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FACULTY OF SCIENCES

DEPARTMENT PHYSICS AND ASTRONOMY
RESEARCH GROUP MATHEMATICAL AND THEORETICAL PHYSICS

REAL-TIME SIMULATION OF THE SCHWINGER EFFECT

Benasque

Boye Buyens

16.02.2017

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TNS FOR 1+1 DIMENSIONAL GAUGE FIELD THEORIES

Matrix product states for gauge field theories

BB, J. Haegeman, K. Van Acoleyen, H. Verschelde, F. Verstraete

Arxiv: 1312.6654 [hep-lat]

Real-time simulation of the Schwinger effect with Matrix Product States,

BB, J. Haegeman, F. Hebenstreit, F. Verstraete and K. Van Acoleyen

Arxiv:1612.00739 [hep-lat]

Main message: MPS provide a tool **to study** non-equilibrium physics

- Simulation of real-time evolution
- Explaining physics behind the results

200 YEARS
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MOTIVATION

STANDARD MODEL

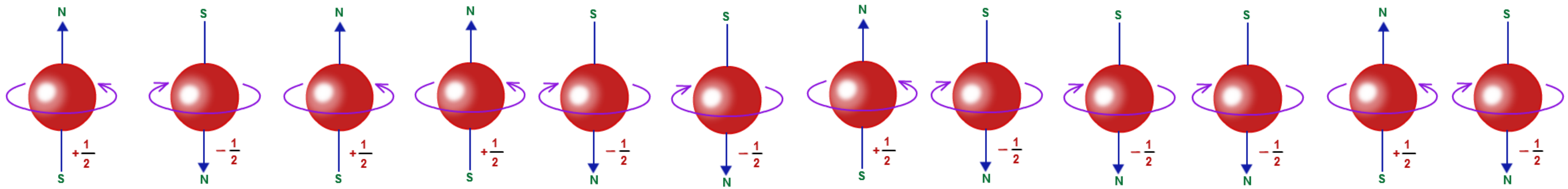
Glashow-Weinberg-Salam model: unifies QED and weak interaction

Perturbation theory (Feynman diagrams)

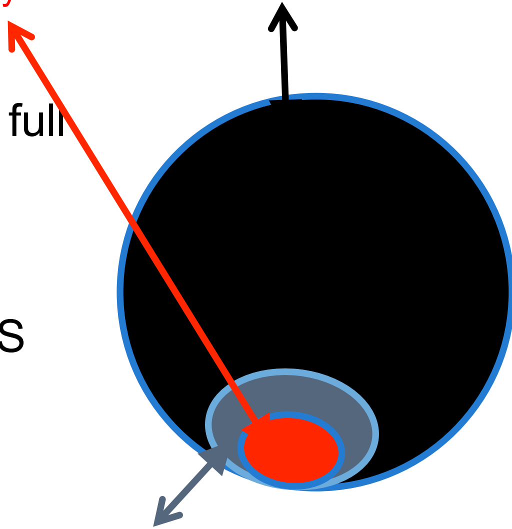
Almost… QCD (strong interactions) is challenging

- Perturbation theory fails at low energy
- Lattice QCD (Bali ’99)
  - By now most successfull method
  - Limited to small baryon densities (~~neutron stars~~)
  - Not formulated for real-time (~~heavy ion collisions~~)
- Hamiltonian framework
  - Overcomes obstacles of lattice QCD
  - Many-body problem

TINY CORNER OF HILBERT SPACE



$$|\uparrow,\downarrow,\uparrow,\uparrow,\downarrow,\downarrow,\uparrow,\downarrow,\downarrow,\downarrow,\uparrow,\downarrow\rangle$$



Low-energy states of local Hamiltonians live in a `tiny corner' of the full Hilbert space (Orus '04, Poulin '11)

In 1D:

- Low-energy states can be approximated efficiently by MPS

  (Hastings '07)

- MPS enable simulations on an ordinary desktop

  (Fannes '92)

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MPS FOR SCHWINGER MODEL

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(MASSIVE) SCHWINGER MODEL

QED in (1+1)-dimensions

$$\mathcal{L} = \bar{\psi}\left(\gamma^\mu(i\partial_\mu + gA_\mu) - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Local U(1)-symmetry

$$\psi(x) \longrightarrow e^{-ig\varphi(x)}\psi(x),$$

$$A_\mu(x) \longrightarrow A_\mu(x) - \partial_\mu\varphi(x)$$

$m/g = 0$: photon acquires mass (Schwinger '62)

$m/g \neq 0$: not exactly solvable, perturbative results for

- $m/g \ll 1$: (strong-coupling regime) (Coleman '75, Adams '97)
- $m/g \gg 1$: (weak-coupling regime) (Coleman '75)
- $m/g \sim \mathcal{O}(1)$: non-perturbative methods (Byrnes '03, Bañuls et al. '13-'16,…)

J. Schwinger, Gauge invariance and mass II, Phys. Rev. 128, 2425 (1962)

LATTICE FORMULATION

Kogut-Susskind discretization (Kogut '75, Banks '76)

$$H = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}\frac{1}{g^2}E(n)^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n[\sigma_z(n)+(-1)] + x\sum_{n=1}^{2N-1}\left(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.\right)\right)$$

$$N \to +\infty$$

$$x = 1/g^2a^2 \to +\infty \qquad E(n) = g[L(n)+\alpha]$$

Fermions: spin-system staggered on the lattice (Jordan-Wigner)

$$\sigma_z\,|s\rangle = s\,|s\rangle\,, s=\pm 1 \qquad \sigma^{\pm}\,|\pm 1\rangle = 0 \qquad \sigma^{\pm}\,|\mp 1\rangle = |\pm 1\rangle$$

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Gauge field: live on links between sites

$$L(n)\,|p\rangle = p\,|p\rangle\,, p\in\mathbb{Z} \qquad e^{\pm i\theta}\,|p\rangle = |p\pm 1\rangle$$

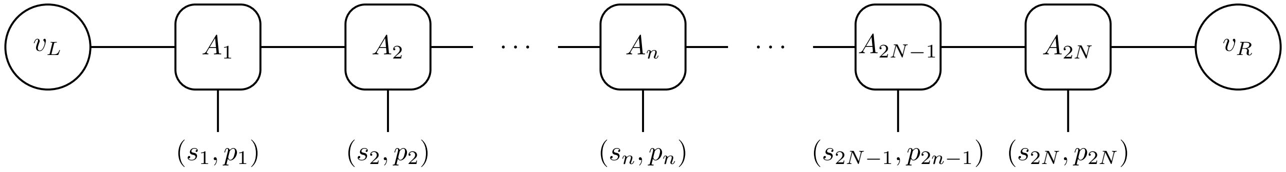
GAUGE INVARIANT MPS-ANSATZ BB '13

Gauss' law

$$G(n) = L(n) - L(n-1) - \frac{\sigma_z(n) + (-1)^n}{2} = 0 \qquad (\partial_z E = g\bar{\psi}\gamma^0\psi)$$

General MPS

$$|\Psi[A]\rangle = \sum_{\{s,p\}} v_L^\dagger A_1^{s_1,p_1} A_2^{s_2,p_2} \dots A_n^{s_n,p_n} \dots A_{2N-1}^{s_{2N-1},p_{2N-1}} A_{2N}^{s_{2N},p_{2N}} v_R \, |s_1, p_1, \dots, s_{2N}, p_{2N}\rangle$$

$=$ 

$$s = \pm 1, p \in \mathbb{Z}$$

Gauge invariant MPS

$$G(n)\,|\Psi[A]\rangle = 0 \Leftrightarrow [A_n^{s,p}]_{(q,\alpha_q);(r,\beta_r)} = \delta_{r,q+(s+(-1)^n)/2}\,\delta_{r,p}\,[a_n^{q,s}]_{\alpha_q,\beta_r}$$

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GAUGE INVARIANT MPS-ANSATZ

$$G(n) = L(n) - L(n-1) - \frac{\sigma_z(n) + (-1)^n}{2} = 0$$

$$G(n)\,|\Psi[A]\rangle = 0 \Leftrightarrow [A_n^{s,p}]_{(q,\alpha_q);(r,\beta_r)} = \delta_{r,q+(s+(-1)^n)/2}\delta_{r,p}[a_n^{q,s}]_{\alpha_q,\beta_r}$$

$$A_1^{1,p} = \begin{array}{c} q\backslash r \\ \vdots \\ p \\ p+1 \\ p+2 \\ \vdots \end{array} \begin{array}{c|c|c|c|c} \cdots & p-1 & p & p+1 & \cdots \\ \ddots & \vdots & \vdots & \vdots & \iddots \\ \hline \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\ \hline \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & a_1^{q,1} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\ \hline \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\ \hline \iddots & \vdots & \vdots & \vdots & \ddots \end{array}$$

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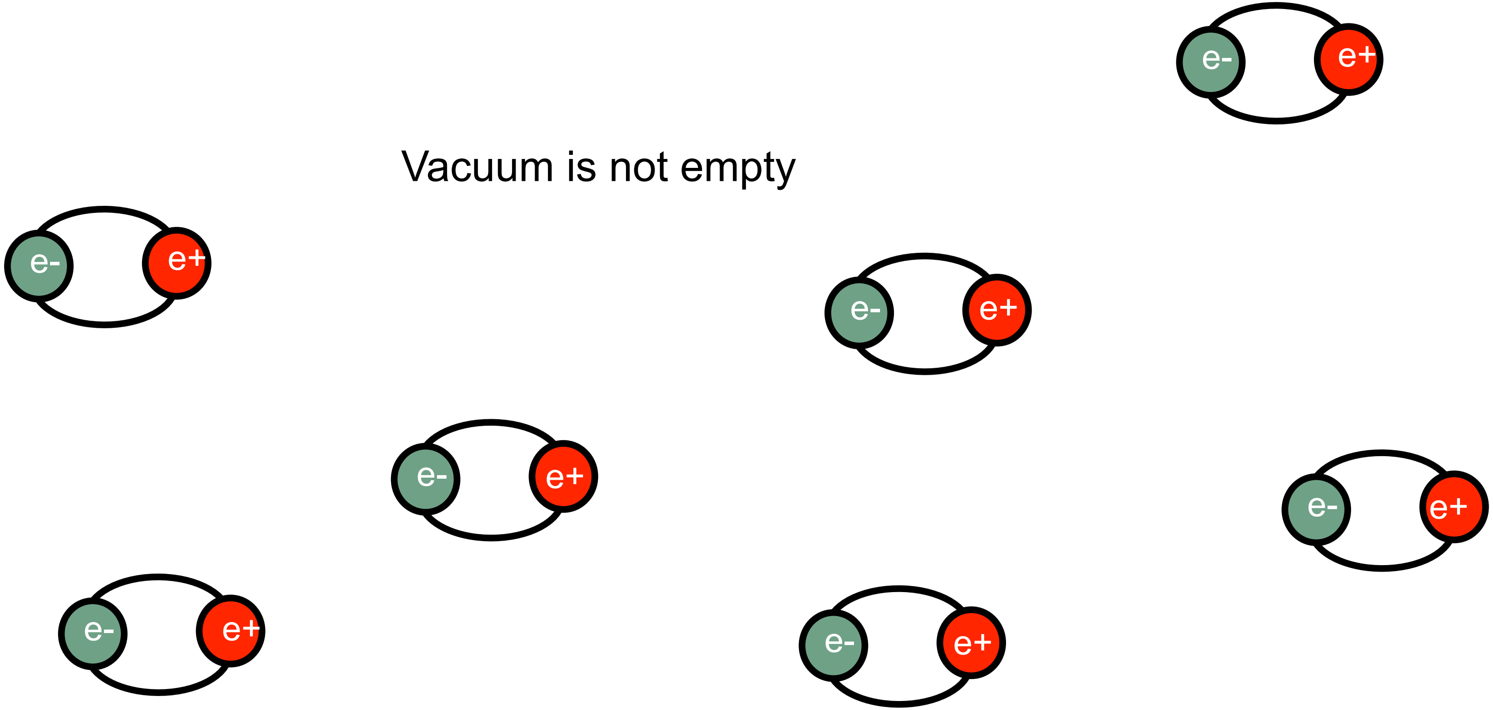
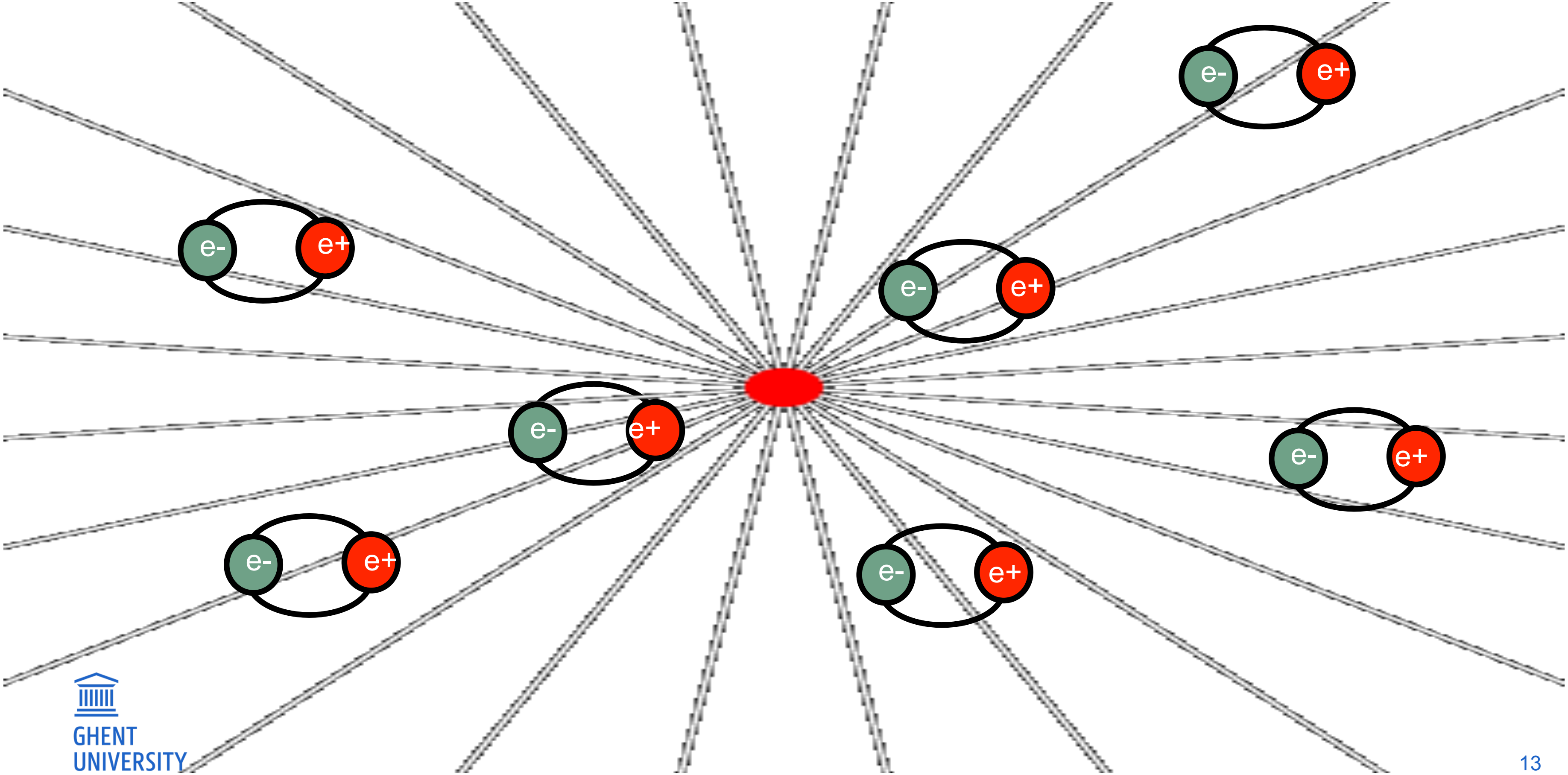
SCHWINGER MECHANISM
SCHWINGER ‘56

Vacuum is not empty



CLASSICAL ELECTRIC BACKGROUND FIELD



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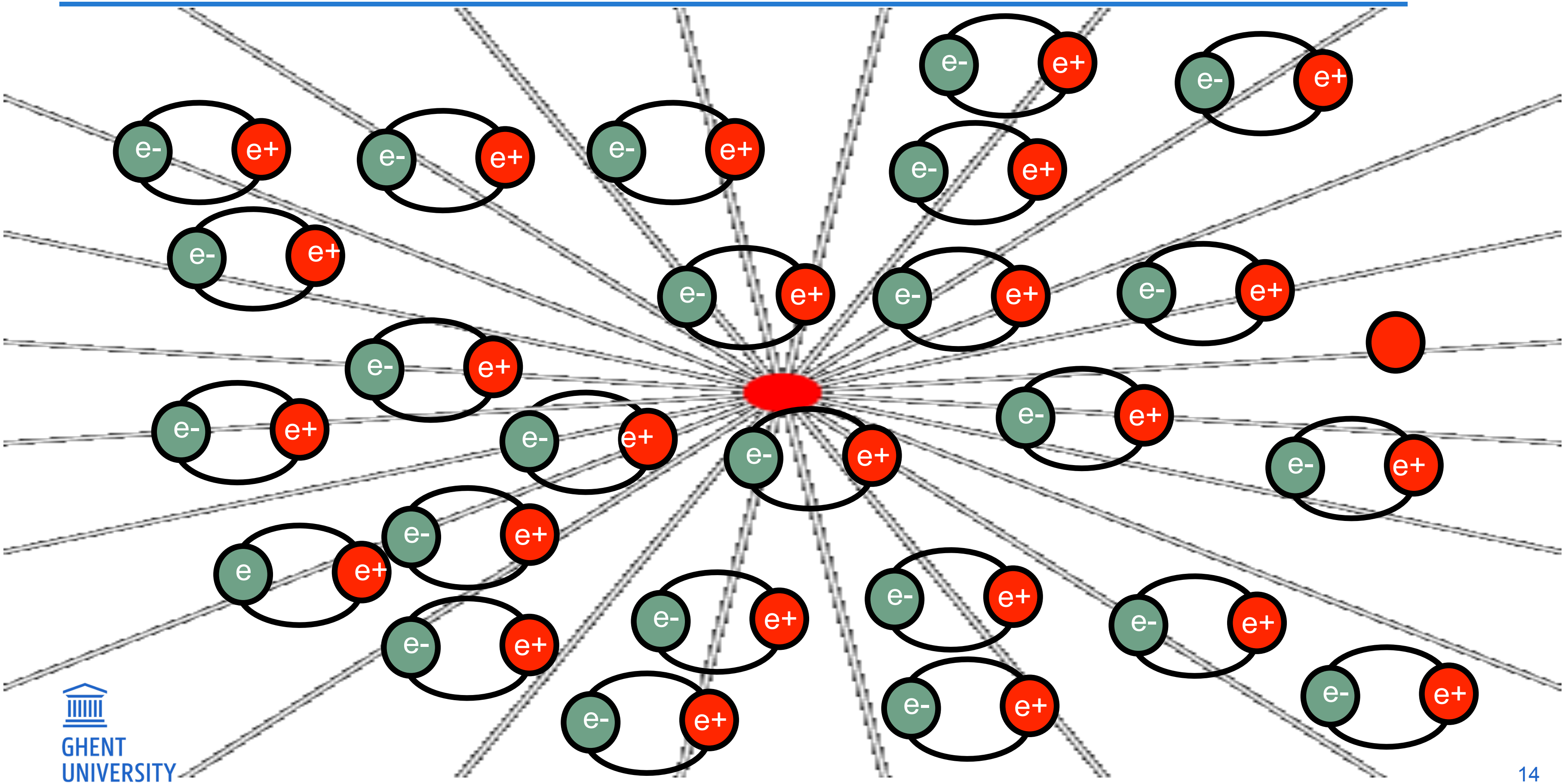
PAIRS ARE CREATED AT A CONSTANT RATE



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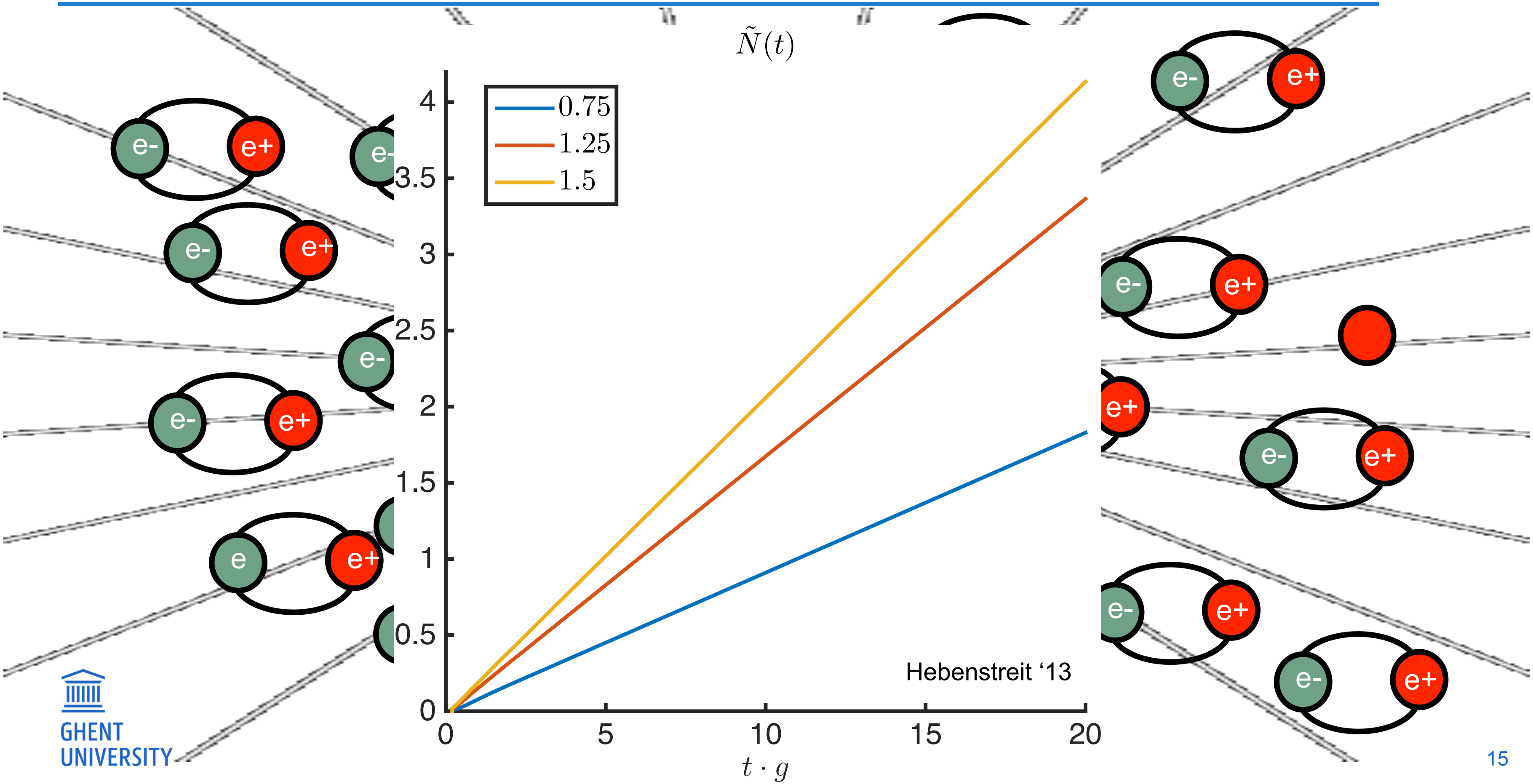
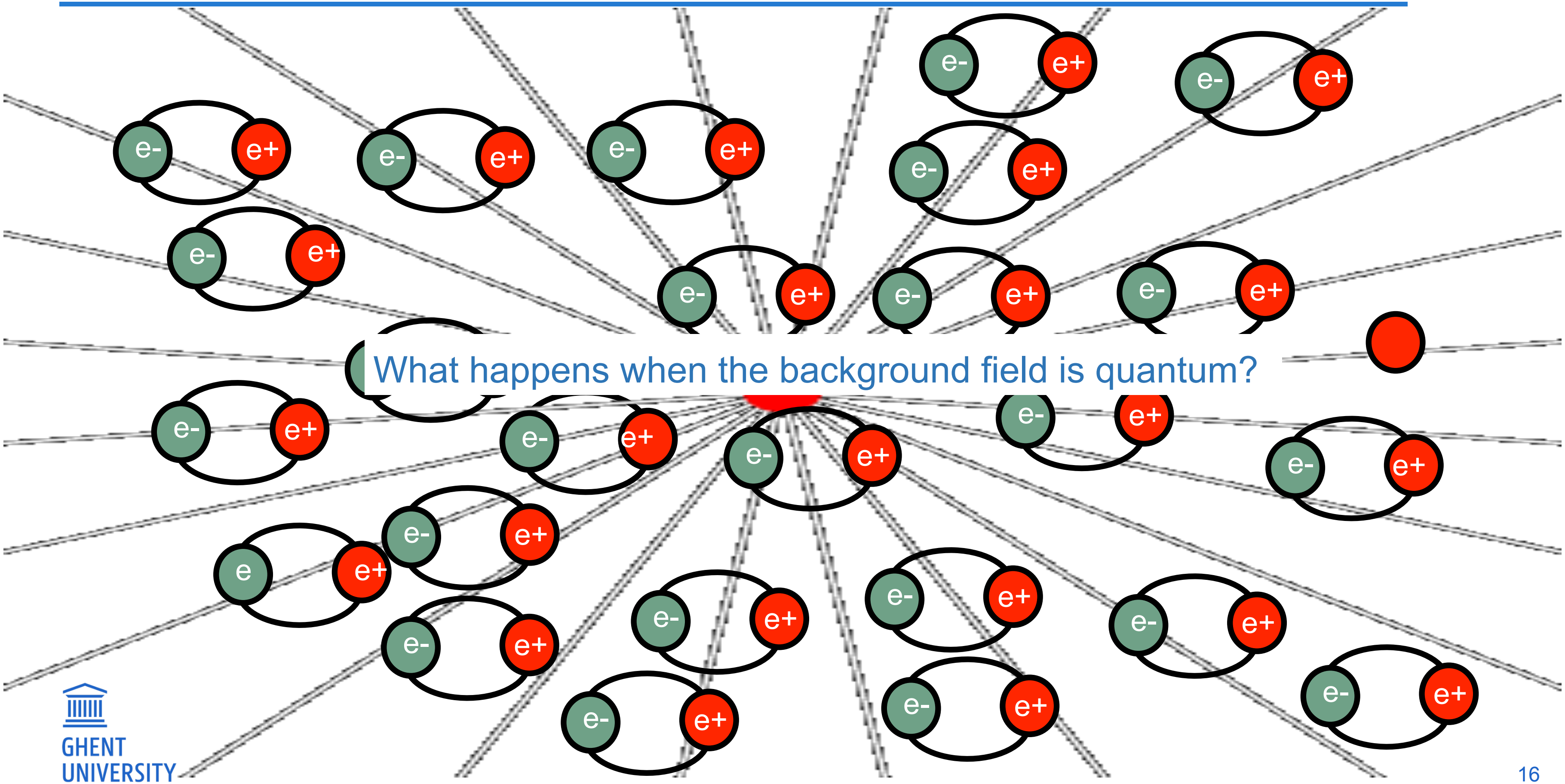
PAIRS ARE CREATED AT A CONSTANT RATE



PAIRS ARE CREATED AT A CONSTANT RATE



What happens when the background field is quantum?

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SET-UP

$$H_\alpha = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}[L(n)+\alpha]^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n[\sigma_z(n)+(-1)] + x\sum_{n=1}^{2N-1}(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.)\right)$$

Initial state $|\Psi(0)\rangle$ is ground state in the abscence of an background charge,

$|\Psi(0)\rangle$: ground state of $H_{\alpha=0}$

After $t=0$ we apply an electric field quench $\alpha \neq 0$

$|\Psi(t)\rangle = \exp(-iH_{\alpha\neq 0}t)|\Psi(0)\rangle$

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TDVP FOR GROUND STATE

SET-UP

$$H_\alpha = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}[L(n)+\alpha]^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n[\sigma_z(n)+(-1)]\right.$$
$$\left. + x\sum_{n=1}^{2N-1}(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.)\right)$$

Initial state $|\Psi(0)\rangle$ is ground state in the abscence of an background charge,

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$|\Psi(t)\rangle = \exp(-iH_{\alpha\neq 0}t)|\Psi(0)\rangle$

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TDVP FOR GROUND STATE

Ground-state ansatz

$$|\Psi[A]\rangle = \sum_{\{s,p\}} v_L^\dagger \left(\prod_{n=1}^{N} A_1^{s_{2n-1},p_{2n-1}} A_2^{s_{2n},p_{2n}} \right) v_R \, |\{s,p\}\rangle \qquad (N \to +\infty)$$

Minimize $\dfrac{\langle \Psi[\bar{A}] | H | \Psi[A] \rangle}{\langle \Psi[\bar{A}] | \Psi[A] \rangle}$ using TDVP (Haegeman '11)

Taking into account gauge invariance

$$G(n)\,|\Psi[A]\rangle = 0 \Leftrightarrow [A_n^{s,p}]_{(q,\alpha_q);(r,\beta_r)} = \delta_{r,q+(s+(-1)^n)/2}\,\delta_{r,p}[a_n^{q,s}]_{\alpha_q,\beta_r}$$

We obtain *global optimal MPS-approximation* $|\Psi[A]\rangle$ for initial state $|\Psi(0)\rangle$

$$|\Psi(0)\rangle \approx |\Psi[A]\rangle$$

for a fixed bond dimension, but how far is this from the real ground state?

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SCHMIDT SPECTRUM

Half-chain cut of the lattice: Schmidt decomposition

$$|\Psi[A]\rangle = \sum_{q\in\mathbb{Z}} \sum_{\alpha_q=1}^{D^q} \sqrt{\sigma_{\alpha_q}^q}\, |\Psi_{q,\alpha_q}^{(1)}\rangle \otimes |\Psi_{q,\alpha_q}^{(2)}\rangle$$

Exact ground state in limit $D^q \to +\infty$

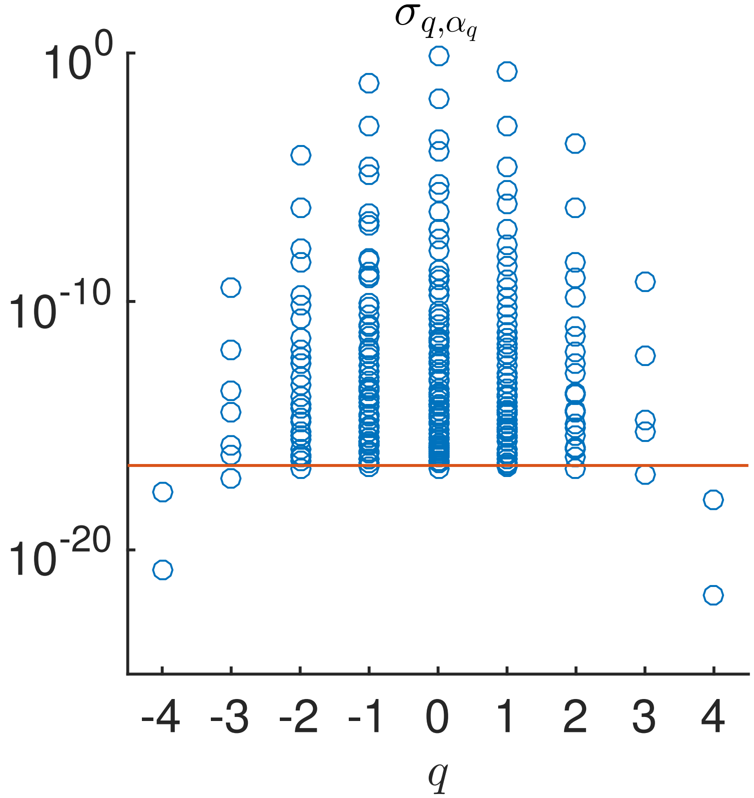
$$H \sim \frac{1}{2\sqrt{x}} \sum_n [L(n)+\alpha]^2$$

At low energies: states with large electric field have small contribution in Schmidt spectrum

$$|q+\alpha| \gtrsim 3 \Rightarrow \sigma_{\alpha_q,q} \ll 1$$

We can safely neglect the high-q sectors, i.e.

$$|q+\alpha| \gtrsim 4 \Rightarrow D^q = 0$$



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TRUNCATING ELECTRIC FIELD

Two interesting quantities for the weight of each sector

- Number of Schmidt values larger than 10^{-16}

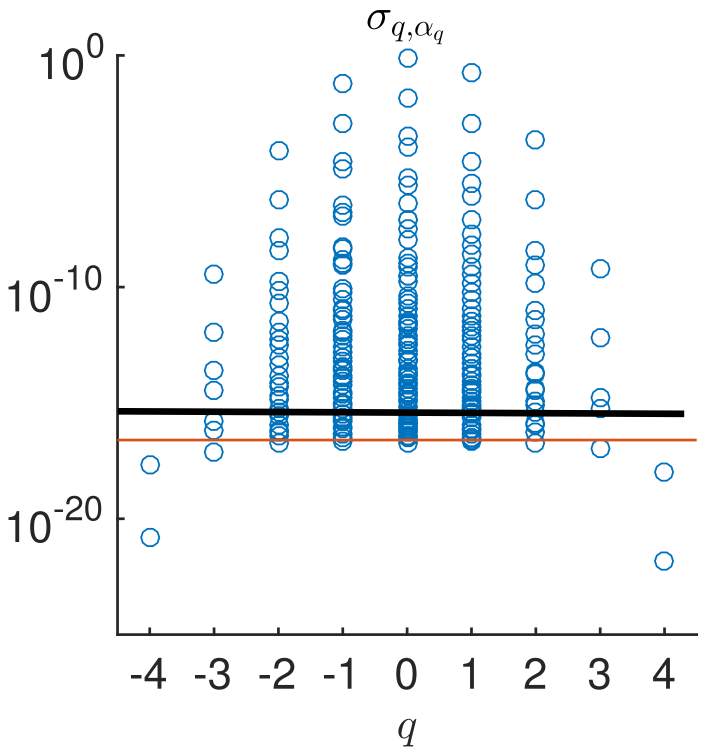
$$\tilde{D}^q = \#\{\sigma^q_{\alpha_q} \geq 10^{-16}\}$$

- Contribution to reduced density matrix

$$\rho_2 = \bigoplus_{q=p_{min}}^{p_{max}} \rho_{2,q}$$

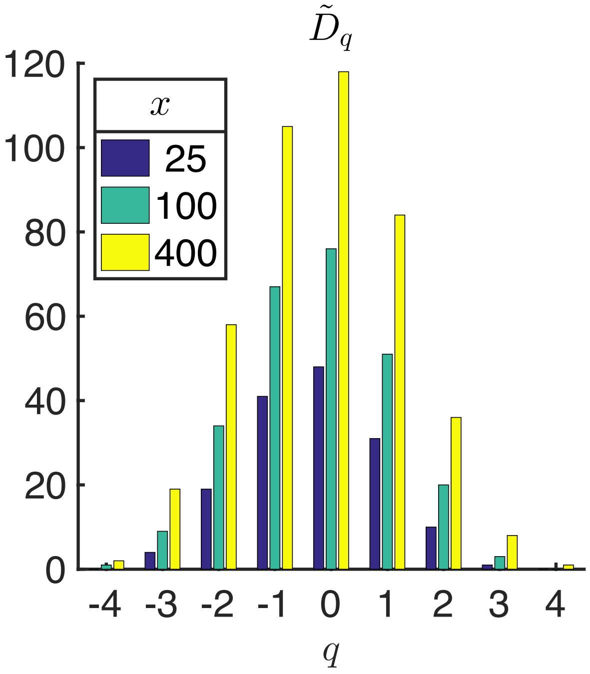
$$\frac{1}{2N} |\langle\Psi[A]|O|\Psi[A]\rangle| = \left| \sum_{q=p_{min}}^{p_{max}} \mathrm{tr}\left(\rho_{2,q} o_q\right) \right|$$

$$\leq ||\rho_{2,q}||_1 \cdot \underbrace{||o_q||}_{\lesssim q^M}$$



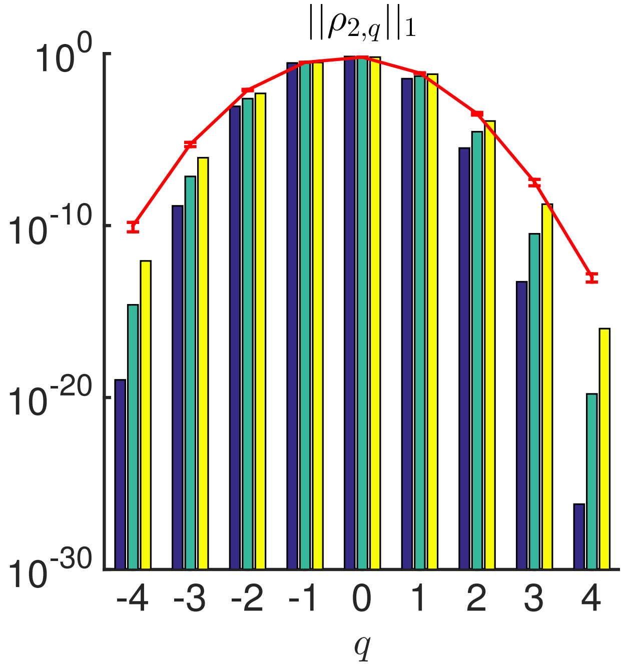
TOWARDS CONTINUUM LIMIT

Continuum limit $1/\sqrt{x} \to 0 \Leftrightarrow x \to +\infty$



$$\tilde{D}^q \sim \sqrt{x}$$

$$S \sim \log(\sqrt{x})$$



$$||\rho_{2,q}||_1 \sim \exp(-Cq^2)(x \to +\infty)$$

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TOWARDS PHASE TRANSITION

Schwinger model has phase transition
(Coleman '75, Byrnes '02)

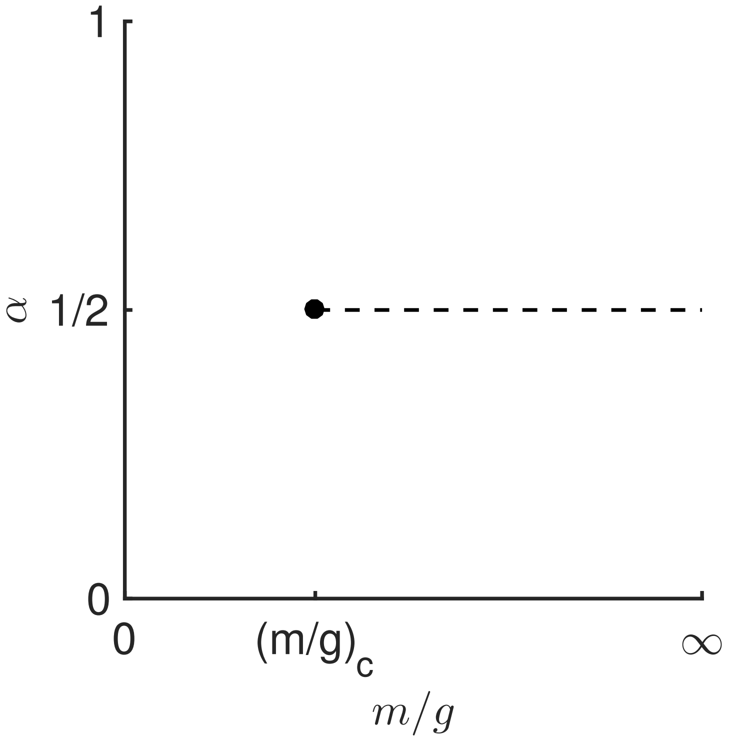
$$(m/g, \alpha) = ((m/g)_c \approx 0.33, 1/2)$$

Correlation length diverges

$$D^q \sim \xi \to +\infty$$

Close to phase transition, simulations
Indeed become harder!

- `Regular' case: a day
- $m/g = 0.3, \alpha = 0.5$: a few weeks



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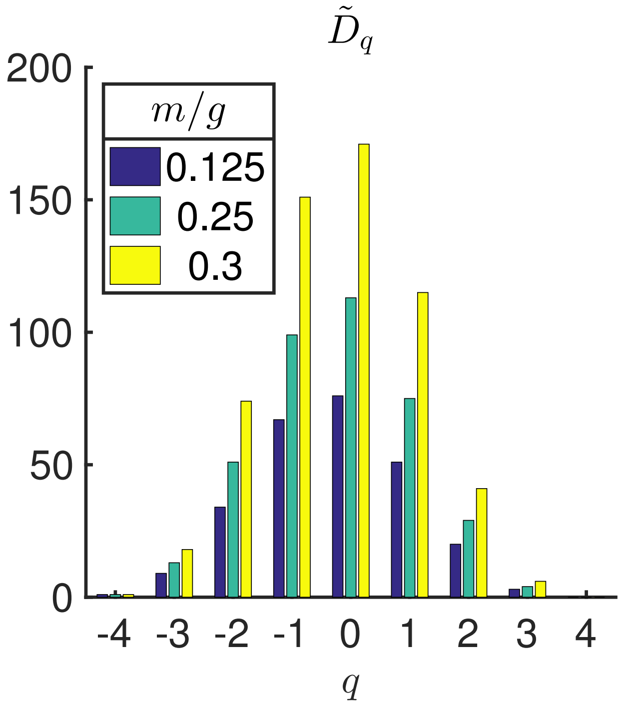
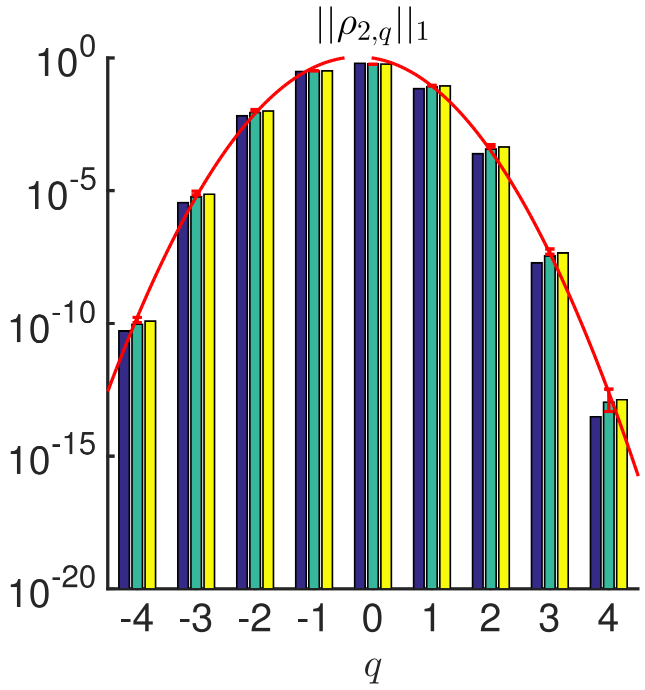
TOWARDS PHASE TRANSITION



$$\alpha = 0.5, x = 100$$



$$||\rho_{2,q}||_1 \sim \exp(-Cq^2)((m/g) \to (m/g)_c)$$

GAUGE-INVARIANT MPS

POS(LATTICE2014)308

$$
A_1^{1,p} = \begin{array}{c|ccccc}
q\backslash r & \cdots & p-1 & p & p+1 & \cdots \\
\hline
\vdots & \ddots & \vdots & \vdots & \vdots & \iddots \\
p & \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\
p+1 & \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & a_1^{q,1} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\
p+2 & \cdots & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix} & \cdots \\
\vdots & \iddots & \vdots & \vdots & \vdots & \ddots
\end{array}
$$

Elitzurs' theorem ('75): ground state is automatically gauge invariant, so why should we care?

	without GI	with GI	without GI	with GI	with GI [1]
p_{max}	2	2	3	3	3
D	29	[2 6 9 8 4]	40	[2 3 7 11 10 4 2]	[5 20 48 70 62 34 10]
steps	9645	278	12417	561	
time	3 h 30 min	2 min	6 h 27min	5 min	
$\langle G^2 \rangle$	3×10^{-9}	0	3×10^{-9}	0	0
e	-3.048961	–3.048961	-3.048961	-3.048961	-3.048961
$E_{1,v}$	1.04252{10}	1.04254	1.04194 {14}	1.04209	1.04207
$E_{2,v}$	2.455 {37}	2.455	2.385 {59}	2.386	2.357
$E_{1,s}$	1.7719{20}	1.7719	1.7559 {31}	1.7565	1.7516

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ITEBD FOR REAL-TIME

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SET-UP

$$H_\alpha = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}[L(n)+\alpha]^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n[\sigma_z(n)+(-1)]\right.$$
$$\left.+x\sum_{n=1}^{2N-1}(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.)\right)$$

Initial state $|\Psi(0)\rangle$ is ground state in the abscence of an background charge,

$|\Psi(0)\rangle$: ground state of $H_{\alpha=0}$ ✓

After $t=0$ we apply an electric field quench $\alpha\neq 0$

$|\Psi(t)\rangle = \exp(-iH_{\alpha\neq0}t)|\Psi(0)\rangle$

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ITEBD FOR REAL-TIME EVOLUTION

$$|\Psi(t)\rangle = \exp(-iH_\alpha t)|\Psi(0)\rangle \approx \exp(-iH_\alpha t)|\Psi[A]\rangle \overset{iTEBD}{\approx} |\Psi[A(t)]\rangle$$

with
$$|\Psi[A(t)]\rangle = \sum_{\{s,p\}} v_L^\dagger \left(\prod_{n=1}^{N} A_1^{s_{2n-1},p_{2n-1}}(t) A_2^{s_{2n},p_{2n}}(t) \right) v_R \, |\{s,p\}\rangle$$

$$[A_n^{s,p}(t)]_{(q,\alpha_q),(r,\beta_r)} = \delta_{r,q+(s+(-1)^n)/2} \delta_{r,p} [a_n^{q,s}(t)]_{\alpha_q,\beta_r}$$ (Gauge invariance!)

ITEBD (Vidal '04) tells us how to determine $a_n^{q,s}(t) \in \mathbb{C}^{D^q(t)\times D^r(t)}$

- Size of matrices changes also over time
- Involves two sources of error (see next slide)

SOURCES OF ERROR IN ITEBD

1. Trotter-error:

   Trotter-decomposition of $\exp(-iH_\alpha t)$: $\exp(A+B) \approx \exp(A)\exp(B)$

   Can be controlled very well: error of order $(dt)^5$

   Exact in limit $dt \to 0$

2. After every step dt: size of $a_n^{q,s}(t) \in \mathbb{C}^{D^q(t)\times D^r(t)}$ is multiplied by two

$$D^q(t+dt) = 2D^q(t) \Rightarrow D^q(t+Mdt) = 2^M D^q(t)$$

   Can be reduced by truncating Schmidt spectrum between site 1 and 2:

$$|\Psi[A(t)]\rangle = \sum_{q\in\mathbb{Z}} \sum_{\alpha_q=1}^{D^q(t)} \sqrt{\sigma^q_{\alpha_q}(t)}\, |\Psi^{(1)}_{q,\alpha_q}(t)\rangle \otimes |\Psi^{(2)}_{q,\alpha_q}(t)\rangle$$

i.e. discarding all $\sqrt{\sigma^q_{\alpha_q}(t)} \leq \epsilon$

   Exact in limit $\epsilon \to 0$

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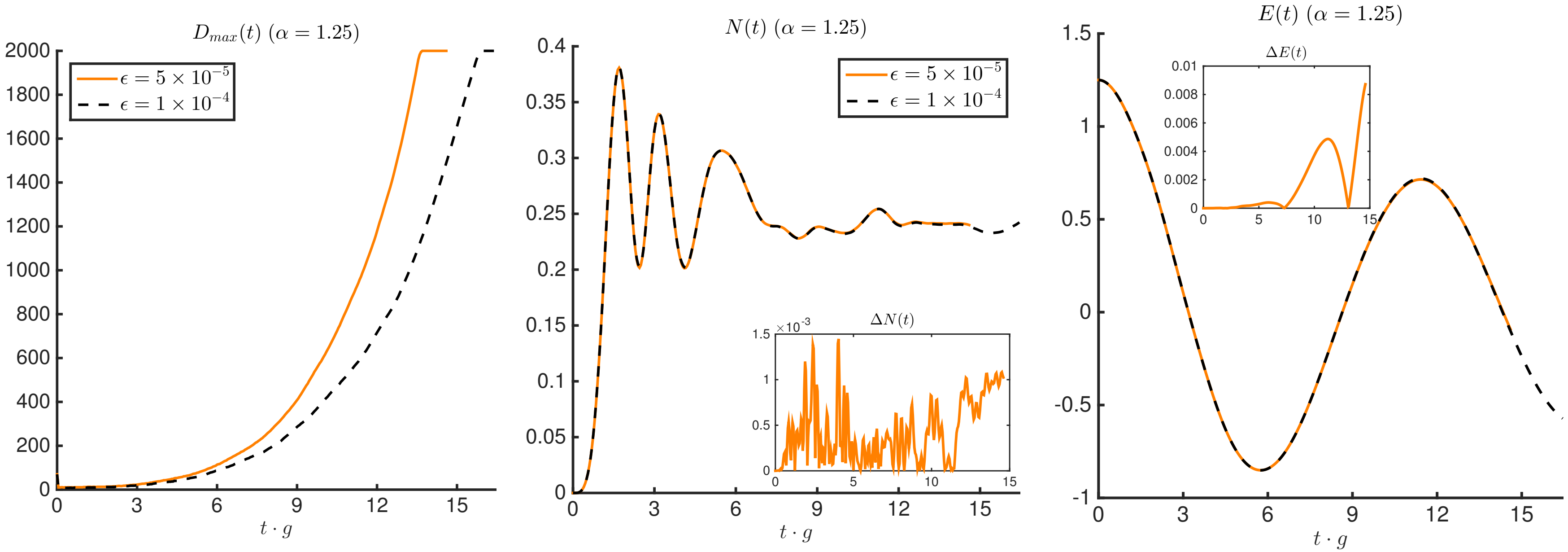
EVOLUTION OF BOND DIMENSION



$$m/g = 0.25, \alpha = 1.25, x = 100$$

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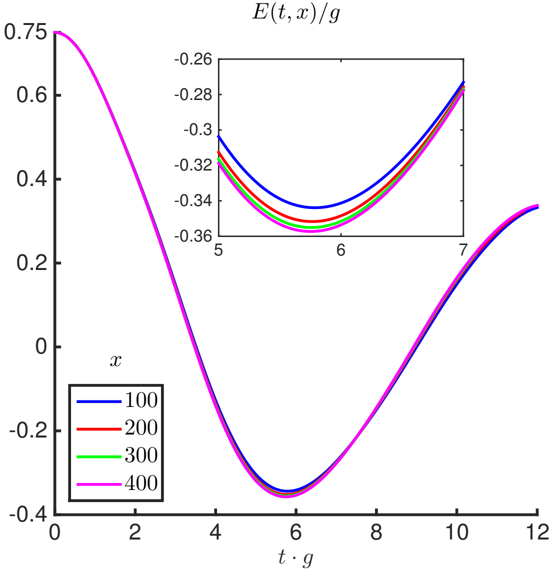
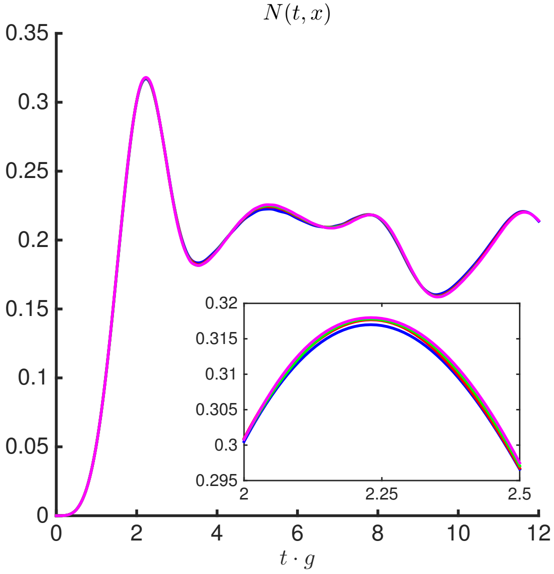
CONTINUUM LIMIT

$$ga = 1/\sqrt{x} \to 0 \Leftrightarrow x \to +\infty$$

$$m/g = 0.25, \alpha = 0.75$$





$$E(t) = g \langle \Psi(t) | L + \alpha | \Psi(t) \rangle$$

$$N(t) = \langle \Psi(t) | \bar{\psi}\psi | \Psi(t) \rangle / g - \langle \Psi(0) | \bar{\psi}\psi | \Psi(0) \rangle / g$$

Our results at x = 100 are very close to the continuum limit!

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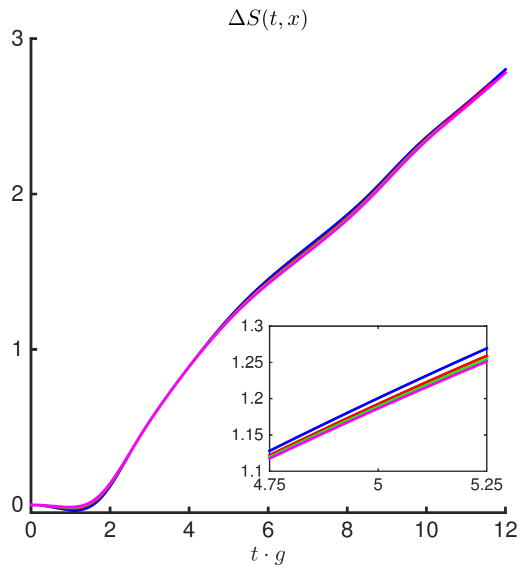
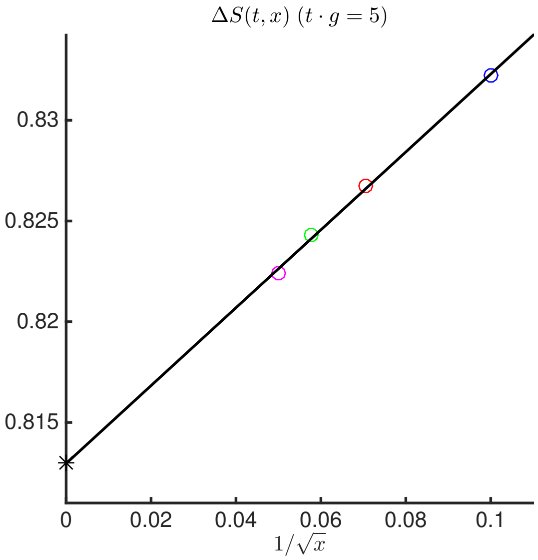
VON NEUMANN ENTROPY

$$ga = 1/\sqrt{x} \to 0 \Leftrightarrow x \to +\infty$$

$$|\Psi[A(t)]\rangle = \sum_{q\in\mathbb{Z}} \sum_{\alpha_q=1}^{D^q(t)} \sqrt{\sigma^q_{\alpha_q}(t)}\, |\Psi^{(1)}_{q,\alpha_q}(t)\rangle \otimes |\Psi^{(2)}_{q,\alpha_q}(t)\rangle$$

Von Neumann entropy: $S(t) = -\sum_{q\in\mathbb{Z}} \sum_{\alpha=1}^{D^q(t)} \sigma^q_{\alpha_q}(t) \log \sigma^q_{\alpha_q}(t)$





$$\Delta S(t) = S(t) - S(0)$$

is UV-finite

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RESULTS: WEAK-FIELD REGIME

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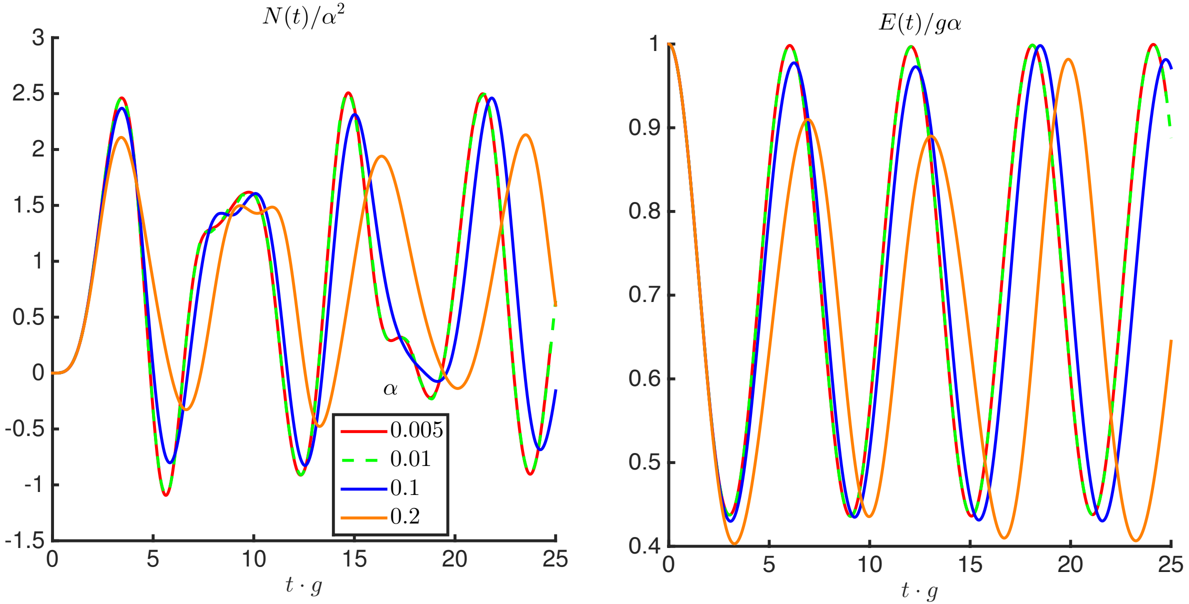
QUASI-PERIODIC OSCILLATIONS



$$N(t) = \langle \Psi(t) | \bar{\psi}\psi | \Psi(t) \rangle / g - \langle \Psi(0) | \bar{\psi}\psi | \Psi(0) \rangle / g \propto \alpha^2 \; (|\alpha| \ll 1)$$

$$E(t) = g \langle \Psi(t) | L + \alpha | \Psi(t) \rangle \propto g\alpha \; (|\alpha| \ll 1)$$

For $\alpha \gtrsim 0.1$ results leave linear response regime

Quasi-periodic behavior electric field: frequencies can be computed from mass gap H_α

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WEAK-FIELD APPROXIMATION

Idea: restricting Hilbert space to ground state and single-particle excitations of H_α

$$H_\alpha \to |0\rangle\,, |\mathcal{E}_m(k)\rangle\,, \ldots$$

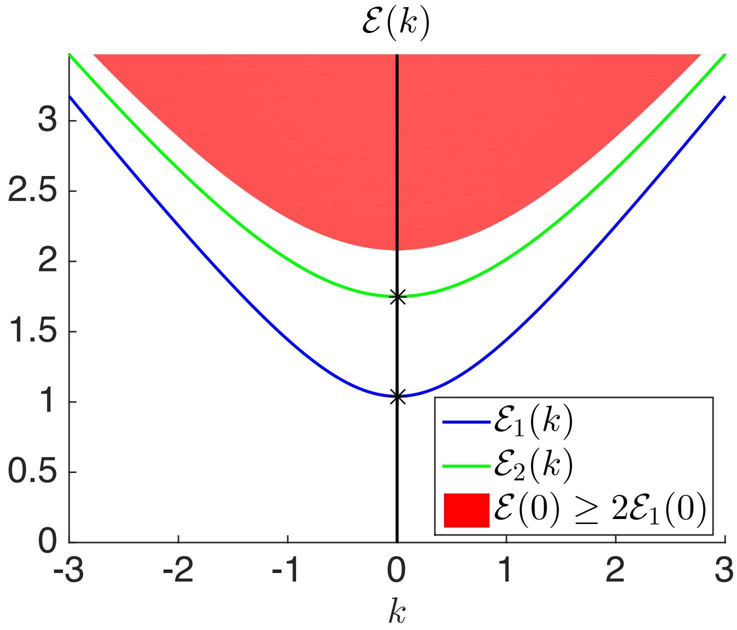
Are all approximated accurately by MPS!
(BB '13, '15; Haegeman '12)

Creation/annihilation operators

$$a_m^\dagger(k)\,|0\rangle = |\mathcal{E}_m(k)\rangle$$

$$a_m(k)\,|0\rangle = 0$$

Expand observables linearly in these operators (Delfino '14)



$$\mathcal{O} \approx \sum_{m,n} \int dk \int dk' O_{m,n}(k,k') a_m^\dagger(k) a_n(k) + \sum_m \left(\int dk\ o_m(k) a_m(k) + h.c \right)$$

WEAK-FIELD APPROXIMATION

$$H_\alpha \rightarrow |0\rangle, |\mathcal{E}_m(k)\rangle, \ldots \qquad a_m^\dagger(k)|0\rangle = |\mathcal{E}_m(k)\rangle \qquad a_m(k)|0\rangle = 0$$

$$\mathcal{O} \approx \sum_{m,n} \int dk \int dk' O_{m,n}(k,k') a_m^\dagger(k) a_n(k) + \sum_m \left(\int dk \; o_m(k) a_m(k) + h.c \right)$$

Expansion identically zero in the many-particle excitations subspace!

$O_{m,n}(k,k')$ and $o_m(k)$ are easily computed within MPS framework

$$O_{m,n}(k,k') = 2\pi\delta(k-k') o_{1,m,n} \qquad O_m(k) = 2\pi\delta(k) o_{2,m}$$

$$\delta(k) = \frac{2N}{2\pi}\delta_{k,0} \; (N \rightarrow +\infty)$$

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WEAK-FIELD APPROXIMATION

$$H_\alpha \to |0\rangle\,, |\mathcal{E}_m(k)\rangle\,, \ldots \qquad a_m^\dagger(k)\,|0\rangle = |\mathcal{E}_m(k)\rangle \qquad a_m(k)\,|0\rangle = 0$$

$$\mathcal{O} \approx \sum_{m,n} \int dk \int dk' O_{m,n}(k,k') a_m^\dagger(k) a_n(k) + \sum_m \left(\int dk\; o_m(k) a_m(k) + h.c \right)$$

$H_0\,|\Psi(0)\rangle = 0 \Rightarrow a_m(k)\,|\Psi(0)\rangle = d'_m \delta(k)\,|\Psi(0)\rangle$: Initial state is coherent state of $a_m(k)$

$$\frac{1}{2N}\langle \Psi(t)|\mathcal{O}|\Psi(t)\rangle = \frac{1}{2\pi}\left[\left(\sum_m o_{2,m} d'_m e^{-i\mathcal{E}_m(0)t} + h.c.\right) + \sum_{m,n} o_{1,m,n} \bar{d}'_m d_n e^{-i[\mathcal{E}_m(0)-\mathcal{E}_n(0)]t}\right]$$

Valid as long as $\rho_m = \dfrac{|d'_m|^2}{2\pi a} \ll \mathcal{E}_m(0)$

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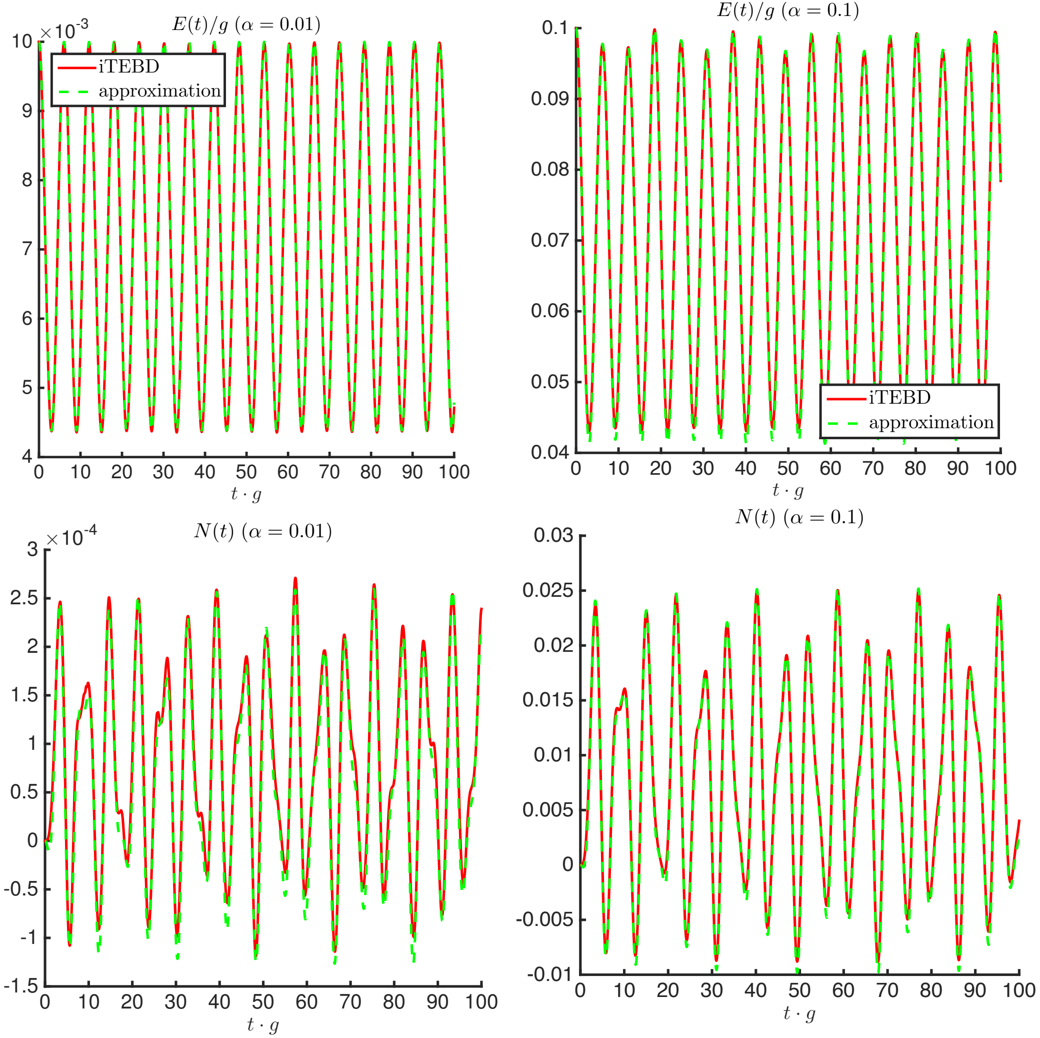

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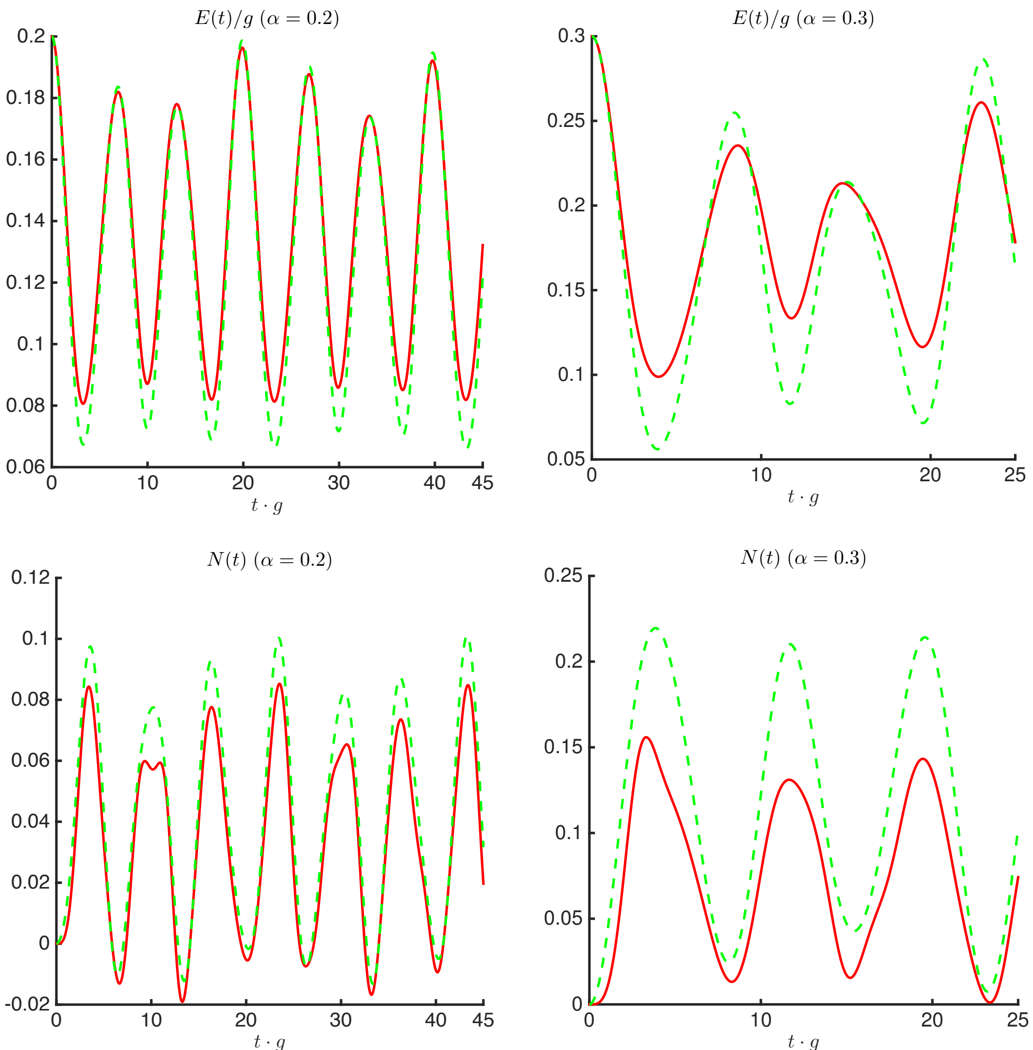

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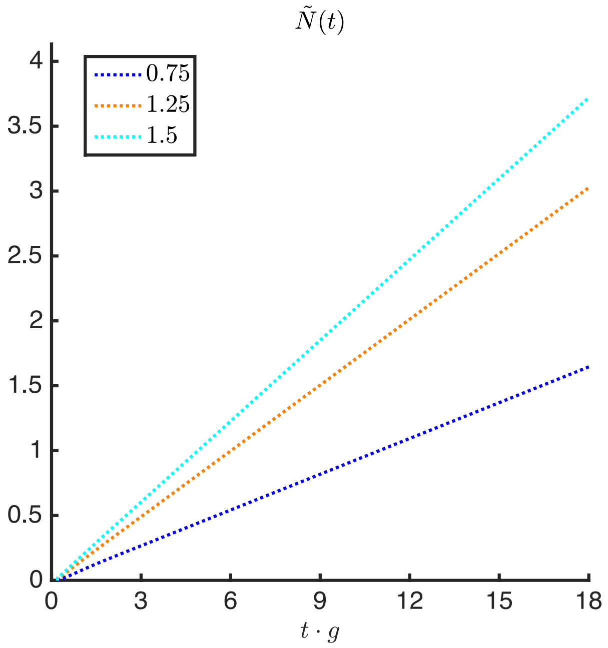
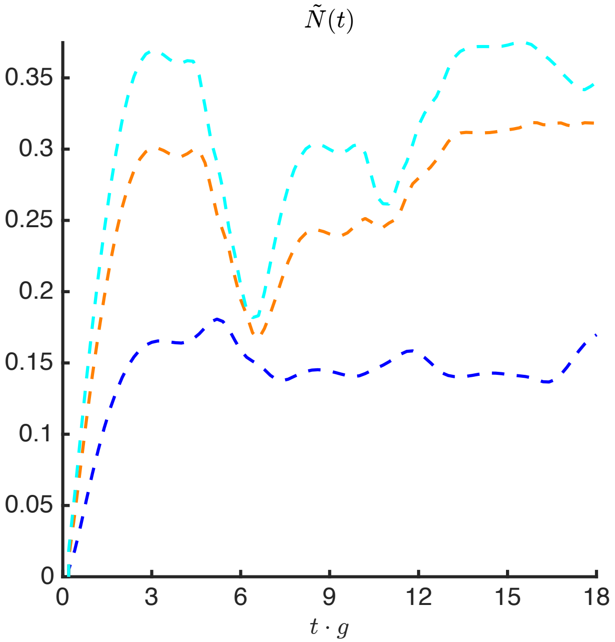
STRONG-FIELD REGIME

SCHWINGER EFFECT





Schwinger effect ('56 Schwinger)

- Classical background field
- Electron-postitron pairs created at constant rate

Semi-classical (Klüger '92, Hebenstreit '13)

- Electric field is treated classically $\langle E^2 \rangle = 0$
- Backreaction of fermions with gauge fields

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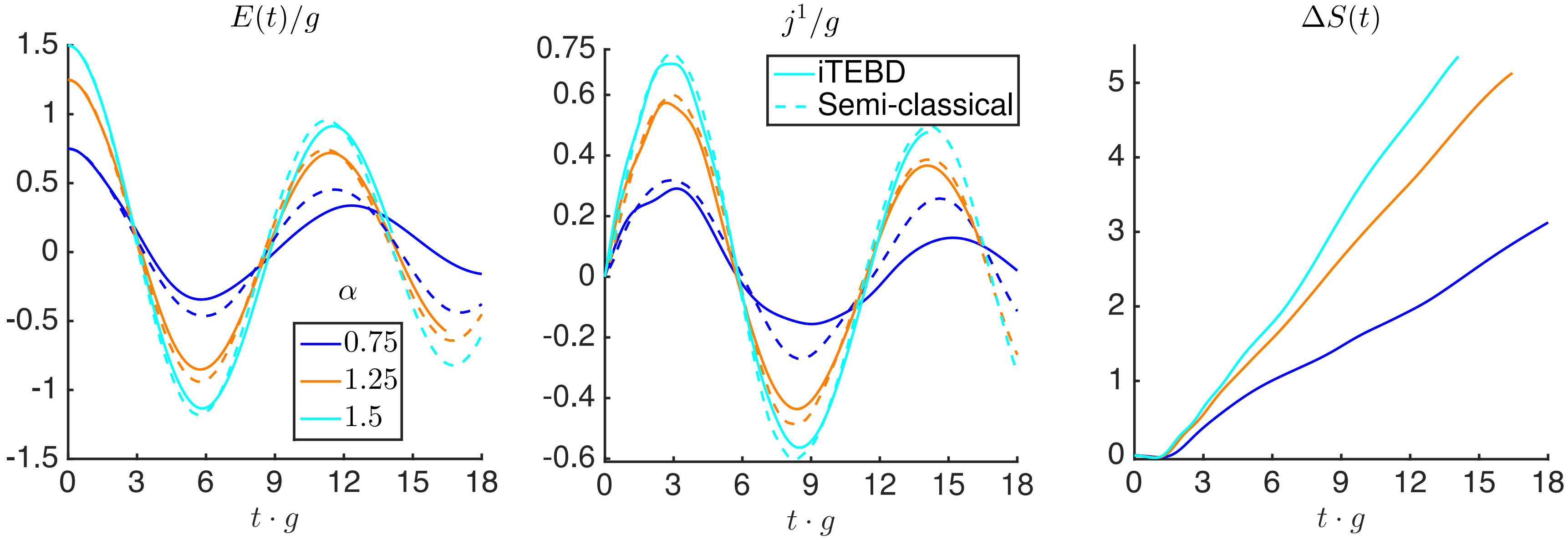
SEMI-CLASSICAL BEHAVIOR



When $\alpha \gtrsim 1.25$: quantitative agreement for electric field improves

- Creation of electron-positron pairs accelerates particles
- Saturation of current $j^1 = -\dot{E}/g$ for $tg \approx 3$ (Klüger '92)
- Increasing half-chain entropy (Calabrese '05)

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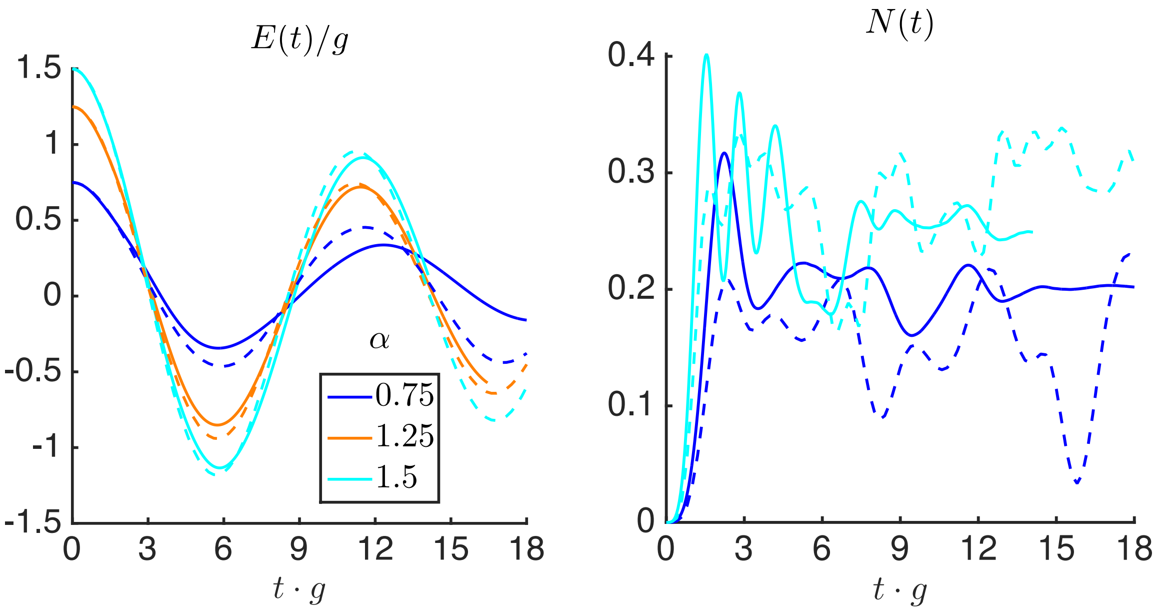
FULL-QUANTUM VS. SEMI-CLASSICAL



Particle number: only qualitative agreement

In general: oscillations are more damped as predicted by semi-classical studies

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EQUILIBRATION

State brought out-of-equilbrium will equilibrate locally (Linden '09)

$$\rho_m(t) = \mathrm{tr}_{1,\ldots,2n,2(n+m+1)+1,\ldots,2N}\left(|\Psi(t)\rangle\langle\Psi(t)|\right)$$

$$\rho_m(t) \to \rho_m^{(asymptotic)}$$

How does $\rho_m^{(asymptotic)}$ look?

Thermalization: $\rho_m^{(asymptotic)} = \mathrm{tr}_{1,\ldots,2n,2(n+m+1)+1,\ldots,2N}\left(\frac{e^{-\beta H_\alpha}}{\mathrm{tr}(e^{-\beta H_\alpha})}\right)$

i.e. Asymptotic state corresponds locally to a thermal state

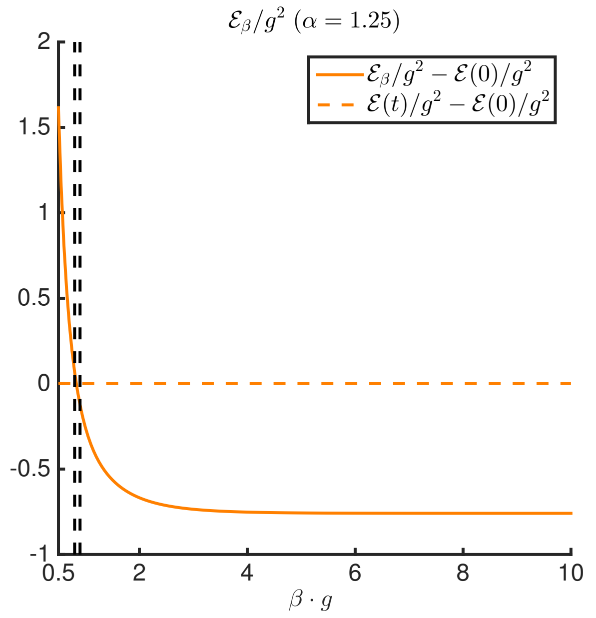
Lot of debate on that: ETH, strong,weak-thermalization, GGE-thermalization, many-body localization (see talk D. Abanin)…

But let's start to investigate whether thermalization could occur

INVERSE TEMPERATURE

$$\rho(t) = |\Psi(t)\rangle\langle\Psi(t)| \xrightarrow{t\to+\infty} \rho_{Gibbs}(\beta) = \frac{e^{-\beta H_\alpha}}{\mathrm{tr}(e^{-\beta H_\alpha})}$$ (local)?

$\rho_{Gibbs}(\beta)$ can be approximated with MPS methods
(Verstraete '04, BB '16)



To determine $\beta = 1/T$

- Energy conservation over time $\mathcal{E}(t) = \mathcal{E}(0)$
- Energy of Gibbs state is $\mathcal{E}_\beta = \frac{\mathrm{tr}\left(e^{-\beta H} H\right)}{\mathrm{tr}\left(e^{-\beta H}\right)}$
- $\beta = 1/T$ follows from $\mathcal{E}_\beta = \mathcal{E}(0)$

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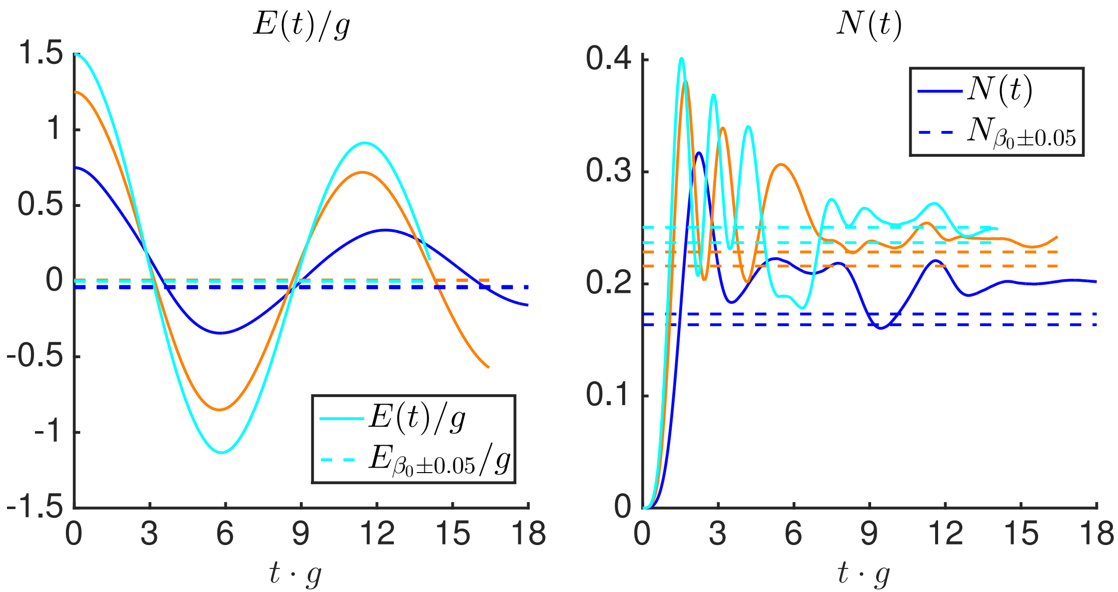
THERMALIZATION?



Electric field oscilates around its predicted thermal value

Particle number is close to thermal value for $\alpha \gtrsim 1.25$

First indications for thermalization, but we should be very cautious!!

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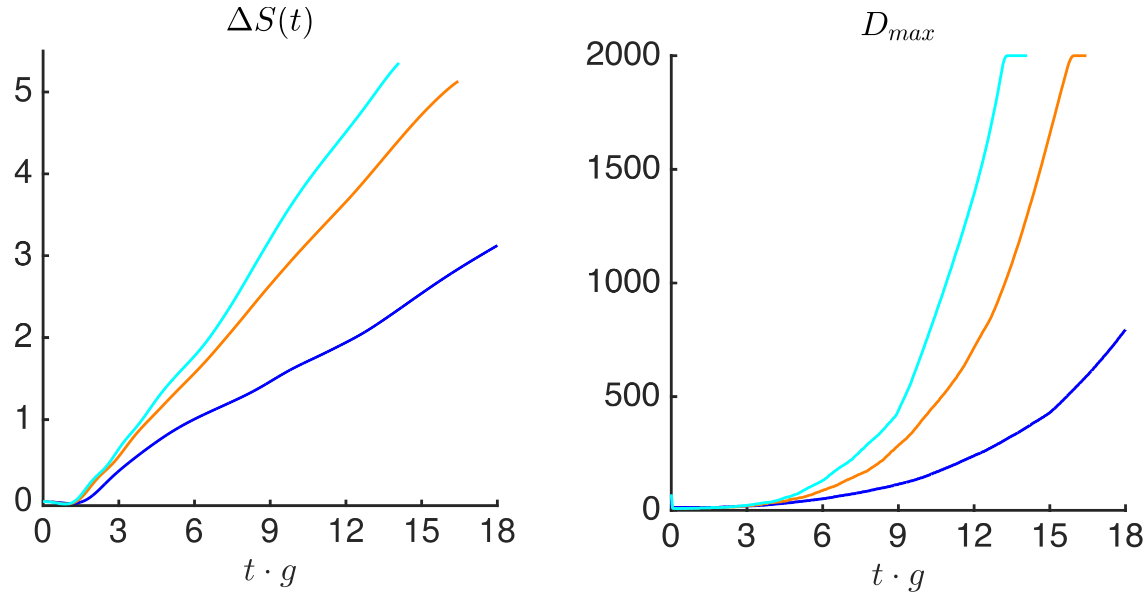
THERMALIZATION?



To reach definite conclusion we should track state for longer times

Linear increase in entanglement

Hence, exponential increase in variational parameters (quite problematic ☹)

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CONCLUSION

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CONCLUSION: MPS FOR REAL-TIME

MPS for the Schwinger effect works good

- Real-time simulation when amount of entanglement is not too large (e.g. Small quench, at early times)
- To understand our results
  - Excitations for weak-field regime to explain oscilatory behavior
  - Thermal states for first evidence of thermalization

Still many interesting things to explore

- Finding ways to approximate real-time at late times for large quenches
- Other setups:
  - Scattering Gaussian wavepackets
  - Confinement with dynamical charges (Hebenstreit '13)
  - Other quenches (e.g. Mass quench, time-dependent background field)
  - ….

MASS QUENCH: SET-UP

$$H_{m/g} = \frac{g}{2\sqrt{x}}\left(\sum_{n=1}^{2N}\frac{1}{g^2}E(n)^2 + \frac{\sqrt{x}}{g}m\sum_{n=1}^{2N}(-1)^n[\sigma_z(n)+(-1)] + x\sum_{n=1}^{2N-1}(\sigma^+(n)e^{i\theta(n)}\sigma^-(n+1)+h.c.)\right)$$

$$E(n) = gL(n)$$

Initial state $|\Psi(0)\rangle$ is ground state in the absence of an background charge,

$|\Psi(0)\rangle$: ground state of $H_{m/g=0.25}$

After $t=0$ we apply a mass quench $\Delta(m/g)$

$$|\Psi(t)\rangle = \exp(-iH_{m/g=0.25+\Delta(m/g)}t)|\Psi(0)\rangle$$

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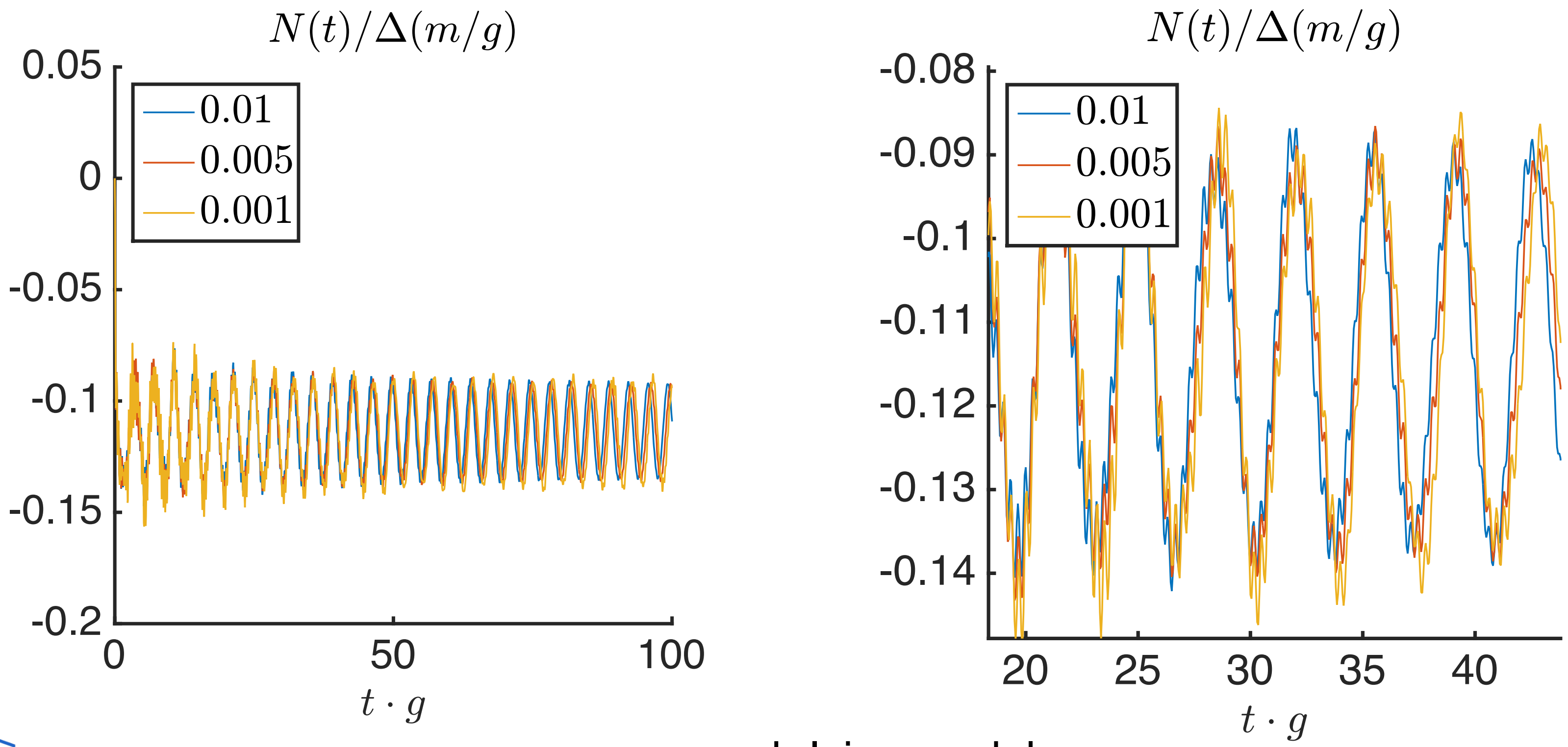
MASS QUENCH: PRELIMINARY RESULTS



~ mass quench Ising model

(Calabrese '11, Schuricht '12, Rakovszky '16)

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MASS QUENCH: PRELIMINARY RESULTS

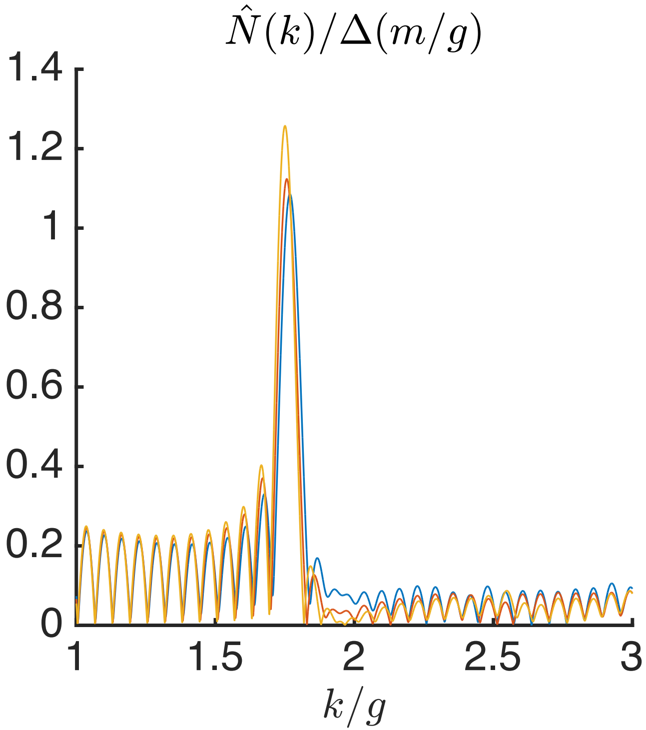
$$\hat{N}(k) = \int_0^{+\infty} dt\, e^{ikt} N(t)$$

Shows (delta)-peak around $k = k_{max}$ corresponding to mass second excited state:

$$k_{max} \approx \mathcal{E}_2$$

State with energy $\mathcal{E}_2$ = bound state of two particles with energy $\mathcal{E}_1$ (Confinement, Kormos '16)

$\Delta(m/g)$	k_{max}	$\mathcal{E}_2$
0.01	1.7682	1.7712
0.005	1.7569	1.7614
0.001	1.7505	1.7526



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CONCLUSION: TNS FOR GAUGE FIELD THEORIES

MPS works good for 1D gauge (field) theories,e.g.,

- Schwinger model (this work, Byrnes '02, Bañuls et al '13-'16.,…)
- U(1) QLM (Rico '13), SU(2) (Silvi '16, Kühn '16 …), ….

But… real world is nor one-dimensional, nor QED

MPS for 1+1 D QED ✓ ⇢ MPS for 1+1 D SU(N)

- One-flavor
- Multi-flavor (Bañuls, Cichy '16)

MPS for 1+1 D SU(N)

- Kühn '15, Silvi '16, Milsted '16

↓

PEPS for 2+1 D QED ⊙ ⇢ PEPS for 2+1 D SU(N)

- Zohar '15
- Variational ⊙

(Corboz '09 - '15, Vanderstraeten '16)

PEPS for 2+1 D SU(N)

- Tagliacozzo '14, Haegeman '15
- Zohar '16

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MPS-ANSATZ EXCITATIONS

Ground- state ansatz

$$|\Psi[A]\rangle = \sum_{s,p} v_L^\dagger \left(\prod_{n=1}^{2N} A^{s_n,p_n} \right) v_R \, |s_1,p_1,\ldots,s_{2N},p_{2N}\rangle \qquad (N \to +\infty)$$

Minimize $\dfrac{\langle\Psi[\bar{A}]|H|\Psi[A]\rangle}{\langle\Psi[\bar{A}]|\Psi[A]\rangle}$ using TDVP (Haegeman '11)

One-particle excitations ansatz (Haegeman '12)

$$|\Phi[B]\rangle = \sum_{m=1}^{2N} e^{ikn} \sum_{s,p} v_L^\dagger \underbrace{A^{s_1,p_1} \ldots A^{s_{m-1},p_{m-1}}}_{(m-1)\times} B^{s_m,p_m} \underbrace{A^{s_{m+1},p_{m+1}} \ldots A^{s_{2N},p_{2m}}}_{(2N-m-1)\times} v_R \, |s_1,p_1,\ldots,s_{2N},p_{2N}\rangle$$

Minimize $\dfrac{\langle\Phi[\bar{B}]|H|\Phi[B]\rangle}{\langle\Phi[\bar{B}]|\Phi[B]\rangle}$ = equivelent with generalized eigenvalue problem

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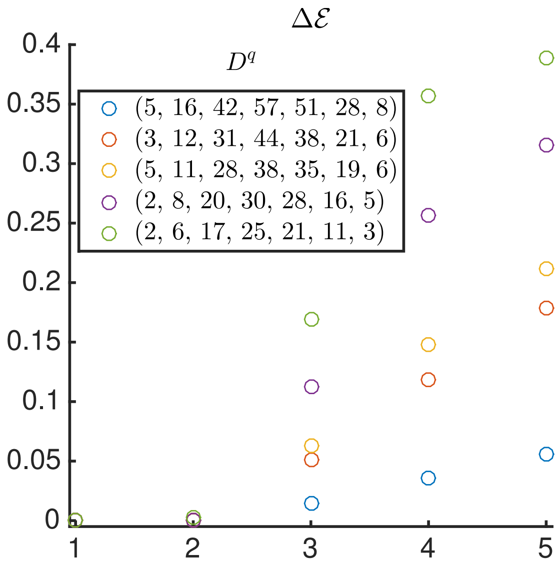
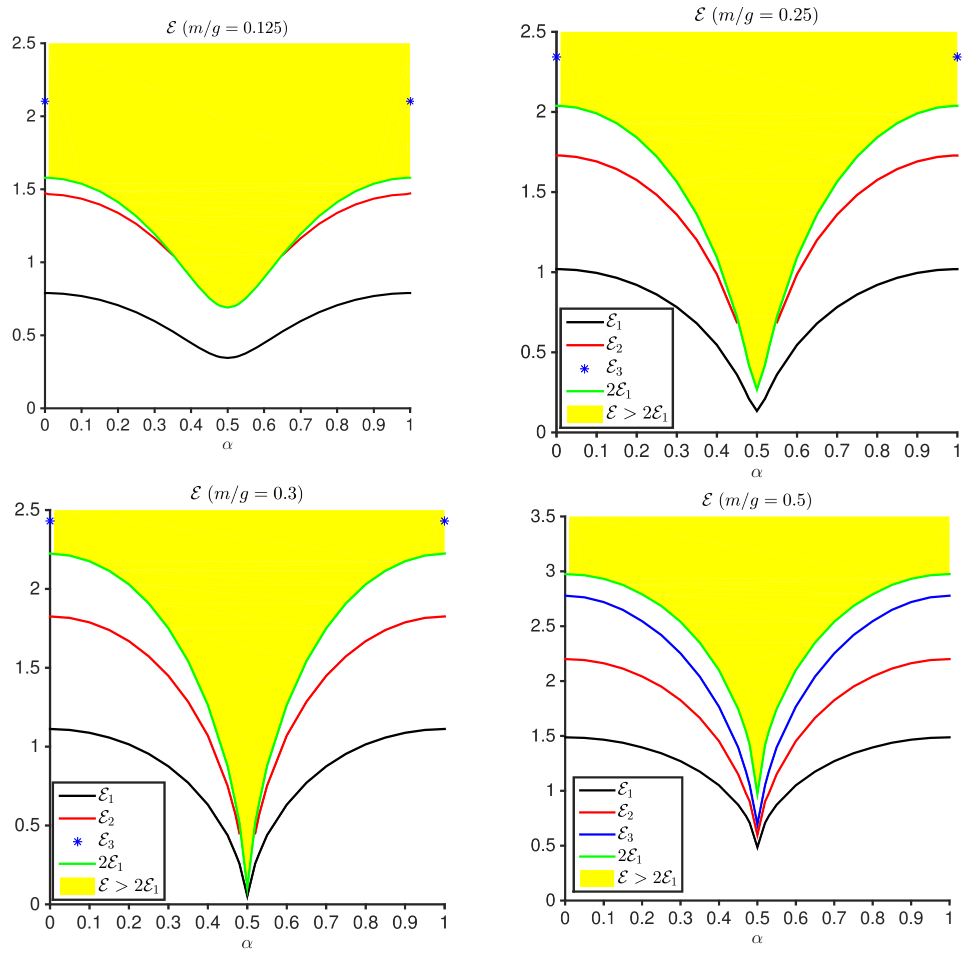
SINGLE-PARTICLE EXCITATIONS





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ITEBD FOR FINITE TEMPERATURE

$$\mathcal{H}_{full} = \bigotimes_{n=1}^{2N} \mathcal{H}_n \otimes \mathcal{H}_n^a$$

$$\rho_{Gibbs}(0) = \mathrm{id}_{GI} = \mathrm{tr}_{\mathcal{H}^a} \left(|\Psi[A]\rangle \langle \Psi[\bar{A}]| \right)$$

$$|\Psi[A]\rangle = \sum_{\vec{\kappa},\vec{\kappa}^a} \mathrm{tr} \left(A_1^{\kappa_1,\kappa_1^a} \ldots A_{2N}^{\kappa_{2N},\kappa_{2N}^a} \right) |\vec{\kappa}, \vec{\kappa}^a\rangle$$

$$[A_n^{(s,p),(s^a,p^a)}]_{(q,\alpha);(r,\beta)} = [a_n]_{\alpha_q,\beta_r} \delta_{r,q+[s+(-1)^n]/2} \delta_{p,r} \delta_{s,s^a} \delta_{p^a,q+[s^a+(-1)^n]/2}$$

$$\rho_{Gibbs}(\beta) = e^{-\beta H} = \mathrm{tr}_{\mathcal{H}^a} \left(|\Psi[A(\beta)]\rangle \langle \Psi[\bar{A}]| \right)$$

$$|\Psi[A(\beta)]\rangle = e^{-\beta H/2} |\Psi[A(0)]\rangle$$ with iTEBD

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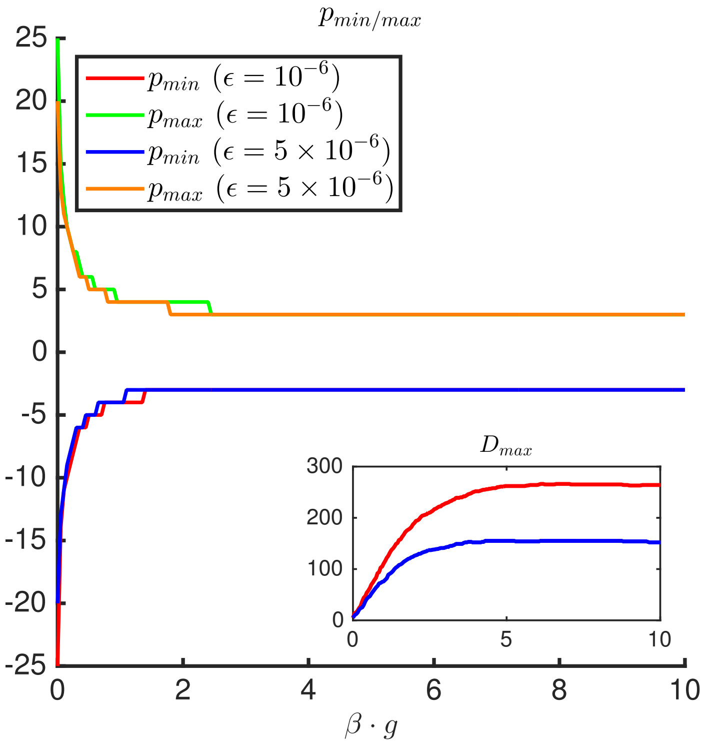
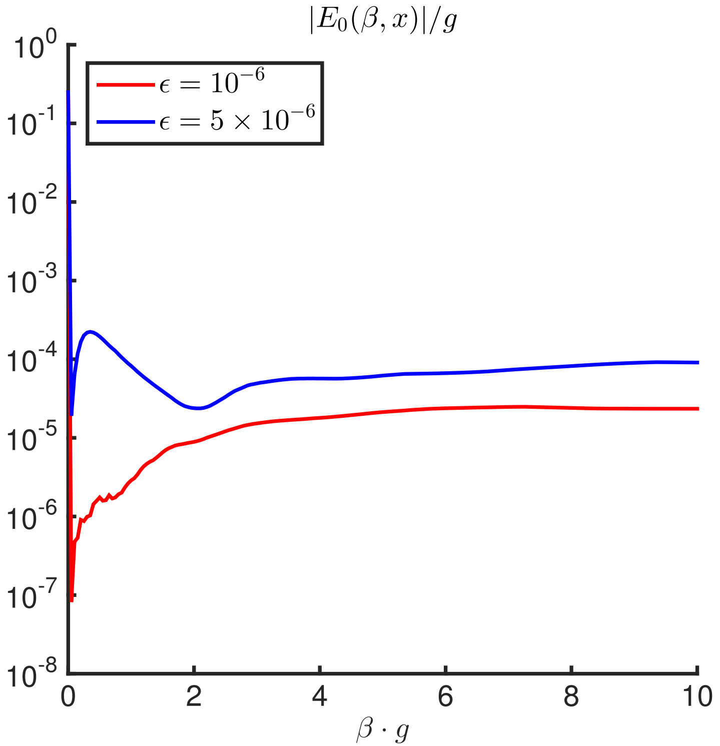
FINITE T SIMULATIONS





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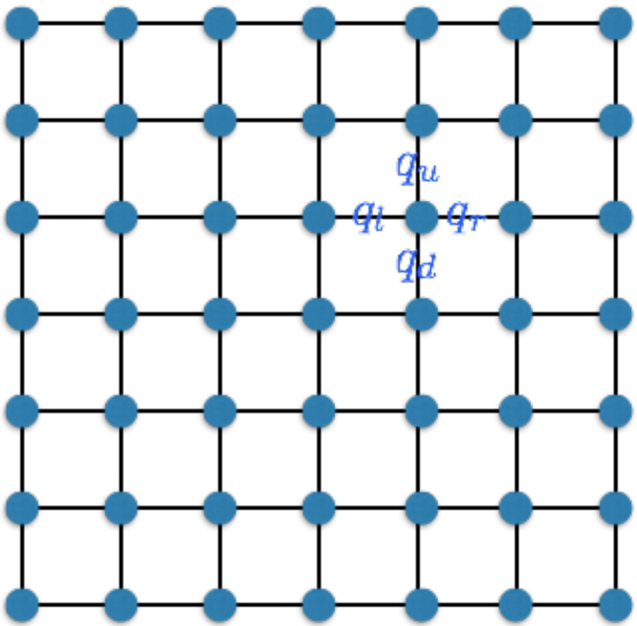
$$\rho(\beta) = e^{-\beta H}$$

TWO DIMENSIONS



Pure QED: Gauge fields live on links

Gauss' law:

$$\vec{\nabla} \cdot \vec{E} = 0 \Leftrightarrow (q_u - q_d + q_r - q_l) \left|\Psi\right\rangle = 0$$

$$q_u, q_d, q_l, q_r \in \mathbb{Z}$$

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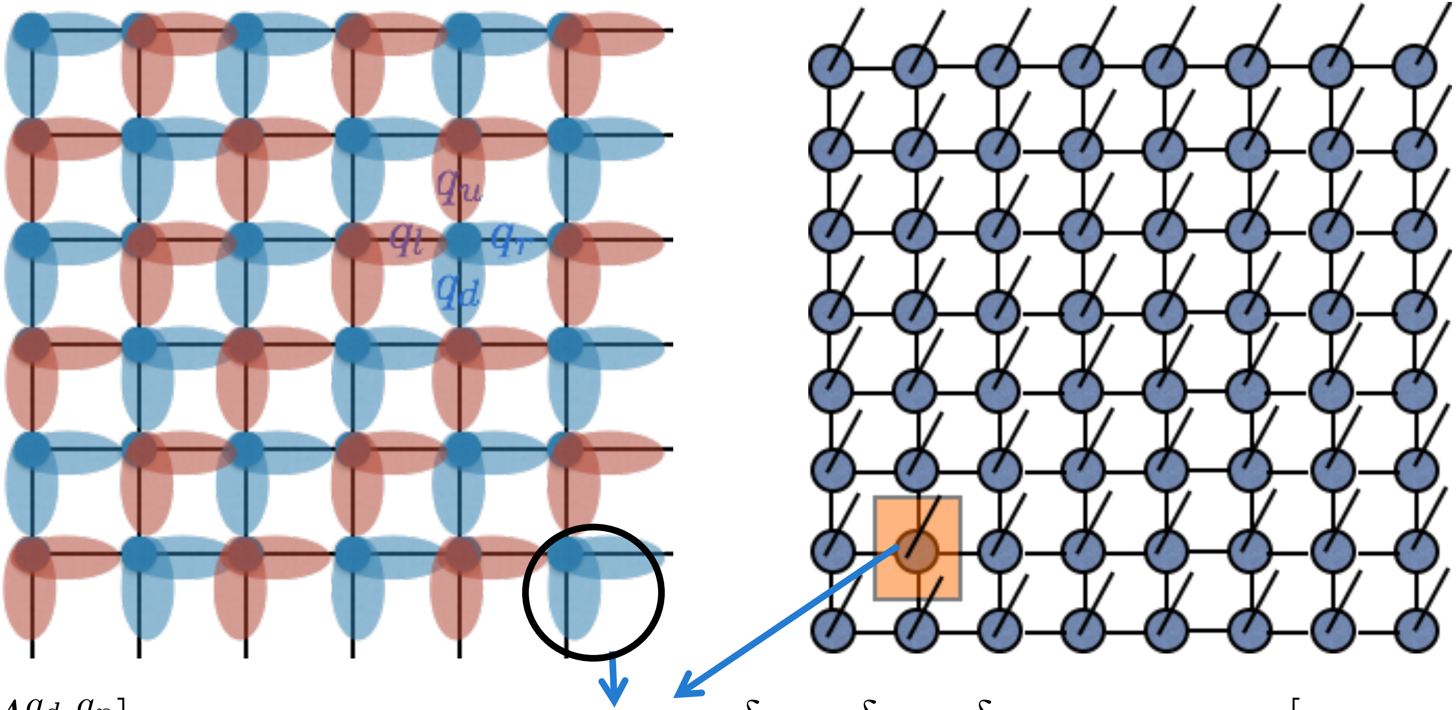
PEPS-ANSATZ

PEPS = 2D analogue of MPS (Verstraete ‘04)



$$[A^{q_d,q_r}]_{(p_l,\alpha_l);(p_d,\alpha_d)(p_r,\alpha_r),(p_u,\alpha_u)} = \delta_{q_d,p_d}\delta_{q_r,p_r}\delta_{p_u-p_d+p_r-p_l,0}\,[a_{q_l,q_d,q_r}]_{(\alpha_l,\alpha_d,\alpha_r,\alpha_u)}$$