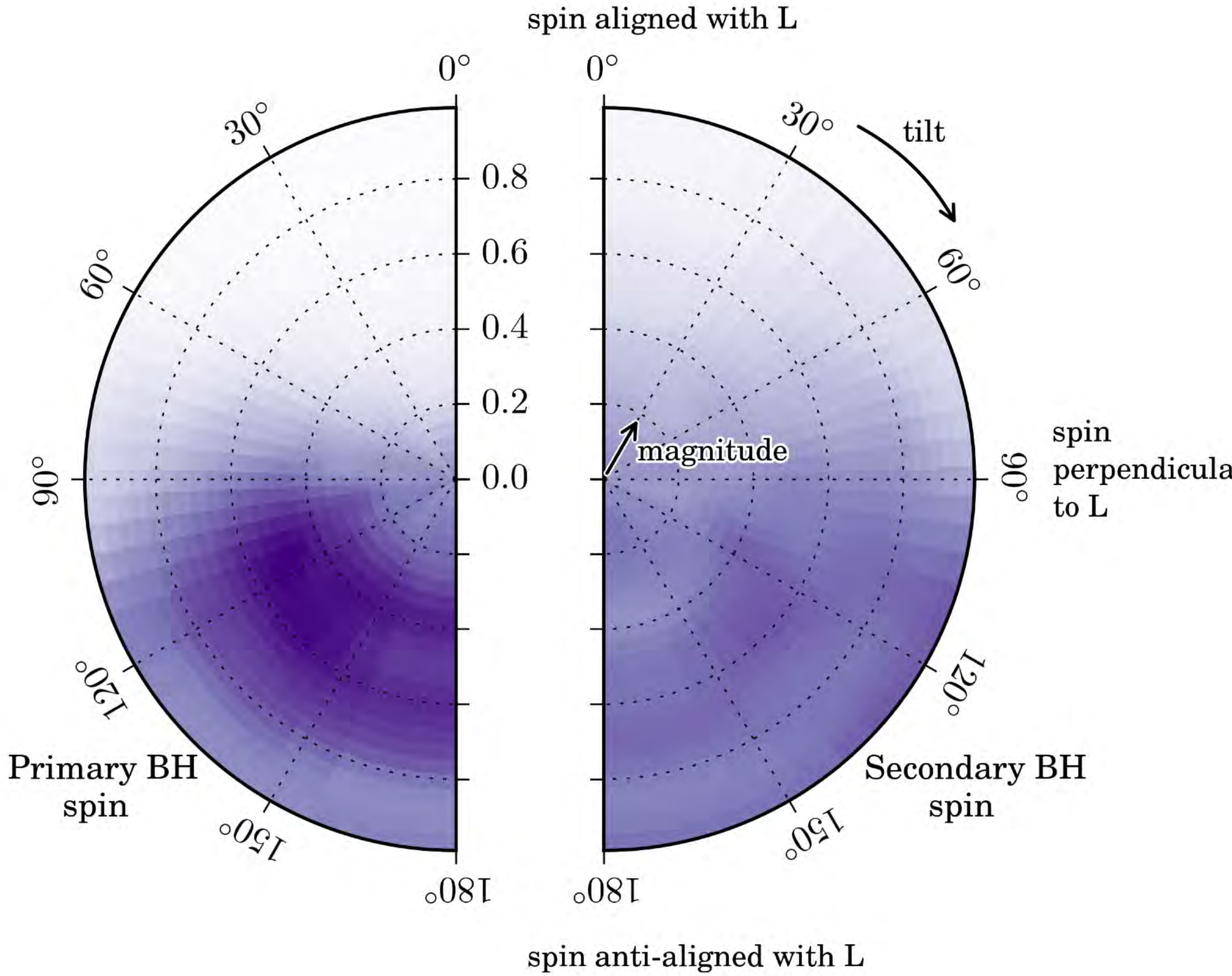
Triangulating the Source



Hanford + Livingston + Virgo

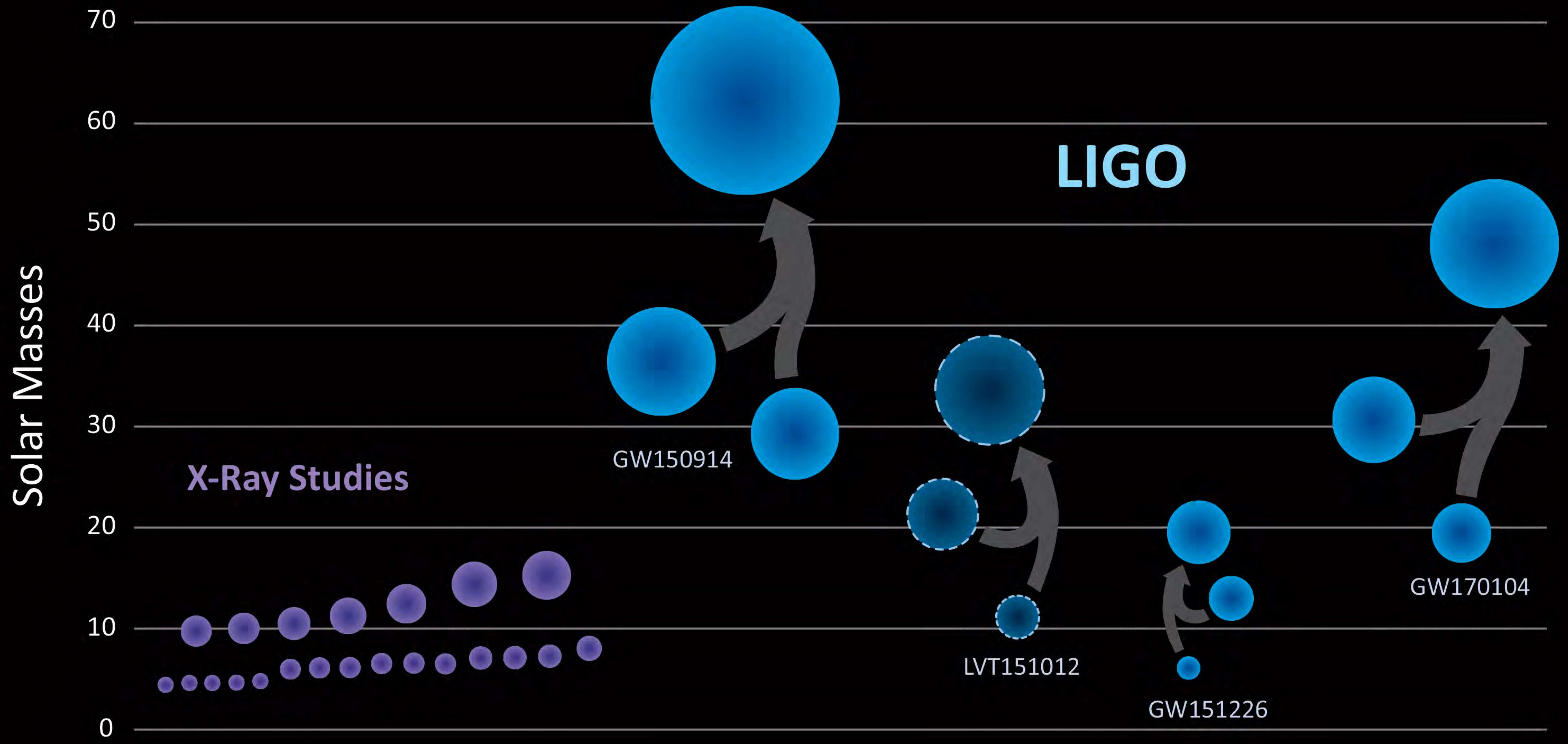
Parameter estimation





GW170104

Black Holes of Known Mass

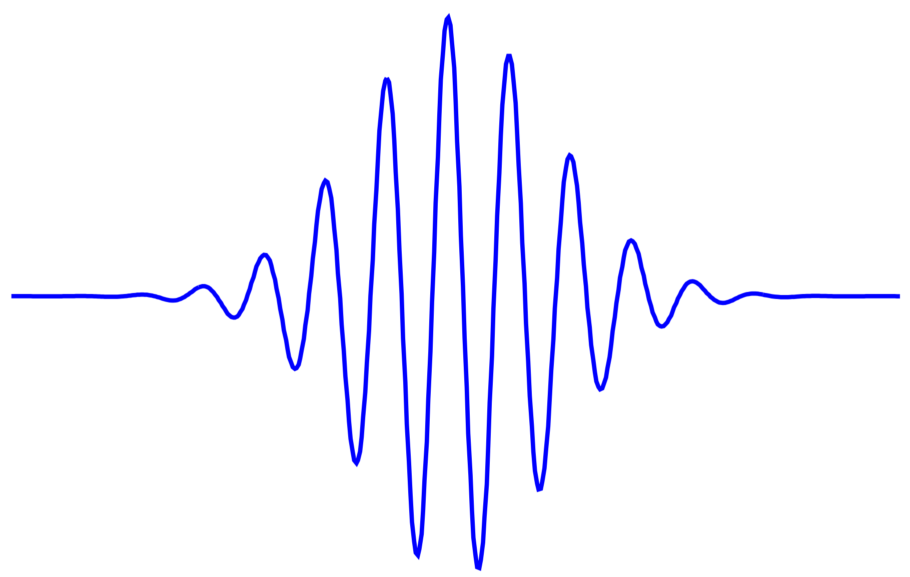


Detection without templates

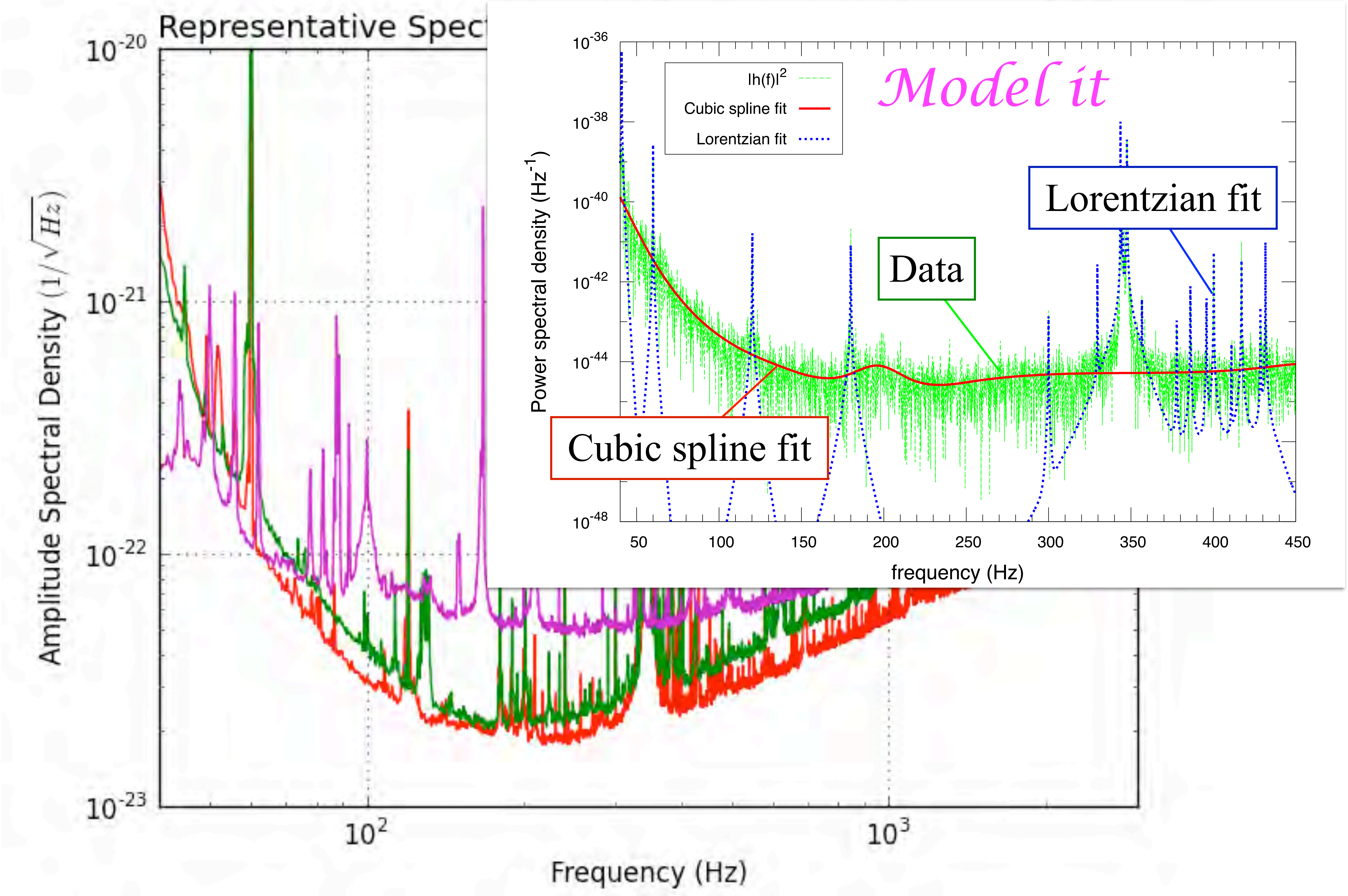
BayesWave   Cornish & Littenberg 2015

- Bayesian model selection
  - Three part model (signal, glitches, gaussian noise)
  - Trans-dimensional Markov Chain Monte Carlo
- Wavelet decomposition
  - Glitch & GW modeled by wavelets
  - Number, amplitude, quality and TF location of wavelets varies

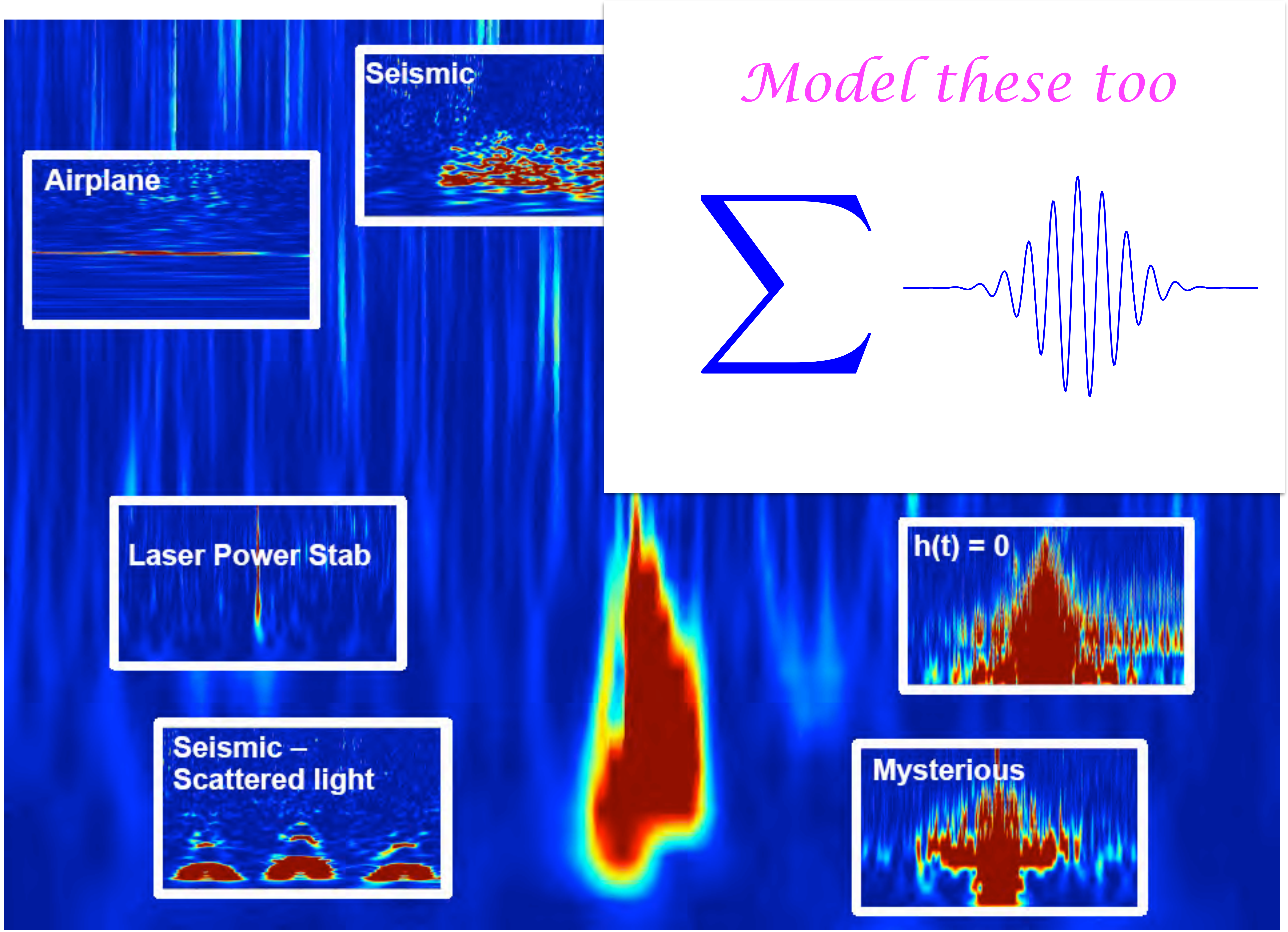
Continuous Morlet/Gabor Wavelets



Lines and a drifting noise floor

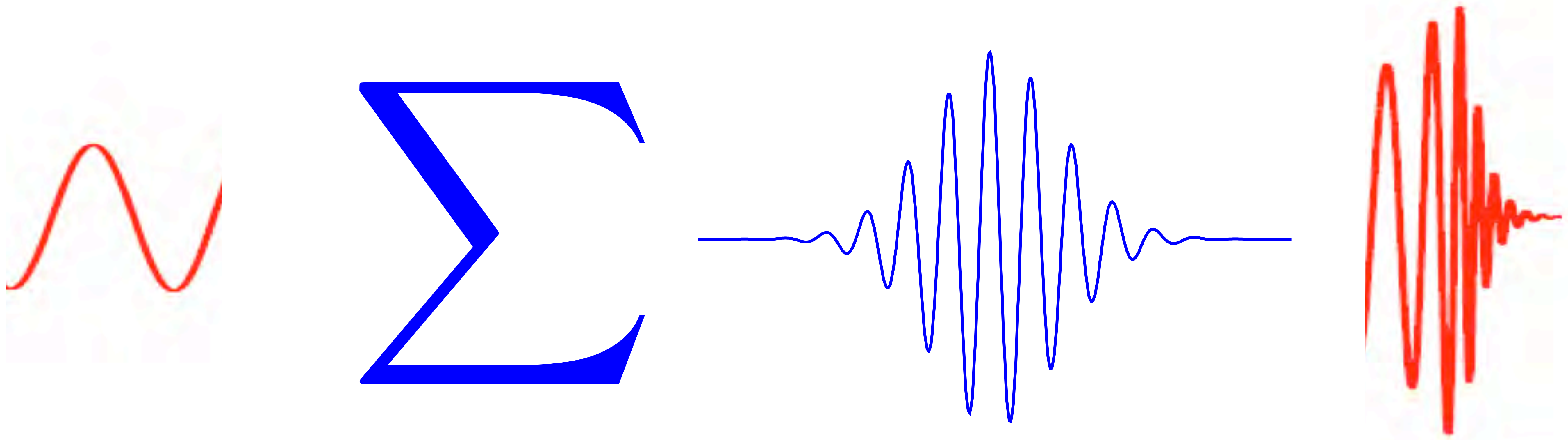


Glitches


Seismic
Airplane
Laser Power Stab
h(t) = 0
Seismic –
Scattered light
Mysterious
Model these too

Gravitational Waves

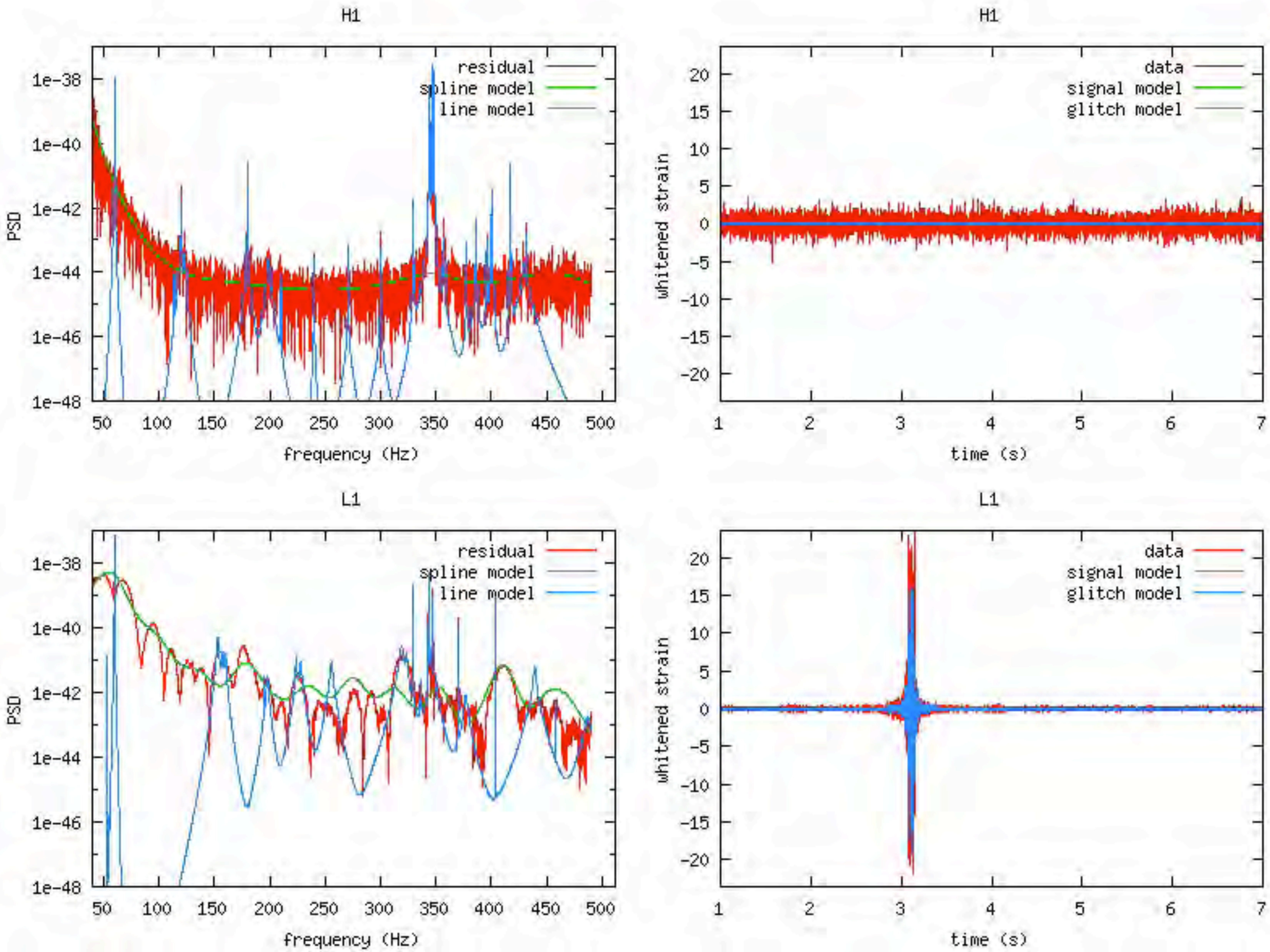
and model these



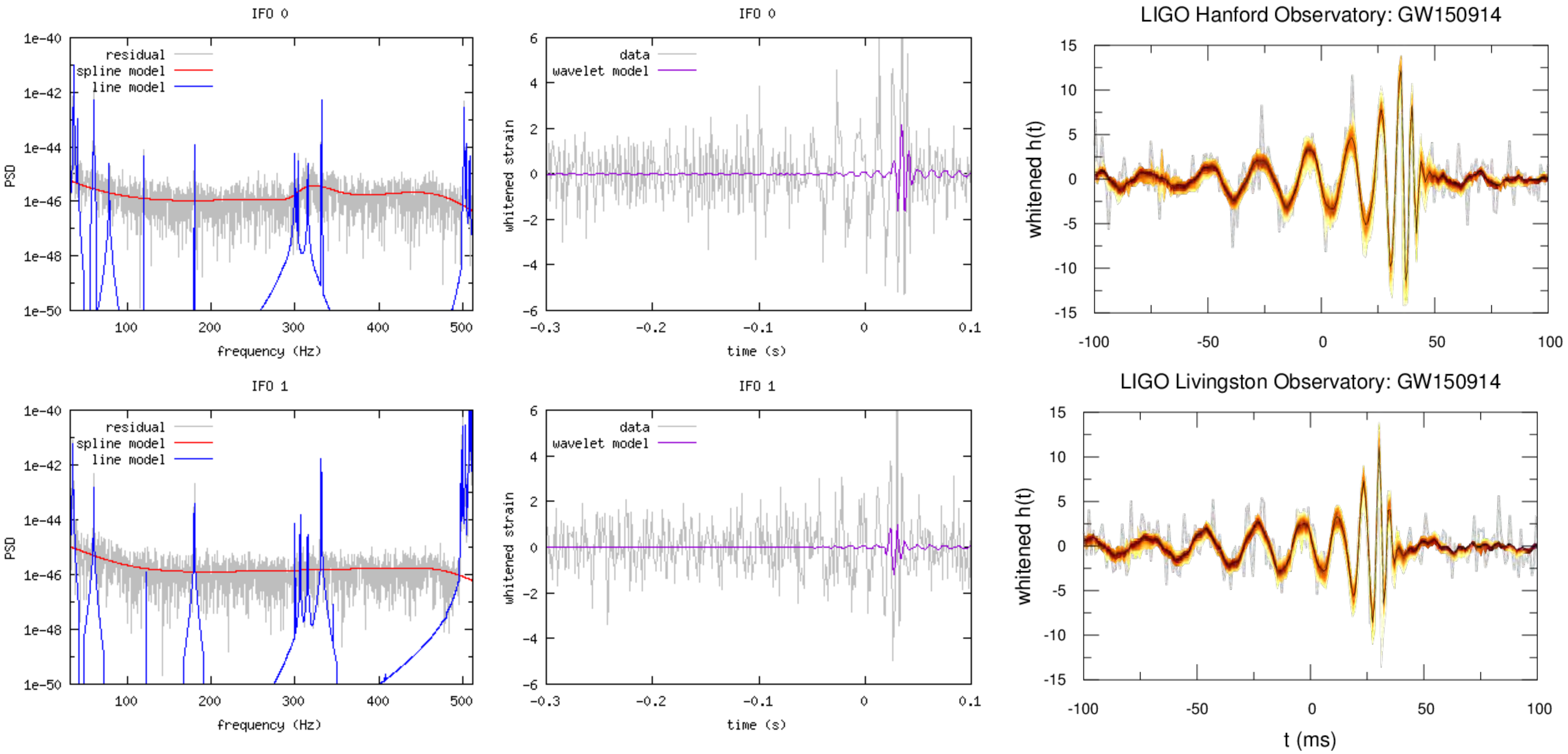
Model everything and let the data sort it out

“Caedite eos. Novit enim Dominus qui sunt eius” Arnaud Amalric
(Kill them all. For the Lord knoweth them that are His.)

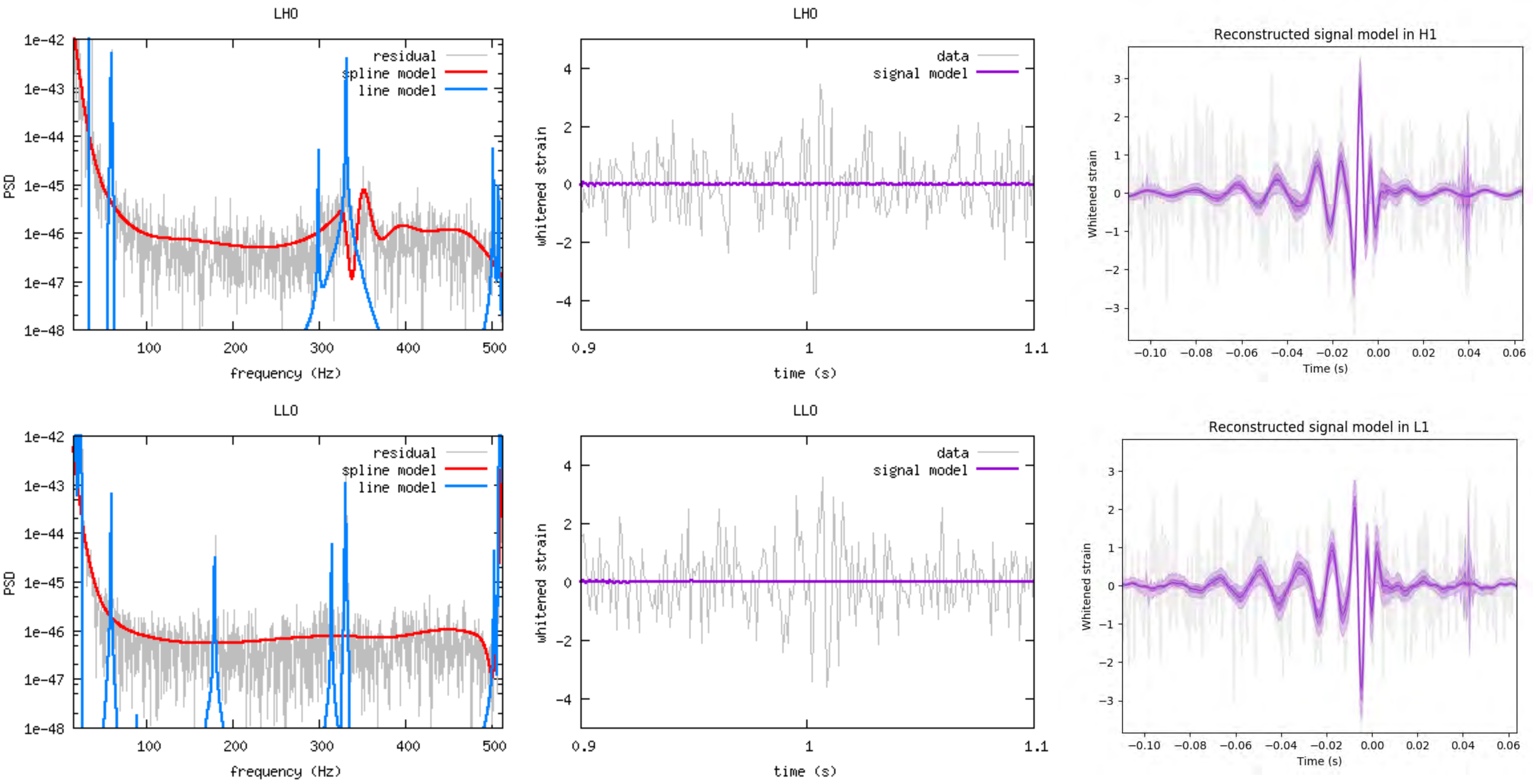
Example from LIGO's S5 science run



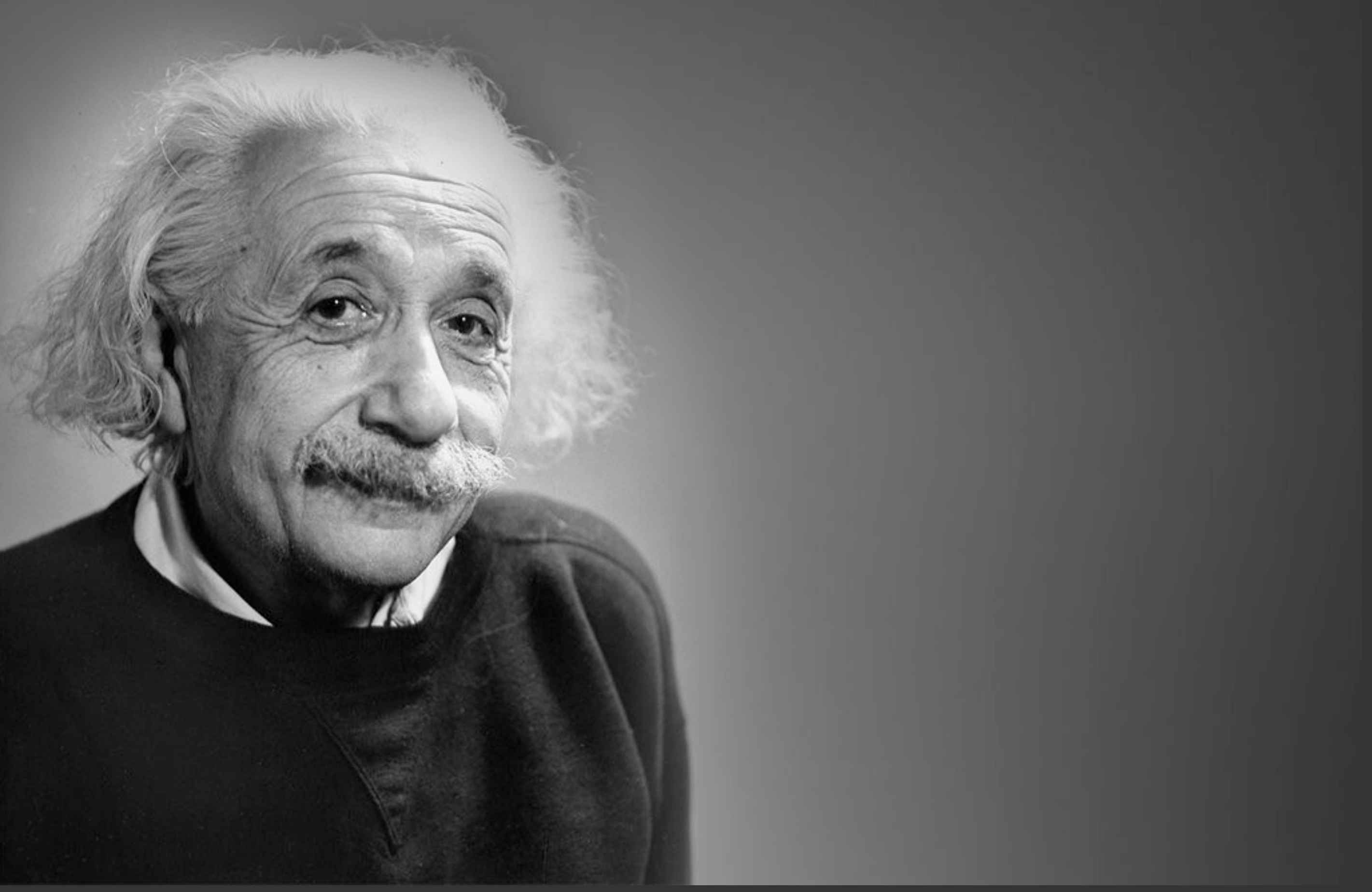
Reconstructing GW150914 with wavelets

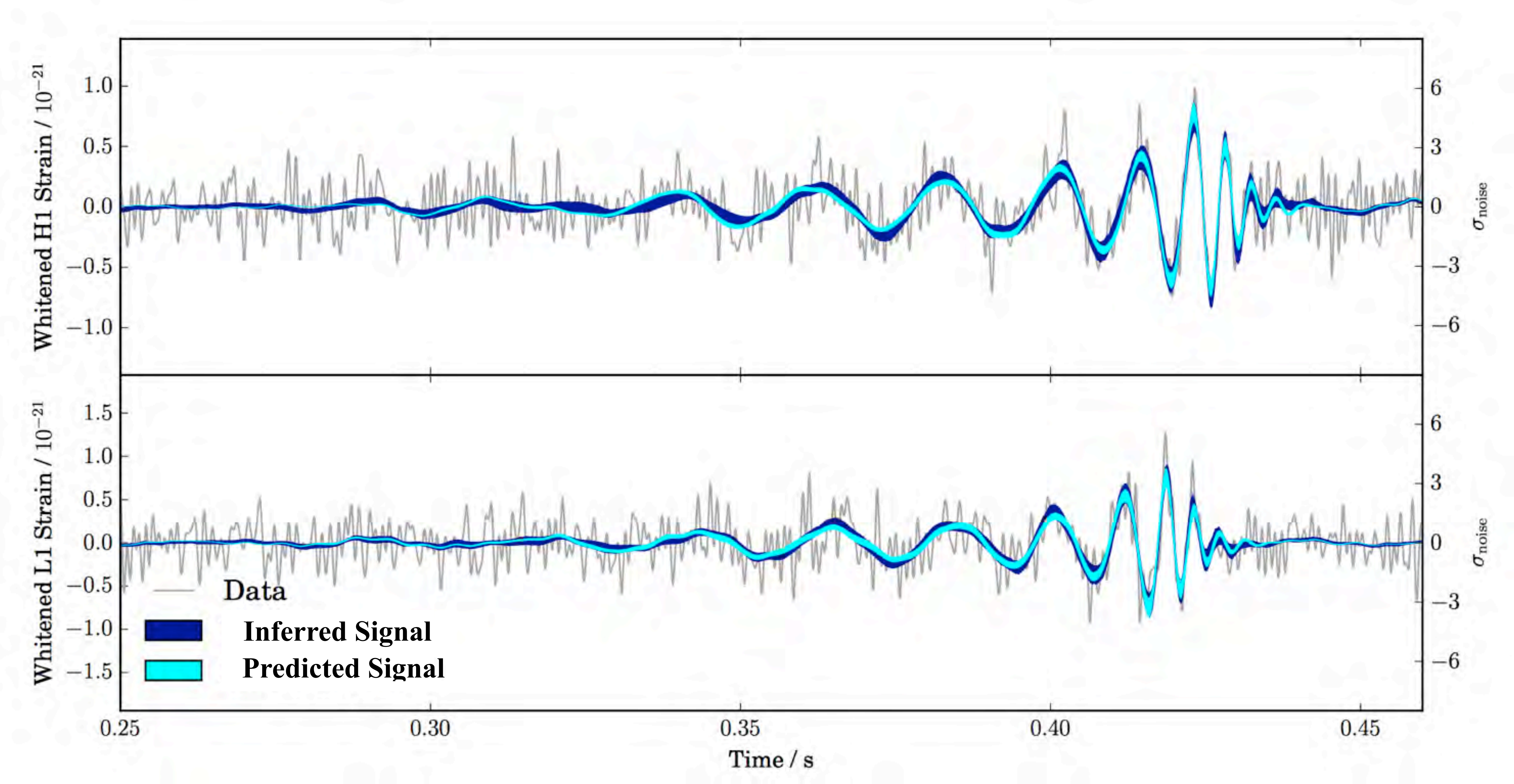


Reconstructing GW170104 with wavelets

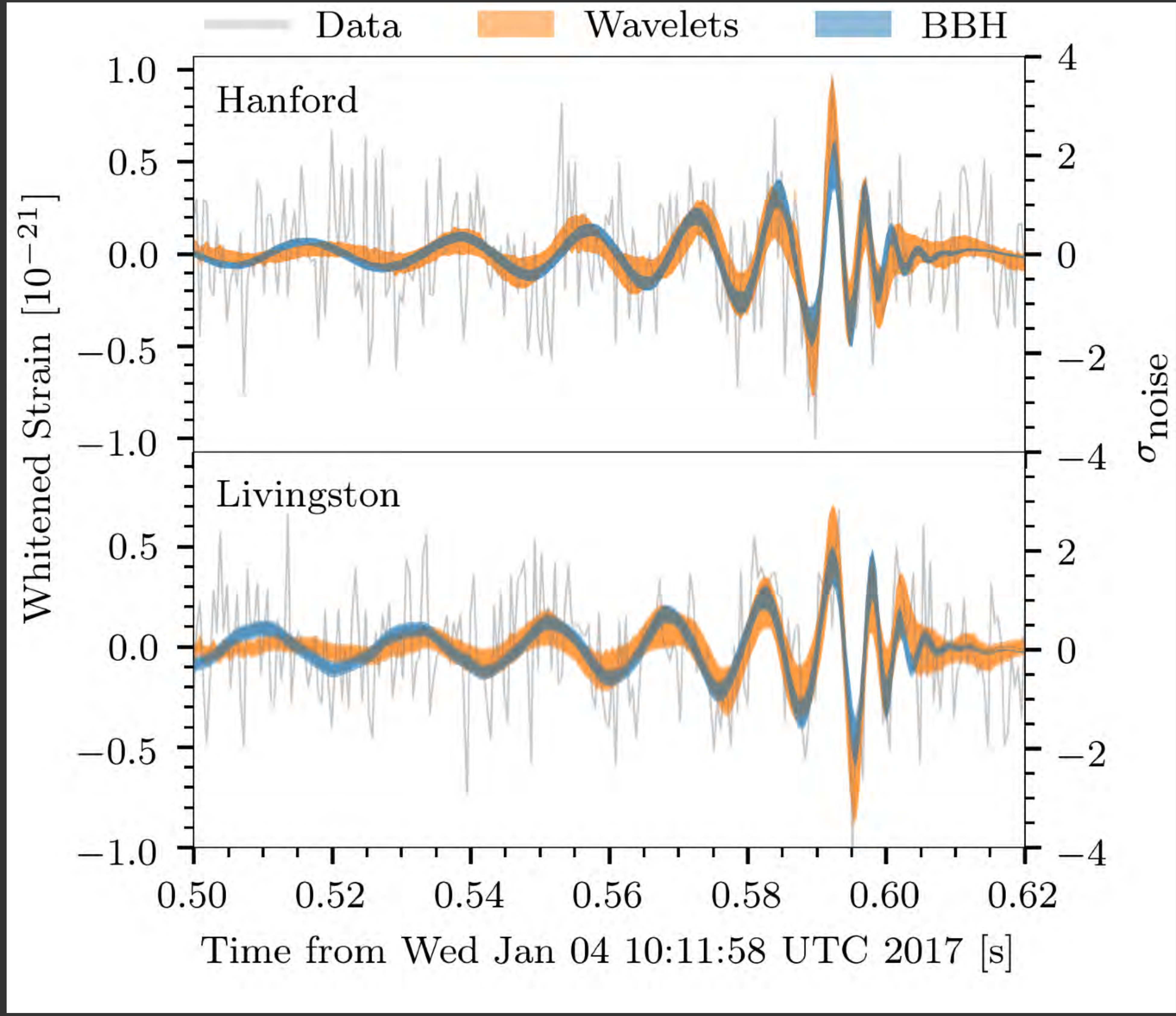


Match GR prediction



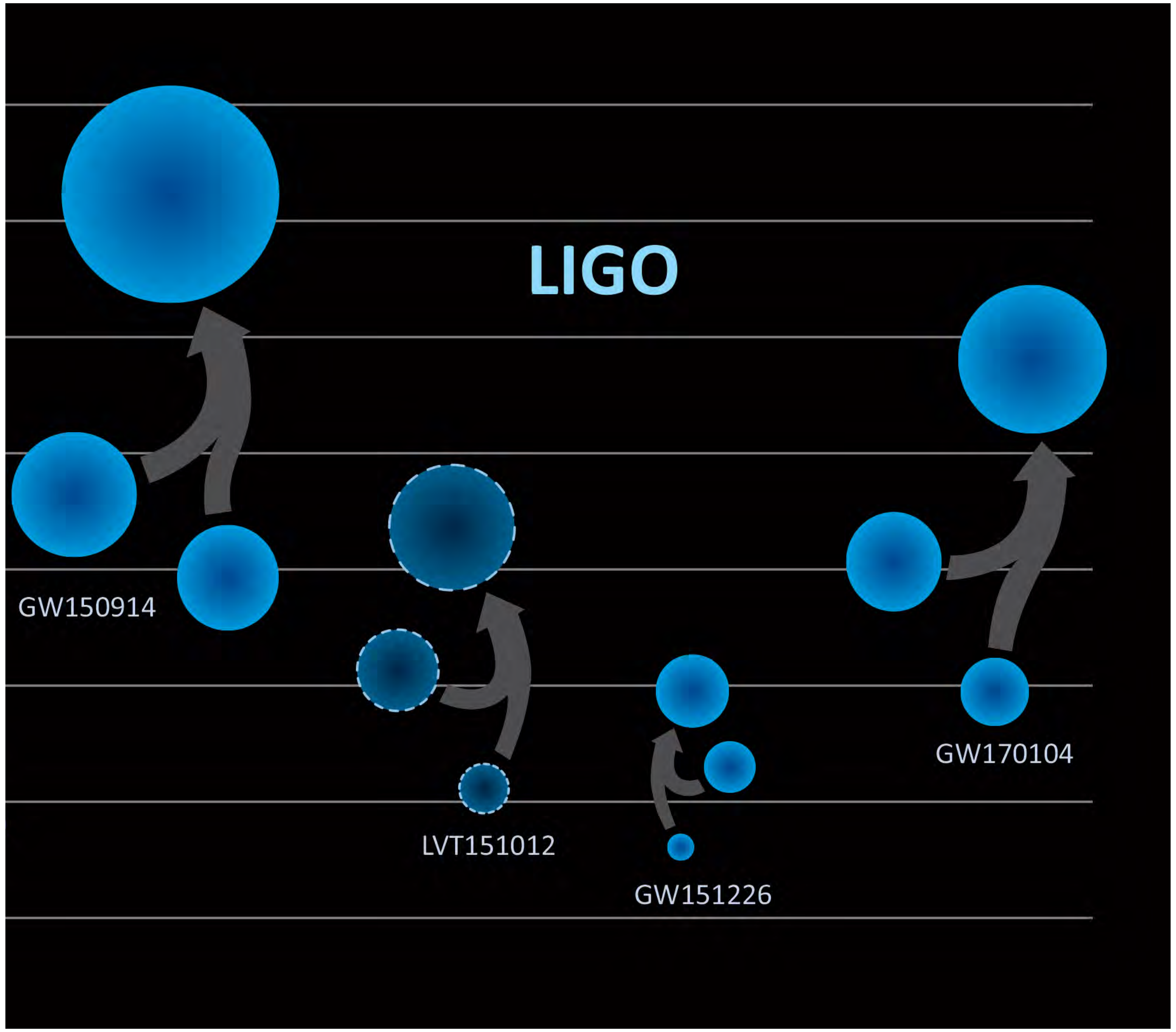


GW150914



GW170104

Astrophysical Inference



Would like to know merger rate to constrain population synthesis models. Even better, would like to know merger rate as a function of mass, spin, redshift etc

Many cool techniques being developed to do this using things like Gaussian processes

Only have time to discuss the total merger rate

Astrophysical Rate Limits

Even without a detection we can produce interesting astrophysical results such as bounds on the binary merger rate for NS-NS

Expect binary mergers to be a Poisson process. If the expected number of events is λ,
then the probability of detecting k events is

$$p(k|\lambda) = \lambda^k e^{-\lambda}/k!$$

If the event rate is $R \ [\mathrm{Mpc}^{-3}\ \mathrm{year}^{-1}]$, and the observable 4-volume is $VT\ [\mathrm{Mpc}^3\ \mathrm{year}]$

$$\lambda = RVT$$

The probability of observing zero events (k=0) is then

$$p(R) = VT\, e^{-RVT}$$

Follows from $p(0|\lambda) = p(R)\, dR = e^{-RVT}$

Astrophysical Rate Limits

$$p(R) = VT\, e^{-RVT}$$

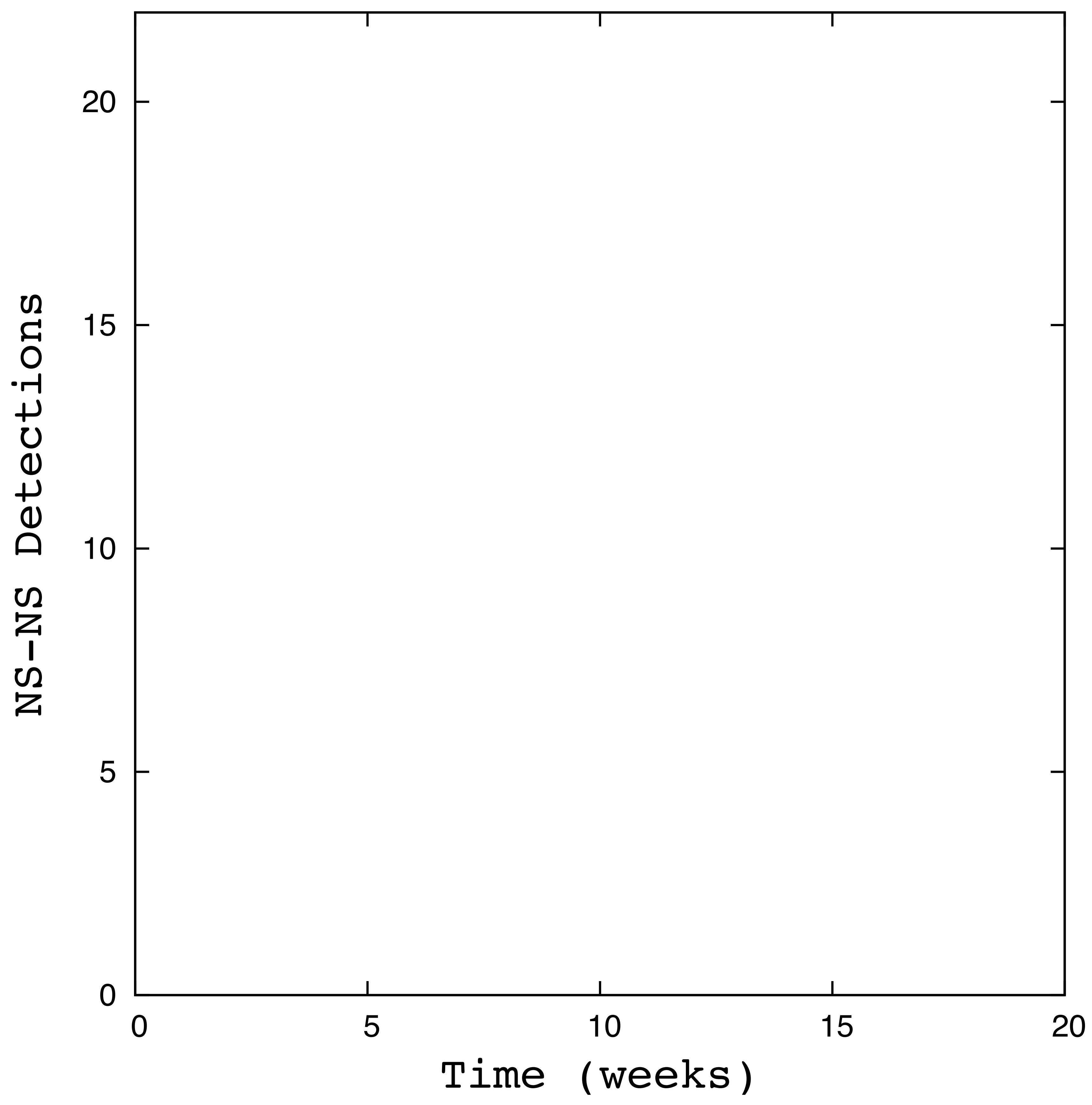
The probability distribution is peaked at a rate of zero. A 90% rate upper limit can be computed:

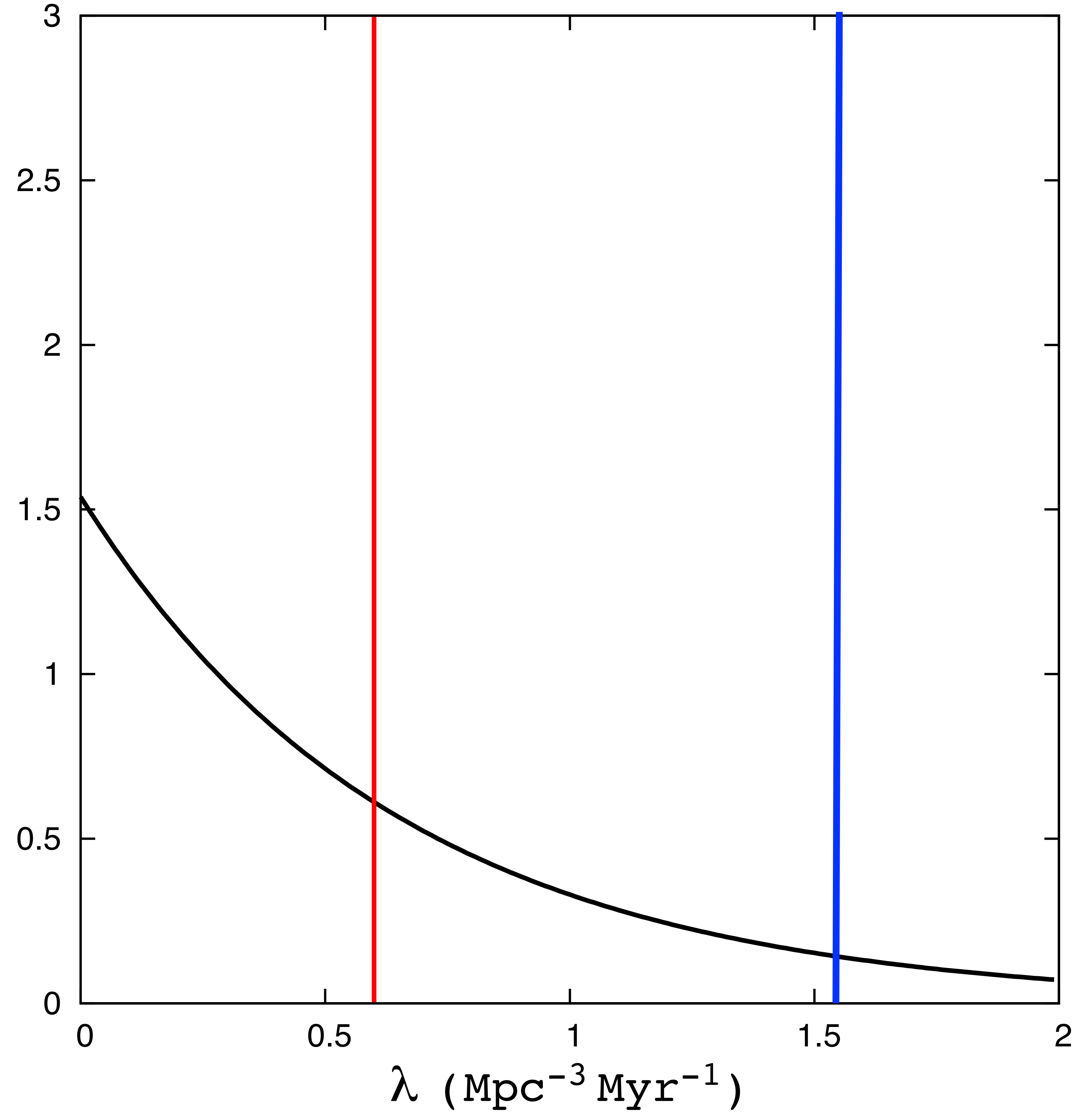
$$\int_0^{R_*} p(R)dR = 1 - e^{-R_* VT} = 0.9$$

$$\Rightarrow \quad R_* VT = \ln(0.1)$$

$$\Rightarrow \quad R_* = \frac{2.3}{VT}$$

e.g. NS-NS Merger Rate, aLIGO at design sensitivity $V = \frac{4\pi}{3}(200\,\mathrm{Mpc})^3$





Truth

90% upper limit

Week 1

Astrophysical Rate Limits

When a signal is detected the probability distribution for the rate is no longer peaked at zero. For example, with a single detection (k=1) we have

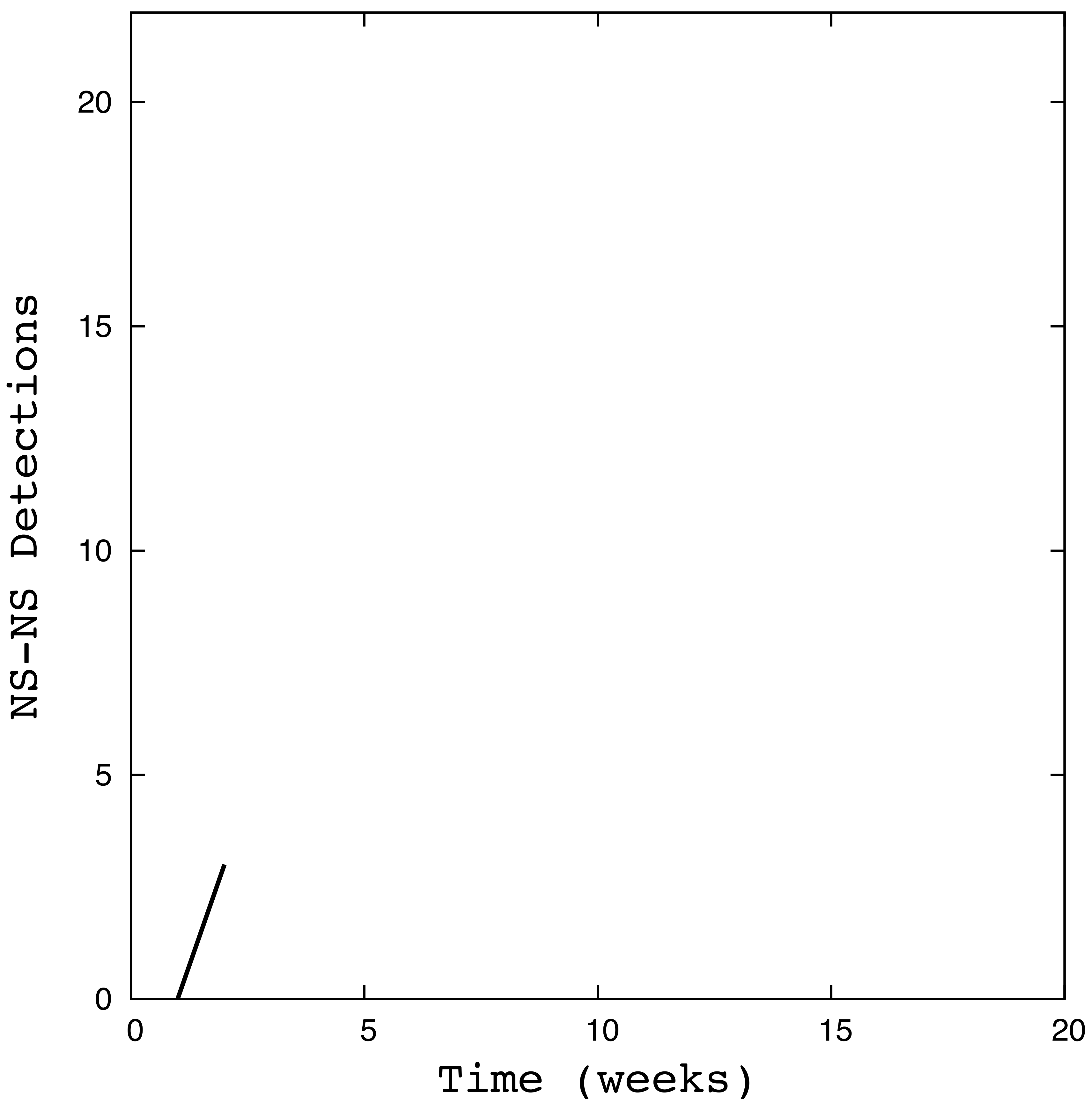
$$p(R) = R(VT)^2\, e^{-RVT}$$

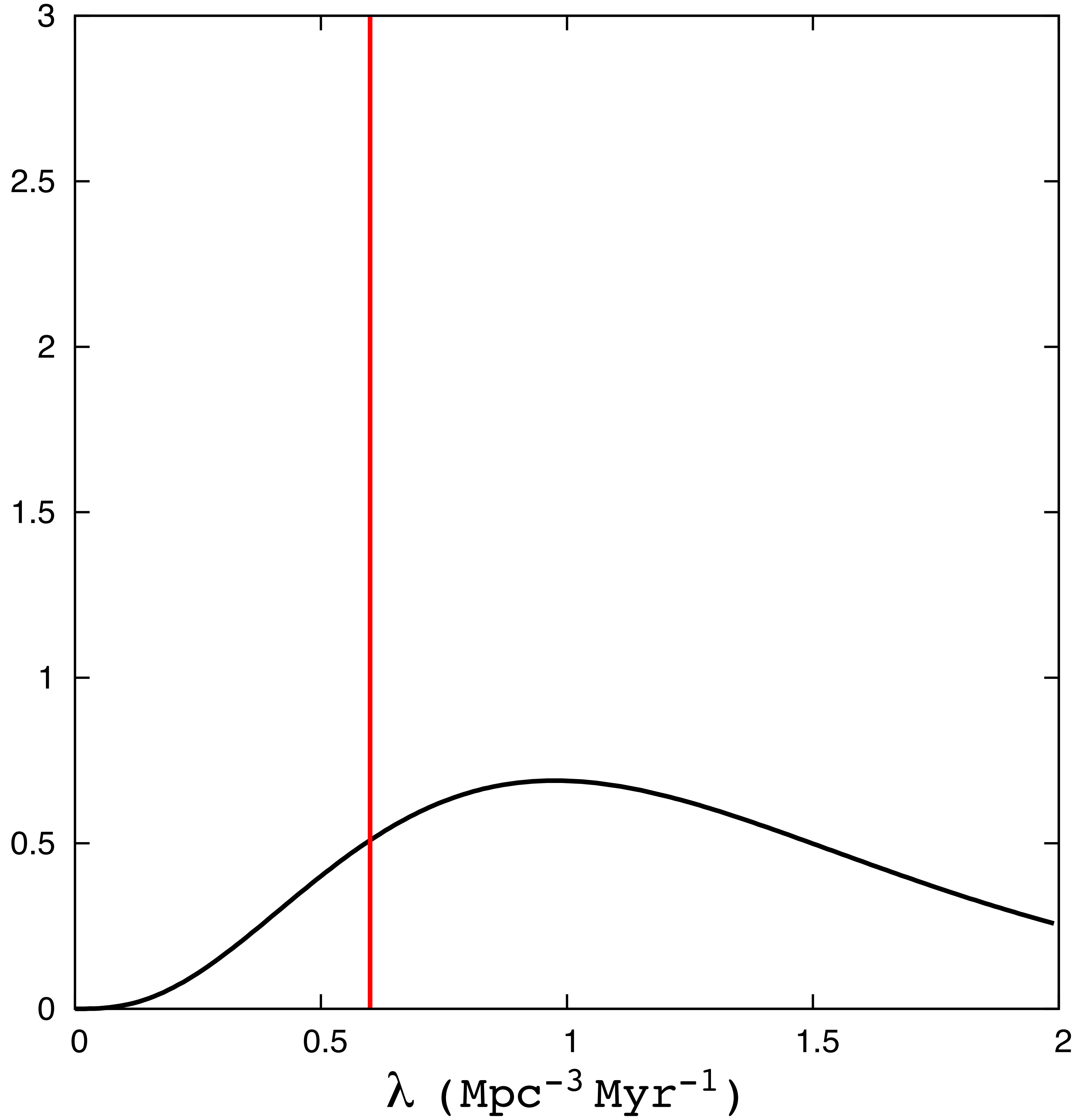
This distribution is peaked at

$$R = \frac{1}{VT}$$

The 90% confidence interval now sets upper ***and*** lower limits on the merger rate.

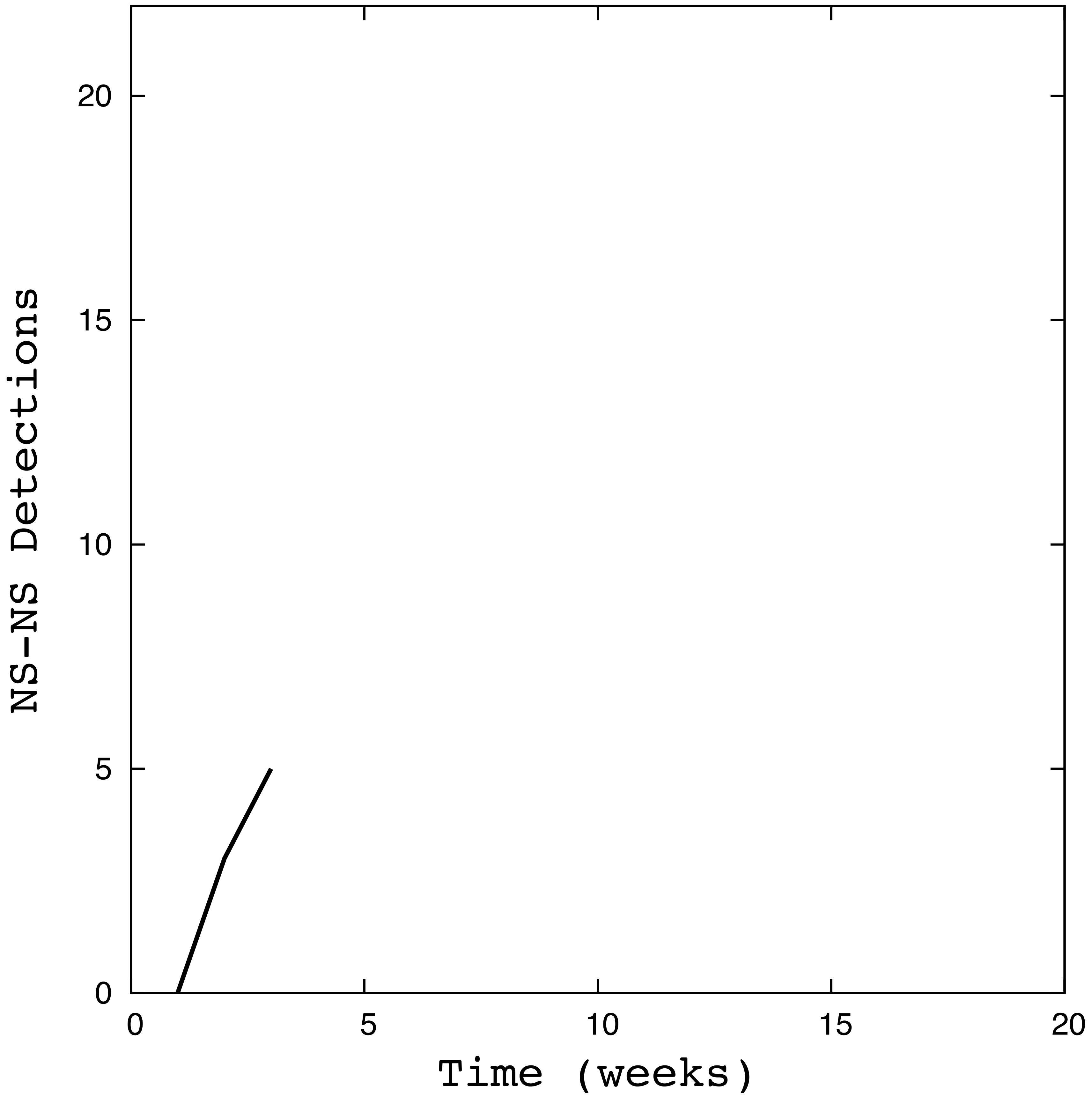
Simulated NS-NS Merger Rate Constraints

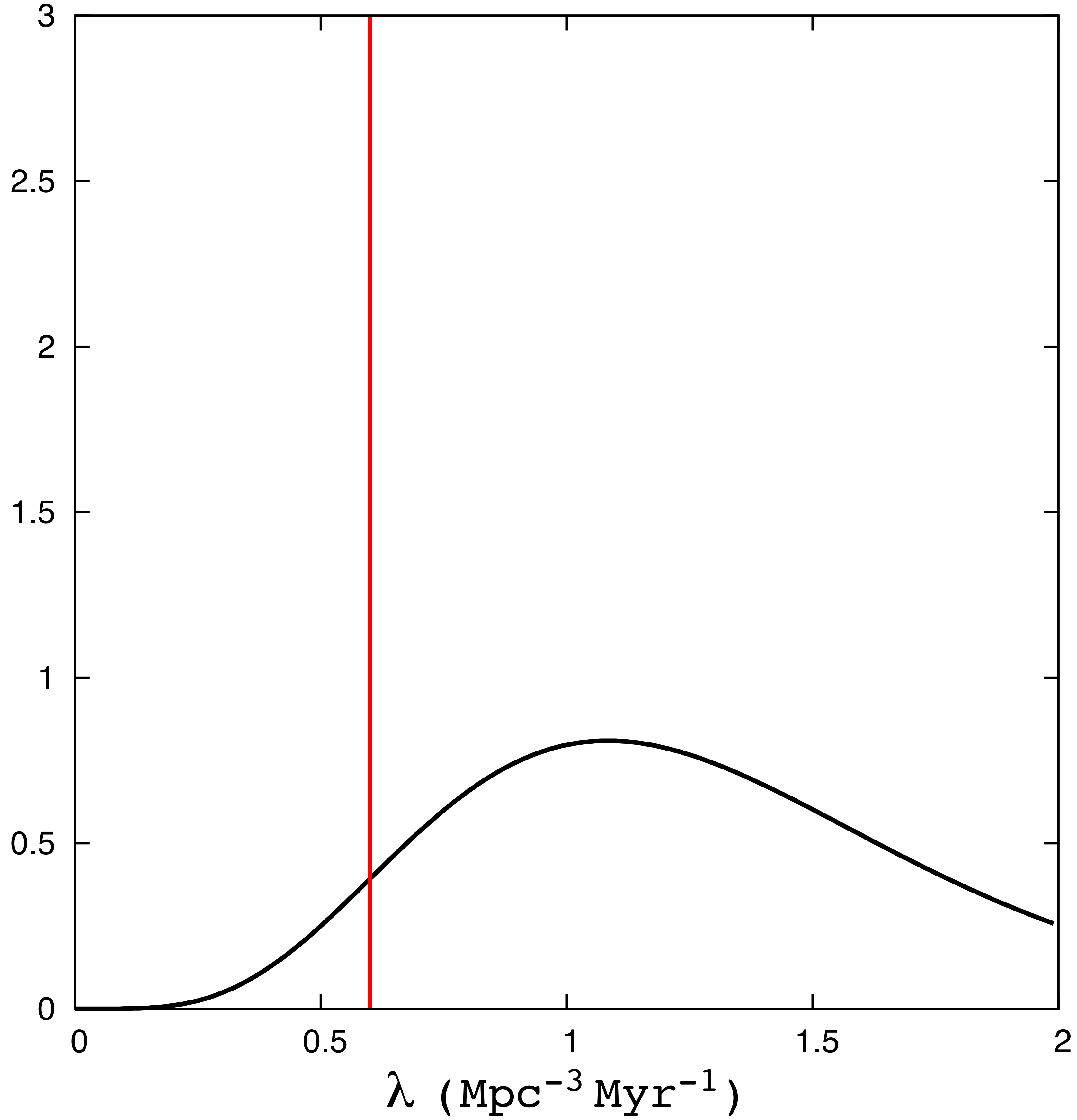




Week 2

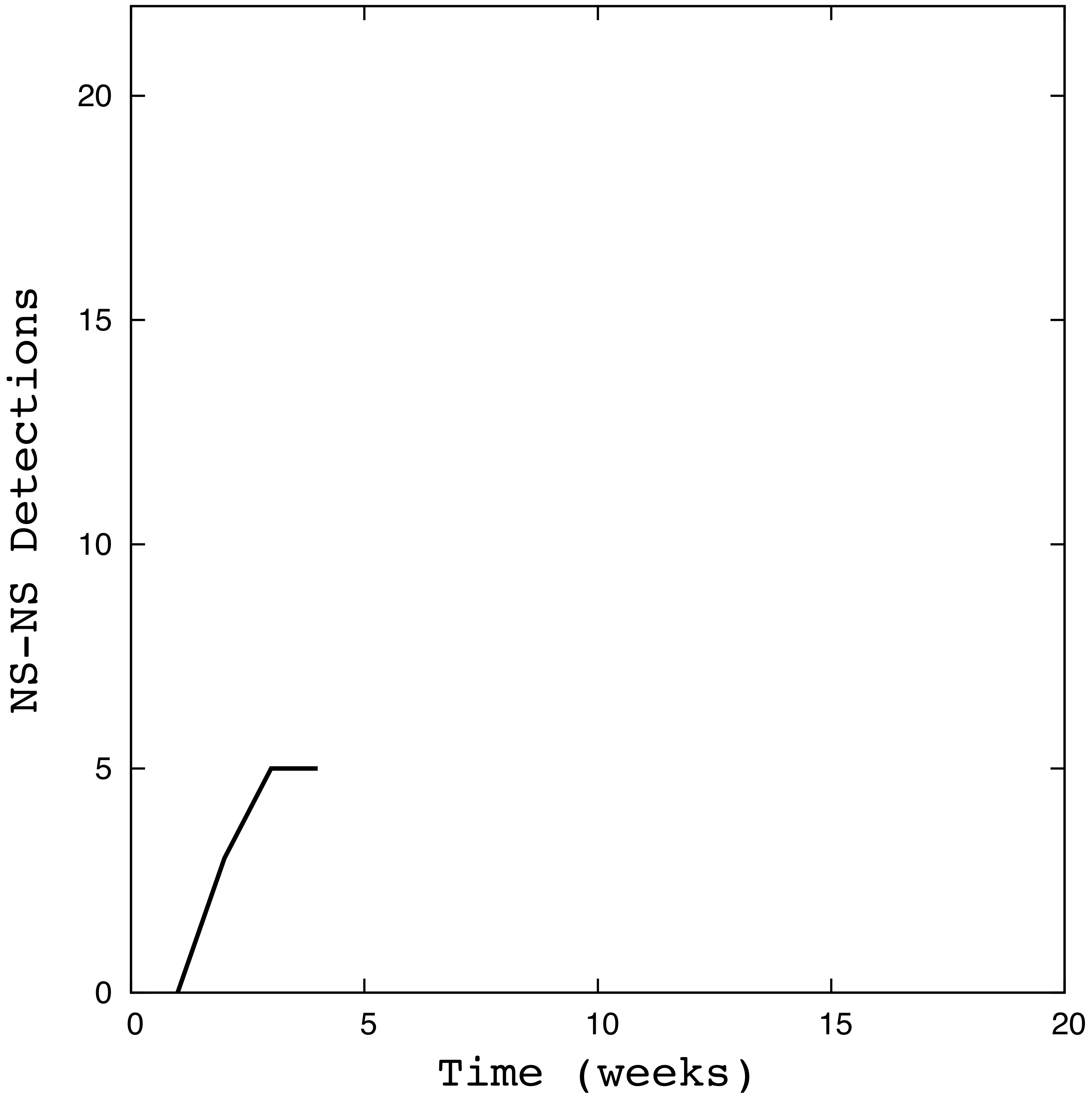
Simulated NS-NS Merger Rate Constraints

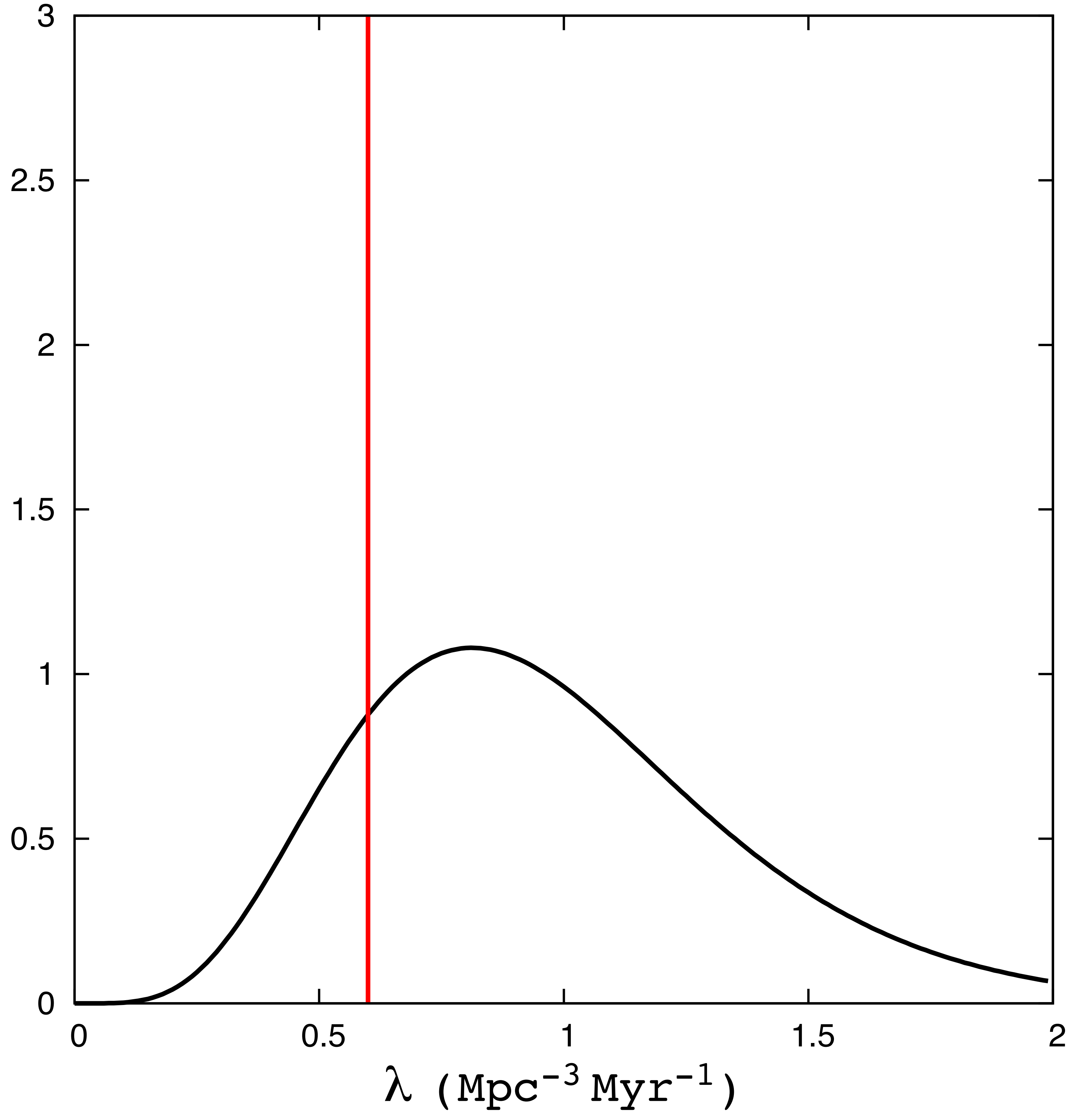




Week 3

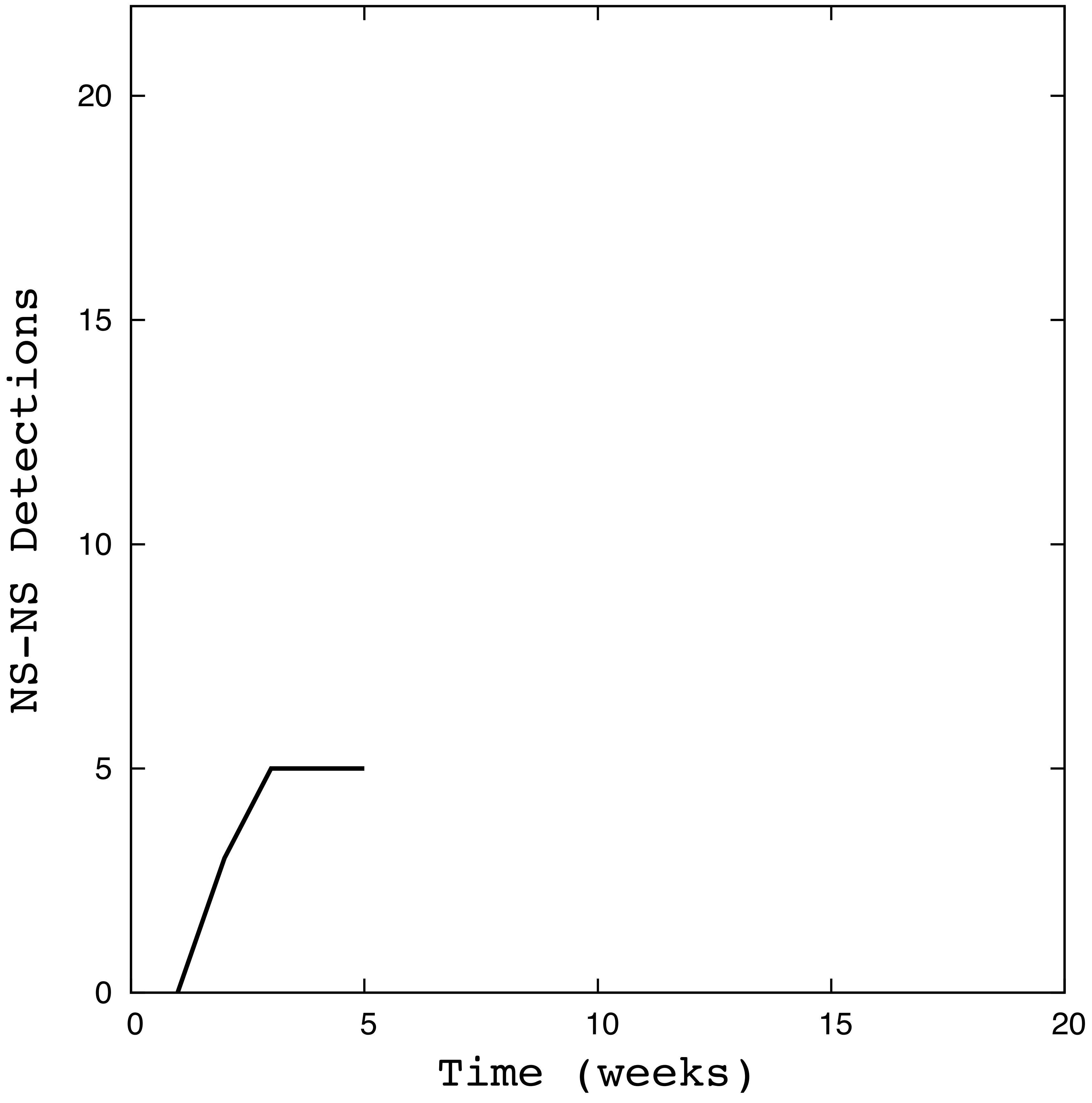
Simulated NS-NS Merger Rate Constraints

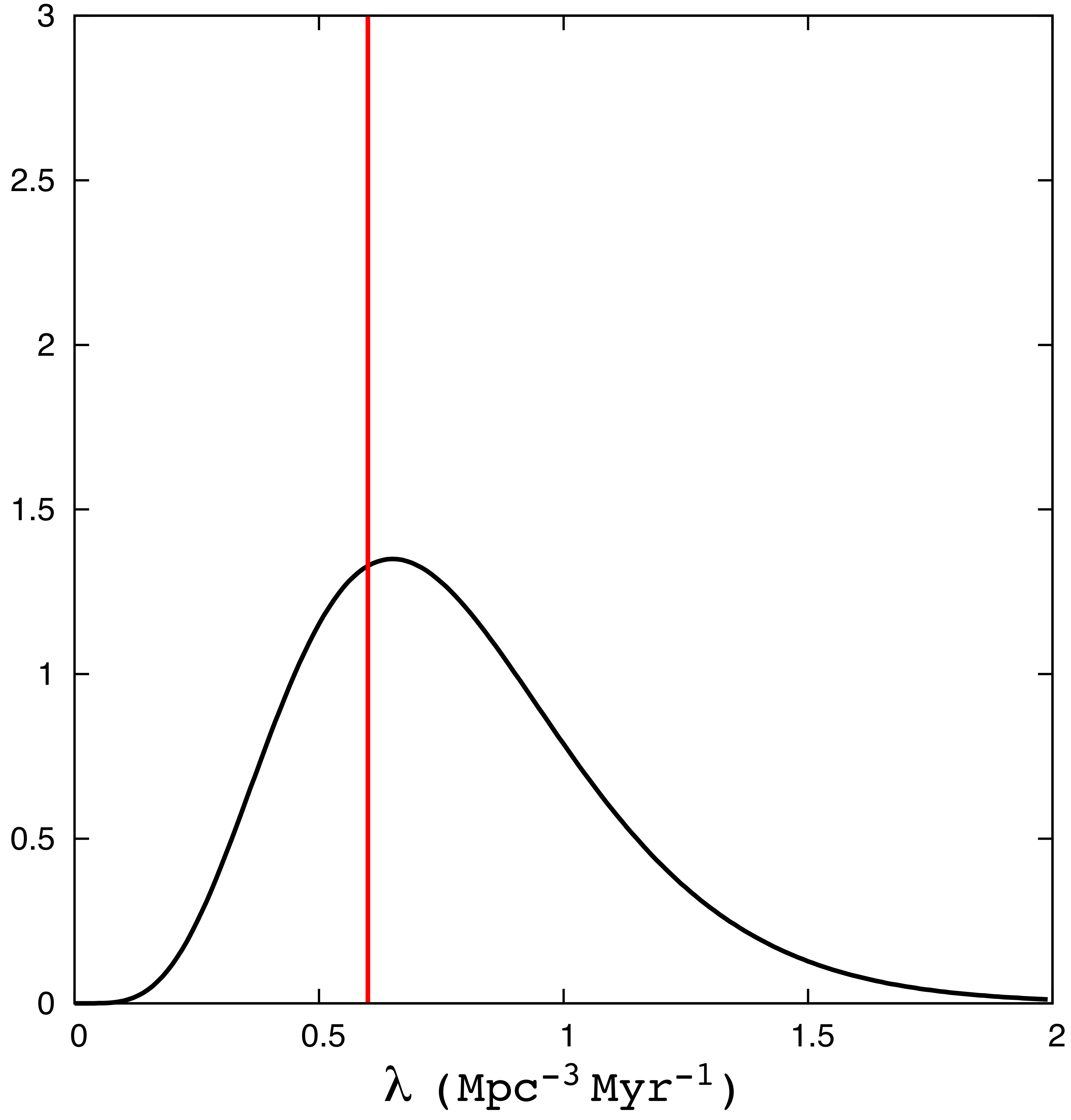




Week 4

Simulated NS-NS Merger Rate Constraints





Week 5

Simulated NS-NS Merger Rate Constraints