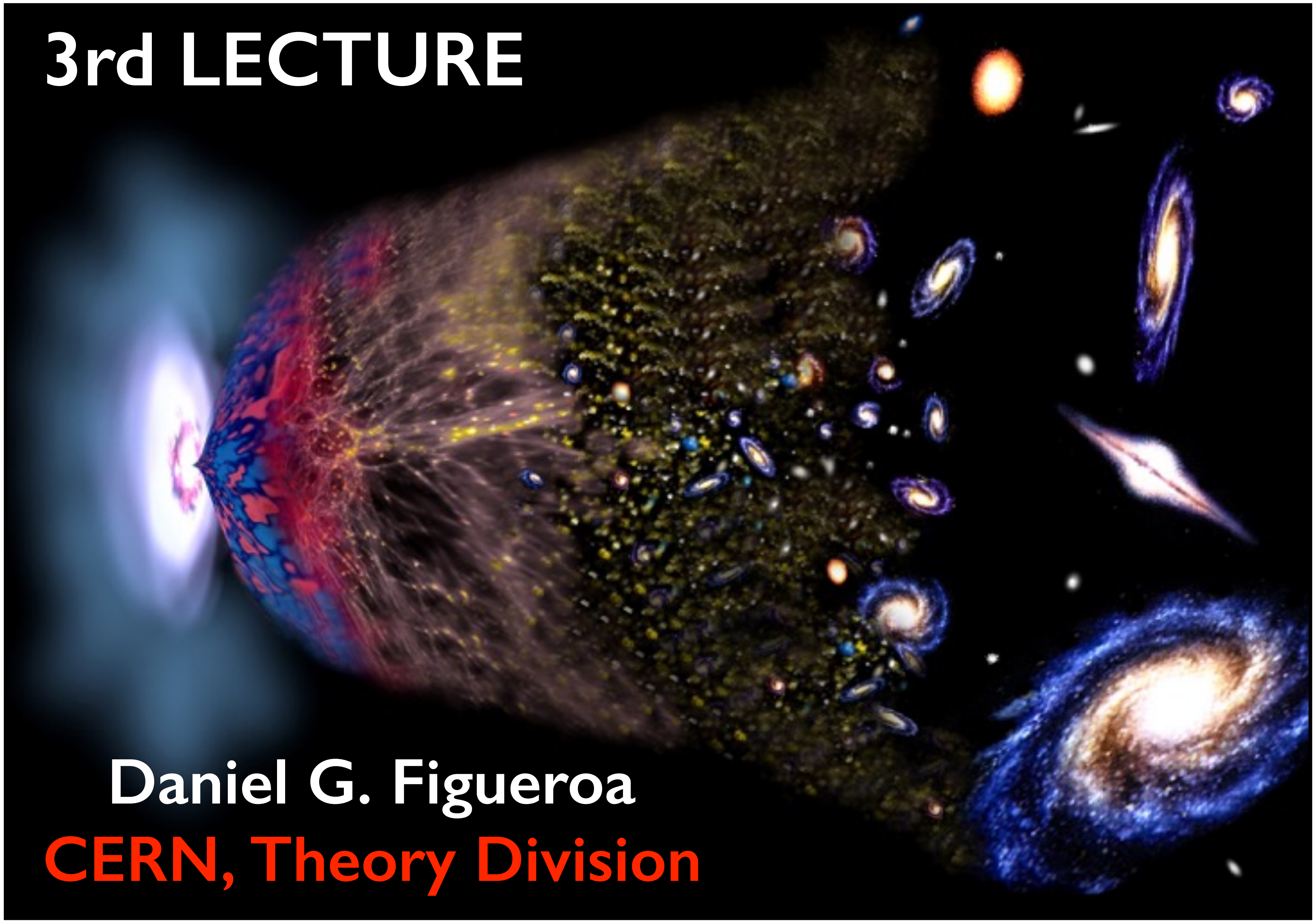
# GRAVITATIONAL WAVES
# PROBE OF THE EARLY UNIVERSE

## 3rd LECTURE



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School on Gravitational Waves for Cosmology and Astrophysics, Benasque, May 28 - June 10, 2017

# INFLATIONARY COSMOLOGY

Inflation

- → 'cures' hBB $\Longrightarrow$ Cosmological Pple ✓
- → $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ = Primordial perturbations $\left\{\begin{array}{l}\text{Scalar ✓}\\ \\ \text{Tensor ✓}\end{array}\right.$
- → Scenarios

# INFLATIONARY COSMOLOGY

'cures' hBB ⟹ Cosmological Pple

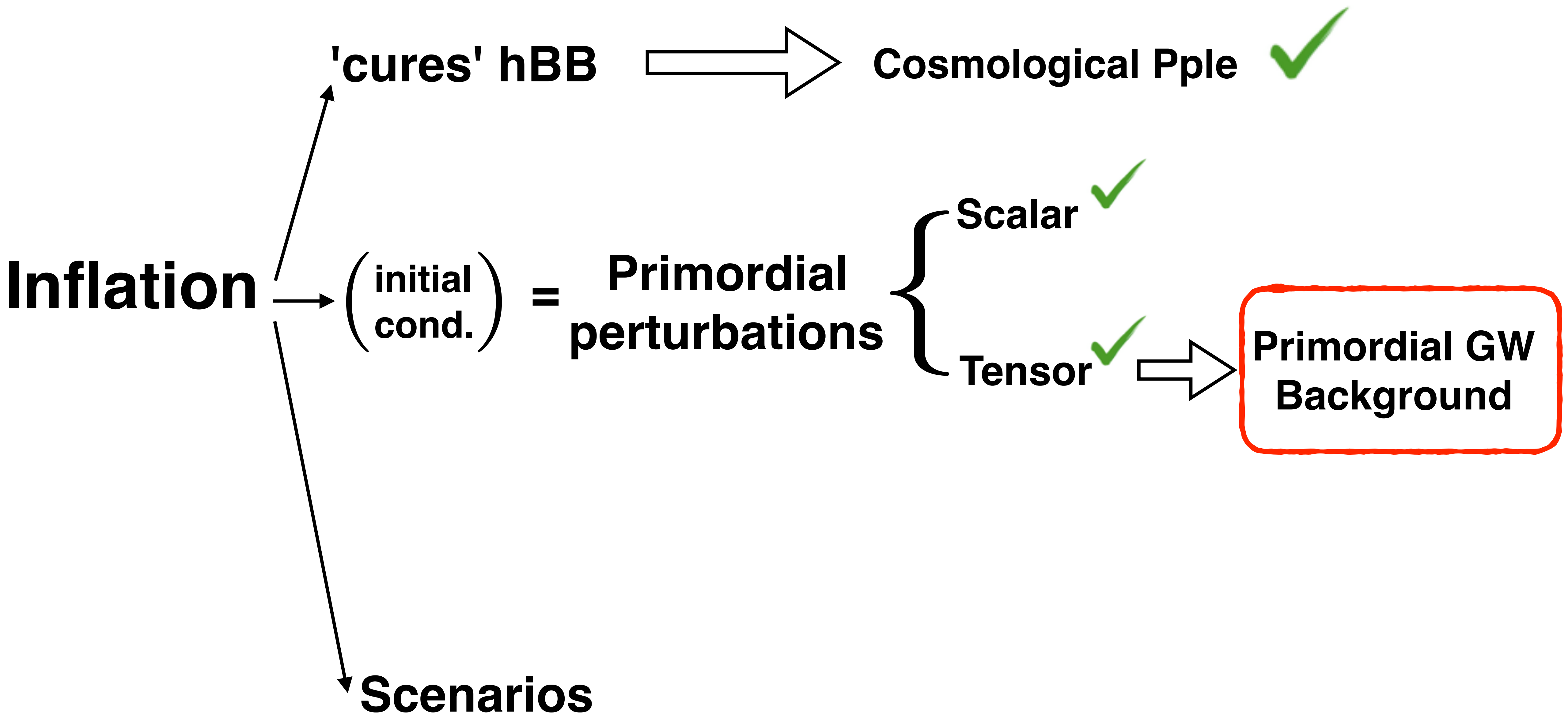
Inflation

(initial cond.) = Primordial perturbations

- Scalar
- Tensor ⟹ Primordial GW Background

Scenarios

# INFLATIONARY COSMOLOGY



# Irreducible GW background from Inflation

H & I

INFLATION

$$g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2 \dot{\phi}^2} \left(\frac{k}{aH}\right)^{n_s - 1}$$

$$n_s - 1 \equiv 2(\eta - 2\epsilon)$$

$$\Delta^2_h(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p}\right)^2 \left(\frac{k}{aH}\right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

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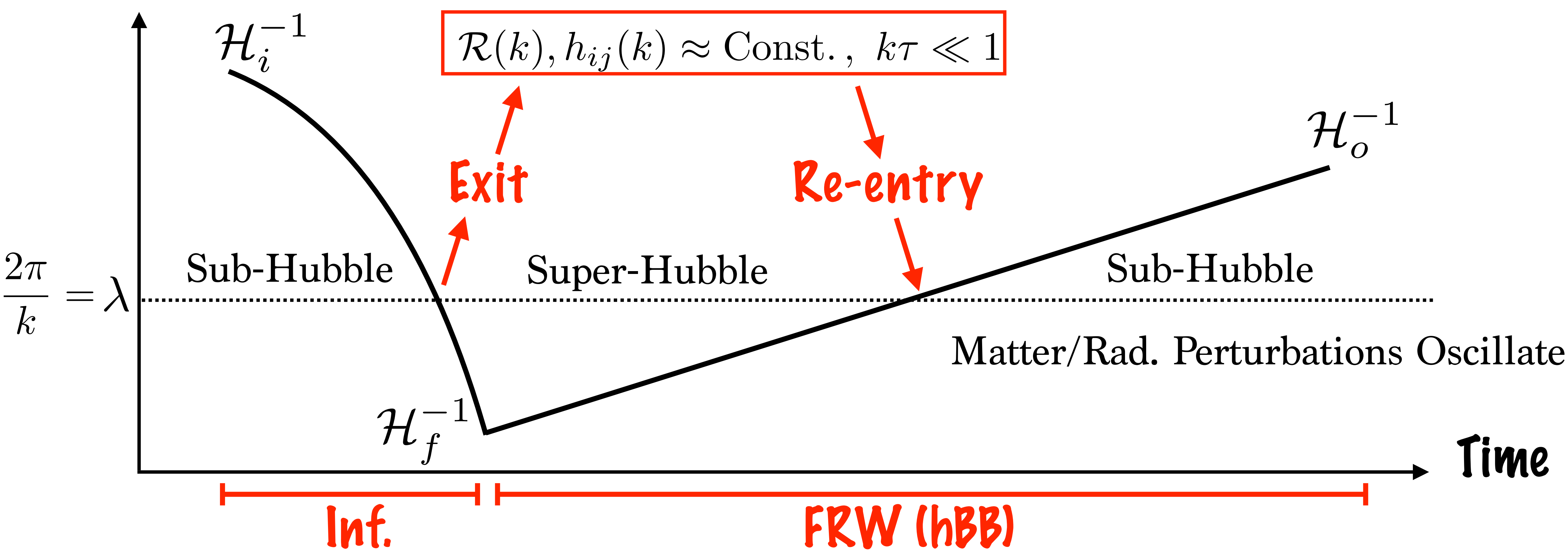

# Irreducible GW background from Inflation

$$\boxed{\begin{gathered} ds^2 = -(1+2\Phi)dt^2 + 2B_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \\ \phi(\vec{x},t) = \phi(t) + \delta\phi(\vec{x},t) \end{gathered}}$$

$$B_i = \partial_i B - S_i$$

$$E_{ij} = 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}$$

$$\partial_i h_{ij} = h_{ii} = 0$$

Tensors *dof*
(Transverse-Traceless)

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Tensors *dof*
(Transverse-Traceless)

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{kx}}\, \hat{a}_{\mathbf{k}r}^{+}\right)\, e_{ij}^{r}(\hat{\mathbf{k}})}$$

quantum fields

Polarizations: +, x

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quantum fields

Polarizations: +, x

$\boxed{t = \eta = \tau}$

conformal time

Bad notation

# Irreducible GW background from Inflation

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quantum fields

Polarizations: +, x

$$\begin{aligned}
\rho_{\rm GW}(t) \;&=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_V \\
&\equiv\; \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)
\end{aligned}$$

# Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\; e^r_{ij}(\hat{\mathbf{k}})}$$

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&= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{d\mathbf{k}'}{(2\pi)^3}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k}',t) \\
&\qquad \times \frac{1}{V} \int_V d\mathbf{x}\; e^{-i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\,,
\end{aligned}$$

# Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^{+}_{\mathbf{k}r}\right)\; e^{r}_{ij}(\hat{\mathbf{k}})}$$

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&\quad \times \frac{1}{V} \boxed{(2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')} \,,
\end{aligned}$$

# Irreducible GW background from Inflation

Tensors = GWs

$$\boxed{\hat{h}_{ij}(\mathbf{x},\eta) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left(h_k(\eta)\, e^{i\mathbf{k}\mathbf{x}}\, \hat{a}_{\mathbf{k}r} + h_k^*(\eta) e^{-i\mathbf{k}\mathbf{x}}\, \hat{a}^+_{\mathbf{k}r}\right)\, e^r_{ij}(\hat{\mathbf{k}})}$$

quantum fields

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$$\equiv \frac{1}{32\pi G a^2(t)} \frac{1}{V} \int_V d\mathbf{x}\, \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)$$

$$= \boxed{\frac{1}{32\pi G a^2(t) V} \int \frac{d\mathbf{k}}{(2\pi)^3}\; \dot{h}_{ij}(\mathbf{k},t)\dot{h}^*_{ij}(\mathbf{k},t)}$$

# Irreducible GW background from Inflation

Tensors = GWs

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Polarizations: +, x

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# Irreducible GW background from Inflation

Tensors = GWs

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quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) \;=\; \frac{1}{32\pi G a^2(t)} \left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t) \right\rangle_{\rm QM} \longrightarrow \text{ensemble average}$$

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$$= \frac{1}{32\pi G a^2(t)} \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}\, e^{i\mathbf{x}(\mathbf{k}-\mathbf{k}')}\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\,\dot{h}_{ij}^*\,(\mathbf{k}',t)\right\rangle$$

# Irreducible GW background from Inflation

Tensors = GWs

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$$\boxed{\left\langle \dot{h}_{ij}\,(\mathbf{k},t)\, \dot{h}^*_{ij}\,(\mathbf{k}',t) \right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot{h}}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

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quantum fields

Polarizations: +, x

$$\rho_{\rm GW}(t) = \frac{1}{(4\pi)^3 G a^2(t)}\int \frac{dk}{k}\,k^3\,\mathcal{P}_{\dot{h}}(k,t) \;=\; \int \frac{d\rho_{\rm GW}}{d\log k}\,d\log k$$

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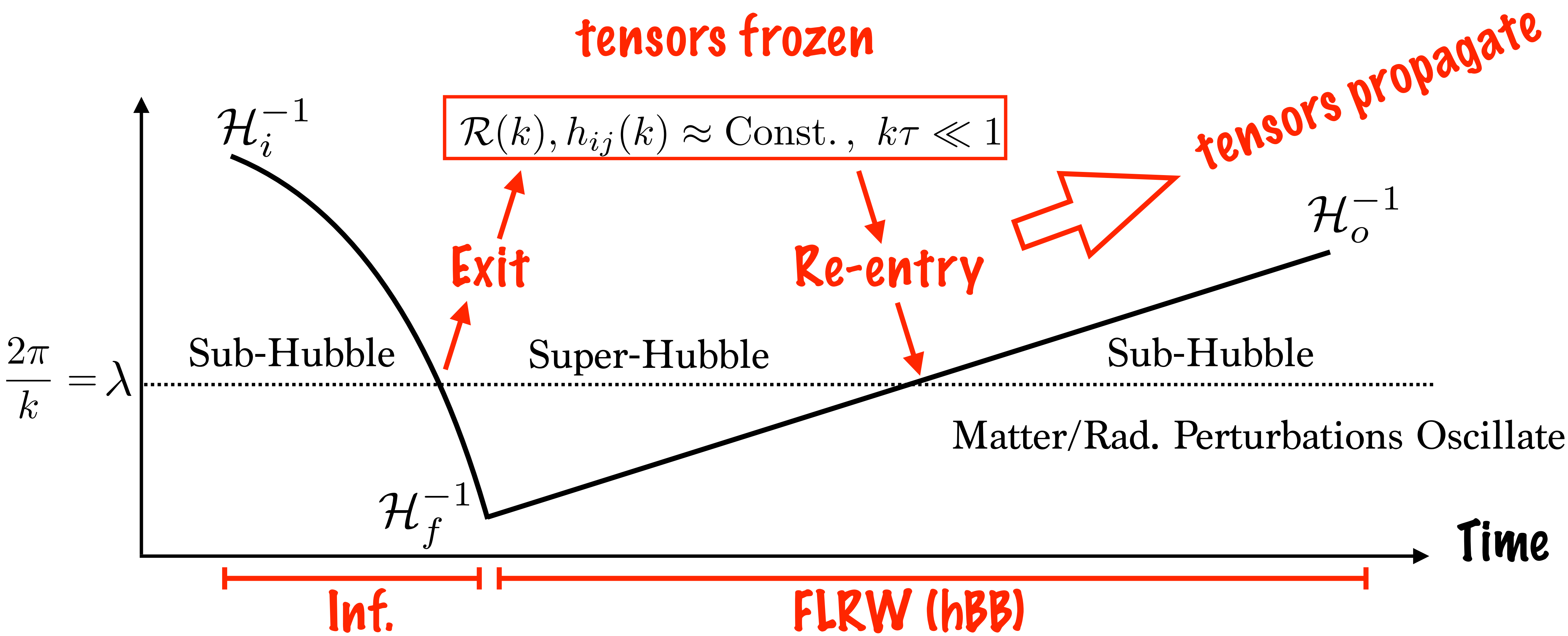

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Horizon Re-entry $\Longrightarrow$ tensors propagate

Rad Dom: $h_r(\mathbf{k},\eta) = \dfrac{A_r(\mathbf{k})}{a(\eta)}\, e^{ik\eta} + \dfrac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ik\eta}$

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$$\text{@ Horizon :} \begin{cases} h = h_* \\ \dot h_* = 0 \end{cases}$$

$$A = B = \frac{1}{2} a_* h_*$$

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$$\left\langle \dot h \dot h \right\rangle = k^2 \langle hh\rangle = \left(\frac{a_*}{a}\right)^2 \frac{k^2}{2}\langle |h_*|^2\rangle = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\frac{2\pi^2}{k^3}\Delta^2_{h_*}$$

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**Inflationary Tensor Spectrum !**

# Irreducible GW background from Inflation

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$$(1+z_*)^{-2}_{\rm RD} = \Omega^{(o)}_{\rm Rad}\frac{a_o^2 H_o^2}{k^2} \quad \Rightarrow \quad \frac{d\log\rho_{\rm GW}}{d\log k} = \frac{\Omega^{(o)}_{\rm Rad}}{24}\left(\frac{a_o}{a}\right)^4 3m_p^2 H_o^2 \Delta^2_{h_*}$$

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$$\boxed{\Omega^{(o)}_{\rm GW} \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega^{(o)}_{\rm Rad}}{24}\Delta^2_{h_*}}$$

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$$\boxed{\frac{d\rho_{\rm GW}}{d\log k}(k,t) = \frac{1}{(4\pi)^3 G\, a^2(t)}\, k^3\, \mathcal{P}_{\dot h}(k,t)}$$

$$\boxed{\left\langle \dot h_{ij}\,(\mathbf{k},t)\, \dot h^*_{ij}\,(\mathbf{k}',t)\right\rangle \equiv (2\pi)^3\, \mathcal{P}_{\dot h}(k,t)\delta^{(3)}(\mathbf{k}-\mathbf{k}')}$$

$$\boxed{\mathcal{P}_{\dot h} = \left(\frac{a_o}{a}\right)^2 \frac{k^2}{2(1+z_*)^2}\frac{2\pi^2}{k^3}\Delta^2_{h_*}} \Longrightarrow \boxed{\frac{d\log\rho_{\rm GW}}{d\log k} = \frac{1}{8}\frac{a_o^2}{a^4}\frac{m_p^2 k^2}{(1+z_*)^2}\Delta^2_{h_*}}$$

$$\boxed{\Omega^{(o)}_{\rm GW}(f) \equiv \frac{1}{\rho_c^{(o)}}\left(\frac{d\log\rho_{\rm GW}}{d\log k}\right)_o = \frac{\Omega^{(o)}_{\rm Rad}}{24}\Delta^2_{h_*}(k)} \qquad (k = 2\pi f)$$

GW normalized energy density spectrum (today)

Inflationary tensor spectrum

# Irreducible GW background from Inflation

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Transfer Funct

$$T(k)\begin{cases} \propto k^0 (\mathrm{RD}) \\ \propto k^{-2} (\mathrm{MD}) \end{cases}$$

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Inflationary tensor spectrum

Inflationary Hubble Rate

$$\boxed{\Delta_h^2(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}}$$

$\boxed{n_t \equiv -2\epsilon}$ Small red-tilt, i.e. (almost-) scale-invariant

# Irreducible GW background from Inflation

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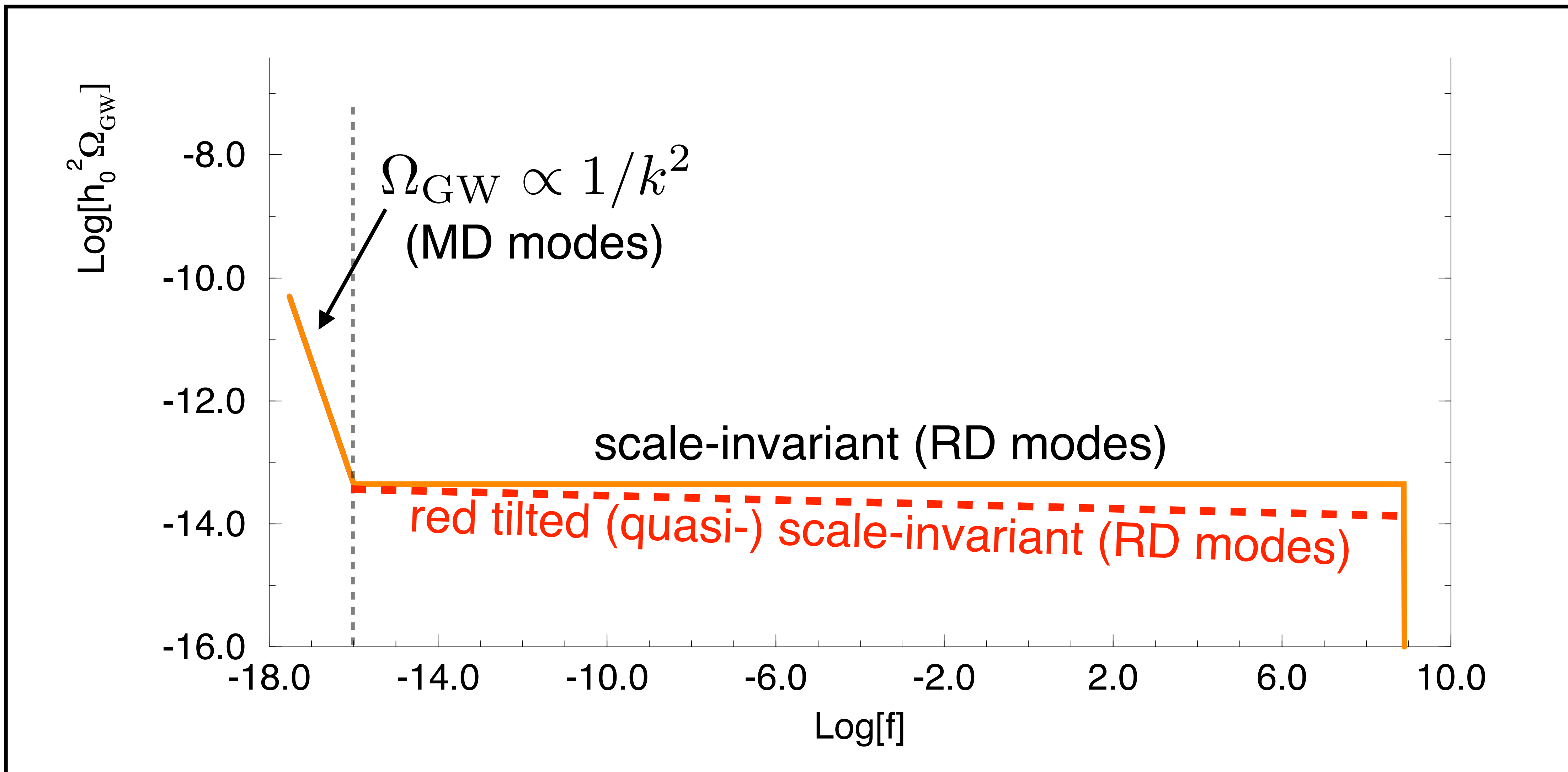

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Slow-Roll Inflation

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$$\boxed{\Delta_{\mathcal{R}}^2(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}} \quad \boxed{n_s - 1 \equiv 2(\eta - 2\epsilon)}$$

$$\Rightarrow \quad \boxed{r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} \simeq 16\epsilon}$$

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**Consistency Relation**

# INFLATIONARY COSMOLOGY

'cures' hBB ⇒ Cosmological Pple

Inflation

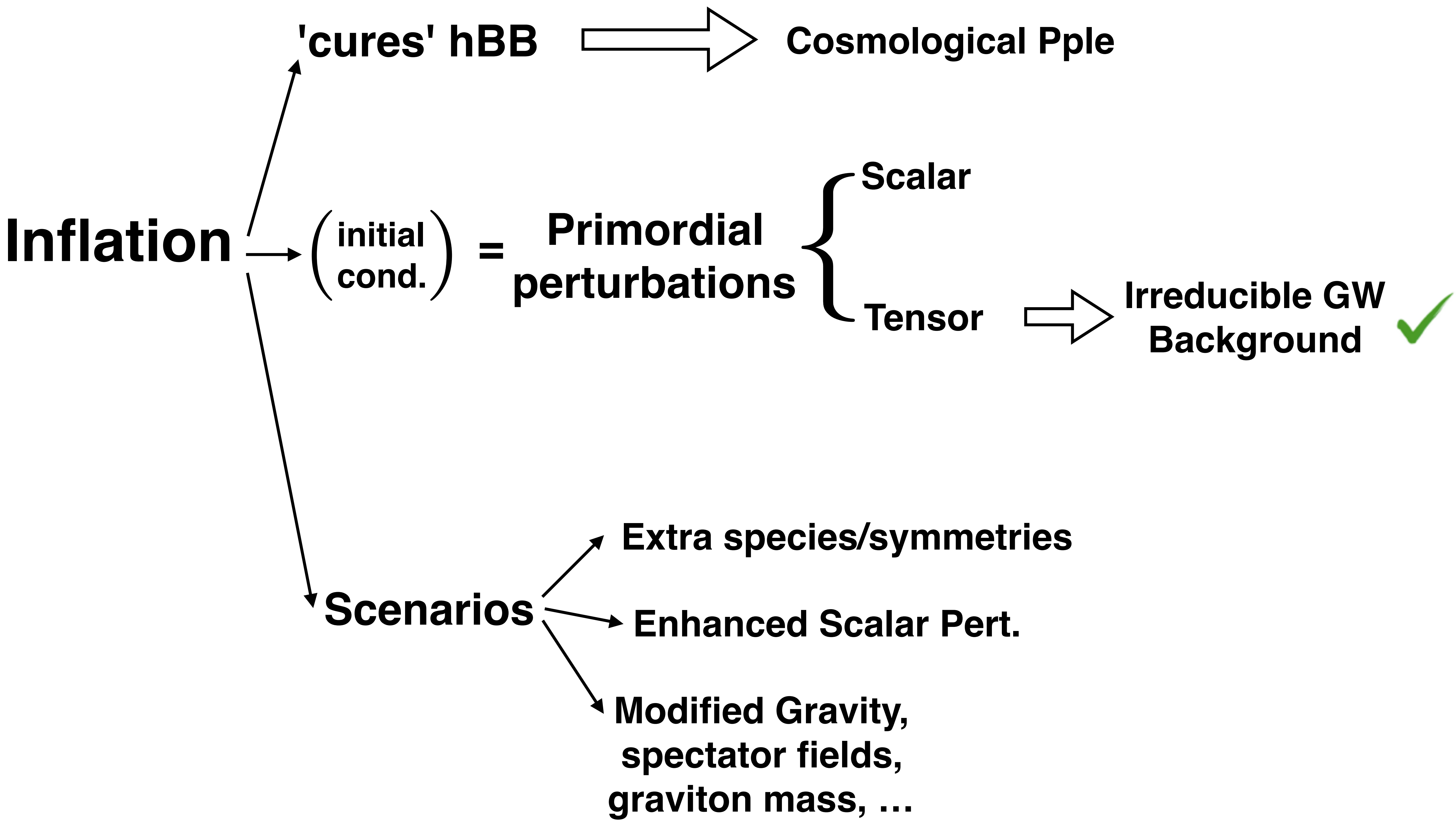
(initial cond.) = Primordial perturbations

Scalar

Tensor ⇒ Irreducible GW Background ✓

Scenarios

# INFLATIONARY COSMOLOGY



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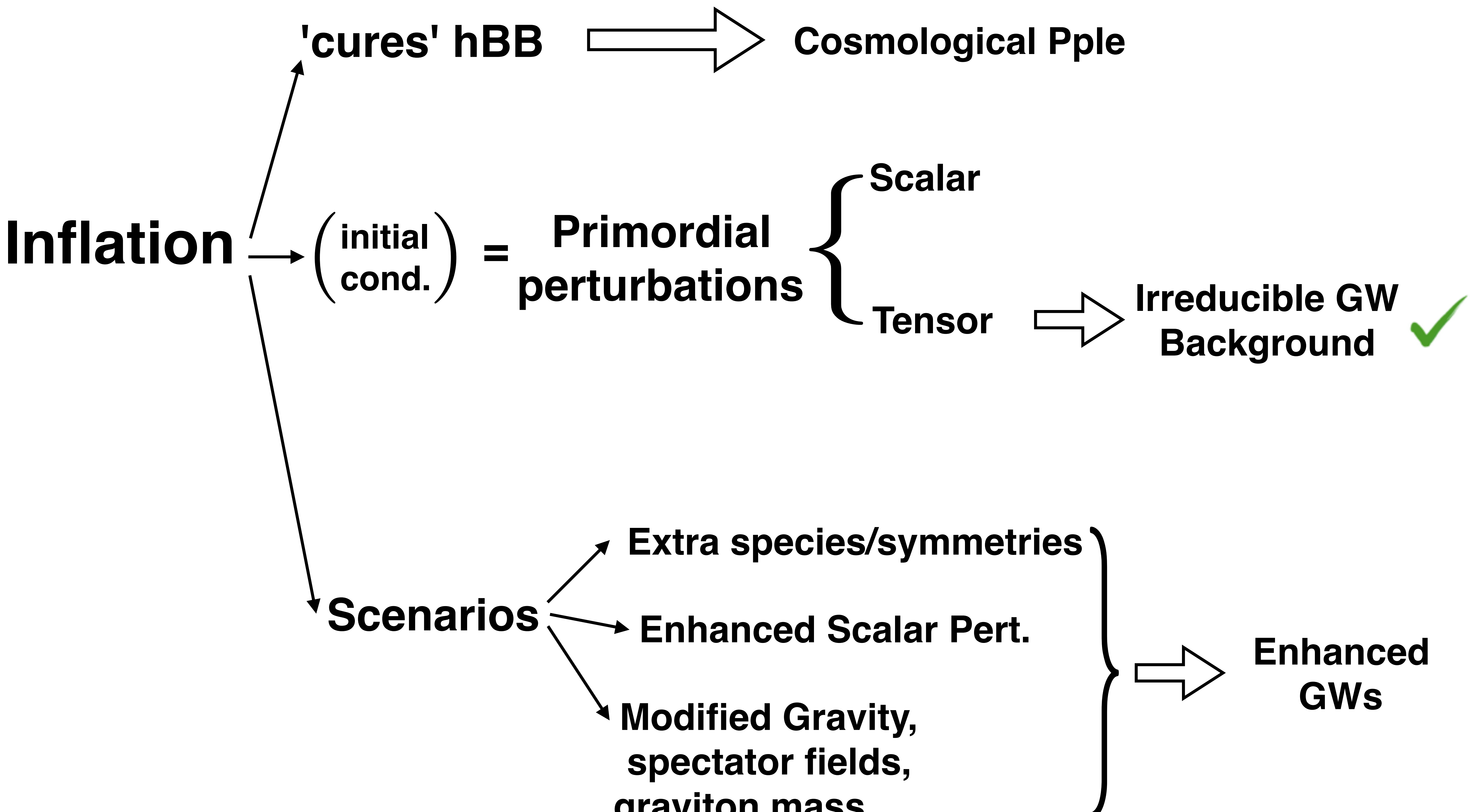

# INFLATIONARY COSMOLOGY

'cures' hBB $\Rightarrow$ Cosmological Pple

Inflation $\rightarrow$ $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ = Primordial perturbations $\begin{cases}\text{Scalar}\\ \text{Tensor} \Rightarrow \text{Irreducible GW Background} \checkmark\end{cases}$

Scenarios

mass, ...

GWs

Before Modelling… let's think!

What is the SOURCE of GWs ?

Let's revisit GW definition & propagation

# INFLATIONARY COSMOLOGY

Inflation

→ 'cures' hBB ⇒ Cosmological Pple

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→ Scenarios

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mass,

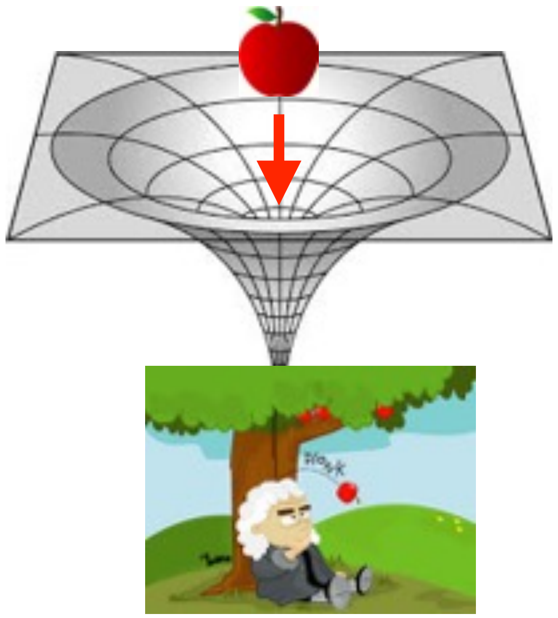
See also BLANCHET Lectures for more details

# Gravitational Wave Definition

**1st approach to GWs**

$$\mathrm{DIFF}: \quad x^\mu \to x'^\mu(x)$$

symmetry



# Gravitational Wave Definition

Minkowski

**1st approach to GWs**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x)$$

fixed frame

$$( \ |h_{\mu\nu}| \ll 1 \ )$$

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$$\mathrm{DIFF}: \quad \cancel{x^\mu \to x'^\mu(x)}$$

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x)$$

$$\boxed{(\ |\partial_\mu \xi_\nu(x)| \lesssim |h_{\mu\nu}|\ )}$$

residual symm.

$$\boxed{h_{\mu\nu}(x) \to h'_{\mu\nu}(x') = h_{\mu\nu}(x) - \partial_{[\mu}\xi_{\nu]}}$$

---

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Trace-reversed

$$\boxed{\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}} \Rightarrow \partial^\alpha \partial_\alpha \bar{h}_{\mu\nu} + \eta_{\mu\nu} \partial^\alpha \partial^\beta \bar{h}_{\alpha\beta} - \partial^\alpha \partial_{(\mu} \bar{h}_{\alpha\nu)} = -\frac{2}{m_p^2} T_{\mu\nu}$$

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residual symm.

$$\boxed{\partial^\nu \bar{h}_{\mu\nu} = 0} \Rightarrow \boxed{\partial_\alpha \partial^\alpha \bar{h}_{\mu\nu} = -\frac{2}{m_p^2} T_{\mu\nu}}$$

**Lorentz gauge**

(10 - 4 = 6 d.o.f. )

---

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$$\boxed{h^{0\mu} = 0 \,, \quad h^i_i = 0 \,, \quad \partial_j h_{ij} = 0}$$

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(6 - 4 = 2 d.o.f. )

(transverse-traceless gauge)

# Gravitational Wave Definition

(TT gauge: 6 - 4 = 2 d.o.f. )

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**Wave Eq. ⟶ Gravitational Waves !**

**(GW can not be 'gauged away' !)**

---

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**Wave Eq. → Gravitational Waves !**

**(GW can not be 'gauged away' !)**

---

**2 *dof* = 2 polarizations**

$$h_{ab}(t, \mathbf{x}) = \int_{-\infty}^{\infty} df \int d\hat{n}\, h_{ab}(f, \hat{n}) e^{-2\pi i f (t - \hat{n}\mathbf{x})}$$

(plane wave)

$$h_{ab}(f, \hat{n}) = \sum_{A=+,\mathrm{x}} h_A(f, \hat{n}) \epsilon^{(A)}_{ab}(\hat{n}) \; = \begin{pmatrix} h_+ & h_x & 0 \\ h_x & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Transverse-Traceles (2 dof)

# Gravitational Wave Definition

(TT gauge: 6 - 4 = 2 d.o.f. )

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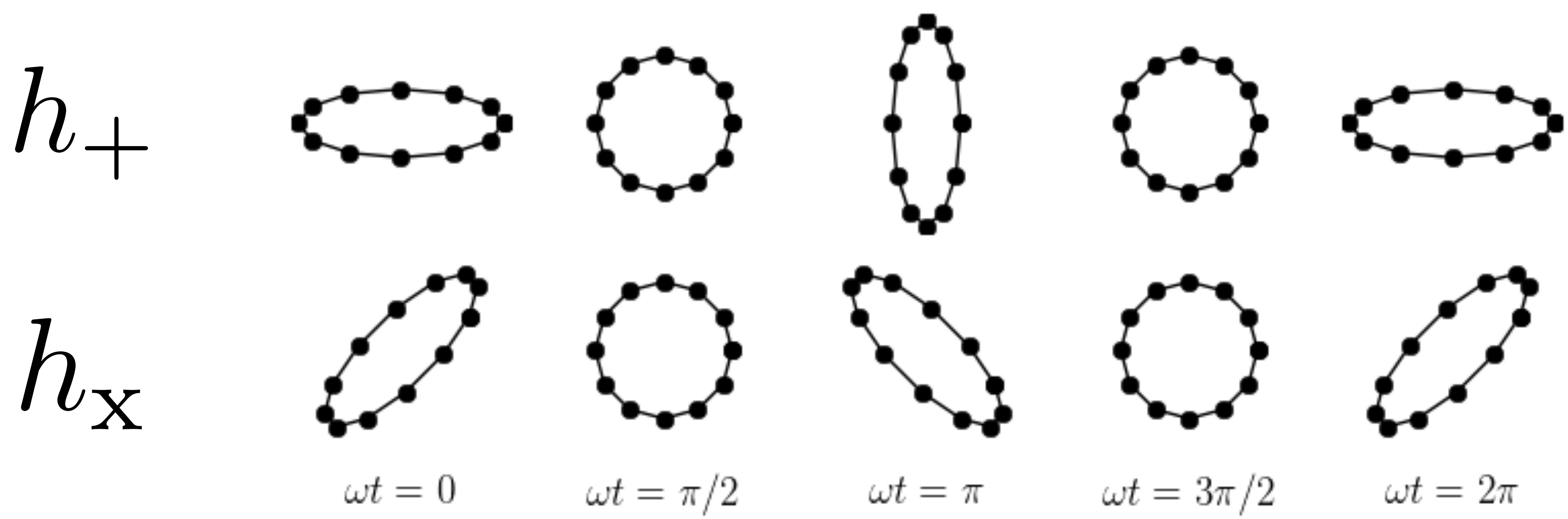

# Gravitational Wave Definition

**2nd approach to GWs**

(for a curved space-time)

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + h_{\mu\nu}(x)\,, \quad |h_{\mu\nu}| \ll 1$$

(separation not well defined)

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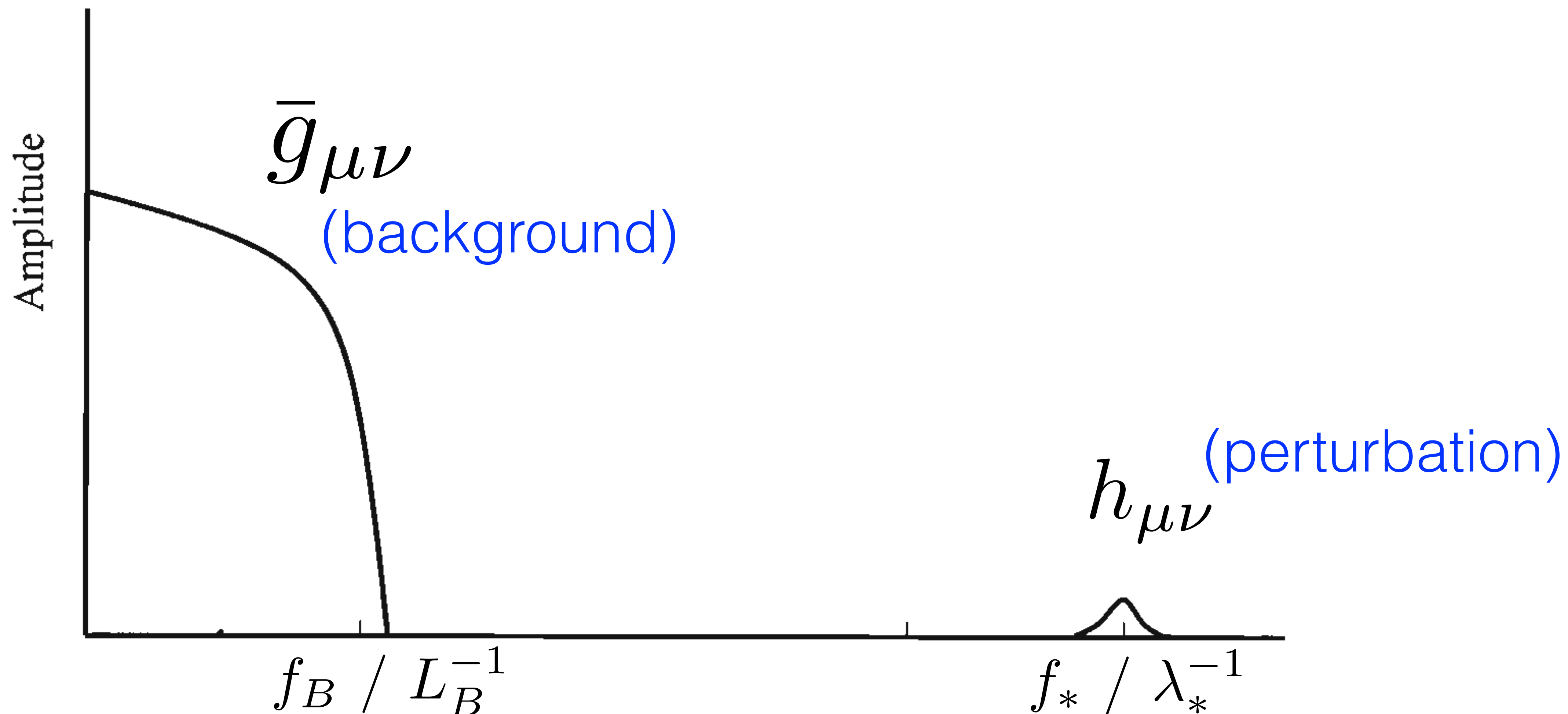
More subtle problem! <u>Solution</u>: Separation of scales !



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See Maggiore's Book on GWs

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High Freq. / Short Scale: $$R^{(1)}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{\rm High} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\rm High}$$

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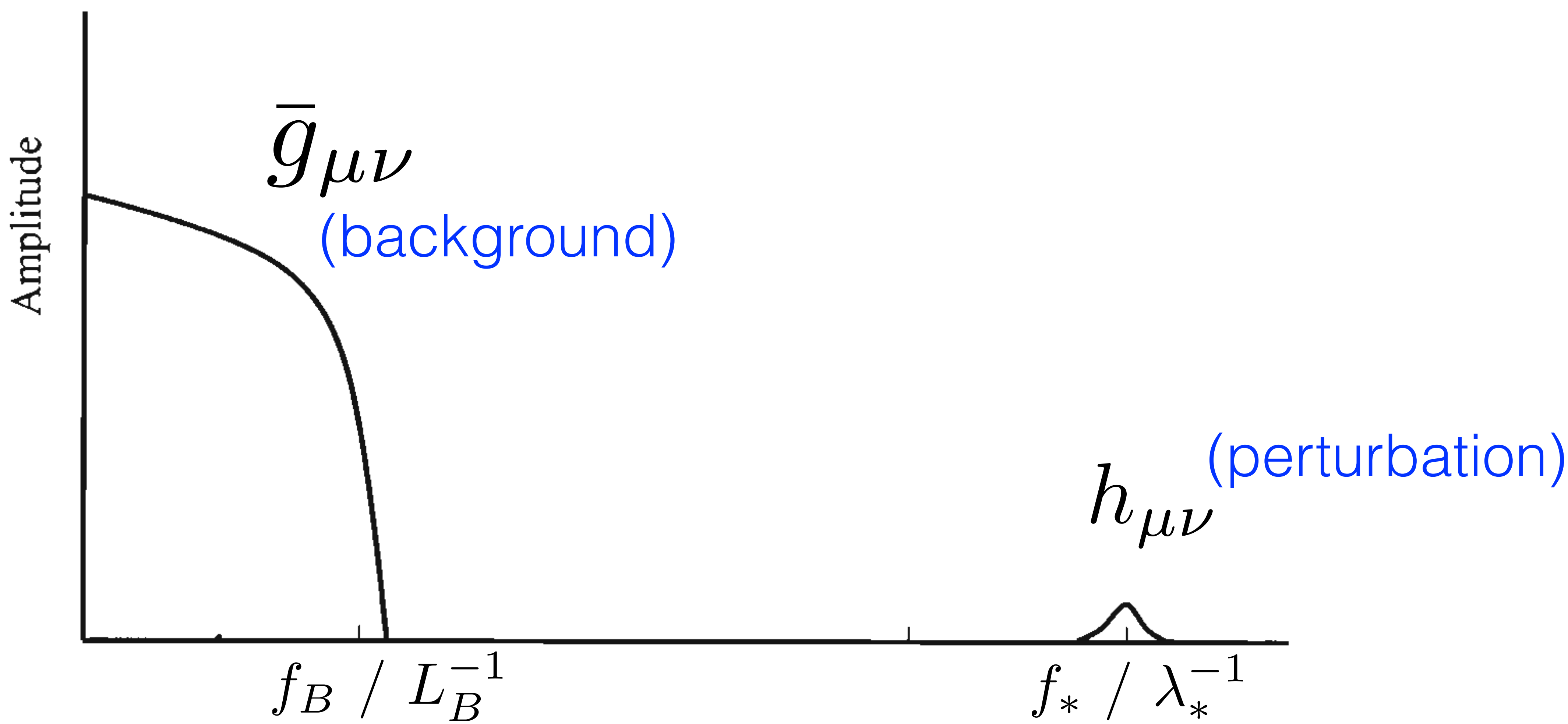
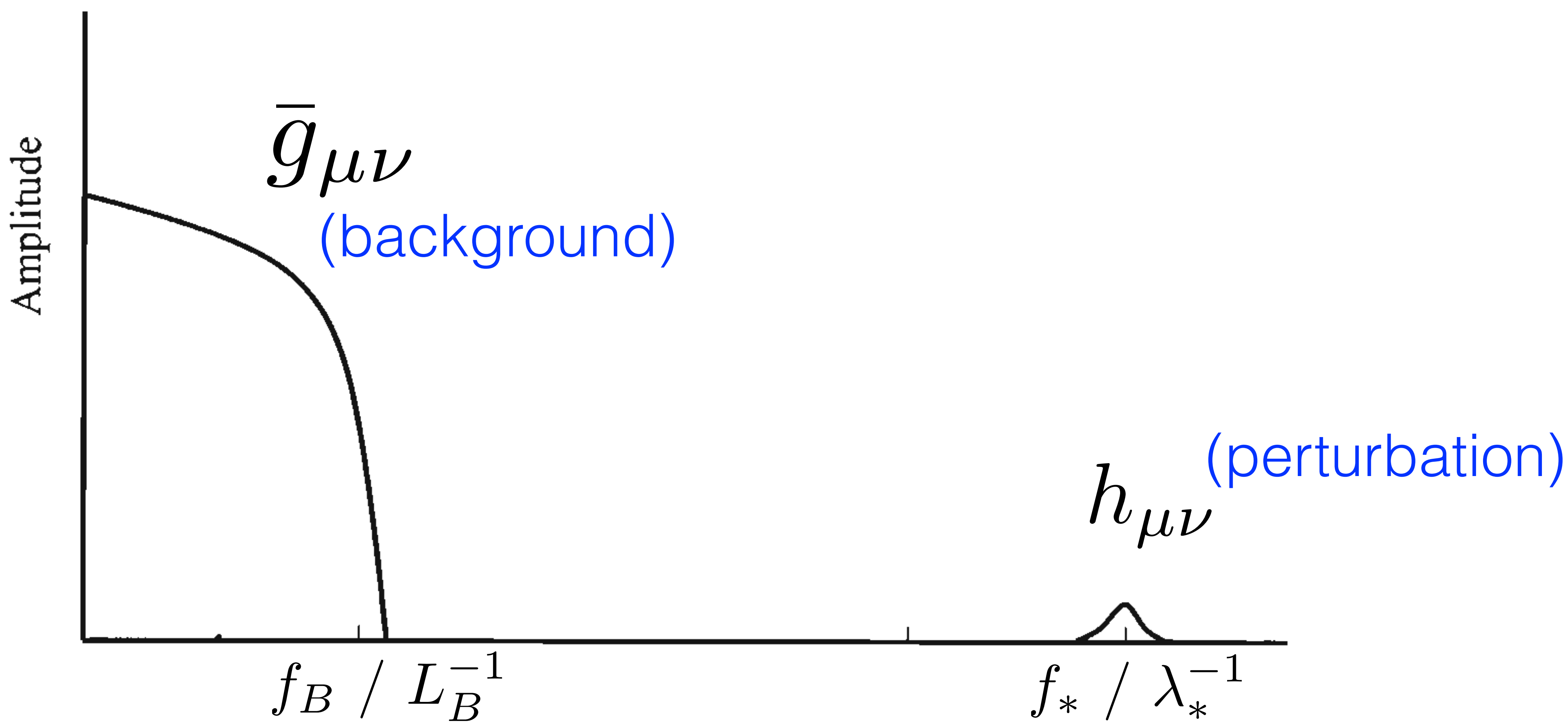


# Gravitational Wave Definition

Low Freq. / Long Scale: $$\bar{R}_{\mu\nu} = -\langle R^{(2)}_{\mu\nu} \rangle + \frac{8\pi G}{c^4} \langle T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \rangle$$ (space/time average)



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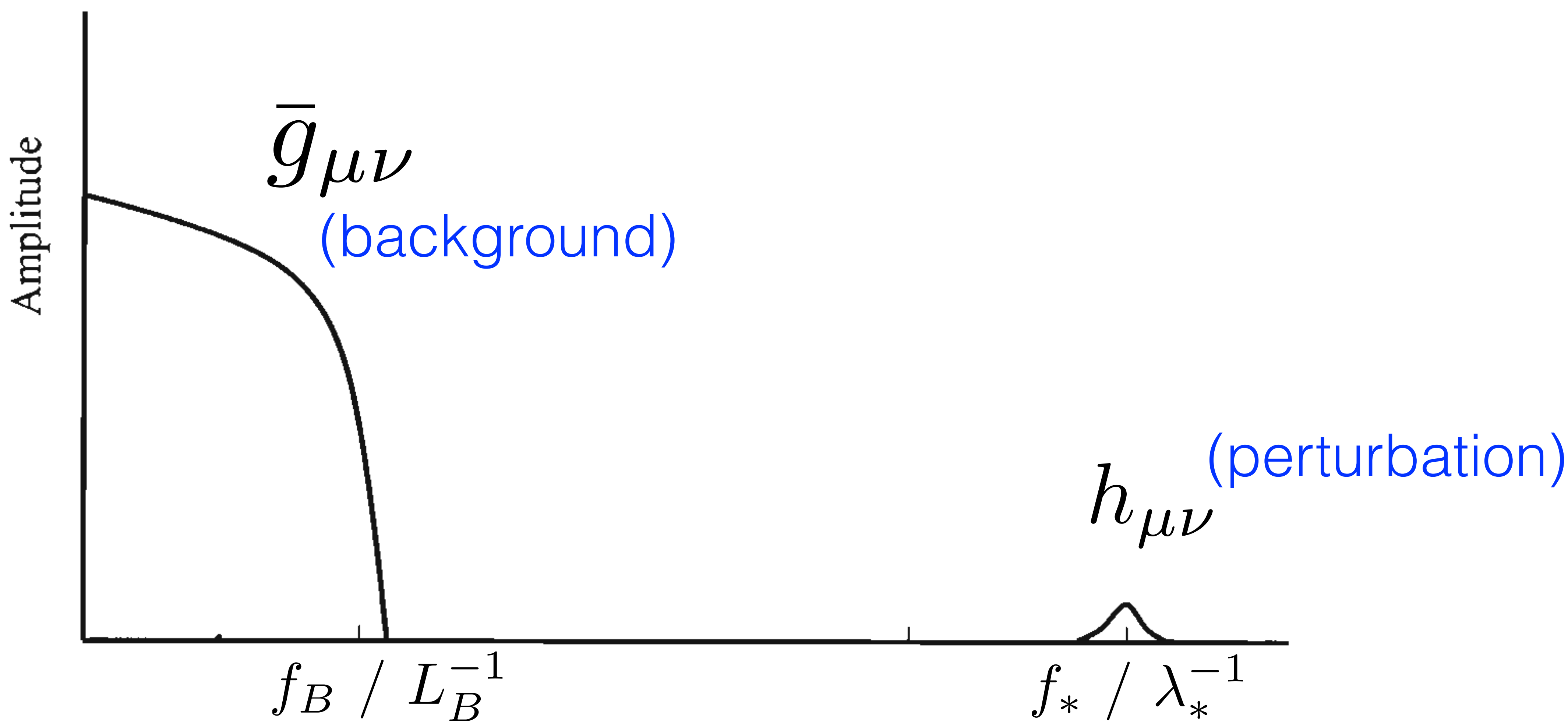

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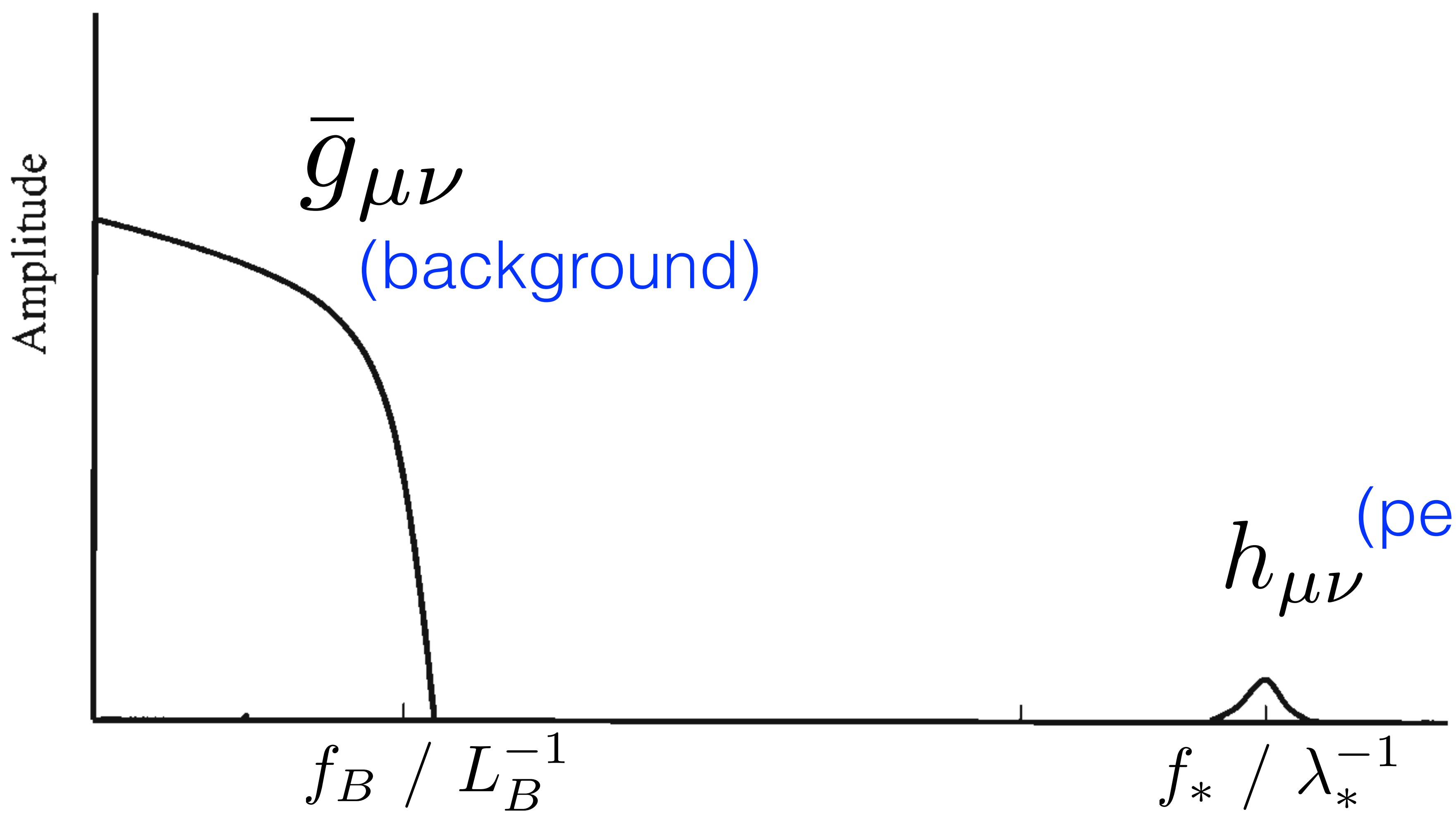

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**It can be shown that only TT *dof* contribute to < … >**

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**It can be shown that only TT *dof* contribute to < … >**

$$\boxed{t_{\mu\nu} = \frac{c^4}{32\pi G}\left\langle \partial_\mu h^{\mathrm{TT}}_{ij}\partial_\nu h^{\mathrm{TT}}_{ij}\right\rangle} \quad \Rightarrow \quad \boxed{\frac{dE}{dAdt} = \frac{c^4}{32\pi G}\left\langle \dot{h}^{\mathrm{TT}}_{ij}\dot{h}^{\mathrm{TT}}_{ij}\right\rangle}$$

GW energy-momentum tensor

GW power/area radiated

# Gravitational Wave Propagation

What about the
High Freq. / Short Scale?

$$R^{(1)}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{\mathrm{High}} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\mathrm{High}}$$

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$$\frac{|R^{(2)}_{\mu}|^{\mathrm{High}}}{|R^{(1)}_{\mu}|} \sim \mathcal{O}\left(\frac{\lambda_*}{L_B}\right) \longrightarrow |R^{(2)}_{\mu}|^{\mathrm{High}}$$ negligible

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$$R^{(1)}_{\mu\nu} = \bar{g}^{\alpha\beta}\left(D_\alpha D_{(\mu}h_{\nu)\beta} - D_\mu D_\nu h_{\alpha\beta} - D_\alpha D_\beta h_{\mu\nu}\right)$$

$$D_\mu \bar{h}_{\mu\nu} = 0 \quad \left(\ \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h\ \right)$$ Lorentz gauge

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$$R^{(1)}_{\mu\nu} = -[R^{(2)}_{\mu\nu}]^{\rm High} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\rm High}$$

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$$\frac{|R^{(2)}_{\mu}|^{\rm High}}{|R^{(1)}_{\mu}|} \sim \mathcal{O}\left(\frac{\lambda_*}{L_B}\right) \longrightarrow |R^{(2)}_{\mu}|^{\rm High}$$ negligible

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$$D_\mu \bar{h}_{\mu\nu} = 0 \quad \left( \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h \right)$$ Lorentz gauge

$$\longrightarrow \quad \boxed{D_\alpha D^\alpha \bar{h}_{\mu\nu} = 0}$$

Propagation of GWs
in curved space-time

# Gravitational Wave Propagation

What about the
High Freq. / Short Scale?

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Creation of GWs
in curved space-time

# Gravitational Wave Propagation in Cosmology

**FLRW:** $ds^2 = a^2(-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \qquad \mathrm{TT}: \begin{cases} h_{ii} = 0 \\ h_{ij,j} = 0 \end{cases}$

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**Creation of GWs in curved space-time**

**Source: Anisotropic Stress**

Eom: $\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi^{\mathrm{TT}}_{ij},}$ $\quad \boxed{\Pi_{ij} = T_{ij} - \langle T_{ij} \rangle_{\mathrm{FRW}}}$

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**Only TT degrees of freedom carry energy out of the source !**

GW Source(s): ( SCALARS , VECTOR , FERMIONS )

$$\Pi^{TT}_{ij} \propto \{\partial_i \chi^a \partial_j \chi^a\}^{TT}, \quad \{E_i E_j + B_i B_j\}^{TT}, \quad \{\bar{\psi}\gamma_i D_j \psi\}^{TT}$$

More on Gravitational Wave Definition & Propagation @

BLANCHET Lectures

# INFLATIONARY COSMOLOGY

Inflation

'cures' hBB ⇒ Cosmological Pple

(initial cond.) = Primordial perturbations

Scalar

Tensor ⇒ Irreducible GW Background

Scenarios

Extra sp

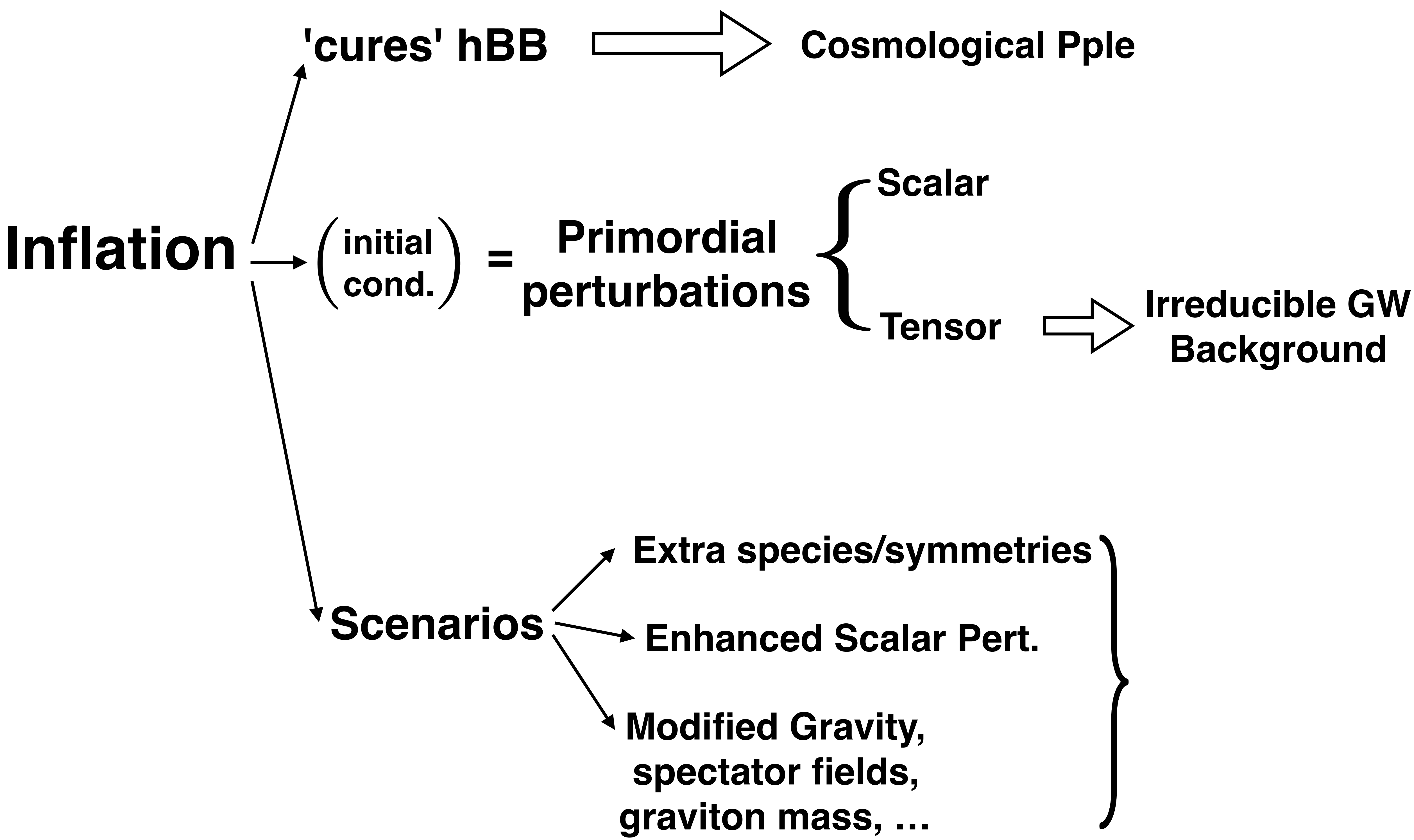
ields,

graviton mass, …

GW definition & propagation

revisited! ✓

# INFLATIONARY COSMOLOGY



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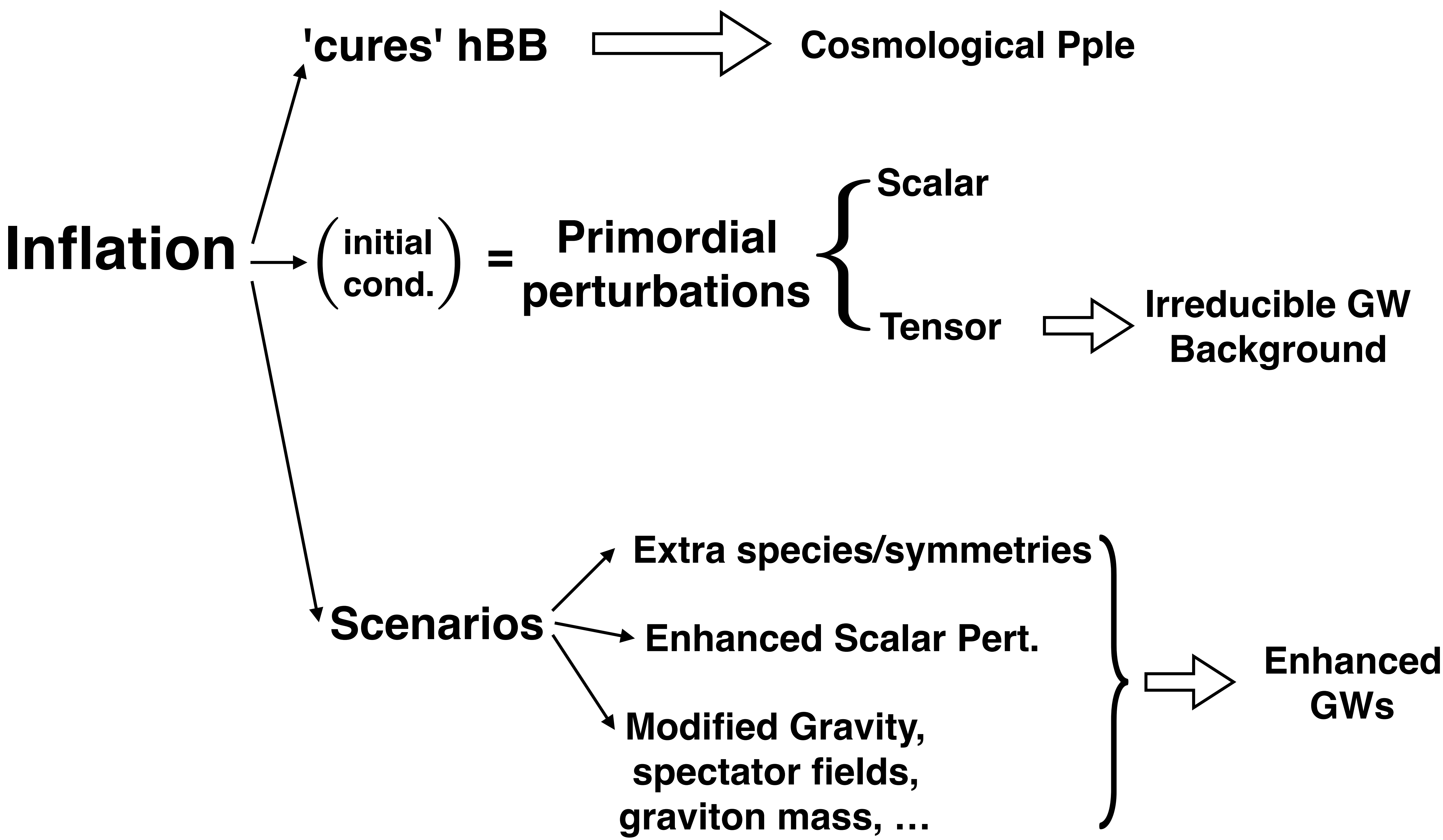

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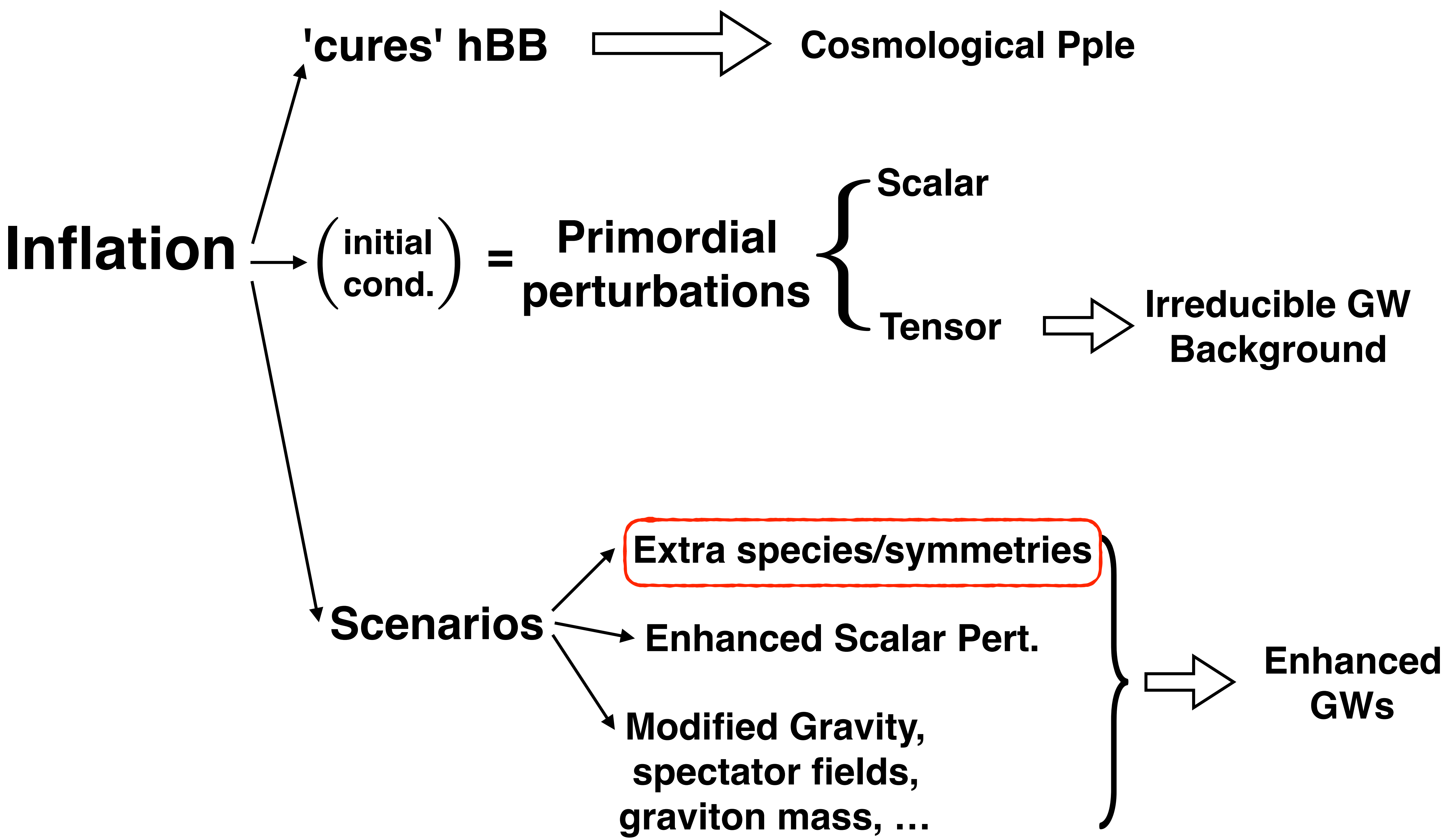

# INFLATIONARY MODELS

B-mode experiments: $\boxed{H_{\rm inf} \approx 10^{14} GeV \Rightarrow E_{\rm inf} \approx 10^{16} GeV}$

- Proof of Inflation ?
- Scale of Inflation !
- Quantum Gravity !
- Super-Planckian Excursion*
- Identification as GW** ✓

(*,**: to be discussed today)

# INFLATIONARY MODELS

## Single field slow-roll models: $V(\phi) \propto \phi^n$

$$\boxed{H_{\rm inf} \approx 10^{14} GeV \Rightarrow E_{\rm inf} \approx 10^{16} GeV} \quad \Rightarrow \quad \text{Super-Planckian Excursion !}$$

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$\Rightarrow$ **Super-Planckian Excursion !**

**Super-Planckian Excursion** $\longrightarrow$ $\Delta\phi \sim \mathcal{O}(10) m_p \times (r/0.1)^{1/2}$

**Operators Not Suppressed**

$$\mathcal{O}_6((\phi/m_p)^2), \mathcal{O}_8((\phi/m_p)^4), \ldots \Rightarrow \Delta m_\phi > H$$

$$\Rightarrow \begin{cases} \text{No } \delta\phi \text{ Fluctuations!} \\ \eta > 1 \Rightarrow \text{Inflation Stops} \end{cases}$$

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**Serious problem for High-Scale Inflationary models ??**

# INFLATIONARY MODELS

## Axion-Inflation

Freese, Frieman, Olinto '90; . . .

Shift symmetry $\phi \to \phi + C$ on couplings to other fields

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**derivative couplings to:** fermions gauge fields

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Not the QCD axion;

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**(derivative couplings)**

$= \partial_\mu K^\mu$

topological term

$[\phi\partial_\mu K^\mu = K^\mu \partial_\mu \phi]$

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$[\phi \partial_\mu K^\mu = K^\mu \partial_\mu \phi]$

With shift symmetry, $\Delta V \propto V_{\cancel{\text{shift}}}$ ⇒ **Protected against radiative corrections !**

# INFLATIONARY MODELS

## Axion-Inflation

Shift symmetry $\phi \to \phi + C$

[J. Cook, L. Sorbo (arXiv:1109.0022)] [N. Barnaby, E. Pajer, M. Peloso (arXiv:1110.3327)]

$$V(\varphi) + \frac{\varphi}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

inflaton $\varphi$ = pseudo-scalar axion

$\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$

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Photon: 2 helicities

$$\vec{A}(\tau, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \vec{\epsilon}_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}) A_\lambda(\tau, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]$$

$$\left[ a_\lambda(\mathbf{k}), a^\dagger_{\lambda'}(\mathbf{k}') \right] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$$

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**Chiral instability**

$$\boxed{A_+(\tau, k) \cong \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi a H}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}}$$

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$$A_+ \propto e^{\pi \xi}\,, \quad |A_-| \ll |A_+|$$

$A_+$ exponentially amplified,

$A_-$ has no amplification

**Gauge field excitation creates chiral GWs !**

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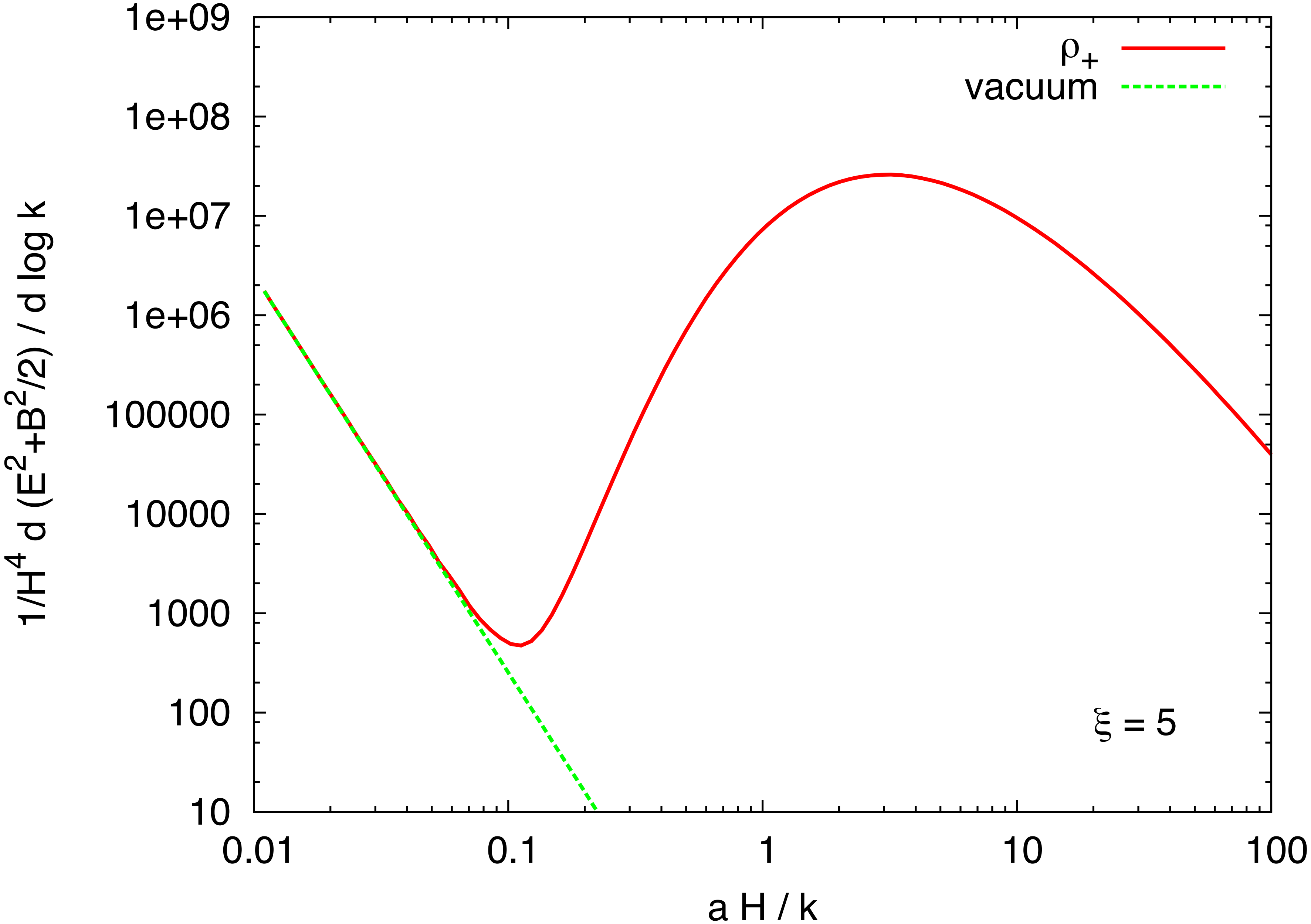
**GW left-chirality only**

$A_\mu$ **Chiral**

# INFLATIONARY MODELS

## Axion-Inflation

Gauge field energy density

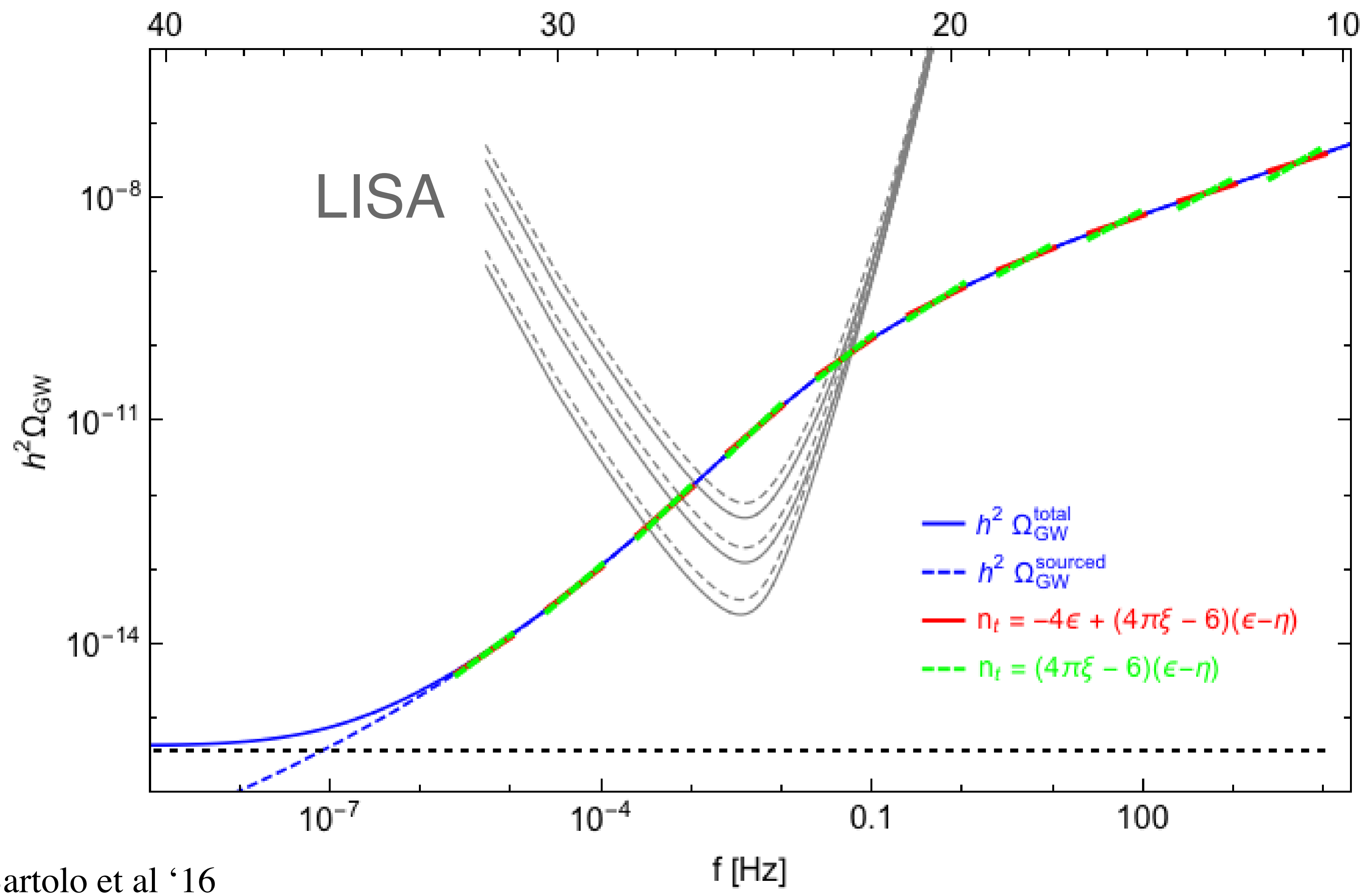



Gauge field (chiral) excitation !

# INFLATIONARY MODELS

## Axion-Inflation

GW energy spectrum today




Bartolo et al '16

# INFLATIONARY MODELS

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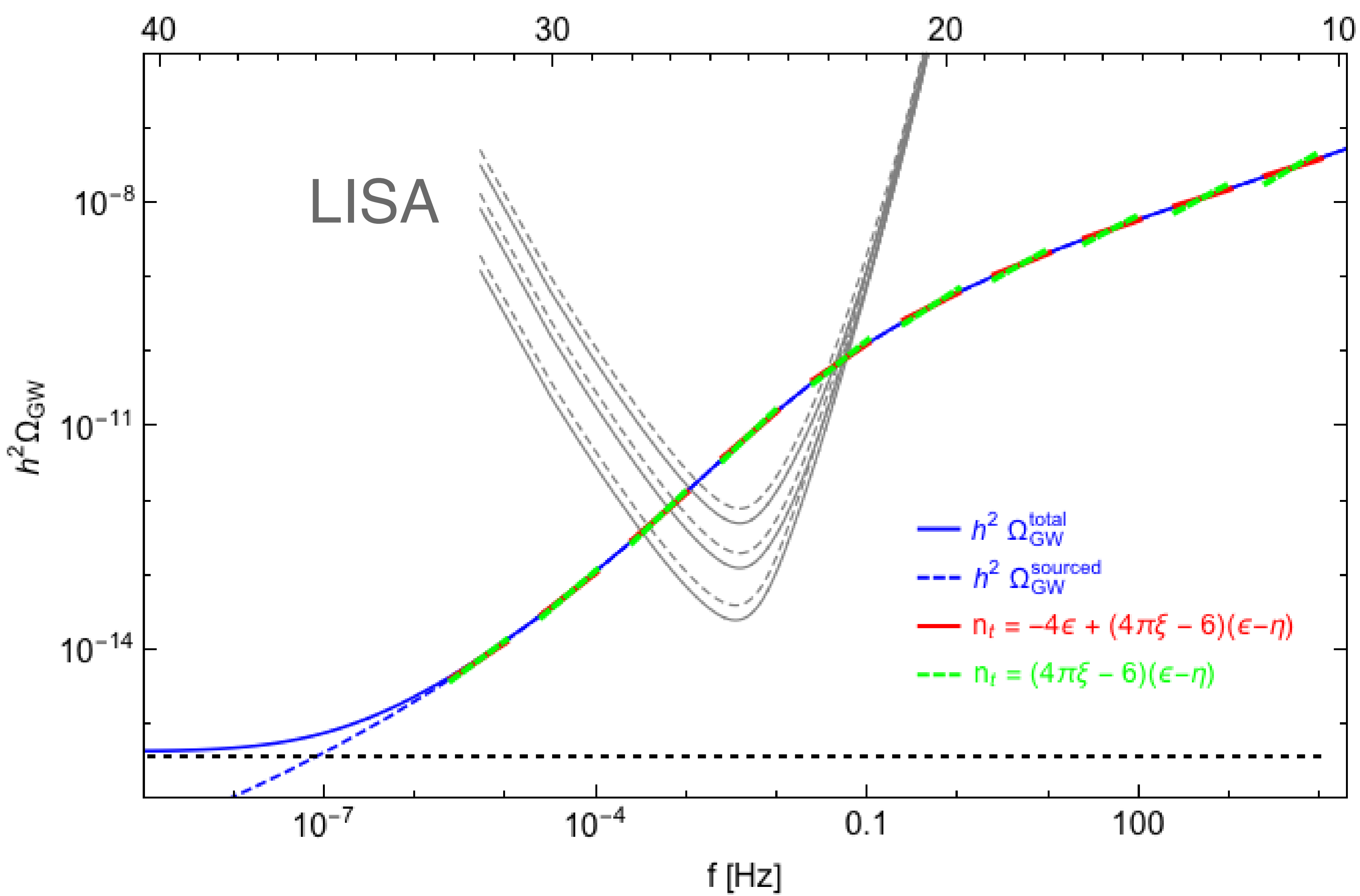


Gauge fields source a blue tilted & chiral GW background

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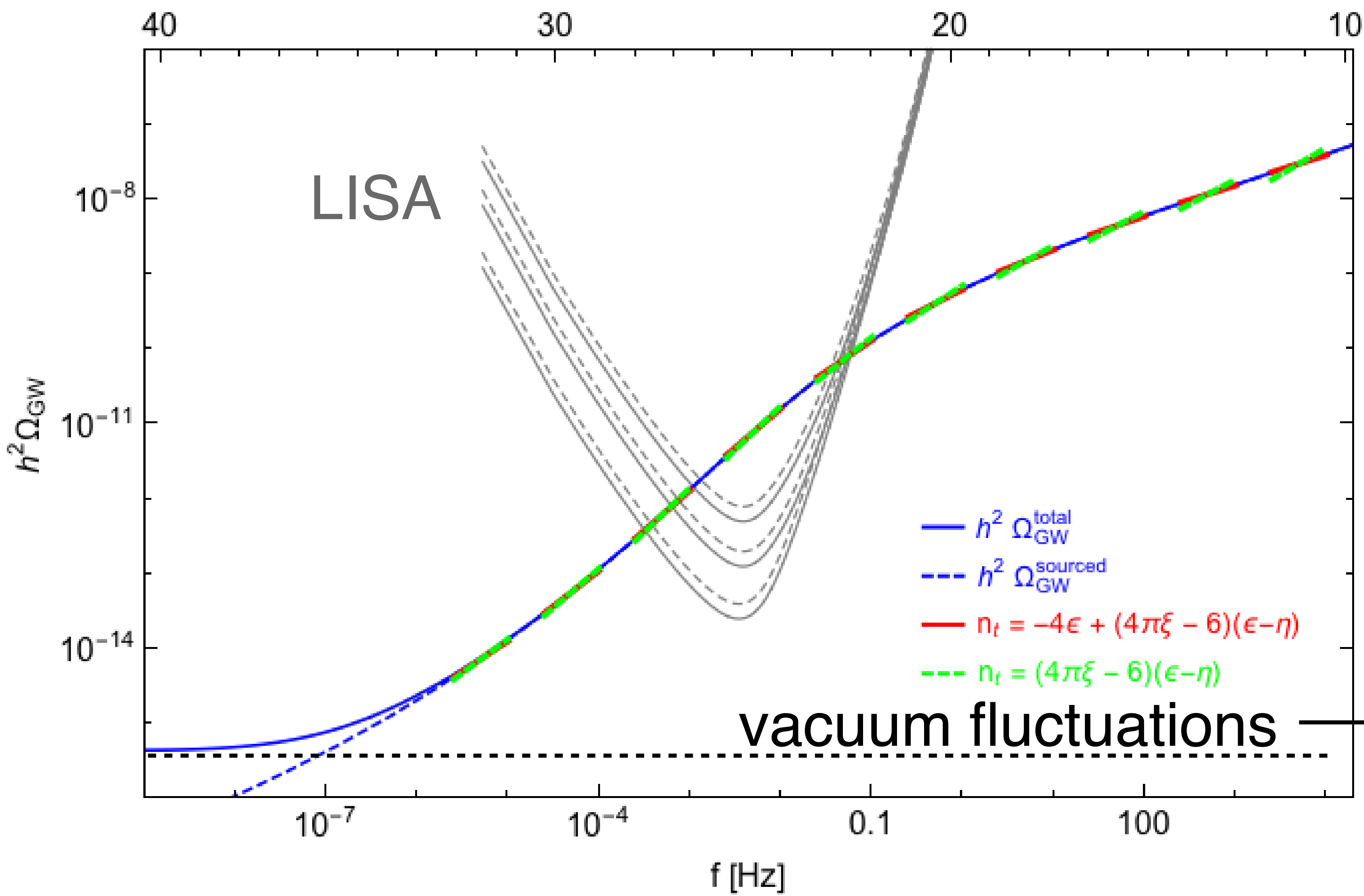



$(\Omega_{\rm Rad}/24)\Delta_h^2$

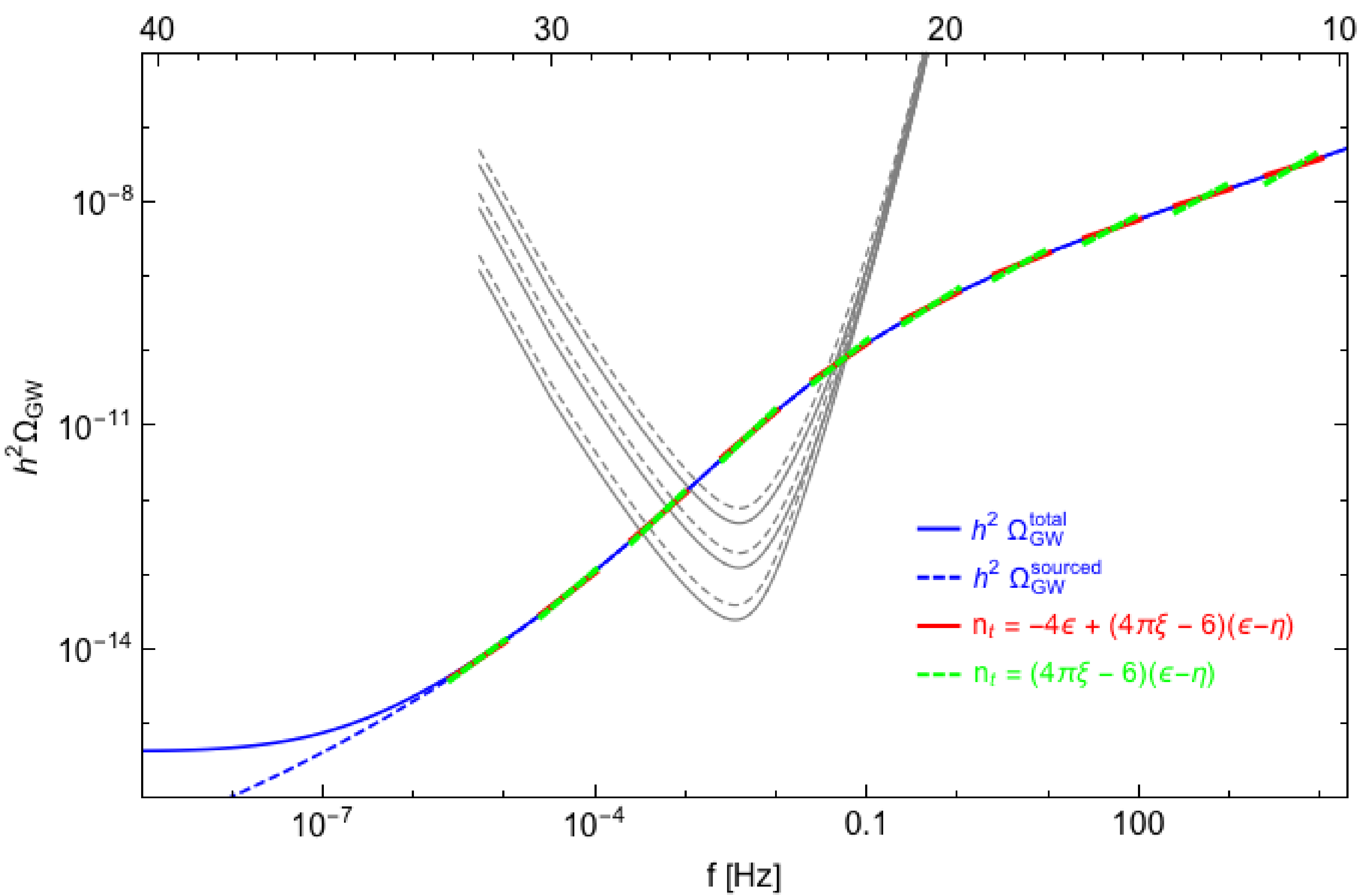
Gauge fields source a blue tilted & chiral GW background

# INFLATIONARY MODELS

## Axion-Inflation

Bartolo et al '16

$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T}$$



# INFLATIONARY MODELS

## Axion-Inflation

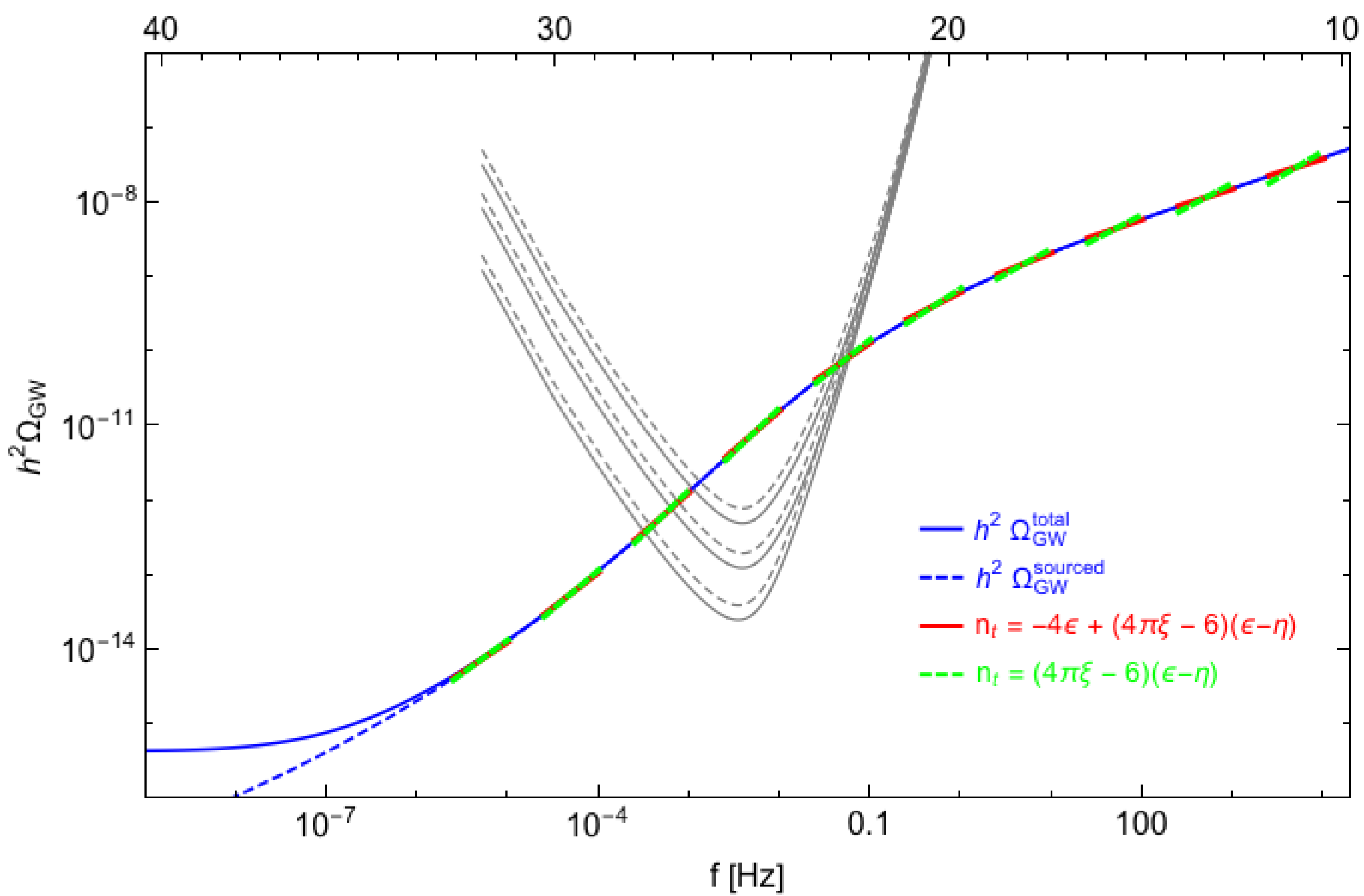
Bartolo et al '16

$$h^2\Omega_{\rm gw} = A_* \left(\frac{f}{f_*}\right)^{n_T} \rightarrow \boxed{n_T \simeq (4\pi\xi - 6)(\epsilon_H - \eta)}$$

$$A_* \rightarrow \boxed{1.5 \cdot 10^{-13} \frac{H^4}{M_{Pl}^4} \frac{e^{4\pi\xi}}{\xi^6}} \,,$$

$$\boxed{\xi \equiv \frac{\alpha\dot{\phi}}{2fH}}$$

$\boxed{H,\, \xi,\, \epsilon_H - \eta}$ **(3 parameters!)**



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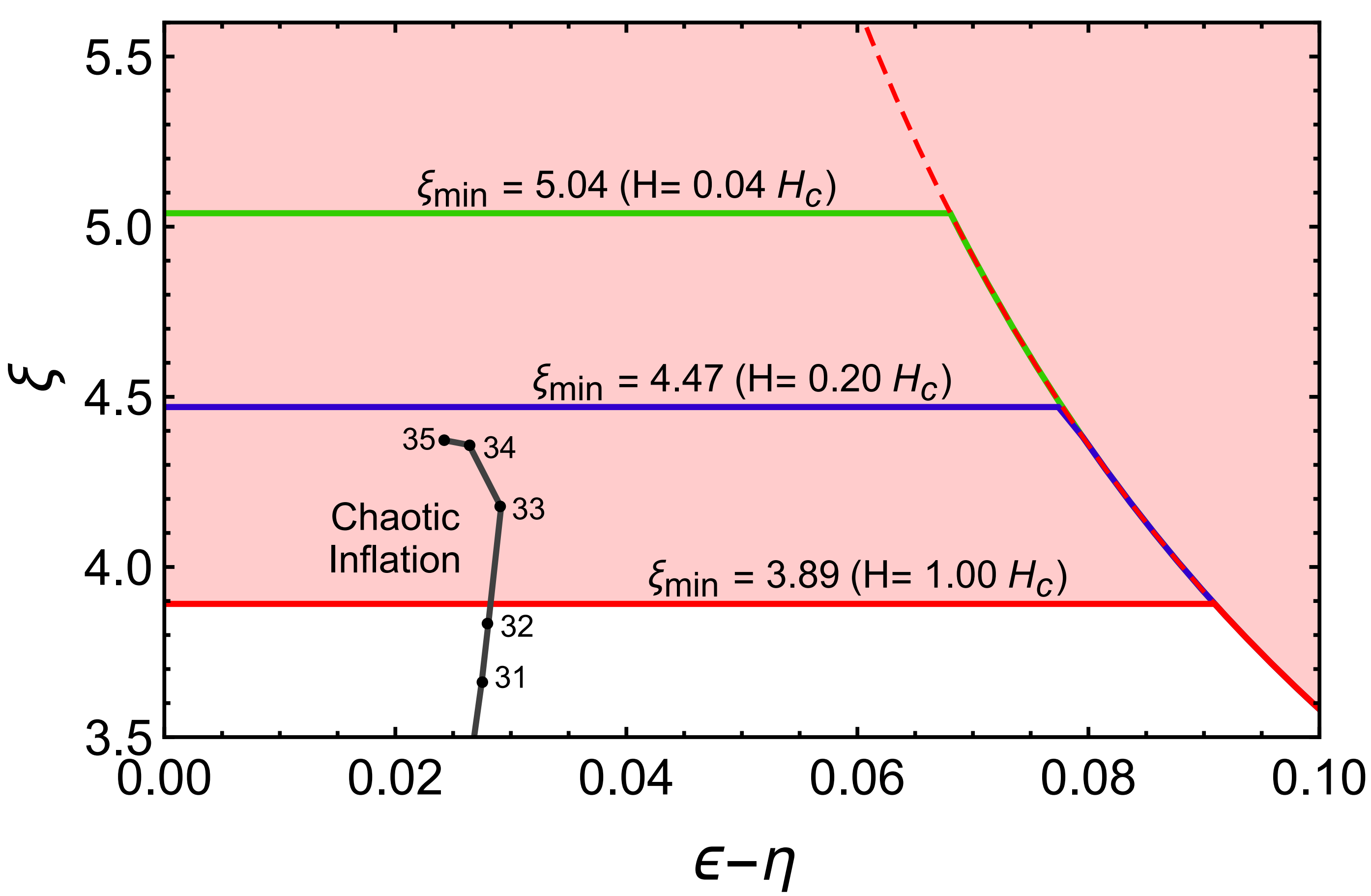
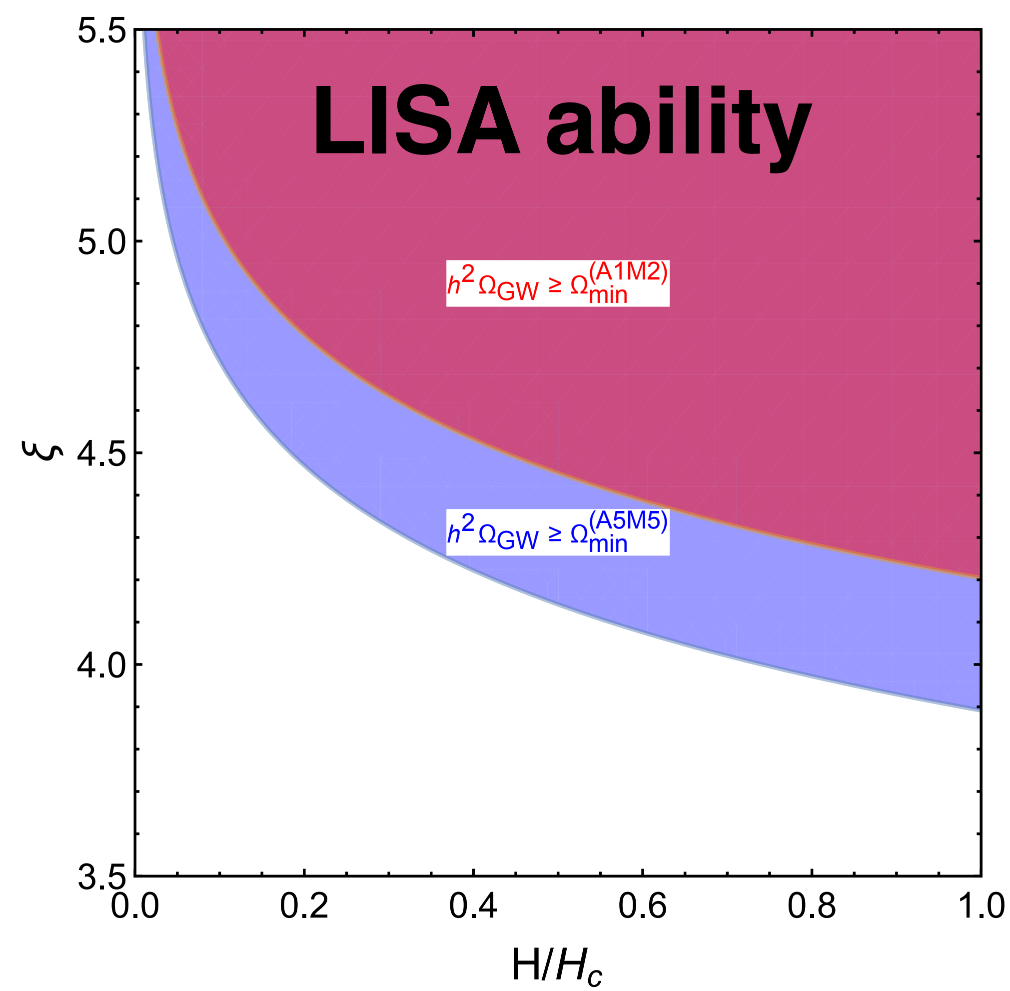
**LISA ability**



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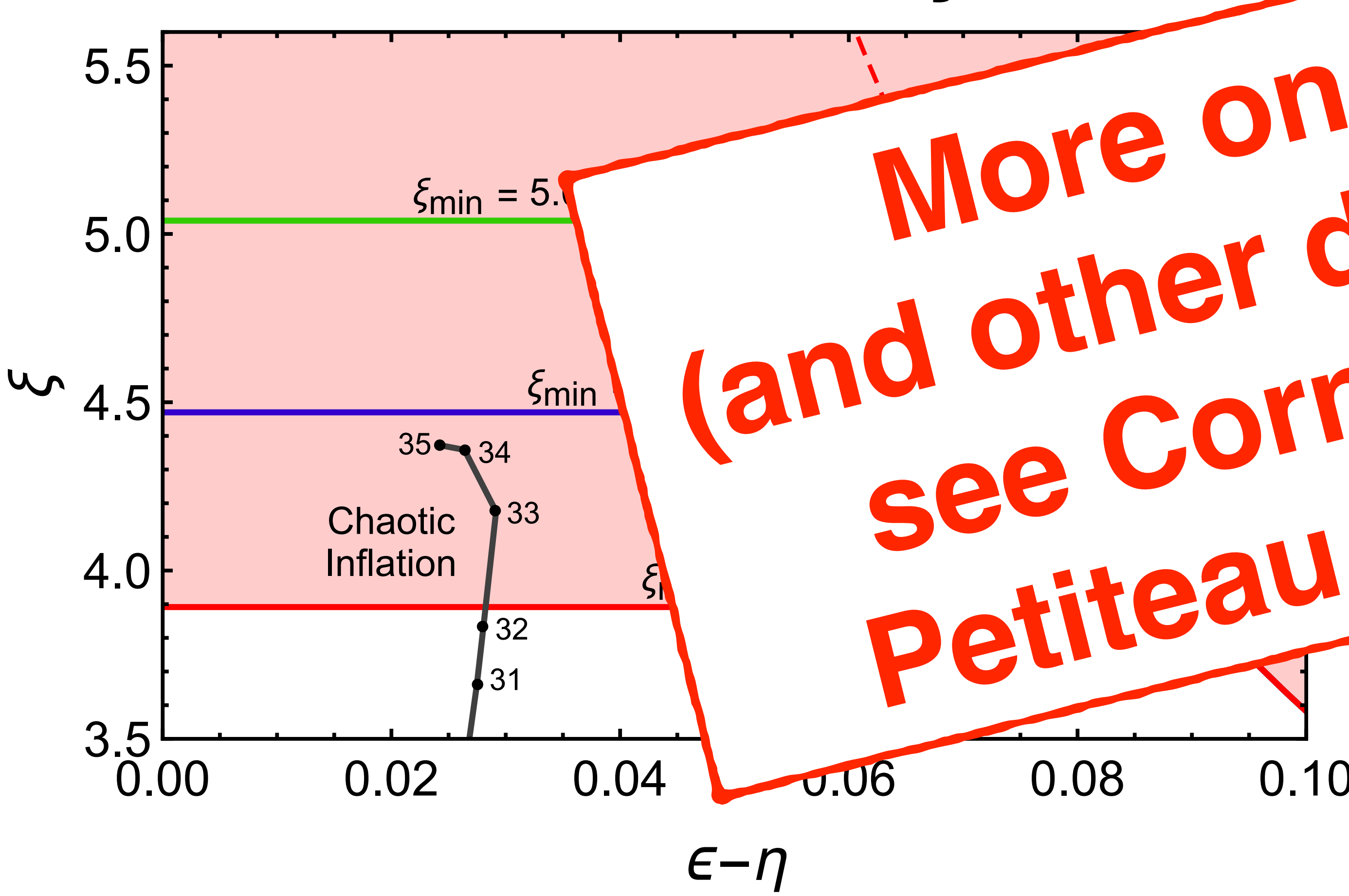
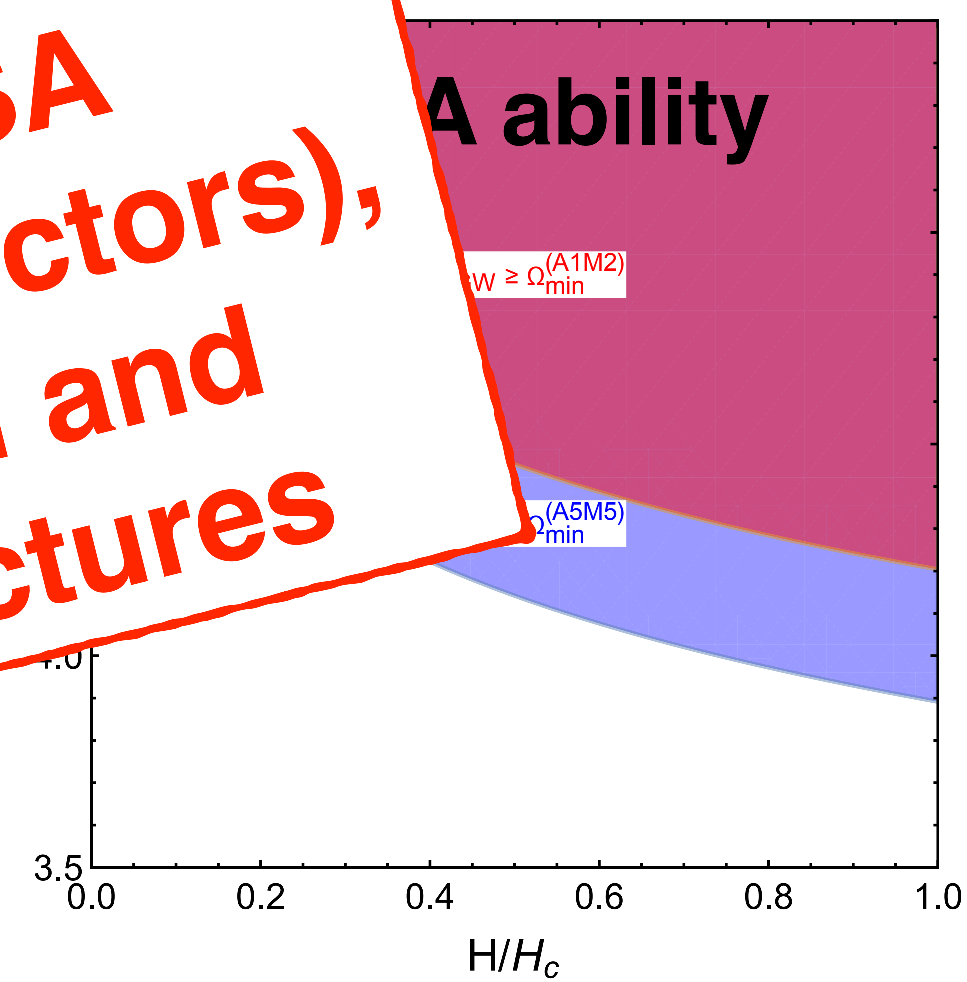
**LISA ability**




**More on LISA (and other detectors), see Cornish and Petiteau Lectures**

# INFLATIONARY MODELS

**Axion-Inflation:** ***Shift*** **symmetry** $\longrightarrow$ **Natural (chiral) coupling to** $A_\mu$

**huge excitation of fields ! (photons)**

# INFLATIONARY MODELS

**Axion-Inflation:** ***Shift*** **symmetry** $\longrightarrow$ **Natural (chiral) coupling to** $A_\mu$

**huge excitation of fields ! (photons)**

**What if there are arbitrary fields coupled to the inflaton ?** $\rightarrow$ **large excitation of these fields !?**

**(i.e. no need of extra symmetry)**

**will they create GWs?**

# INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ?

(i.e. no need of extra symmetry)

GW generation !?

# INFLATIONARY MODELS

**fields coupled to the inflaton ?** → **large excitation ?**

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$$\text{inflaton } \phi \longrightarrow V(\phi)$$

$$-\mathcal{L}_\chi = (\partial\chi)^2/2 + g^2(\phi - \phi_0)^2\chi^2/2$$ **Scalar Fld**

$$-\mathcal{L}_\psi = \bar{\psi}\gamma^\mu\partial_\mu\psi + g(\phi - \phi_0)\bar{\psi}\psi$$ **Fermion Fld**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - gA_\mu)\Phi)|^2 - V(\Phi^\dagger\Phi)$$ **Gauge Fld ($\Phi = \phi e^{i\theta}$)**

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All 3 cases:

$$m = g(\phi(t) - \phi_0) \;\Rightarrow\; \dot{m} \overset{\text{non-adiabatic}}{\gg} m^2 \quad \text{during} \quad \Delta t_{\rm na} \sim 1/\mu\,, \qquad \boxed{\mu^2 \equiv g\dot{\phi}_0}$$

$$\boxed{n_k = \mathrm{Exp}\{-\pi(k/\mu)^2\}}$$ **Non-adiabatic field excitation (particle creation)**

# INFLATIONARY MODELS

fields coupled to the inflaton ? → large excitation ✓

(i.e. no need of extra symmetry)

GW generation !?

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**Non-adiabatic field excitation (particle creation !)**
**(spin-independent)**

# INFLATIONARY MODELS

**fields coupled to the inflaton ?** → **large excitation** ✓

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$$\boxed{n_k \;=\; \mathrm{Exp}\{-\pi(k/\mu)^2\}}$$

**Non-adiabatic field excitation (particle creation !)**
**(spin-independent)**

In all three cases (scalars, fermions, and vectors)

GWs generated by anisotropic distribution of the created species

(Only $k \ll \mu$ long-wave modes excited)

# INFLATIONARY MODELS

**fields coupled to the inflaton ?** → **large excitation** ✓

**(i.e. no need of extra symmetry)** **GW generation !?**

$$\boxed{n_k \;=\; \mathrm{Exp}\{-\pi(k/\mu)^2\}}$$

**Non-adiabatic field excitation (particle creation !)**
**(spin-independent)**

In all three cases (scalars, fermions, and vectors)

**GWs power spectrum:** $\mathcal{P}_h^{(\mathrm{tot})}(k) \;=\; \mathcal{P}_h^{(\mathrm{vac})}(k) + \mathcal{P}_h^{(\mathrm{pp})}(k)$ **from particle production**

GW Source(s): ( SCALARS , VECTOR , FERMIONS )

$$\Pi_{ij}^{TT} \;\propto\; \{\partial_i\chi^a\partial_j\chi^a\}^{TT},\;\; \{E_iE_j + B_iB_j\}^{TT},\;\; \{\bar{\psi}\gamma_i D_j\psi\}^{TT}$$

# INFLATIONARY MODELS

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$\boxed{(W \lesssim 0.5)}$


N. Barnaby *et al.*, Phys. Rev. **D86**, 103508 (2012), [1206.6117].
J. L. Cook and L. Sorbo, Phys. Rev. **D85**, 023534 (2012), [1109.0022].

# INFLATIONARY MODELS

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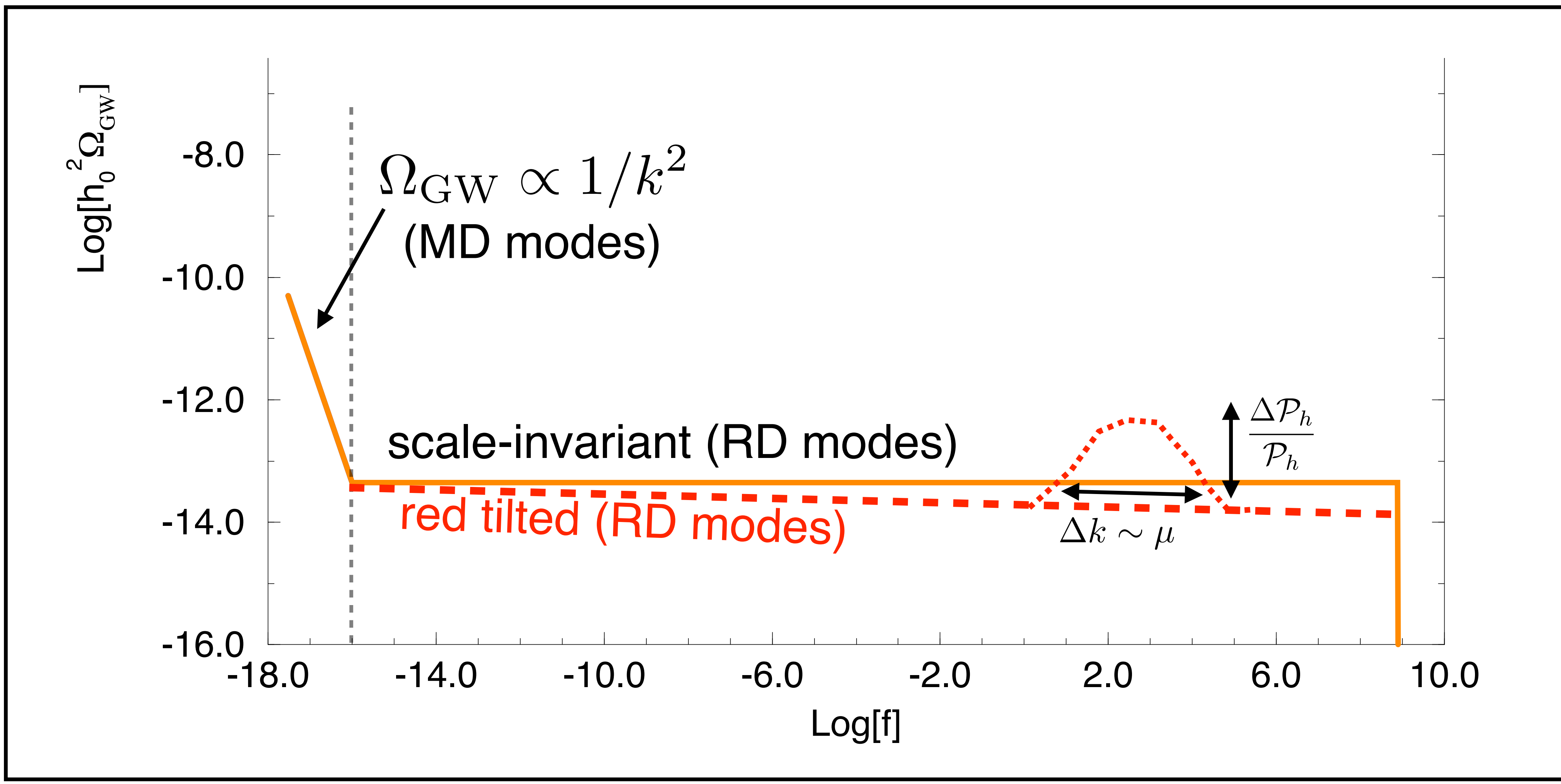
$$\mu^2 \equiv g\dot{\phi}_0$$



# INFLATIONARY MODELS

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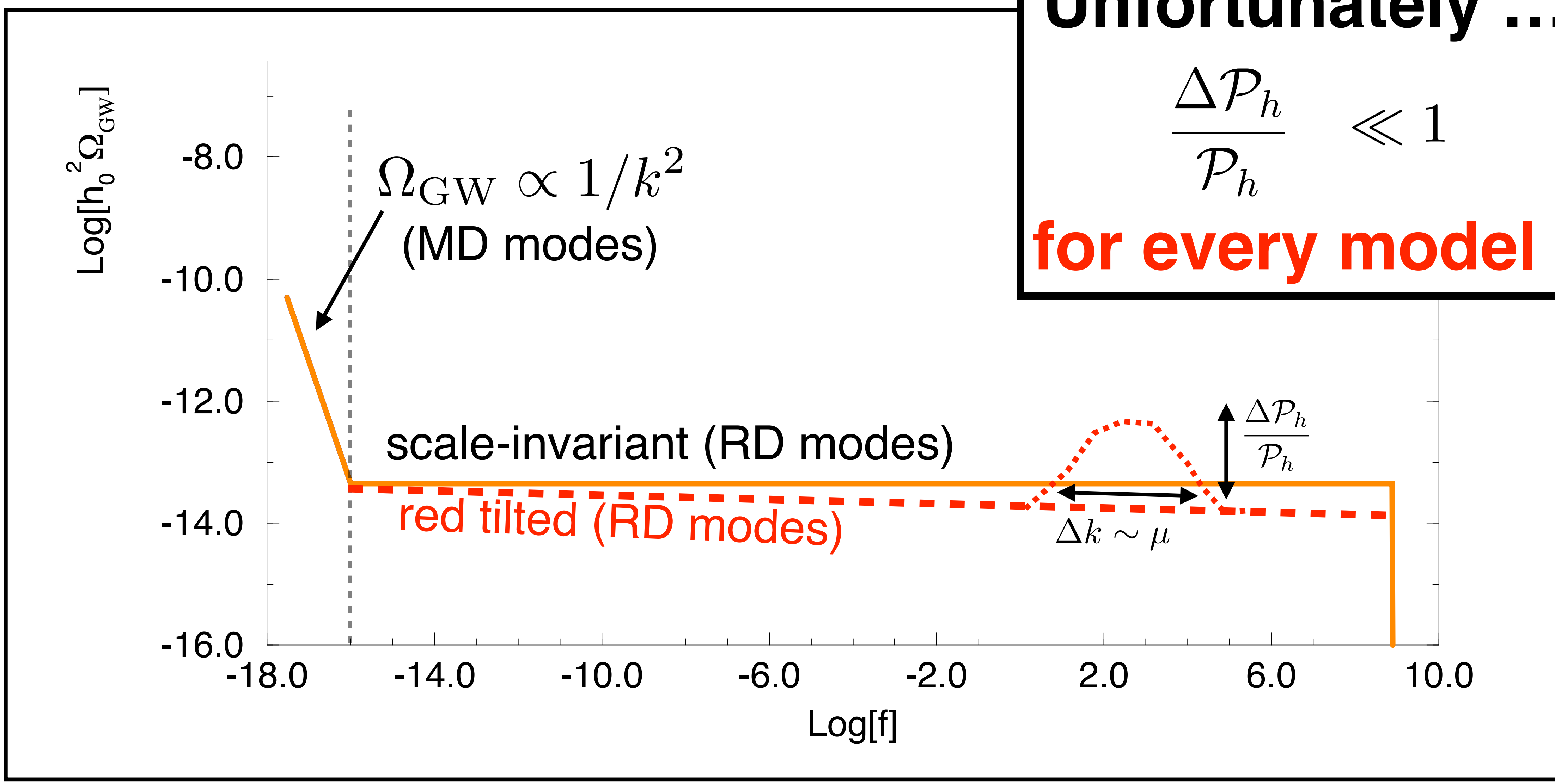
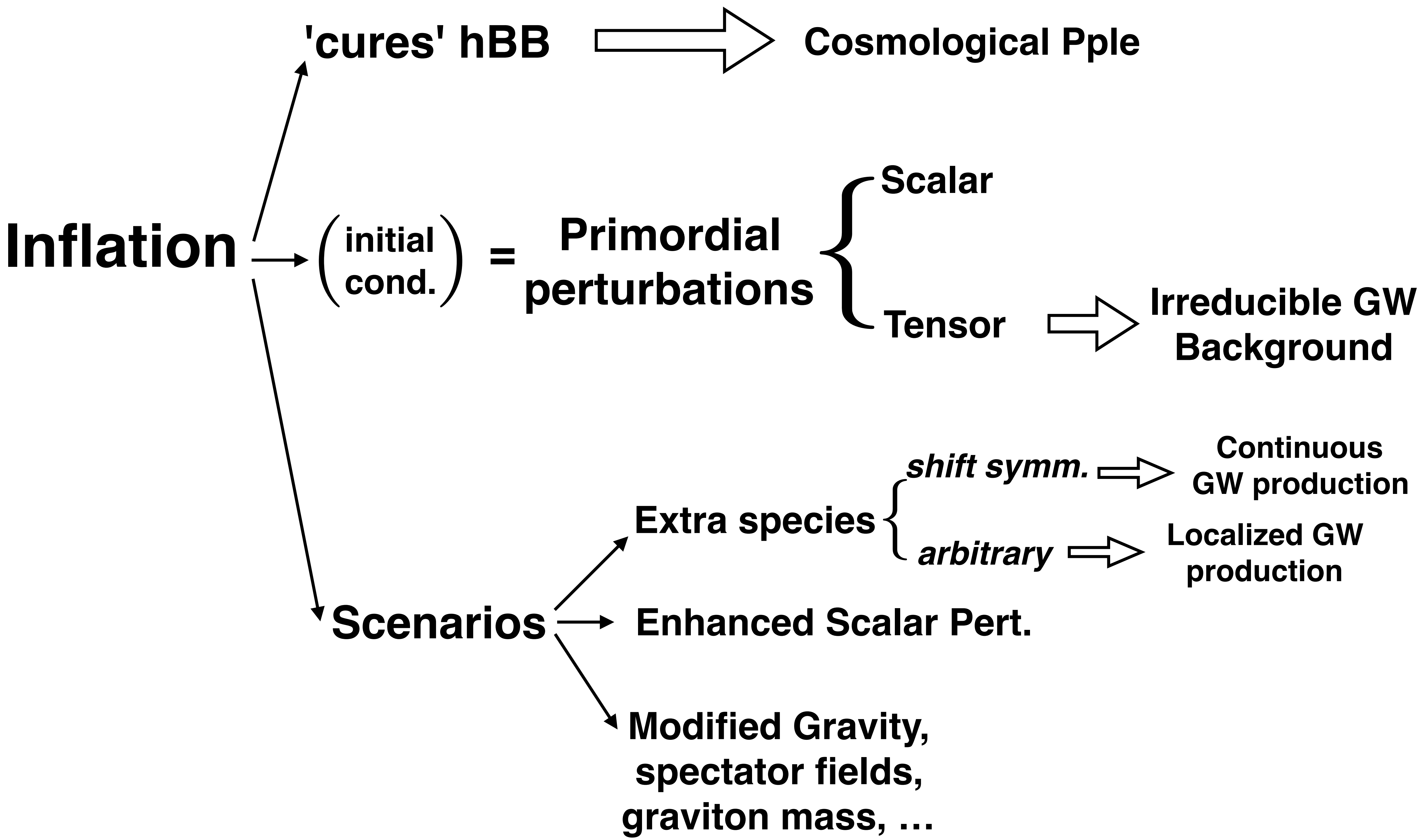



**Unfortunately …**

$$\frac{\Delta \mathcal{P}_h}{\mathcal{P}_h} \ll 1$$

**for every model !**

# INFLATIONARY COSMOLOGY



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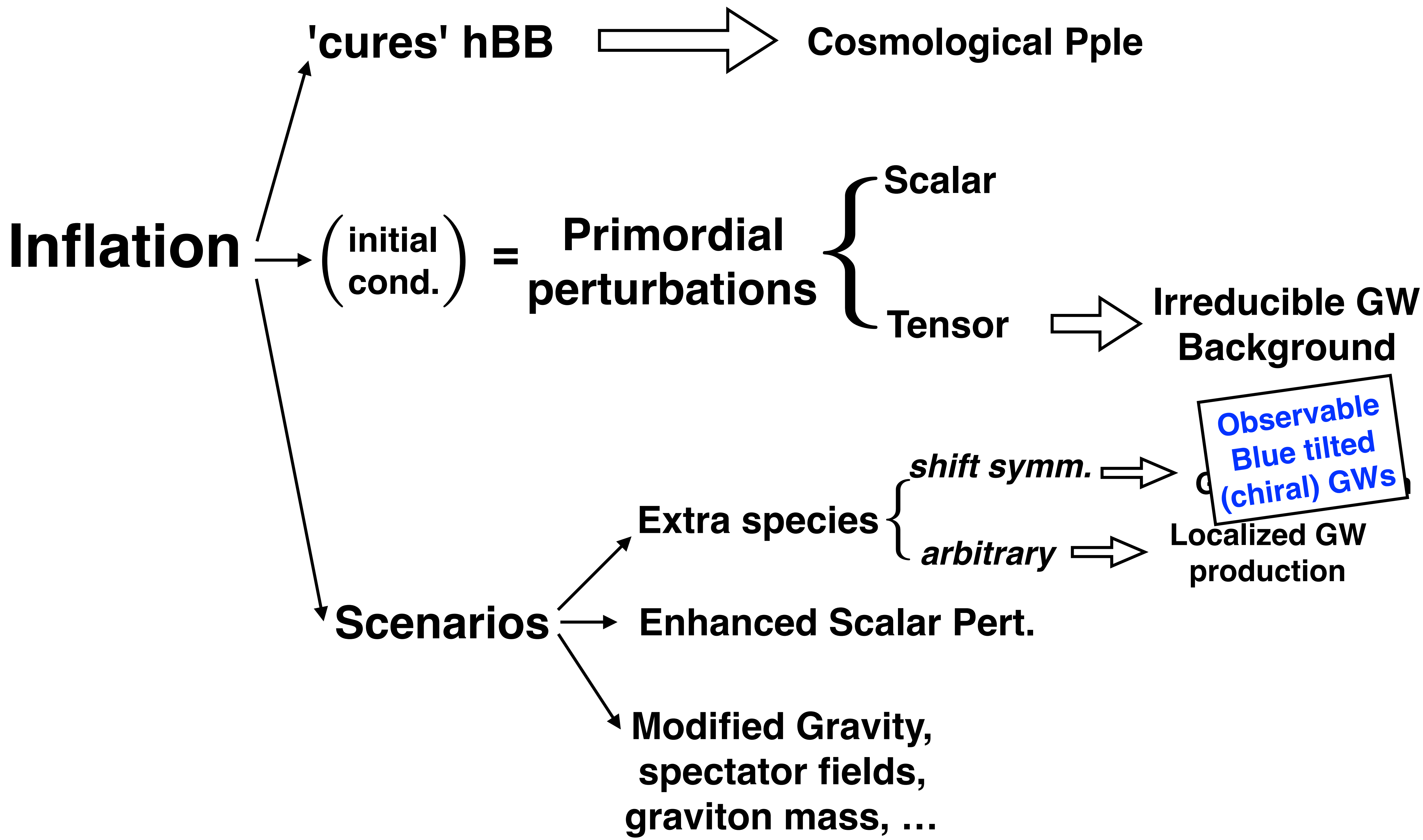


# INFLATIONARY COSMOLOGY

Inflation

'cures' hBB ⇒ Cosmological Pple

$\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ = Primordial perturbations

Scalar

Tensor ⇒ Irreducible GW Background

Scenarios

Extra species

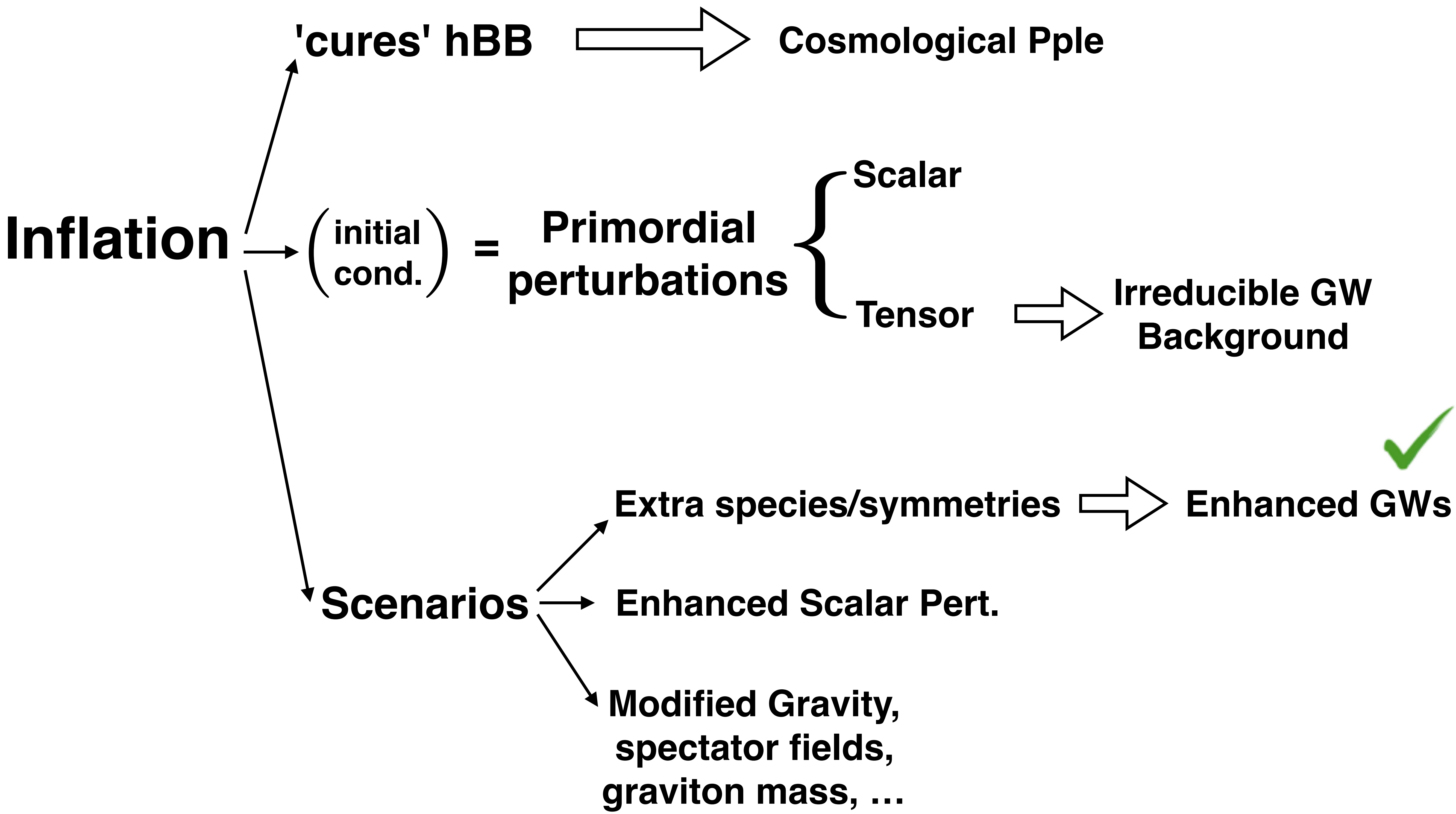
*shift symm.* ⇒ Observable Blue tilted (chiral) GWs

*arbitrary* ⇒ negligible GW production

Enhanced Scalar Pert.

Modified Gravity, spectator fields, graviton mass, …

# INFLATIONARY COSMOLOGY



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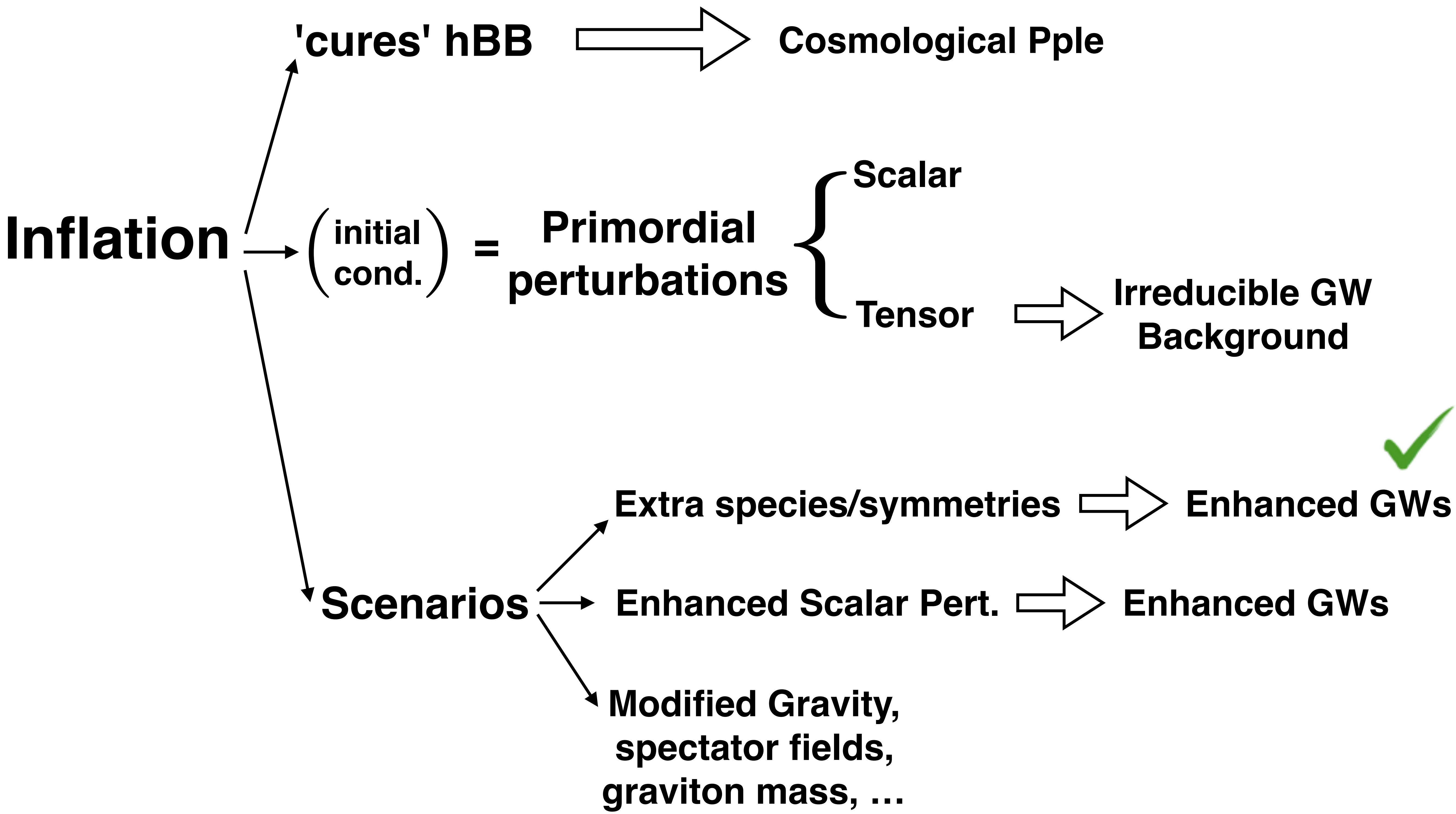

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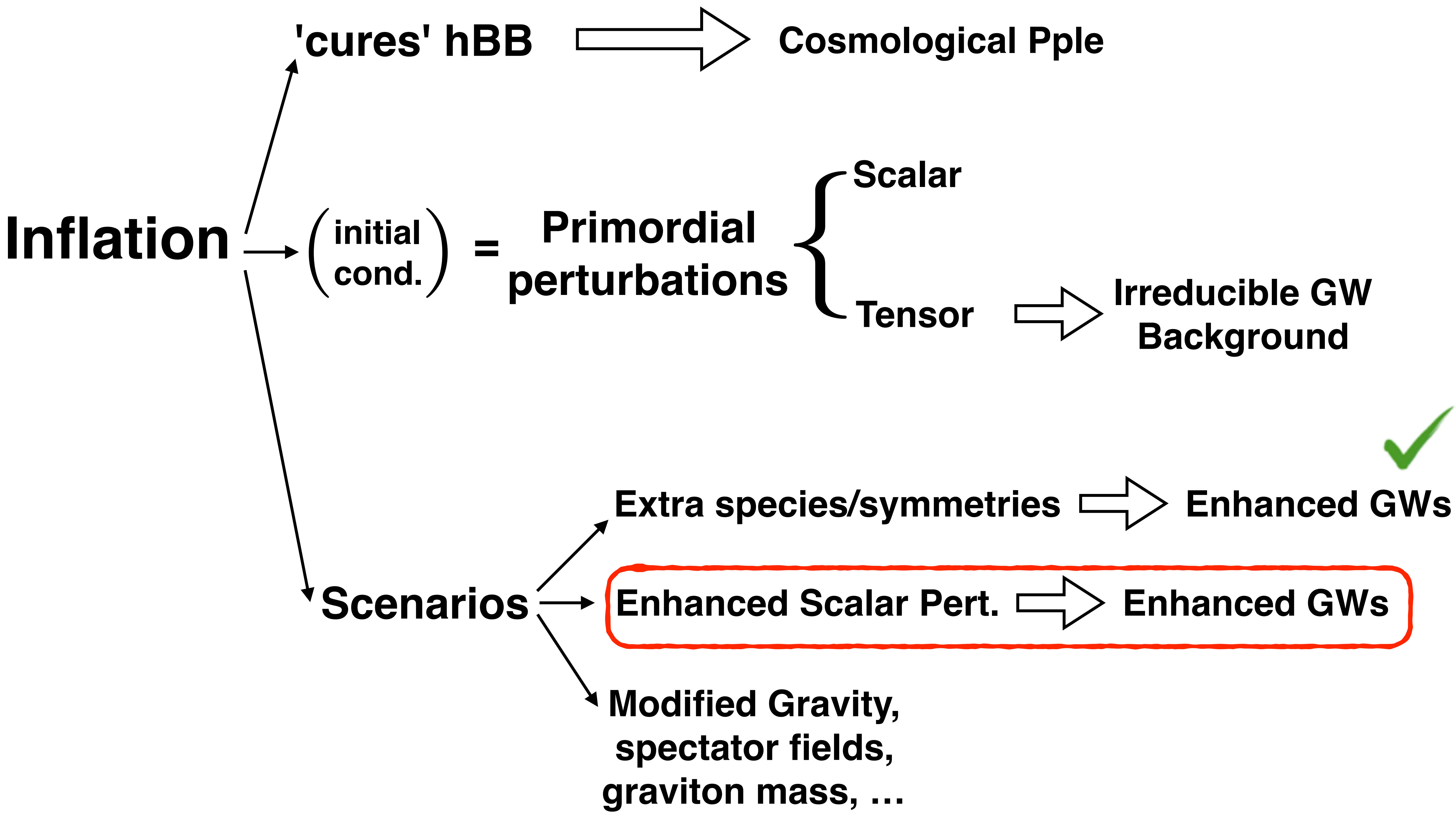

# INFLATIONARY MODELS

H & I

INFLATION ➡ $g_{\mu\nu} = g_{\mu\nu}^{FRW} + \delta g_{\mu\nu}[\mathcal{R}, h_{ij}]$

$$\Delta^2_{\mathcal{R}}(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$n_s - 1 \equiv 2(\eta - 2\epsilon)$

$$\Delta^2_h(k) = \frac{2}{\pi^2}\left(\frac{H}{m_p}\right)^2\left(\frac{k}{aH}\right)^{n_t}$$

$n_t \equiv -2\epsilon$

(quasi-)scale invariance $\longleftrightarrow$ Slow roll monotonic potentials

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$\Delta^2_{\mathcal{R}}(k)$

$- 2\epsilon)$

$\Delta^2_h($

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$\boxed{n_t \equiv -2\epsilon}$

**Monotonic Single Field Slow Roll**

(quasi-)scale invariance $\longleftrightarrow$ Slow roll monotonic potentials

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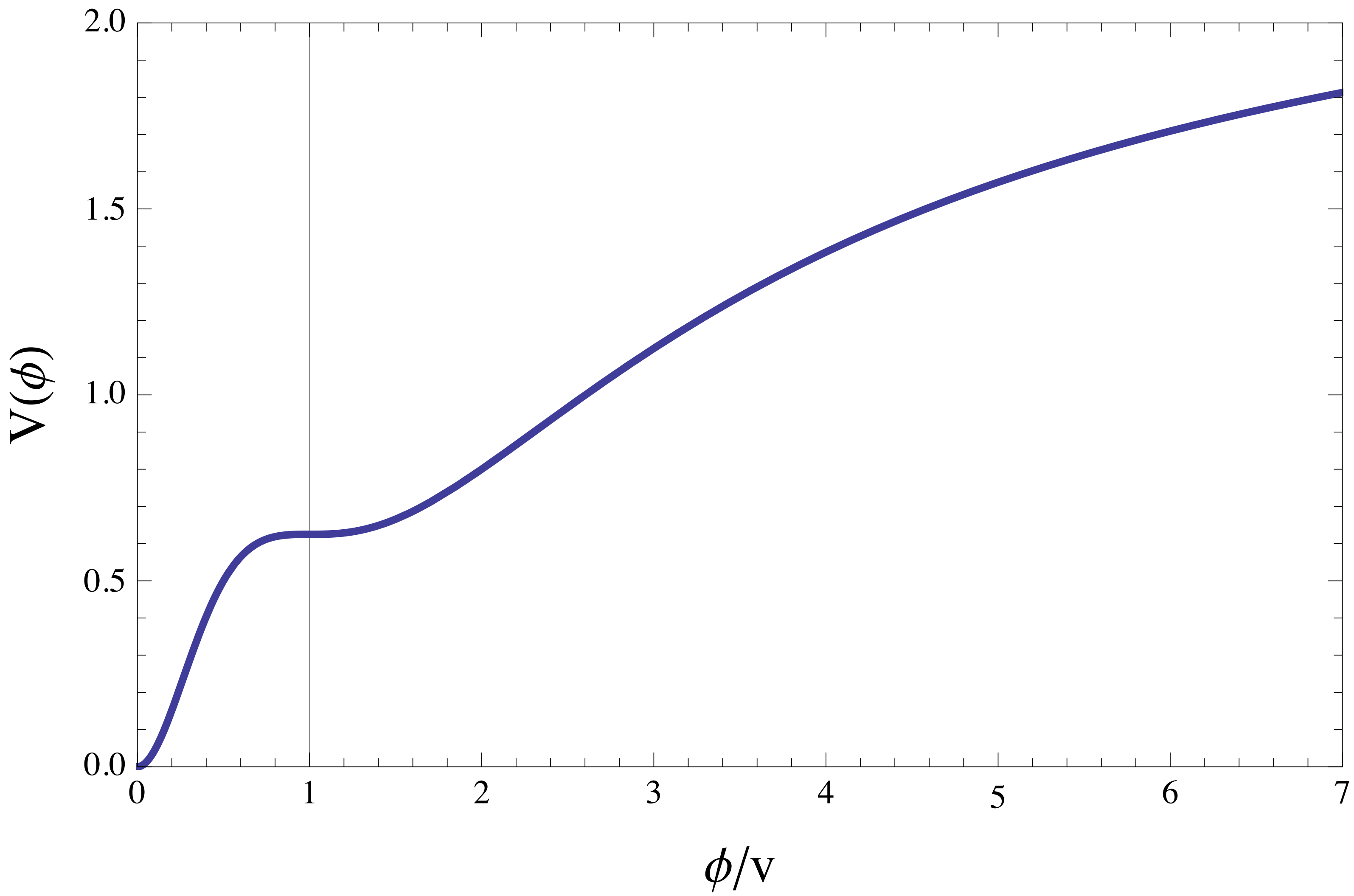


What if the potential is not monotonic ?

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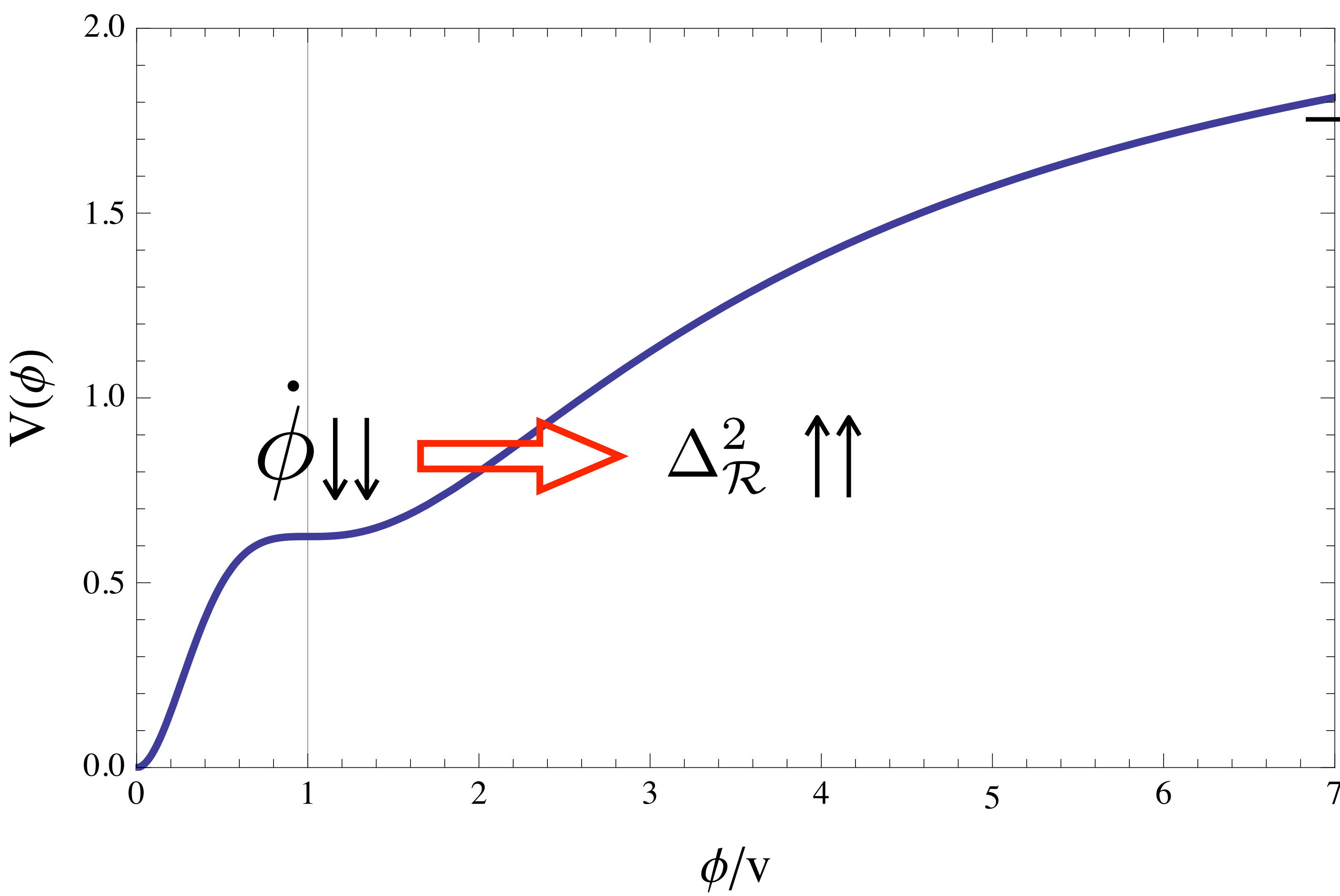


$$\Delta^2_{\mathcal{R}}\big|_{\mathrm{CMB}} \sim 3 \cdot 10^{-9}$$

**What if the potential is not monotonic ?**

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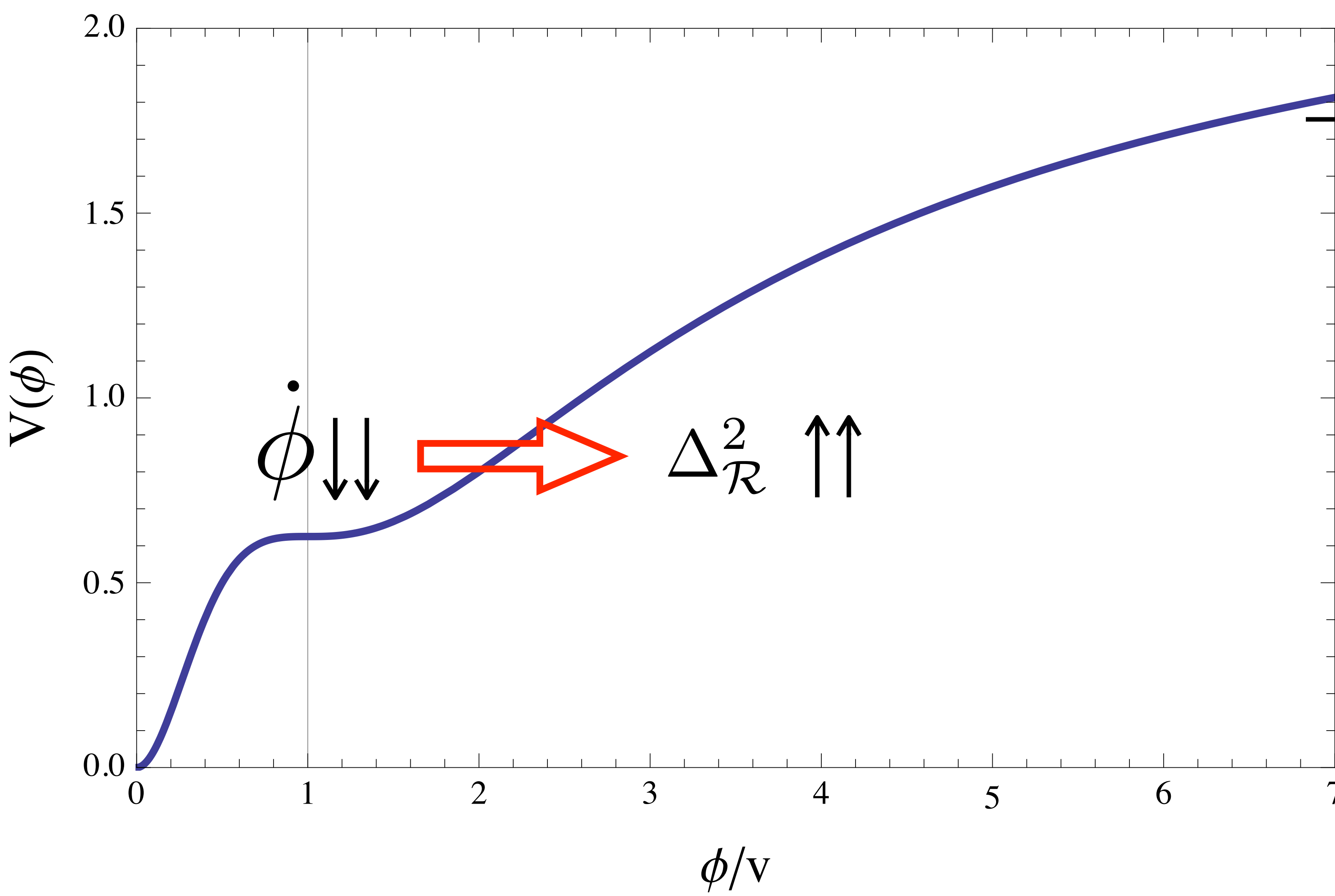


$$\Delta^2_{\mathcal{R}}\big|_{\mathrm{CMB}} \sim 3\cdot 10^{-9}$$

What if the potential is not monotonic ?

$\Delta^2_{\mathcal{R}}$ **greatly enhanced!**
**(at small scales)**

Garcia-Bellido, Morales, 2017

# INFLATIONARY MODELS

H & I

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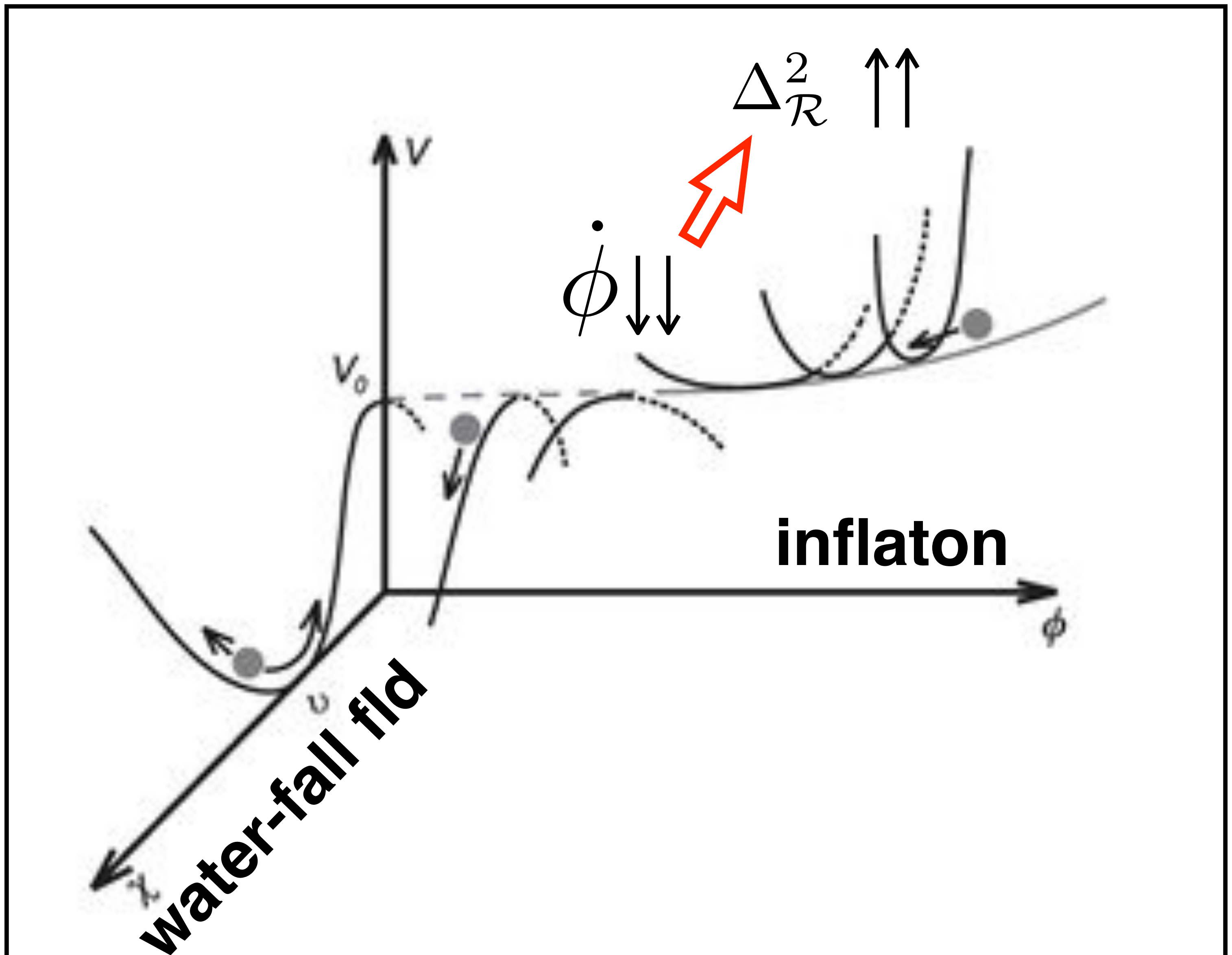


What if it is multi-field inflation ?

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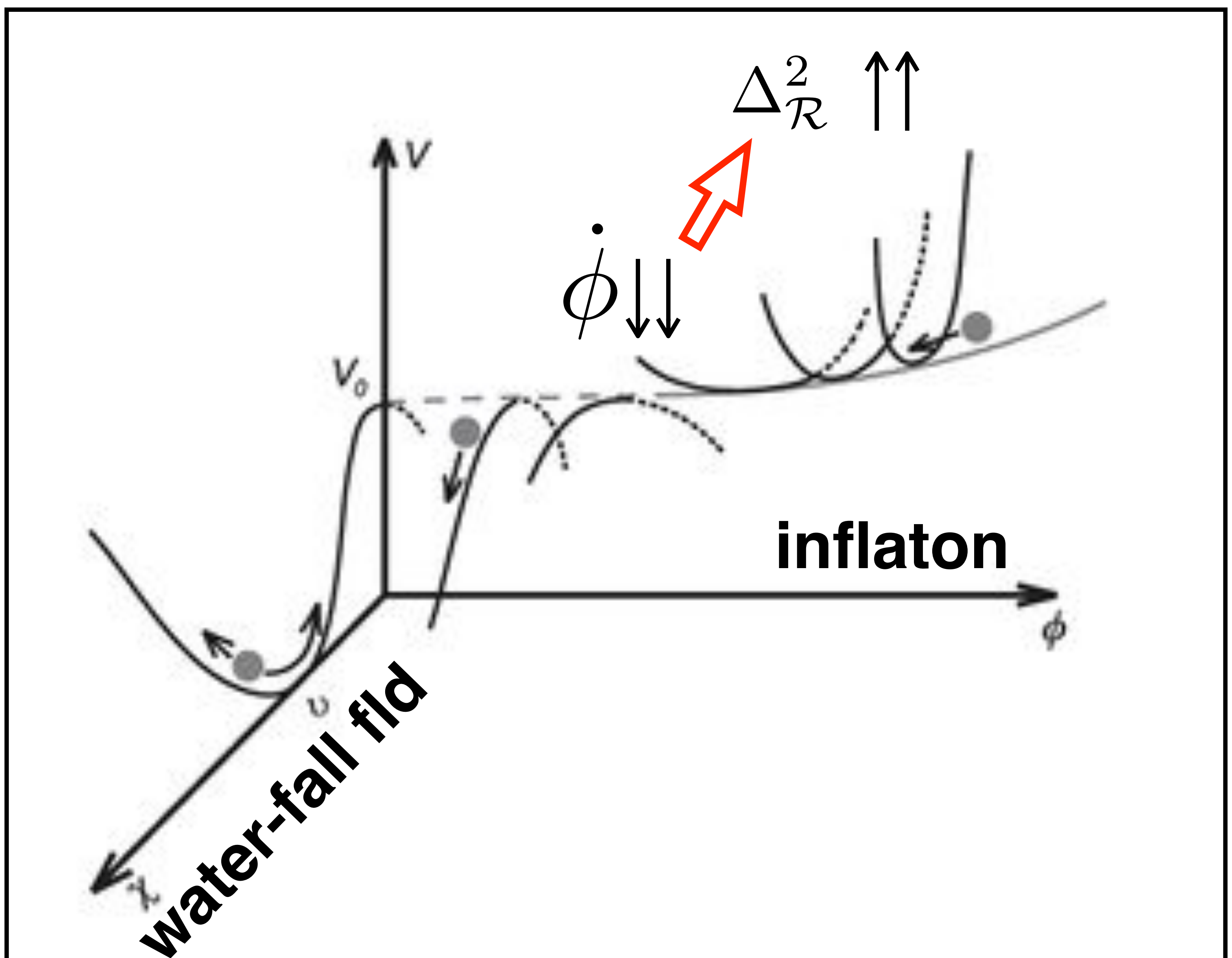


## What if it is multi-field inflation ?

**also possible to greatly enhance $\Delta^2_{\mathcal{R}}$ (at small scales)**

Linde 1992, Barnaby et al 2006
Clesse, Garcia-Bellido 2015/16

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INFLATION

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$\Delta^2_{\mathcal{R}}(k)$ ... $H^4$ ...

... $- 2\epsilon)$

$\Delta^2_h($ ... $\left(aH\right)$

$n_t \equiv -2\epsilon$

**Monotonic Single Field Slow Roll**

# INFLATIONARY MODELS

INFLATION

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**Monotonic Single Field Slow Roll**

**IF** $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ **possible to enhance** $\Delta^2_{\mathcal{R}}$ **(at small scales)**

# INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\} \Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

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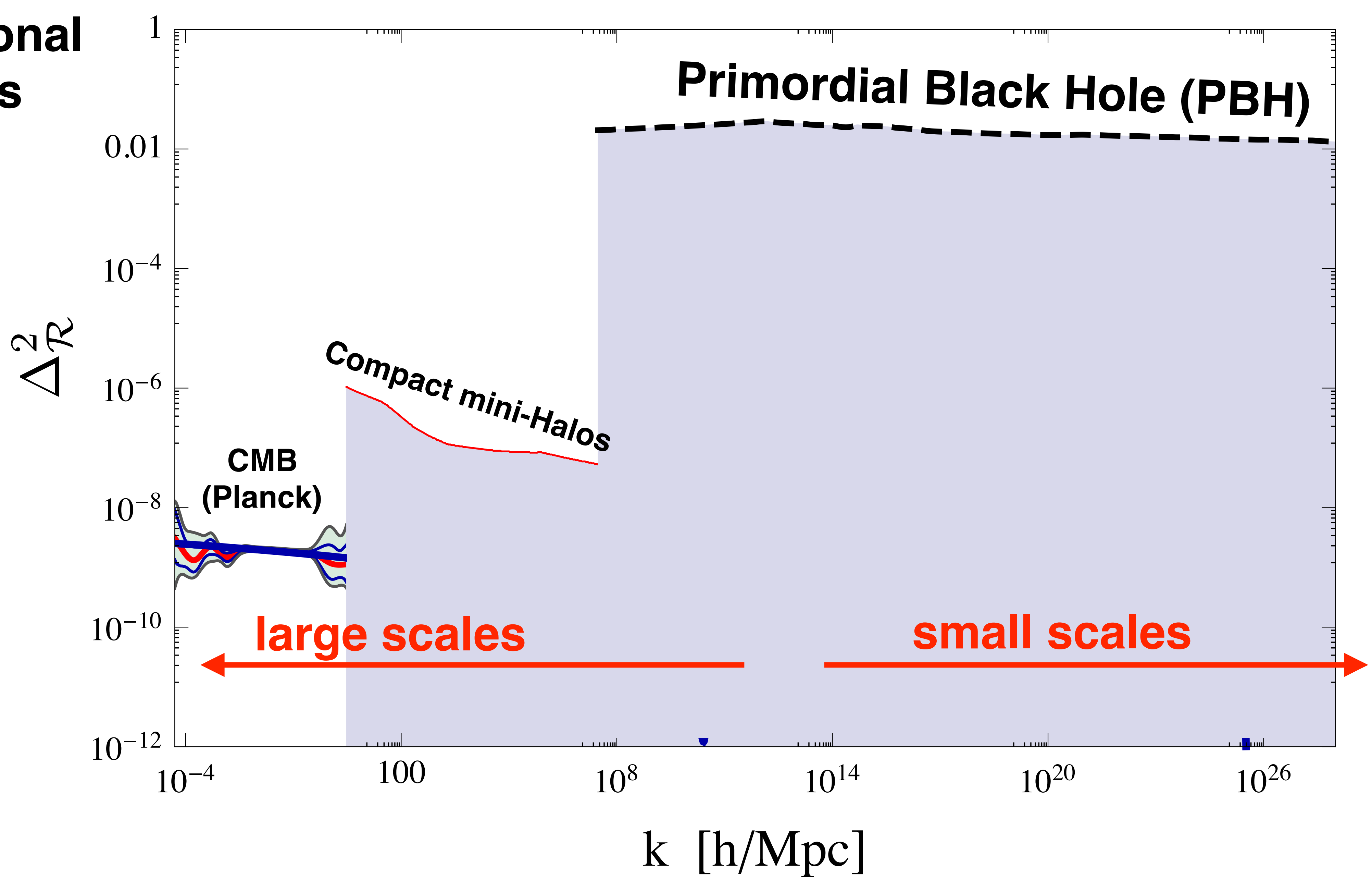
INFLATION → IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ ⇨ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

Observational constraints



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**Let us suppose** $\boxed{\Delta^2_{\mathcal{R}} \gg \Delta^2_{\mathcal{R}}\big|_{\rm CMB} \sim 3 \cdot 10^{-9}}$, @ small scales

$$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta^2 + [(1-2\Psi)\delta_{ij} + 2F_{(i,j)} + h_{ij}]dx^i dx^j]$$

# INFLATIONARY MODELS

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$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = S^{TT}_{ij}}$$

**2nd Order Pert.**

$$\begin{aligned} S_{ij} \;=\; & 2\Phi\partial_i\partial_j\Phi - 2\Psi\partial_i\partial_j\Phi + 4\Psi\partial_i\partial_j\Psi + \partial_i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) \\ & - \frac{2c_s^2}{3w\mathcal{H}}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi) \end{aligned}$$

**Phys.Rev. D81 (2010) 023527**

**Phys.Rev. D75 (2007) 123518**

D. Wands et al, 2006-2010

# INFLATIONARY MODELS

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**2nd Order Pert.**

$$S_{ij} = \sim \Phi * \Phi$$

# INFLATIONARY MODELS

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**2nd Order Pert.**

$$\boxed{\Omega_{gw,0}(k) = F_{\rm rad}\,\Omega_{\gamma,0}\,\Delta^4_{\mathcal{R}}(k)}$$

$$\boxed{F_{\rm rad} = \frac{8}{3}\left(\frac{216^2}{\pi^3}\right) 8.3 \times 10^{-3} f_{ns} \sim 30}$$

$f_{ns} \sim 1$

**Phys.Rev. D81 (2010) 023527** **Phys.Rev. D75 (2007) 123518** D. Wands et al, 2006-2010

# INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

**BBN** $\Omega_{gw,0} < 1.5 \times 10^{-5}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 0.1 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

Phys.Rev. D81 (2010) 023527

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**LIGO** $\Omega_{gw,0} < 6.9 \times 10^{-6} \longrightarrow \triangle^2_{\mathcal{R}} < 0.07 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

Phys.Rev. D81 (2010) 023527

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**PTA** $\Omega_{gw,0} < 4 \times 10^{-8}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 5 \times 10^{-3} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

**Phys.Rev. D81 (2010) 023527**

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INFLATION $\Rightarrow$ IF $\left\{\begin{array}{l}\text{non-monotonic}\\ \text{multi-field}\end{array}\right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

**BBN** $\Omega_{gw,0} < 1.5 \times 10^{-5}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 0.1 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

**LIGO** $\Omega_{gw,0} < 6.9 \times 10^{-6}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 0.07 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

**PTA** $\Omega_{gw,0} < 4 \times 10^{-8}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 5 \times 10^{-3} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

**LISA** $\Omega_{gw,0} < 10^{-13}$ $\longrightarrow$ $\triangle^2_{\mathcal{R}} < 1 \times 10^{-5} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$

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# INFLATIONARY MODELS

INFLATION $\Rightarrow$ IF $\left\{\begin{array}{l}\text{non-monotonic} \\ \text{multi-field}\end{array}\right\} \Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

| | | | |
|---|---|---|---|
| **BBN** | $\Omega_{gw,0} < 1.5 \times 10^{-5}$ | $\longrightarrow$ | $\triangle^2_{\mathcal{R}} < 0.1 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$ |
| **LIGO** | $\Omega_{gw,0} < 6.9 \times 10^{-6}$ | $\longrightarrow$ | $\triangle^2_{\mathcal{R}} < 0.07 \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$ |
| **PTA** | $\Omega_{gw,0} < 4 \times 10^{-8}$ | $\longrightarrow$ | $\triangle^2_{\mathcal{R}} < 5 \times 10^{-3} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$ |
| **LISA** | $\Omega_{gw,0} < 10^{-13}$ | $\longrightarrow$ | $\triangle^2_{\mathcal{R}} < 1 \times 10^{-5} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$ |
| **BBO** | $\Omega_{gw,0} < 10^{-17}$ | $\longrightarrow$ | $\triangle^2_{\mathcal{R}} < 3 \times 10^{-7} \left(\frac{F_{rad}}{30}\right)^{-\frac{1}{2}}$ |

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INFLATION $\Rightarrow$ IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ $\Rightarrow$ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

IF $\Delta^2_{\mathcal{R}}$ very enhanced $\Rightarrow$ Primordial Black Holes (PBH) may be produced!

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INFLATION → IF $\left\{ \begin{array}{l} \text{non-monotonic} \\ \text{multi-field} \end{array} \right\}$ ⇨ possible to enhance $\Delta^2_{\mathcal{R}}$ (at small scales)

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**PBH still a candidate for Dark Matter**
(though there is debate)

Clesse & Garcia-Bellido, 2015-2017
Kamionkowski et al 2016-2017

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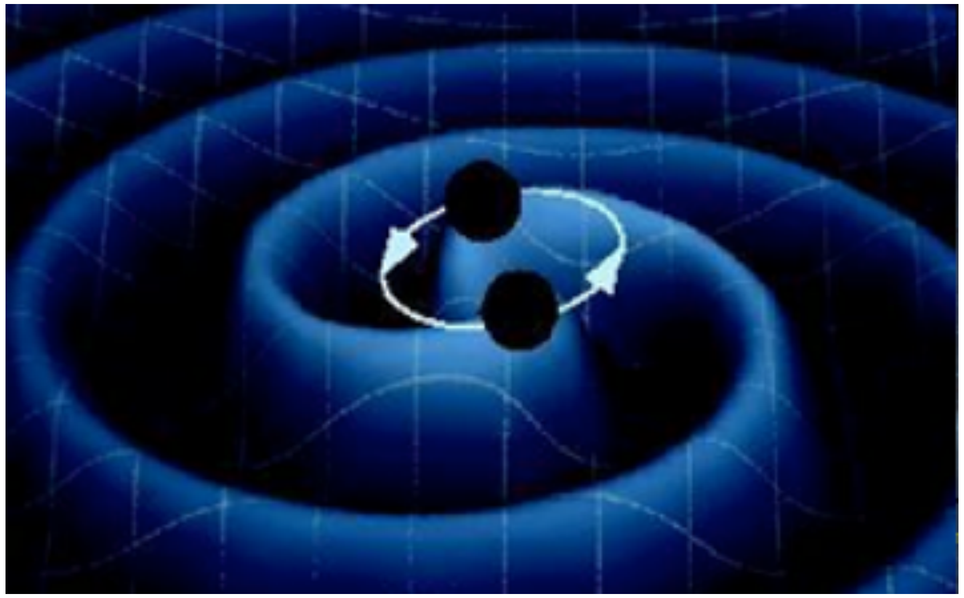

Has LIGO detected PBH's ?

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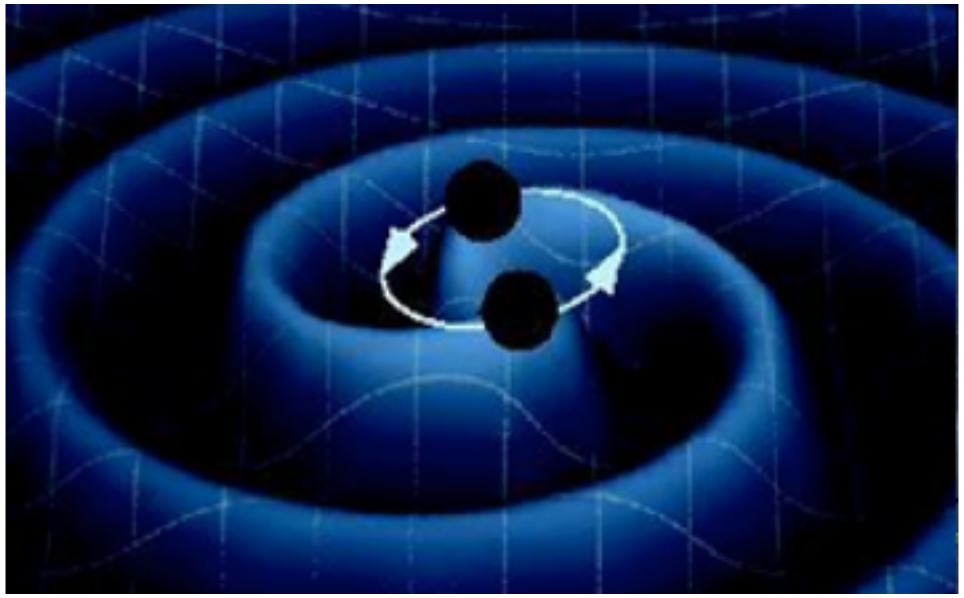

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**Will LIGO differentiate astrophysical BH's from PBH's?**

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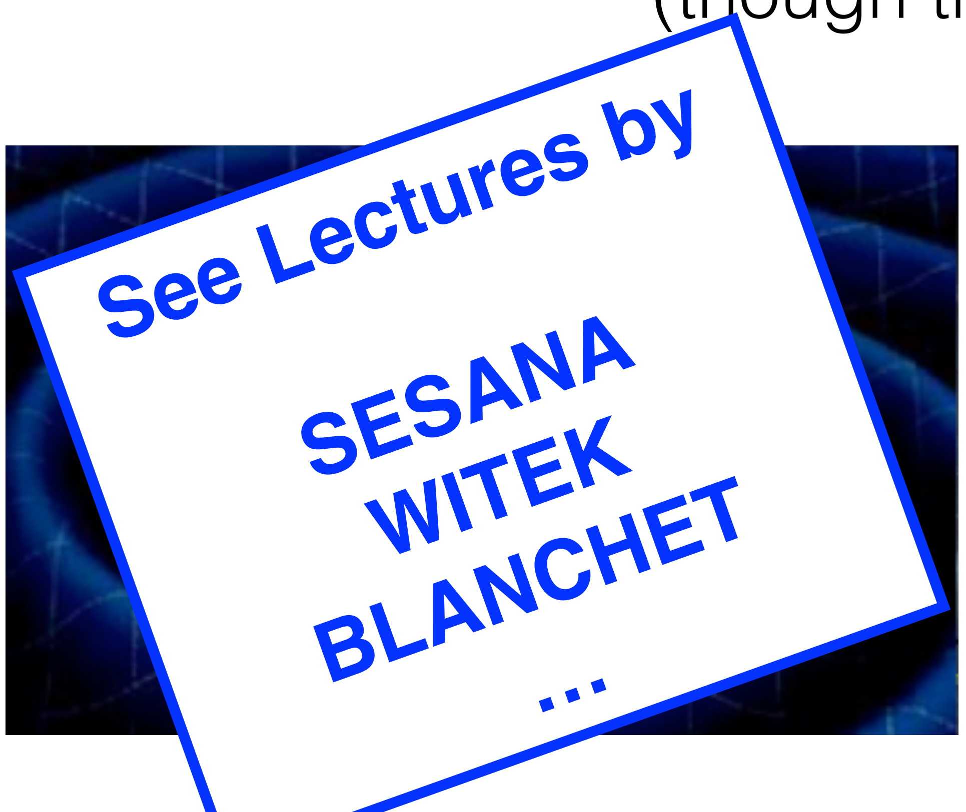


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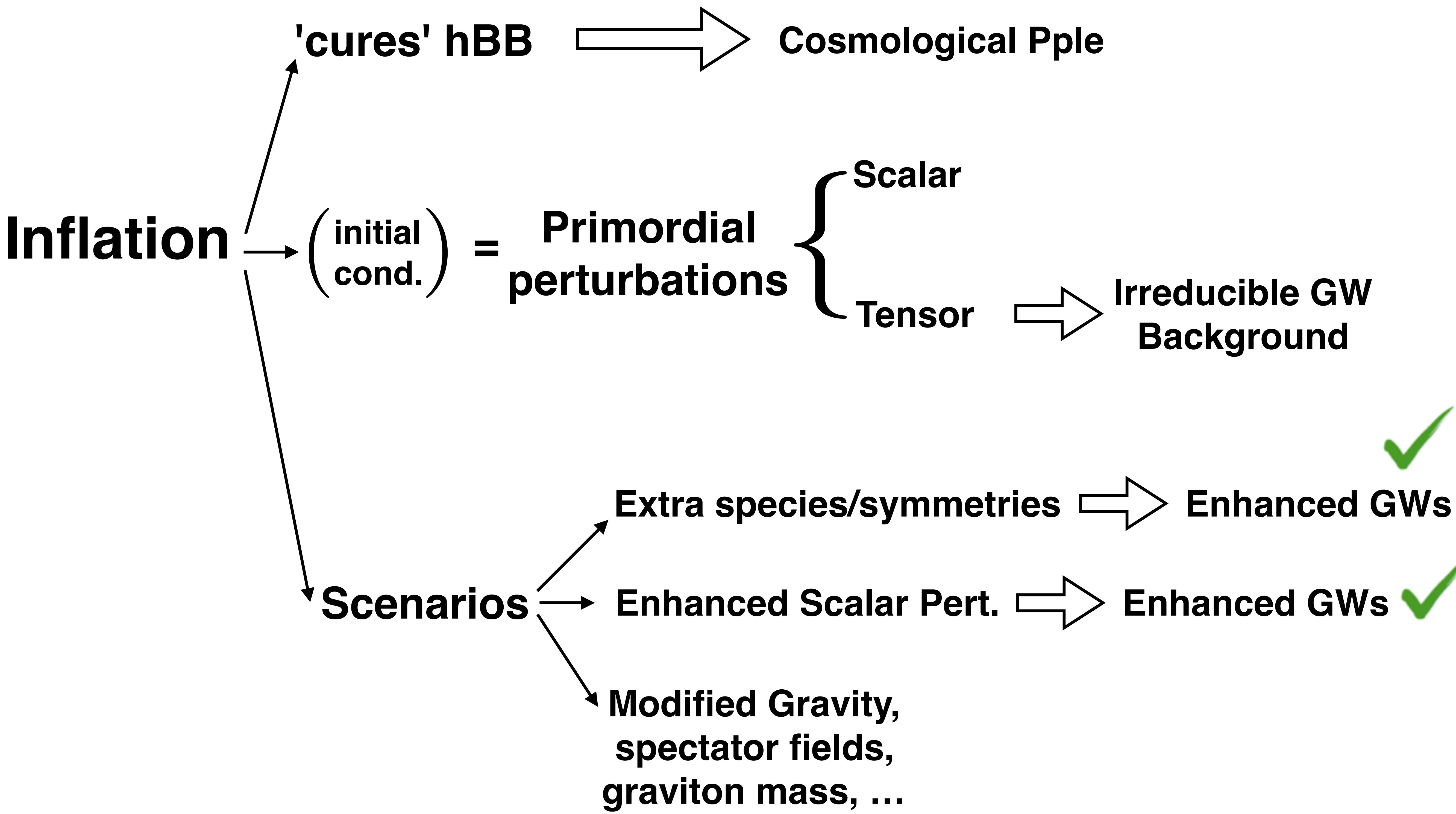
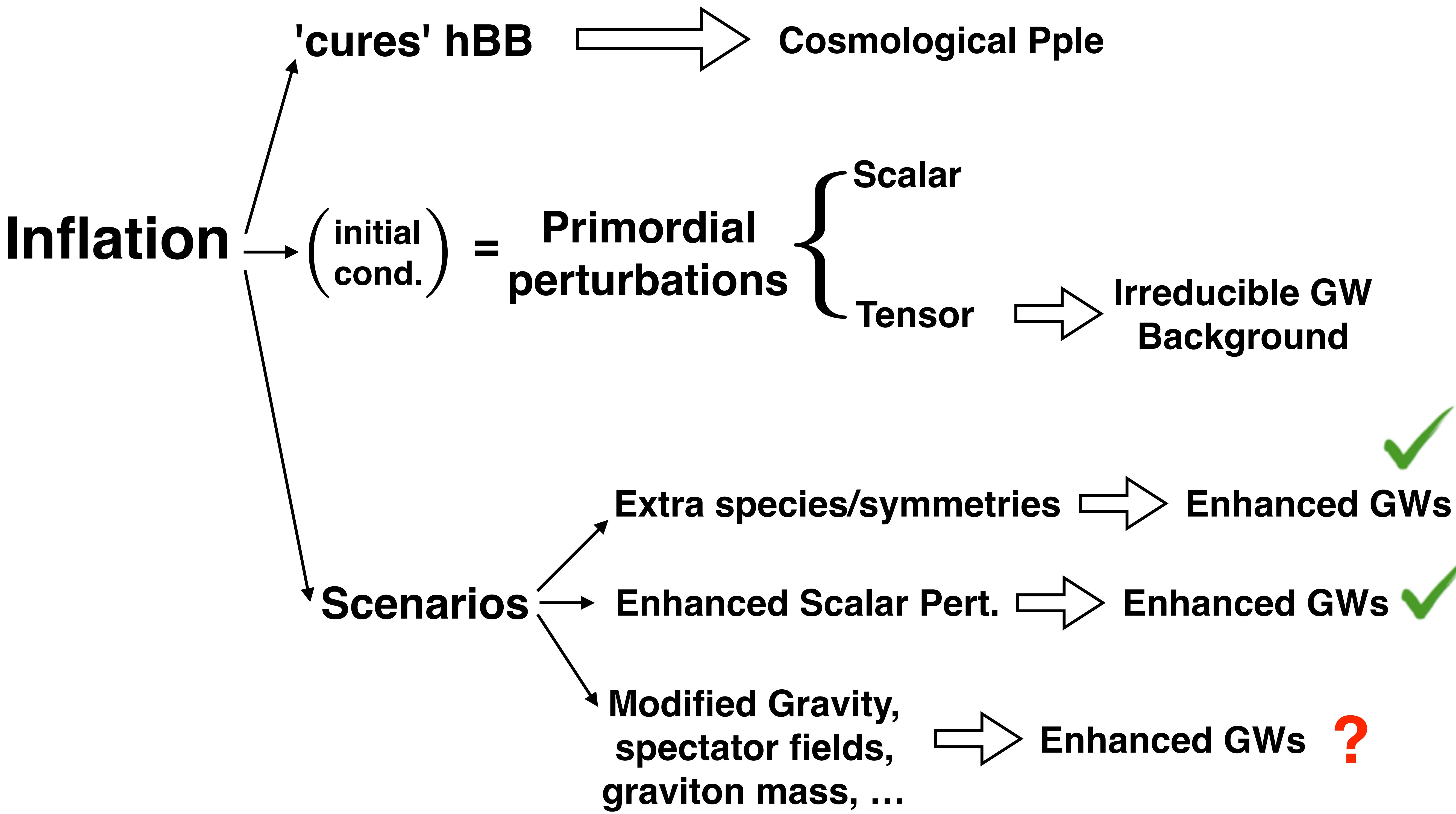
# INFLATIONARY COSMOLOGY



# INFLATIONARY COSMOLOGY



# INFLATIONARY COSMOLOGY