CLASSIFICATION OF SU(2)-SYMMETRIC TENSOR NETWORKS

Matthieu Mambrini

Laboratoire de Physique Théorique

CNRS & Université de Toulouse

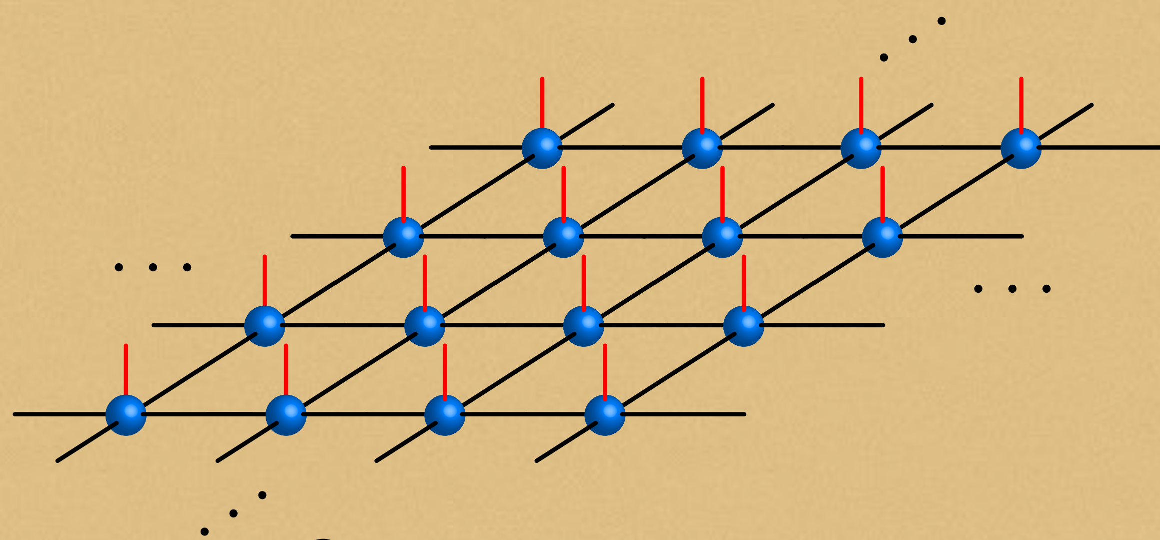
M.M., R. Orús, D. Poilblanc, Phys. Rev. B **94**, 205124 (2016)
D. Poilblanc, M.M., arXiv:1702.05950

OUTLINE

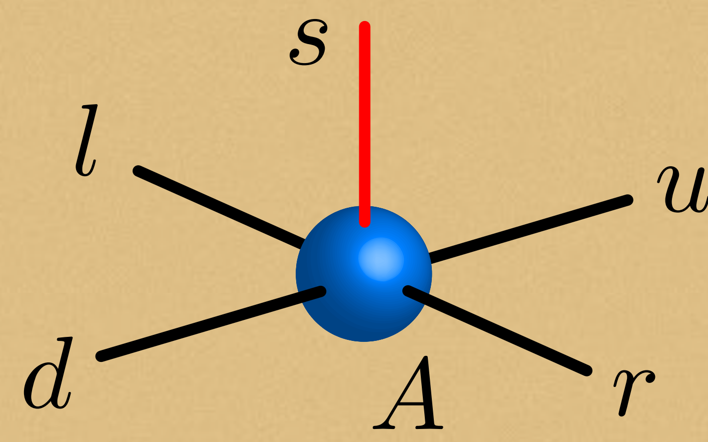
- SU(2) symmetric PEPS on the square lattice
  Why ? Construction / Classification scheme
- Some remarkable PEPS
- $J_1 - J_2$ model on the square lattice. PEPS description of the QCP
- Perspectives

TENSOR CLASSIFICATION

PEPS & SYMMETRIES : STRATEGY



Symmetry matters !



Symmetry ??

Strategy : systematic building of SU(2) invariant and linearly independent tensors organized in classes $\mathcal{C}$ with identical physical S and virtual V degrees of freedom and identical IRREP of the lattice point group

State design

$$\sum_{i\in\mathcal{C}} \lambda_i A^{(i)}$$

natural route to spin liquids manifolds

$$\sum_{i\in\mathcal{C},\mathcal{C}'} \lambda_i A^{(i)}$$

controlled symmetry breaking (lattice/spin nematics, chiral spin liquids...)

State optimization

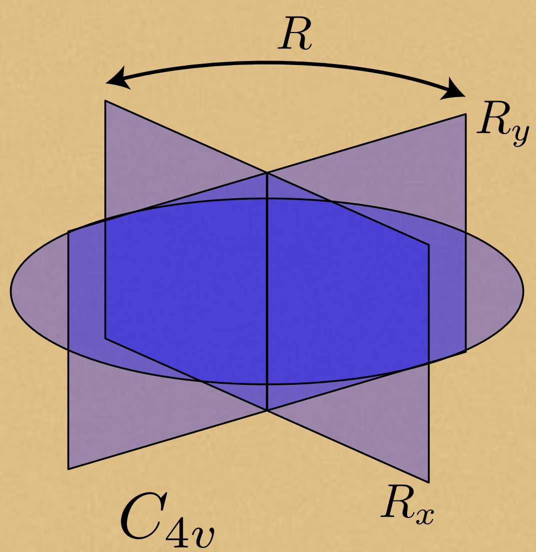
$$\sum_{i\in\mathcal{C}} \lambda_i A^{(i)}$$

Optimized ansätze for frustrated Heisenberg model

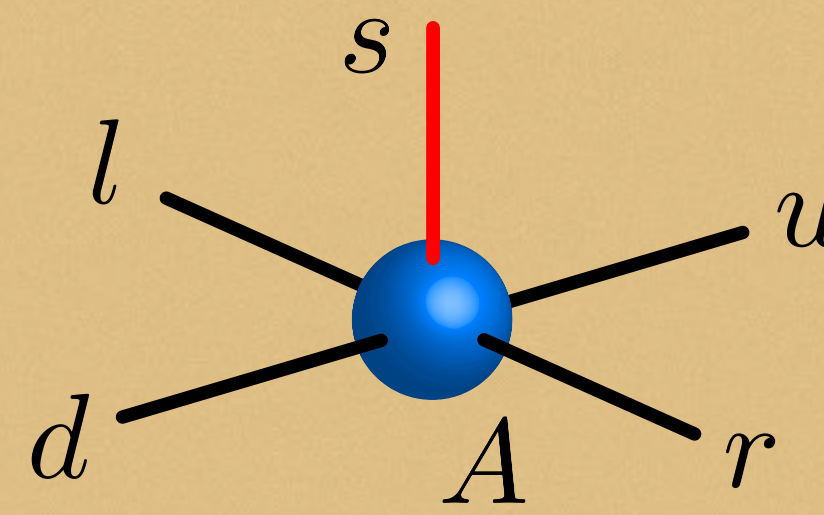
TENSOR CLASSIFICATION

SYMMETRIES

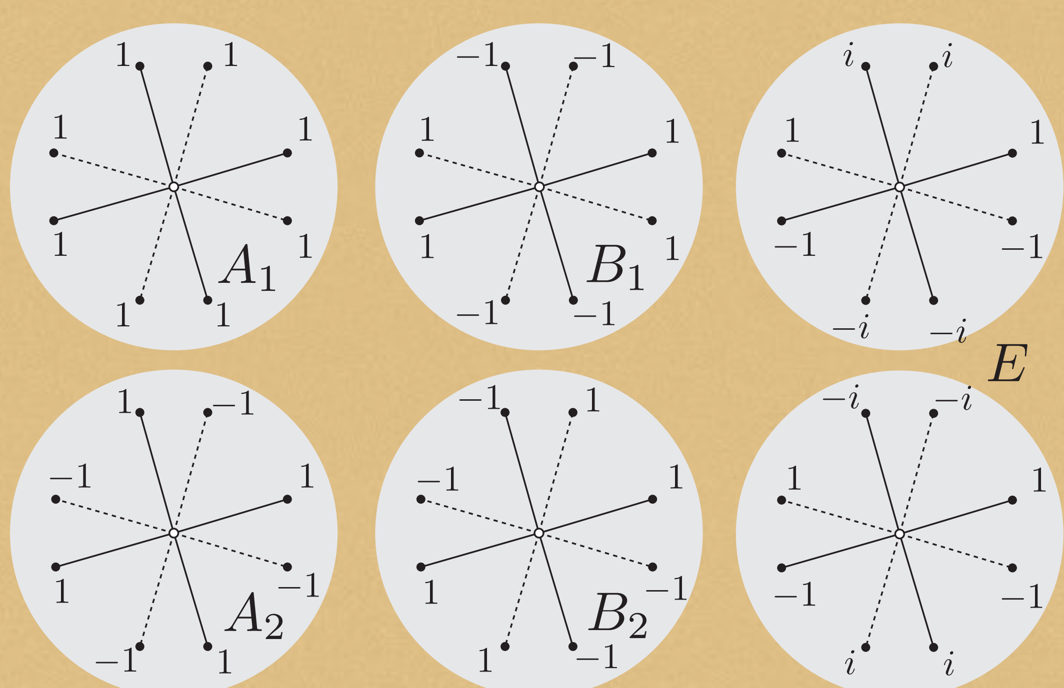
Discrete symmetries



- Translation invariance
- Point group C_{4v}



$$(s, u, l, d, r) \in S \otimes V^{\otimes 4}$$



SU(2)-Spin rotational invariance

A_s encodes a projector P_s

$$V^{\otimes 4} \longrightarrow S$$
$$(u, l, d, r) \longmapsto |s\rangle = |S, S_z = s\rangle$$

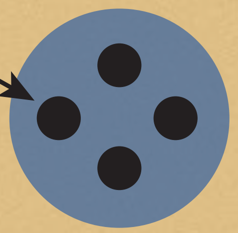
TENSOR CLASSIFICATION

PRINCIPLE

virtual spins

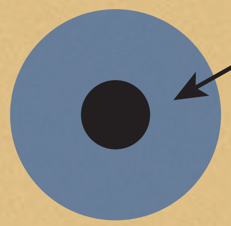
e.g. $2 \oplus \frac{1}{2} \oplus 0$

$D = 5 + 2 + 1$



$$V^{\otimes 4} \longrightarrow S$$

$$(u, l, d, r) \longmapsto |s\rangle = |S, S_z = s\rangle$$



physical spin

e.g.

$S = 3/2, Sz = -1/2$

finding all projectors P_s

$\Leftrightarrow$

enumerate all D^4 orthogonal spin-S wave functions that can be constructed out of the $(\oplus V_i)^{\otimes 4}$ basis states

$$|s\rangle = \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} A_s(\alpha_1, \alpha_2, \alpha_3, \alpha_4) |\alpha_1, \alpha_2, \alpha_3, \alpha_4\rangle$$

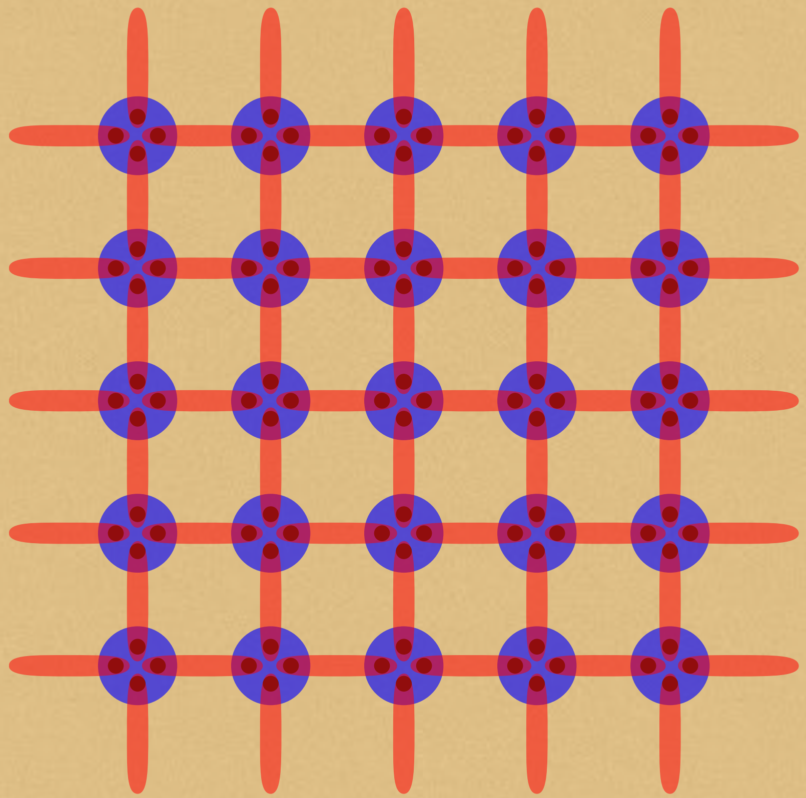
orthogonality

$$\sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} [A_s(\alpha_1, \alpha_2, \alpha_3, \alpha_4)]^* B_{s'}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \delta_{ss'} \delta_{AB}$$

TENSOR CLASSIFICATION

PRINCIPLE

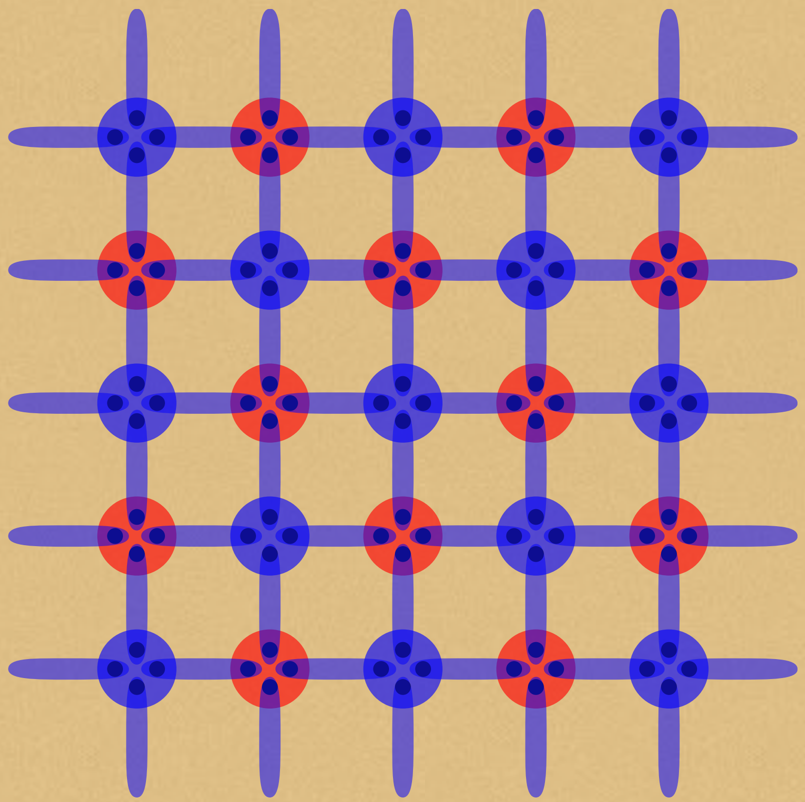
SU(2) invariance
global singlet



contracting NN virtual spins
virtual singlets

Bipartite lattice
π rotation around Y-axis
on one sublattice
bond singlet becomes

$$|\mathcal{S}\rangle = \sum_{\alpha=1}^{D} |v_\alpha v_\alpha\rangle$$



TENSOR CLASSIFICATION

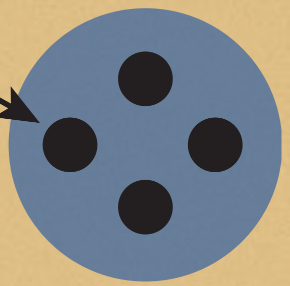
CONSERVED QUANTITIES

- SU(2) : total spin and z-component

$$\mathbf{S}^2 = (\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4)^2$$
$$S^z = S^z_1 + S^z_2 + S^z_3 + S^z_4$$

Composite spin

$$V = \oplus V_i$$
$$D = \sum_i (2V_i + 1)$$



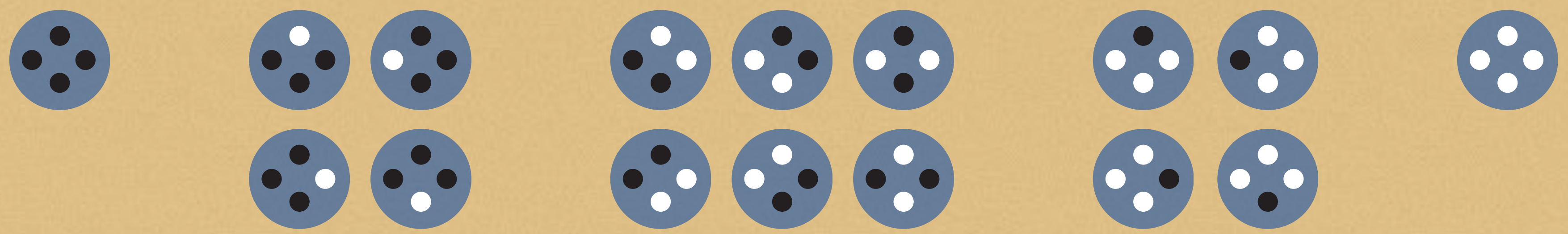
e.g. $2 \oplus \frac{1}{2} \oplus 0$

$D = 5 + 2 + 1$

- Point group : $\mathcal{R}_{\pi/2}$ rotation (A,B,E) and Δ_x reflection (1,2)
- Occupation numbers $\{n_i\}$ such as $\sum_i n_i = 4$

Example

$$V = \frac{1}{2} \oplus 0$$
$$D = 2 + 1$$



$\{4,0\}, S=2$
$\{4,0\}, S=1$
$\{4,0\}, S=0$

$\{3,1\}, S=3/2$
$\{3,1\}, S=1/2$

$\{2,2\}, S=1$
$\{2,2\}, S=0$

$\{1,3\}, S=1/2$

$\{0,4\}, S=0$

TENSOR CLASSIFICATION
IN PRACTICE

Fix $V = \oplus V_i$
Generate the D^4 basis states with $D = \sum_i (2V_i + 1)$

$$\mathcal{O}_{\sigma,\sigma_z,\rho,\delta,\nu} = \sigma \mathbf{S}^2 + \sigma_z S_z + \rho \mathcal{R} + \delta \Delta + \nu \mathcal{N}$$

Find all D^4 eigenvalues and eigenvectors

Problems

- we want analytical expressions of tensors (Mathematica)
- for e.g. $D = 7$, too large Hilbert space $D^4 = 2401$

TENSOR CLASSIFICATION

IN PRACTICE

$$\mathcal{O}_{\sigma,\sigma_z,\rho,\delta,\nu} = \sigma \mathbf{S}^2 + \sigma_z S_z + \rho \mathcal{R} + \delta \Delta + \nu \mathcal{N}$$

Key points

- Numerical diagonalization (eigenvalues and eigenvectors) possible
- The analytical form of the eigenvalues is known

Recipe

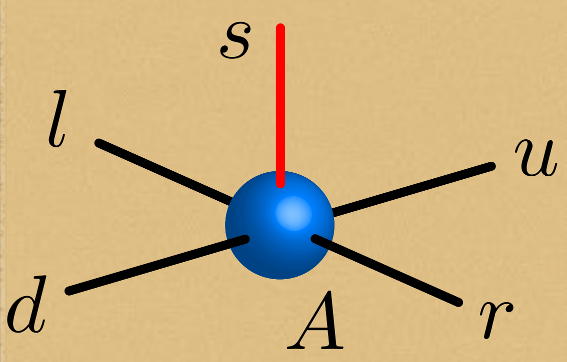
1. Numerical diagonalization
2. Exact eigenvalues and degeneracies
3. Eigenvector masks (discard 0 components)
4. Exact target eigenvalue λ + eigenvector mask (subspace $\mathcal{E}$),find $\ker_{\mathcal{E}}(\mathcal{O} - \lambda.\mathrm{Id})$
5. Check degeneracy

TENSOR CLASSIFICATION

EXAMPLE

$$V = \frac{1}{2} \oplus 0$$

$$D = 2 + 1$$



$\mathbf{S = 2,\ n_{occ.} = \{4, 0\}}$

State	1	2	3	4	5
Sz	2	1	0	-1	-2
Rx	1	1	1	1	1
Ry	1	1	1	1	1
R	1	1	1	1	1
Landau	A1	A1	A1	A1	A1

$\mathbf{S = 3/2,\ n_{occ.} = \{3, 1\}}$

State	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Sz	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
Rx	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
Ry	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
R	1	1	1	1	-1	-1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$	$-i$
Landau	A1	A1	A1	A1	B1	B1	B1	B1	E	E	E	E	E	E	E	E

$\mathbf{S = 1,\ n_{occ.} = \{4, 0\}}$

State	22	23	24	25	26	27	28	29	30
Sz	1	0	-1	1	0	-1	1	0	-1
Rx	1	1	1	0	0	0	0	0	0
Ry	1	1	1	0	0	0	0	0	0
R	-1	-1	-1	i	i	i	$-i$	$-i$	$-i$
Landau	B1	B1	B1	E	E	E	E	E	E

$\mathbf{S = 1,\ n_{occ.} = \{2, 2\}}$

State	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Sz	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1
Rx	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	-1	-1	-1
Ry	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	-1	-1	-1
R	1	1	1	1	1	1	-1	-1	-1	i	i	i	$-i$	$-i$	$-i$	-1	-1	-1
Landau	A1	A1	A1	A1	A1	A1	B1	B1	B1	E	E	E	E	E	E	B2	B2	B2

$\mathbf{S = 1/2,\ n_{occ.} = \{3, 1\}}$

State	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
Sz	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
Rx	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
Ry	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
R	1	1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$	$-i$	1	1	-1	-1
Landau	A1	A1	B1	B1	E	E	E	E	E	E	E	E	A2	A2	B2	B2

$\mathbf{S = 1/2,\ n_{occ.} = \{1, 3\}}$

State	65	66	67	68	69	70	71	72
Sz	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
Rx	1	1	1	1	0	0	0	0
Ry	1	1	1	1	0	0	0	0
R	1	1	-1	-1	i	i	$-i$	$-i$
Landau	A1	A1	B1	B1	E	E	E	E

$\mathbf{S = 0,\ n_{occ.} = \{4, 0\}}$

State	73	74
Sz	0	0
Rx	1	-1
Ry	1	-1
R	1	-1
Landau	A1	B2

$\mathbf{S = 0,\ n_{occ.} = \{2, 2\}}$

State	75	76	77	78	79	80
Sz	0	0	0	0	0	0
Rx	1	0	0	0	0	-1
Ry	1	0	0	0	0	-1
R	-1	i	i	$-i$	$-i$	1
Landau	B1	E	E	E	E	A2

$\mathbf{S = 0,\ n_{occ.} = \{0, 4\}}$

State	81
Sz	0
Rx	1
Ry	1
R	1
Landau	A1

Tensor

$$T_0(0,0,1,1) = -\frac{1}{2\sqrt{3}}$$

$$T_0(0,1,0,1) = \frac{1}{\sqrt{3}}$$

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$$T_0(1,1,0,0) = -\frac{1}{2\sqrt{3}}$$

Index convention

Virtual state (S_z)	Tensor index
S_1	0
$S_1 - 1$	1
$\ldots$	$\ldots$
$-S_1$	$2S_1$
S_2	$2S_1 + 1$
$S_2 - 1$	$2S_1 + 2$
$\ldots$	$\ldots$
$-S_p$	$2\,(S_1 + \ldots S_p) + (p-1)$

Wavefunction

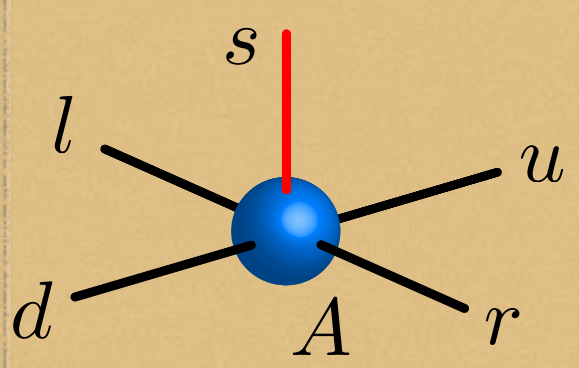
$$-\frac{|\{S_1,S_1,S_1,S_1,-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}\}\rangle}{2\sqrt{3}}+\frac{|\{S_1,S_1,S_1,S_1,-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}\}\rangle}{\sqrt{3}}-\frac{|\{S_1,S_1,S_1,S_1,-\frac{1}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2}\}\rangle}{2\sqrt{3}}-$$
$$\frac{|\{S_1,S_1,S_1,S_1,\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\}\rangle}{2\sqrt{3}}+\frac{|\{S_1,S_1,S_1,S_1,\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\}\rangle}{\sqrt{3}}-\frac{|\{S_1,S_1,S_1,S_1,\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\}\rangle}{2\sqrt{3}}$$

TENSOR CLASSIFICATION

EXAMPLE

$$V = \frac{1}{2} \oplus 0$$

$$D = 2 + 1$$



$\mathbf{S = 2,\ n_{occ.} = \{4, 0\}}$

State	1	2	3	4	5
Sz	2	1	0	-1	-2
Rx	1	1	1	1	1
Ry	1	1	1	1	1
R	1	1	1	1	1
Landau	A1	A1	A1	A1	A1

$\mathbf{S = 3/2,\ n_{occ.} = \{3, 1\}}$

State	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Sz	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
Rx	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
Ry	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
R	1	1	1	1	-1	-1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$	$-i$
Landau	A1	A1	A1	A1	B1	B1	B1	B1	E	E	E	E	E	E	E	E

$\mathbf{S = 1,\ n_{occ.} = \{4, 0\}}$

State	22	23	24	25	26	27	28	29	30
Sz	1	0	-1	1	0	-1	1	0	-1
Rx	1	1	1	0	0	0	0	0	0
Ry	1	1	1	0	0	0	0	0	0
R	-1	-1	-1	i	i	i	$-i$	$-i$	$-i$
Landau	B1	B1	B1	E	E	E	E	E	E

$\mathbf{S = 1,\ n_{occ.} = \{2, 2\}}$

State	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
Sz	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1
Rx	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	-1	-1	-1
Ry	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	-1	-1	-1
R	1	1	1	1	1	1	-1	-1	-1	i	i	i	$-i$	$-i$	$-i$	-1	-1	-1
Landau	A1	A1	A1	A1	A1	A1	B1	B1	B1	E	E	E	E	E	E	B2	B2	B2

$\mathbf{S = 1/2,\ n_{occ.} = \{3, 1\}}$

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Rx	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
Ry	1	1	1	1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
R	1	1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$	$-i$	1	1	-1	-1
Landau	A1	A1	B1	B1	E	E	E	E	E	E	E	E	A2	A2	B2	B2

$\mathbf{S = 1/2,\ n_{occ.} = \{1, 3\}}$

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Sz	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
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Ry	1	1	1	1	0	0	0	0
R	1	1	-1	-1	i	i	$-i$	$-i$
Landau	A1	A1	B1	B1	E	E	E	E

$\mathbf{S = 0,\ n_{occ.} = \{4, 0\}}$

State	73	74
Sz	0	0
Rx	1	-1
Ry	1	-1
R	1	-1
Landau	A1	B2

$\mathbf{S = 0,\ n_{occ.} = \{2, 2\}}$

State	75	76	77	78	79	80
Sz	0	0	0	0	0	0
Rx	1	0	0	0	0	-1
Ry	1	0	0	0	0	-1
R	-1	i	i	$-i$	$-i$	1
Landau	B1	E	E	E	E	A2

$\mathbf{S = 0,\ n_{occ.} = \{0, 4\}}$

State	81
Sz	0
Rx	1
Ry	1
R	1
Landau	A1

Tensor

$$T_{\frac{1}{2}}(0,0,2,1) = \frac{1}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(0,1,0,2) = -\frac{i}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(0,2,0,1) = \frac{i}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(0,2,1,0) = -\frac{i}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(1,0,0,2) = \frac{i}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(1,0,2,0) = -\frac{1}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(2,0,1,0) = \frac{1}{2\sqrt{2}}$$

$$T_{\frac{1}{2}}(2,1,0,0) = -\frac{1}{2\sqrt{2}}$$

Index convention

Virtual state (S_z)	Tensor index
S_1	0
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$-S_1$	$2S_1$
S_2	$2S_1 + 1$
$S_2 - 1$	$2S_1 + 2$
$\dots$	$\dots$
$-S_p$	$2(S_1 + \dots S_p) + (p-1)$

Wavefunction

$$\frac{i|\{S_1,S_1,S_1,S_2,-\frac{1}{2},\frac{1}{2},\frac{1}{2},0\}\rangle}{2\sqrt{2}} - \frac{i|\{S_1,S_1,S_1,S_2,\frac{1}{2},-\frac{1}{2},\frac{1}{2},0\}\rangle}{2\sqrt{2}} - \frac{|\{S_1,S_1,S_2,S_1,-\frac{1}{2},\frac{1}{2},0,\frac{1}{2}\}\rangle}{2\sqrt{2}} +$$
$$\frac{|\{S_1,S_1,S_2,S_1,\frac{1}{2},\frac{1}{2},0,-\frac{1}{2}\}\rangle}{2\sqrt{2}} - \frac{i|\{S_1,S_2,S_1,S_1,\frac{1}{2},0,-\frac{1}{2},\frac{1}{2}\}\rangle}{2\sqrt{2}} + \frac{i|\{S_1,S_2,S_1,S_1,\frac{1}{2},0,\frac{1}{2},-\frac{1}{2}\}\rangle}{2\sqrt{2}} -$$
$$\frac{|\{S_2,S_1,S_1,S_1,0,-\frac{1}{2},\frac{1}{2},\frac{1}{2}\}\rangle}{2\sqrt{2}} + \frac{|\{S_2,S_1,S_1,S_1,0,\frac{1}{2},-\frac{1}{2},\frac{1}{2}\}\rangle}{2\sqrt{2}}$$

TENSOR CLASSIFICATION

(AWFULL) EXAMPLE

$$\frac{\left(\frac{115}{4}-\frac{23i}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{5}{2},\frac{3}{2},\frac{5}{2}\}\rangle}{\sqrt{689997}}+\left(\frac{5}{2}i\sqrt{\frac{7}{98571}}-\frac{1}{4\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{5}{2},\frac{5}{2},\frac{3}{2}\}\rangle+$$

$$\left(\frac{43\sqrt{\frac{5}{1379994}}}{4}+\frac{5}{4}i\sqrt{\frac{35}{197142}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{3}{2},\frac{1}{2},\frac{5}{2}\}\rangle-\left(\frac{53}{4}+15i\right)\sqrt{\frac{3}{229999}}|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{3}{2},\frac{3}{2},\frac{3}{2}\}\rangle$$

$$\left(\frac{205}{4}i\sqrt{\frac{5}{1379994}}+\frac{1}{\sqrt{6899970}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{3}{2},\frac{5}{2},\frac{1}{2}\}\rangle+\left(\frac{73}{2}i\sqrt{\frac{5}{689997}}+\frac{3\sqrt{\frac{165}{20909}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{1}{2}$$

$$\left(\frac{7\sqrt{\frac{7}{98571}}}{4}-\frac{5}{4}i\sqrt{\frac{77}{8961}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2}\}\rangle+\frac{\left(\frac{79}{2}+\frac{125i}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{1}{2},\frac{3}{2},\frac{1}{2}\}\rangle}{\sqrt{689997}}+$$

$$\left(-\frac{13\sqrt{\frac{3}{1149995}}}{2}-\frac{5}{2}i\sqrt{\frac{5}{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},-\frac{1}{2},\frac{5}{2},-\frac{1}{2}\}\rangle+\left(\frac{113\sqrt{\frac{5}{1379994}}}{4}+3i\sqrt{\frac{15}{459998}}\right)|\{S_1,S_1,S_1,S_1,-$$

$$\left(-\frac{15}{2}i\sqrt{\frac{3}{229999}}-\frac{47}{2\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{1}{2},-\frac{1}{2},\frac{3}{2}\}\rangle-(23-15i)\sqrt{\frac{3}{459998}}|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\}\rangle+$$

$$\left(-\frac{25}{2}i\sqrt{\frac{7}{98571}}-\frac{41}{2\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{1}{2},\frac{3}{2},-\frac{1}{2}\}\rangle+\left(-\frac{155}{4}i\sqrt{\frac{5}{1379994}}+\frac{\sqrt{\frac{77}{89610}}}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{1}{2}$$

$$\left(-\frac{15}{4}i\sqrt{\frac{3}{229999}}-\frac{5\sqrt{\frac{21}{32857}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},-\frac{5}{2},\frac{5}{2}\}\rangle+\left(\frac{15}{4}i\sqrt{\frac{3}{229999}}-\frac{43\sqrt{\frac{7}{98571}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},$$

$$\left(-\frac{15}{4}i\sqrt{\frac{3}{229999}}+\frac{\sqrt{\frac{11}{62727}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},-\frac{1}{2},\frac{1}{2}\}\rangle+\left(\frac{65}{2}i\sqrt{\frac{3}{229999}}+\frac{107}{2\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},\frac{1}{2},$$

$$\left(\frac{\sqrt{\frac{3}{229999}}}{2}+\frac{305i}{4\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},\frac{3}{2},-\frac{3}{2}\}\rangle-\frac{\left(\frac{23}{2}+\frac{115i}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{3}{2},\frac{5}{2},-\frac{5}{2}\}\rangle}{\sqrt{689997}}+$$

$$\left(\frac{15}{4}i\sqrt{\frac{3}{229999}}+\frac{5\sqrt{\frac{21}{32857}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{5}{2},-\frac{5}{2},\frac{3}{2}\}\rangle+\left(\frac{67\sqrt{\frac{5}{1379994}}}{2}+\frac{15}{4}i\sqrt{\frac{15}{459998}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{5}{2},$$

$$\left(\frac{47}{4}-\frac{79i}{2}\right)\sqrt{\frac{5}{689997}}|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{5}{2},-\frac{1}{2},-\frac{1}{2}\}\rangle+\left(-\sqrt{\frac{145}{47586}}-\frac{1}{2}i\sqrt{\frac{435}{15862}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{5}{2},\frac{1}{2},-\frac{3}{2}\}\rangle+$$

$$\left(\frac{5\sqrt{\frac{7}{98571}}}{2}+\frac{i}{4\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{5}{2},\frac{5}{2},\frac{3}{2},-\frac{5}{2}\}\rangle+\left(\frac{155\sqrt{\frac{5}{1379994}}}{4}+\frac{1}{2}i\sqrt{\frac{77}{89610}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{5}{2},\frac{1}{2},$$

$$\left(\frac{1}{2}i\sqrt{\frac{3}{229999}}-\frac{305}{4\sqrt{689997}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{5}{2},\frac{3}{2},\frac{3}{2}\}\rangle+\left(-i\sqrt{\frac{145}{47586}}+\frac{\sqrt{\frac{435}{15862}}}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{5}{2},\frac{5}{2},\frac{1}{2}$$

$$\frac{\left(\frac{415}{2}-\frac{71i}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{5}{2}\}\rangle}{\sqrt{1379994}}+\left(\frac{1}{2}i\sqrt{\frac{5}{1379994}}+\sqrt{\frac{30}{229999}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{3}{2},\frac{1}{2},\frac{3}{2}\}\rangle+$$

$$\left(\frac{37\sqrt{\frac{5}{1379994}}}{4}+\frac{7}{4}i\sqrt{\frac{35}{197142}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{3}{2},\frac{3}{2},\frac{1}{2}\}\rangle-\frac{\left(\frac{289}{4}+\frac{325i}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{3}{2},\frac{5}{2},-\frac{1}{2}\}\rangle}{\sqrt{1379994}}+$$

$$\left(-13i\sqrt{\frac{11}{125454}}-\frac{5}{\sqrt{1379994}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{1}{2},-\frac{3}{2},\frac{5}{2}\}\rangle-\left(\frac{19}{2}-\frac{79i}{4}\right)\sqrt{\frac{5}{689997}}|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{1}{2},-\frac{1}{2},$$

$$\left(\frac{19\sqrt{\frac{5}{1379994}}}{4}+5i\sqrt{\frac{30}{229999}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}\}\rangle+\left(\frac{19}{2}-\frac{25i}{2}\right)\sqrt{\frac{5}{689997}}|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{1}{2},\frac{3}{2},-\frac{1}{2}\}$$

$$\frac{\left(\frac{71}{4}+\frac{415i}{2}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},-\frac{1}{2},\frac{5}{2},-\frac{3}{2}\}\rangle}{\sqrt{1379994}}+\left(-\frac{67\sqrt{\frac{5}{1379994}}}{2}-\frac{15}{4}i\sqrt{\frac{15}{459998}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},-\frac{5}{2},\frac{5}{2}\}\rangle+$$

$$\left(\frac{15}{4}i\sqrt{\frac{15}{459998}}+\frac{5\sqrt{\frac{105}{65714}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},-\frac{3}{2},\frac{3}{2}\}\rangle+\left(\frac{185\sqrt{\frac{5}{1379994}}}{4}-\frac{15}{4}i\sqrt{\frac{15}{459998}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},$$

$$\left(-\frac{95}{4}i\sqrt{\frac{5}{1379994}}-4\sqrt{\frac{30}{229999}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},\frac{1}{2},-\frac{1}{2}\}\rangle+\left(\frac{\sqrt{\frac{5}{1379994}}}{2}-i\sqrt{\frac{30}{229999}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},$$

$$\left(-\frac{43}{4}i\sqrt{\frac{5}{1379994}}+\frac{5\sqrt{\frac{35}{197142}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{1}{2},\frac{5}{2},-\frac{5}{2}\}\rangle+\left(-\frac{15}{4}i\sqrt{\frac{3}{229999}}+\frac{43\sqrt{\frac{7}{98571}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2}$$

$$\left(-\frac{15}{4}i\sqrt{\frac{15}{459998}}-\frac{5\sqrt{\frac{105}{65714}}}{4}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{3}{2},-\frac{3}{2},\frac{1}{2}\}\rangle+\left(-2\sqrt{\frac{5}{689997}}-\frac{7}{4}i\sqrt{\frac{35}{98571}}\right)|\{S_1,S_1,S_1,S_1,-\frac{3}{2},\frac{3}{2},$$

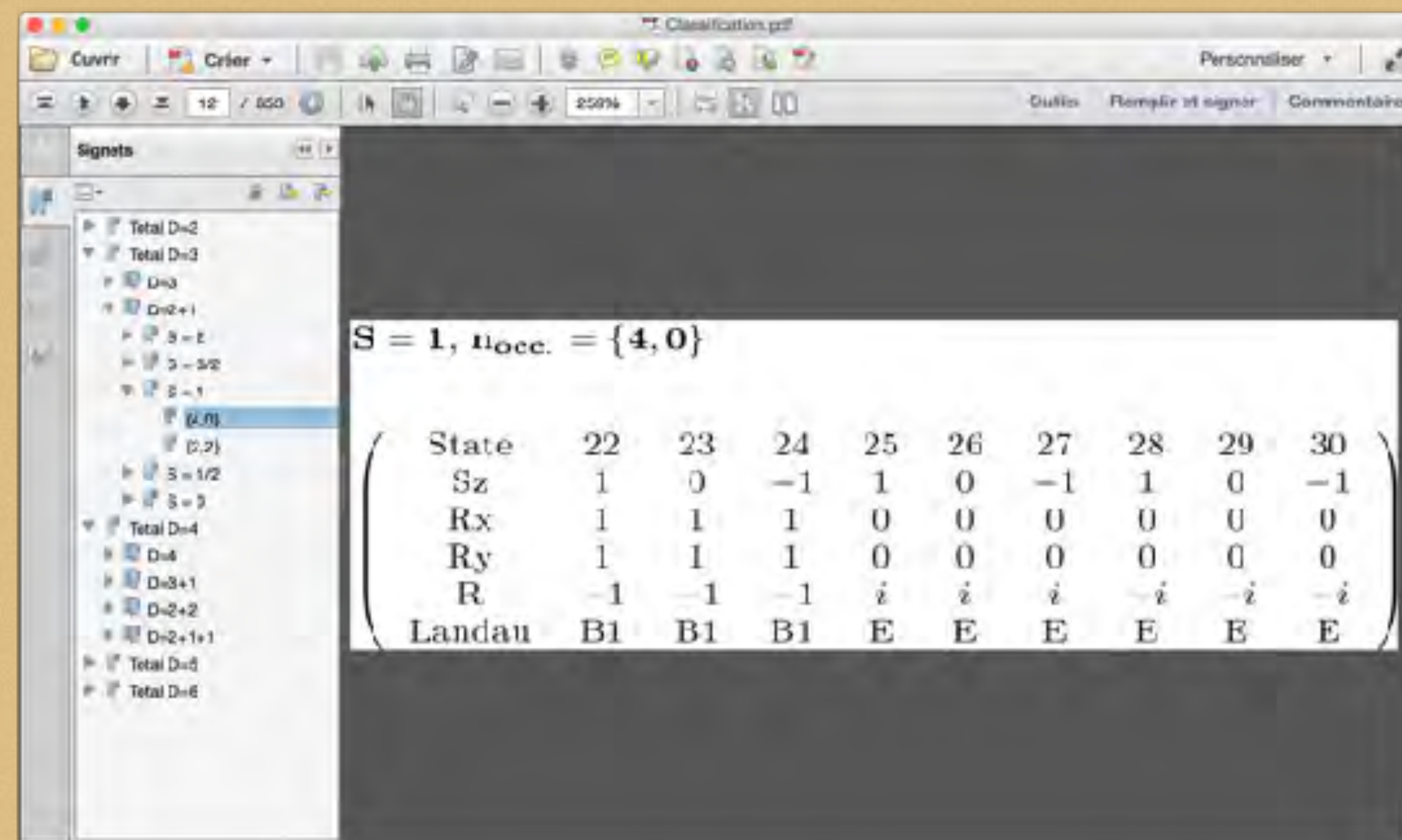
TENSOR CLASSIFICATION

USER MANUAL

Classification completed up to $D=6$ ($D=7$ also accessible)

$D=2$	$D=3$	$D=4$	$D=5$	$D=6$
$\frac{1}{2}$	$\frac{1}{2}\oplus 0$	$\frac{1}{2}\oplus 0\oplus 0$	$\frac{1}{2}\oplus 0\oplus 0\oplus 0$	$\frac{1}{2}\oplus 0\oplus 0\oplus 0\oplus 0$
	1	$\frac{1}{2}\oplus\frac{1}{2}$	$\frac{1}{2}\oplus\frac{1}{2}\oplus 0$	$\frac{1}{2}\oplus\frac{1}{2}\oplus 0\oplus 0$
		$1\oplus 0$	$1\oplus 0\oplus 0$	$\frac{1}{2}\oplus\frac{1}{2}\oplus\frac{1}{2}$
		$\frac{3}{2}$	$1\oplus\frac{1}{2}$	$1\oplus 0\oplus 0\oplus 0$
			$\frac{3}{2}\oplus 0$	$1\oplus\frac{1}{2}\oplus 0$
			2	$1\oplus 1$
				$\frac{3}{2}\oplus 0\oplus 0$
				$\frac{3}{2}\oplus\frac{1}{2}$
				$2\oplus 0$
				$\frac{5}{2}$

More than 20000 tensor expressions !



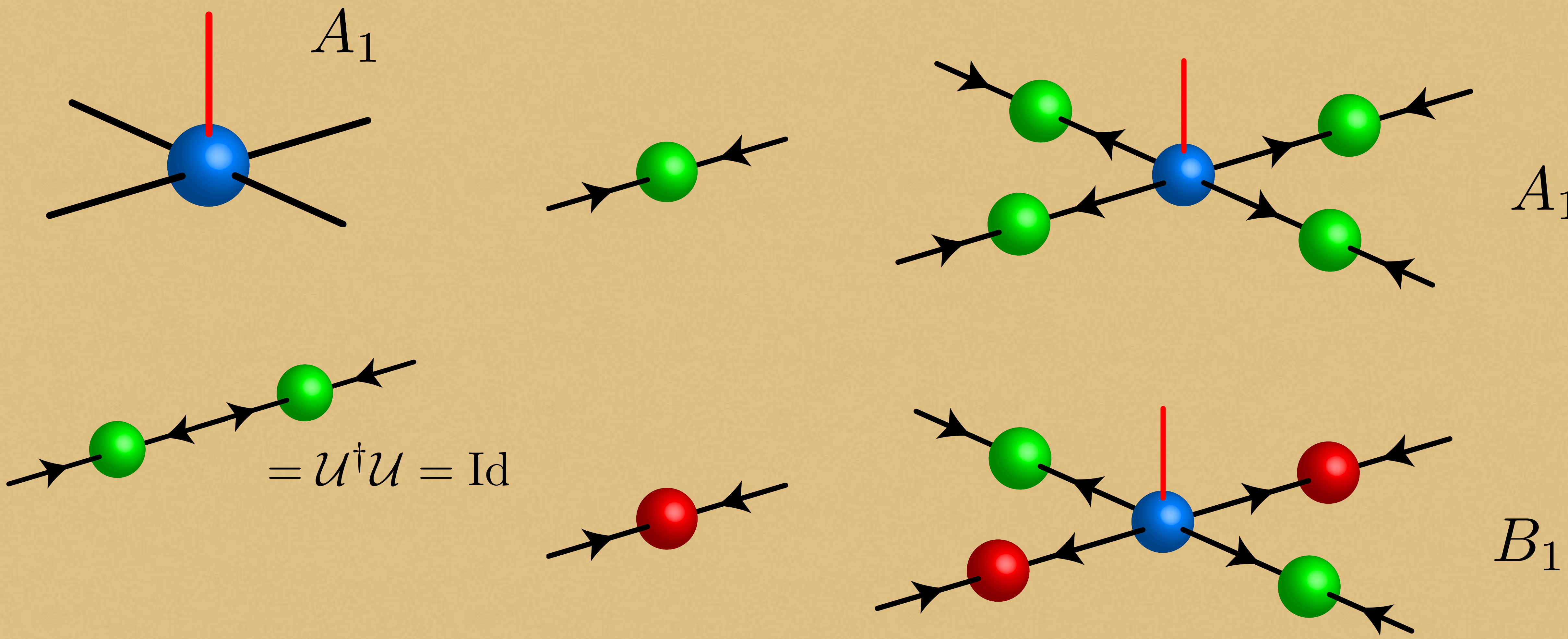
$\mathbf{S=1},\ \mathbf{n_{occ.}}=\{4,0\}$

State	22	23	24	25	26	27	28	29	30
Sz	1	0	−1	1	0	−1	1	0	−1
Rx	1	1	1	0	0	0	0	0	0
Ry	1	1	1	0	0	0	0	0	0
R	−1	−1	−1	i	i	i	$-i$	$-i$	$-i$
Landau	B1	B1	B1	E	E	E	E	E	E

Symmetry tables and tensor expressions available as supplementary materials

M.M., R. Orús, D. Poilblanc, Phys. Rev. B **94**, 205124 (2016)

GAUGE EQUIVALENT TENSORS



Tensors with different PG symmetries may lead to the same TN state after contraction

GAUGE EQUIVALENT TENSORS

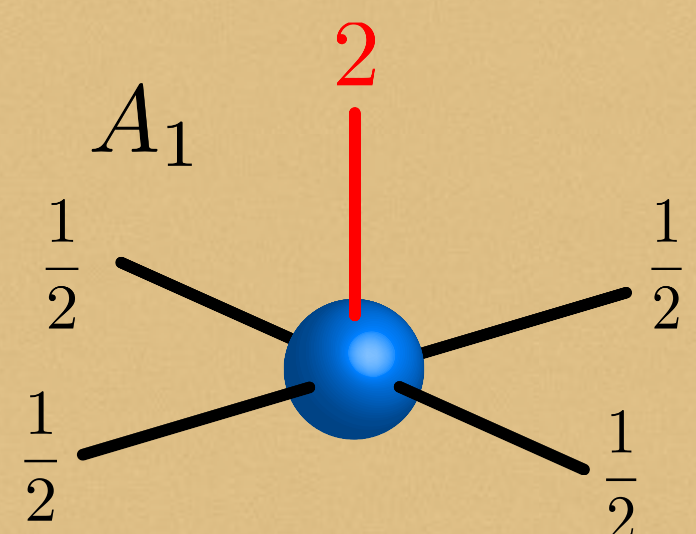
V \ S	1/2	1
$\frac{1}{2}$		$\mathbf{B_1}\ E$
$\frac{1}{2}\oplus 0$		
$n_{\text{occ.}}=\{1,3\}$	$\mathbf{A_1[B_1]\ E}$	
$n_{\text{occ.}}=\{2,2\}$		$\mathbf{A_1^{(a)}\ A_1^{(b)}[B_1]}\ E\ B_2$
$n_{\text{occ.}}=\{3,1\}$	$\mathbf{A_1[B_1]}\ E^{(a,b)}\ \mathbf{A_2[B_2]}$	
1		$B_1\ E^{(a,b)}\ A_2$
$\frac{1}{2}\oplus 0\oplus 0$		
$n_{\text{occ.}}=\{1,1,2\}$	$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$	
$n_{\text{occ.}}=\{2,1,1\}$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$
$\frac{1}{2}\oplus\frac{1}{2}$		

$n_{\text{occ.}}=\{1,2,1\}$	$A_1^{(a-c)}[B_1^{(a-c)}]\ E^{(a-f)}\ A_2^{(a-c)}[B_2^{(a-c)}]$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$	
$n_{\text{occ.}}=\{1,1,2\}$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$		
$1\oplus 0\oplus 0$				
$n_{\text{occ.}}=\{1,1,2\}$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$		
$n_{\text{occ.}}=\{2,1,1\}$		$A_1[B_1]\ E^{(a-c)}\ A_2^{(a,b)}[B_2^{(a,b)}]$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$
$1\oplus\frac{1}{2}$				
$n_{\text{occ.}}=\{1,3\}$	$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-c)}\ A_2[B_2]$	
$n_{\text{occ.}}=\{2,2\}$		$A_1^{(a-c)}[B_1^{(a-c)}]\ A_1^{(d,e)}\ E^{(a-e)}\ A_2^{(a,b)}[B_2^{(a,b)}]\ B_2^{(c,d)}$		$A_1^{(a,b)}[B_1^{(a,b)}]\ B_1^{(c)}\ E^{(a-e)}\ A_2[B_2]\ B_2$
$n_{\text{occ.}}=\{3,1\}$	$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-d)}\ A_2^{(a,b)}[B_2^{(a,b)}]$		$A_1^{(a-c)}[B_1^{(a-c)}]\ E^{(a-e)}\ A_2^{(a,b)}[B_2^{(a,b)}]$	
$\frac{3}{2}\oplus 0$				
$n_{\text{occ.}}=\{1,3\}$			$\mathbf{A_1[B_1]}\ E$	
$n_{\text{occ.}}=\{2,2\}$		$A_1^{(a)}\ A_1^{(b)}[B_1]\ E\ B_2$		$B_1[A_2]\ E^{(a,b)}$
$n_{\text{occ.}}=\{3,1\}$	$A_1[B_1]\ E^{(a,b)}\ A_2[B_2]$		$A_1^{(a,b)}[B_1^{(a,b)}]\ E^{(a-d)}\ A_2^{(a,b)}[B_2^{(a,b)}]$	
2		$B_1^{(a,b)}[A_2^{(a,b)}]\ E^{(a-d)}$		$A_1^{(a,b)}[B_1^{(a,b)}]\ A_1^{(c,d)}\ E^{(a-c)}\ A_2[B_2^{(a)}]\ B_2^{(b,c)}$

REMARKABLE PEPS

REMARKABLE PEPS

SPIN-S (S EVEN INTEGER) AKLT STATES

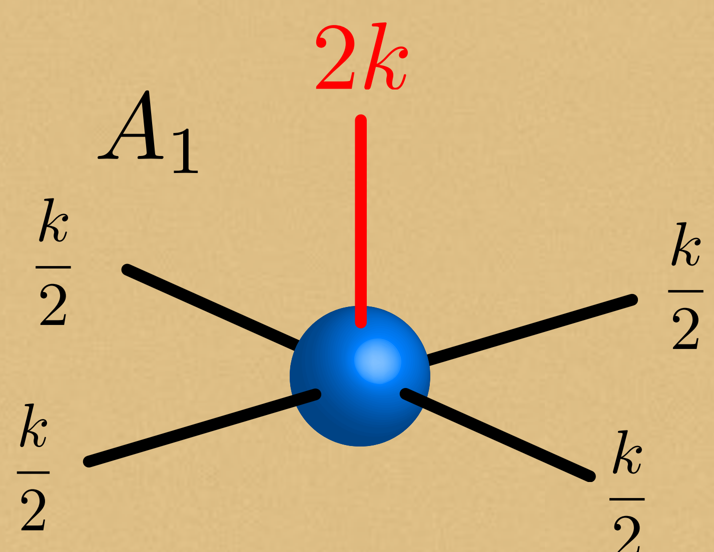


$S = 2$

$V = \frac{1}{2}$

$D = 2$

Generalization

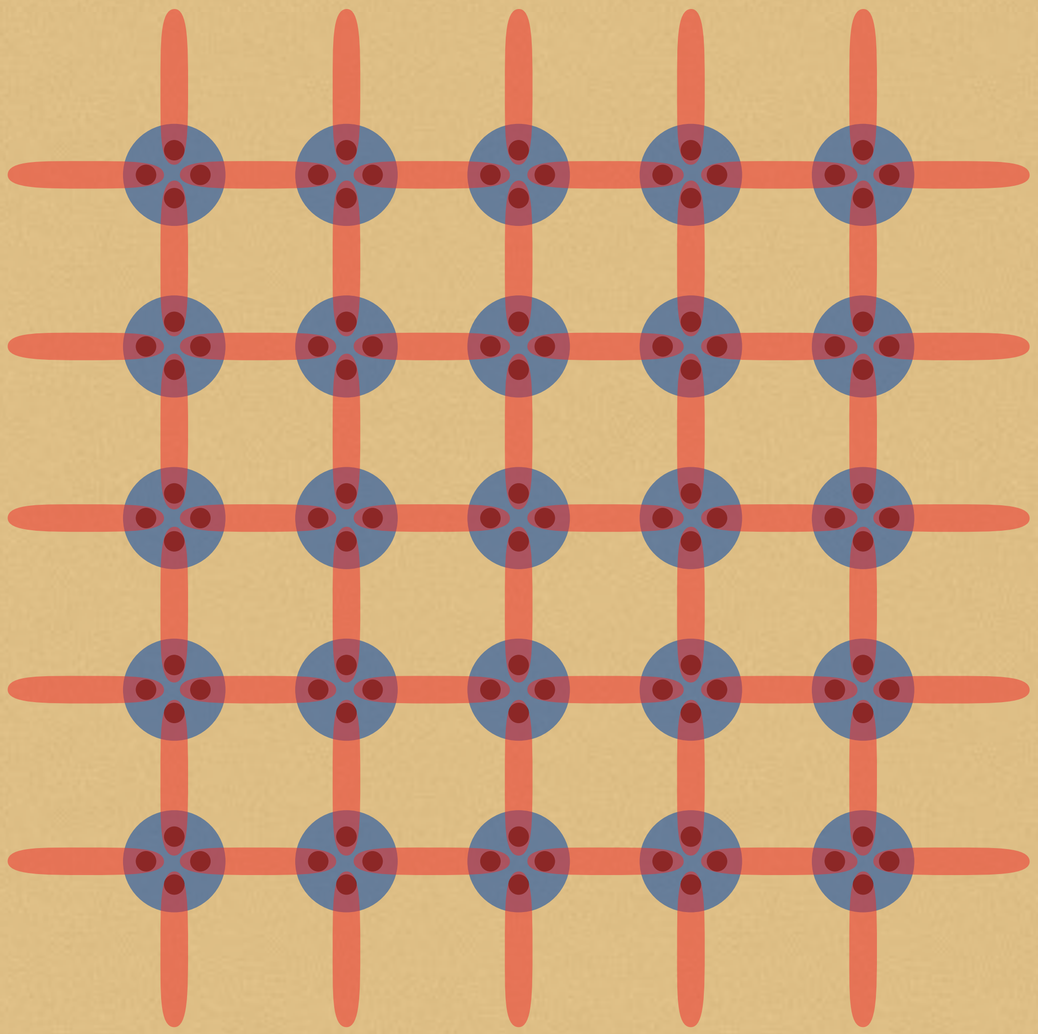


$S = 2k$

$V = \frac{k}{2}$

$D = k + 1$

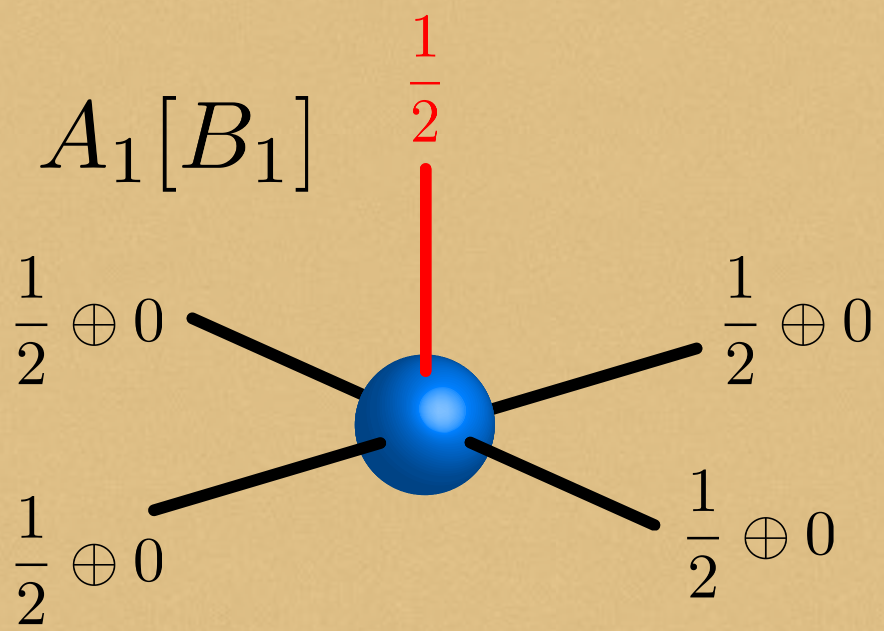
k integer



REMARKABLE PEPS

SPIN-1/2 RVB STATE

$A_1[B_1]$



$$S = \frac{1}{2}$$

$$V = \frac{1}{2} \oplus 0$$

$$D = 3$$

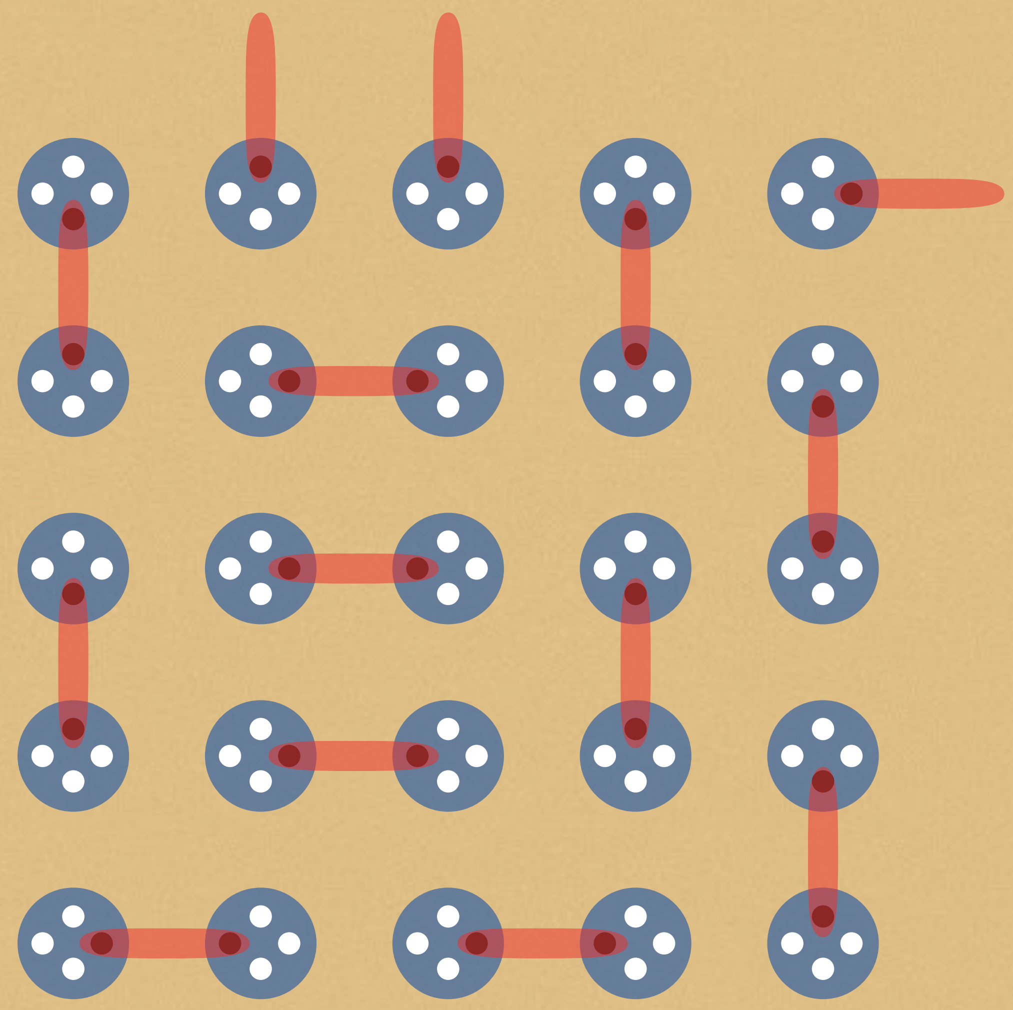
$$n_{\text{occ.}} = \{1, 3\}$$

A_1

$$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$$
$$T_{\frac{1}{2}}(2,0,2,2) = \frac{1}{2}$$
$$T_{\frac{1}{2}}(2,2,0,2) = \frac{1}{2}$$
$$T_{\frac{1}{2}}(2,2,2,0) = \frac{1}{2}$$

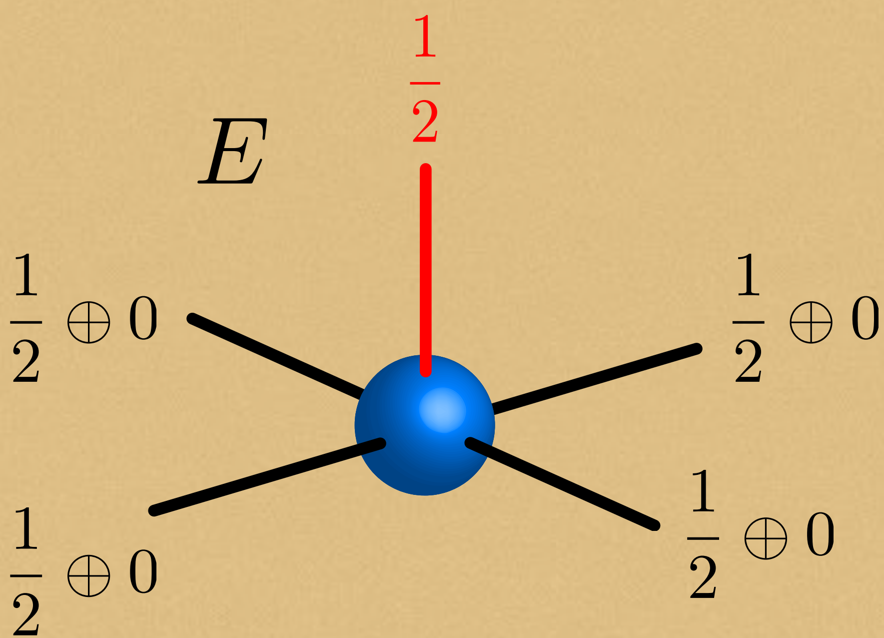
B_1

$$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$$
$$T_{\frac{1}{2}}(2,0,2,2) = -\frac{1}{2}$$
$$T_{\frac{1}{2}}(2,2,0,2) = \frac{1}{2}$$
$$T_{\frac{1}{2}}(2,2,2,0) = -\frac{1}{2}$$



REMARKABLE PEPS

FERMIONIC SPIN-1/2 RVB STATE



$$S = \frac{1}{2}$$

$$V = \frac{1}{2} \oplus 0$$

$$D = 3$$

$$n_{\text{occ.}} = \{1, 3\}$$

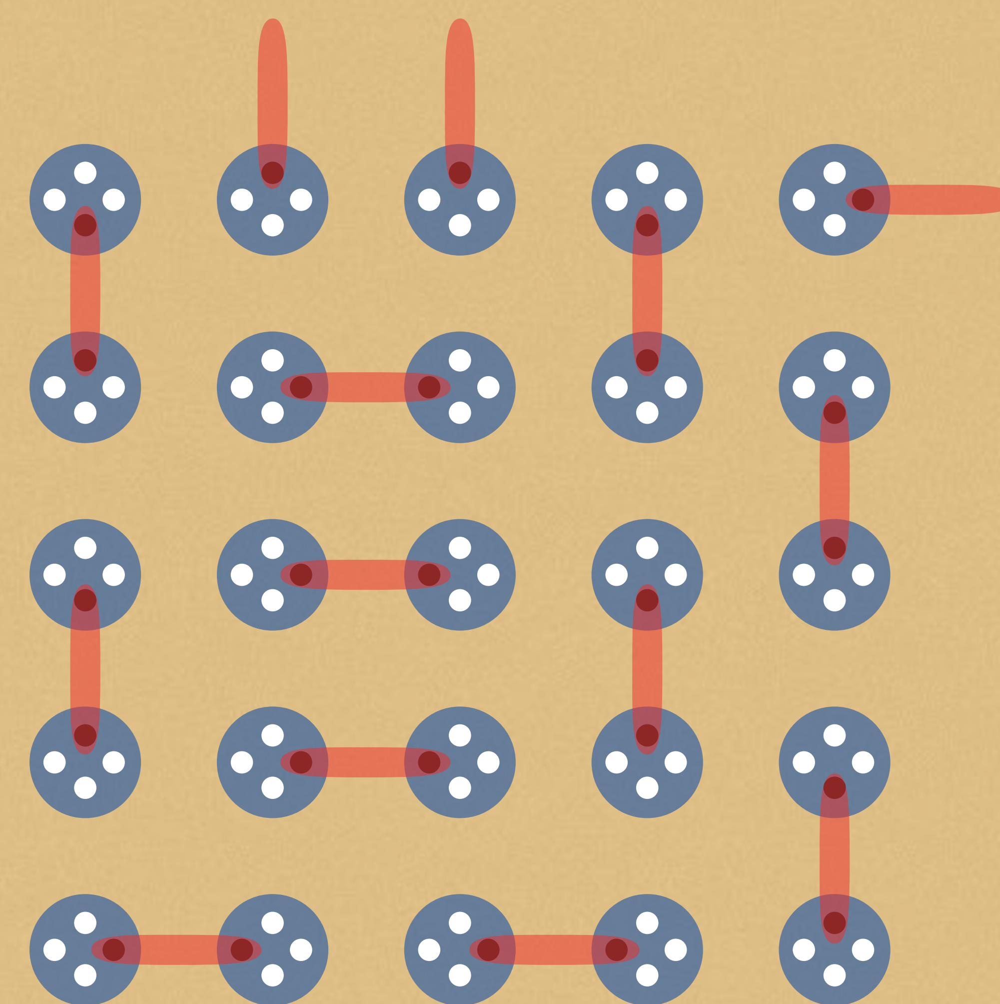
$$E$$

$$T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2} \qquad T_{\frac{1}{2}}(0,2,2,2) = \frac{1}{2}$$

$$T_{\frac{1}{2}}(2,0,2,2) = \frac{i}{2} \qquad T_{\frac{1}{2}}(2,0,2,2) = -\frac{i}{2}$$

$$T_{\frac{1}{2}}(2,2,0,2) = -\frac{1}{2} \qquad T_{\frac{1}{2}}(2,2,0,2) = -\frac{1}{2}$$

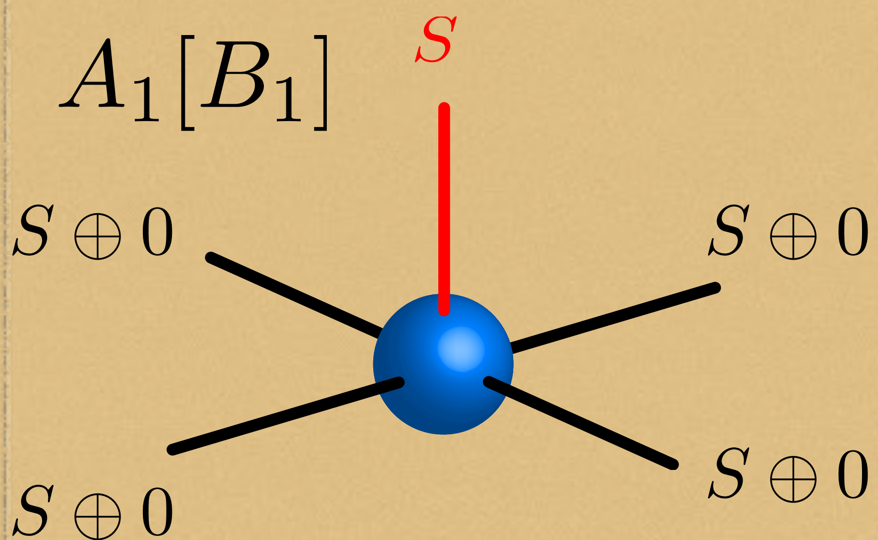
$$T_{\frac{1}{2}}(2,2,2,0) = -\frac{i}{2} \qquad T_{\frac{1}{2}}(2,2,2,0) = \frac{i}{2}$$



REMARKABLE PEPS

GENERALIZED SPIN-S RVB STATE

$A_1[B_1]$

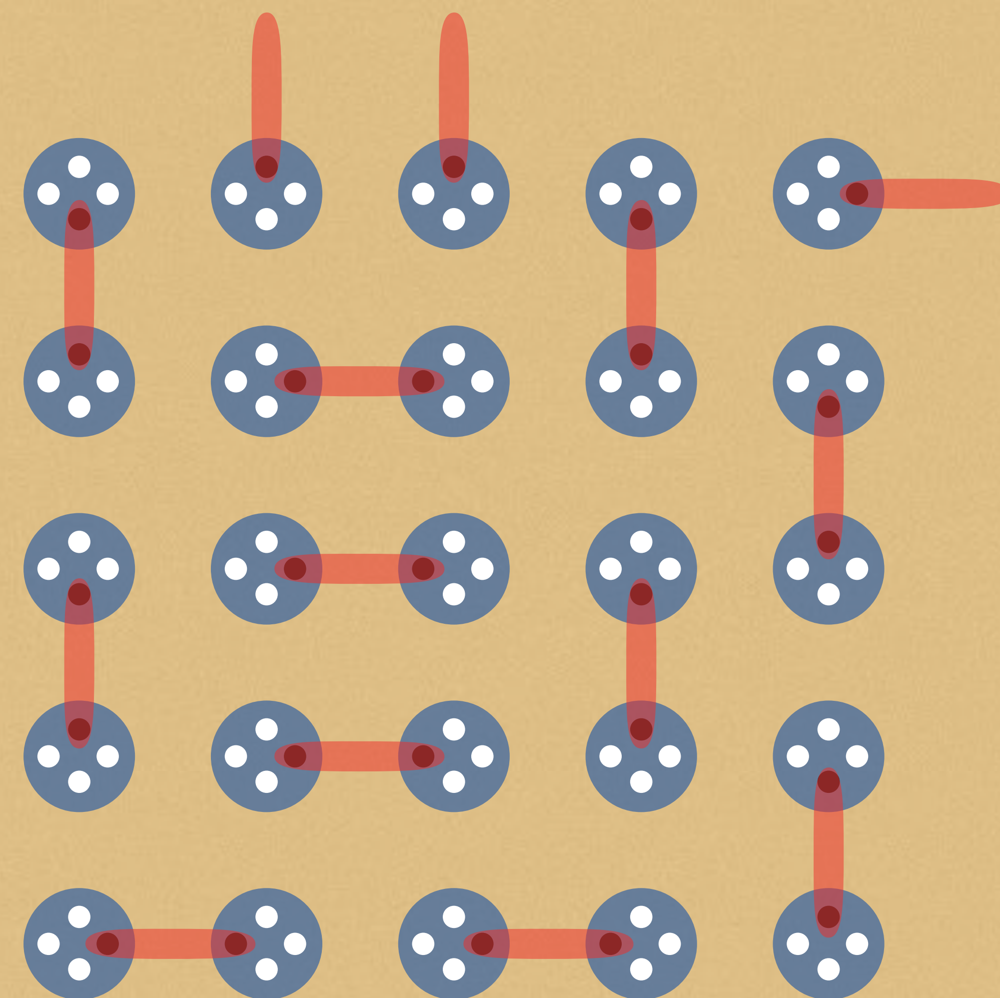


$n_{\mathrm{occ.}} = \{1, 3\}$

S

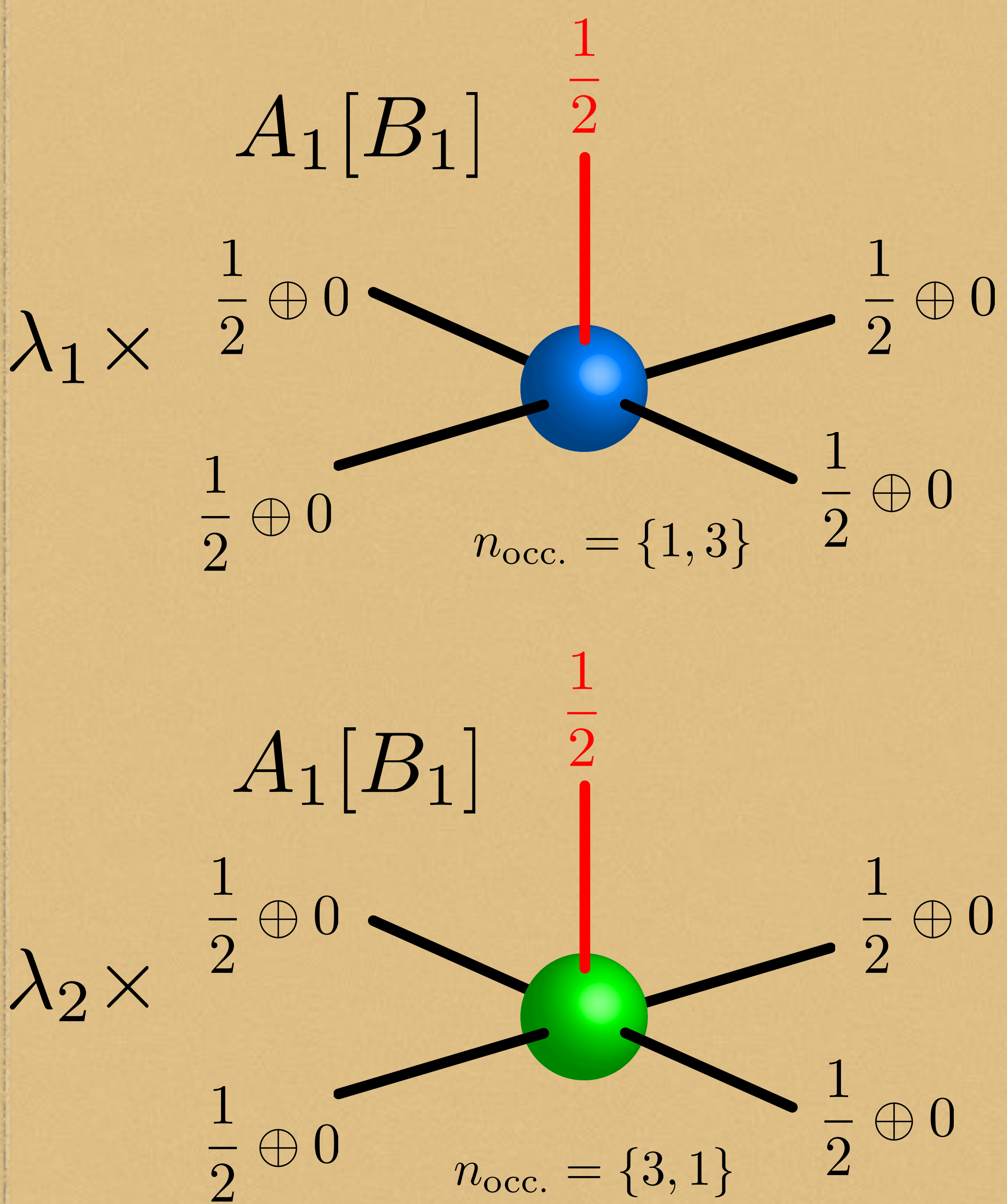
$$V = S \oplus 0$$

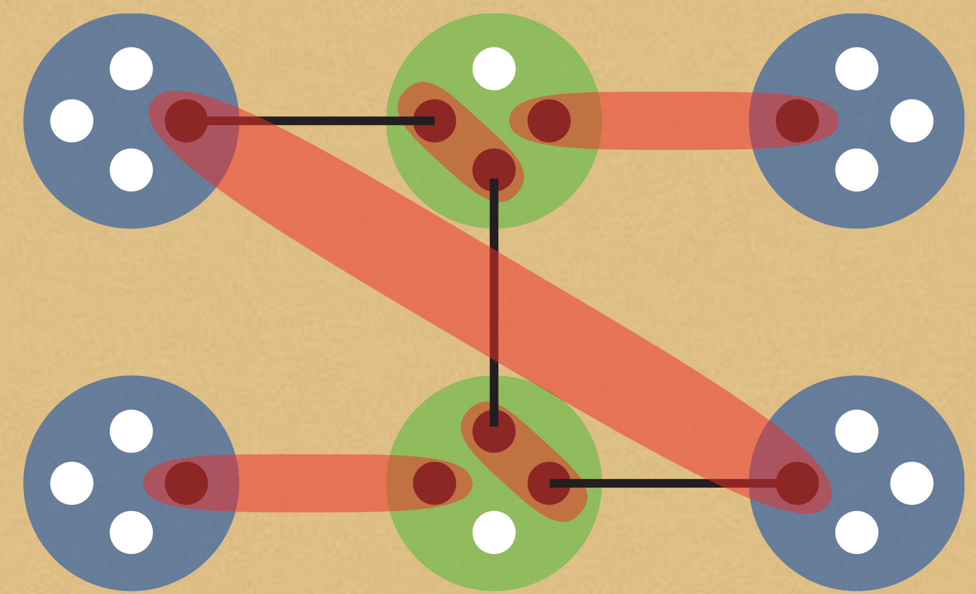
$$D = 2S + 2$$



REMARKABLE PEPS

LONG RANGE SPIN-1/2 RVB STATE

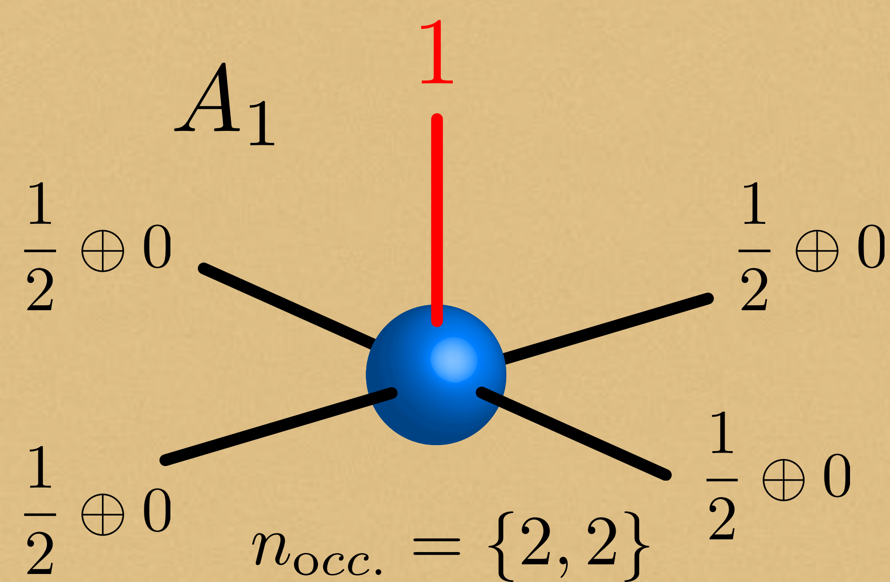




Singlet bond distribution controlled by the ratio λ_2/λ_1

REMARKABLE PEPS

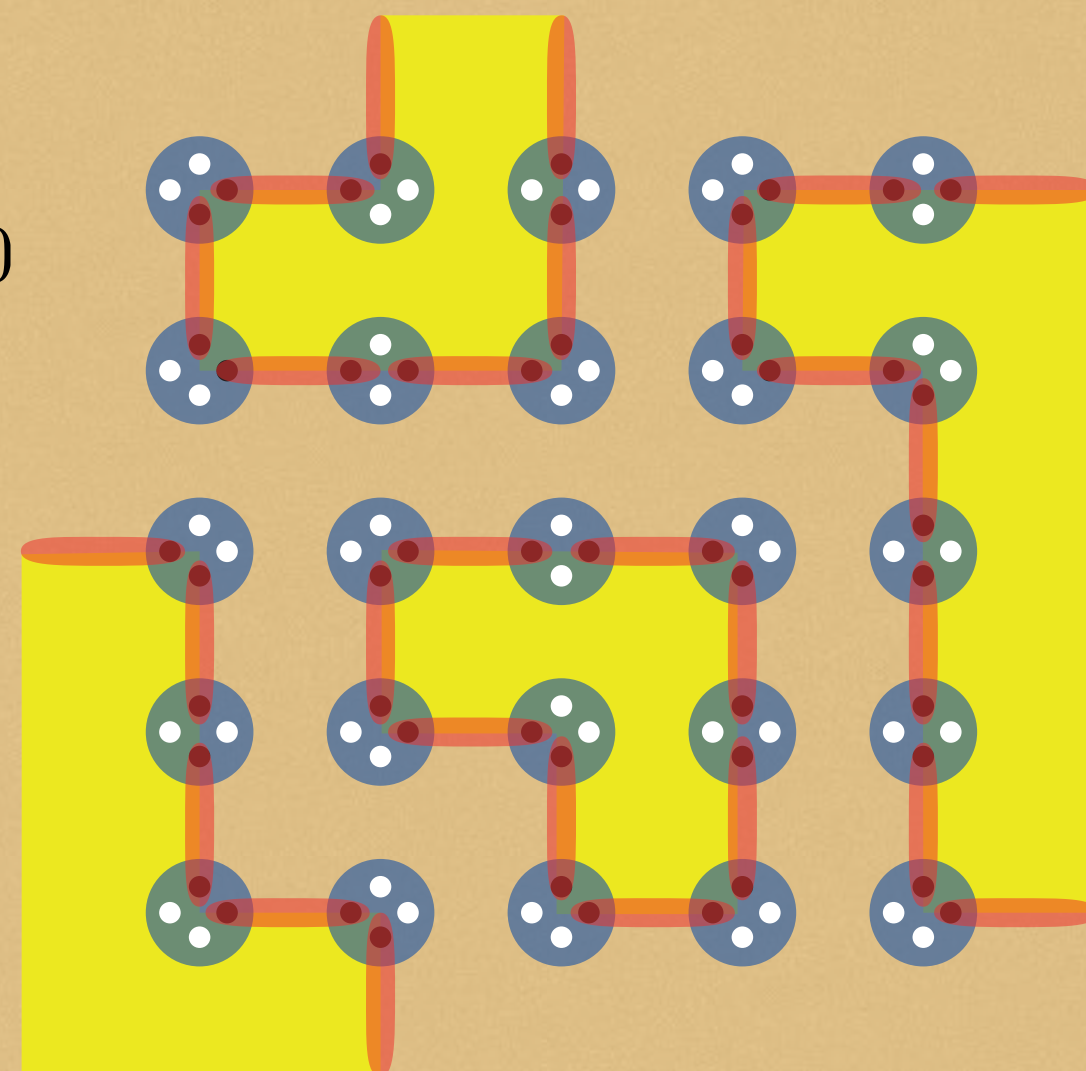
SPINS-S RESONATING AKLT LOOP



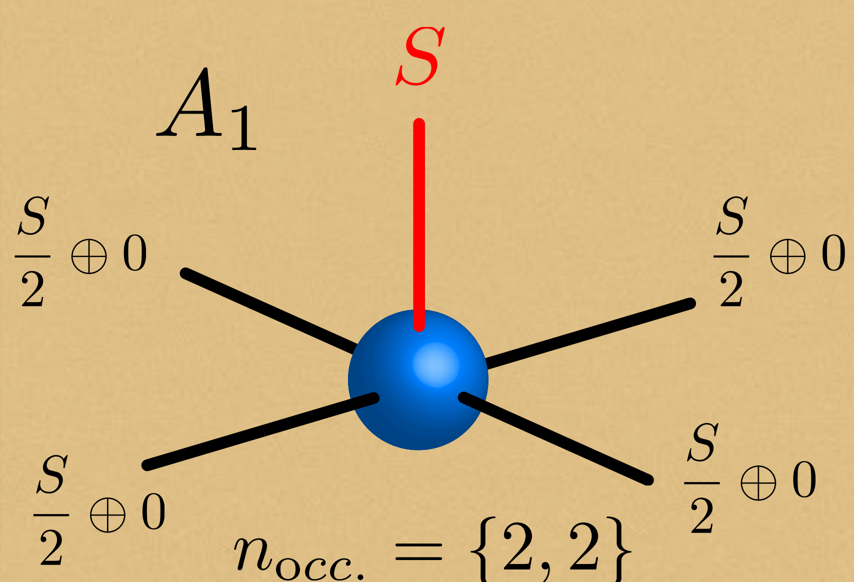
$S = 1$

$$V = \frac{1}{2} \oplus 0$$

$$D = 3$$



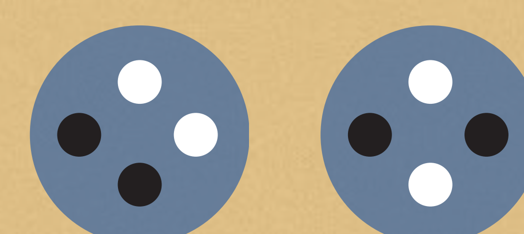
Generalization



S

$$V = \frac{S}{2} \oplus 0$$

$$D = S + 2$$



GENERIC SPIN LIQUIDS

Linearly combine (real coefficients) tensors with :

- same physical and virtual degrees of freedom
- same IRREP of C4v

$$\sum_{i \in \mathcal{C}} \lambda_i A^{(i)}$$

Class with $\mathcal{D}$ tensors

$\mathcal{D}-1$ **dimensional family of completely symmetric spin liquids which (potentially) do not break any symmetry, neither SU(2) or point group symmetries**

V \ S	1/2	1	3/2	2
$\frac{1}{2}$	0/0/0/0/0	0/1/0/0/1	0/0/0/0/0	1/0/0/0/0
$\frac{1}{2} \oplus 0$	2/2/1/1/3	2/2/0/1/2	1/1/0/0/1	1/0/0/0/0
1	0/0/0/0/0	0/1/1/0/2	0/0/0/0/0	2/1/0/1/1
$\frac{1}{2} \oplus 0 \oplus 0$	8/8/4/4/12	6/5/1/3/6	2/2/0/0/2	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2}$	0/0/0/0/0	6/9/4/3/13	0/0/0/0/0	6/3/0/1/3
$1 \oplus 0$	0/0/0/0/0	3/5/3/1/8	0/0/0/0/0	5/3/1/3/4
$\frac{3}{2}$	0/0/0/0/0	0/2/1/0/3	0/0/0/0/0	3/1/1/2/2
$\frac{1}{2} \oplus 0 \oplus 0 \oplus 0$	21/21/12/12/33	12/10/3/6/13	3/3/0/0/3	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0$	10/10/8/8/18	12/13/5/6/18	6/6/2/2/8	6/3/0/1/3
$1 \oplus 0 \oplus 0$	0/0/0/0/0	11/14/9/6/23	0/0/0/0/0	10/7/3/6/10
$1 \oplus \frac{1}{2}$	4/4/3/3/7	5/5/3/4/8	5/5/3/3/8	5/4/2/2/6
$\frac{3}{2} \oplus 0$	1/1/1/1/2	2/3/1/1/4	3/3/2/2/5	3/2/2/2/4
2	0/0/0/0/0	0/2/2/0/4	0/0/0/0/0	4/2/1/3/3
$\frac{1}{2} \oplus 0 \oplus 0 \oplus 0 \oplus 0$	44/44/28/28/72	20/17/6/10/23	4/4/0/0/4	1/0/0/0/0
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0 \oplus 0$	28/28/20/20/48	25/24/11/14/35	12/12/4/4/16	6/3/0/1/3
$\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$	0/0/0/0/0	33/39/24/21/63	0/0/0/0/0	21/15/3/6/18
$1 \oplus 0 \oplus 0 \oplus 0$	0/0/0/0/0	27/31/22/18/53	0/0/0/0/0	17/13/6/10/19
$1 \oplus \frac{1}{2} \oplus 0$	11/11/8/8/19	13/13/8/9/21	11/11/7/7/18	10/8/4/5/12
$1 \oplus 1$	0/0/0/0/0	9/13/13/9/26	0/0/0/0/0	19/15/7/11/22
$\frac{3}{2} \oplus 0 \oplus 0$	2/2/2/2/4	6/6/2/3/8	10/10/6/6/16	4/4/5/4/9
$\frac{3}{2} \oplus \frac{1}{2}$	0/0/0/0/0	7/12/8/5/20	0/0/0/0/0	15/10/7/10/17
$2 \oplus 0$	0/0/0/0/0	1/4/5/2/9	0/0/0/0/0	10/7/3/6/10
$\frac{5}{2}$	0/0/0/0/0	0/3/2/0/5	0/0/0/0/0	5/2/2/4/4

PLAYING WITH SYMMETRIES

Another strategy : explicitly break some symmetries by combining tensors belonging to different classes and/or making complex linear combinations

$$\sum_{i \in \mathcal{C}, \mathcal{C}'} \lambda_i A^{(i)}$$

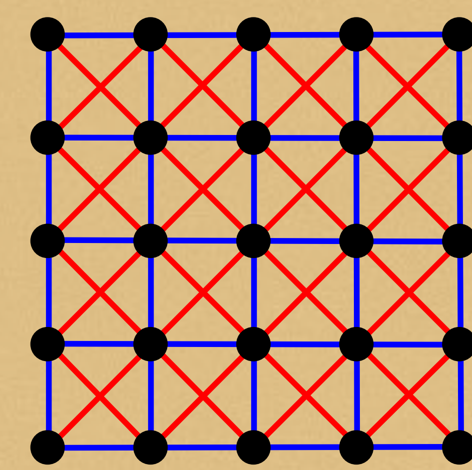
- Real combinations of e.g. A_1/B_1 horizontal/vertical bonds become inequivalent. **Lattice nematics**

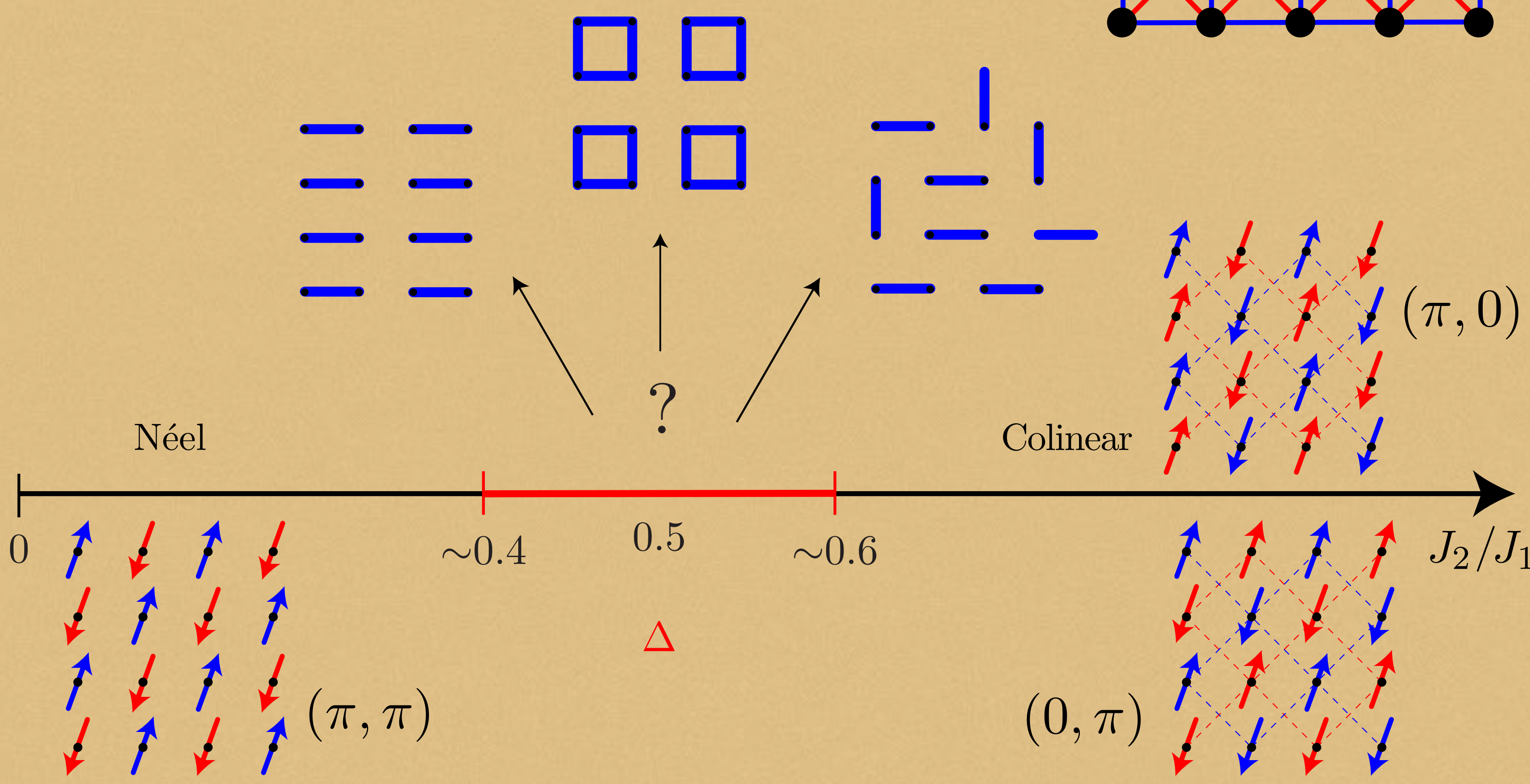
- Partially break SU(2) down to U(1) by combining tensors with different S_z but with $\lambda_i = f(|S_z|)$. **Spin nematics**
- Using complex tensors (E) or making complex linear combination of real tensors. **Time reversal symmetry breaking**

$J_1 - J_2$ MODEL ON THE SQUARE LATTICE

$J_1 - J_2$ MODEL ON THE SQUARE LATTICE

$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} \vec{S}_i.\vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i.\vec{S}_j$$





SU(2)-SYMMETRIC PEPS

Variational optimization

$$\sum_{i \in \mathcal{C}} \lambda_i A^{(i)}$$

Key feature Uniform A_1 ansatz leads to a small number of variational parameters to optimize

V	D	D^4	SU(2) **tensors**
$\frac{1}{2} \oplus 0$	3	81	2
$\frac{1}{2} \oplus 0 \oplus 0$	4	256	8
$1 \oplus \frac{1}{2}$	5	625	4
$\frac{1}{2} \oplus \frac{1}{2} \oplus 0$	5	625	10
$\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0$	7	2401	30

OPTIMIZATION

Conjugate gradient minimization

- Small number of parameters allow for a brute force appraoch
- Requires efficient computation of the variational energy

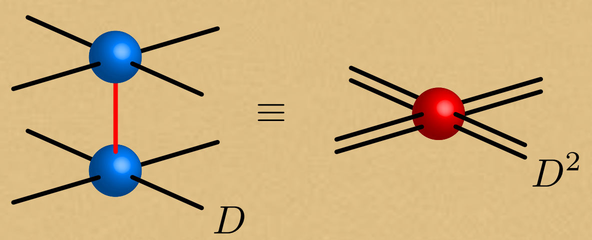
iPEPS Corner Transfer Matrix method and RG algorithm

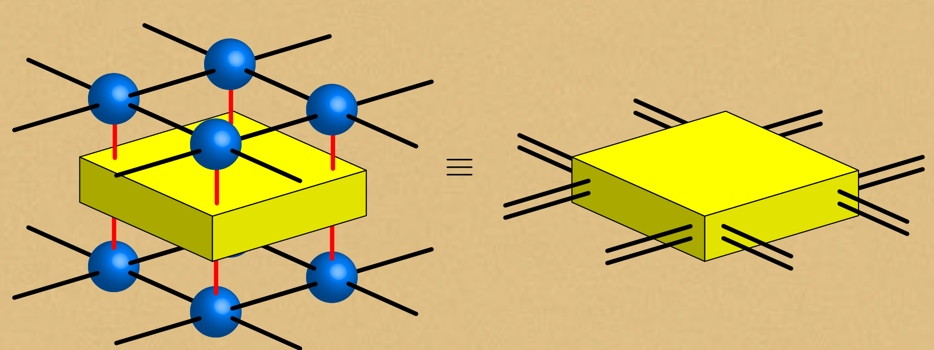
J. Jordan, R. Orús, G. Vidal, F. Verstraete, J.I. Cirac, PRL **101**, 250602 (2006)

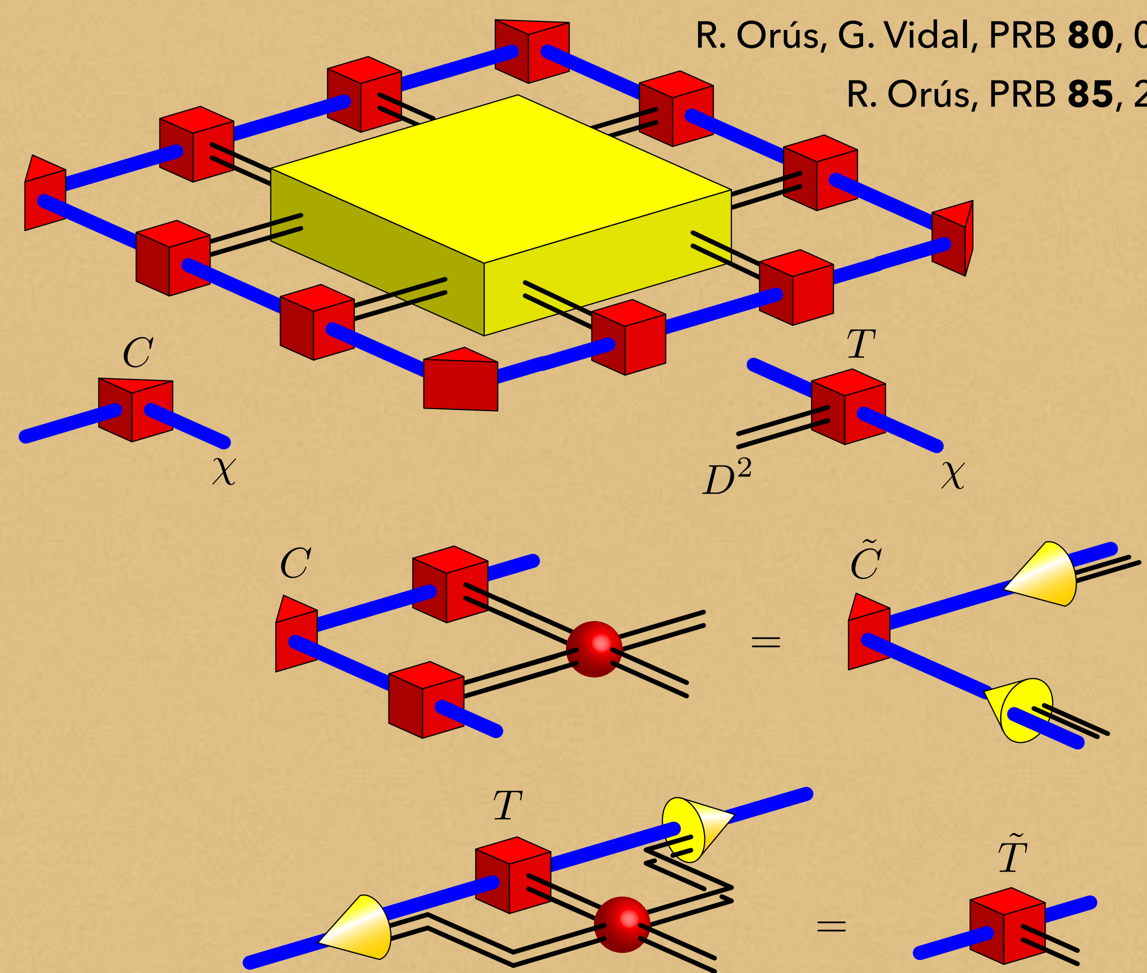
T. Nishino, K. Okunishi, JPSP **65**, 891-894 (1996)

R. Orús, G. Vidal, PRB **80**, 094403 (2009)

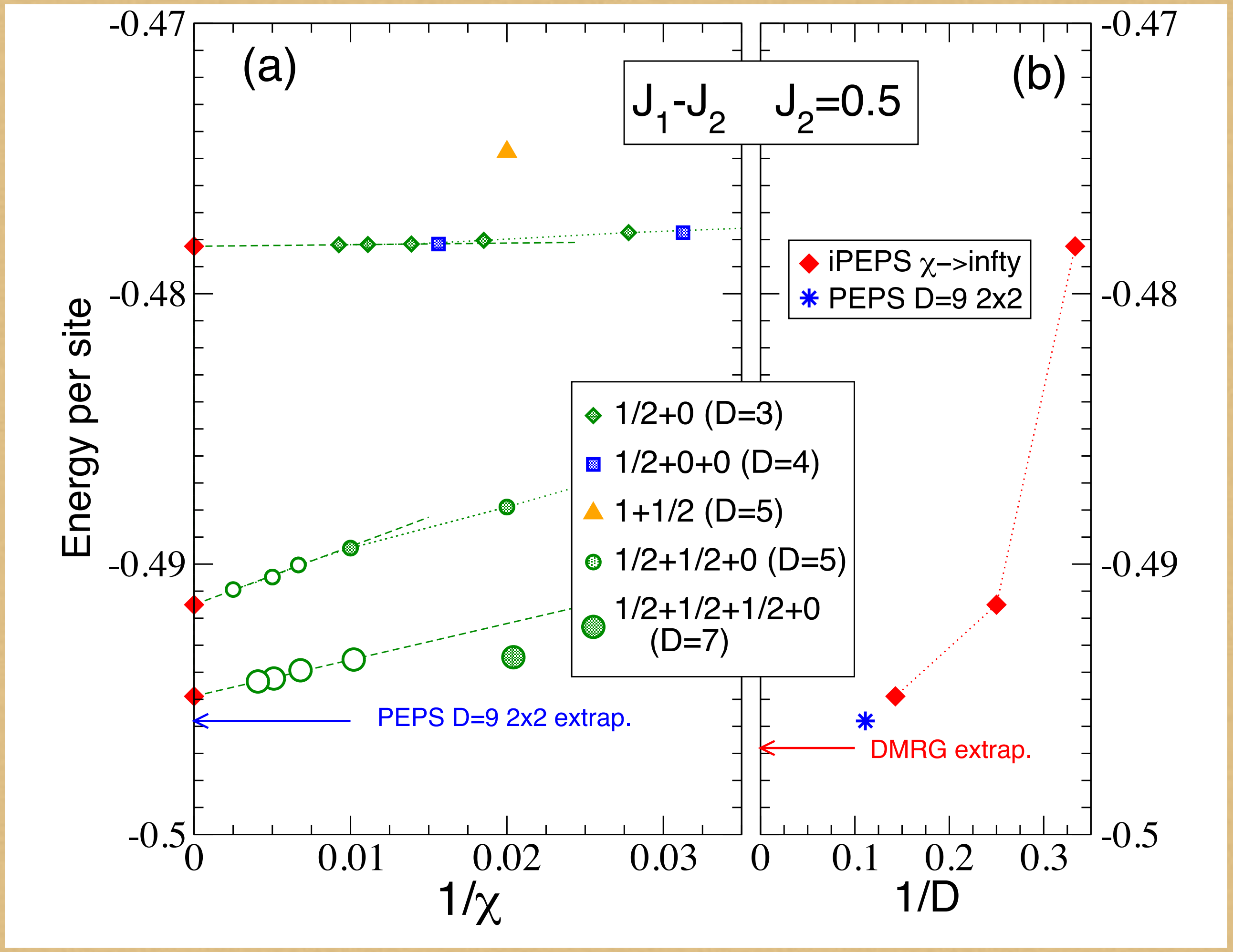
R. Orús, PRB **85**, 205117 (2012)







ENERGY

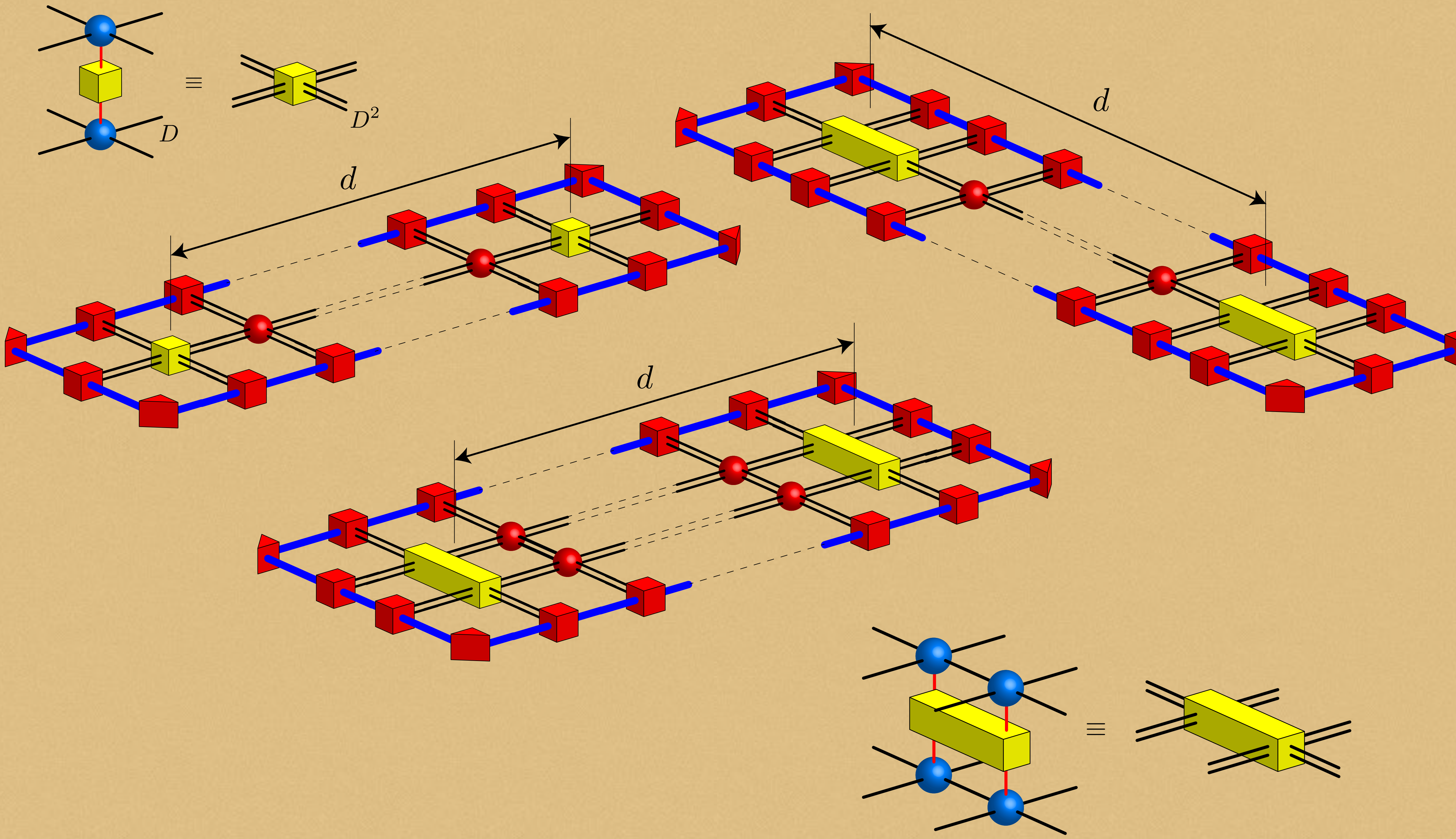


Remarkable consistency

Unique tensor depending on a small number of parameters

J	0.5	
DMRG	-0.4968	S-S. Gong *et al.*, PRL **113**, 027201 (2014)
VMC	$-0.4970(5)$	W-J. Hu *et al.*, PRB **88**, 060402 (2013)
$D = 9$ PEPS	$-0.4958(3)$	L. Wang *et al.*, PRB **94**, 075143 (2016)
$D = 7$ iPEPS	-0.4949	This work

CORRELATIONS

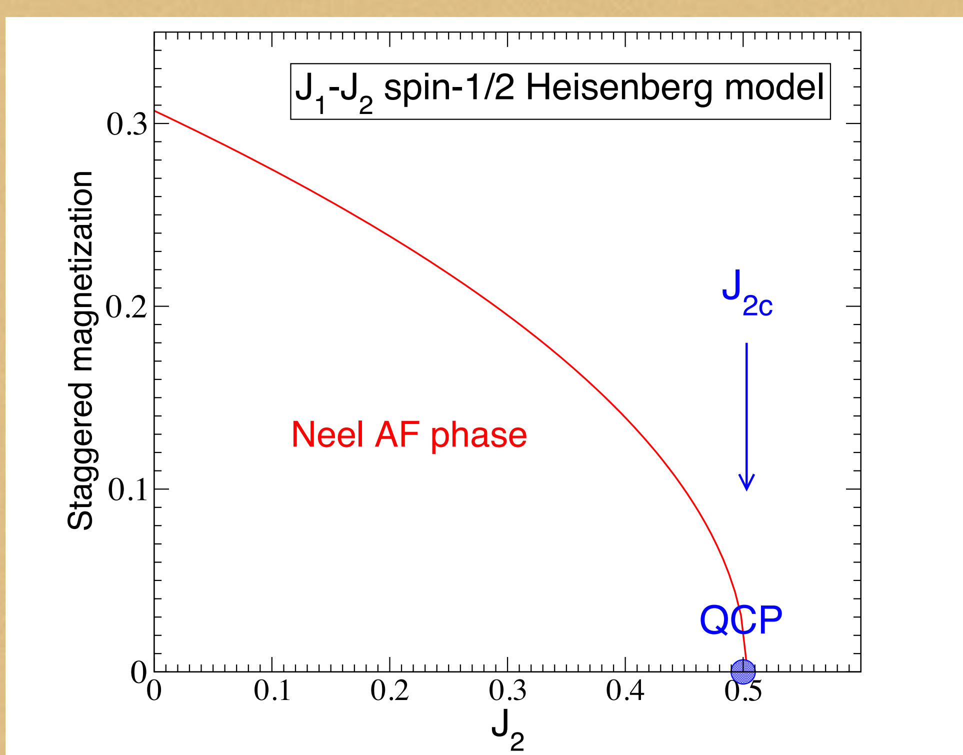


CORRELATIONS

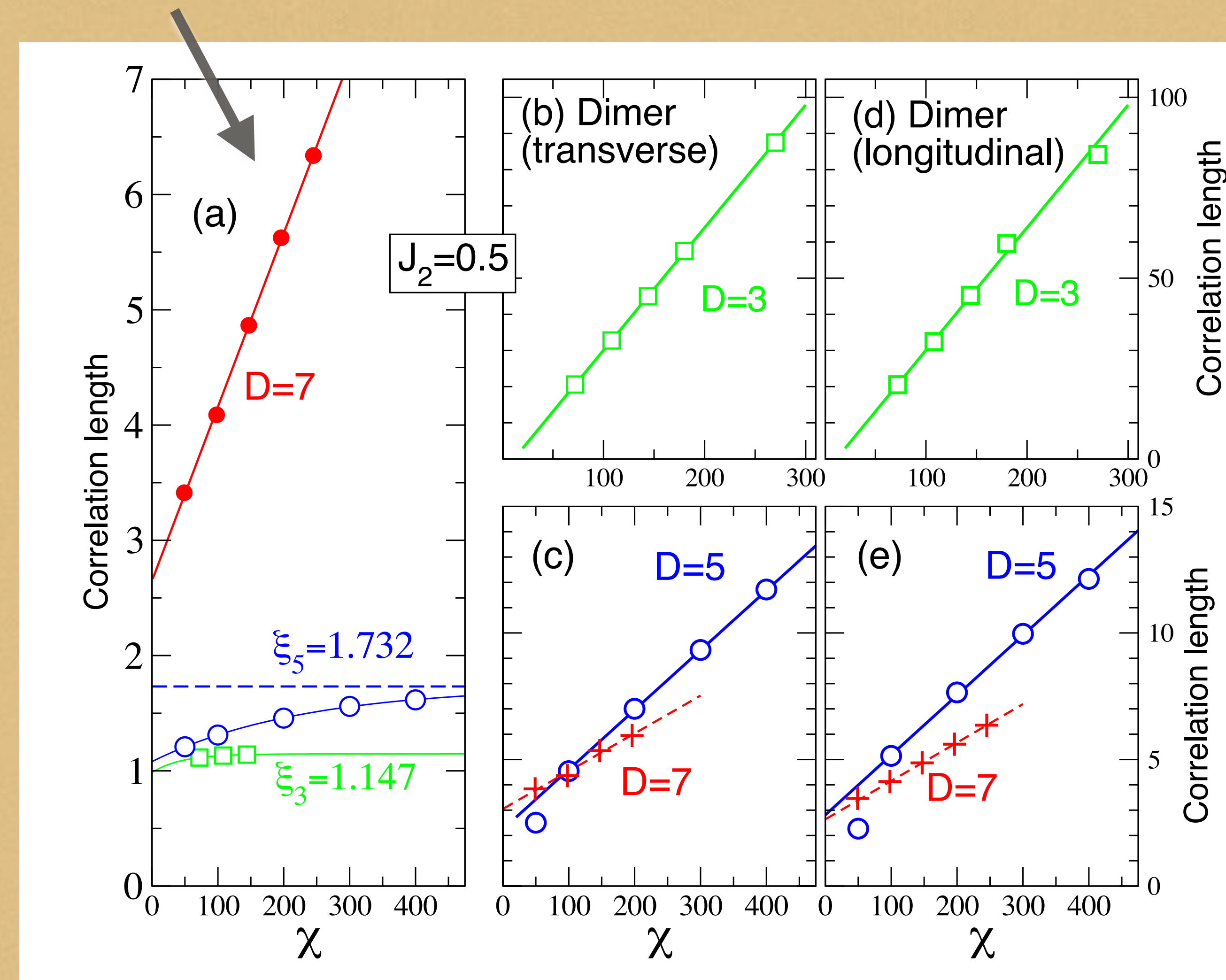
Linear divergence of correlation lengths with environment dimension

Algebraic (critical) behaviors for spin-spin and dimer-dimer correlations

Staggered magnetization vanishes for $\chi \to \infty$



Qualitative change for D=7



$J_2 = 0.5$ **very close to** J_{2c}

This PEPS give a good description of the QCP

CONCLUSIONS

- Simple classification scheme of all rank-5 SU(2)-spin rotational symmetric tensors (S,V) and C4v IRREP
- Remarkable PEPS (AKLT, RVB, Loops) are naturally recovered
- Playing with local bricks with *controlled symmetry properties* to design interesting quantum states

$$\sum_{i \in \mathcal{C}} \lambda_i A^{(i)}$$

natural route to spin liquids manifolds

$$\sum_{i \in \mathcal{C}, \mathcal{C}'} \lambda_i A^{(i)}$$

controlled symmetry breaking (lattice/spin nematics, chiral spin liquids…)

$$\sum_{i \in \mathcal{C}} \lambda_i A^{(i)}$$

Optimized ansätze for frustrated Heisenberg model

- Description of non trivial states (QCP) with PEPS
- Direct extension to other point groups / lattices (triangular, kagome…)