Creating a bosonic fractional quantum Hall state by pairing fermions

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with C. Repellin (MPIPKS) and T. Yefsah (ENS)

Topological phases and cold atomic gases

Topological phases have been at the center of research in condensed matter for almost forty years

- Fundamental interests:
  - Beyond Laundau theory of phase transition: no local order parameter, no symmetry breaking
  - Possible realization of anyonic statistics
- Applications:
  - quantum memories
  - topologically protected quantum computing
- Experimental progress:
  - Solid states physics: topological insulator, Weyl semi-metals, Majorana fermions (?), Chern insulator
  - Cold atomic systems: realization of topological band structure

Interest of cold atomic systems:

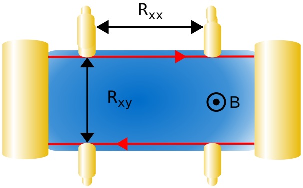
- More controlled, clean and tunable systems
- Access to more observables, possibility of local manipulations

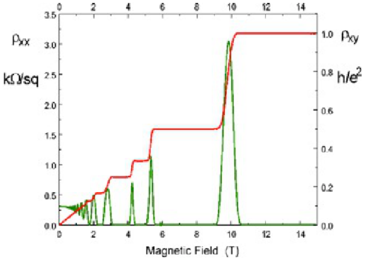
Outline

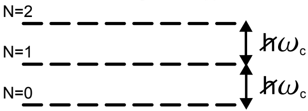
- Introduction to the Fractional Quantum Hall Effect
- FQHE without magnetic field: Fractional Chern Insulators
- Creating a bosonic fractional quantum Hall liquid by pairing fermions

Introduction to the Fractional Quantum Hall Effect

First topological phase: Integer Quantum Hall Effect



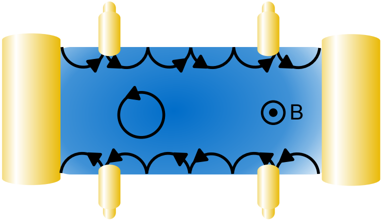




On each plateau:

$$\sigma_{xy} = C\frac{e^2}{h}\ ,\ \sigma_{xx} = 0$$

- Cyclotron frequency: $\omega_c = \frac{eB}{m}$
- Filling: $\nu = \frac{hC}{eB} = \frac{N}{N_\phi}$
- For $\nu = C$, C filled level and a gap $\hbar\omega_c$: insulator
- Transverse conductance: existence of chiral edge modes

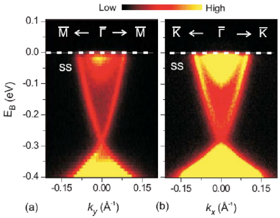


Topological phases

81 - 83: Integer and Fractional Quantum Hall Effects
06 - 08: Topological Insulators

- Different phases with the same symmetries
- some physical quantities are related to a topological invariant ($\sim$ surface genus)
- example: transverse conductance in IQHE (Chern Number)
- Insensitive to local perturbations
- Gapped systems in the bulk
- Topological characteristic at the edge (edge modes)

3D TI:



Two flavors of topological phases

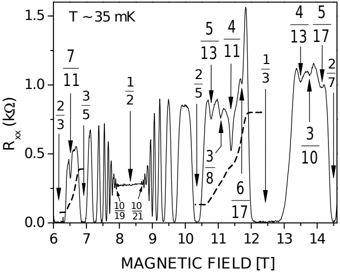
No topological order

- Unique groundstate on every surface
- No anyons in the bulk, but fractionalized edge excitations
- Examples: Free fermion TI, AKLT

Topological order

- Interactions needed
- Groundstate degeneracy depends on the surface genus
- Anyonic excitations in the bulk
- Examples: FQHE, toric code

The fractional quantum Hall effect



- Partial filling: extensive groundstate degeneracy
- Only explanation: Interactions

Model wave function at filling $\nu = \frac{1}{m}$

$$\Psi_L(z_1, ...z_N) \quad = \quad \prod_{i<j}(z_i - z_j)^m e^{-\sum_i \frac{|z_i|^2}{4l^2}}$$

- Very good approximation of the ground state at $\nu = \frac{1}{3}$
- **Topological state**: the degeneracy of the model state depends on the surface genus (sphere: deg = 1 , torus: deg = m)

Quantum Hall effects: Fermions vs Bosons

While only observed for fermions (so far!), QH physics can also appear for bosons

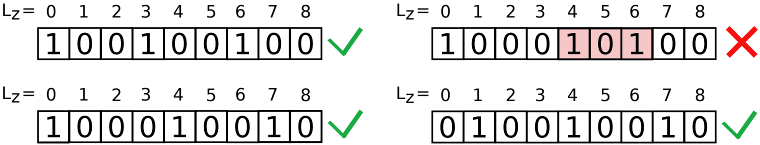
- IQHE can appear for bosons when ν is even and with interaction

- in the lowest Landau level: $\Psi_B = \Psi_F / \prod_{i<j}(z_i - z_j)$
- filling $\nu_B^{-1} = \nu_F^{-1} - 1$
- example: $\nu = 1/3$ Laughlin state $\rightarrow$ $\nu = 1/2$ Laughlin state
- shorter range interaction to realize the bosonic state
- example: $H_{m=3} = \sum_{i,j} \delta(z_i - z_j) \nabla \delta(z_i - z_j)$ vs $H_{m=2} = \sum_{i,j} \delta(z_i - z_j)$

Main candidate experimental systems: Optical lattices, rotating trap, Chern insulator

Quasihole state counting: generalized Pauli principle

- Quasiholes: excitations with fractional charge and anyonic statistic
- The number of groundstates and quasihole states (i.e. zero enery states of the parent Hamiltonians) can be predicted by a generalization of the Pauli principle
- Laughlin $\nu = 1/m$: *no more than* 1 *particle in m consecutive orbitals* (including periodic boundary conditions on the torus)
- Example: Laughlin $\nu = 1/3$ state with 8 flux quanta



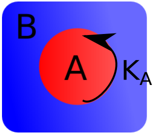
These numbers are a fingerprint of the phase (related to the statistics of the excitations).

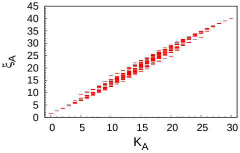
How can we probe the emergence of a topological phase numerically?

No local order parameter, a tool is thus needed!

- system in state $|\Psi\rangle$, cut the system in two parts A et B
- Reduced density matrix $\rho_A = \mathrm{Tr}_B |\Psi\rangle \langle\Psi| = \exp(-H_\xi)$
- Entanglement spectrum = spectrum of H_ξ

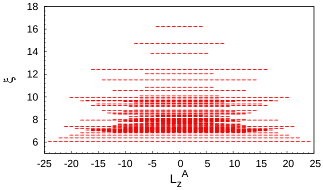
Real space





chiral mode: linear dispersion
$\xi \propto K_A$ and state counting
edge physics

Particle space

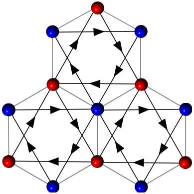


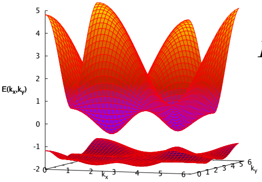
Quasihole state counting $\longrightarrow$
fingerprint of excitation statistics
bulk physics

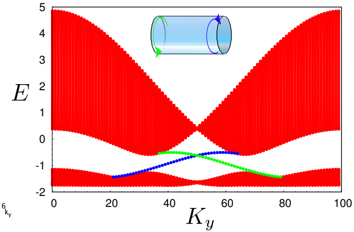
FQHE without magnetic field: fractional Chern insulators

Chern insulators

- Quantum Hall Effect without magnetic field: Chern insulator (Haldane, PRL 88)







- Topological properties emerge from the band structure:
  - Berry connection and potential:

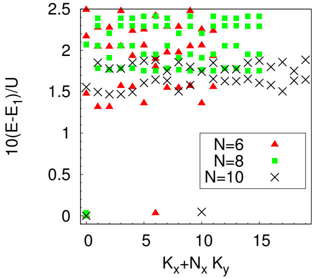
$$\mathbf{A}(\mathbf{k}) = i \langle u_{\mathbf{k}} | \nabla_{\mathbf{k}} | u_{\mathbf{k}} \rangle \; ; \; F(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}(\mathbf{k})$$

  - Chern number: $C_{1B} = \frac{1}{2\pi} \int_{BZ} F(\mathbf{k})$
- Gapped system with chiral edge modes and a band with non-zero Chern number

Fractional Chern insulators: Laughlin state at $\nu = 1/2$ when $C_{1B} = 1$

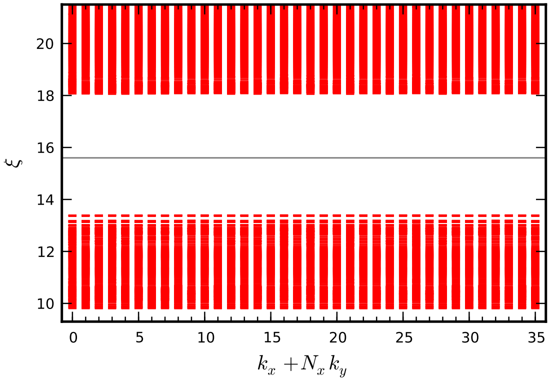
- finite system with $N_x N_y$ unit cells and N_B bosons
- partial filling of the lowest band: $\nu = \frac{N_B}{N_x N_y} = \frac{1}{2}$

2-body on-site Hubbard interaction (analogue of the delta interaction in real space): $H_{\text{int}} = U \sum_i : n_i^2 :$



- correct groundstate degeneracy
- groundstates appear in the correct momentum sector
- Not enough to prove Laughlin physics
- can correspond to a breaking of translation symmetry

FCI: Laughlin state at $\nu = 1/2$ when $C_{1B} = 1$

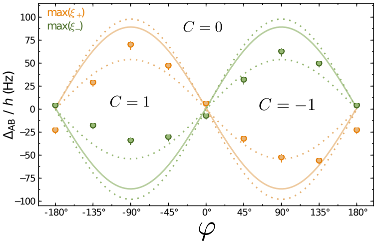


- ES exhibits an entanglement gap
- Part below the gap has the same fingerprint as the Laughlin state
- Strongly model and interaction dependent

Experimental realization of Chern insulators

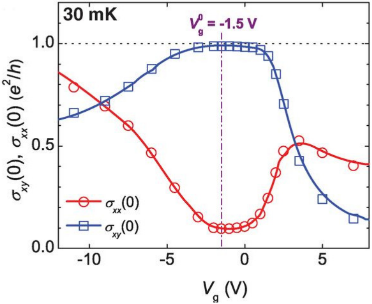
Many experimental developments in the last years

Fermionic cold atomic gases using periodic driving



Jotzu et al. 2014

Solid state:



Chang et al. 2013, Bestwick et al. 2014

Creating a bosonic fractional quantum Hall liquid by pairing fermions

Pairing

What prevents the realization of fermionic FQH state in cold atomic systems?

- fermionic FQH states require longer-range interaction (difficult to engineer)
- solution: use atoms with strong dipolar interaction (experimentally less mastered)

idea: use pairing between spinful fermions to obtain bosonic molecules that would form bosonic FQH state

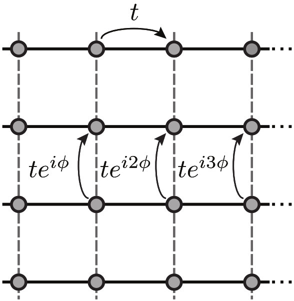
- Preparing spinful fermionic Mott insulator is now common practice (very recent measurements of anti-ferromagnetic correlations)
- Tuning the interaction to be attractive and strong can be done using Feschbach resonance
- Bose-condensation of such pairs was observed (Zwierlein *et al.*, PRL 2004)

Hofstadter model

Square lattice with a magnetic field:

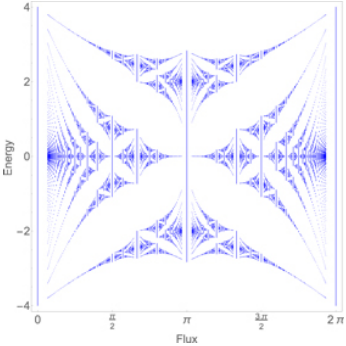
$$\mathcal{H} = -t \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left(e^{2\pi i \alpha e_{\mathbf{r}\mathbf{r}'}} c_{\mathbf{r}}^{\dagger} c_{\mathbf{r}'} + h.c. \right)$$

- $e_{\mathbf{r}\mathbf{r}'}$ gauge dependent and field independent
- $\phi = 2\pi\alpha$, α flux density



Hofstadter model

One-body spectrum given by the fractal Hofstadter butterfly



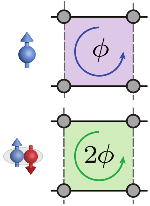
- At small flux density α one recovers the Landau levels
- This model was realized in two experiments recently (Aidelsburger *et al.*, Miyake *et al.*; PRLs 2013)

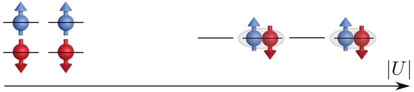
Pairing spinful fermions on Hofstadter model

Spinful Fermi-Hubbard model on a square lattice with a magnetic field and attractive interaction ($U < 0$):

$$\mathcal{H} = -t \sum_{\langle \mathbf{r},\mathbf{r}' \rangle, \sigma=\uparrow,\downarrow} \left(e^{2\pi i \alpha e_{\mathbf{r}\mathbf{r}'}} c^{\dagger}_{\mathbf{r},\sigma} c_{\mathbf{r}',\sigma} + h.c. \right) + U \sum_{\mathbf{r}} n_{\mathbf{r},\uparrow} n_{\mathbf{r},\downarrow}$$

- At large $|U|$ bosonic molecules form
- Half as many but feel twice the flux: $\nu_{BM} = \nu_F/4$
- $\nu_F = 2$ IQH state $\rightarrow \nu_{BM} = 1/2$ Laughlin state



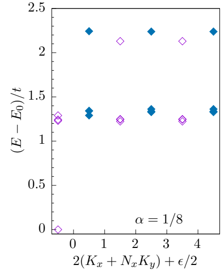


Phase transition between a fermionic topological phase and a bosonic intrinsic topological phase (studied in the continuum using effective Chern-Simmons theory by K.Yang *et al.* PRL 2008)

Phase transition: energy results

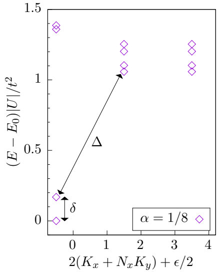
- Quantum numbers of the system: K_x, K_y and S_z
- in $S_z = 0$ sector, spin inversion symmetry: $\mathcal{P}_{\mathcal{SI}} |\psi\rangle = \epsilon |\psi\rangle$ and $\epsilon = \pm 1$

$U/t = -1, N_F = 6, S_z = 0$



- Unique groundstate
- low-energy states: even or odd under spin inversion

$U/t = -30, N_F = 6, S_z = 0$



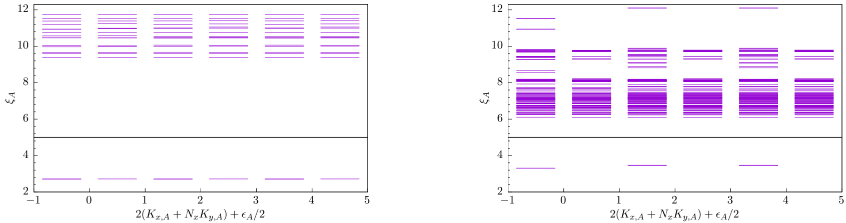
- quasi 2-fold degenerate groundstates for $\alpha \leq \frac{1}{8}$
- only states with $\epsilon = (-1)^{\frac{N_F}{2}}$

Phase transition: entanglement spectrum results

PES very different in $\nu_F = 2$ IQH state and $\nu_{BM} = 1/2$ Laughlin state

ex: $N_F = 6, \alpha = 1/8, N_A = 2, S_{z,A} = 0$

$U/t = -1$ | $U/t = -30$



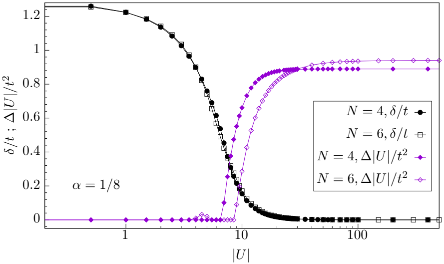
9 states below gap
= choose 1 up among 3 * choose 1 down among 3

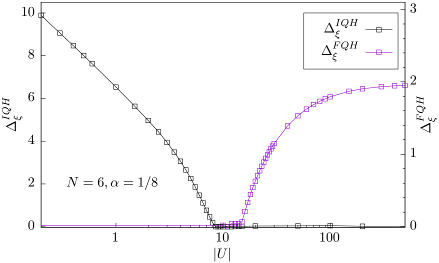
6 states below the gap
= number of flux quanta for the bosonic molecules

states odd under spin inversion

PES at $N_A = 4$ shows that Laughlin state is realized at large $|U|$

Monitoring the phase transition using both energy gap and the entanglement gap



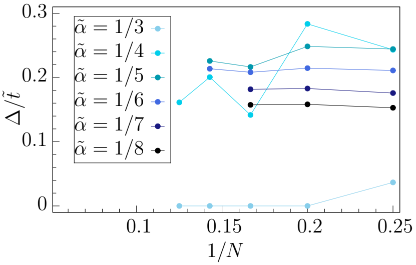


- Transition occurs around $U_c \approx -10t$ (for these system sizes)
- Typically the range of the 1-body spectrum
- Cannot access bigger system ($N = 8$ dim $\sim 10^9$)
- Unfortunately we cannot tell more about the nature of the phase transition

Large $|U|$ limit: Mapping to hardcore bosons

To access bigger system sizes we can map the bosonic molecules to hardcore bosons

$$\mathcal{H}_{\text{bos}} = -\tilde{t} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left(e^{2\pi i \tilde{\alpha} e_{\mathbf{r}\mathbf{r}'}} a_{\mathbf{r}}^{\dagger} a_{\mathbf{r}'} + h.c. \right) \; ; \tilde{\mathbf{t}} = \frac{\mathbf{t}^2}{|\mathbf{U}|}, \; \tilde{\alpha} = \mathbf{2}\alpha$$



- Matches the fermionic computation
- PES also displays Laughlin state counting
- large fluctuations for $\tilde{\alpha} = 1/4$
- $\tilde{\alpha} < 1/4$: gap is expected to be finite in the thermo. limit

Conclusion

While chiral topologically ordered phases have not been yet realized outside of 2DEG, recently a vast research effort to realize them on lattice systems and in cold atom gases

- Attractive interaction can also lead to FQHE effect!
- Pairing leads to a QPT between two topological phases of different nature
- First microscopic model where this effect is observed
- Experimental ingredients have been implemented

Open questions:

- Does this also occur in the continuum? Universality class?
- Is it possible to reach states beyond usual FQHE?

Thank you for your attention !