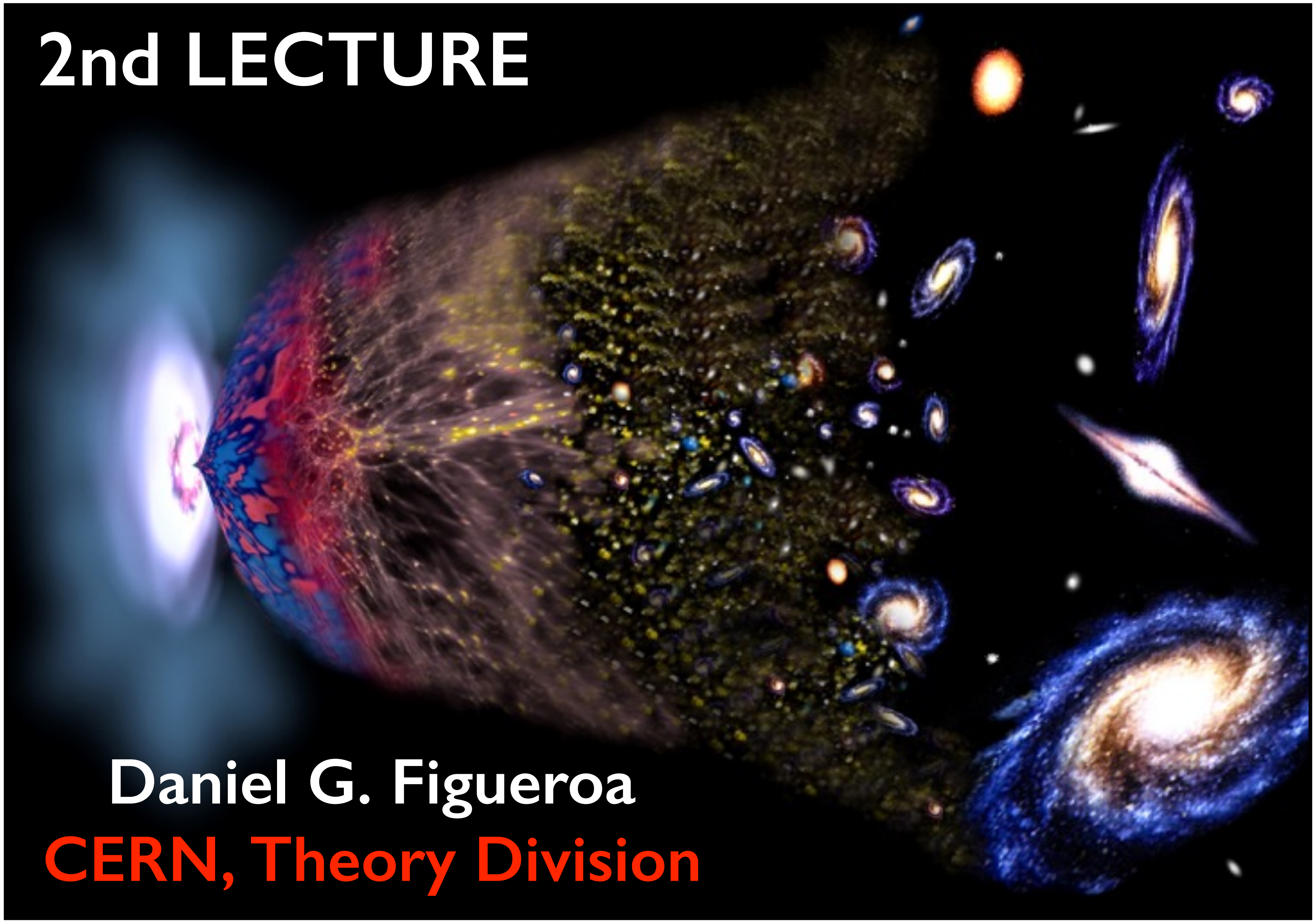
GRAVITATIONAL WAVES
PROBE OF THE EARLY UNIVERSE



2nd LECTURE

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CERN, Theory Division

School on Gravitational Waves for Cosmology and Astrophysics, Benasque, May 28 - June 10, 2017

BASICS of COSMOLOGY

- **Cosmology**
  - **hot Big Bang (hBB)** ✓
    (evolution of the Universe)
    - **theoretical pillars**
      - **General Relativity**
      - **Cosmological Pple**
    - **observational pillars**
      - **Expansion**
      - **CMB**
      - **BBN**
  - **Inflation = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ 'cures' hBB**

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Cosmology

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    - **Expansion**
    - **CMB**
    - **BBN**
- Problems with the hBB !

Shortcomings of the hBB framework

hBB shortcomings
(motivation for inflation)

UNIVERSE

1) Gravity Th: GENERAL RELATIVITY (**GR**)

$$G_{\mu\nu}[g_{**}] = \frac{1}{m_p^2} T_{\mu\nu}[\psi]$$

2) P. Symmetry: HOMOGENEITY & ISOTROPY (**H & I**)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - K r^2} + r^2 d\Omega^2\right]$$

(FLRW)

Scale Factor

Curvature

hBB shortcomings
(motivation for inflation)

UNIVERSE → $\boxed{\text{1) GR} + \text{2) H \& I}}$ ⇒

Friedmann Equations

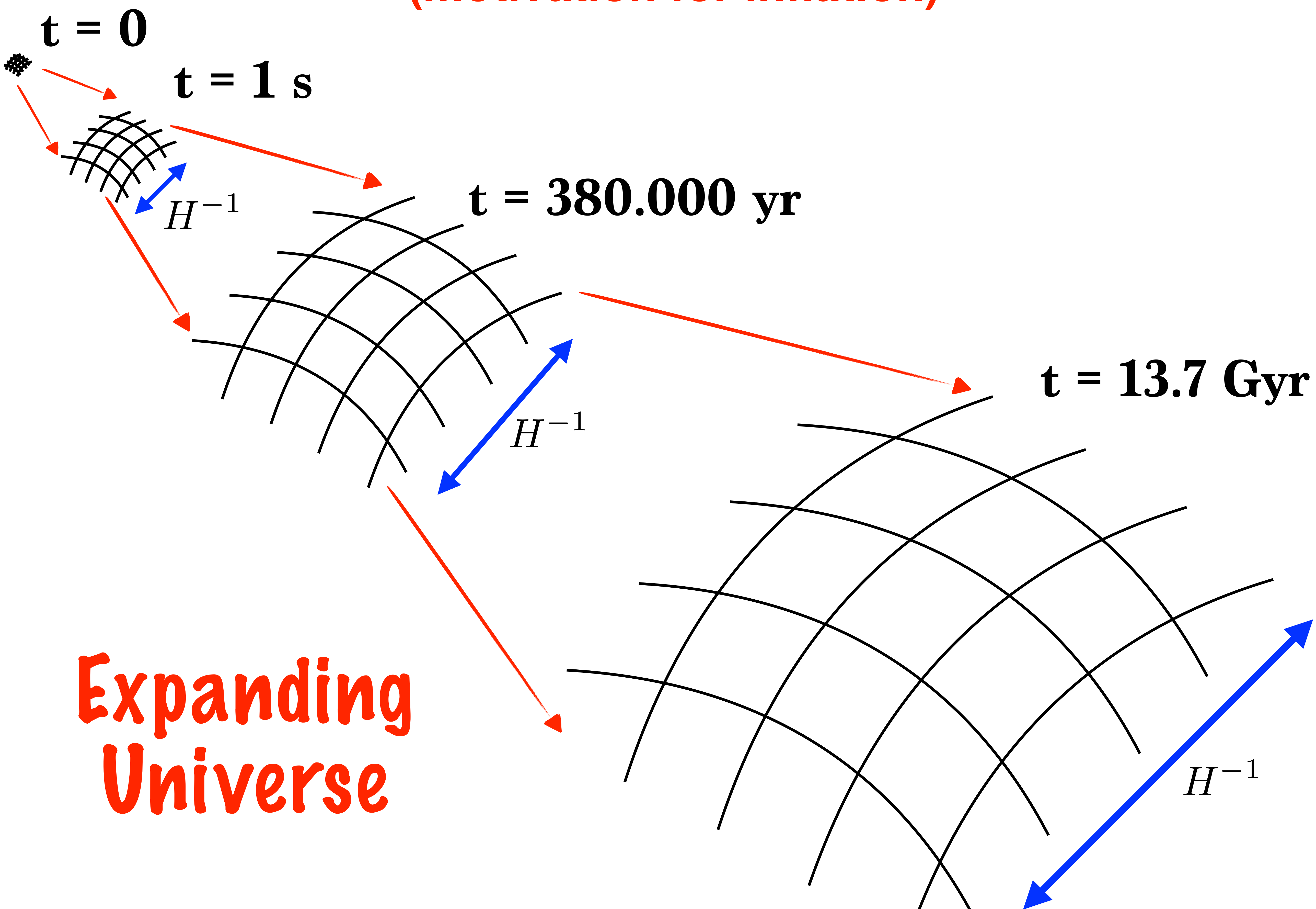
$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w)$$

$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

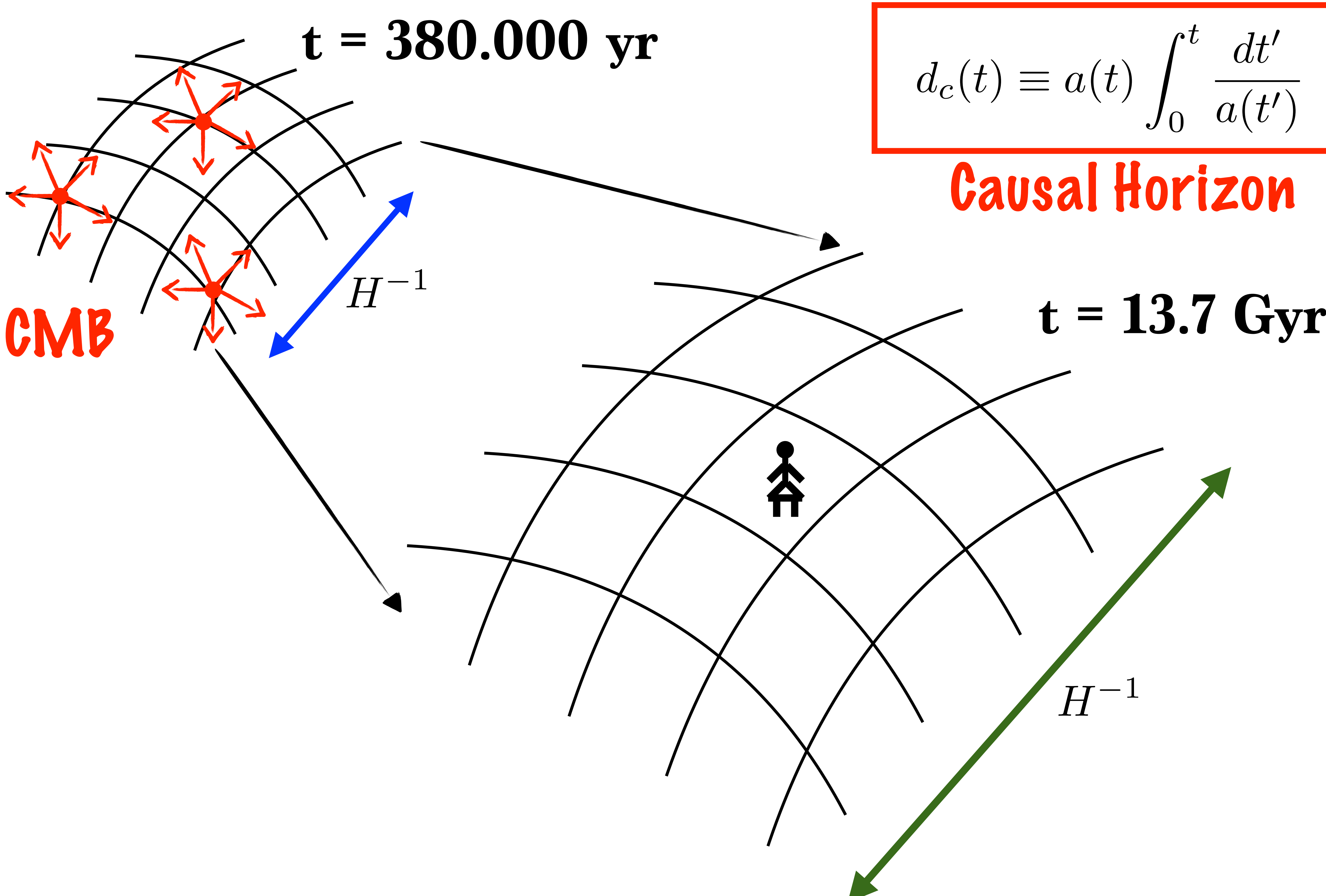
hBB shortcomings
(motivation for inflation)



Expanding Universe

hBB shortcomings

(motivation for inflation)



$$d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}$$

Causal Horizon

hBB shortcomings

(motivation for inflation)

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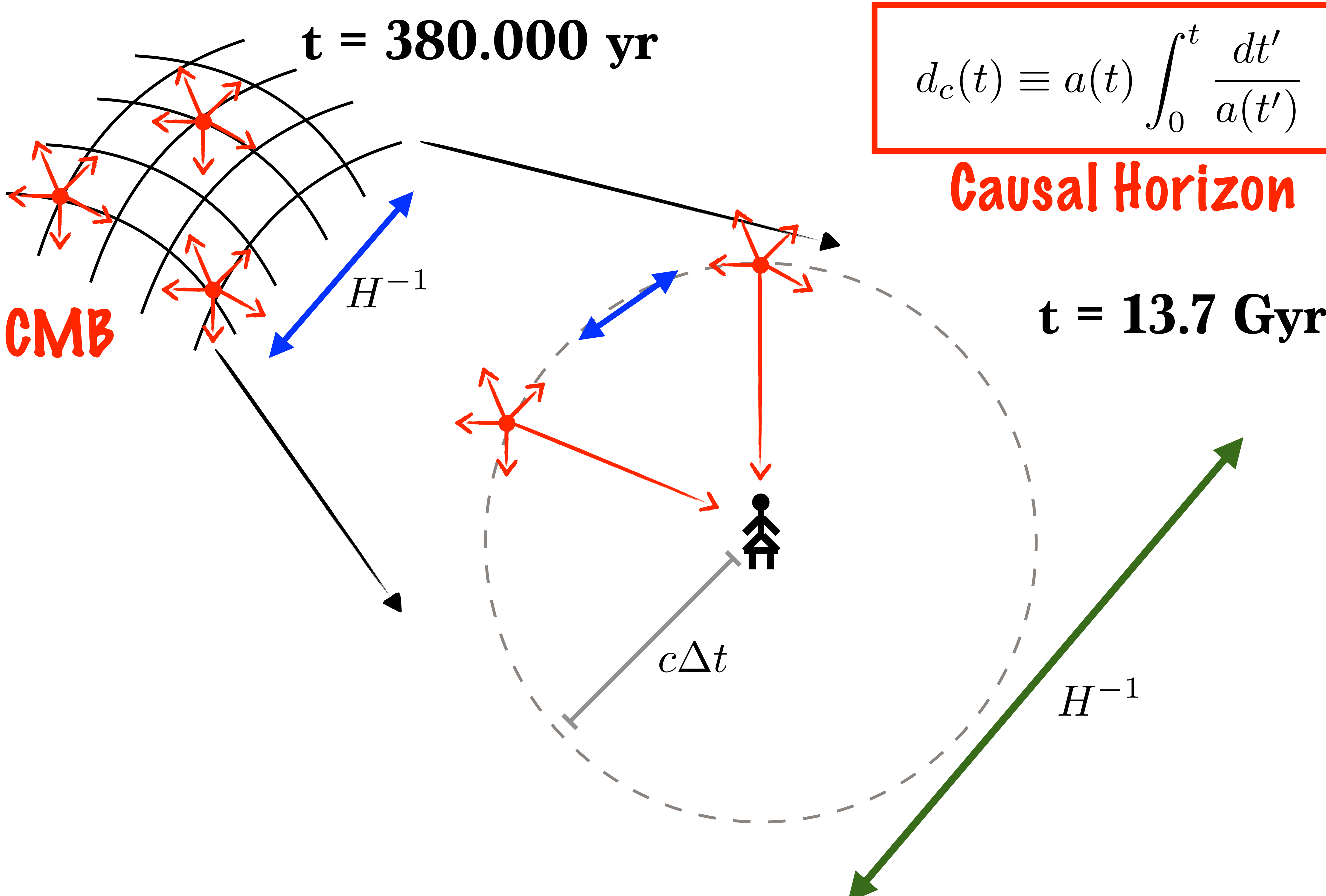
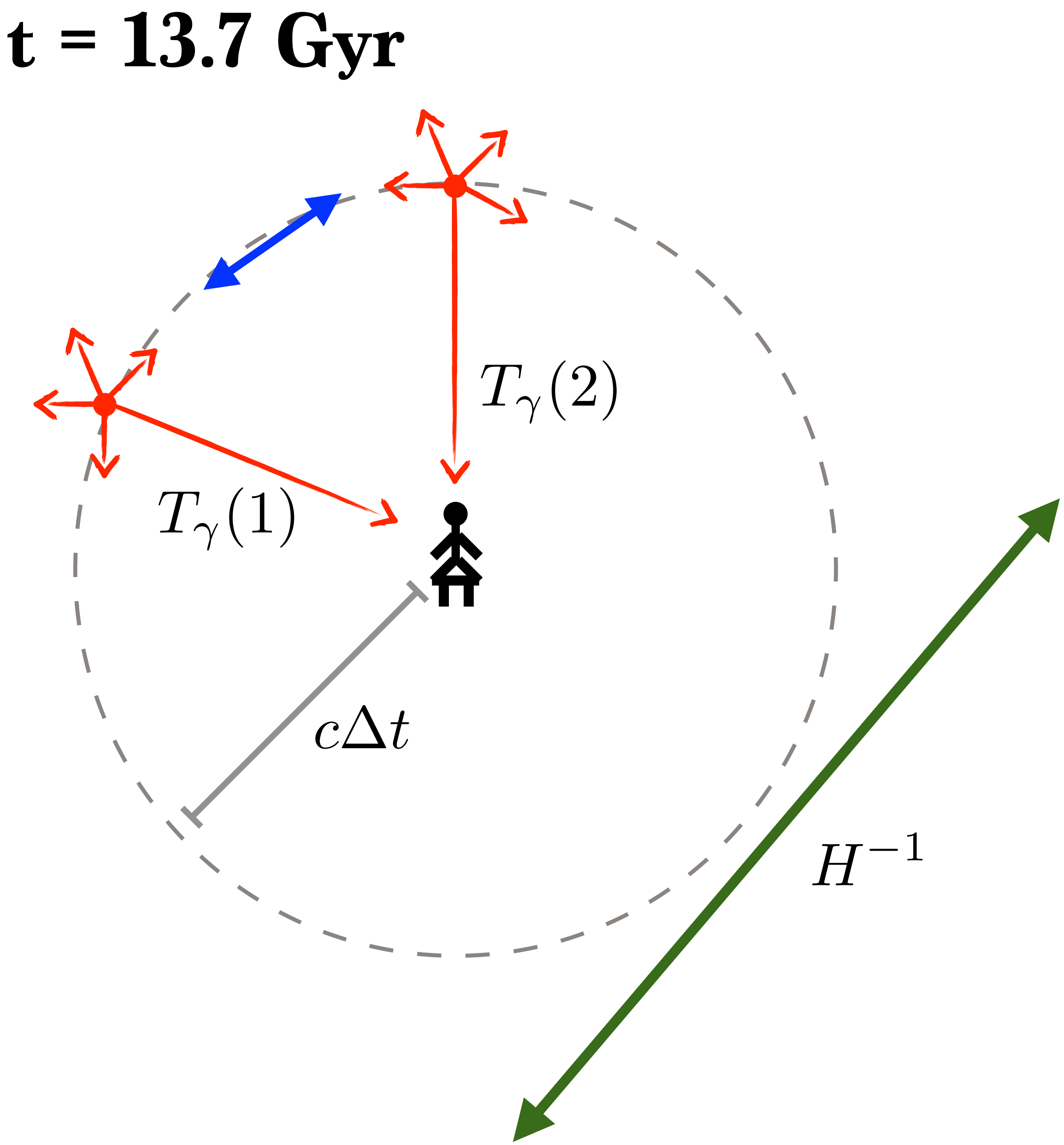
Causal Horizon



hBB shortcomings

(motivation for inflation)



IF $\boxed{T_\gamma(1) = T_\gamma(2)}$

CAUSALITY VIOLATION !

hBB:

H&I @ Scales >> 1/H

iLL-defined!

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

hBB shortcomings

(motivation for inflation)

hBB

1) **Horizon Problem** $\longrightarrow$ **Causality Violation !**

2) **Curvature Problem**

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2H^2}\begin{cases}\propto a^2, & \text{RD}\\ \propto a, & \text{MD}\end{cases}$$

$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \boxed{|\Omega - 1|_{\rm BBN} \lesssim 10^{-18}} \ \boxed{|\Omega - 1|_{\rm GUT} \lesssim 10^{-56}}$$

It might well be that k = 0 ...

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings
(motivation for inflation)

hBB

1) Horizon Problem $\longrightarrow$ Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

Need extra 'Ingredient' ! $\longrightarrow$ INFLATION !

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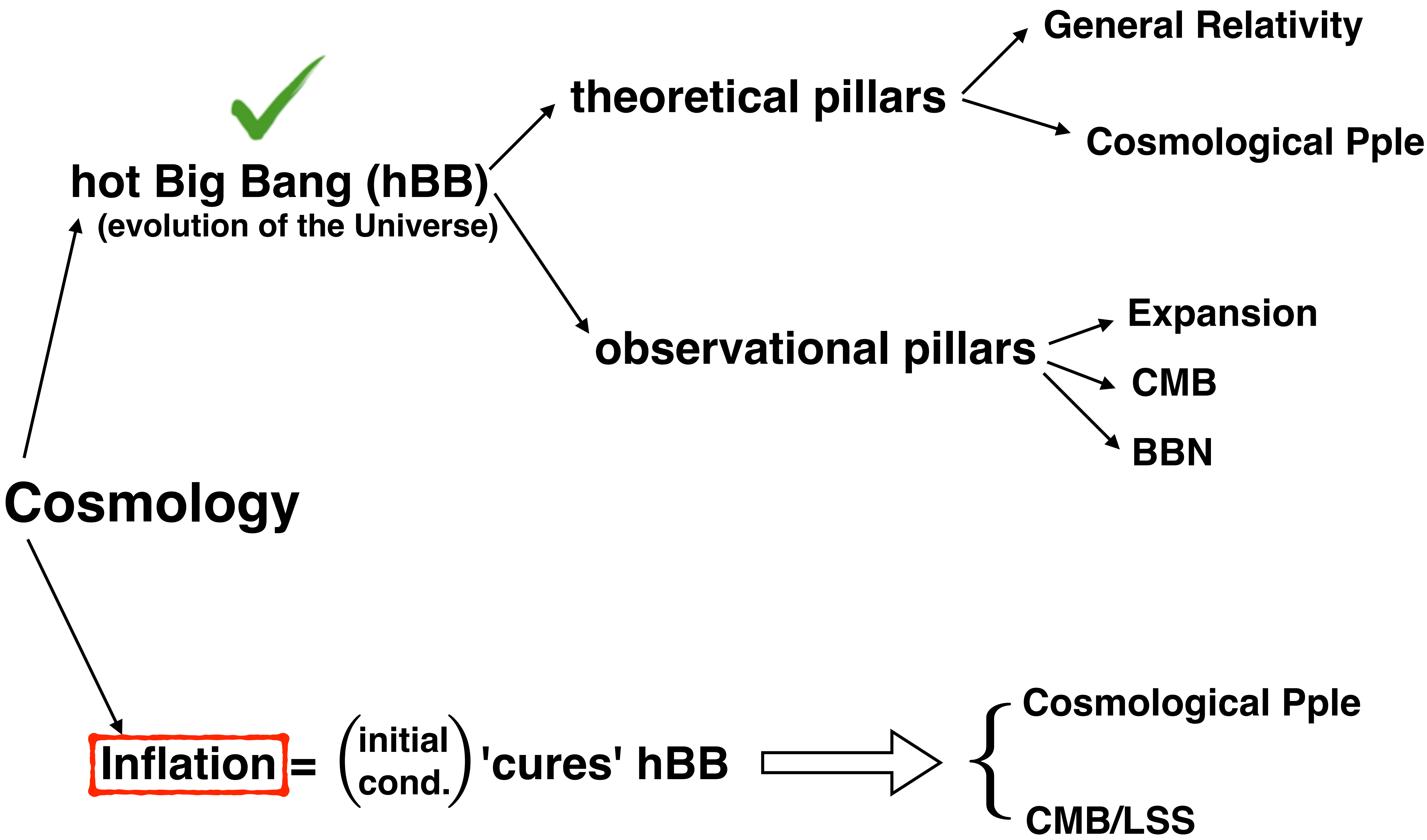

Inflation: Definition + Implementation

Comoving Hubble Radious

$$\mathcal{H}^{-1} \equiv \frac{1}{aH} \sim \begin{cases} a^2\,, & \text{hBB} \end{cases}$$ **(increasing)**

Inflation: Definition + Implementation

INF → * **Definition:** $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

Comoving Hubble Radious $$\mathcal{H}^{-1} \equiv \frac{1}{aH} \sim \begin{cases} a^2 \;, & \mathrm{hBB} \\ a^{-1} \;, & \mathrm{Inf.} \end{cases}$$ **(increasing)** / **(decreasing)**

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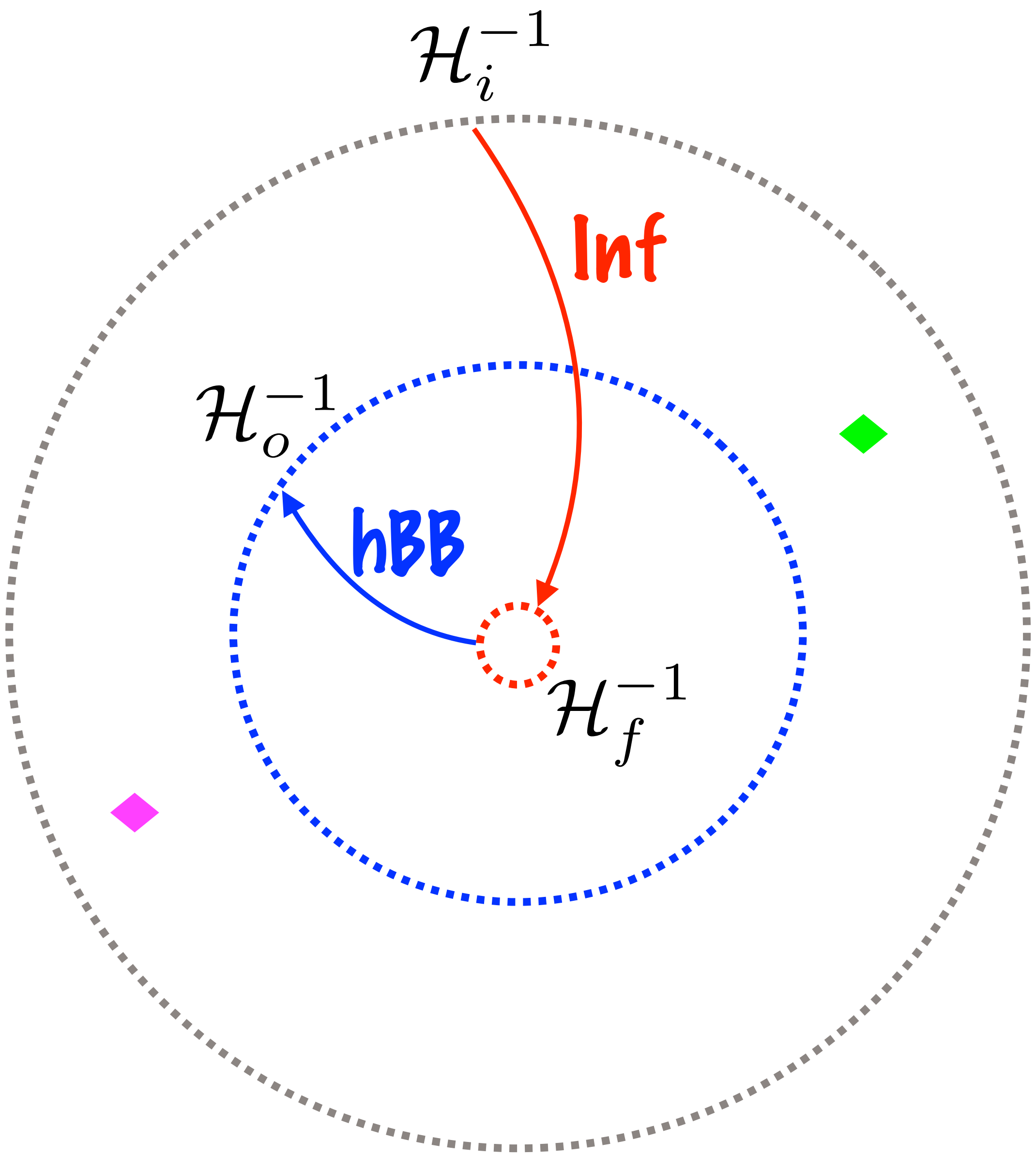
INF → * Definition: $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$



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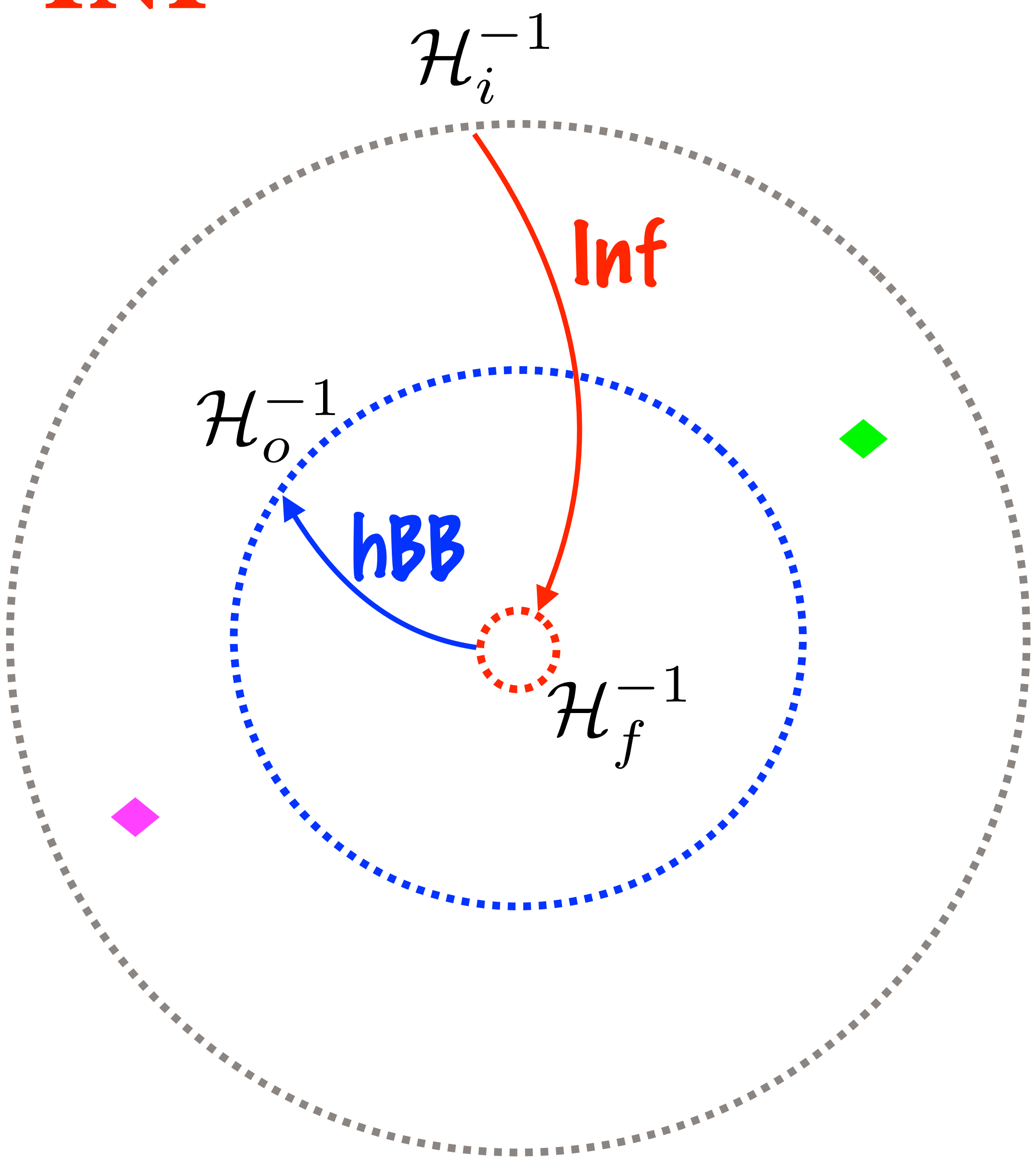

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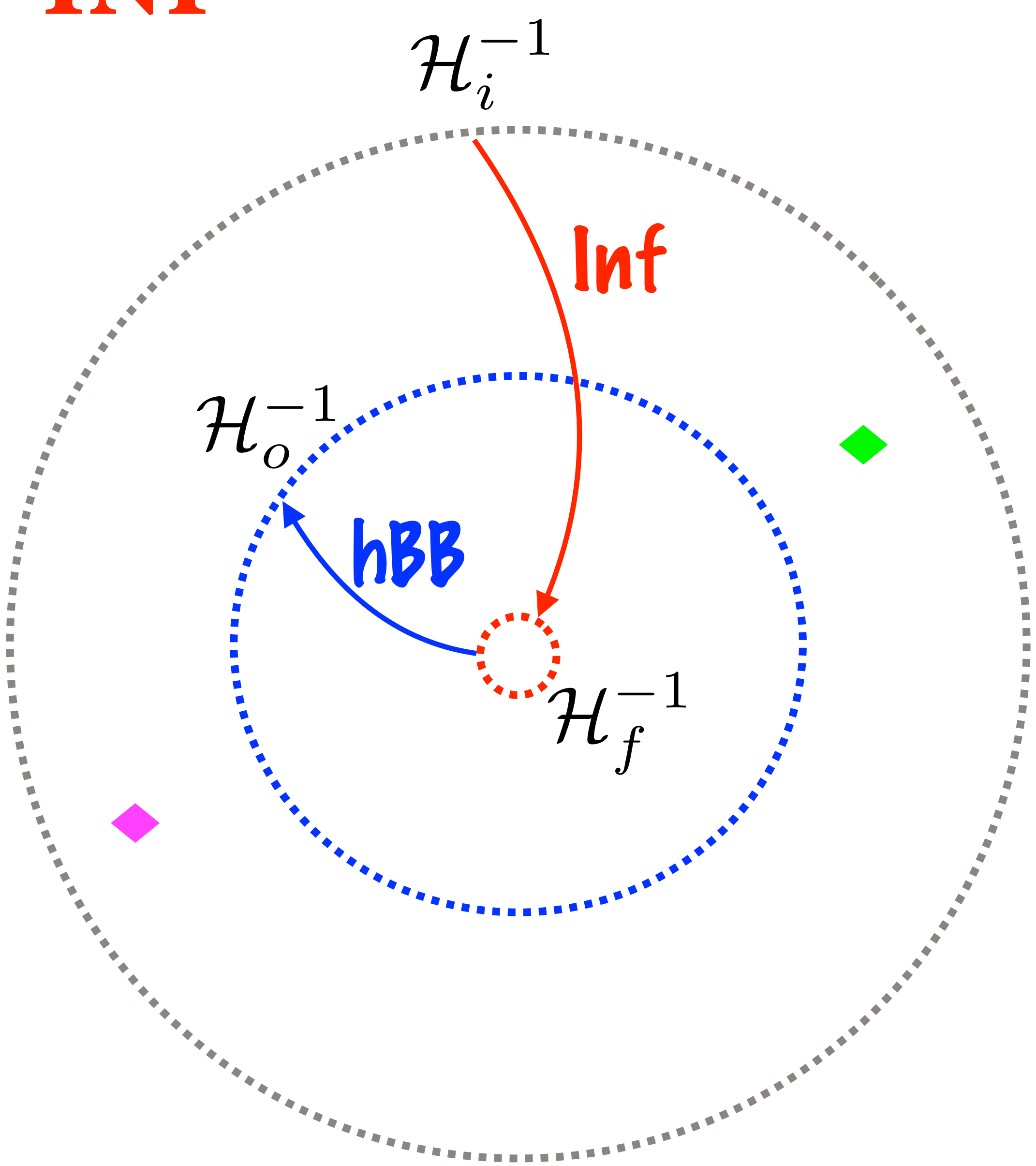

$$\frac{a_f}{a_i} \equiv e^N \quad (\text{\# e-folds})$$

$$\begin{aligned} \mathcal{H}_i^{-1} &= \frac{a_f}{a_i}\,\mathcal{H}_f^{-1} \\ &= e^N\,\mathcal{H}_f^{-1} \geq \mathcal{H}_o^{-1} \end{aligned}$$

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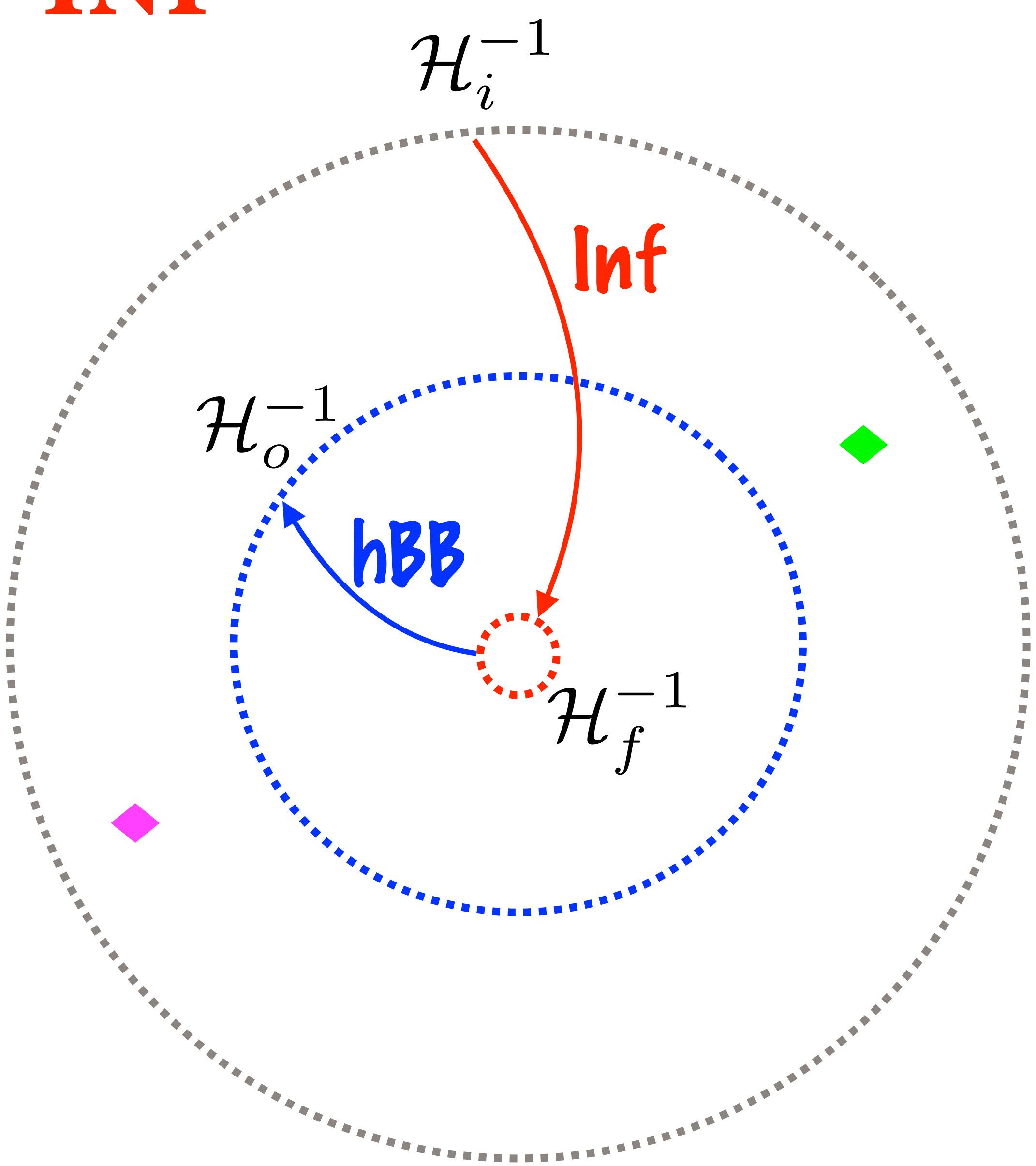

$$\frac{a_f}{a_i} \equiv e^N \quad (\#\text{ e-folds})$$

$$\begin{aligned}\mathcal{H}_i^{-1} &= \frac{a_f}{a_i}\,\mathcal{H}_f^{-1} \\ &= e^N\,\mathcal{H}_f^{-1} \geq \mathcal{H}_o^{-1}\end{aligned}$$

$$\begin{aligned} N &\geq \log(\mathcal{H}_f/\mathcal{H}_o) = \log(E_f/E_o) \\ &\gtrsim 60 + \log(E_f[GeV]/10^{16})\end{aligned}$$

Inflation: Definition + Implementation

INF

* Definition: $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

* Consequences: If N $\gtrsim 60$

Inflation: Definition + Implementation

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Horizon Problem Solved !

Inflation: Definition + Implementation

INF

* Definition: $$\boxed{\frac{d^2a}{dt^2} > 0 \Leftrightarrow \frac{d}{dt}\mathcal{H}^{-1} < 0}$$

* Consequences: If N $\gtrsim$ 60 → Horizon Problem Solved ! *

* Cosmological Principle (H & I) explained !

Inflation: Definition + Implementation

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Horizon Problem Solved !

Bonus: Null Curvature

$$\boxed{\left(\left|\frac{K}{\mathcal{H}^2}\right| \sim |K/H_i^2| e^{-2N} = |K/H_i^2| e^{-120} \ll 1 \right)}$$

Inflation: Definition + Implementation

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* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

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$$w \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}(\nabla\phi)^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)} \simeq \frac{-V(\phi)}{V(\phi)} \simeq -1$$

Inflation: Definition + Implementation

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IF $V(\phi) \gg \frac{1}{2}\dot{\phi}^2, \frac{1}{2}(\nabla\phi)^2$ $\Rightarrow$ $\boxed{w \simeq -1 \; (EoS)}$ $\Rightarrow$ [Friedmann Equations]

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$$\Rightarrow \boxed{\begin{array}{llll} \text{i)} & \dfrac{d\rho_\phi}{dt} & \simeq & 0 \\ \text{ii)} & H^2 & \simeq & \dfrac{V(\phi)}{3m_p^2} \\ \text{iii)} & \dfrac{1}{a}\dfrac{d^2a}{dt^2} & \simeq & +\dfrac{V(\phi)}{3m_p^2} \end{array}}$$

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$\rightarrow$

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$\rightarrow$ $\boxed{a(t) \simeq a_i e^{\int_t H(\phi)dt'}}$

(Quasi) de Sitter

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$$\boxed{\frac{\ddot{a}}{a} = \frac{1}{3m_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \simeq H^2(1-\epsilon)} \longleftrightarrow \boxed{\epsilon = -\frac{\dot{H}}{H^2}}$$

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$\Updownarrow$

Can $\epsilon \ll 1$ be sustained for $\Delta N = 60$? No, unless $V(\phi)$ is "flat" !

Inflation: Definition + Implementation

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$\Updownarrow$

Can $\epsilon \ll 1$ **be sustained for** $\Delta N = 60$**?** **No, unless** $V(\phi)$ **is "flat" !**

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \quad \Rightarrow \quad \phi \text{ accelerates!} \Rightarrow \dot{\phi} \uparrow\uparrow \Rightarrow \epsilon \uparrow\uparrow$$

Inflation: Definition + Implementation

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$\Updownarrow$

Can $\epsilon \ll 1$ **be sustained for** $\Delta N = 60$ **?** **No, unless** $V(\phi)$ **is "flat" !**

$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ $\Rightarrow$ ϕ accelerates! $\Rightarrow \dot{\phi} \uparrow\uparrow \Rightarrow \epsilon \uparrow\uparrow$

Needed: $|\ddot{\phi}| \ll 3H\dot{\phi}, V'(\phi)$ $\Rightarrow$ $\boxed{\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1}$

Inflation: Definition + Implementation

* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Slow Roll (SR) Regime:

$$\boxed{\epsilon = \frac{\dot{\phi}^2}{2m_p^2 H^2} \ll 1 \; ; \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1}$$

Inflation: Definition + Implementation

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$$\boxed{\begin{aligned} \epsilon_V &\equiv \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta_V &\equiv m_p^2\left(\frac{V''}{V}\right) \end{aligned}}$$

$$(\epsilon \simeq \epsilon_V \; , \quad \eta \simeq \eta_V - \epsilon_V)$$

Inflation: Definition + Implementation

$\ast$ **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Slow Roll (SR) Regime:

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$(\epsilon \simeq \epsilon_V \, , \quad \eta \simeq \eta_V - \epsilon_V)$

$\Rightarrow$

$$\boxed{\text{If } \epsilon_V , \eta_V \ll 1 \;\; \Rightarrow \;\; \epsilon, \eta \ll 1}$$

SR $\Longrightarrow$ quasi dS for $\Delta N = 60$

$$\boxed{N(\phi) \simeq \int_{\phi_f}^{\phi} \frac{d\phi'}{\sqrt{2\epsilon(\phi',\dot\phi')}}}$$

Inflation: Definition + Implementation

* **Implementation:** $\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi)}$ (ϕ Inflaton)

Case of Study: $\boxed{V(\phi) = \frac{1}{2} m_\phi^2 \phi^2}$

$$\epsilon_V(\phi) = \eta_V(\phi) = 2(m_p/\phi)^2$$

$$N(\phi) = (\phi/2m_p)^2 - 1/2$$

Inflation: Definition + Implementation

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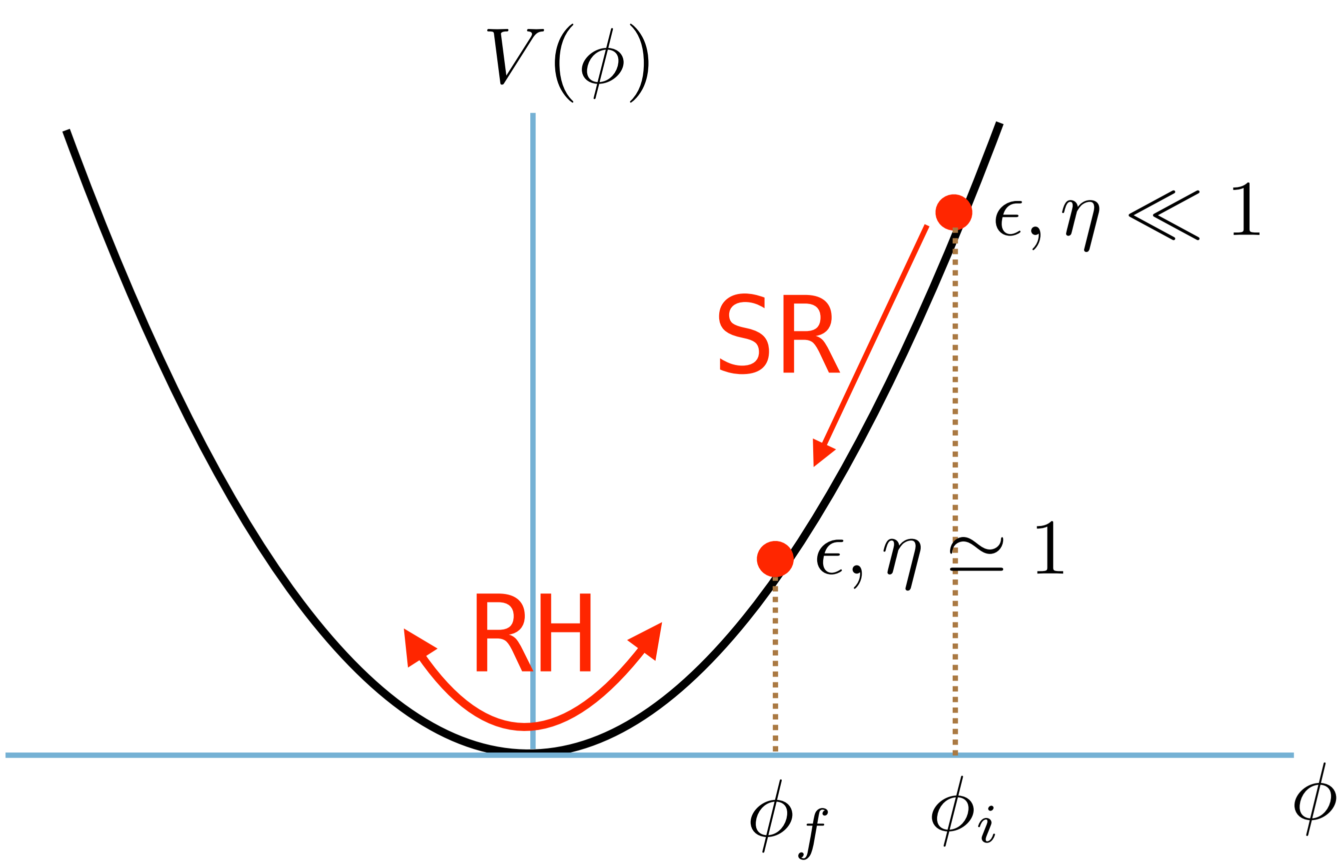

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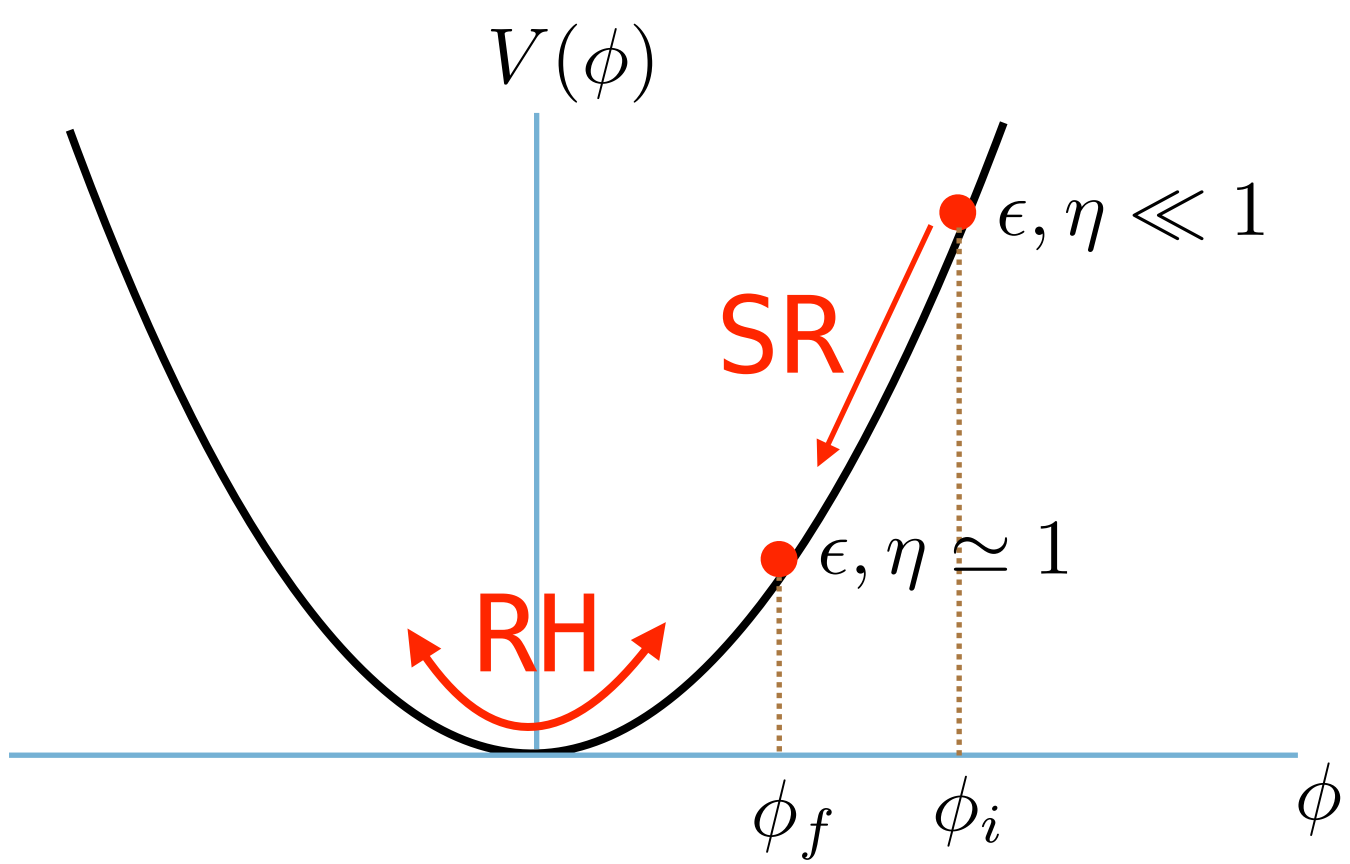

'Inflating ' is easy with any potential of the type $V(\phi) \propto \phi^p$

BASICS of COSMOLOGY

Cosmology

hot Big Bang (hBB)
(evolution of the Universe)

- **theoretical pillars**
  - General Relativity
  - Cosmological Pple
- **observational pillars**
  - Expansion
  - CMB
  - BBN

Inflation = (initial cond.) 'cures' hBB $\Longrightarrow$
- Cosmological Pple
- CMB/LSS

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Cosmology

Inflation = (initial cond.) 'cures' hBB ⟹ { Cosmological Pple, CMB/LSS ?

?

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple ✓

Inflation $\rightarrow$ $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

INFLATIONARY COSMOLOGY

'cures' hBB $\Longrightarrow$ Cosmological Pple ✓

Inflation $\rightarrow$ $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ = Primordial perturbations $\left\{\begin{array}{l}\text{Scalar} \Longrightarrow \text{CMB } \Delta T \text{ anisotropies (eventually Galaxies)}\\ \\ \end{array}\right.$

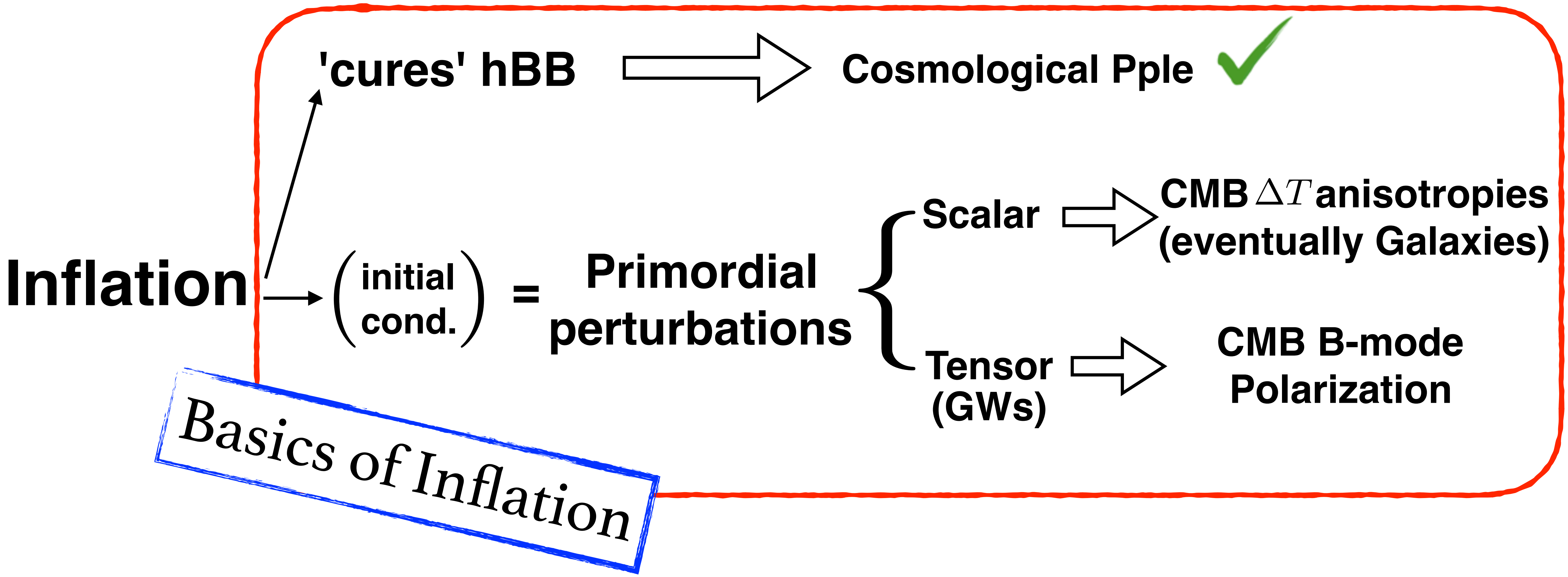
INFLATIONARY COSMOLOGY

Inflation $\longrightarrow$ 'cures' hBB $\Longrightarrow$ Cosmological Pple

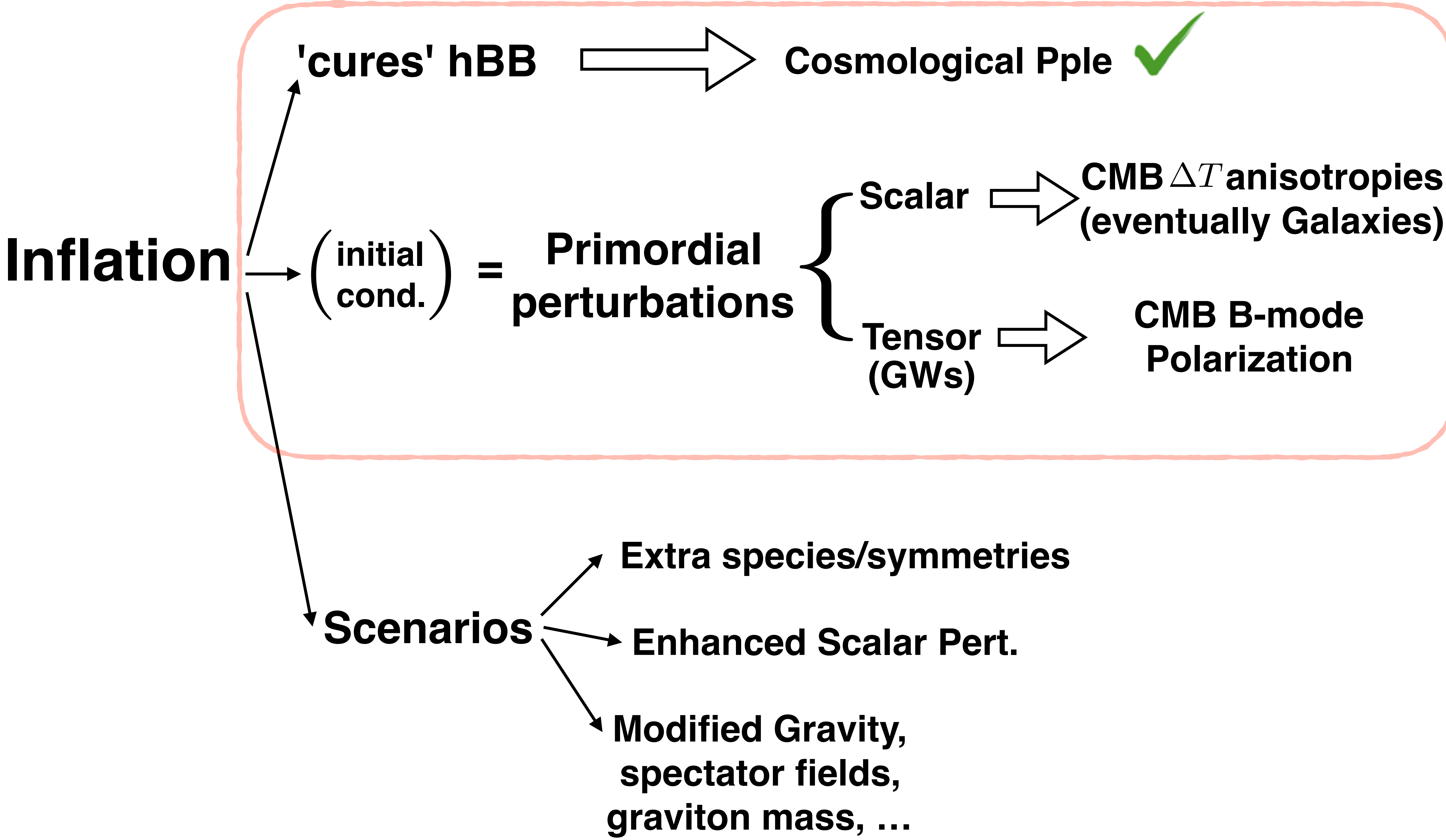
Inflation $\longrightarrow$ $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

- Scalar $\Longrightarrow$ CMB ΔT anisotropies (eventually Galaxies)
- Tensor (GWs) $\Longrightarrow$ CMB B-mode Polarization

INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY

Inflation

'cures' hBB $\Rightarrow$ Cosmological Pple ✓

$\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

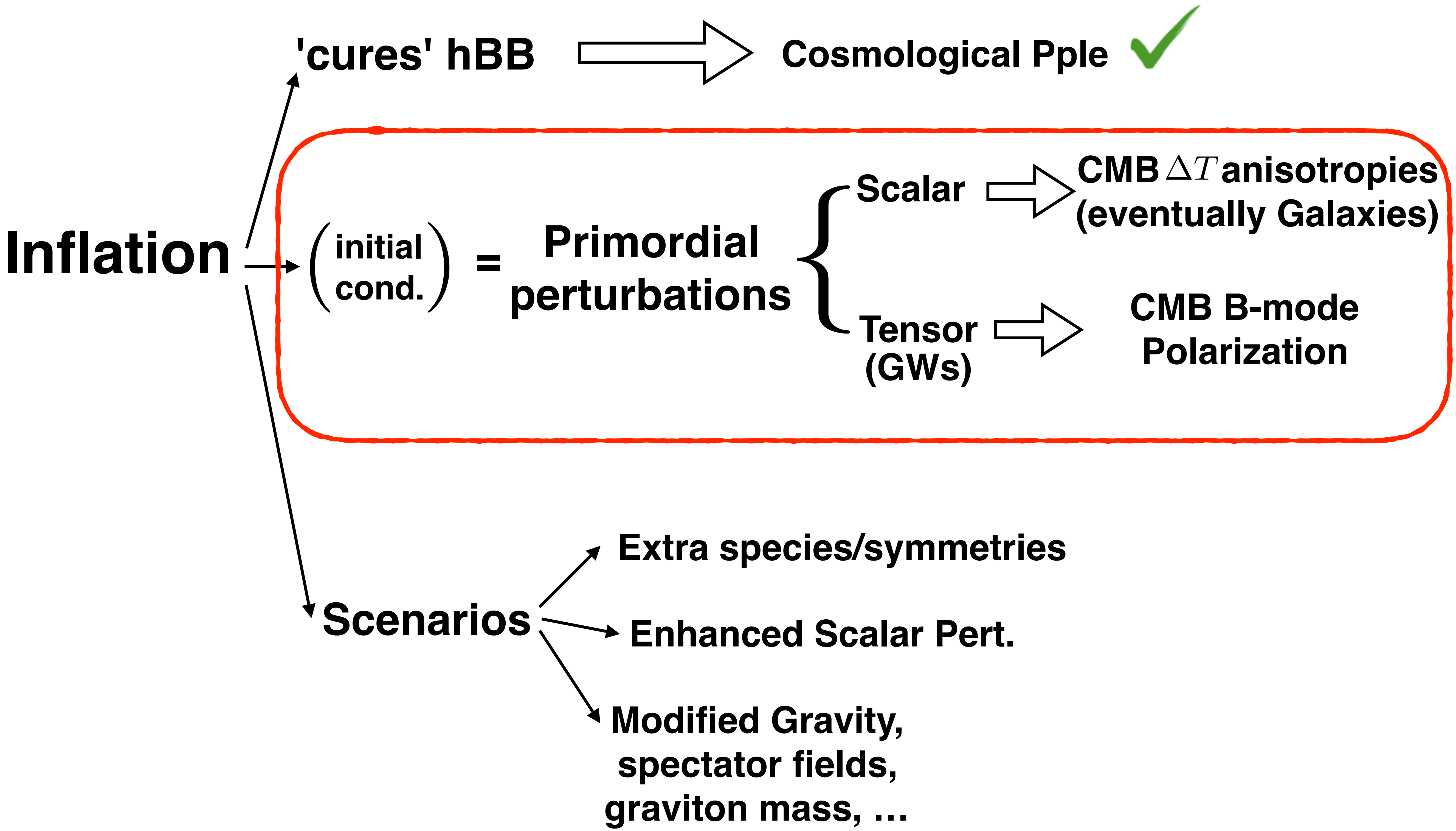
- Scalar $\Rightarrow$ CMB ΔT anisotropies (eventually Galaxies)
- Tensor (GWs) $\Rightarrow$ CMB B-mode Polarization

Scenarios

- Extra species/symmetries
- Enhanced Scalar Pert.
- Modified Gravity, spectator fields, graviton mass, …

Model dependent

INFLATIONARY COSMOLOGY



Inflation & Primordial Perturbations

INF → **SR:** $\boxed{\epsilon, \eta \ll 1 \rightarrow \epsilon, \eta \simeq 1}$

(Start) **(End)**

$\boxed{a \sim e^{\int H dt'} \gtrsim e^{60}}$ (qdS)

Flat Universe !

No Hor. Problem !

Inflation & Primordial Perturbations

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$\boxed{a \sim e^{\int H dt'} \gtrsim e^{60}}$ (qdS)

No Hor. Problem !

Flat Universe !

*** Is that ALL?** **NO!**

$$\boxed{\begin{matrix} \phi(t) \\ \\ g_{\mu\nu}(t) \end{matrix}}$$

(Background)

Inflation & Primordial Perturbations

INF → **SR:** $\epsilon, \eta \ll 1 \rightarrow \epsilon, \eta \simeq 1$

(Start) — **(End)**

$a \sim e^{\int H dt'} \gtrsim e^{60}$ (qdS)

Flat Universe !

No Hor. Problem !

*** Is that ALL? NO!**

$\phi(t) \rightarrow \phi(t) + \delta\phi(\vec{x}, t)$

$g_{\mu\nu}(t) \rightarrow g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

(Background) **(Fluctuations)**

INF → **Primordial fluctuations !**

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\begin{matrix}\phi(t) \\ g_{\mu\nu}(t)\end{matrix}} \begin{matrix}\longrightarrow \\ \longrightarrow\end{matrix} \boxed{\begin{matrix}\phi(t)+\delta\phi(\vec{x},t) \\ g_{\mu\nu}(t)+\delta g_{\mu\nu}(\vec{x},t)\end{matrix}}$$

(Background) **(Fluctuations)**

but WHY fluctuations ?

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$\phi(t)$	$\longrightarrow$	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$	$\longrightarrow$	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$
(Background)		**(Fluctuations)**

but WHY fluctuations ?

because of...

Quamtum Mechanics !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

(Background)		(Fluctuations)
$\phi(t)$	$\rightarrow$	$\phi(t) + \delta\phi(\vec{x}, t)$
$g_{\mu\nu}(t)$	$\rightarrow$	$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x}, t)$

but WHY fluctuations ?

because of...

Quamtum Mechanics !

VeV

Vacuum Quam. Fluct.

QM:

$$\hat{\phi}(\vec{x}, t) \;\rightarrow\; \langle\hat{\phi}(\vec{x}, t)\rangle = \phi(t) \quad \Rightarrow \quad \hat{\phi}(\vec{x}, t) = \phi(t) + \hat{\delta\phi}(\vec{x}, t)$$

$$\langle\hat{\delta\phi}(\vec{x}, t)\rangle = 0$$

but...

$$\left\langle\, [\hat{\delta\phi}(\vec{x}, t)]^2 \,\right\rangle \neq 0$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t) \quad \rightarrow \quad \langle \hat{\delta\phi}^2(\vec{x},t) \rangle \neq 0}$$

but ... ~~Minkowski~~ → Curved Space: (quasi)dS

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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but … ~~Minkowski~~ → Curved Space: (quasi)dS

$$S = \frac{m_p^2}{2} \int d^4x \sqrt{-g} \{ R - (\partial\phi)^2 - 2V(\phi) \} \begin{cases} \phi(t) + \delta\phi(\vec{x},t) \\ g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t) \end{cases}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

$$\phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$ds^2 \quad = \quad g^{\text{tot}}_{\mu\nu}dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t))dx^\mu dx^\nu$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$\boxed{\hat{\phi}(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t) \quad \to \quad \langle \hat{\delta\phi}^2(\vec{x},t)\rangle \neq 0}$$

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$$S = \frac{m_p^2}{2}\int d^4x\sqrt{-g}\{R - (\partial\phi)^2 - 2V(\phi)\}$$

$$\phi(t) + \delta\phi(\vec{x},t)$$

$$g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t)$$

$$\begin{aligned} ds^2 &= g^{\rm tot}_{\mu\nu}dx^\mu dx^\nu = (g_{\mu\nu}(t) + \delta g_{\mu\nu}(\vec{x},t))dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \end{aligned}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

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Expanding U. $\longrightarrow$ Vector Perturbations $S_i, F_i \propto \dfrac{1}{a}$

Inflation & Primordial Perturbations

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$$\partial_i h_{ij} = h_{ii} = 0$$

(tensors = GWs)

Inflation & Primordial Perturbations

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$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

$$\text{dof} \xrightarrow{\text{scalars}} \delta\phi + \{\Phi, B, \Psi, E\}$$

$$\text{dof} \xrightarrow{\text{tensors}} h_{ij}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\} \xrightarrow{\text{Diff.}} \delta\phi' + \{\Phi', B', \Psi', E'\}$ ~~Gauge Inv.~~

tensors: $h_{ij} \xrightarrow{\text{Diff.}} h_{ij}$ Gauge Inv. !

Diff.: $\boxed{x^\mu \rightarrow x^\mu + \xi^\mu}$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

$$ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j$$

$$\phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t)$$

dof

scalars: $\delta\phi + \{\Phi, B, \Psi, E\}$

tensors: h_{ij}

$$\zeta \equiv -[\Psi + (H/\dot{\rho})\delta\rho_\phi] \xrightarrow{\text{Diff.}} \zeta$$

$$\mathcal{R} \equiv [\Psi + (H/\dot{\phi})\delta\phi] \xrightarrow{\text{Diff.}} \mathcal{R}$$

$$Q \equiv [\delta\phi + (\dot{\phi}/H)\Psi] \xrightarrow{\text{Diff.}} Q$$

All Gauge Inv. !

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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All Gauge Inv. !

Curvature Pert.

Tensor Pert. (GW)

Fixing Gauge: e.g. $E, \delta\phi = 0 \Rightarrow g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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Inflation & Primordial Perturbations

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Inflation & Primordial Perturbations

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$$\Rightarrow \quad \boxed{S = S_{(0)} + S^{(s)}_{(2)} + S^{(t)}_{(2)}}$$

$$S^{(s)}_{(2)} = \frac{1}{2}\int d^4x\; a^3\frac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]$$

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dtdx^3a^3\left[(\dot h_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

Background
Inflationary dynamics

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$\boxed{S^{(s)}_{(2)} = \frac{1}{2}\int d^4x \; a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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$d\tau \equiv dt/a(t)$

$=$

?

$\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]$ (Mukhanov variable)

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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(F.T.: $v(\mathbf{x},t) = \int d\mathbf{k}\, e^{-i\mathbf{k}\mathbf{x}}\, v_{\mathbf{k}}(t)$ **)**

$$\Rightarrow \boxed{v''_{\vec{k}} + (k^2 - z''/z)v_{\vec{k}} = 0}$$

with $$\boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\,, \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Inflation & Primordial Perturbations

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Inflation & Primordial Perturbations

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$$\Rightarrow \boxed{v''_{\vec k} + (k^2 - z''/z)v_{\vec k} = 0} \quad \text{with} \quad \boxed{\frac{z''}{z} = \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right) \;, \quad \nu \equiv \frac{3}{2} + 2\epsilon - \eta}$$

Quantization: $\boxed{v_{\vec k}(t) \to v_k(t)\hat a_{\vec k} + v^*_k(t)\hat a^\dagger_{-\vec k} \;, \quad [a_{\vec k}, a^\dagger_{\vec k'}] = (2\pi)^3\delta(\vec k - \vec k')}$ $\Rightarrow$

Inflation & Primordial Perturbations

$$\boxed{\text{Inflation: A generator of Primordial Fluctuations}}$$

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Quantization: $\boxed{v_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}} \;\; , \quad [a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta(\vec{k} - \vec{k}')}$ $\rightarrow$

$$\rightarrow \boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

(Bunch-Davies)
Vacuum Fluct.

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$\boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2} (-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

$$\boxed{\hat{v}_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}}}$$

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(Bunch-Davies) Vacuum Fluct.

$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]}$$

$$\Rightarrow$$

$$\boxed{\left\langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \right\rangle \equiv \frac{1}{z^2} \left\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2} \left| v_k(\eta) \right|^2} \delta(\vec{k} + \vec{k}')}$$

Inflation & Primordial Perturbations

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$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]}$$

$$\left\langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \right\rangle \equiv \frac{1}{z^2} \left\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2} \left|v_k(\eta)\right|^2} \delta(\vec{k} + \vec{k}')$$

$$\boxed{\equiv P_{\mathcal{R}}(k, \eta)}$$

Scalar Power Spectrum

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluct:

$$\boxed{v_k(\tau) = \frac{\sqrt{\pi}}{2} e^{i\pi(\nu+1/2)/2}(-\tau)^{1/2} H_\nu^{(1)}(-k\tau)}$$

$$\boxed{\hat{v}_{\vec{k}}(t) \to v_k(t)\hat{a}_{\vec{k}} + v_k^*(t)\hat{a}^\dagger_{-\vec{k}}}$$

(Bunch-Davies) Vacuum Fluct.

$$\boxed{\left[v \equiv z\mathcal{R}\,, \quad z \equiv a\frac{\dot{\phi}}{H}\right]}$$

$$\boxed{\left\langle \hat{\mathcal{R}}_{\vec{k}} \hat{\mathcal{R}}_{\vec{k}'} \right\rangle \equiv \frac{1}{z^2}\left\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \boxed{\frac{H^2}{a^2\dot{\phi}^2}|v_k(\eta)|^2} \delta(\vec{k}+\vec{k}')}$$

$$\boxed{\equiv P_{\mathcal{R}}(k,\eta)}$$

Scalar Power Spectrum

$$\boxed{\Delta^2_{\mathcal{R}}(k,\tau) \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k,\tau)}$$

Dimensionless Scalar PS

$$\xrightarrow[\text{(Super-Horizon)}]{k\tau \ll 1}$$

$$\boxed{\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{2\eta-4\epsilon}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]}$$

Inflation & Primordial Perturbations

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$d\tau \equiv dt/a(t)$

Tensor Fluctuations:

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]} \quad = \quad \boxed{?}$$

$$h_{ij}(\vec{k},\tau) = \epsilon^{(s)}_{ij} h^{(s)}_{\vec{k}} \longrightarrow v^{(s)} \equiv \frac{a}{2} m_p h^{(s)}_{\vec{k}}$$

Inflation & Primordial Perturbations

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Tensor Fluctuations:

$$d\tau \equiv dt/a(t)$$

$$S^{(t)}_{(2)} = \frac{m_p^2}{8} \int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2} (\partial_l h_{ij})^2 \right] = \sum_s \frac{1}{2} \int \mathrm{d}\tau \mathrm{d}^3\mathbf{k} \left[(v_{\mathbf{k}}^{s\,\prime})^2 - \left(k^2 - \frac{a''}{a} \right) (v_{\mathbf{k}}^s)^2 \right]$$

$$h_{ij}(\vec{k}, \tau) = \epsilon^{(s)}_{ij} h^{(s)}_{\vec{k}} \longrightarrow v^{(s)} \equiv \frac{a}{2} m_p h^{(s)}_{\vec{k}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Tensor Fluctuations:

$$\boxed{S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3 \left[(\dot{h}_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]} = \boxed{\sum_s \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{k}\left[(v^{s\,\prime}_{\mathbf{k}})^2 - \left(k^2 - \frac{a''}{a}\right)(v^s_{\mathbf{k}})^2\right]}$$

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$\Rightarrow$ [Same Procedure as with Scalar Pert.

Quantize → Bunch-Davies → Power Spectrum]

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Quantize $\rightarrow$ Bunch-Davies $\rightarrow$ Power Spectrum]

Quantization of Gravity dof !

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Quantize → Bunch-Davies → Power Spectrum]

Quantization of Gravity dof !

$$\boxed{\Delta_h^2(k,\tau) \equiv \frac{k^3}{2\pi^2}P_h(k,\tau)} \xrightarrow[\text{(Super-Horizon)}]{k\tau \ll 1} \boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{-2\epsilon}}$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

Scalar Fluctuations:

$$S^{(s)}_{(2)} = \tfrac{1}{2}\int d^4x\; a^3 \tfrac{\dot\phi^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot\phi^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$$n_s - 1 \equiv 2(\eta - 2\epsilon)$$

Tensor Fluctuations:

$$S^{(t)}_{(2)} = \frac{m_p^2}{8}\int dt dx^3 a^3\left[(\dot h_{ij})^2 - a^{-2}(\partial_l h_{ij})^2\right]$$

$$\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

Inflation & Primordial Perturbations

Inflation: A generator of Primordial Fluctuations

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@ Super-Horizon Scales:

(i.e. Super-Hubble radius)

$$\mathcal{R}(k), h_{ij}(k) \approx \text{Const.}\,, \quad k\tau \ll 1$$

Exercise: demonstrate that GWs are frozen at Super-Horizon scales

Inflation & Primordial Perturbations

H & I

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2 \dot{\phi}^2} \left(\frac{k}{aH} \right)^{n_s - 1}$$

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Inflation & Primordial Perturbations

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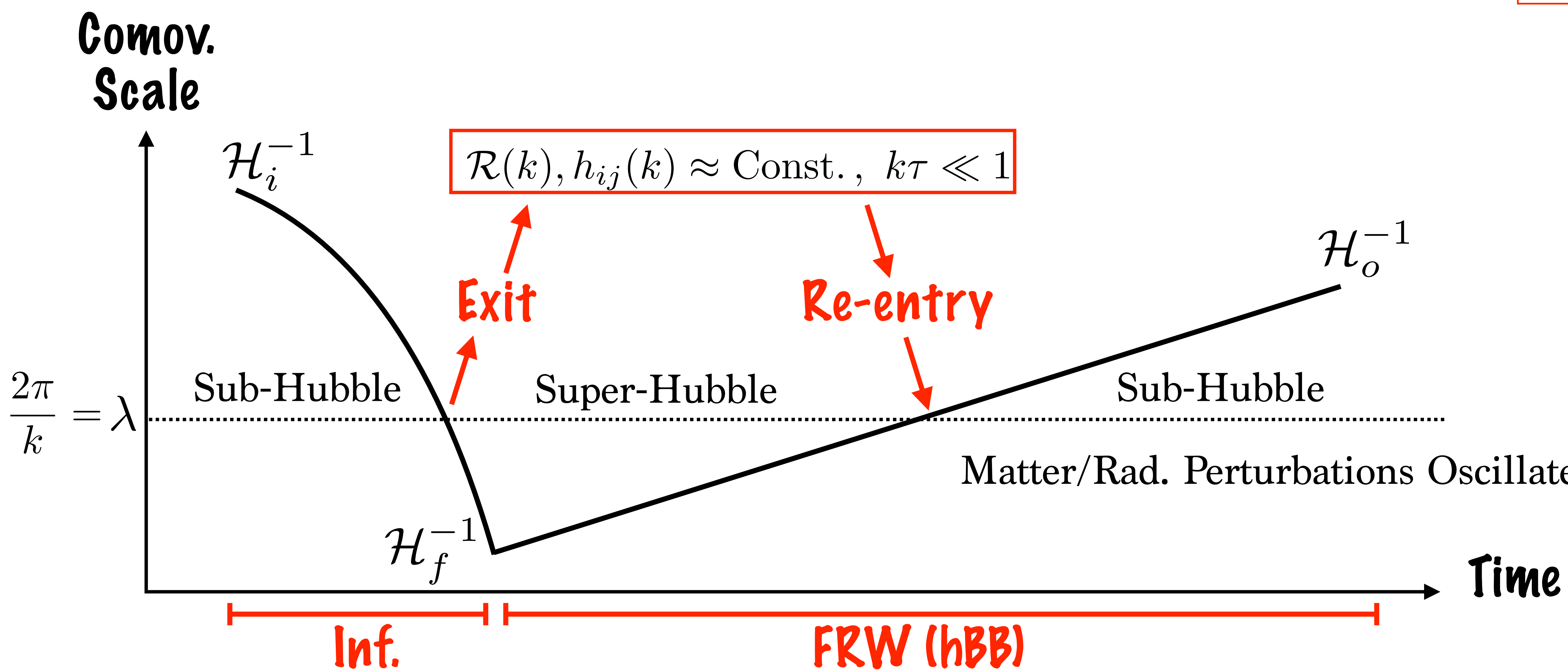
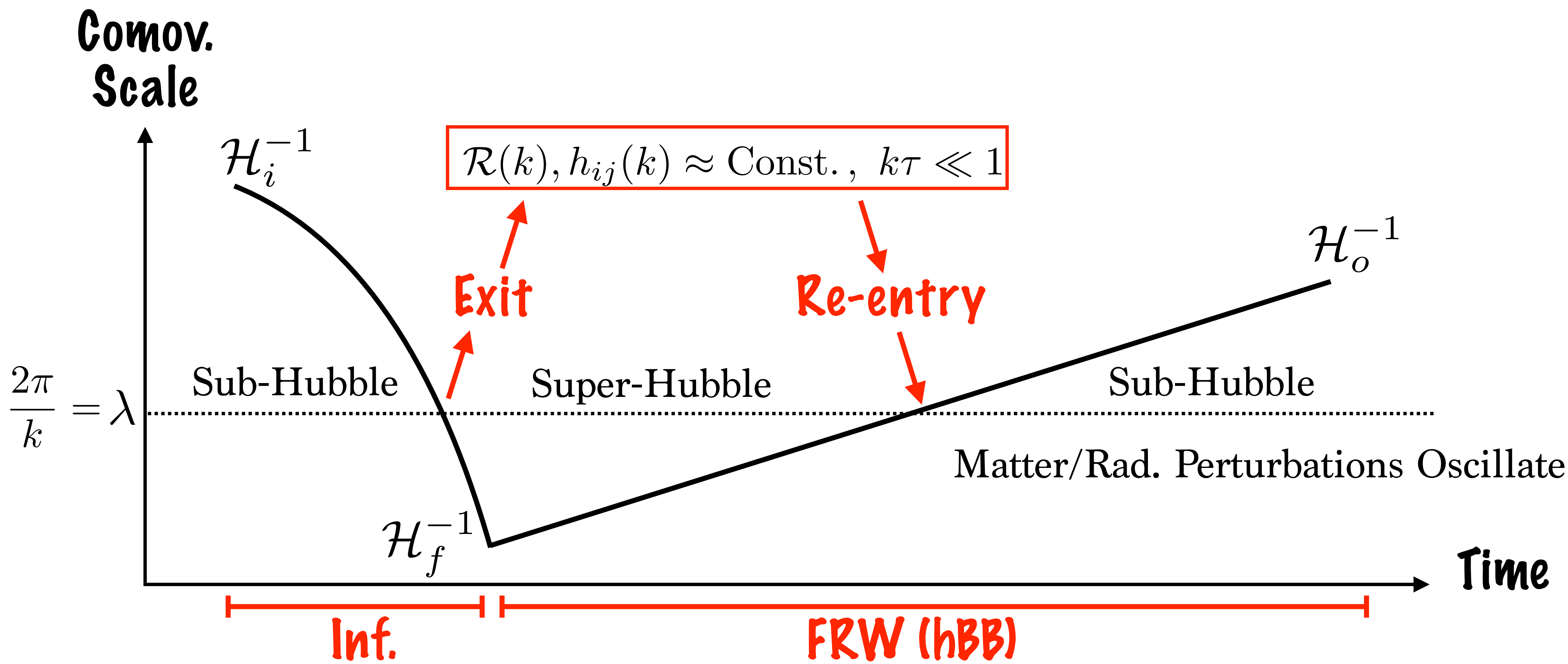


Inflation & Primordial Perturbations

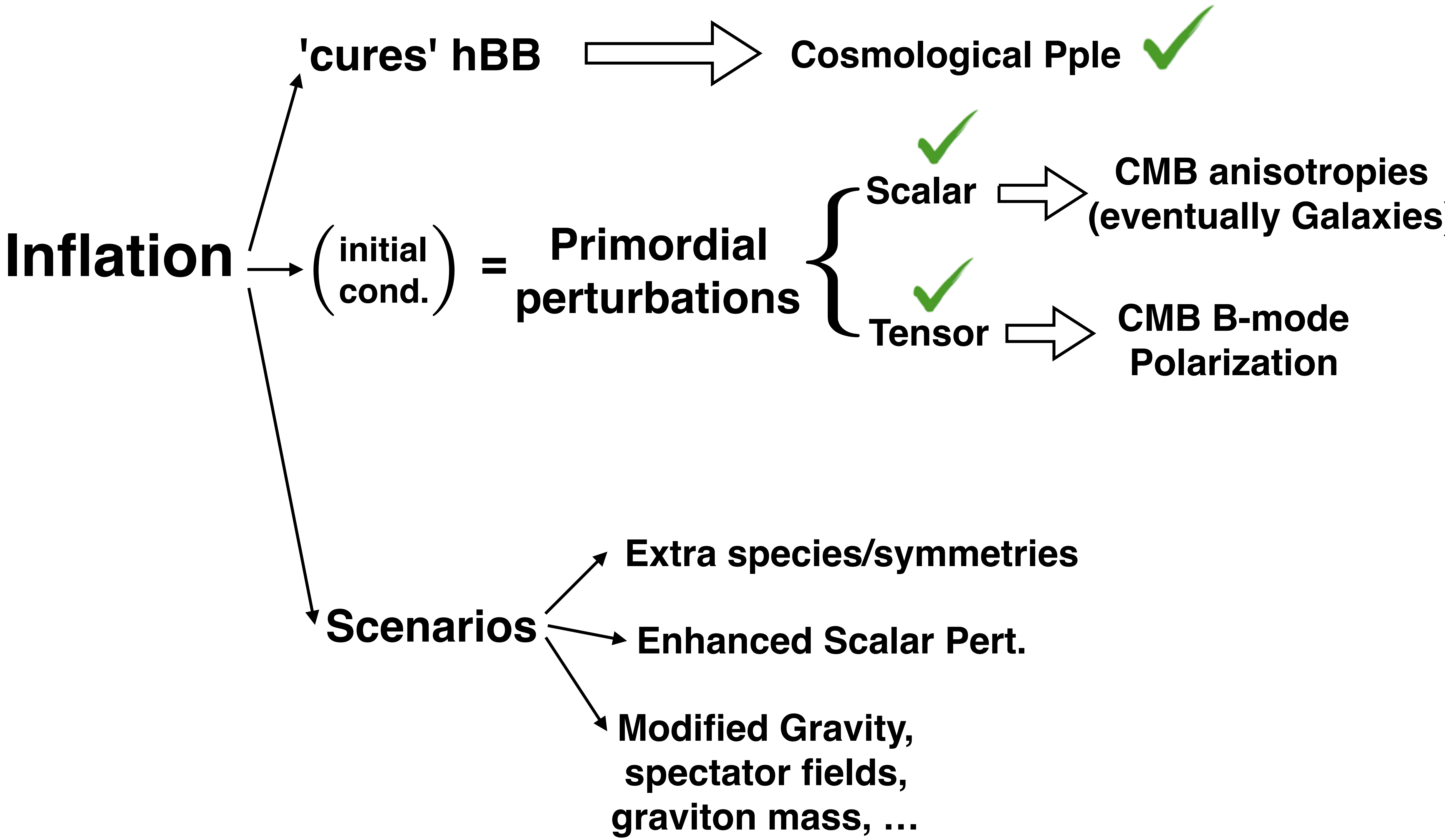
Inflation → **Reheating** → **FLRW (hBB)**

$a \propto e^{Ht}$ | $\rho_\phi \to \rho(\psi, A_\mu, \ldots)$ | $\rho_\gamma, \rho_b, \rho_{DM}, \ldots$

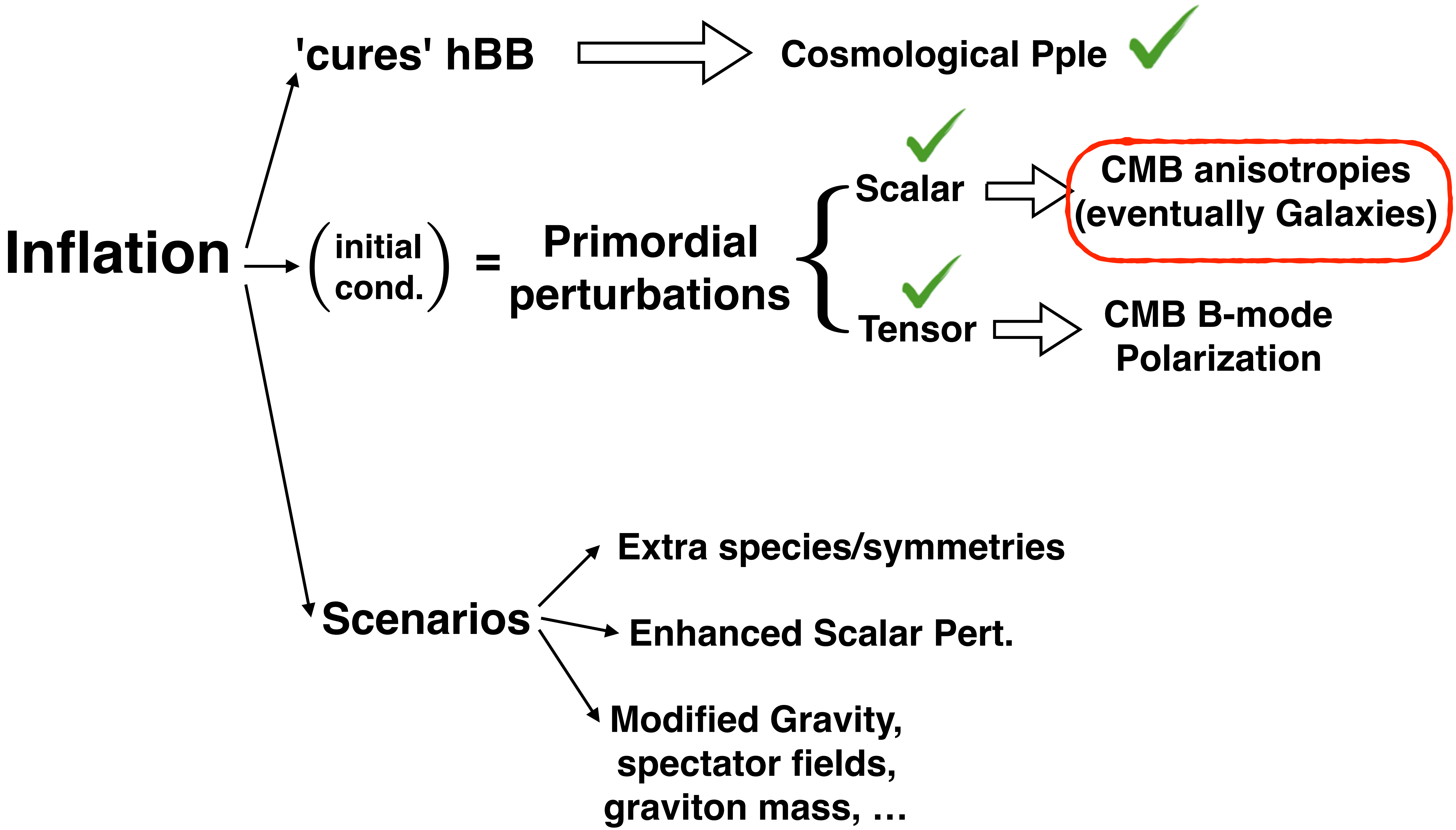
$$g_{\mu\nu} = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}] \xrightarrow{\text{GR}} \rho_i + \delta\rho_i \xrightarrow{\text{GR, Int.}} \delta\rho_i(t)\ \text{[Evolution]}$$



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



Inflation: Observables

Inflation $\longrightarrow$ **Reheating** $\longrightarrow$ **FLRW (hBB)**

$\boxed{a \propto e^{Ht}}$ $\qquad$ $\boxed{\rho_\phi \to \rho(\psi, A_\mu, \ldots)}$ $\qquad$ $\boxed{\rho_\gamma, \rho_b, \rho_{DM}, \ldots}$

$\downarrow$

$$\boxed{g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]} \xrightarrow[\text{GR}]{} \boxed{\rho_i + \delta\rho_i} \xrightarrow[\text{GR , Int.}]{} \boxed{\delta\rho_i(t)\ [\text{Evolution}]}$$

Inflation: Observables

Inflation → Reheating → FLRW (hBB)

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(Pressure waves
= Acoustic Peaks)

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(Pressure waves
= Acoustic Peaks)

↕

characteristic
scale(s) !

$$\frac{L_*(\Omega_M, \Omega_R, \Omega_k, \ldots)}{d_A(z_{\text{dec}})} \equiv \Delta\theta_*$$

Inflation: Observables

Inflation → Reheating → FLRW (hBB)

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(Pressure waves
= Acoustic Peaks)

$\updownarrow$

$$\rho_\gamma + \delta_\gamma \;\Rightarrow\; T_\gamma(\hat{n}) = \bar{T} + \delta T(\hat{n})$$

$\Leftarrow$

$$\left\{ \begin{array}{l} \text{characteristic scale(s) !} \\ \dfrac{L_*(\Omega_M, \Omega_R, \Omega_k, \ldots)}{d_A(z_{\text{dec}})} \equiv \Delta\theta_* \end{array} \right.$$

Inflation: Observables

Inflation → Reheating → FLRW (hBB)

$a \propto e^{Ht}$

$\rho_\phi \to \rho(\psi, A_\mu, \ldots)$

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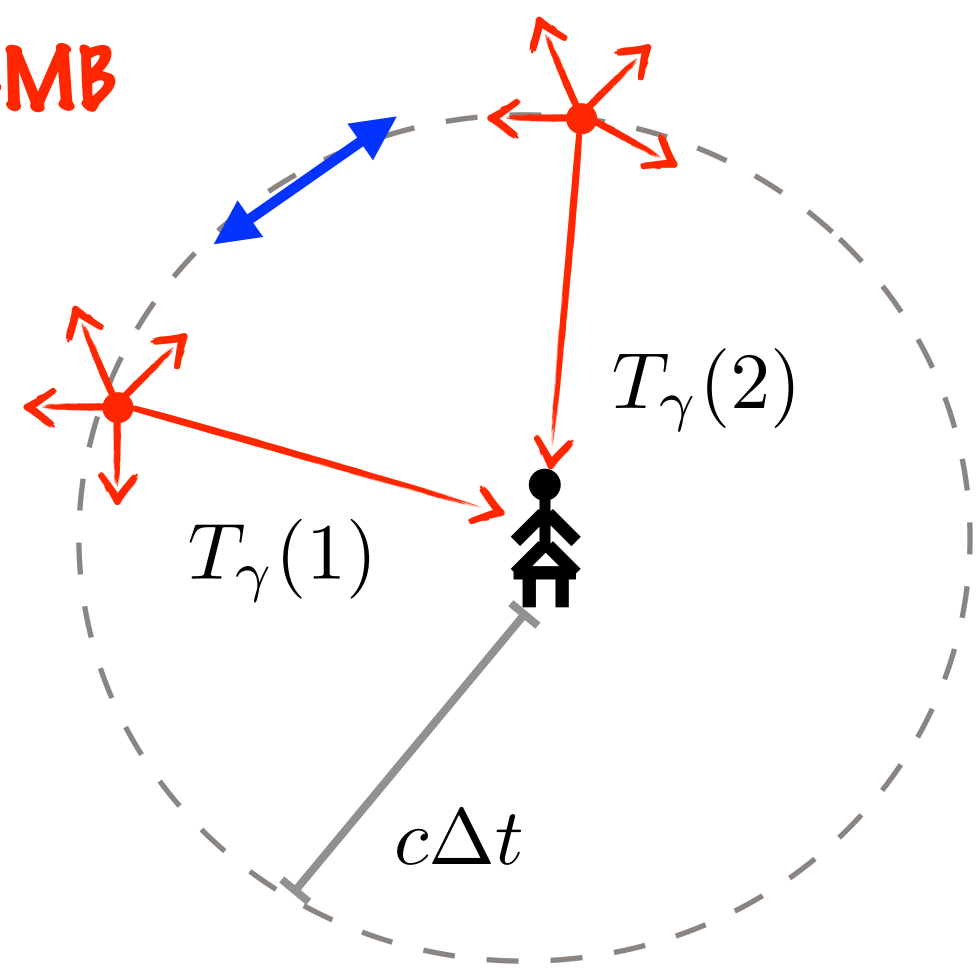
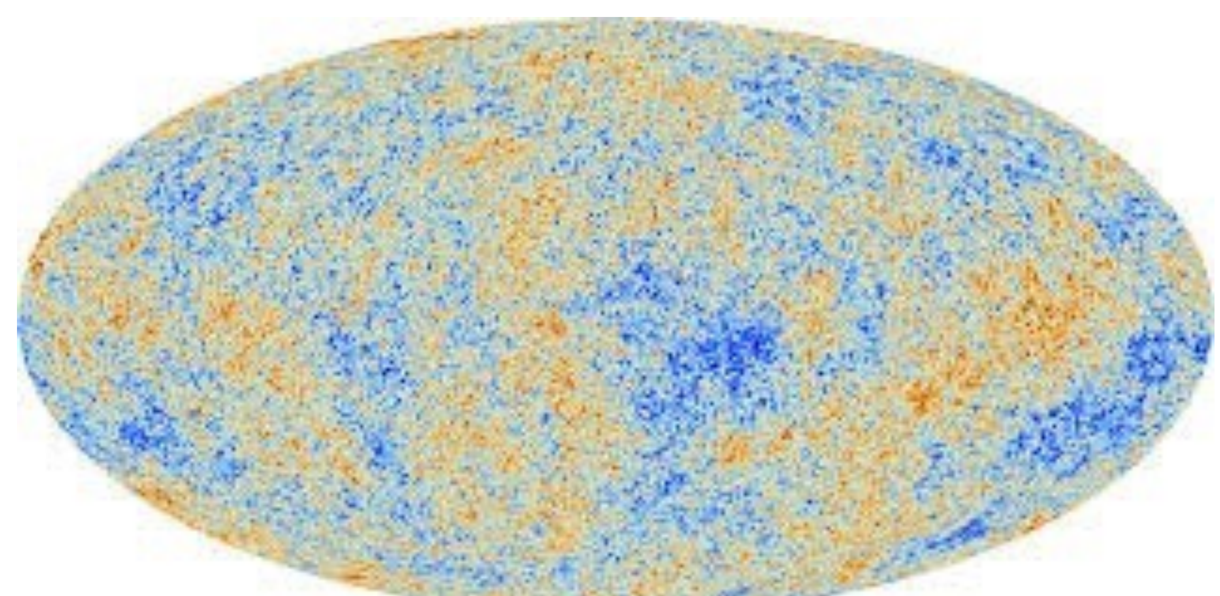
(Pressure waves
= Acoustic Peaks)

CMB



$$\rho_\gamma + \delta_\gamma \;\Rightarrow\; T_\gamma(\hat{n}) = \bar{T} + \delta T(\hat{n})$$



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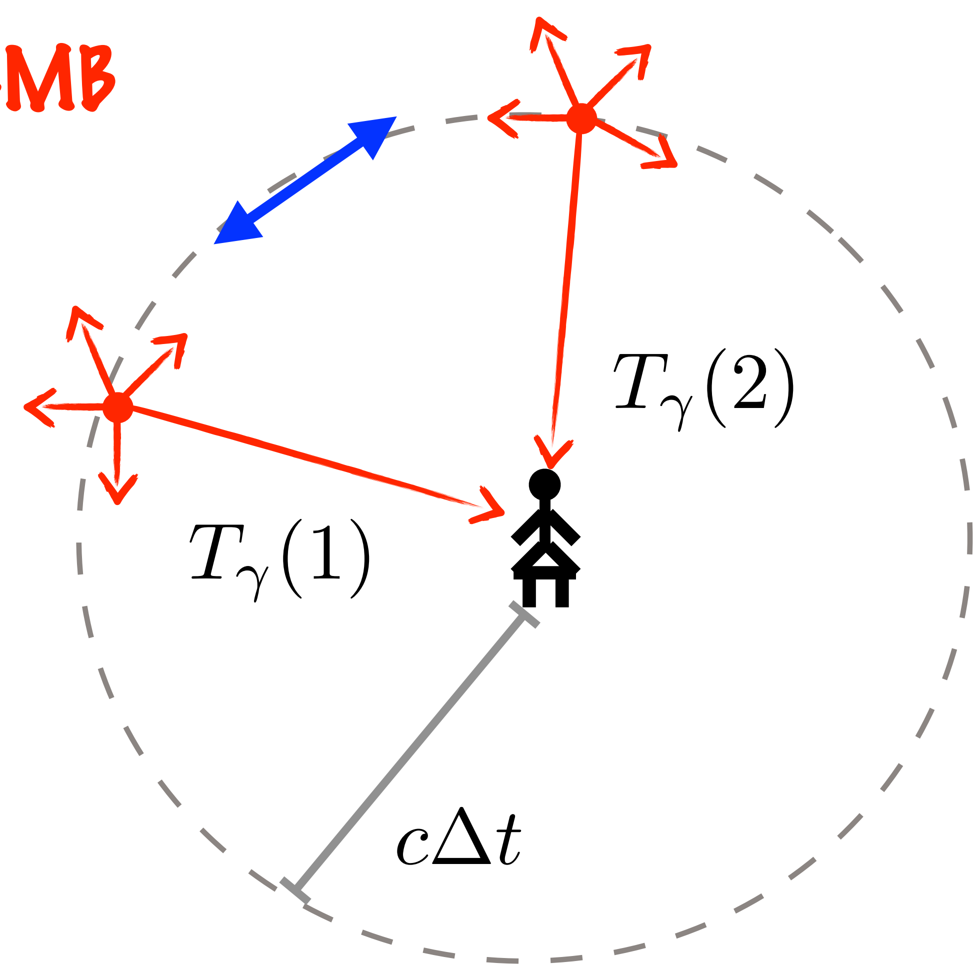

$$\rho_\gamma + \delta_\gamma \Rightarrow T_\gamma(\hat{n}) = \bar{T} + \delta T(\hat{n})$$

Temperature Angular
Power Spectrum

$$\delta T(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n}) \Rightarrow \langle [\delta T]^2 \rangle \to \boxed{\langle |a_{lm}|^2 \rangle \equiv C_l}$$

Inflation: Observables

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Temperature Angular Power Spectrum

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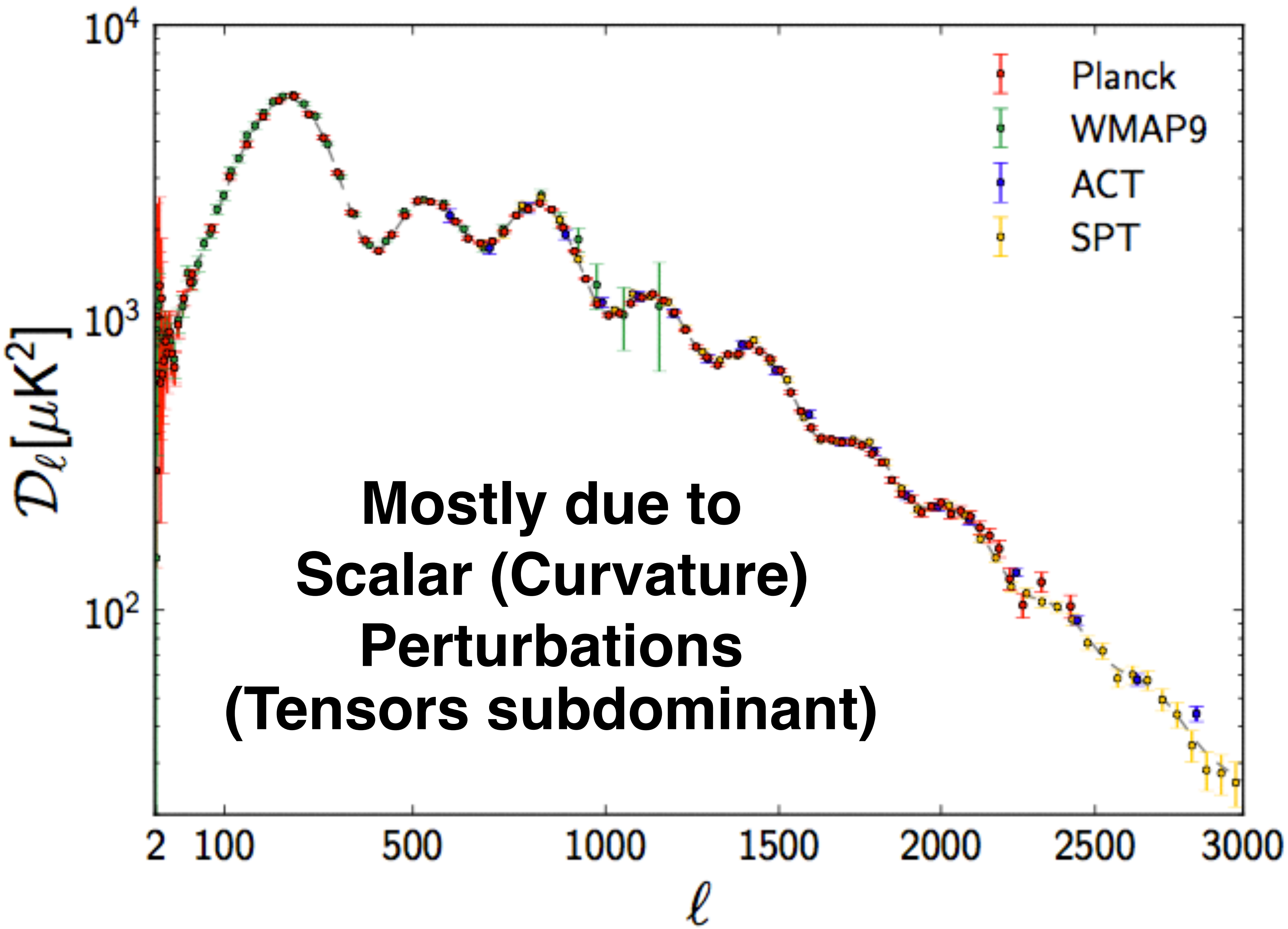
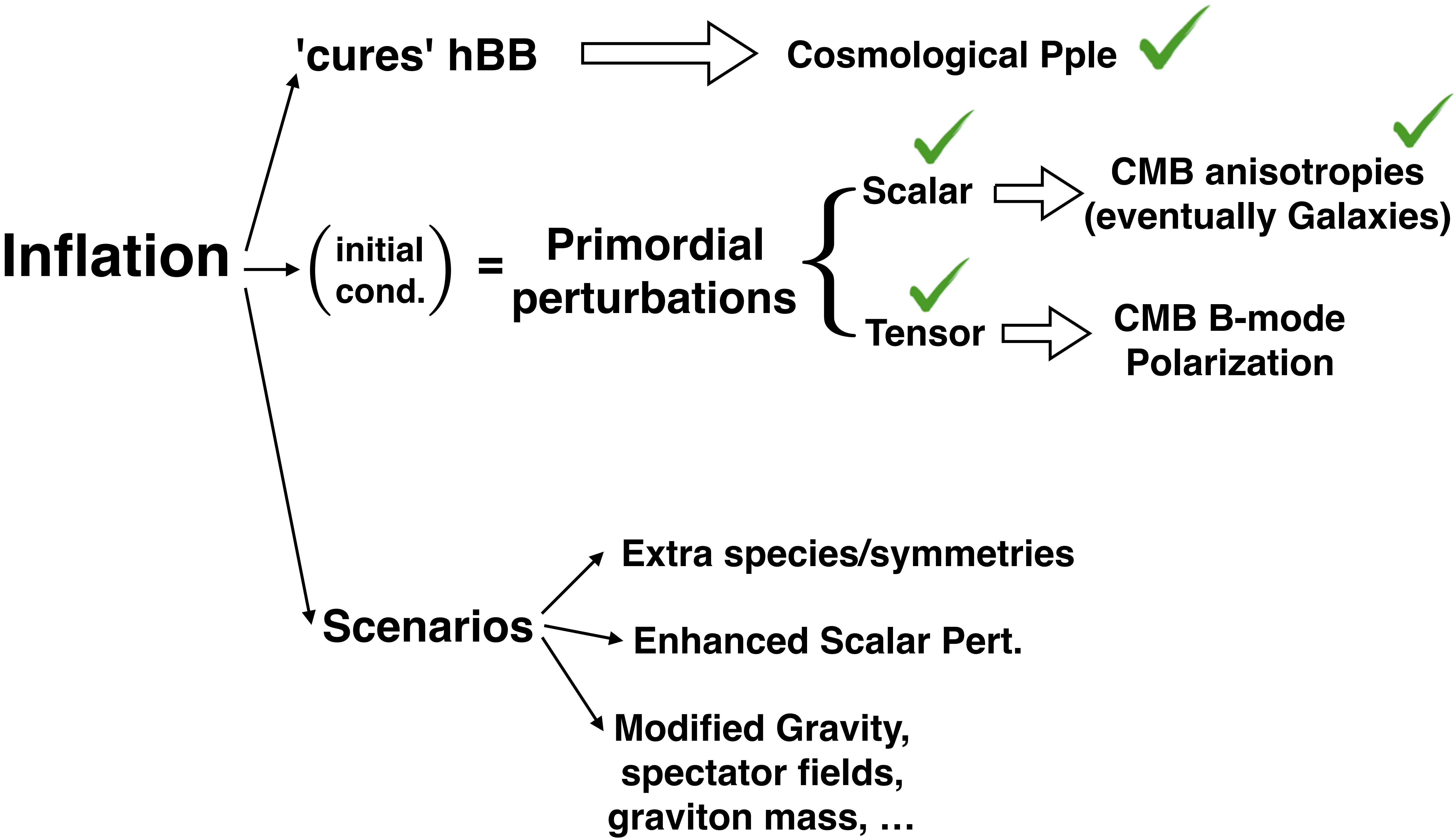
Temperature Angular Power Spectrum



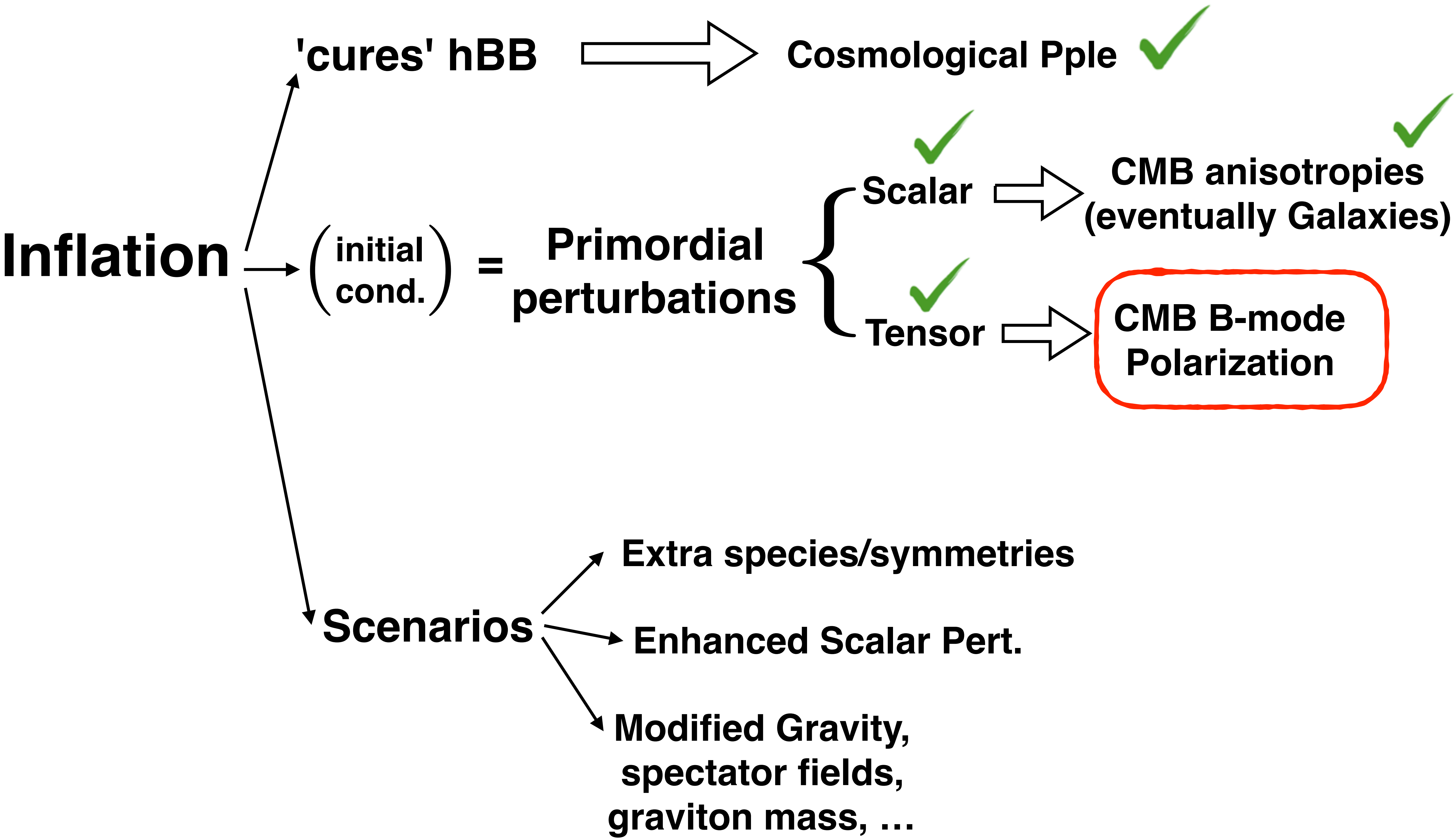
Dashed Line Theoretical Expectation from Inflation !

$$|\Omega_k| \ll 1$$
$$n_s \approx 0.96, \Delta^2_{\mathcal{R}} \simeq 2 \cdot 10^{-9}$$
Adiabatic, Gaussian

INFLATIONARY COSMOLOGY



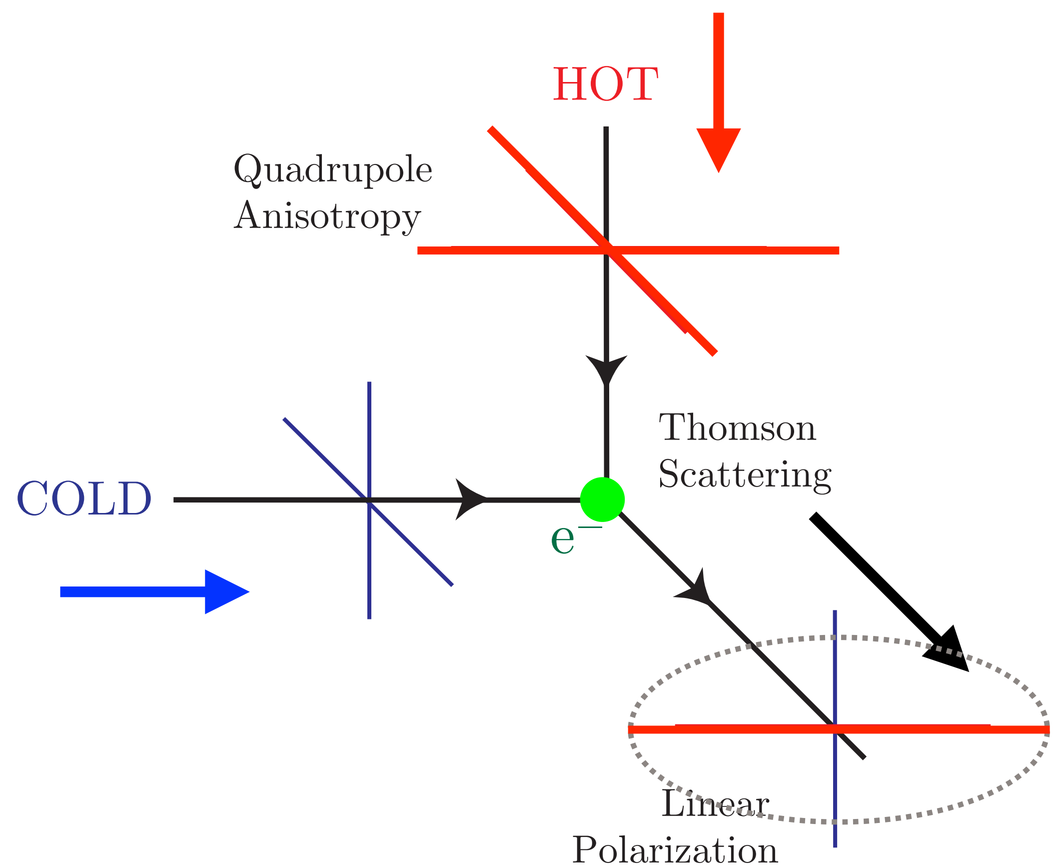
INFLATIONARY COSMOLOGY



Inflation: Observables

$$\boxed{\delta\rho_\gamma, \delta\rho_e \Rightarrow [\text{Thomson Scattering}] \Rightarrow \text{Linear Polarization}}$$

CMB must be polarized !



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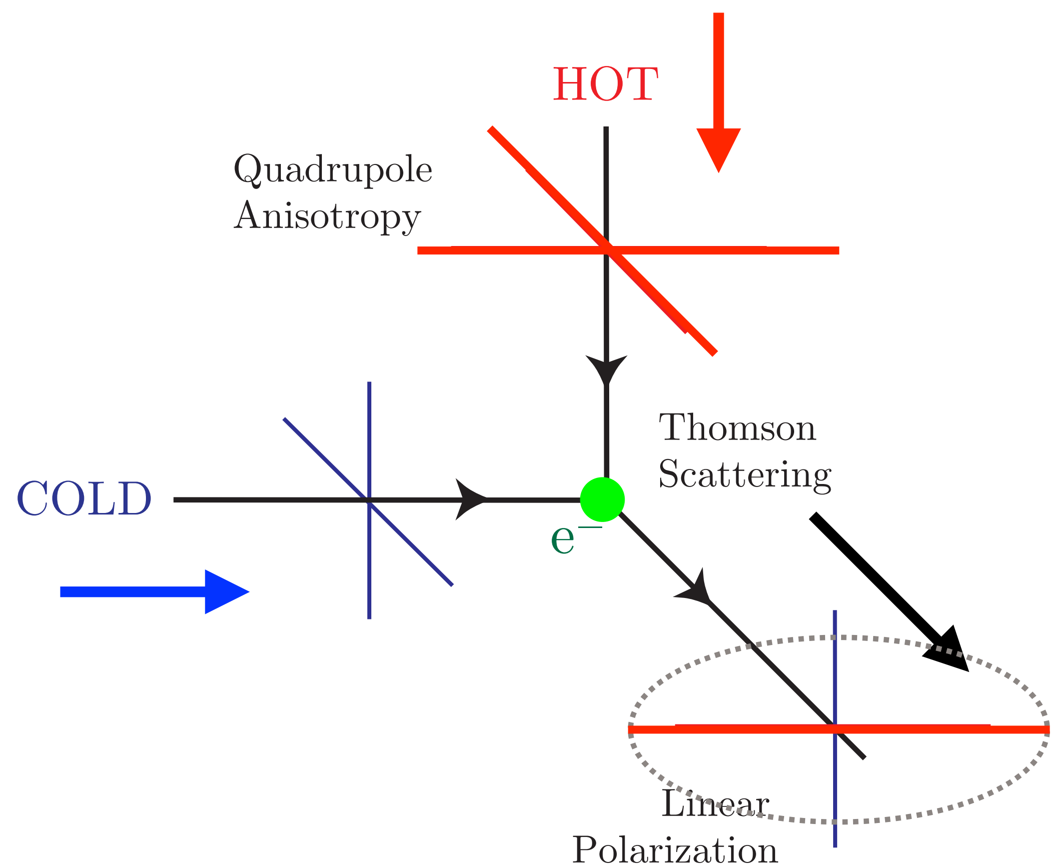

$$\boxed{\sigma_T = \frac{8\pi}{3}\frac{e^4}{(4\pi)^2 m^2}}$$

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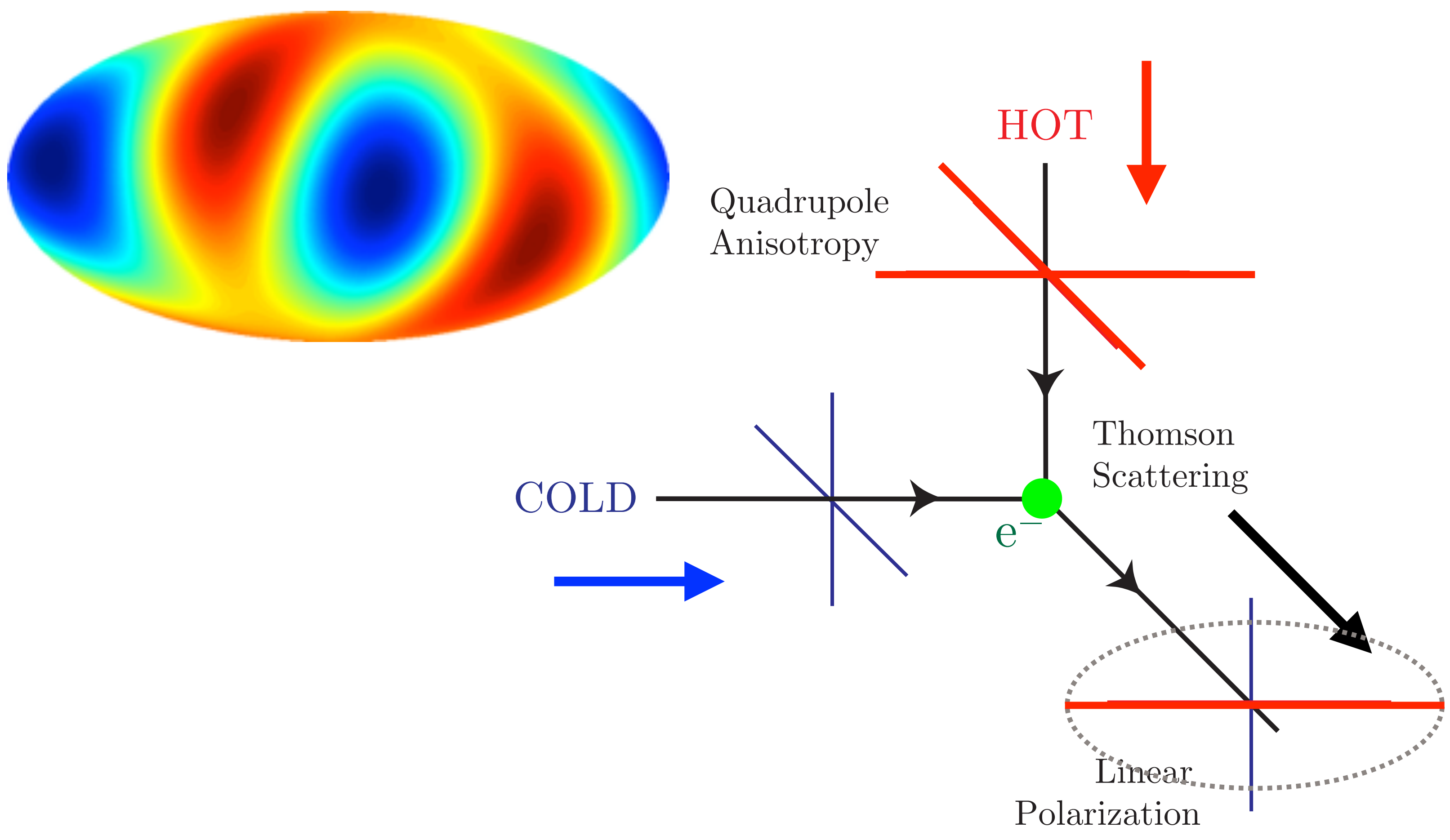

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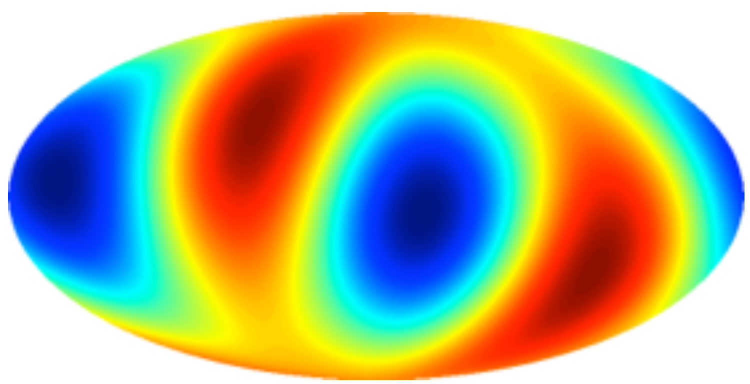

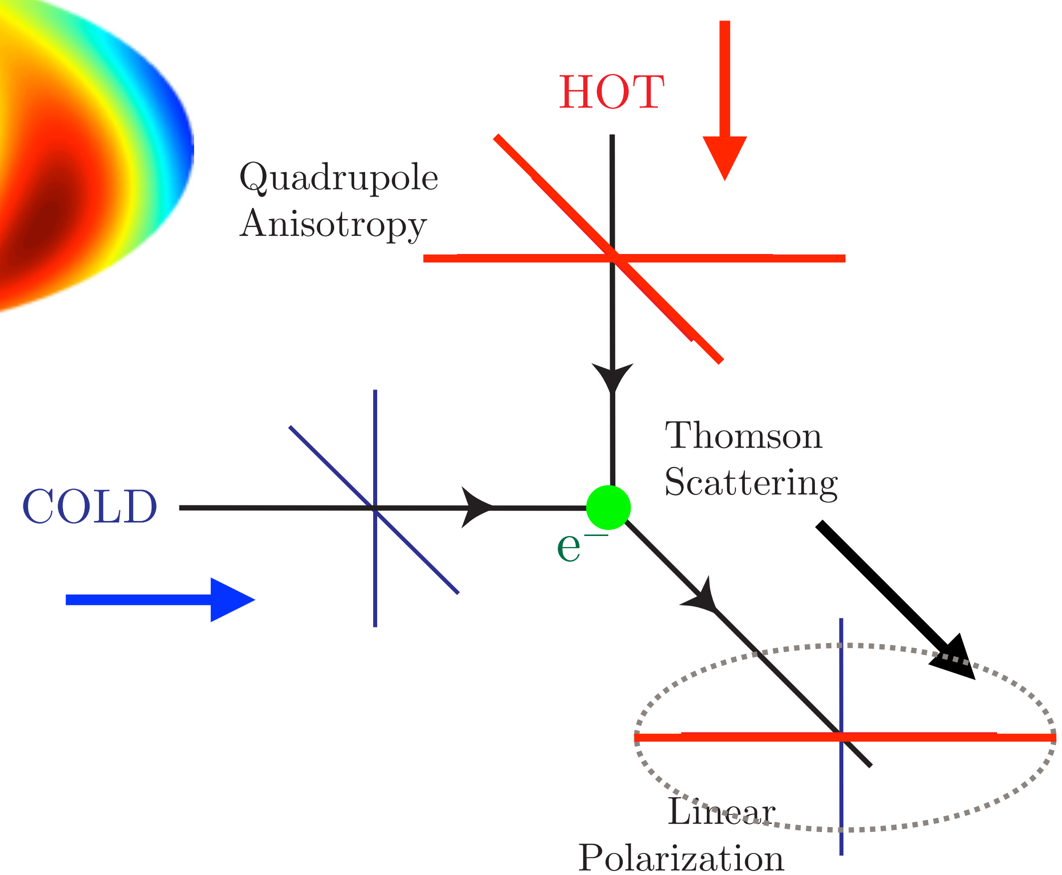

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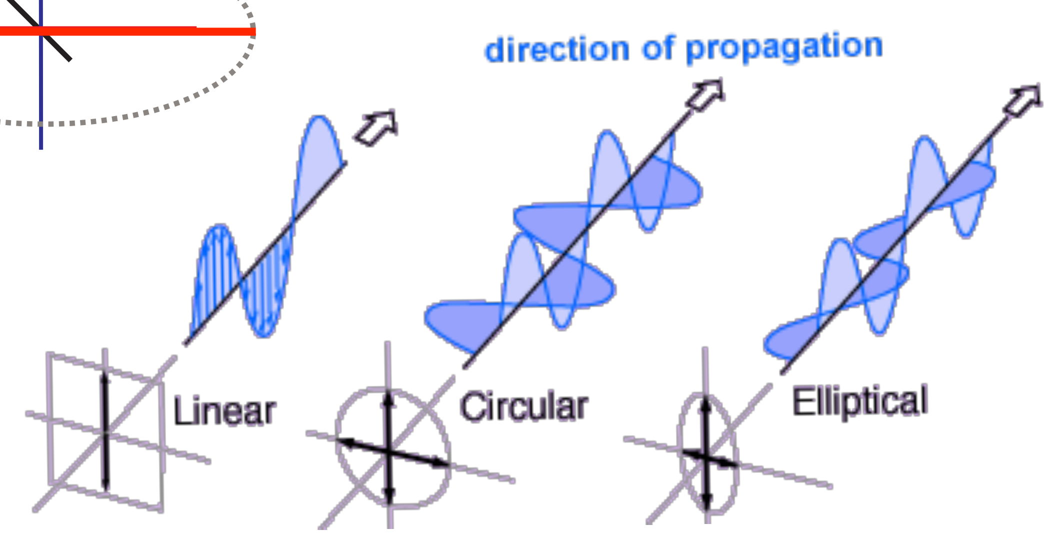

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$$\boxed{\text{Linear Polarization} \rightarrow Q, U \ \ (\text{Stokes Parameters})}$$

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$$(Q \pm iU)(\hat{n}) = \sum_{l,m} a_{lm}^{(\pm 2)} Y_{lm}^{(\pm 2)} = \sum_{l,m} (e_{lm} \pm i b_{lm}) Y_{lm}^{(\pm 2)}(\hat{n})$$

tensorial spherical harmonics

$$\mathcal{E}(\hat{n}) = \sum_{l,m} e_{lm} Y_{lm}(\hat{n}) \, , \ \ \mathcal{B}(\hat{n}) = \sum_{l,m} b_{lm} Y_{lm}(\hat{n})$$

E, B modes

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E, B modes

$$\boxed{\langle \mathcal{E}^2 \rangle \,, \;\; \langle \mathcal{B}^2 \rangle \;\; \rightarrow \;\; \langle |e_{lm}|^2 \rangle \equiv C_l^E, \;\;\; \langle |b_{lm}|^2 \rangle \equiv C_l^B}$$

E-mode, B-mode angular power spectra

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E-mode, B-mode angular power spectra

Depends on Scalar (also tensor) Perturbations

Depends only on Tensor Perturbations !

Inflation: Observables

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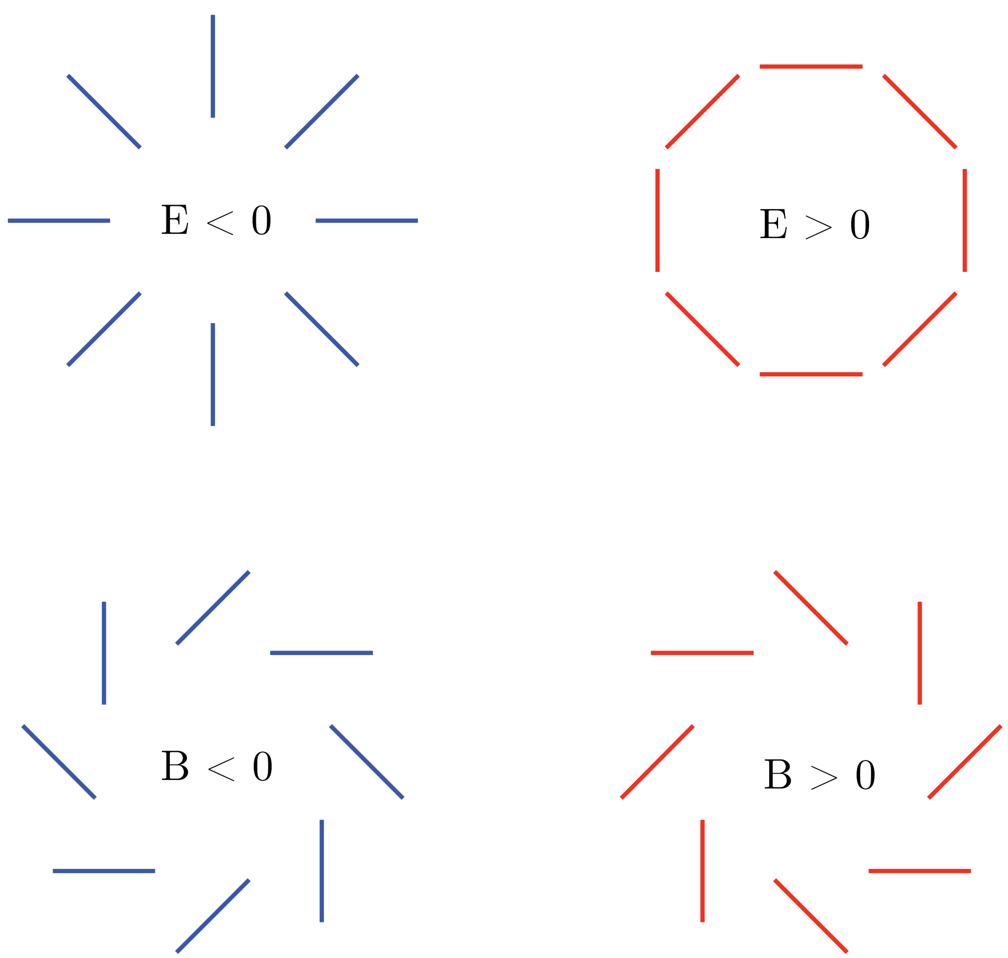
Polarization Angular Power Spectrum

Depends on Scalar (also tensor) Perturbations

Depends only on Tensor Perturbations !



Inflation: Observables

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Polarization Angular Power Spectrum

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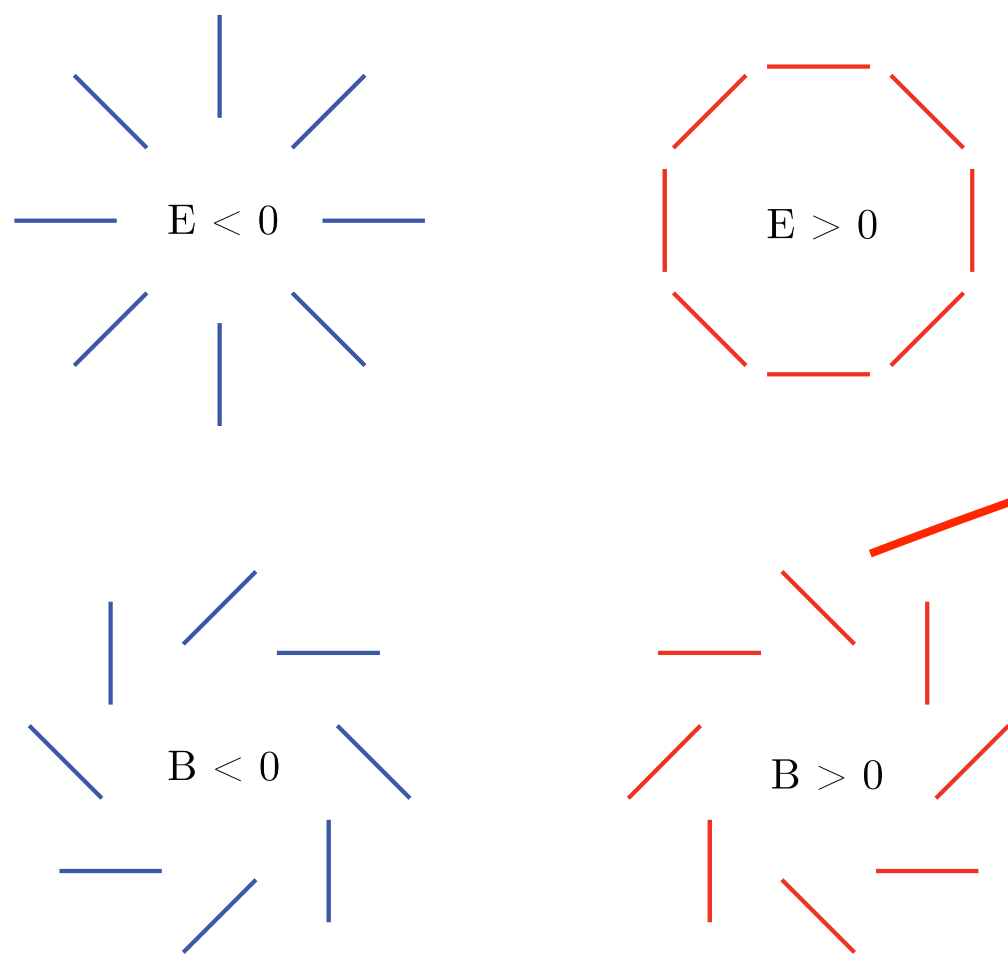

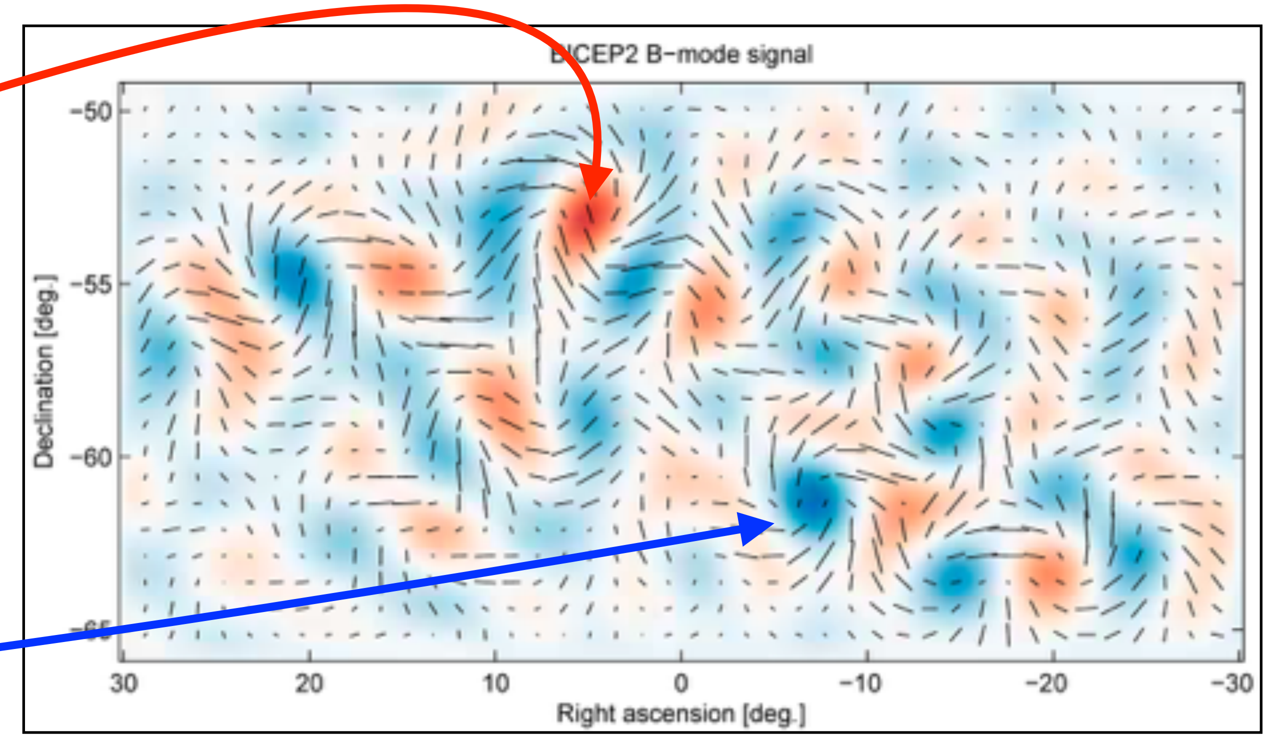

Inflation: Observables

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Polarization Angular
Power Spectrum

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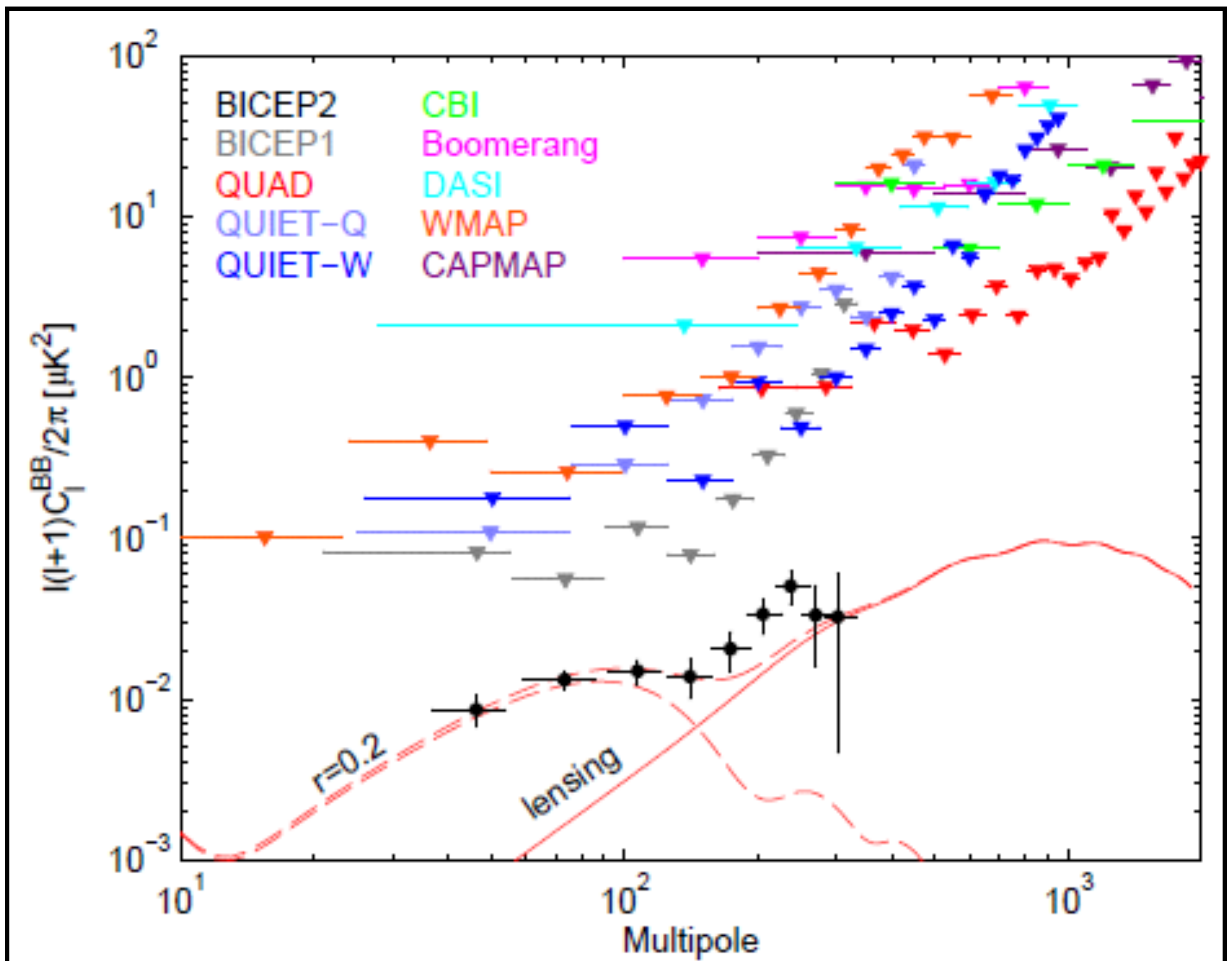

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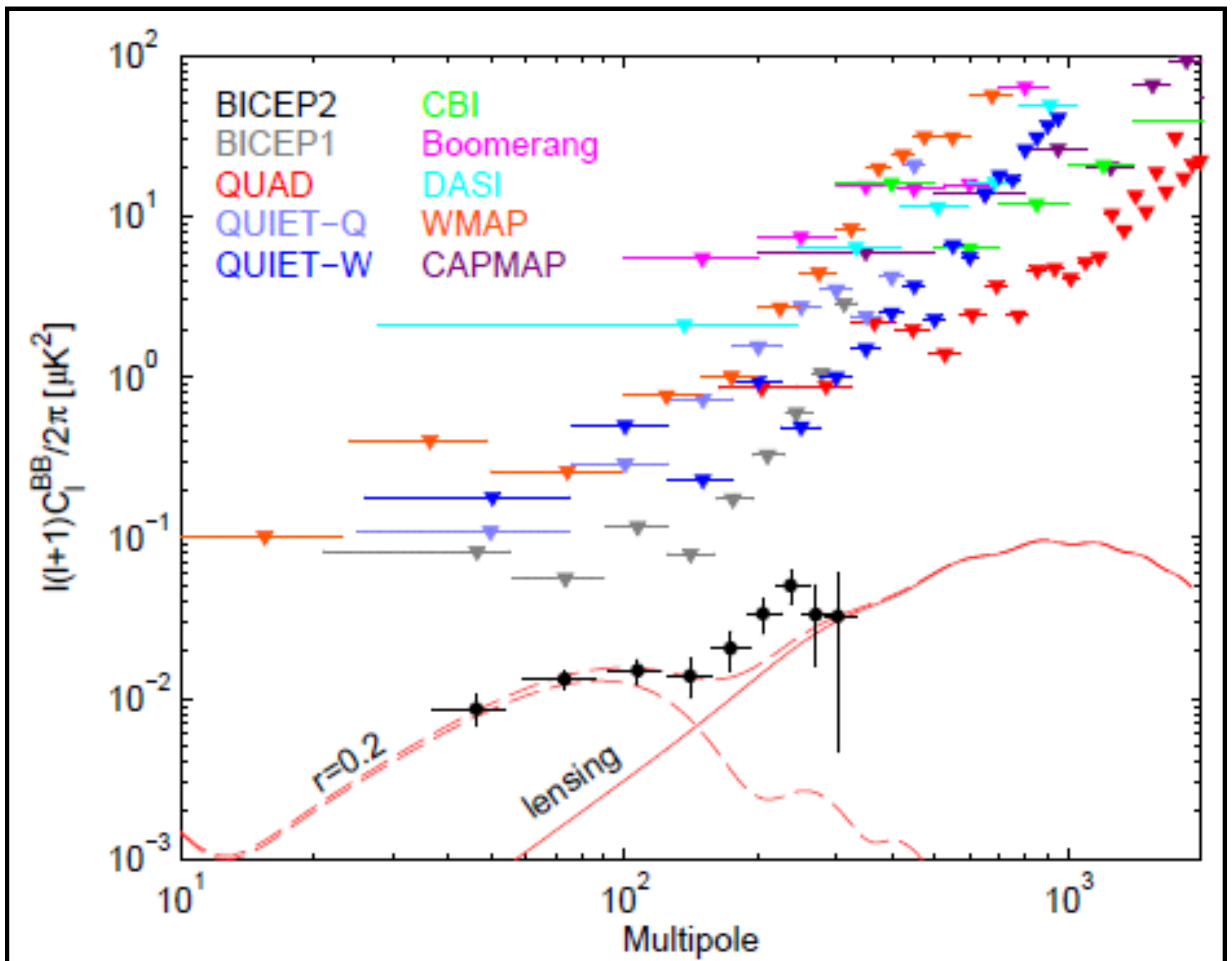

Dashed Line Theoretical Expectation from Inflation

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Polarization Angular Power Spectrum

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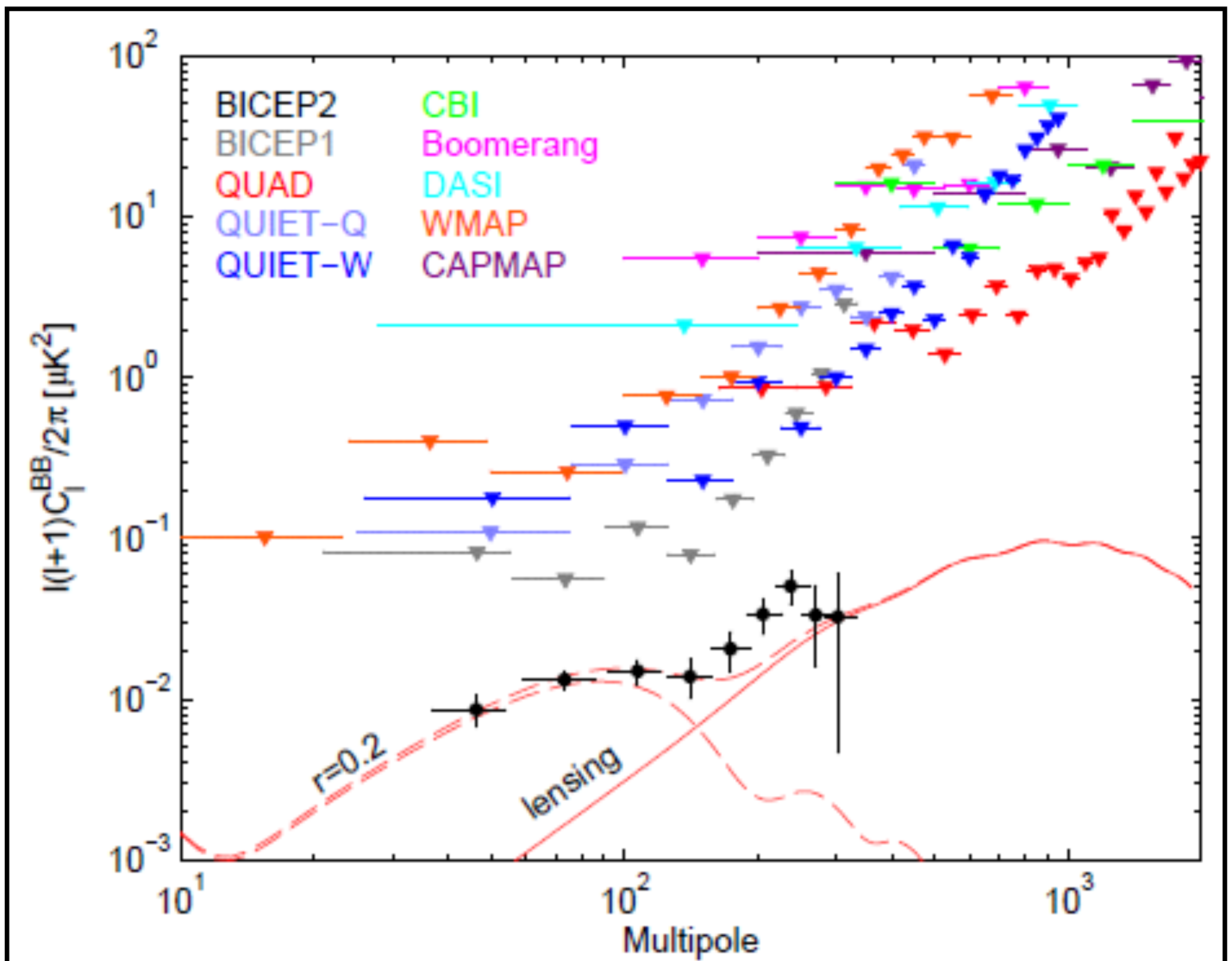

Dashed Line Theoretical Expectation from Inflation

BICEP-2 thought "we found B-modes due to Inflationary tensors"

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Polarization Angular Power Spectrum

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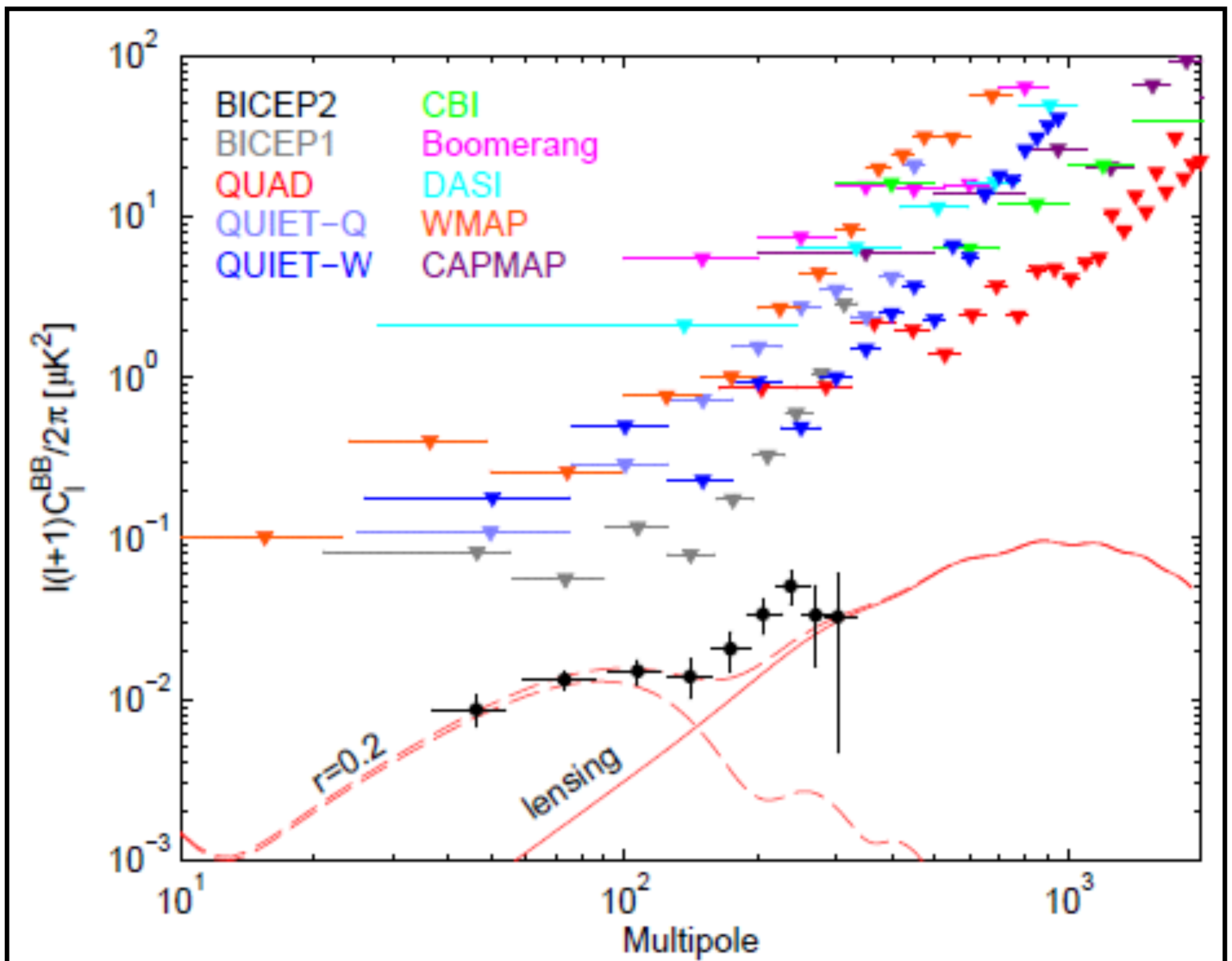


Dashed Line Theoretical Expectation from Inflation

but ...
signal (even real) was only due [at least mostly dominated by] ... dust contamination

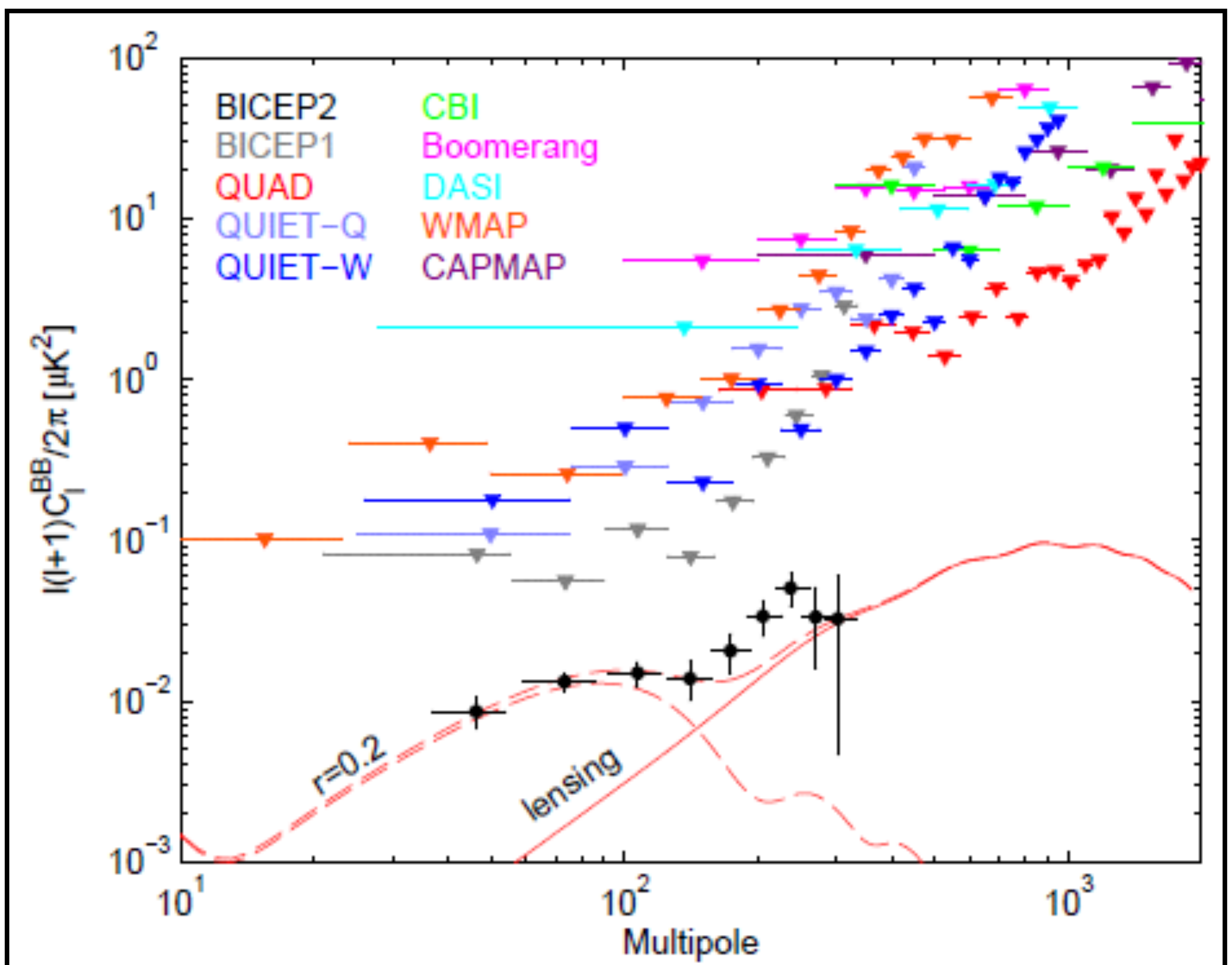
Inflation: Observables

$$\boxed{\langle \mathcal{E}^2 \rangle, \ \langle \mathcal{B}^2 \rangle \ \rightarrow \ \boxed{\langle |e_{lm}|^2 \rangle \equiv C_l^E, \quad \langle |b_{lm}|^2 \rangle \equiv C_l^B}}$$

Polarization Angular Power Spectrum

Depends on Scalar (also tensor) Perturbations

Depends only on Tensor Perturbations !



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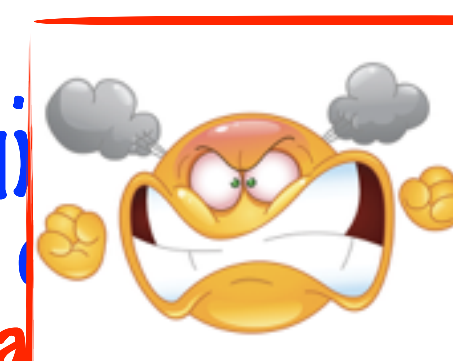

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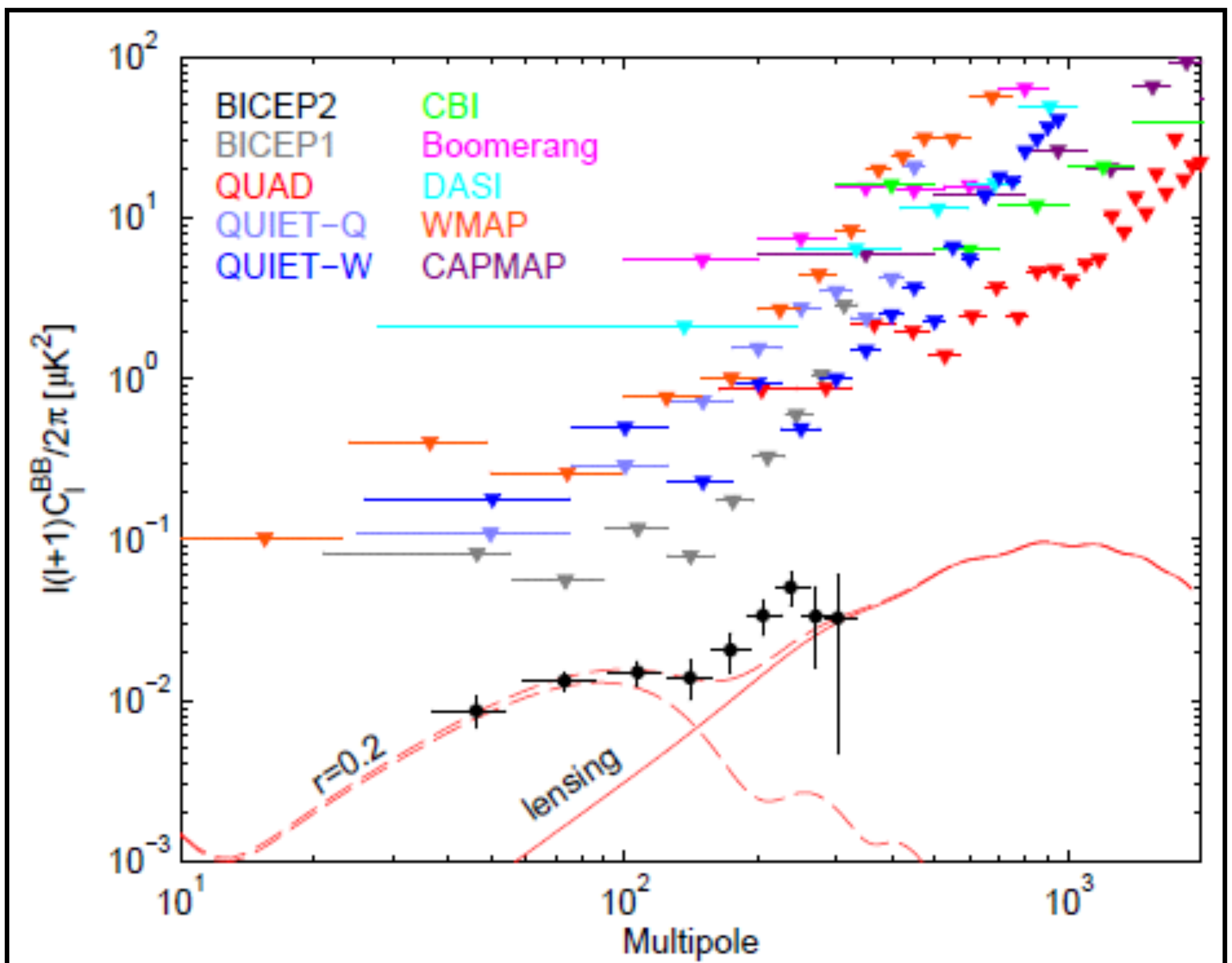

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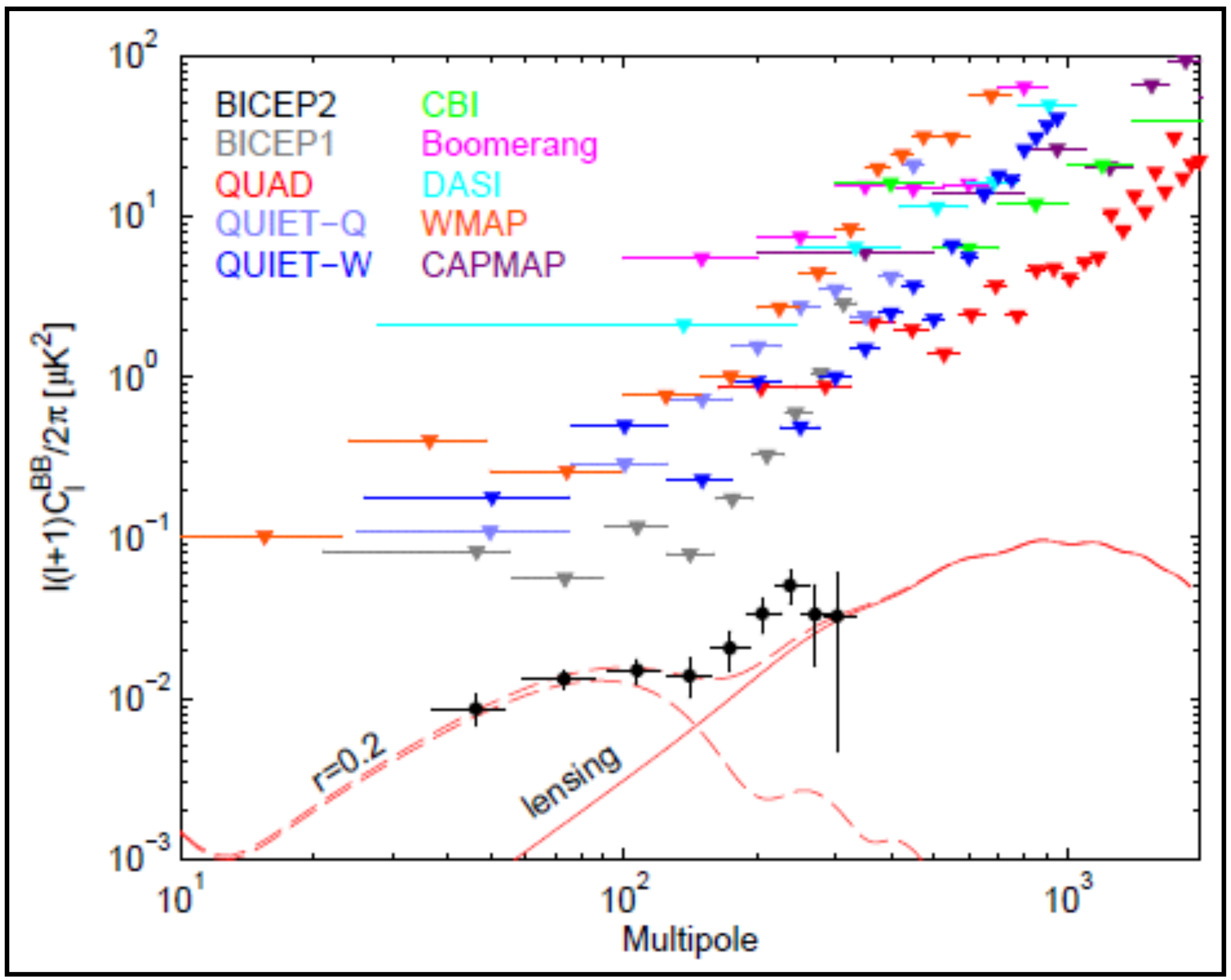

Dashed Line Theoretical Expectation from Inflation

$$r \equiv \Delta_t^2 / \Delta_s^2 < 0.07 \ (2\sigma)$$

Planck/Keck

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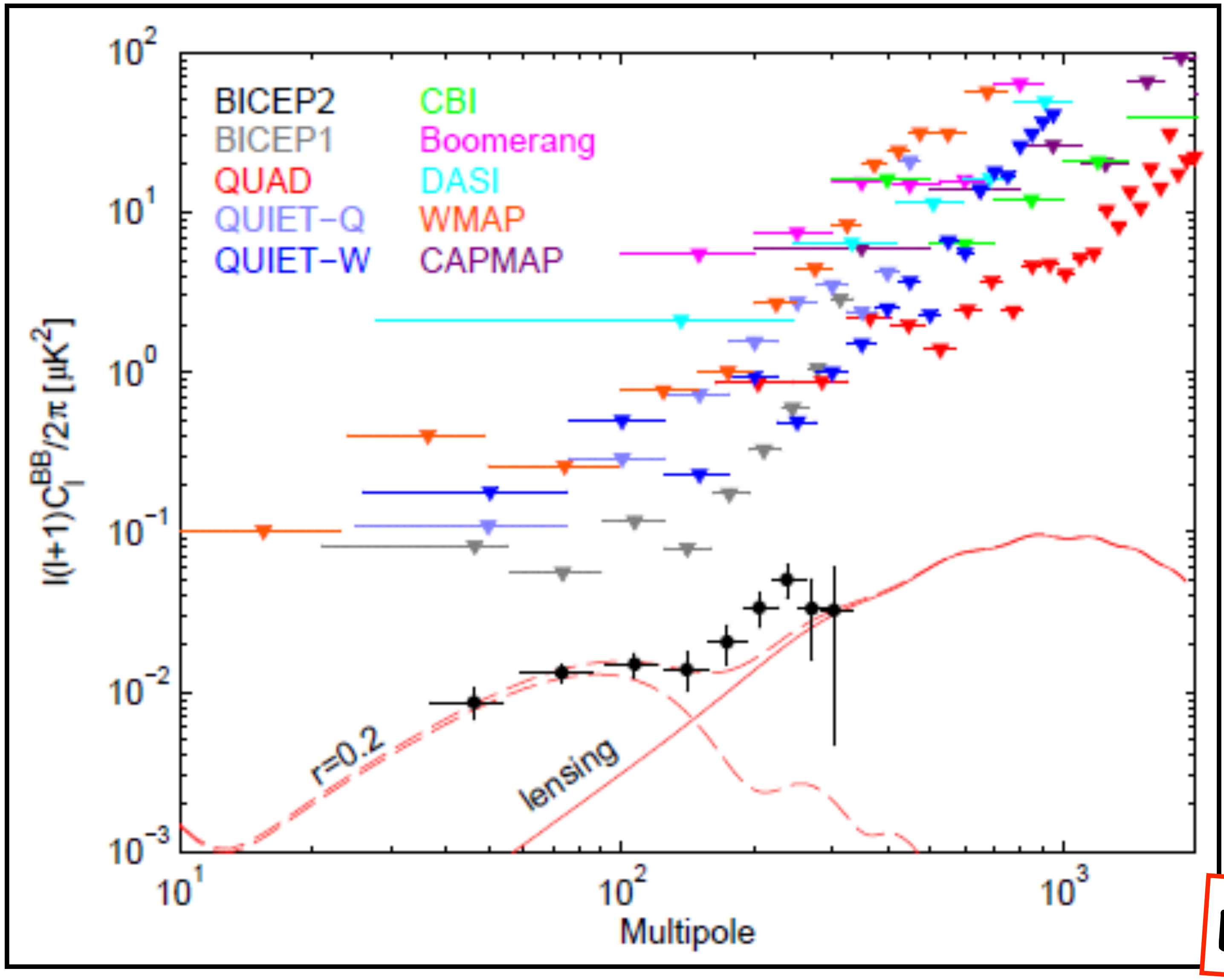

Dashed Line Theoretical Expectation from Inflation

Planck/Keck

$$r \equiv \Delta_t^2/\Delta_s^2 < 0.07 \ (2\sigma)$$

$$r \sim 10^{-2} - 10^{-3} \Rightarrow \boxed{E_* \lesssim 5 \cdot 10^{15} \mathrm{GeV}} \ (!)$$

next generation

Inflation: Observables

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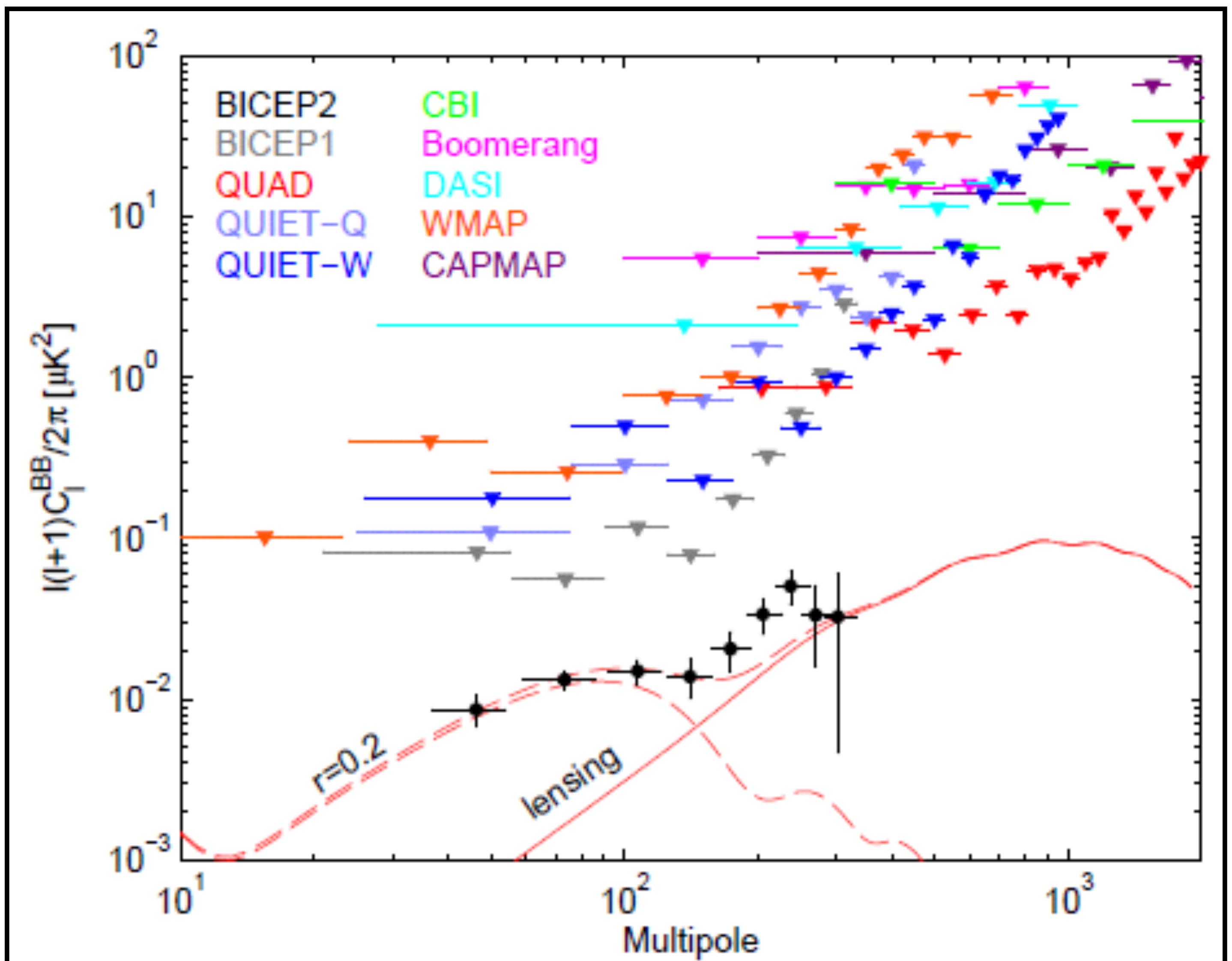

Search of B-modes @ CMB, might be only change to detect Inflationary Tensors !

Inflation: Observables

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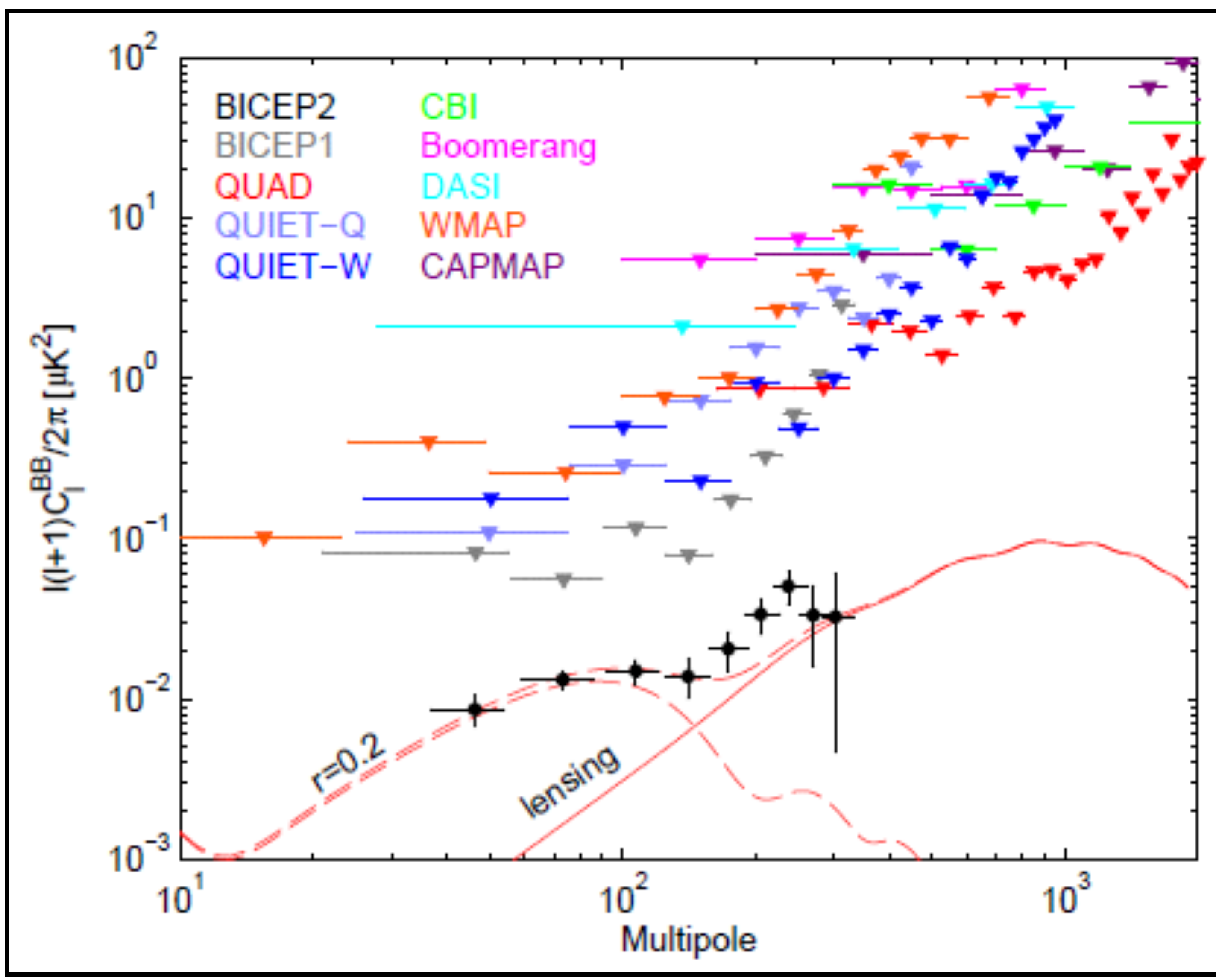
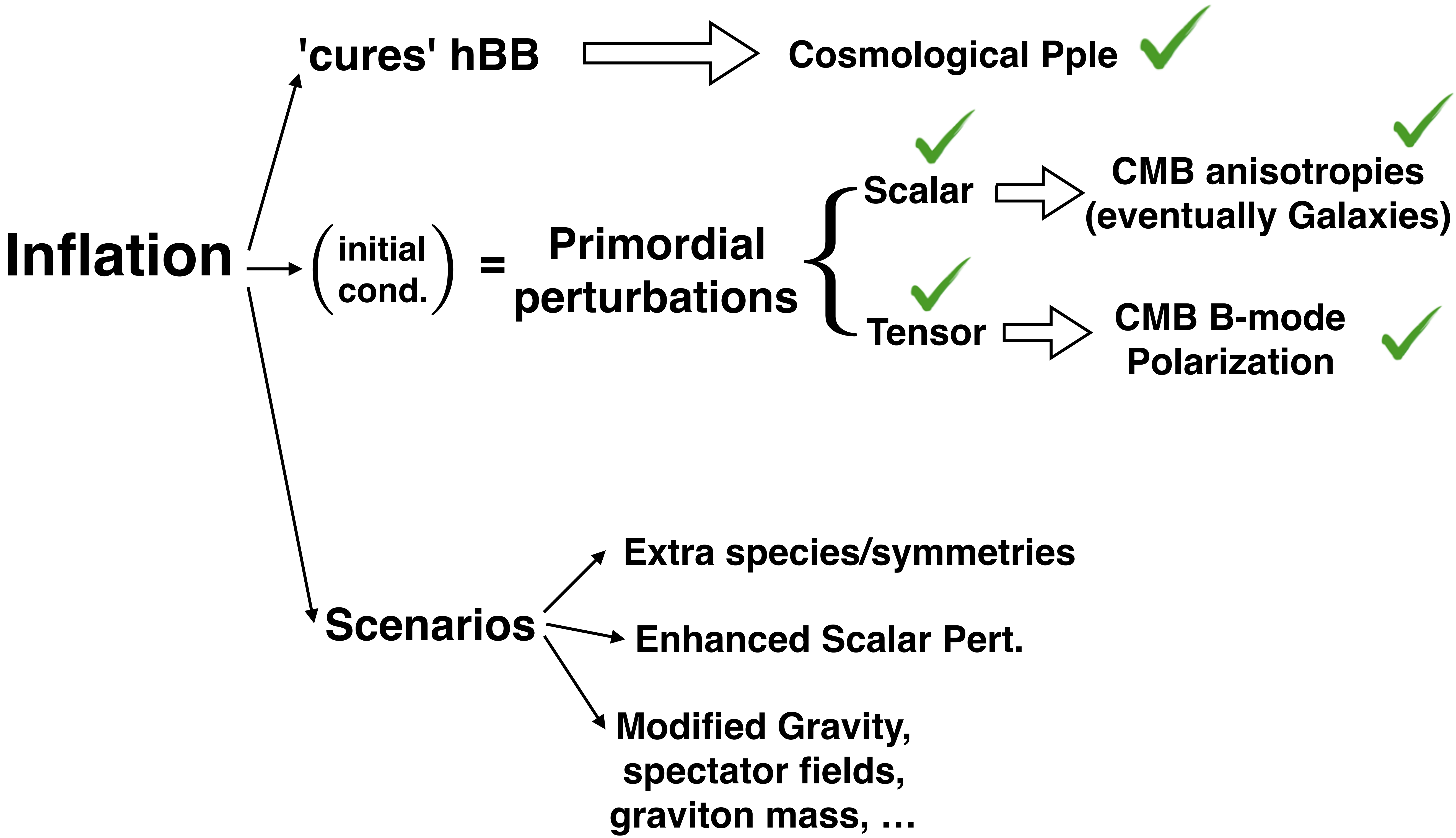


Holy Grial of Inflation, great effort put forward by CMB community

INFLATIONARY COSMOLOGY



Inflation: Summary

Inflation: Solves Causality Problem. Bonus: Universe Flat

$\boxed{a \propto e^{Ht}}$ → $\boxed{g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]}$ → $\boxed{\Delta T,\ \mathcal{E},\ \mathcal{B} \quad [\text{also } \delta\rho]}$

quasi dS (Slow-Roll)

Quantum Origin of Fluctuations

Angular Temperature/ Polarization Anisotropies

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quasi dS (Slow-Roll) | Quantum Origin of Fluctuations | Angular Temperature/ Polarization Anisotropies

Observations:

$\boxed{|\Omega_k| \ll 1}$ Locally Flat

$\boxed{n_s - 1 \sim -0.04}$ Almost Scale-Inv

$\boxed{\langle \mathcal{R}^3 \rangle \approx 0}$ Gaussian

$\boxed{4\delta_m = 3\delta_\gamma}$ Adiabatic

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$\lvert\Omega_k\rvert \ll 1$	$n_s - 1 \sim -0.04$	$\langle \mathcal{R}^3 \rangle \approx 0$	$4\delta_m = 3\delta_\gamma$
Locally Flat	Almost Scale-Inv	Gaussian	Adiabatic

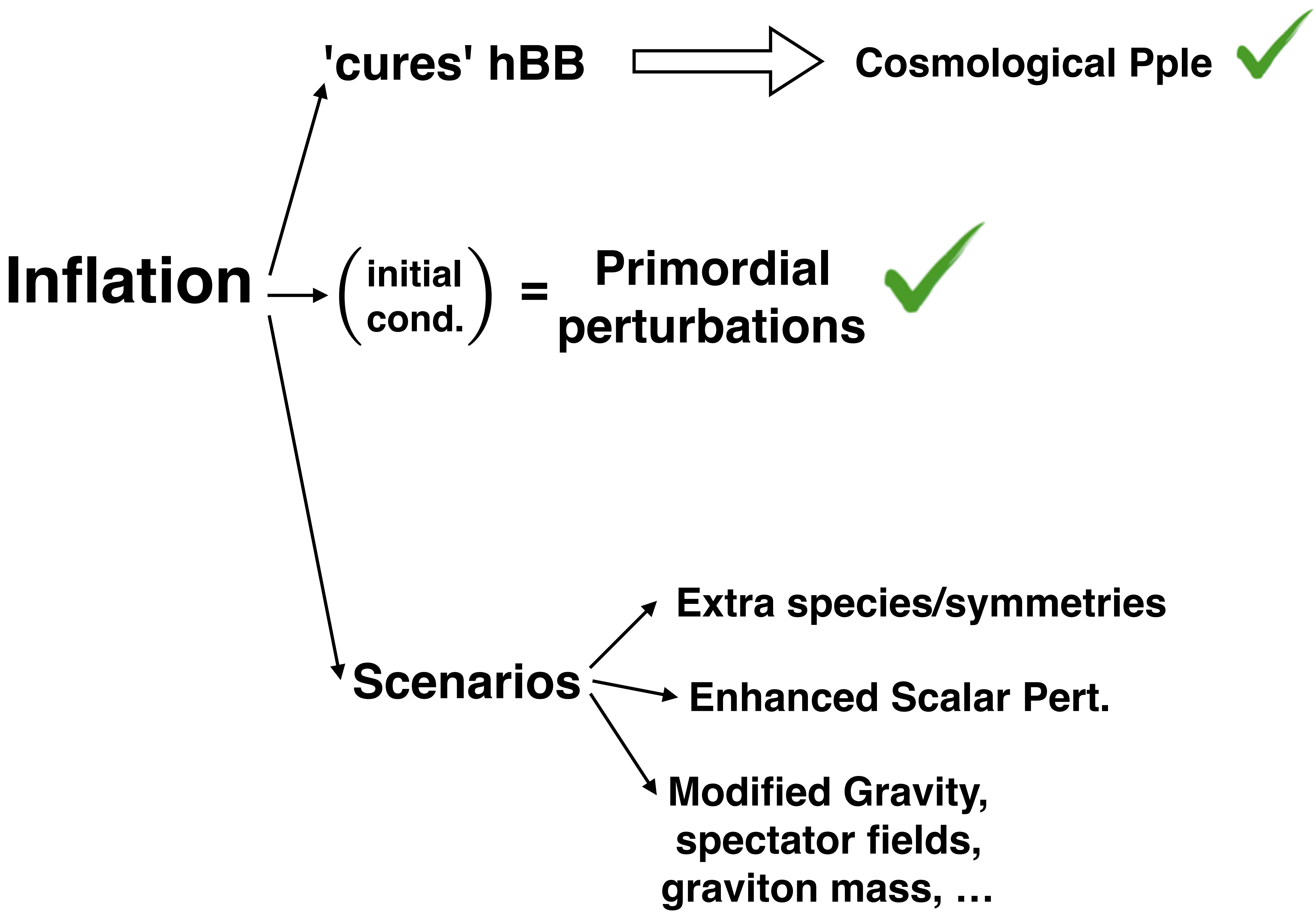
B-mode experiments:

$\boxed{H_{\text{inf}} \approx 10^{14} GeV \Rightarrow E_{\text{inf}} \approx 10^{16} GeV}$

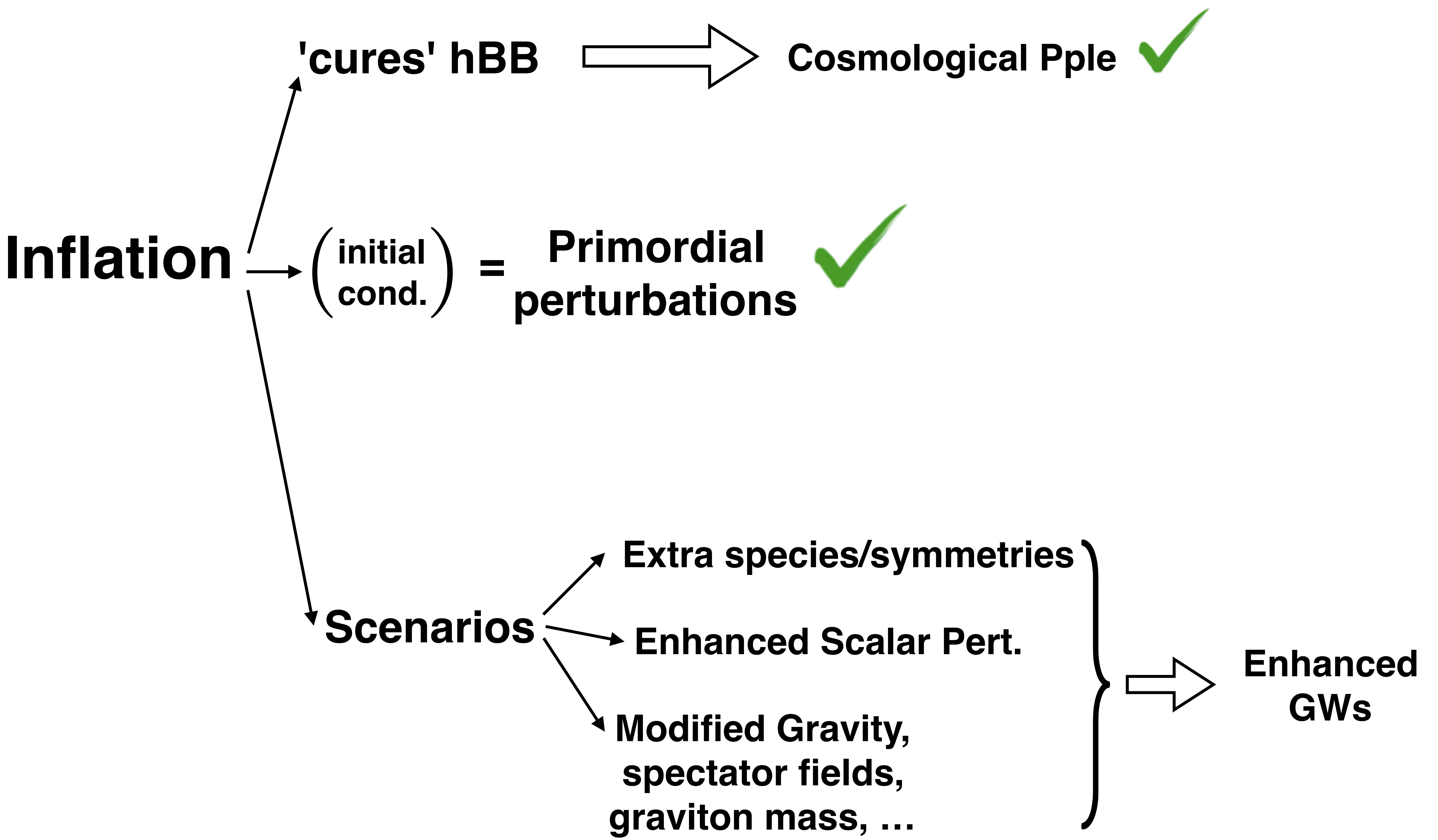
- Proof of Inflation ?
- Scale of Inflation !
- Quantum Gravity !
- Super-Planckian Excursion*
- Detection of GW**

(*,**: to be discussed tomorrow)

INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY



INFLATIONARY COSMOLOGY

Inflation

'cures' hBB ⟹ Cosmological Pple ✓

$\begin{pmatrix}\text{initial} \\ \text{cond.}\end{pmatrix}$ = Primordial perturbations ✓

Scenarios

Extra species/symmetri

GWs

mass, ...

To Be Continued