PDE Control, Optimization and Design of Engineered Systems (Progress and Future Directions)

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VII Partial Differential Equations, Optimal Design and Numerics

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People and Background

- Virginia Tech - ICAM
  - Jeff Borggaard, Gene Cliff, Terry Herdman, Lizette Zietsman
- Texas Tech - Math
  - Eugenio Aulisa, David Gilliam
- Carnegie Mellon - Chemical Engineering
  - Larry Biegler
- United Technologies Corporation (UT Aerospace, Carrier, UTRC)
  - Trevor Bailey (CCS), Rui Huang, Clas Jacobson (UTRC)
- MIT - Aeronautics & Astronautics
  - Boris Kramer
- Oklahoma State - Math
  - Weiwei Hu

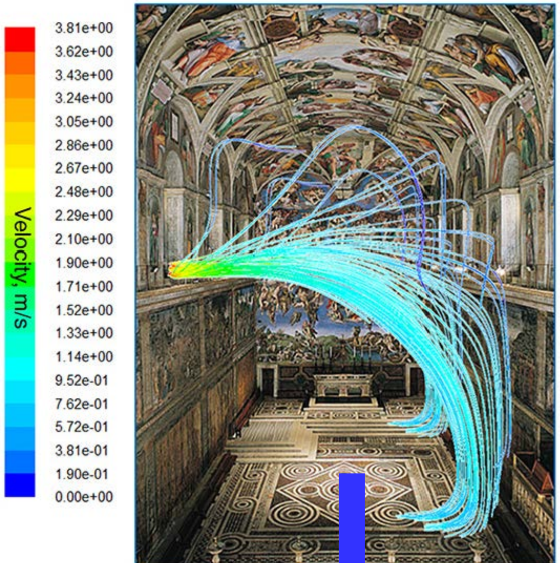
Modeling for "X"

"GOOD" approximations are essential

- **Applications** to energy efficient buildings, Environmental Control Systems (ECS) & Thermal Management Systems (TMS) for airplanes
- Need good dynamic models FOR control, optimization & design

Energy Management & Control Systems

BUILDINGS



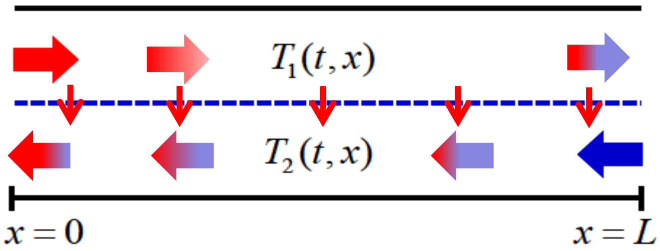
ENVIRONMENTAL & THERMAL MANAGEMENT SYSTEMS

TRANSPORTATION

Outline

Part I) Start with some motivating applications

Part II) Discuss modeling and approximation of a (dynamic) HX to illustrate the ideas & issues



Part III) Discuss research problems and challenges

Part IV) Some advice for students

Thermal Fluid Systems (VCS)

*"The **vapor compression systems** **(VCS)** for the Boeing 787 environmental control system (ECS) produces enough cooling to cool more than 25 typical New England homes".* - Tom Pelland, VP, UTC AM Systems.

Interconnected Complex Physical Systems

- mechanical pumps – electrical valves (ODEs) & compressors (empirical maps)
- thermal-fluid systems in heat exchangers (HX) - coupled DAEs & PDES
- dynamic models needed for passing ANSI/AHRI certification tests (& control)

If any component is infinite dimensional, then the system is infinite dimensional.

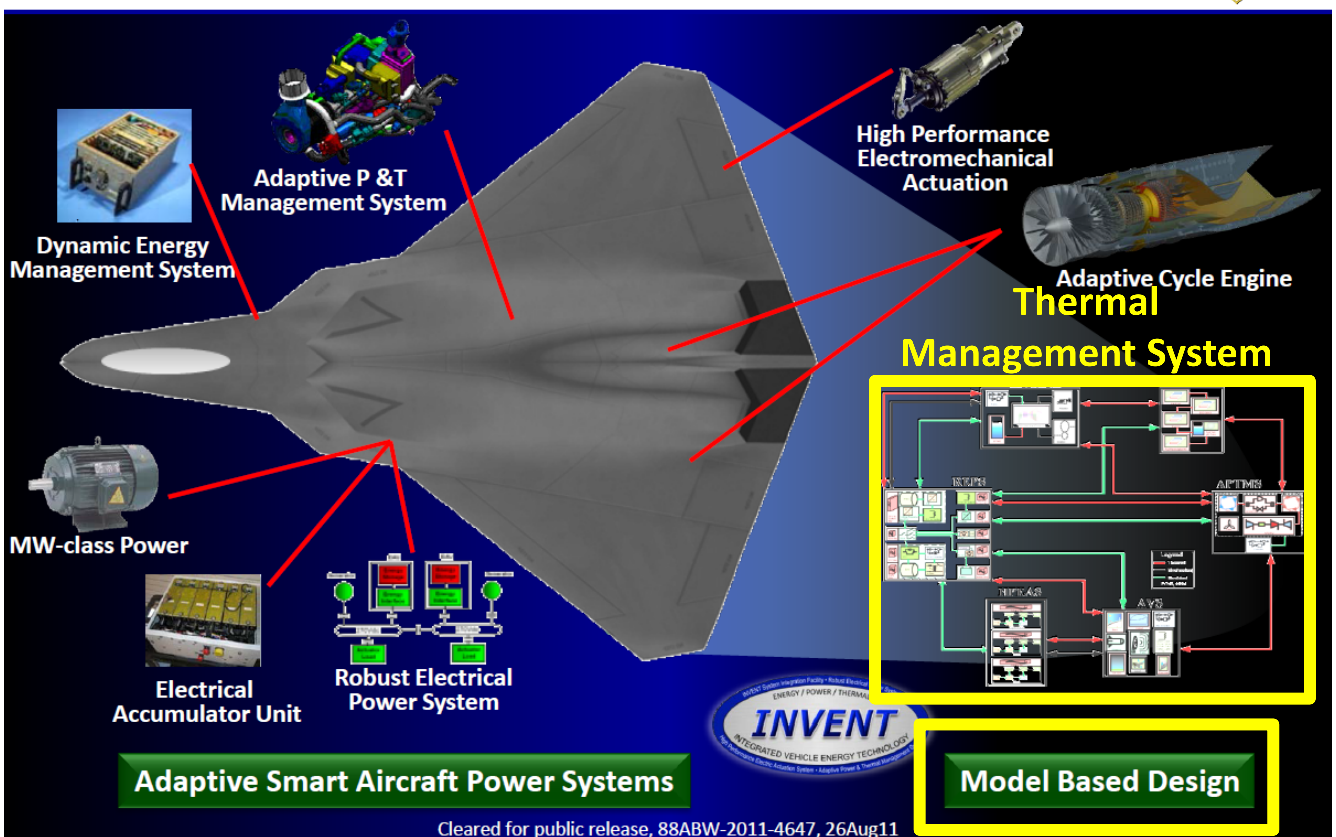
HEAT EXCHANGER (HX) IS A CRITICAL TECHNOLOGY

Thermal Fluid Control: More Electric Aircraft

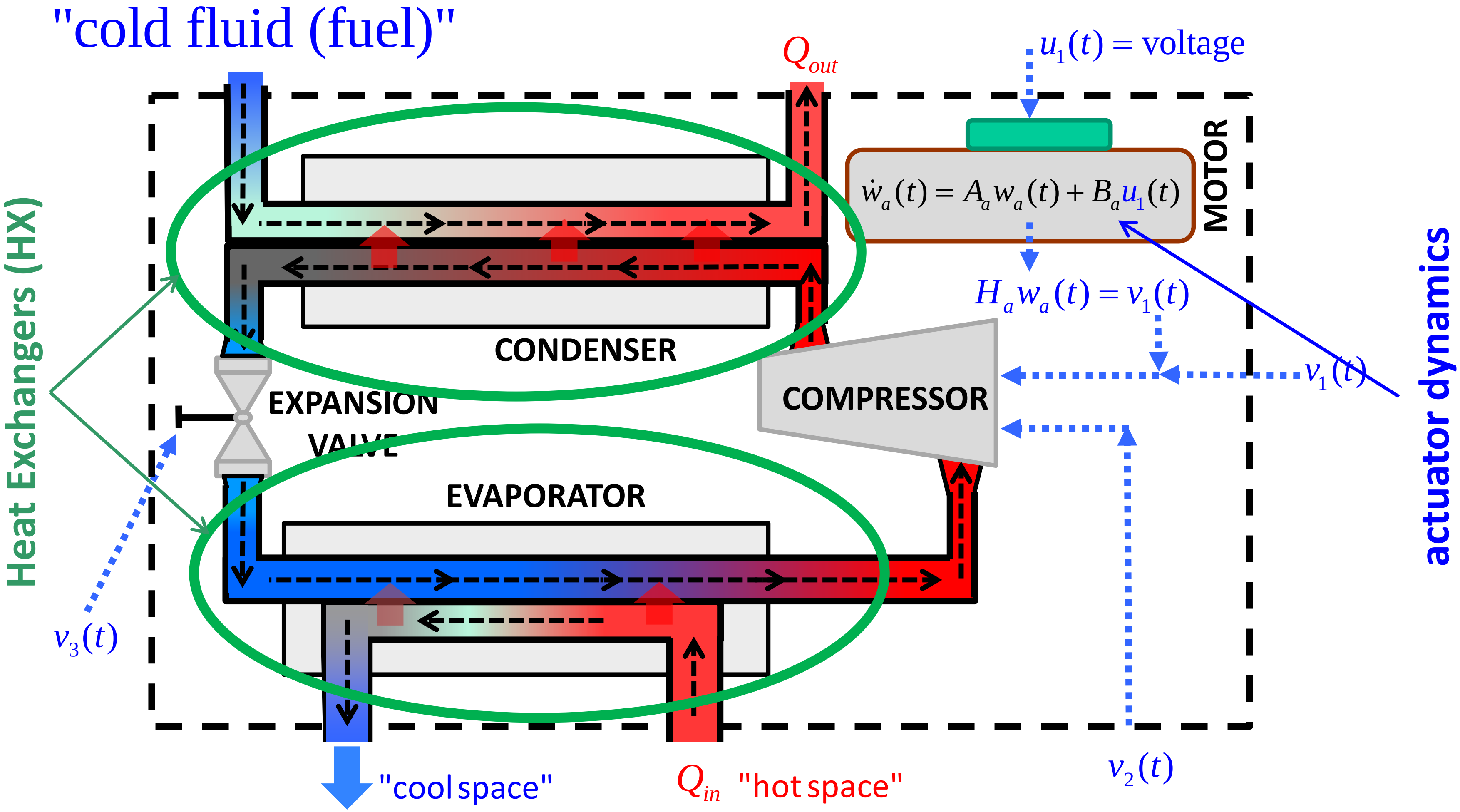
"DUMPS" HEAT ENERGY INTO FUEL

Integrated Vehicle Energy Technology (INVENT)

The Next System Revolution



Basic Vapor Compression System



$v_1(t) = P(t)$ - compressor input power (kw)

$v_2(t) = V_{des}(t)$ - compressor volume flow rate (m^2 / s)

$v_3(t) = C_f(t)$ - venturi area valve (m^2)

DYNAMIC MODELING OF HXs IS A CHALLENGE

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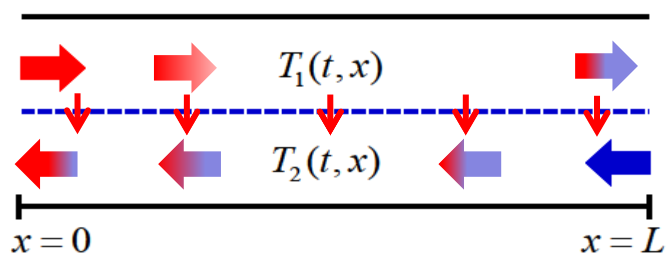
Part II) Discuss modeling and approximation of a (dynamic) HX to illustrate the ideas & issues



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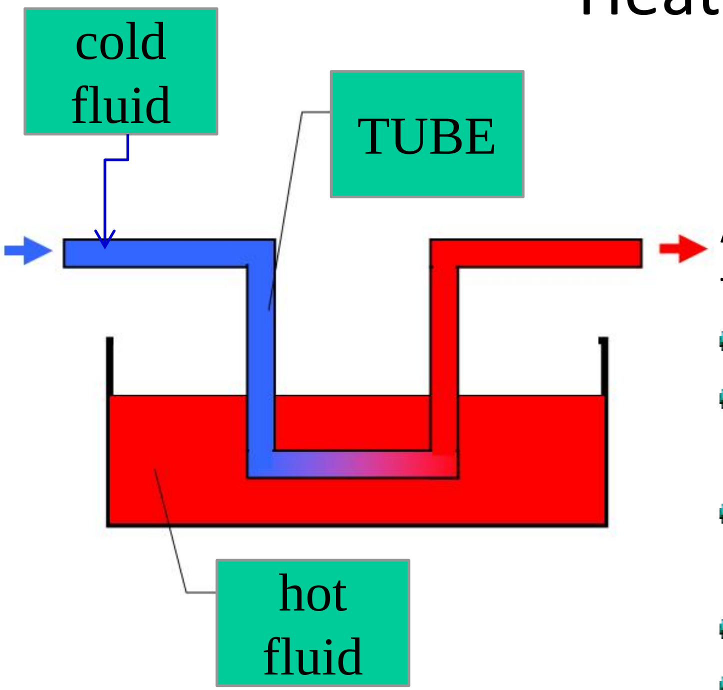
Start: Simple Dynamic Modeling of HXs

Heat Exchangers (HX)



A basic HX is really "very simple" … the temperature of the fluid exiting the HX depends on a number of things:

- The temperature of the hot fluid in the pipe
- The surface area of the tube in the hot liquid pipe
  = $2\pi r(r+L)$
- The temperature of the cold fluid when it enters the tube
- How fast the fluids flow
- Axial conduction in fluids is **"small"** (often **neglected**)
- ….

spatial property

"small" axial conduction assumption

PHYSICS BASED MODELING STARTING POINT

Starting Point & “Fundamental Laws”

J. Ward MacArthur and Eric W. Grald, *Unsteady compressible two-phase flow model for predicting cyclic heat pump performance and a comparison with experimental data*. Int. J. of Refrigeration, Vol. 12, (1989), 29–41.

continuity equation (1D) $\frac{\partial}{\partial t}\rho(t,x)+\frac{\partial}{\partial x}[\rho(t,x)u(t,x)]=0$

momentum equation (1D)

$$\frac{\partial}{\partial t}(\rho(t,x)u(t,x))+\frac{\partial}{\partial x}\left((\rho(t,x)[u(t,x)]^2\right)=-\frac{\partial}{\partial x}P(t,x)+\frac{\partial}{\partial x}\left(\frac{4}{3}\mu\frac{\partial}{\partial x}u(t,x)\right)$$
$$-F_f(t,x)+\rho(t,x)f_x(t,x)$$

energy equation (1D)

$$\frac{\partial}{\partial t}(\rho(t,x)E(t,x))+\frac{\partial}{\partial x}[(\rho(t,x)u(t,x)E(t,x)]$$
$$=-\frac{\partial}{\partial x}[u(t,x)P(t,x)]+\frac{\partial}{\partial x}\left[k\frac{\partial}{\partial x}T_r(t,x)\right]+\frac{\partial}{\partial x}\left(u(t,x)\left(\frac{4}{3}\mu\frac{\partial}{\partial x}u(t,x)\right)\right)$$
$$+u(t,x)F_f(t,x)+[u(t,x)\dot{q}(t,x)]+\rho(t,x)u(t,x)f_x(t,x)$$

$$E(t,x)=\left(e(t,x)+(u(t,x))^2/2\right)$$

Starting Point & “Simplifying Assumptions”

J. Ward MacArthur and Eric W. Grald, *Unsteady compressible two-phase flow model for predicting cyclic heat pump performance and a comparison with experimental data*. Int. J. of Refrigeration, Vol. 12, (1989), 29–41.

- √ fluid flow is one-dimensional (horizontal);
- viscous dissipation is negligible; “small … so ignore
- spatial variations in pressure are negligible; “small … so ignore
- axial conduction is negligible; “small … so ignore
- √ work associated with the rate of change of pressure with respect to time is negligible;
- √ cross-sectional area of the flow stream is constant
- √ pressure drop along the heat exchanger are negligible,

$$\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x}[\rho u]=0$$

$$\cancel{\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}\left((\rho[u]^2\right)-\cancel{\frac{\partial}{\partial x}P}+\frac{\partial}{\partial x}\left(\cancel{\frac{4}{3}\mu\frac{\partial}{\partial x}}u\right)-F_f+\rho f_x}$$

$$\frac{\partial}{\partial t}(\rho E)+\frac{\partial}{\partial x}[(\rho uE]=-\frac{\partial}{\partial x}[uP]+\frac{\partial}{\partial x}\left[\cancel{k\frac{\partial}{\partial x}}T_r\right]+\frac{\partial}{\partial x}\left(u\left(\cancel{\frac{4}{3}\mu\frac{\partial}{\partial x}}u\right)\right)$$
$$+uF_f+[u\dot{q}]+\rho uf_x$$

Simplified Model & Numerical Approximations

J. Ward MacArthur and Eric W. Grald, *Unsteady compressible two-phase flow model for predicting cyclic heat pump performance and a comparison with experimental data*. Int. J. of Refrigeration, Vol. 12, (1989), 29–41.

$$\dot{m}(t,x) = \rho(t,x)u(t,x)A \quad \text{---} \quad \text{mass flow rate}$$

$$h(t,x) = e(t,x) + P(t,x)V \quad \text{---} \quad \text{enthalpy}$$

(1) $$\frac{\partial}{\partial t}[\rho(t,x)A] + \frac{\partial}{\partial x}[\dot{m}(t,x)] = 0$$

(2) $$\frac{\partial}{\partial t}[\rho(t,x)Ah(t,x)] + \frac{\partial}{\partial x}[(\dot{m}(t,x)h(t,x)] = -HP\left[T_r(t,x) - T_{HXw}(t,x)\right]$$

(3) $$(\rho(t,x)Ac_p)\frac{\partial T_{HXw}(t,x)}{\partial t} = \lambda A_w \cancel{\frac{\partial^2 T_{HXw}(t,x)}{\partial x^2}} + \alpha_1\left(T_r(t,x) - T_{HXw}(t,x)\right) + \alpha_a\left(T_a(t,x) - T_{HXw}(t,x)\right)$$

NOW ONE APPLIES

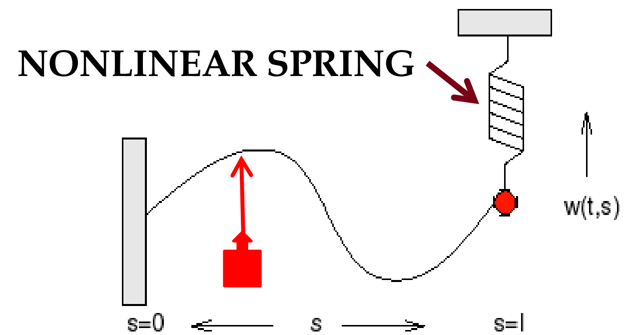
A FINITE VOLUME OR MOVING BOUNDARY APPROXIMATION

The modeling "simplifications" above are the root cause of many problems with existing models used for control and optimization.

Mathematicians can be guilty too …

“Simplifying” Assumptions Not Always Good

We have lost credibility with design engineers



$$\rho \frac{\partial^2}{\partial t^2} w(t,s) = \frac{\partial}{\partial s}\left(\tau \frac{\partial}{\partial s} w(t,s) + \gamma \frac{\partial^2}{\partial t \partial s} w(t,s) \right)$$

$$m \frac{\partial^2}{\partial t^2} w(t,l) = -\left(\tau \frac{\partial}{\partial s} w(t,l) + \gamma \frac{\partial^2}{\partial t \partial s} w(t,l) \right) - \alpha_1 w(t,l) - \cancel{\alpha_2 [w(t,l)]^3} + u(t) + \cancel{\eta(t)}$$

RATINGS

	Math Geek	**Design Engr.**
Assumption (A): Consider only the linearized system	√	√
Assumption (B): No disturbance	√	“OK”
Assumption (C): Point displacement sensor $y(t) = w(t,\hat{s})$	√	MAYBE
Beautiful mathematics (unbounded output operators)	wonderful	X
Not realistic (no physical device)	who cares?	I care X
Produces “silly results” like … “a fly lands on the cable and observability changes”	even more wonderful	XXX goodbye

UNDERSTANDING WHEN “SMALL” PARAMETERS MATTER IS IMPORTANT

VCS: Issues & Some Research Challenges

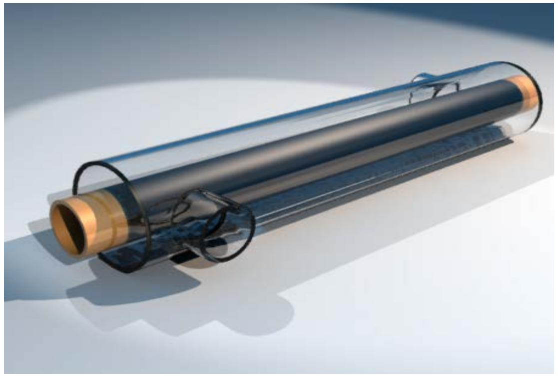
- Models fail at low (or zero flows) -- demonstrated with simple HX
- Control system properties are lost … making the models unsuitable for control design -- as demonstrated with simple HX
- "Actuator dynamics" are not in any of the models
- Empirical functions & equations of state are not always smooth
- Since all existing dynamic models are very <u>crude reduced ODE models</u>, the benefits of nonlinear geometric PDE control are lost (at least diminished)

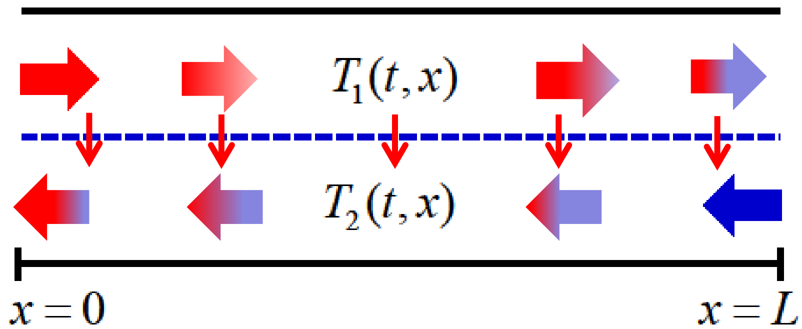
"REDUCED MODEL" **IS NOT** A REDUCED ORDER MODEL!

<u>OUR CURRENT EFFORT</u>

- Return to "full flux" models (full conservation laws) to construct physics models with actuator dynamics for complete VCS
- Develop "property preserving approximations" of complete VCSs and apply model reduction to develop finite dimensional hierarchical design & control models

Simple Counter Flow Heat Exchangers





$$\frac{\partial T_1(t,x)}{\partial t} = k_1 \cancel{\frac{\partial^2 T_1(t,x)}{\partial x^2}} - v_1 \frac{\partial T_1(t,x)}{\partial x} + h_1\left(T_2(t,x) - T_1(t,x)\right), \quad T_1(t,0) = \boxed{g(t),} \quad k_1 \cancel{\frac{\partial T_1(t,L)}{\partial x}} = 0$$

$$\frac{\partial T_2(t,x)}{\partial t} = k_2 \cancel{\frac{\partial^2 T_2(t,x)}{\partial x^2}} + v_2 \frac{\partial T_2(t,x)}{\partial x} + h_2\left(T_1(t,x) - T_2(t,x)\right), \quad T_2(t,L) = 0, \quad -k_2 \cancel{\frac{\partial T_2(t,0)}{\partial x}} = 0$$

“small” axial conduction assumption

possible controls - disturbances

actuator dynamics

Full Flux Model

$u(t) \longrightarrow$

$$\dot{w}(t) = \mathbf{A}_a w(t) + \mathbf{B}_a u(t)$$

$$g(t) = \mathbf{H}_a w(t) - r)$$

$\longrightarrow g(t)$

An Optimal Control Problem

$$\frac{\partial T_1(t,x)}{\partial t} = k_1 \frac{\partial^2 T_1(t,x)}{\partial x^2} - v_1 \frac{\partial T_1(t,x)}{\partial x} + h_1\left(T_2(t,x) - T_1(t,x)\right), \; T_1(t,0) = g(t), \qquad k_1 \frac{\partial T_1(t,L)}{\partial x} = 0$$

$$\frac{\partial T_2(t,x)}{\partial t} = k_2 \frac{\partial^2 T_2(t,x)}{\partial x^2} + v_2 \frac{\partial T_2(t,x)}{\partial x} + h_2\left(T_1(t,x) - T_2(t,x)\right), \; T_2(t,L) = 0, \qquad -k_2 \frac{\partial T_2(t,0)}{\partial x} = 0$$

$$J = \int_0^{+\infty} \left\{ \int_0^L q_1(x)[T_1(s,x)]^2\,dx + \int_0^L q_2(x)[T_2(s,x)]^2\,dx + r[g(s)]^2 \right\} ds$$

ADD ACTUATOR DYNAMICS

$$\dot{w}(t) = \mathbf{A}_a w(t) + \mathbf{B}_a u(t)$$

$$g(t) = \mathbf{H}_a w(t)$$

$$J = \int_0^{+\infty} \left\{ \int_0^L q_1(x)[T_1(s,x)]^2\,dx + \int_0^L q_2(x)[T_2(s,x)]^2\,dx + r[\mathbf{H}_a w(s)]^2 \right\} ds + \int_0^{+\infty} \left\{ r_a [u(s)]^2 \right\} ds$$

ADD COST OF ACTUATOR CONTROL

An Optimal Control Problem

CHANGE OF VARIABLES

$$\theta_1(t,x) = T_1(t,x) - g(t) \qquad \theta_2(t,x) = T_2(t,x)$$

$$\cancel{\frac{\partial \theta_1(t,x)}{\partial t}} = k_1 \cancel{\frac{\partial^2 \theta_1(t,x)}{\partial x^2}} - \boldsymbol{v_1} \frac{\partial \theta_1(t,x)}{\partial x} + h_1\big(\theta_2(t,x) - \theta_1(t,x)\big) + F_1 w(t) + B_1 u(t) \qquad \theta_1(t,0) = k_1 \cancel{\frac{\partial \theta_1(t,L)}{\partial x}} = 0$$

$$\cancel{\frac{\partial \theta_2(t,x)}{\partial t}} = k_2 \cancel{\frac{\partial^2 \theta_2(t,x)}{\partial x^2}} + \boldsymbol{v_2} \frac{\partial \theta_2(t,x)}{\partial x} + h_2\big(\theta_1(t,x) - \theta_2(t,x)\big) + F_2 w(\mathrm{t}) \qquad -k_1 \cancel{\frac{\partial \theta_2(t,0)}{\partial x}} = \theta_2(t,L) = 0$$

$$\dot{w}(t) = \mathbf{A}_a w(t) + \mathbf{B}_a u(t)$$

$$F_1 = -[\kappa_1 \mathbf{H}_a + \mathbf{H}_a \mathbf{A}_a] \qquad F_2 = +\kappa_2 \mathbf{H}_a \qquad B_1 = -\mathbf{H}_a \mathbf{B}_a$$

INTRODUCE APPROXIMATIONS TO OBTAIN ODE MODELS

STATE OF THE ART IN INDUSTRY: FINITE VOLUME METHOD on "SIMPLIFIED MODEL"

A Combined FE-FV Approximation

$$\frac{\partial\theta_1(t,x)}{\partial t} = k_1 \frac{\partial^2\theta_1(t,x)}{\partial x^2} - \boldsymbol{v_1}\frac{\partial\theta_1(t,x)}{\partial x} + h_1\big(\theta_2(t,x)-\theta_1(t,x)\big) + F_1 w(t) + B_1 u(t) \qquad \theta_1(t,0) = k_1\frac{\partial\theta_1(t,L)}{\partial x} = 0$$

$$\frac{\partial\theta_2(t,x)}{\partial t} = k_2 \frac{\partial^2\theta_2(t,x)}{\partial x^2} + \boldsymbol{v_2}\frac{\partial\theta_2(t,x)}{\partial x} + h_2\big(\theta_1(t,x)-\theta_2(t,x)\big) + F_2 w(\mathrm{t}) \qquad -k_1\frac{\partial\theta_2(t,0)}{\partial x} = \theta_2(t,L) = 0$$

$$\dot{w}(t) = \mathbf{A}_a w(t) + \mathbf{B}_a u(t)$$

The differential operators $P(k_j) = k_j \dfrac{\partial^2}{\partial x^2}$ are elliptic (and selfadjoint)

The differential operators $H(v_j) = v_j \dfrac{\partial}{\partial x}$ are not selfadjoint.

Basic Idea : Approximate the selfadjoint operators $P(k_j)$ using finite elements and the non - selfadjoint operators $H(v_j)$ using a upwind finite volume scheme.

P. Deuring, R. Eymard, and M. Mildner, "*L^2-stability independent of diffusion for a finite element - finite volume discretization of a linear convection-diffusion equation*", SIAM Journal on Numerical Analysis, 53 (2015), 508--526

A Combined FE-FV Approximation

$$\boldsymbol{k} = (k_1, k_2), \quad \boldsymbol{v} = (v_1, v_2) \quad \boldsymbol{h} = (h_1, h_2) \qquad 0 \le \varepsilon = |k_1| + |k_2| << 1$$

$$(\Sigma_{\boldsymbol{k},\boldsymbol{v},\boldsymbol{h}}) \quad \dot{z}(t) = [\mathcal{P}(\boldsymbol{k}) + \mathcal{H}(\boldsymbol{v}) + \mathcal{K}(\boldsymbol{h})]z(t) + \mathcal{B}u(t) = \mathcal{A}(\boldsymbol{k},\boldsymbol{v},\boldsymbol{h})z(t) + \mathcal{B}u(t)$$

FE-FV

METHOD

$$(\Sigma_{\boldsymbol{k},\boldsymbol{v},\boldsymbol{h}})^N \quad \dot{z}^N(t) = [P_{FE}^N(\boldsymbol{k}) + H_{FV}^N(\boldsymbol{v}) + K^N(\boldsymbol{h})]z^N(t) + B_A^N u(t)$$

1. **"Modeling for control" includes the step of producing appropriate approximations of the control input operator.**
2. **People who only use the model for simulation can (and do) ignore this step …BUT**
3. **If the model is to be used for control one must be careful to construct the "B" operator correctly.**

$$B_A^N = (1-\varepsilon)B_{FV}^N + \varepsilon B_{FE}^N$$

There is a Rigorous Theory ...

Theorem. The PDE system with actuator dynamics is stabilizable for all $\boldsymbol{k} > 0$ and any $\boldsymbol{v} \geq 0$. If $\boldsymbol{k} = 0$, then the systen is stabilizable for all $\boldsymbol{v} > 0$.

Theorem. The combined FE-FV scheme is dual convergent and satisfies POES.

$$u_{opt}(t) = -\int_0^L k_1(x)\theta_1(t,x)dx - \int_0^L k_2(x)\theta_2(t,x)dx - k_w w(t)$$

These are the "functional" gains we need to compute ...

A numerical example ....

$$\dot{w}(t) = -1_a w(t) + 1_a u(t), \quad g(t) = 1_a w(t)$$

$$L = 1, \quad h_1 = 15.933, \quad h_2 = 16.483,$$

$$q_1(x) = q_2(x) = 5, \quad r = 1, \quad r_a = 15$$

FE-FV Method - Optimal Functional Gains

$$v_1 = \beta = 0.01,$$

$$v_2 = (1.1)\beta = 0.011$$

Stabilizable

for $N > 2$!

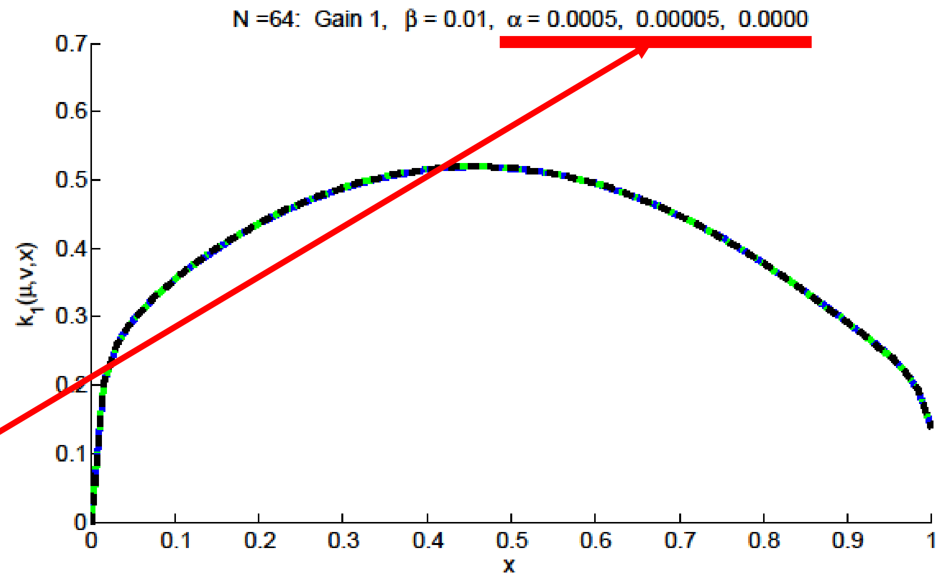
$$k_1 = \alpha,$$

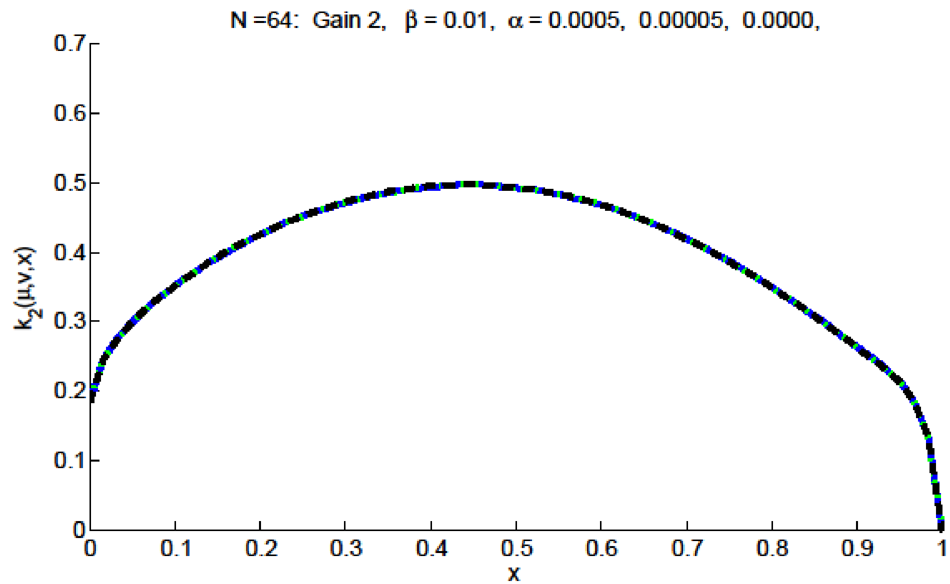
$$k_2 = (1.1)\alpha \ ,$$

$$\alpha \to 0$$

WHAT ABOUT

LOW / ZERO FLOW ?





FE-FV Method - Optimal Functional Gains

$$k_1 = \alpha = 5e^{-5},$$

$$k_2 = (1.1)\alpha = 5.5e^{-5}$$

Stabilizable

for $N > 2$!

$$v_1 = \beta,$$

$$v_2 = (1.1)\beta\ ,$$

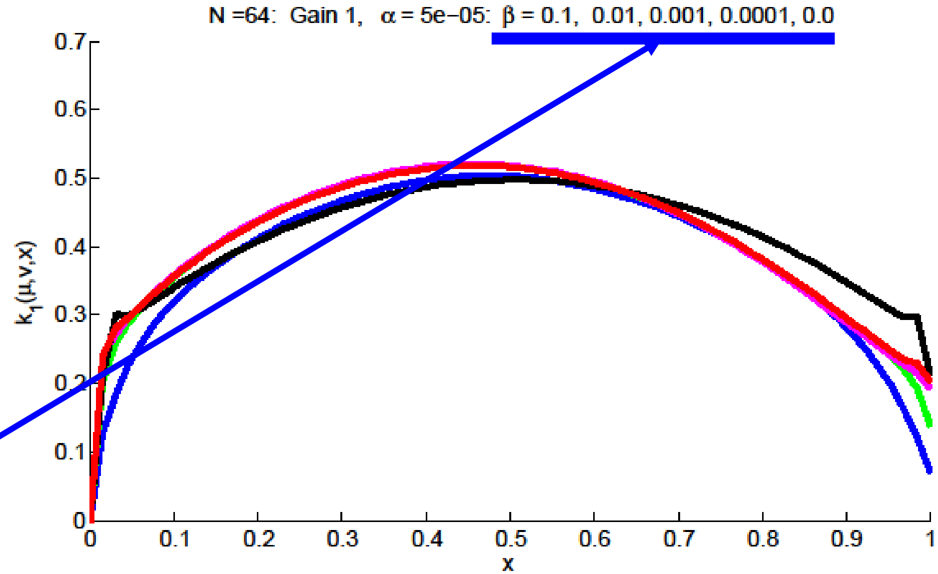
$$\beta \to 0$$

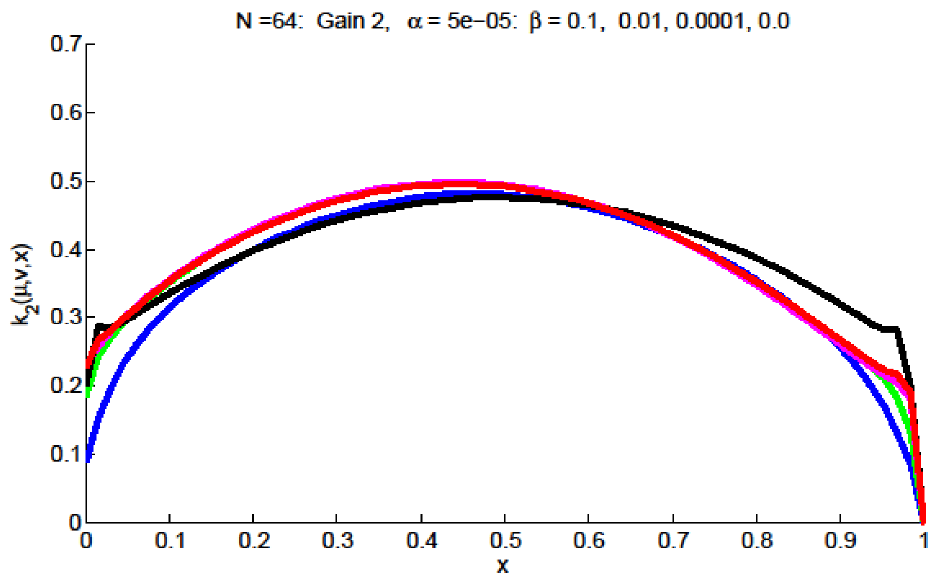
What happens when

$$\boldsymbol{k} = (k_1, k_2) = 0$$

and

$$\boldsymbol{v} = (v_1, v_2) = 0?$$





Zero Flux & Zero Flow?

"Control of a Thermal Fluid Heat Exchanger with Actuator Dynamics",

J. A. Burns and L. Zietsman, 2016 CDC.

$$\dot{w}_a(t) = -x(t) + u(t), \quad g(t) = w(t)$$

Setting $v_1 = v_2 = k_1 = k_2 = 0$ produces the ODE system

$$\frac{d}{dt}\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} -h_1 & +h_1 & -h_1+1 \\ +h_2 & -h_2 & +h_2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ w(t) \end{bmatrix} A + \begin{bmatrix} -1 \\ 0 \\ +1 \end{bmatrix} u(t)$$

$$\frac{d}{dt}\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} -15.9330 & +15.9330_1 & -14.9330 \\ +16.4830 & -16.4830 & +16.4830 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ w(t) \end{bmatrix} A + \begin{bmatrix} -1 \\ 0 \\ +1 \end{bmatrix} u(t)$$

NOT controllable !

IMPORTANT TAKEAWAY: Neglecting axial diffusion and viscous terms (no matter how small) is a root cause of problems with model development for control (and optimization)

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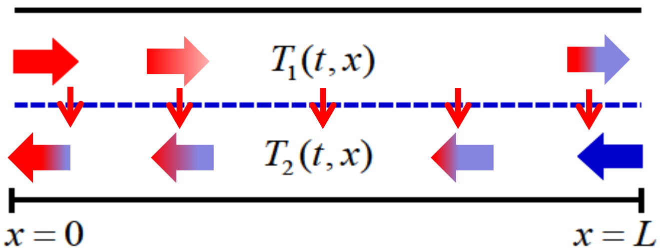


Part III) Discuss research problems and challenges

Part IV) Some advice for students

Problems: Modeling for Control & Optimization

- For practical control - need to add actuator dynamics to "full flux" models (**nonlinea**r conservation laws)

$$z(t,x) = \begin{bmatrix} \rho & \rho u & \rho u E \end{bmatrix}^T$$

$$\frac{\partial}{\partial t} z(t,x) = \frac{\partial}{\partial x} \mathcal{P}(\mathbf{k}, z(t,x)) - \frac{\partial}{\partial x} \mathcal{H}(\mathbf{v}, z(t,x)) + \mathcal{K}(\mathbf{h}, z(t,x)) + \mathcal{B}(z(t,x)) u(t)$$

- For "good physics" - need to develop "property preserving approximations" of complete VCSs

$$\dot{z}^N(t) = [P_{FE}^N(\mathbf{k}, z^N(t)) + H_{FV}^N(\mathbf{v}, z^N(t)) + K^N(\mathbf{h}, z^N(t))] z^N(t) + B_A^N(z^N(t)) u(t)$$

- For optimal design and optimization - need to deal with non-smooth equations of state and correlations …

$$P = P(\rho, T), \quad e = e(T), \quad k = k(\rho, T), \quad \mu = \mu(T)$$

$$F_f = F_f(\mathrm{Re}, \rho, u, \mu)$$

Mathematical & Computational Challenges

- Develop "property preserving approximations" of complete VCSs

$$(\Sigma_{\boldsymbol{k},\boldsymbol{v},\boldsymbol{h}}) \quad \dot{z}(t) = [\mathcal{P}(\boldsymbol{k}) + \mathcal{H}(\boldsymbol{v}) + \mathcal{K}(\boldsymbol{h})]z(t) + \mathcal{B}u(t) = \mathcal{A}(\boldsymbol{k},\boldsymbol{v},\boldsymbol{h})z(t) + \mathcal{B}u(t)$$

NUMERICAL APPROXIMATION

$$(\Sigma_{\boldsymbol{k},\boldsymbol{v},\boldsymbol{h}})^N \quad \dot{z}^N(t) = [P_{FE}^N(\boldsymbol{k}) + H_{FV}^N(\boldsymbol{v}) + K^N(\boldsymbol{h})]z^N(t) + B_A^N u(t)$$

★ Need <u>convergence</u> and <u>dual convergence</u> for optimization and optimal control (combined FE-FV and DG methods important here)

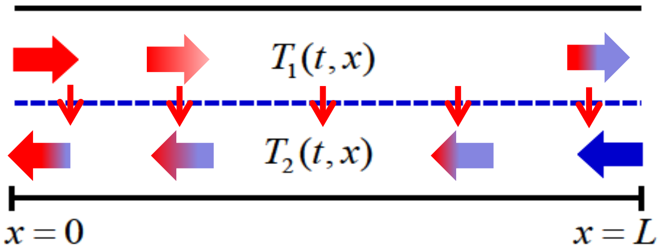
- Develop a theoretical framework to build well-posed interconnected hybrid systems from heterogeneous components (PDEs, ODEs,…)
- Develop practical "non-smooth" optimization algorithms AND software to deal with non-smooth correlations.
- Develop reduced order models (ROMs) that preserve system & essential physical properties (e.g., stability, controllability, 2nd law of thermodynamics …)

Why should a young person work on problems like this?

Outline

Part I) Start with some motivating applications

Part II) Discuss modeling and approximation of a (dynamic) HX to illustrate the ideas & issues

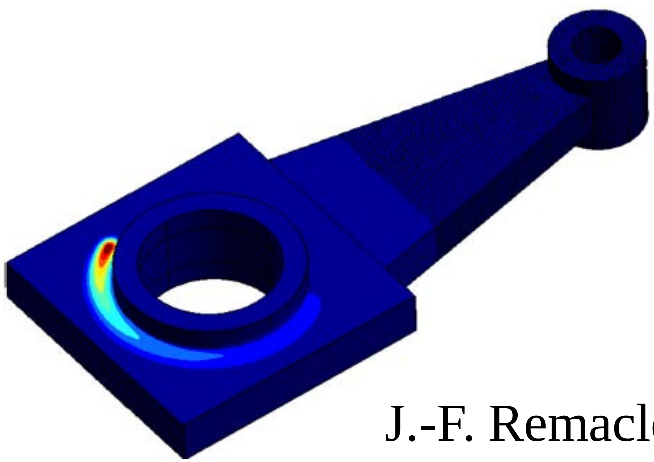


Part III) Discuss research problems and challenges

Part IV) Some advice for students “FREE” advice

Comments and Suggestions

1. Mathematical modeling should be part of the mathematician's work – after all, it is called **mathematical** modeling.
2. Get your hands "dirty" and work with engineers to develop models suitable for control and optimization. As a control "expert" you know what is essential for a good control model.
3. Sometimes "*small parameters*" are extremely important and should not be neglected without careful analysis.
4. Never forget that the mathematical and computational world has changed dramatically in the past decade …



This computation (a 3D heat equation) involved more than 20 million equations and was solved in **a few seconds on a single $500 GPU** processor by using a DG method!

J.-F. Remacle, R. Gandham and T. Warburton, "*GPU accelerated spectral finite elements on all-hex meshes*", Journal of Computational Physics 324 (2016), 246–257.

Advice for Young(er) Mathematicians

"If you do not work on important problems, it's unlikely that you'll do important work. It's perfectly obvious."

Richard Hamming, You and Your Research,
New School Economic Review, Vol. 31, (2008), 5--26.

- Look for big and important problems with impact
- Look for problems in …
  - Engineering
  - Traditional Sciences
  - Life Sciences & Medicine
- Look at industry …
- Examples
  - Cure cancer (at least reduce the risk
  - Save the planet
  - Build a better air conditioner ← **are you serious ?**

? Modern air conditioning / refrigeration ?

- In 1902, the first modern electrical air conditioning unit was invented. This invention is incredibly significant.
- It has and continues to save lives … more than the CT scanner
- Dramatically improved work effort/output in the past 100 years
- Rebecca Rosen, Associate Editor at *The Atlantic*: July 18, 2011 …
  - *"Air conditioning hasn't just cooled our rooms -- it's changed where we live, what our houses look like, and what we do on a hot summer night".*
  - *"Many of the central changes in our society since World War II would not have been possible were air conditioning not keeping our homes and workplaces cool.*
- The ability to preserve food and medicine changed the world
- AC is needed for many medical, scientific laboratories, industrial processes and for keeping computers cool.
- AC is required for modern all-electric airplanes (787, F22, F35, …)

Often listed as one of the top 10 inventions that changed the world forever.

Save the Planet & Reduce Health Risks

- Thinning of ozone layer means getting direct in touch with ultra violet rays which can cause skin cancer or skin irritation which can lead to death. A decrease in 1% of ozone layer can cause 5% increase in cases of skin cancer. Also, increased the cases of cataracts, blindness and causes DNA damage
- Freon (also known as R-22) has been the refrigerant of choice for residential heat pump and air-conditioning systems for more than four decades. It has been described as one of the most significant environmental pollutants and causes of ozone depletion.
- R-22 was totally phased out in 2015 by climate regulations & standards
- All new VCSs must use other refrigerants & still meet energy efficiency and emission standards
- These regulations & standards are forcing industry to …

DESIGN, CONTROL & OPTIMIZE NEXT GENERATION AIR CONDITIONERS

Big and important problems with huge impact & requiring new applied and computational mathematics …

Finally …

Recall Hamming's Advice

"If you do not work on important problems, it's unlikely that you'll do important work. It's perfectly obvious."

Richard Hamming, You and Your Research,
New School Economic Review, Vol. 31, (2008), 5--26.

Don't make the mistake that I did …

John Burns research -- 1972-77

Optimization (Personal History)

X **----- Hilbert Space**

Y **----- Hilbert Space**

$G : X \to Y$ **----- Constraint**

$J : X \to \mathbb{R}^1$ **----- Cost Function**

PROBLEM: Find $x \in X$ to minimize J subject to $G(x)=0$

Impact of the Research (Personal History)

I derived very general existence and necessary conditions
- under very weak assumptions
- that generalized standard Lagrange multiplier rules

Actually, it was worse than that ...

I “relaxed” X and Y to be topological vector spaces and assumed G (not necessarily continuous) was “*co-compact*” with respect to J (not assumed continuous).

This is a rare case where it is possible to precisely measure the importance and impact of my research

IMPACT OF THIS WORK

0^{-}

“Cliff” Factor

Impact and Overview

WHY ZERO (OR WORSE) IMPACT?

NONE OF THESE RESULTS HELPED ACTUALLY SOLVE AN INTERESTING AND IMPORTANT REAL WORLD PROBLEM

I SHOULD HAVE PAID ATTENTION
TO WHAT HAPPENED IN THE
CLASSICAL CALCULUS OF VARIATIONS

"The problem of Bolza was the most general of the single-integral problems of the calculus of variations. Its mastery gave us the power to answer many deep and complicated questions that ***no one was asking****".*

E. J. McShane

The Calculus of Variations From the Beginning Through Optimal Control Theory, in Optimal Control and Differential Equations, A. Schwarzkopf, W. Kelley and S. Eliason, Eds., Academic Press, **1978.**

Looking Back (Pay Attention to Applications)

"In my mind, the greatest difference between the Russion approach and ours was in mental attitude. … Pontryagin and his students encountered some problems in engineering and in economics that urgently asked for answers. … Like most mathematicians in the United States, ***I was not paying attention to the problems of engineers****".* *E. J. McShane*

Actually, they were not paying attention to "problems" inside mathematics. As McShane explained why his fundamental papers on the classical Bolza problem in the calculus of variations *"... burst on the mathematical world with the éclat of a buttery's hiccough."*

"The whole subject was introverted. We who were working in it were striving to advance the theory of the calculus of variations as an end in itself, ***without attention to its relation with other fields of activity****."*

More real wisdom …

Some General Wisdom (not mine)

"Without education, we are in a horrible and deadly danger of taking educated people seriously."

G. K. Chesterton

"It isn't that they can't see the solution. It is that they can't see the problem."

G. K. Chesterton Issue when dealing with industry

"Results! Why, man, I have gotten a lot of results. I know several thousand things that won't work."

Thomas Edison

"… In brief, the flight into abstract generality must start from and return again to the concrete and specific."

Richard Courant

"A theory has only the alternative of being right or wrong. A model has a third possibility: it may be right, but irrelevant." Same for math results.

Manfred Eigen

All advice and predictions have issues ….

Some Wisdom about Predictions

Clarke's Three Laws are three "laws" of prediction formulated by the British science fiction writer Arthur C. Clarke. They are**:**

1. *When a distinguished but elderly scientist states that something is possible, he is almost certainly right. When he states that something is impossible, he is very probably wrong.*
2. *The only way of discovering the limits of the possible is to venture a little way past them into the impossible.*
3. *Any sufficiently advanced technology is indistinguishable from magic.*

"Radio has no future. Heavier-than-air flying machines are impossible. X-rays will prove to be a hoax."

William Thomson, Lord Kelvin, British scientist (1899)

"I think there is a world market for maybe five computers."

Thomas Watson, President of IBM (1943)

"Two years from now, spam will be solved."

Bill Gates, founder of Microsoft (2004)

"It's tough to make predictions, especially about the future."

Yogi Berra

THANKS FOR YOUR ATTENTION