PHILIPP SCHMOLL & ROMÁN ORÚS
JOHANNES GUTENBERG UNIVERSITY MAINZ

ENTANGLEMENT IN STRONGLY CORRELATED SYSTEMS
BENASQUE, 17. FEBRUARY 2017

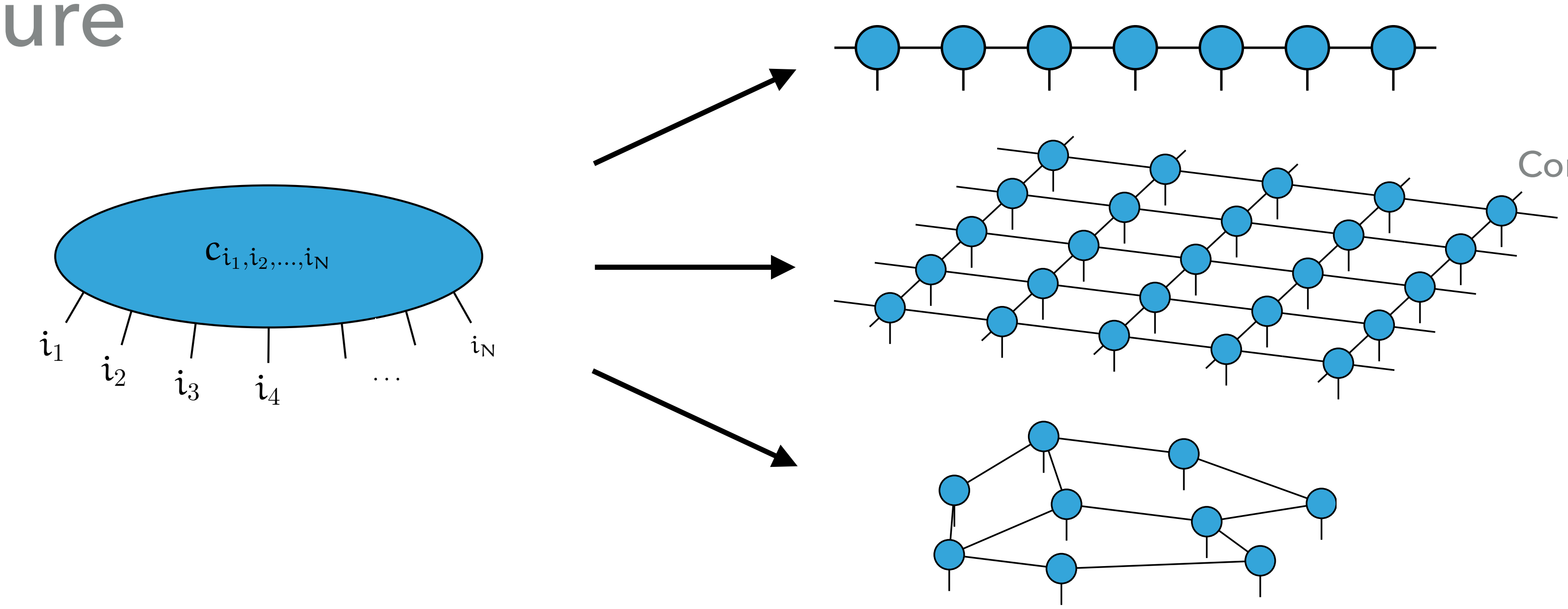
EXACT TENSOR NETWORK STATES FOR THE KITAEV HONEYCOMB MODEL

TENSOR NETWORKS & QUANTUM MANY-BODY ENTANGLEMENT

- Quantum state of many-body system with N particles

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} c_{i_1,i_2,\ldots,i_N} |i_1, i_2, \ldots, i_N\rangle$$

- Decompose wave function using the entanglement structure



MPS

Fannes, Nachtergaele and Werner
Comm. Math. Phys. Volume 144, Number 3

PEPS

Verstraete and Cirac
arXiv:cond-mat/0407066

arbitrary TN

- Kitaev honeycomb model obeys area-law in all phases

Orús, Annals of Physics 349 (2014)
Yao & Qi, PRL 105 080501 (2010)

FERMIONIC TENSOR NETWORKS

- Fundamental difference

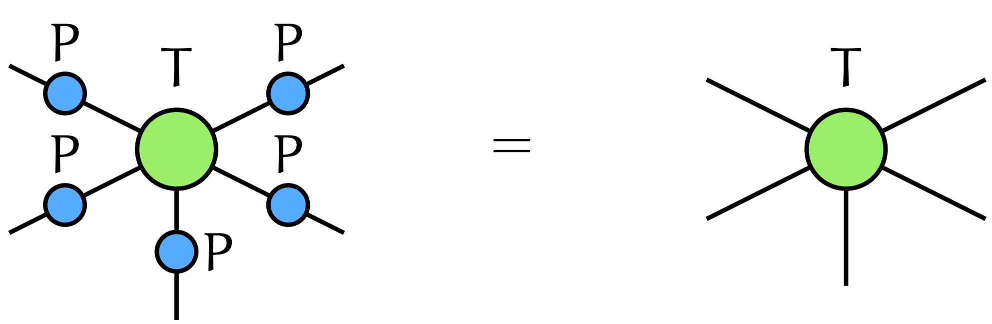
Spins

$$c_i c_j = + c_j c_i$$

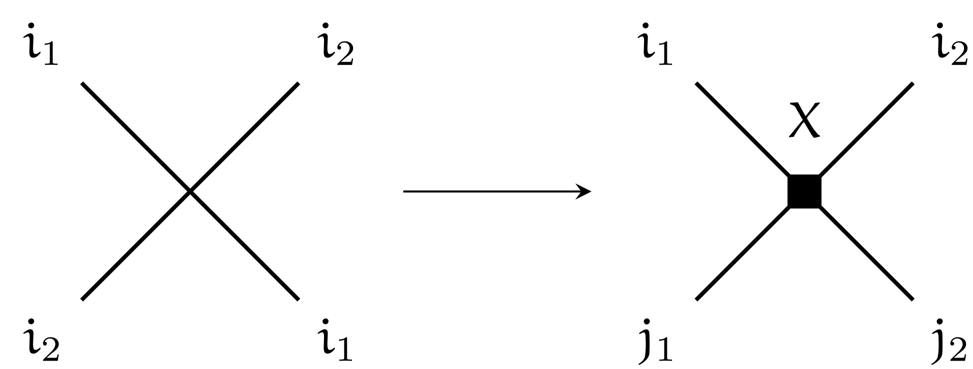
Fermions

$$c_i c_j = - c_j c_i$$

- Fermionization rules for tensor networks



$$T_{i_1,i_2,\ldots,i_M} = 0 \quad \text{if } P(i_1)P(i_2)\ldots P(i_M) \neq 1$$



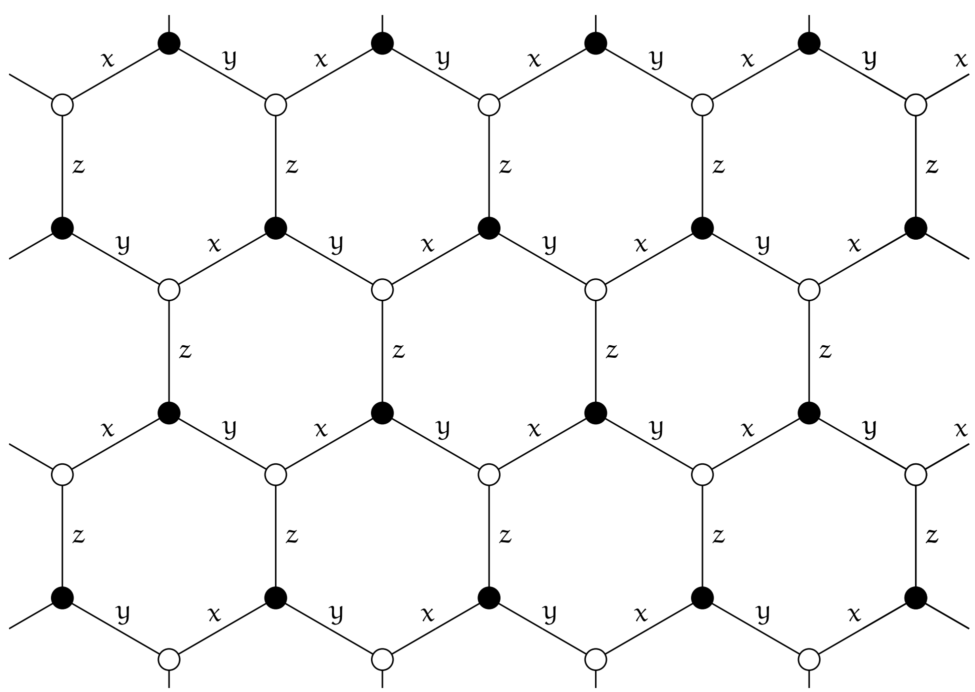
$$X_{i_1,i_2,j_1,j_2} = \delta_{i_1 j_2}\delta_{i_2 j_1} S\left(P(i_1)P(i_2)\right)$$

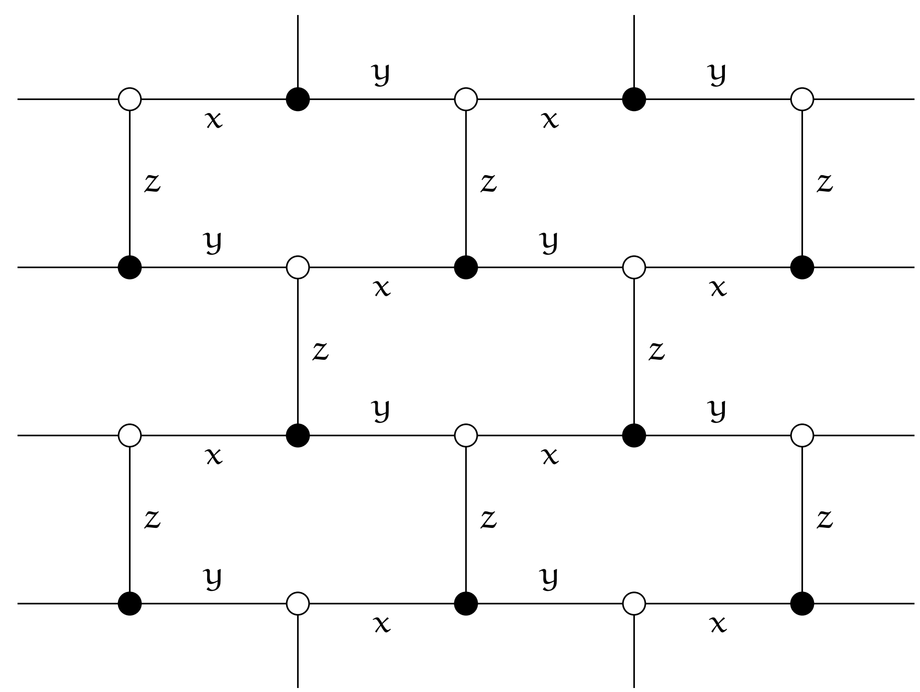
$$S\left(P(i_1)P(i_2)\right) = \begin{cases} -1 & \text{if } P(i_1) = P(i_2) = -1 \\ +1 & \text{otherwise} \end{cases}$$

Corboz, Orús, Bauer, Vidal, Phys. Rev. B 81, 165104 (2010)

THE KITAEV HONEYCOMB MODEL

- Model of spin-1/2 on the sites of a 2d honeycomb lattice
- Fully anisotropic coupling along x, y and z bonds





- Hamiltonian

$$H = -J_x \sum_{\text{x-links}} \sigma_i^x \sigma_j^x - J_y \sum_{\text{y-links}} \sigma_i^y \sigma_j^y - J_z \sum_{\text{z-links}} \sigma_i^z \sigma_j^z$$

Kitaev, Annals of Physics 321 (2006)

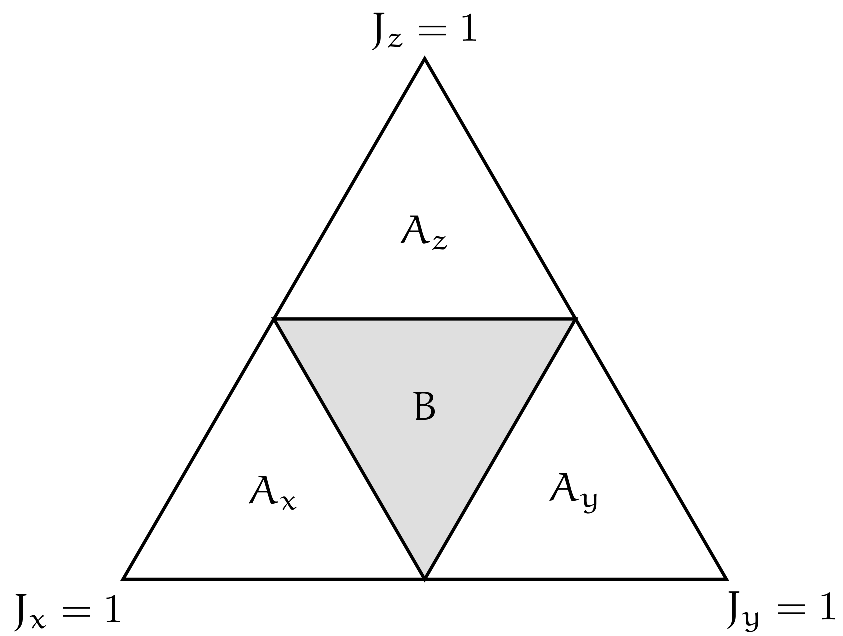
THE SOLUTION

- Mapping to a free fermion model in various steps
  - Jordan-Wigner transformation
  - Introduction of Majorana fermions
  - Recombination of Majorana fermions
  - Fourier transformation
  - Bogoliubov transformation

Feng, Zhang & Xiang, Phys. Rev. Lett. 98, 087204 (2007)
Chen & Nussinov, Journal of Physics A 41 075001 (2008)

PHASE DIAGRAM

- Phase diagram from dispersion relations

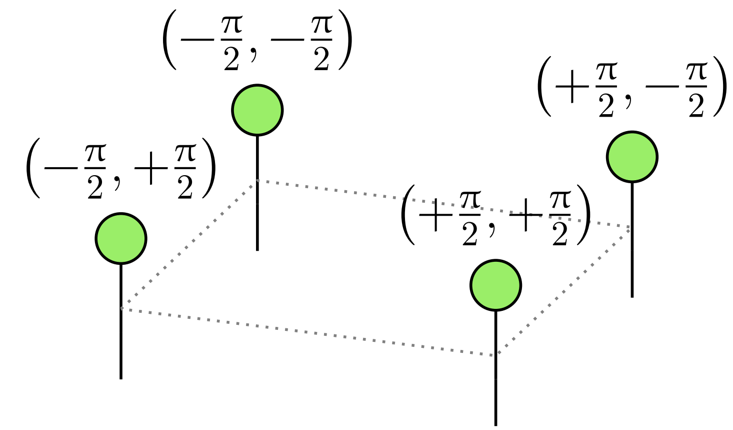


- Gapless B phase for

$$|J_x| \leqslant |J_y| + |J_z|$$
$$|J_y| \leqslant |J_z| + |J_x|$$
$$|J_z| \leqslant |J_x| + |J_y|$$

Kitaev, Annals of Physics 321 (2006)

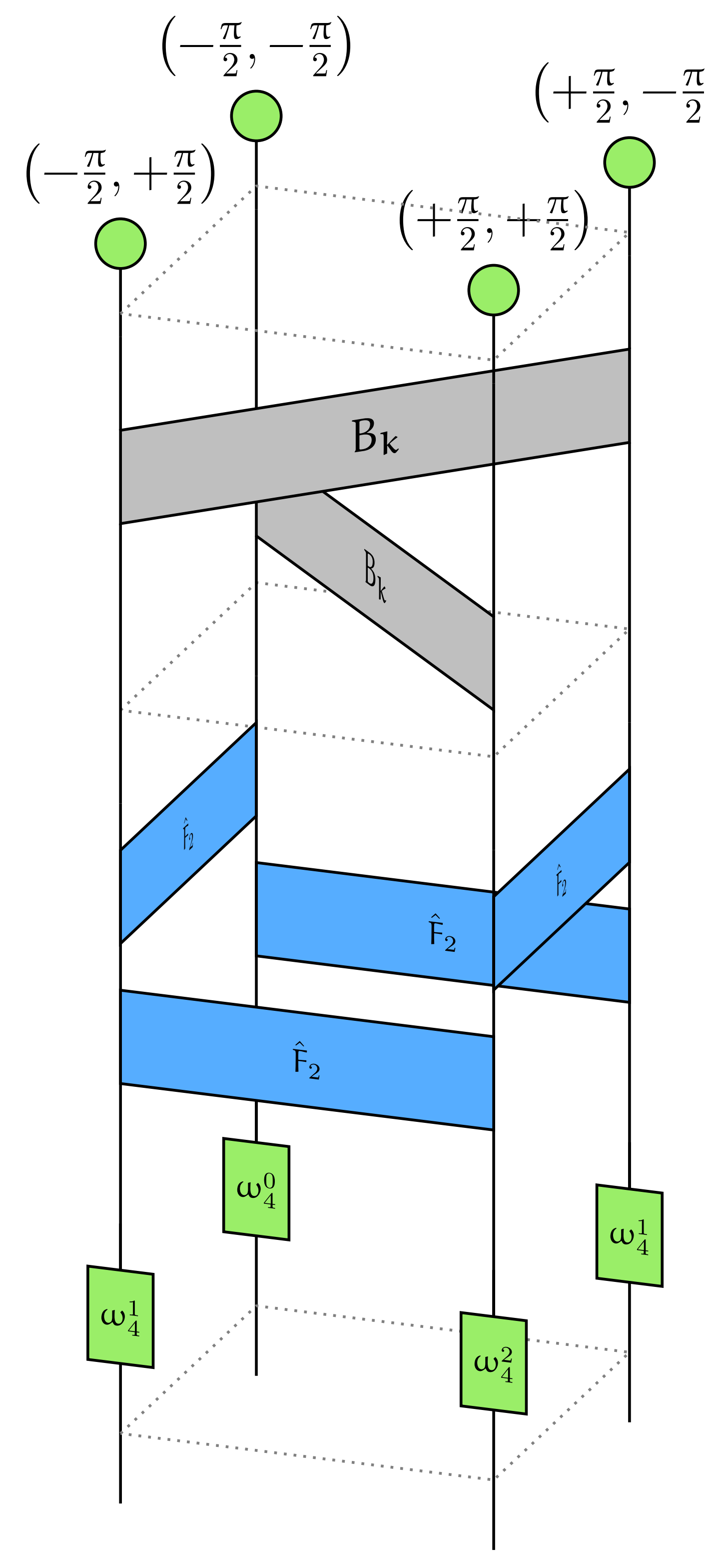
GENERAL PROCEDURE



- Start with eigenstates of diagonal Hamiltonian
- Ground state is $|\psi\rangle = |0\rangle^{\otimes N}$, Bogoliubov vacuum
- Procedure to build TN
  - Reverse steps in the solution of the model
  - Find respective tensor network representation
  - Transform quantum state step-by-step

INTERMEDIATE TENSOR NETWORK

- Bogoliubov + Fourier transformation
- Fourier transformation as 2d Spectral TN plus additional twiddle factors
- Eigenstates of translational invariant Hamiltonian in real space
- Model on the 2d square lattice



Ferris, PRL 113 010401 (2014)

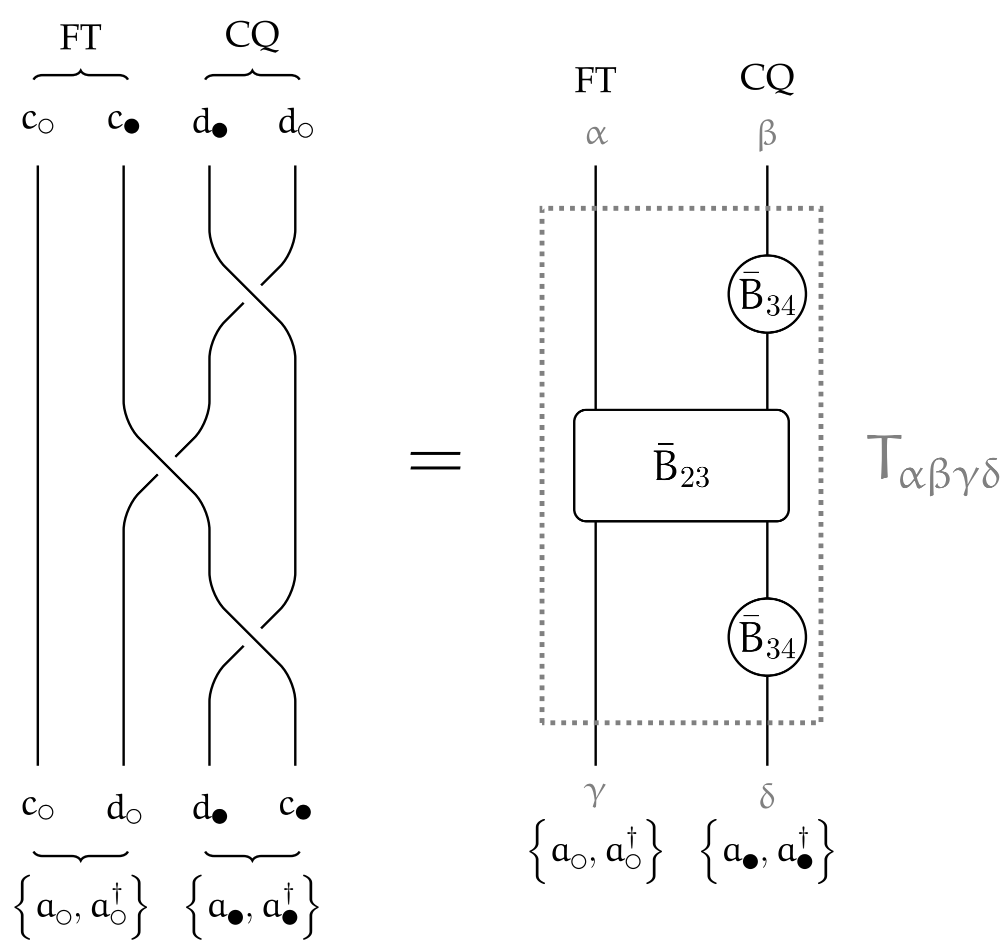
MAJORANA FERMIONS

- Reintroduce conserved quantities
- Recombine Majorana fermions from FT and CQ
  - Need to be treated as non-Abelian anyons
  - Exchange leads to braiding of anyons
- No occupation number for Majorana fermions
- How to treat them in the language of TNs?

MAJORANA FERMIONS

- Majorana transformation ⟶ action on the Fock space
- Define (anti-)clockwise swap operators to braid anyons

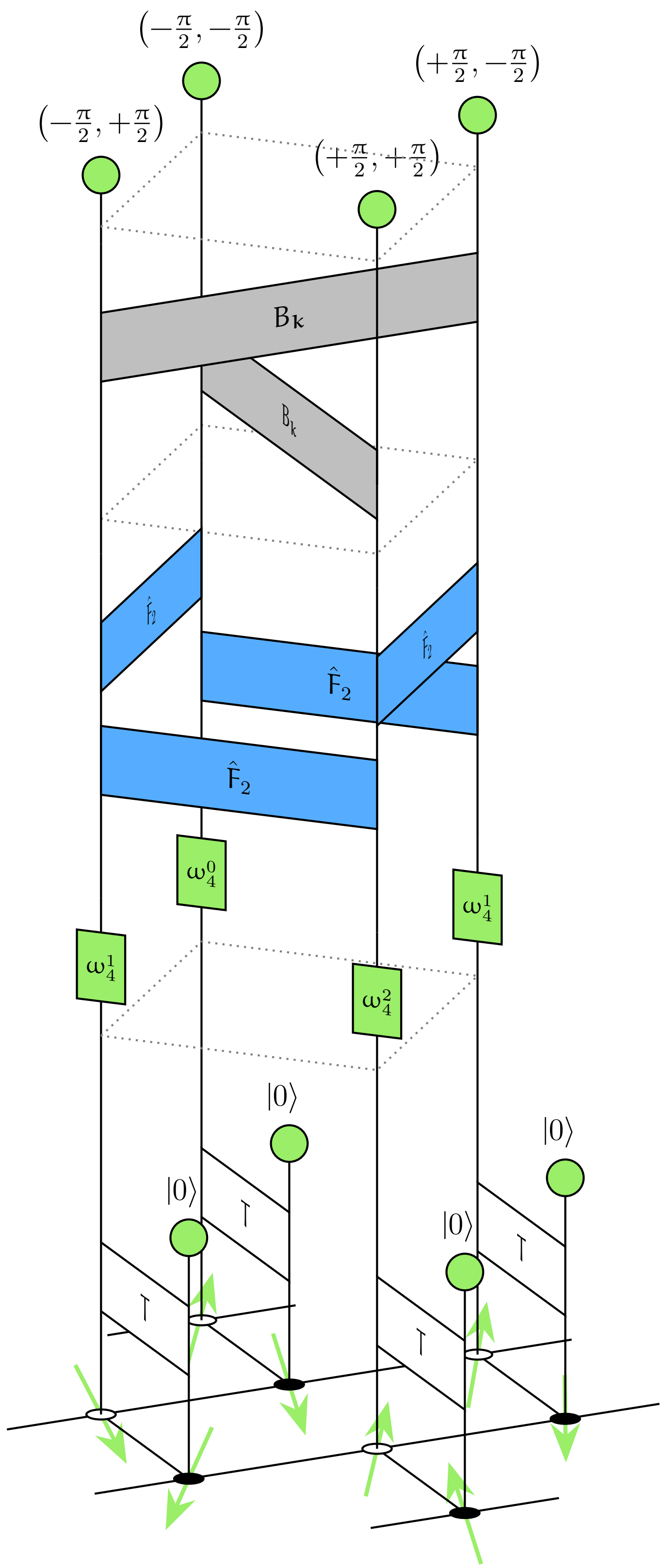
$$B_{ij} = 1/\sqrt{2}\,(1+\gamma_i\gamma_j) \qquad \bar{B}_{ij} = 1/\sqrt{2}\,(1-\gamma_i\gamma_j)$$



Leijnse & Flensberg, Semicond. Sci. Technol. 27, 124003 (2012)

OVERALL TENSOR NETWORK CONSTRUCTION

- Some remarks
  - Fermionic tensor network
  - Numerical checks up to 32 spins
  - Modification of boundary terms necessary
  - All excited states in the vortex-free sector possible
  - Reliable scaling to the thermodynamic limit



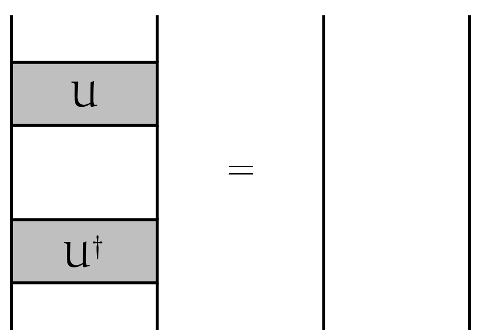
GROUND STATE FIDELITY

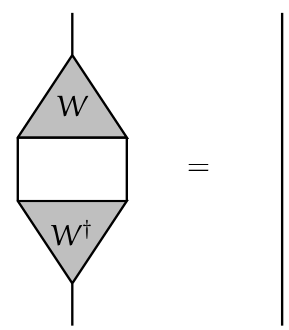
GROUND STATE FIDELITY

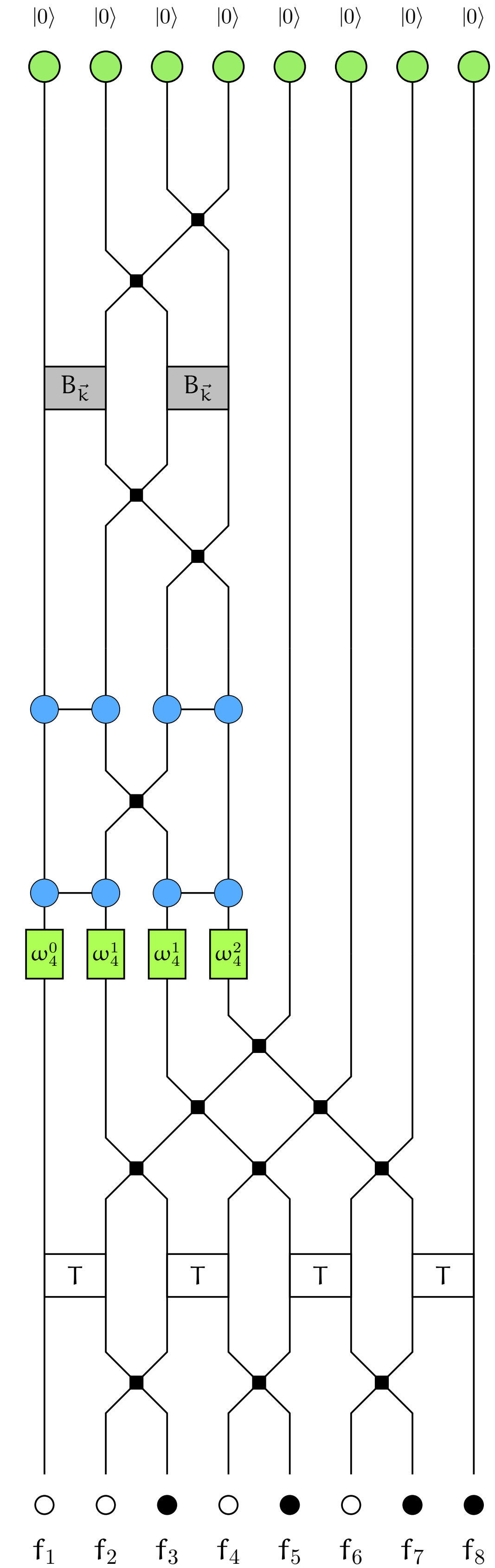
- Ground state fidelity defined as

$$F\left(\tilde{J}_z, J_z\right) = \left| \left\langle \psi\left(\tilde{J}_z\right) \middle| \psi\left(J_z\right) \right\rangle \right|^2$$

- Reduces to a simple form







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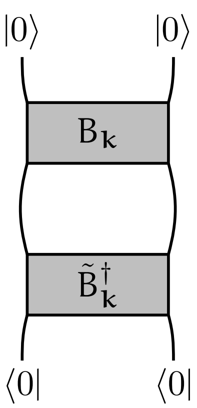
  $$F\left(\tilde{J}_z, J_z\right) = \left| \langle \psi\left(\tilde{J}_z\right) | \psi\left(J_z\right) \rangle \right|^2$$
- Reduces to a simple form

$$\langle \psi\left(\tilde{J}_z\right) | \psi\left(J_z\right) \rangle = \prod_{\mathbf{k}}$$



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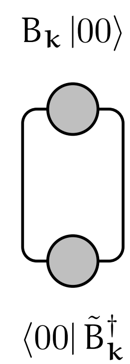
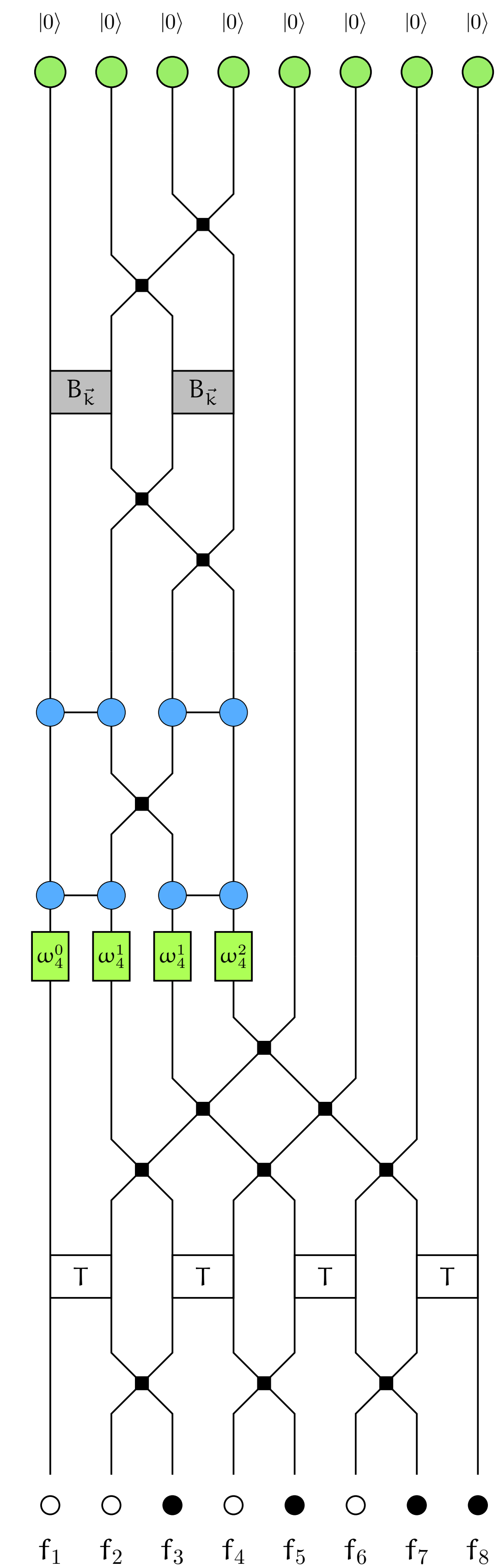
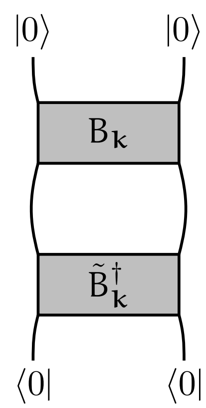




GROUND STATE FIDELITY

- Ground state fidelity defined as
  $$F\left(\tilde{J}_z, J_z\right) = |\langle\psi\left(\tilde{J}_z\right)|\psi\left(J_z\right)\rangle|^2$$
- Reduces to a simple form

$$\langle\psi\left(\tilde{J}_z\right)|\psi\left(J_z\right)\rangle = \prod_{\mathbf{k}}$$



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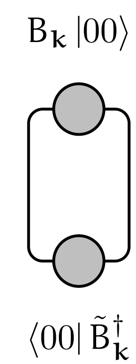
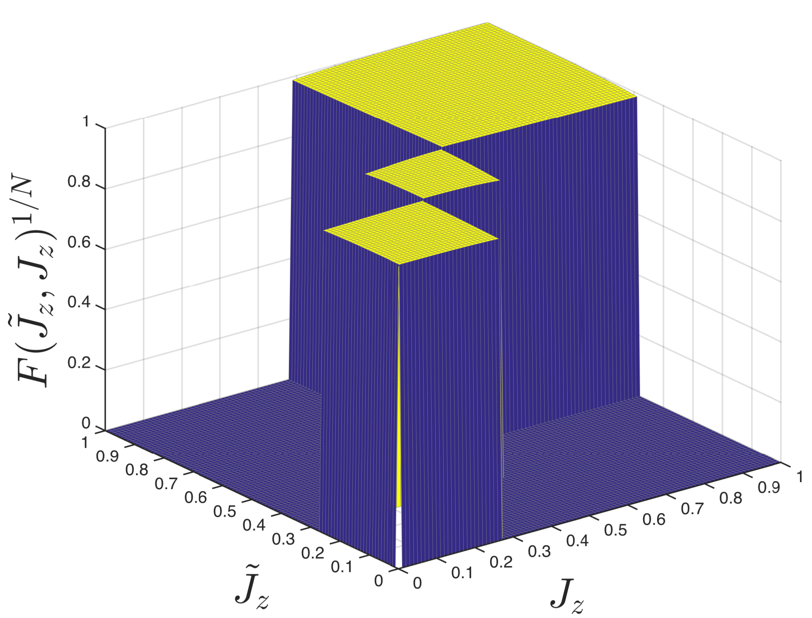
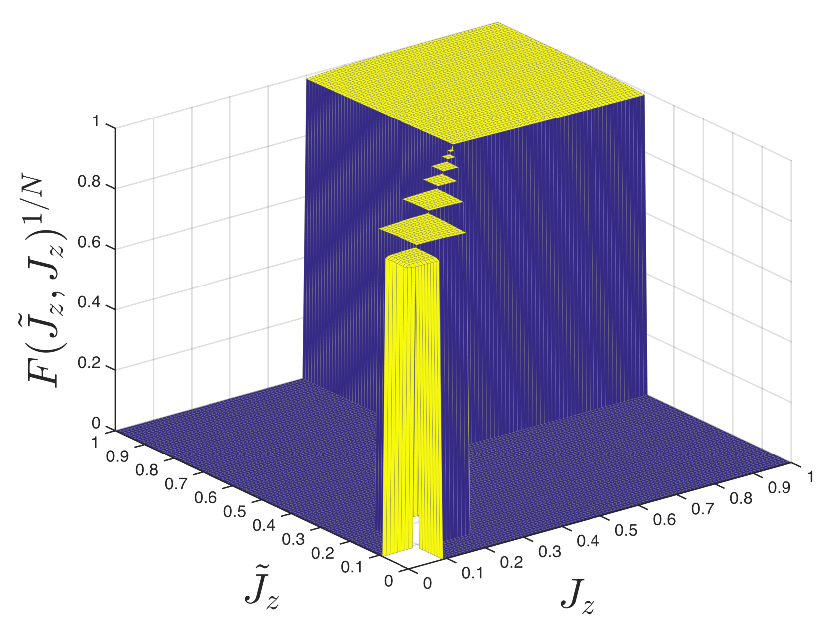


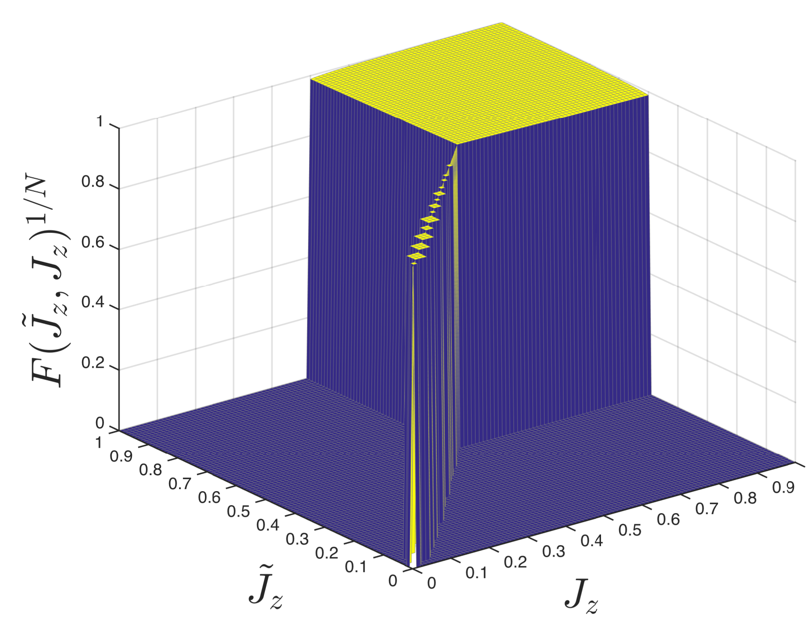
128 spins



2048 spins



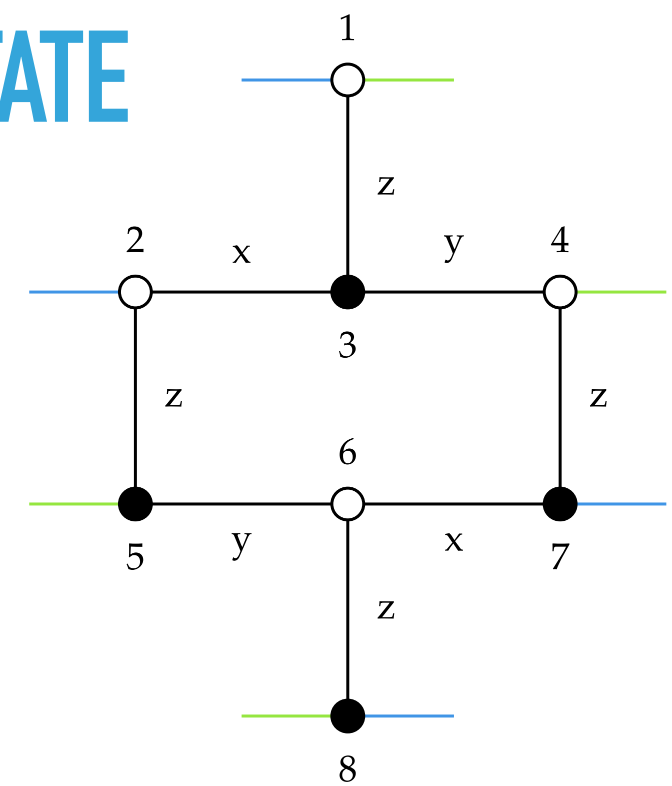
32768 spins



TWO-POINT CORRELATORS FOR THE GROUND STATE

G. Baskaran, S. Mandal, R. Shankar, Phys. Rev. Lett. 98, 247201 (2007)

TWO-POINT CORRELATORS FOR THE GROUND STATE



$$\left\langle \sigma_i^\alpha \sigma_j^\beta \right\rangle = \begin{cases} 0 & \text{if } i, j \text{ not nearest neighbours} \\ g_\alpha \cdot \delta_{\alpha\beta} & \text{if } i, j \text{ are nearest neighbours} \end{cases}$$

$g_\alpha \neq 0$ only for an α-type link

G. Baskaran, S. Mandal, R. Shankar, Phys. Rev. Lett. 98, 247201 (2007)

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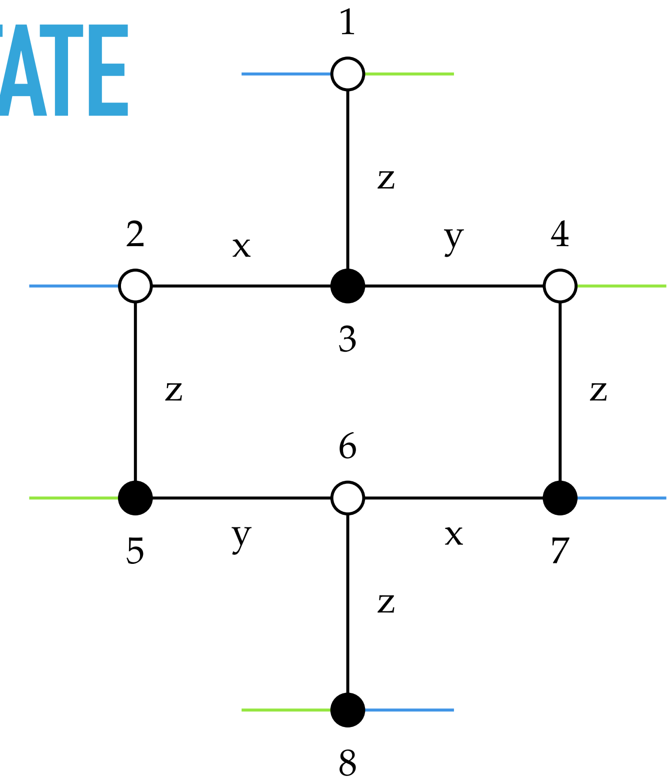

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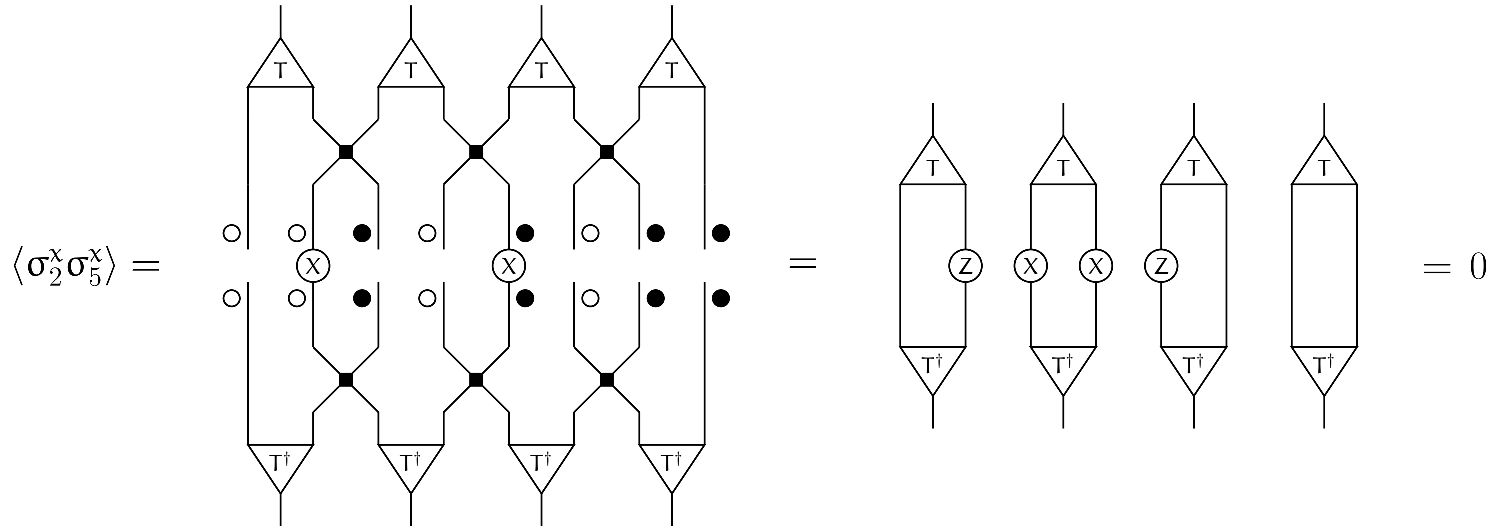

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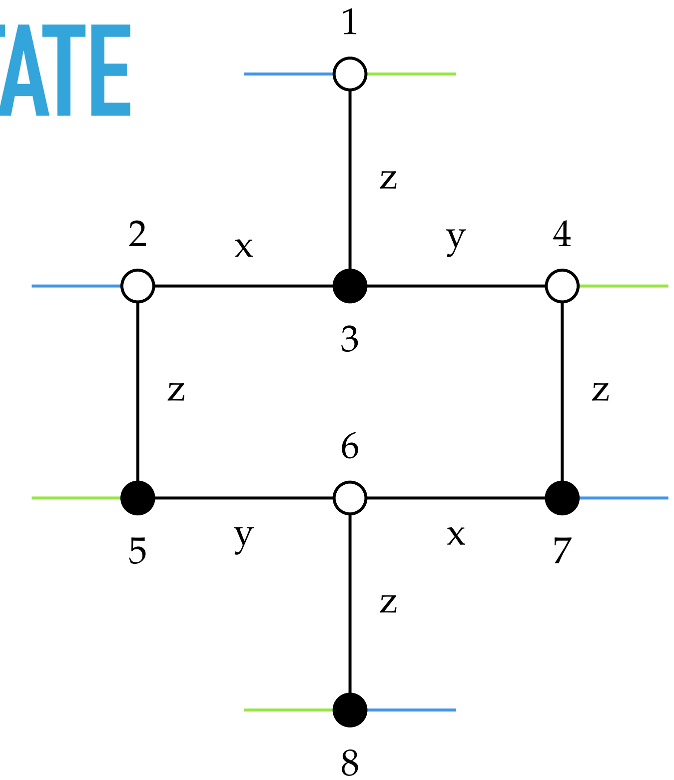

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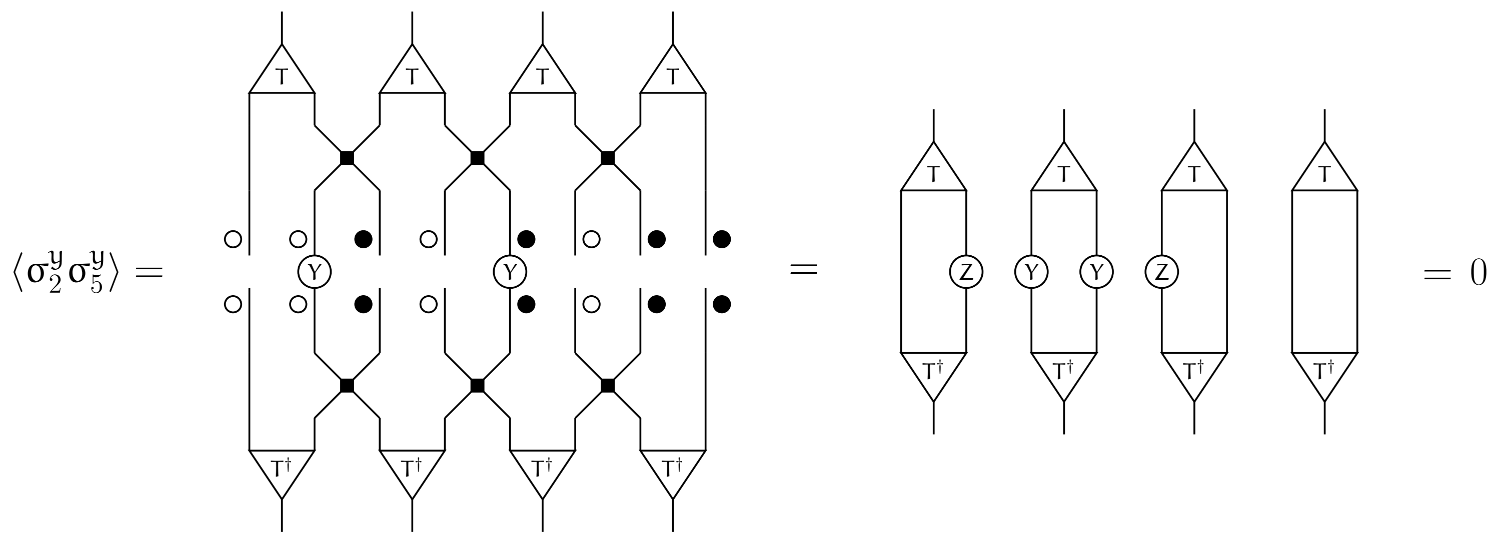

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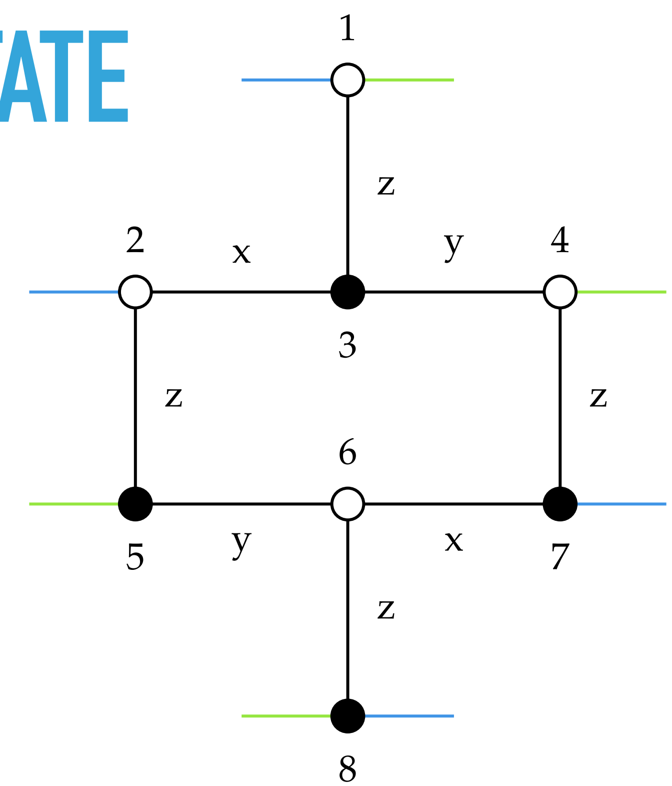

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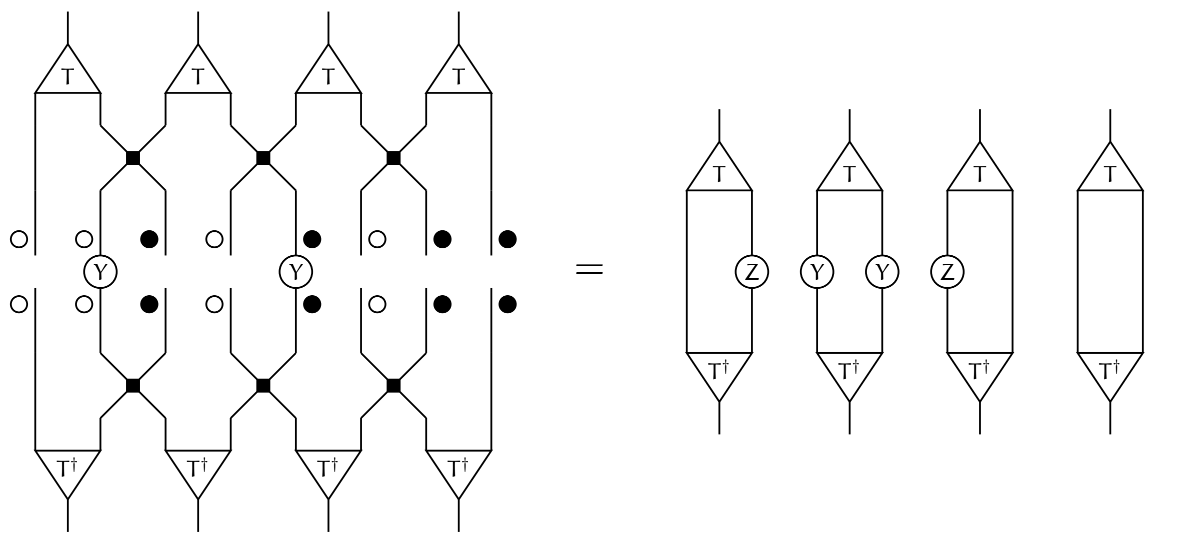
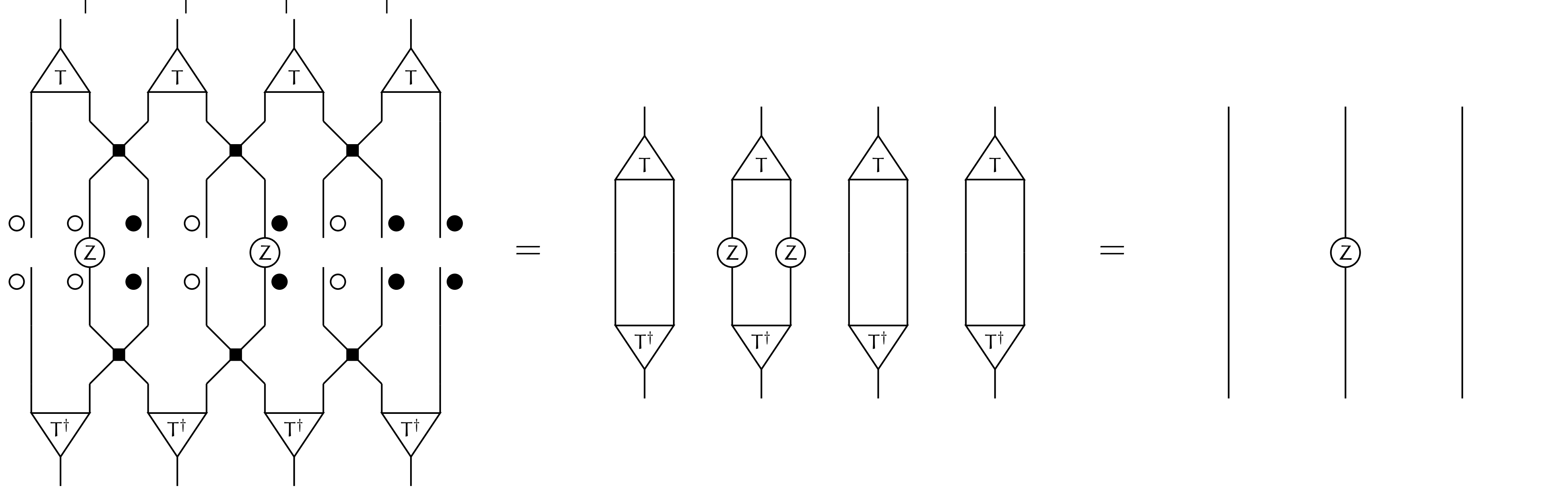
$\langle \sigma_2^y \sigma_5^y \rangle =$



$= 0$

$\langle \sigma_2^z \sigma_5^z \rangle =$



G. Baskaran, S. Mandal, R. Shankar, Phys. Rev. Lett. 98, 247201 (2007)

CONCLUSION

- Exact tensor network for the eigenstates in vortex-free sector
- Scaling to larger systems for all steps possible
- Structure reveals access to physical properties
- Tensor network as quantum circuit to generate many-body wave function
- TN should provide good initial wave function for numerical simulations
- Unclear if contractible to a PEPS with finite bond dimension

P. Schmoll, R. Orús, Phys. Rev. B 95, 045112

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