A Tunable Quantum Anomaly with Ultracold Atoms

Small and Medium Sized Cold Atom Systems – Centro de Ciencias Benasque, Spain

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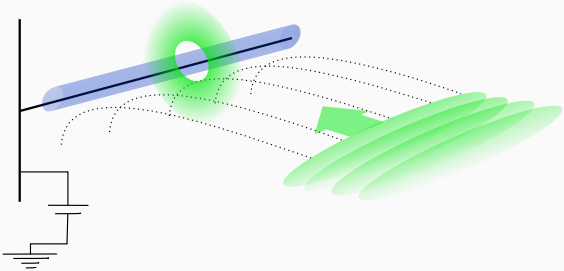

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Cold atoms & and a charged wire

Fall to the centre and the inverse square potential

Effective centrifugal barrier

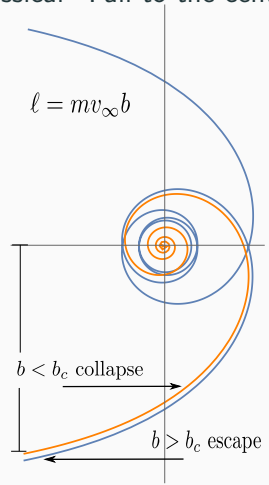
$$V_\ell(r) = \frac{\ell^2 - 2mg}{2mr^2} \quad (d = 2)$$

$$V_\ell(r) := \frac{\ell_{\text{eff}}^2}{2mr^2}$$

- Introduces effective angular momentum ℓ_{eff}
- If $\ell_{\text{eff}}^2 < 0$ then particle falls to the centre
- A critical impact parameter b_c separates scattering from absorption.

$$b_c = \frac{\ell_c}{\ldots} \longrightarrow b_c = \frac{\sqrt{2mg}}{\ldots}$$

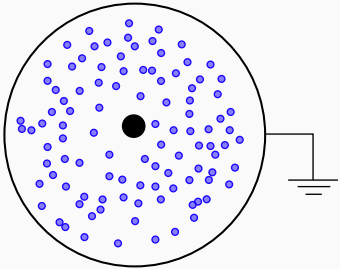
Classical "Fall to the centre"



Motivating experiment

Neutral atoms & charged wire

- Thin charged wire
- $\mathbf{E} = \frac{\lambda}{2\pi r}\,\hat{\mathbf{r}}$
- Electric field induces dipole moment $\mathbf{d} = \alpha \mathbf{E}$
- This induces an attractive inverse square potential.
- Atoms will be absorbed by the wire.



Denschlag, Umshaus, & Schmiedmayer
PRL 1998

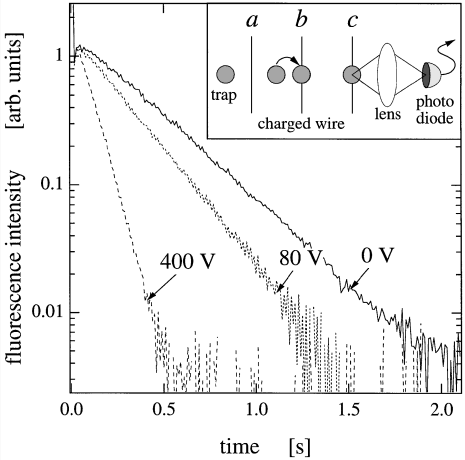
$$\boxed{V = -\frac{1}{2}\mathbf{d}\cdot\mathbf{E} = -\frac{\alpha\lambda^2}{2(2\pi)^2}\frac{1}{r^2} := -\frac{g}{r^2}}$$

Motivating experiment

Experimental protocol

- Tune voltage to induce charge on wire,
- Bring atoms into contact with the wire.
- Measure fluoresence.
- Find rate of absorption.
- Extract σ_{abs} from kinetic theory.

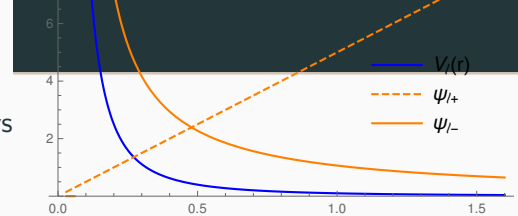
$$\sigma_{\text{abs}}^{(cl)} = \sqrt{\frac{8g}{mv^2}} \quad \text{(good agreement)}$$



Denschlag, Umshaus, & Schmiedmayer
PRL 1998

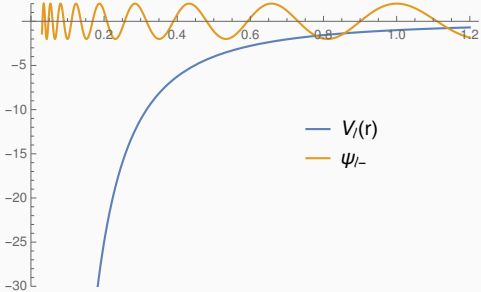
Qua

- $\ell_{\text{eff}}^2 > 0$
- $\psi_\pm \propto r^{\pm\ell_{\text{eff}}}$ at small r
- One solution grows & the other decays



Super-critical coupling

- $\ell_{\text{eff}}^2 < 0$
- $\psi_\pm \propto r^{\pm i|\ell_{\text{eff}}|} = \exp[\pm i|\ell_{\text{eff}}| \ln r]$ at small r
- Both solutions oscillatory



Quantum "fall to the centre"

- $\psi_\pm(t) \propto \exp[i(\pm|\ell_{\text{eff}}| \ln r - \omega t)]$
- Infalling and outgoing waves in logarithmic coordinates

Quick Recap

General features of $1/r^2$

- If $2mg > \ell^2$ a particle (or partial wave) "falls to the centre".
- Choosing $C_+/C_- = 0$ implies that all waves "fall in".

Laboratory realization

- Schmiedmayer & Denschlag saw only classical physics.
- The wire acts as a sink of atoms and was a "perfect absorber".

Quantum anomalies

What are they?

Quantum effects can break classical symmetries...

Particle physics

- QED breaks axial symmetry $\partial^\mu J^5_\mu \neq 0$
- This leads to rapid neutral pion decay $\pi^0 \to \gamma\gamma$

Condensed matter

- Dirac quasi-particles with Coulomb potential
- "Efimov tower" of bound states seen in graphene (Ovdat 2017)

Ultra-cold atoms

- Trapped gas in $d = 2$ has a hidden SO(2,1) symmetry (Pitaevskii and Rosch 1997)
- Anomaly for $d = 2$ (Olshanii 2010) recently observed! (Holten 2018)

Quantum mechanics

- $1/r^2$ potential is scale invariant.
- Short-distance "quantum fluctuations" can destroy scale invariance.

...when a regulator does not respect the classical symmetries

Quantum anomaly of the $1/r^2$ potential

Pre-Denschlag & Schmiedmayer

- Jackiw (1991) [on delta-functions not $1/r^2$]
- Gupta, Rajeev (1993)

Post Denschlag & Schmiedmayer

- Camblong, Epel, *et. al.* (2000)
- Camblong, Ordonez (2000)
- Coon, Holstein (2002)
- Goldberger, Wise (2002)
- Bawin, Coon (2003)
- Mueller, Ho (2004)
- Braaten, Philips (2004)
- Long, van Kolck (2008)
- Hammer, Higa (2009)
- Kaplan *et. al.* (2009)

Scale anomaly and the inverse square potential

The radial Schrödinger equation for $k^2 = 2mE$ in $d = 2$ is

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\ell^2 - 2mg}{r^2}\right]\psi_\ell = k^2\psi_\ell \quad \text{invariant under} \quad \mathbf{r} \to s\mathbf{r}$$

$$\psi_\ell = C_{\ell-}\psi_{\ell-} + C_{\ell+}\psi_{\ell+}$$

Typical Story ($d = 3$)

- $\psi_{\ell+} \propto r^\ell$ and $\psi_{\ell-} \propto r^{-(\ell+1)}$.
- Therefore $C_{\ell-} = 0$

Typical Story ($d = 2$)

- $\psi_{\ell+} \propto r^\ell$ and $\psi_{\ell-} \propto r^{-\ell}$
- $\psi_{\ell-}$ is singular as $r \to 0$ therefore

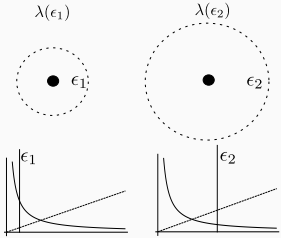
With inverse square potential

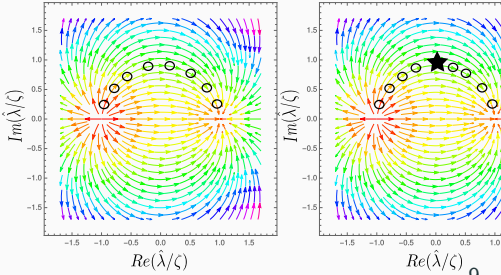
- $\ell \to \ell_{\text{eff}}$ and ℓ_{eff} can be imaginary.
- $\psi_{\pm\ell} \propto \exp[\mathrm{i}|\ell_{\text{eff}}| \ln \mathrm{r}]$
- What determines $C_{\ell-}/C_{\ell+}$?

Regularization, renormalization and RG flow

How is this related to regulators?

- "Zero-range" regulator (i.e. $\delta^{(2)}(r)|\psi|^2$)?.
- Delta-function provides boundary condition at $r = 0$.
- $\psi_{\ell\pm}$ ill-defined at origin!
- "Cut-off" (i.e. delta-shell) regulator $h(\epsilon)\delta(r-\epsilon)|\psi_\ell|^2$.
- Cut-off ϵ is arbitrary so physics cannot depend on ϵ
- This implies an RG-flow of $h(\epsilon)$.





RG-Flow

Discrete scale-invariance

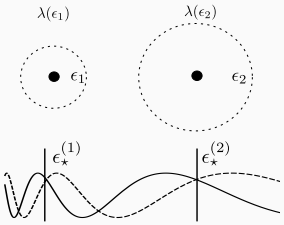
Fall to the centre

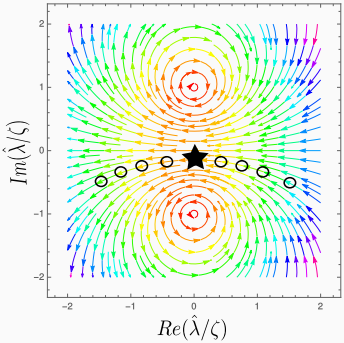
- Mode functions $\psi_{\pm\ell} \sim \exp[\pm|\ell_{\text{eff}}| \log r]$
- Oscillatory instead of monotonic.

RG-Flow

- Physically-equivalent families $\{h(\epsilon)\}$.
- Label these by "RG-invariants" $(\epsilon_\star, y_\star)$.
- Periodic flow implies an infinite tower of $\{\epsilon_\star^{(n)}\}$.

RG limit cycle $\implies$ discrete scale invariance





Quick Recap

Quantum anomalies

- The inverse square potential has a quantum anomaly.
- For "fall to the centre" a continuous scaling symmetry is broken to a discrete one.

RG-Flow

- Regulator leads to RG-flow and introduces. RG-invariant scales
- Scale invariance is restored if you live at a fixed point.

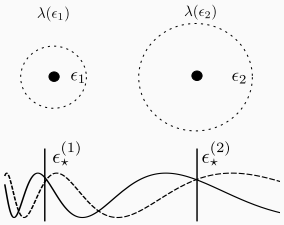
Observable consequences

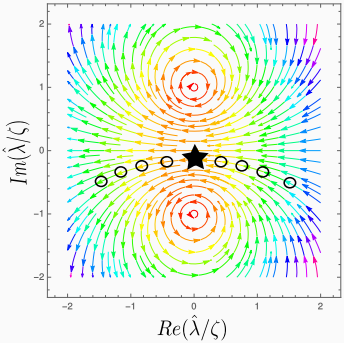
Discrete scale-invariance

From RG flows to the lab

- Before we discussed RG-flow w.r.t. ϵ.
- We really care about ratios of scales.
- E.g. $k\epsilon$ for scattering.
- Tuning k with ϵ fixed is the same as tuning ϵ with k fixed.
- Therefore RG-flow behaviour is inherited by laboratory observables

RG limit cycle $\implies$ discrete scale invariance

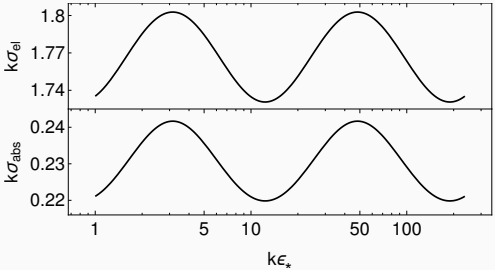




Scattering observables

Absorptive cross section

- Vanishes for $y_\star = 0$ (Hermitian source).
- Scale invariant for $y_\star = -1$ (perfect absorber).



$$\sigma_{\text{abs}}^{(\ell)} = \frac{1}{k}\left[\frac{2(R_\star^2 - 1)\sinh(\pi|\ell|_{\text{eff}})}{\mathrm{e}^{-\pi|\ell_{\text{eff}}|} + 2R_\star \cos \mathfrak{a}_\ell + \mathrm{e}^{\pi|\ell_{\text{eff}}|} R_\star^2}\right]$$

$$\boxed{\mathfrak{a}_\ell = \tfrac{1}{2}|\ell_{\text{eff}}| \ln(k\epsilon_\star) + \Delta\phi}$$

$$R_\star = \frac{1 - y_\star}{1 + y_\star} \quad y_\star = -0.0625 \text{ and } |\ell_{\text{eff}}| = 1.2$$

Elastic cross section displays the same behaviour, *but does not vanish for* $y_\star \to 0$ *(i.e. unitary physics at* $r = 0$*).*

Quick Recap

Signatures of the quantum anomaly

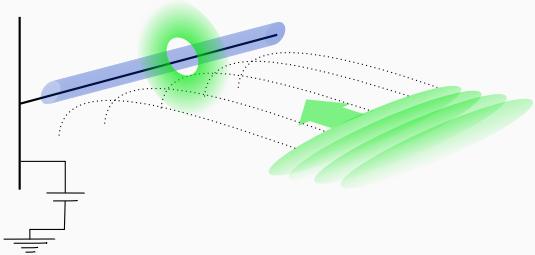
- Discrete scaling symmetry of cross-section under $k \to e^{-\pi/|\ell_{\text{eff}}|}$. **Discrete scaling symmetry of cross-section under $k \to e^{-\pi/|\ell_{\text{eff}}|}$.**
- Need $k_{\text{max}}/k_{\text{min}} > \exp[\pi/|\ell_{\text{eff}}|]$ to observe discrete scale invariance.

Cross section signatures

- If $|\ell_{\text{eff}}|$ is small need many decades of k to tune through.
- If $|\ell_{\text{eff}}|$ is large quantum effects are mostly lost
- No quantum effects if $y_\star = -2\ell_{\text{eff}}$ i.e. if the system is a perfect absorber.

Experimental proposal

Conceptually simple scattering experiment



Necessary ingredients

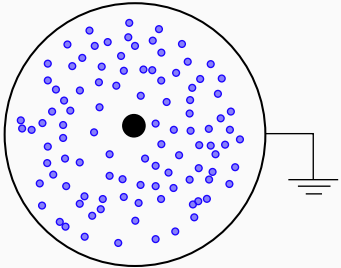
1. Inverse square potential.
2. Ability to prevent "perfect absorber".
3. High precision scale resolution.

Inverse square potential

Earlier we considered $|\ell_{\text{eff}}| \sim \mathcal{O}(1)$. In dimensionful units this is $2mg \sim \mathcal{O}\left(\hbar^2\right)$.

Is this achievable with a charged wire?

$$V_w^{(\hbar)} \approx \left(0.25 \text{ Volts}\right) \ln\left(\frac{R_c}{R_w}\right)\sqrt{\left(\frac{a_0^3}{\alpha}\right)\left(\frac{m_{\text{Li}}}{m}\right)}$$



Answer: Yes!

Inhibiting absorption

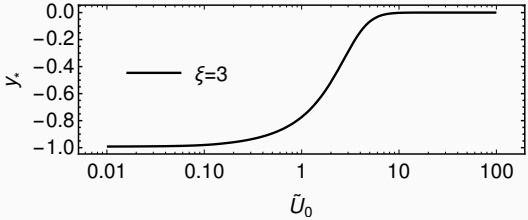
What is needed to limit absorption at the origin?

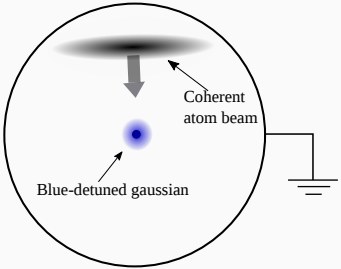
Semi-classical argument

A particle will "fall to the centre" and gain kinetic energy g/r^2. We take $mg \sim \hbar$

$$\left\langle \frac{p^2}{2m} \right\rangle \approx k_B \times (20\ \mathrm{nK}) \times \left(\frac{m_{\mathrm{Li}}}{m} \right) \qquad V(r) = \tilde{U}_0 \times \left(\frac{(\hbar/[1\ \mu\mathrm{m}])^2}{2m_{\mathrm{Li}}} \right) \exp\left[-\tfrac{1}{2} r^2/(1\ \mu\mathrm{m})^2\right]$$

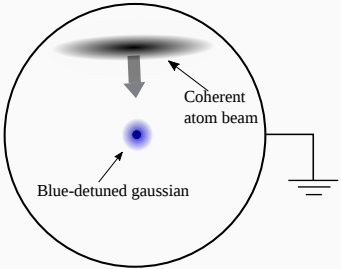
Can be achieved with $200 - 400$ nm laser with 10^{-2} mW

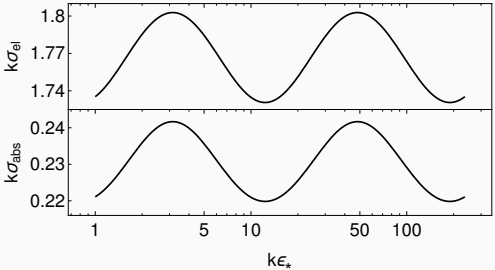




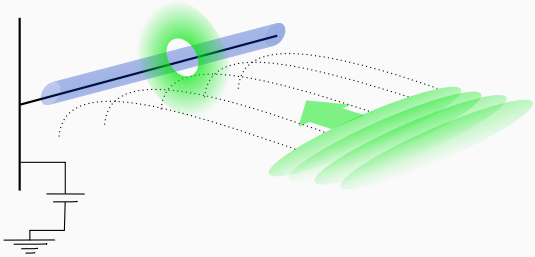
High precision scale resolution

- For a "zero-range" treatment of the wire we need $k < 1/(R_w) \approx 1/(1\ \mu\text{m})$.
- Momentum spread of wave-packet will limit the lowest values of k achievable.
- Delta-kicked cooling can reach $\Delta k \sim 1/(40\ \mu\text{m})$ (arXiv:1407.6995)
- Recall that discrete scaling symmetry $k \to \mathrm{e}^{\pi/|\ell_{\text{eff}}|}k$.
- Need $\Delta k R_w \approx 40 < \mathrm{e}^{-\pi/|\ell_{\text{eff}}|}$.





Alternative methods



Inverse square potential

1. "Painted" inverse square potential with time averaged laser intensity.
2. Beam of electrons.
3. Charged nano-fiber

Tunable atom-wire interactions

1. Evenescent wave around nano-fiber.
2. Donut mode laser.
3. Rapid laser transport.

Quick Recap

Laboratory parameters (orders of magnitude)

- $2mg \sim \mathcal{O}(\hbar^2)$ corresponds to voltage on wire of $V_w \sim$ Volts.
- Absorption can be inhibited with a weak (10^{-2} mW) laser ($V_{\text{barrier}} \sim 20$ nK).
- $R_w \sim 1$ μm and $\Delta k \gtrsim 1/(40\ \mu\text{m})$.

Observing the anomaly

- Large $|\ell_{\text{eff}}|$ suppresses the amplitude of variations in $\sigma(k)$.
- Small $|\ell_{\text{eff}}|$ requires large range of k to see anomaly.
- Effective temperature of atom beam and size of wire set the natural range of k.

Conclusions

Take home messages

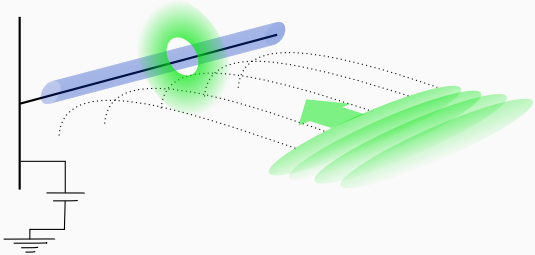
Quantum anomaly

- $1/r^2$ potential exhibits quantum anomaly with single atom physics.
- Signature includes discrete invariance in scattering cross sections.

Observing the anomaly

- Some ambitious (but demonstrated) experimental techniques are required.
- $\Delta k R_w \gg 1$ is necessary to observe anomaly.

Thank you for listening!



Extra points

- More details at arXiv:1804.10324.
- Other observables (e.g. bound states).
- Feedback welcome.
- plestird@mcmaster.ca.

Acknowledgements

- Mavin Holten.
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