A (short) Tour on techniques for exact controllability for entropy solutions

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Outline

Generalized Characteristics

2 Vanishing Viscosity

3 Wave Front Tracking

4 Lyapunov Functionals

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Generalized Characteristics

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4 Lyapunov Functionals

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The idea

- Classical characteristics.
- Generalized characteristics
- Extremal backward characteristics.

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Some articles

- Ancona-Marson (1998) : Reachable states, half-line, 0 initial condition.
- Horsin (1998) : sufficient conditions on interval for Burgers.

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Advantages and Limitations

- A posteriori analysis.
- Very precise description.
- Only 1d.
- Much more difficult with non convex flux.
- Much more difficult with system.

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2 Vanishing Viscosity

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The idea

- Shock selection and viscosity.
- Boundary layer.

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Some articles

- Glass-Guerrero (2007) : Burgers constant target.
- Imanuvilov-Puel (2009) : 2D Burgers.
- Leautaud (2012) : More general flux.

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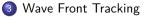
Advantages and limitations

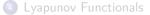
- Work with smoother functions.
- Real boundary.
- Uniform controllability (good and bad).
- More difficult with system (Bianchini Bressan Ancona).
- Reachable states?

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The idea

- Riemann problem.
- Piecewise constant approximations.
- Non physical fronts.
- Backward solver.

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Some Articles

- Bressan-Coclite (2002), counter-example, no linear test.
- Ancona-Coclite (2005), Temple systems.
- Glass (2007, 2014), Euler isentropic and non-isentropic.
- Li Tatsien- Lei Yu (2016), Linearly degenerate systems.

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Advantages and limitations

- Any scalar flux.
- Work with system.
- Numerically implementable (at least theoretically).
- No black box, even for obvious results.
- Reachable states.

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Generalized Characteristics

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The case of the transport equation

$$\partial_t y + c \partial_x y = 0,$$
 $(t, x) \in (0, T) \times (0, L)$
 $y(t, 0) = 0,$ $t \in (0, T).$

Using the method of characteristics :

$$y(t,x) = \begin{cases} y_0(x-ct) & \text{if } x > ct, \\ 0 & \text{otherwise.} \end{cases}$$

For $t \ge L$, y(t, .) = 0.

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A Family of Lyapunov Functionals

• For $\nu > 0$:

$$J_{\nu}(t):=\int_0^L y^2(t,x)e^{-\nu x}dx.$$

• Formally at least :

$$\begin{split} \dot{J}_{\nu}(t) &= \int_{0}^{L} 2y_{t}(t,x)y(t,x)e^{-\nu x}dx \\ &= \int_{0}^{L} -2cy_{x}(t,x)y(t,x)e^{-\nu x}dx \\ &= [-cy^{2}(t,x)e^{-\nu x}]_{0}^{L} - c\nu J_{\nu}(t) \\ &\leq -c\nu J_{\nu}(t). \end{split}$$

• Using Gronwall :

$$J_{\nu}(t) \leq e^{-c\nu t} J_{\nu}(0).$$

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Return on the L^2 norm

Norm equivalence

$$\forall t \geq 0, \qquad e^{-\nu L} ||y(t,.)||^2_{L^2(0,L)} \leq J_{\nu}(t) \leq ||y(t,.)||^2_{L^2(0,L)}.$$

• Inequality on L^2

$$||y(t,.)||^2_{L^2(0,L)} \le e^{-\nu c(t-\frac{L}{c})}||y_0||^2_{L^2(0,L)},$$

• For $t \geq \frac{l}{c}$, letting $\nu \to +\infty$ we get y(t,.) = 0.

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A simple geometric condition

Definition

Let Ω be an smooth open set of \mathbb{R}^d and I be a segment of \mathbb{R} . We say that they satisfy **the replacement condition in time** T > 0 if there exists a vector $w \in \mathbb{R}^d$ and a positive number c such that

$$L := \sup_{x \in \Omega} \langle w | x \rangle - \inf_{x \in \Omega} \langle w | x \rangle < +\infty.$$

$$\forall u \in I, \qquad \langle f'(u) | w \rangle \geq c,$$

and $T = \frac{L}{c}$.

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and its simple corresponding result

Theorem (Donadello, P.)

Let $v \in L^{\infty}((0, +\infty) \times \Omega)$ be an entropy solution to

 $\partial_t u + \operatorname{div}(f(u)) = 0$

and u_0 be a function in $L^{\infty}(\Omega)$.

Suppose that both u_0 and v take value in a segment I such that Ω , I and f satisfy the replacement condition in time T. Then for any times T_1 and T_2 greater than T we have an entropy solution u of the previous equation satisfying

$$u(0,x) = u_0(x),$$
 $u(T_1,x) = v(T_2,x)$ for almost all $x \in \Omega$.

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THANK YOU FOR YOUR ATTENTION

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