Estimates of the minimal time for the controllability of the Grushin operator

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This talk deals with the minimal null controllability time for the following degenerate parabolic equation of Grushin type

$$\begin{cases} \partial_t f - \partial_{xx} f - x^2 \partial_{yy} f = \mathbf{1}_\omega(x,y) u(t,x,y), & (t,x,y) \in (0,T) \times \Omega, \\ f(t,x,y) = 0, \quad (t,x,y) \in (0,T) \times \partial\Omega, \\ f(0,x,y) = f_0(x,y), \quad (x,y) \in \Omega, \end{cases} \tag{G}$$

with $\Omega = (-1,1) \times (0,1)$.

I will briefly recall the ideas used in [2] (transmutation of observability, sideways energy estimates) to prove that the minimal time for null controllability of (G) is $T_{min} = \frac{a^2}{2}$ when $\omega = \big[(-b,-a) \cup (a,b)\big] \times (0,1)$.

Then I will prove that in this setting T_{min} is also the minimal time for the necessary quantitative Fattorini-Hautus test introduced in [3] to hold. Thus in this setting the validity of such a quantitative Fattorini-Hautus test is equivalent to null controllability. This analysis is extracted from a joint work with F. Ammar Khodja, A. Benabdallah and M. González-Burgos [1].

References

[1] F. Ammar Khodja, A. Benabdallah, M. González-Burgos, and M. Morancey, *Quantitative Fattorini-Hautus test and minimal null control time for parabolic problemes*, (2017) https://hal.archives-ouvertes.fr/hal-01557933.

[2] K. Beauchard, L. Miller, and M. Morancey, *2D Grushin-type equations: minimal time and null controllable data*, J. Differential Equations, **259** (2015)(11):5813–5845.

[3] T. Duyckaerts and L. Miller, *Resolvent conditions for the control of parabolic equations*, J. Funct. Anal., **263**(2012) (11):3641–3673.

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