Andreev-reflection and Aharonov-Bohm dynamics in atomtronic circuits
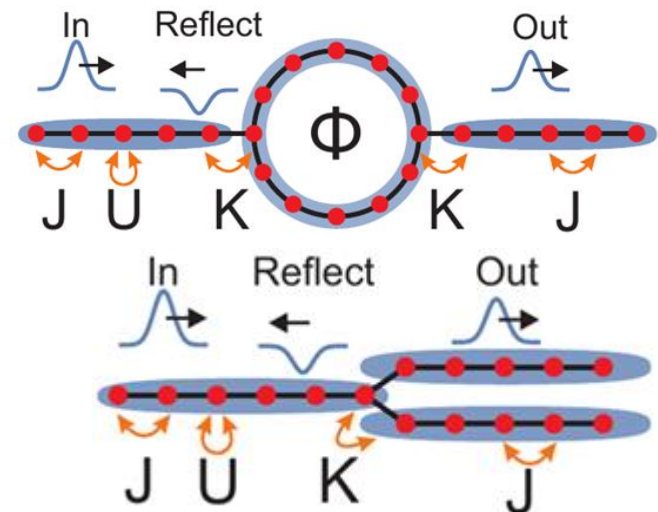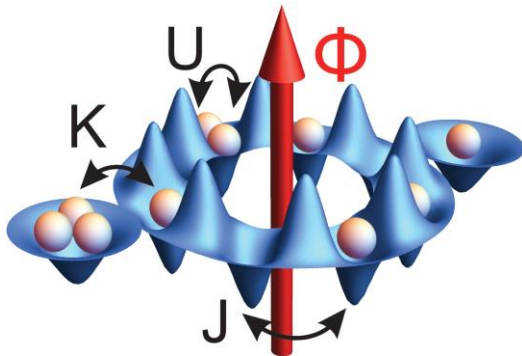
arXiv:1807.03616

arXiv:1706.05180

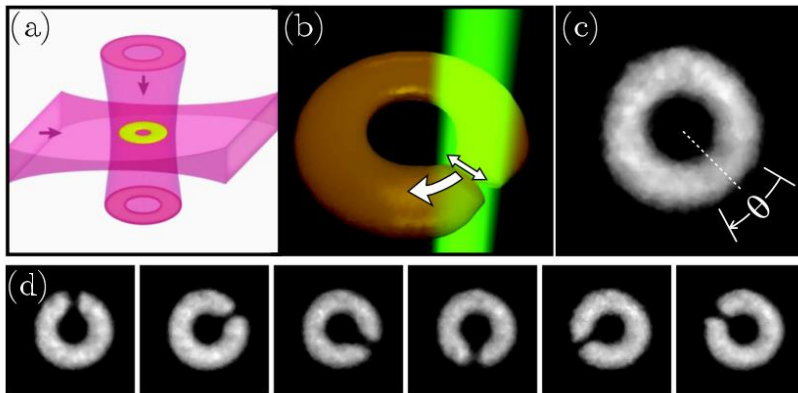Tobias Haug, Rainer Dumke, Leong-Chuan Kwek, Luigi Amico
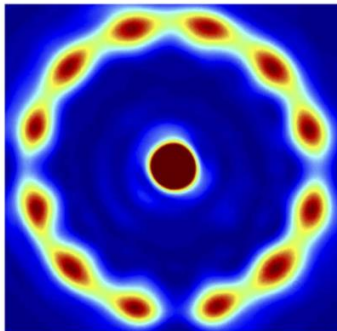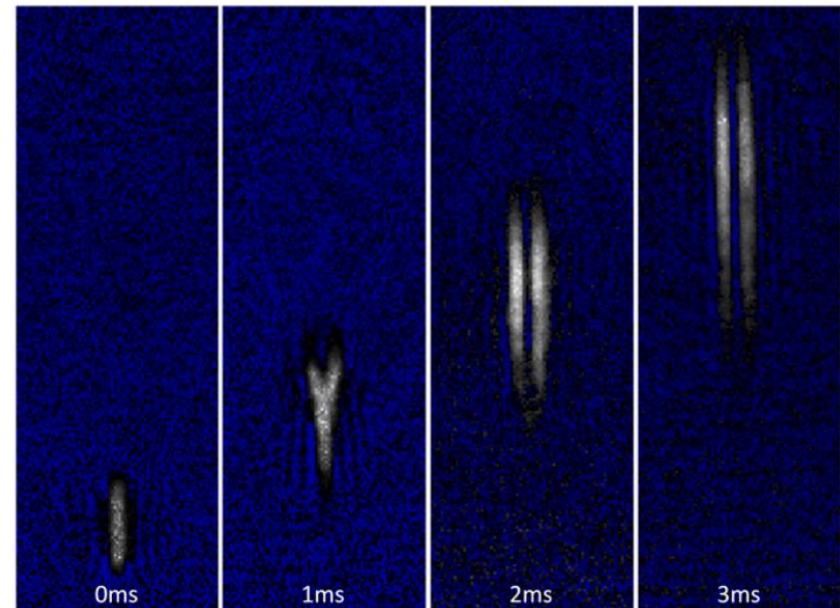
02.08.2018

Cold atom ring and Y-junction

Create rotation/artificial magnetic field in atomic ring

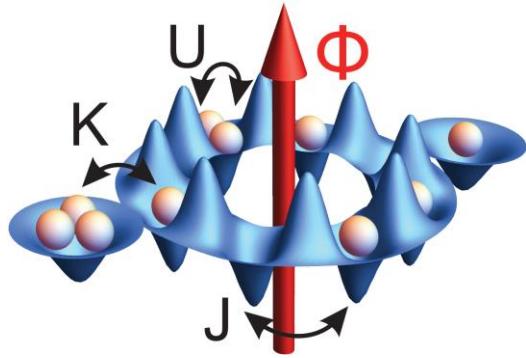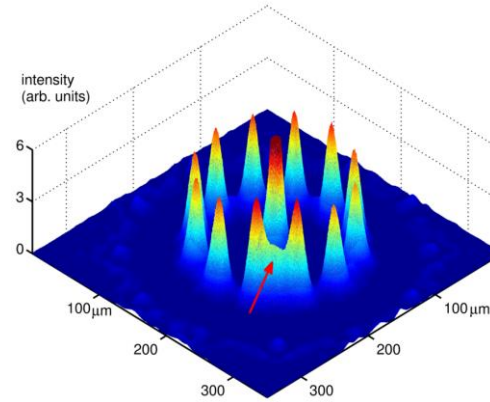G. Campbell, W. Phillips, C. Clark and co-workers@NIST, (2014–2015)

Amico, Aghamalyan, Aukstol, Crepatz, Kwek, Dumke SREP 2014

Control BEC in a Y-junction

C. Ryu and M. G. Boshier, New Journal of Physics 17, 092002 (2015).
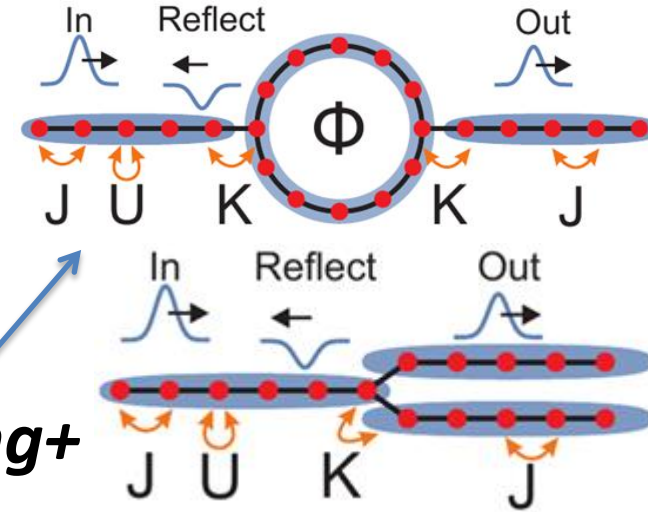
Centre for Quantum Technologies

Aharonov-Bohm effect

TH, H. Heimonen, R. Dumke, L.-C. Kwek, L. Amico
arXiv:1706.05180

Cold atoms + potential shaping + control currents
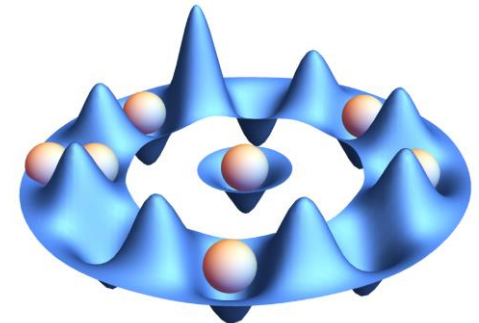
Transport

TH, R. Dumke, L.-C. Kwek, L. Amico
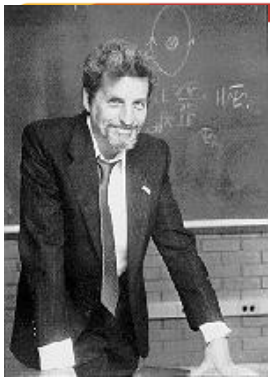arXiv:1807.03616

Quantum phases
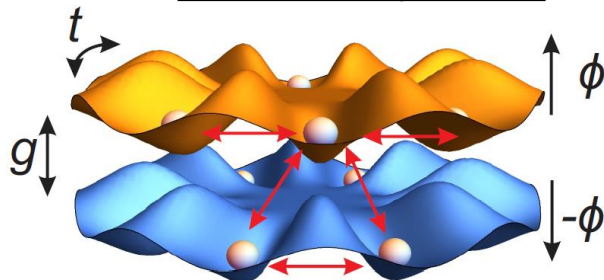
TH, L. Amico, R. Dumke, L.-C. Kwek
Quantum Sci. Technol. (2018)
arXiv:1612.09109

AQUID read-out

TH, J. Tan, M. Theng, R. Dumke, L.C. Kwek, L. Amico
Phys. Rev. A 97 (2018)
arXiv: 1707.09184

L. Amico, D. Aghamalyan, F. Auksztol, H. Crepaz, R. Dumke, L.C. Kwek *Scientific reports 4* (2014)

Aharonov-Bohm effect
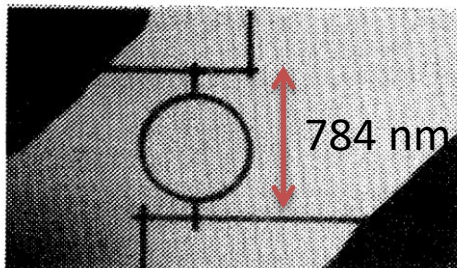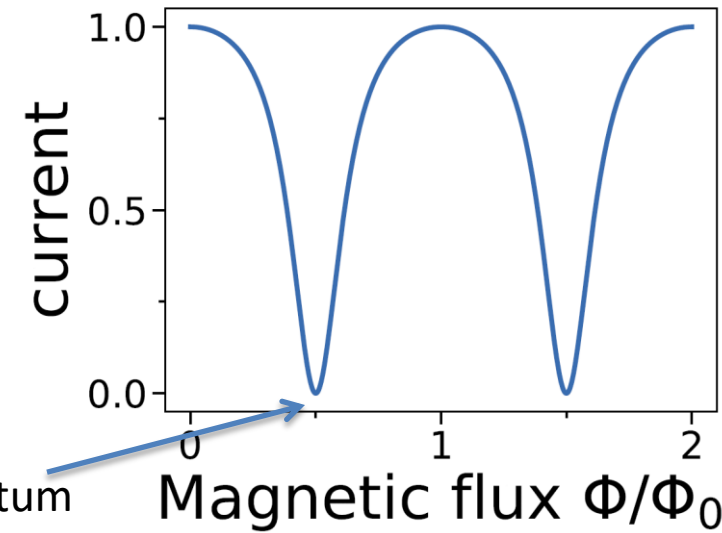
- Charged particle enclosing a region with magnetic field

$$\Delta\phi = \frac{e}{\hbar}\oint_C \mathbf{A}(\mathbf{r})d\mathbf{r} \propto \Phi$$

- Phase shift by magnetic field changes interference pattern/current

- Minimal current at

  $\Phi/\Phi_0 = 1/2$ (destructive interference)

½ flux quantum

current

Magnetic flux Φ/Φ_0

784 nm

Washburn Web 1986
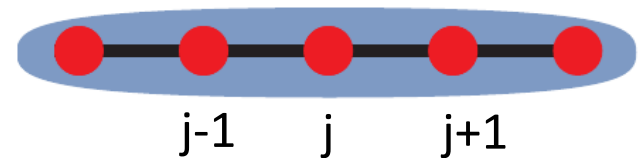
Many-body symmetry

- Two types of particles
- Fermion anti-symmetric
  - Exchange gives minus sign
  - Pauli principle
- Bosons symmetric
  - Exchange gives plus sign
  - Can occupy same space
- Bosons with infinite repulsion: Pauli principle, but still symmetric
- Jordan-Wigner to map fermion to hard-core boson
- Mapping not exact beyond 1D →break 1D a little?

$$|\Psi(r_a, r_b)\rangle = \pm |\Psi(r_b, r_a)\rangle$$

$$a_j^\dagger = e^{\left(+i\pi \sum_{k=1}^{j-1} f_k^\dagger f_k\right)} \cdot f_j^\dagger$$

$$a_j = e^{\left(-i\pi \sum_{k=1}^{j-1} f_k^\dagger f_k\right)} \cdot f_j$$
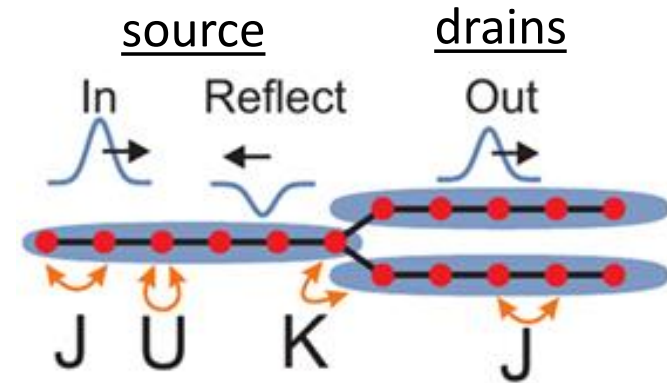
j-1   j   j+1

Models

- Bose-Hubbard model (atoms on lattice with tunneling)
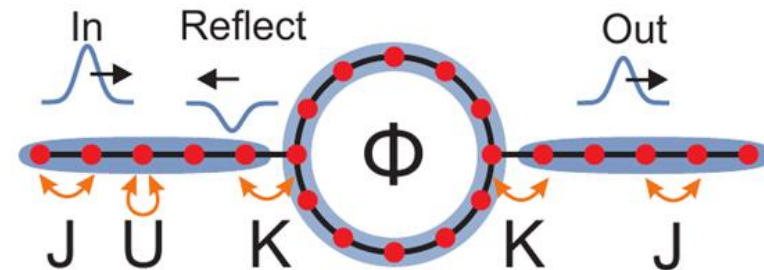- Simulate with DMRG and TEBD

Y junction: three 1D chains coupled with K

$$\mathcal{H}_S = -\sum_{j=1}^{L_S-1}\left(J\hat{s}_j^\dagger\hat{s}_{j+1} + \text{H.C.}\right) + \sum_{j=1}^{L_S}\frac{U}{2}\hat{n}_j^s(\hat{n}_j^s - 1)$$

$$\mathcal{H}_I = -K\hat{s}_1^\dagger\left(\hat{d}_1 + \hat{f}_1\right) + \text{H.C.} \quad \text{(junction coupling)}$$

source   drains

Ring with leads and artificial magnetic field Φ

$$\mathcal{H}_R = -\sum_{j=1}^{L_R}\left(Je^{i2\pi\Phi/L}\hat{a}_j^\dagger\hat{a}_{j+1} + \text{H.C.}\right) + \frac{U}{2}\sum_{j=1}^{L_R}\hat{n}_j^a(\hat{n}_j^a - 1)$$
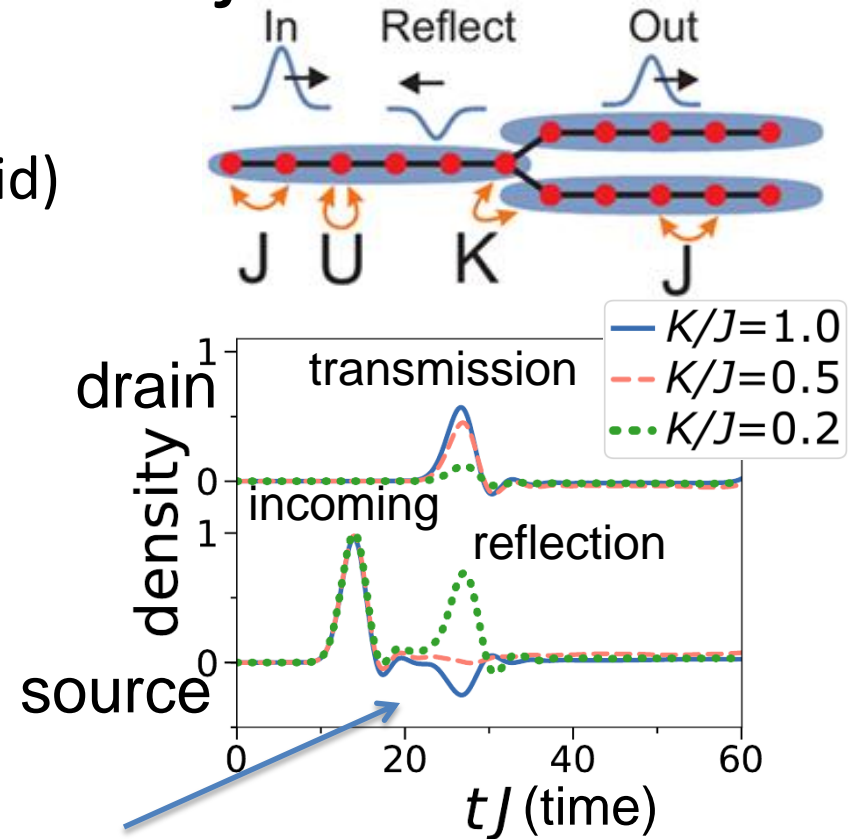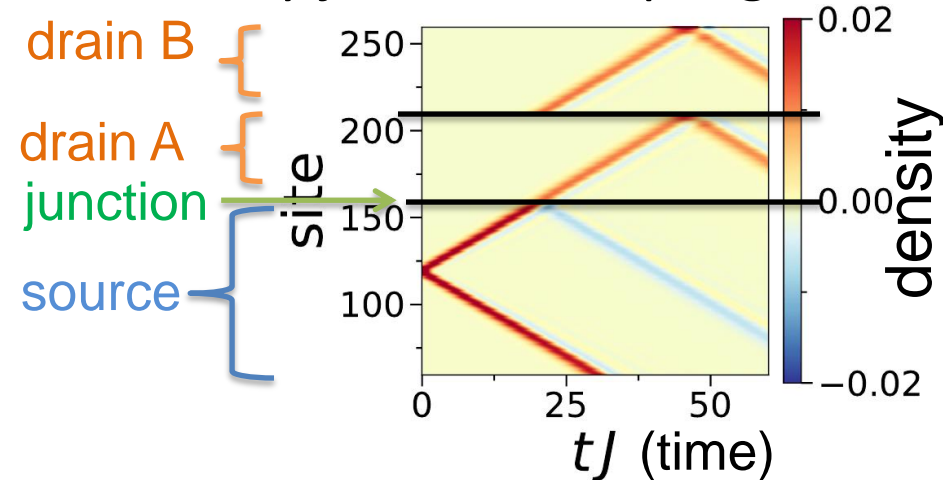

- Add localized potential offset in source lead, calculate ground state, then switch off →density wave

$$\mathcal{H}_P = -\epsilon_D\sum_{j=1}^{L_S}\exp\left(-\frac{(j-j_0)^2}{2\sigma^2}\right)\hat{n}_j$$

Hard-core boson: Y junction

- On-site interaction U→ ∞
- 260 sites, half-filling (superfluid)
- Vary junction coupling K/J

drain B

drain A

junction

source

Three regimes (J=1)

- $K≈1$: negative reflection (similar to Andreev reflection)
  (transmission 4/3, reflection -1/3)
- $K≈0.5$: no/positive-negative reflection
- $K≈0.2$: positive reflection

Also at bosonic interfaces: I. Zapata and F. Sols, Phys. Rev. Lett. 102, 180405 (2009)

Fermion vs hard-core boson

- Spinless fermions **do not have Andreev-reflection peak** for strongly coupled junction

- Jordan-Wigner cannot be applied on Y-junction
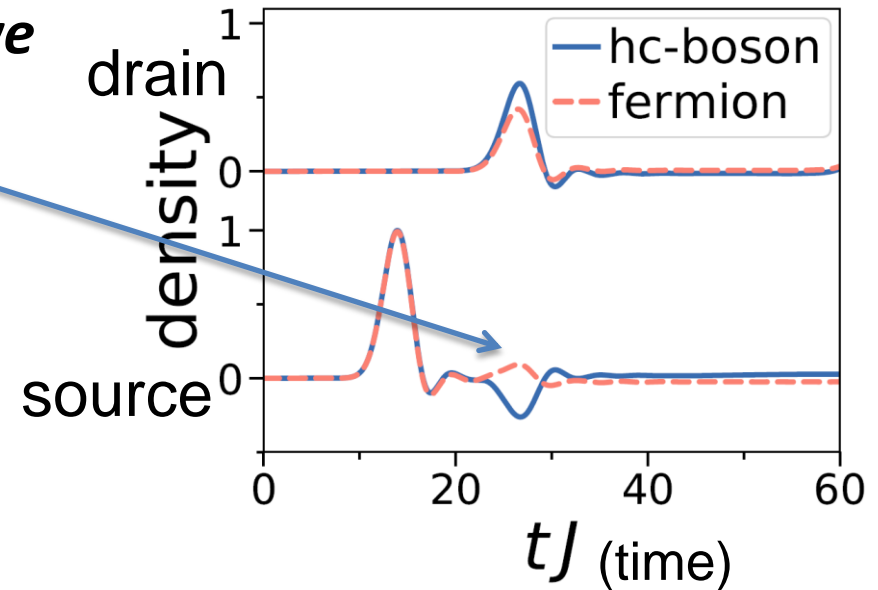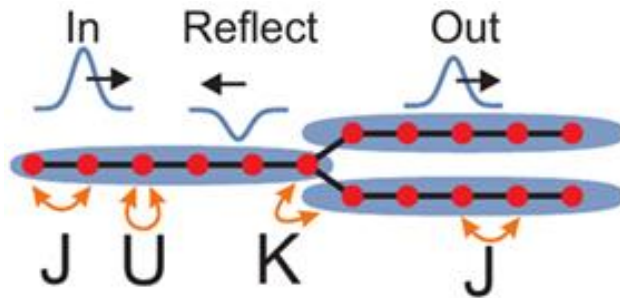
Gross-Pitaevskii

- Limit of weak interaction and many atoms
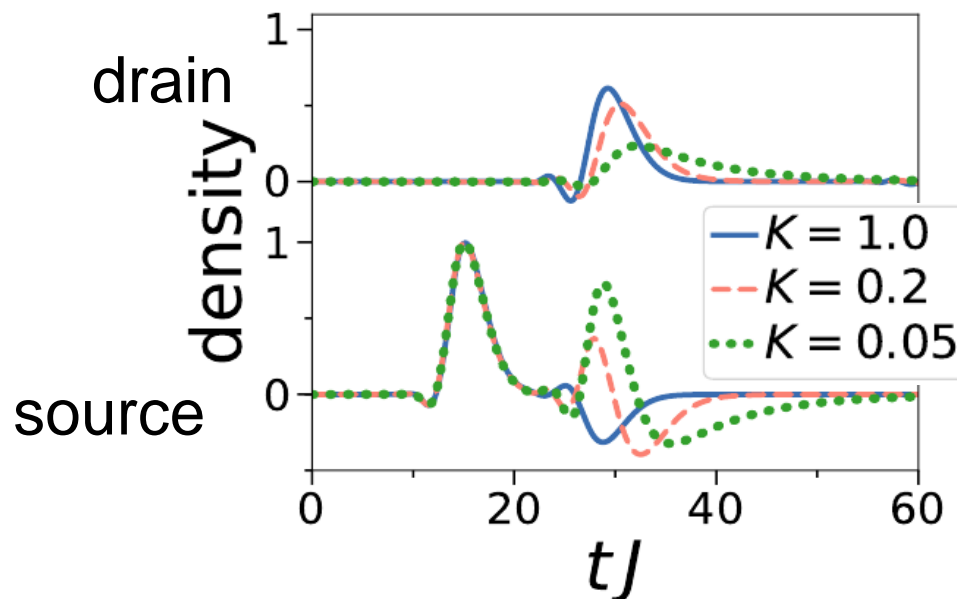
- →Gross-Pitaevskii equation (GPE)

$$i\partial_t \Psi(x,t) = \left[\frac{1}{2}(-i\partial_x - A(x,t))^2 + V(x) + g\,|\Psi(x,t)|^2\right]\Psi(x,t)$$

- Linearize equation $\quad \partial_t^2 \delta n = g n_0 \partial_x^2 \delta n$

    (small excitation→ sound waves)

- **Y-junction**: solve at a boundary between three chains, assume
    - current conserved
    - Density/ derivative of phase continuous at boundary

→ **4/3 transmission**, **-1/3 reflection** for strong coupling (GPE). Also valid for Bose-Hubbard in strong-coupling
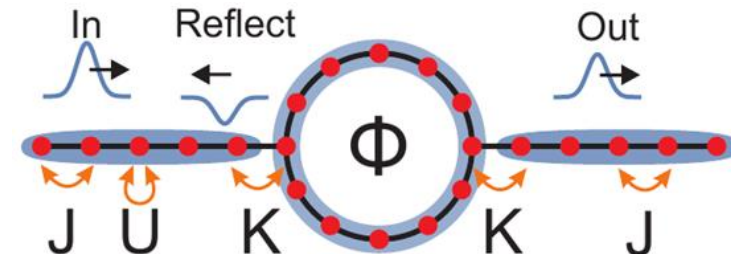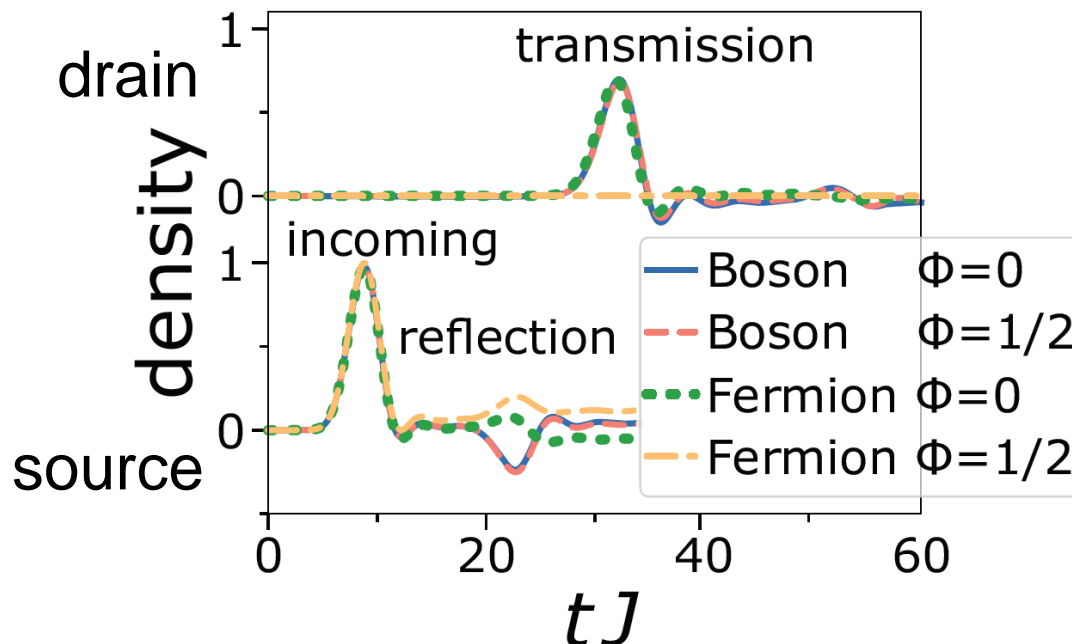
Gross-Pitaevskii Y-junction

- For **GPE**: Total transmission is always 4/3, reflection -1/3, **independent of coupling K** (dynamics just stretched longer in time) →contrast to Bose-Hubbard (coefficient depend on K)

- Even for weakest coupling K there is no purely positive reflection → No weak coupling regime

- Nonlinear wave (GPE) vs many-body dynamics (BHM)?

Ring dynamics

- Simulate full closed system ring + long leads (hard-core bosons)
- Initialize small density excitation in source lead
- Spinless fermions: Aharonov-Bohm effect, total reflection Φ=1/2
- **Interacting bosons** *independent of flux* (no Aharonov-Bohm!)

Also for Luttinger Liquid: A. Tokuno, M. Oshikawa, and E. Demler, Phys. Rev. Lett. 100, 140402 (2008).

Gross-Pitaevskii

- Ring with Gross-Pitaesvkii equation (GPE) with flux A

$$i\partial_t \Psi(x,t) = \left[\frac{1}{2}(-i\partial_x - A(x,t))^2 + V(x) + g\,|\Psi(x,t)|^2 \right]\Psi(x,t)$$

- Linearize equation on top of condensate

$$\partial_t^2 \delta n = \left[gn_0 - (A - v_0)^2 \right]\partial_x^2 \delta n + 2(A - v_0)\partial_t\partial_x \delta n$$
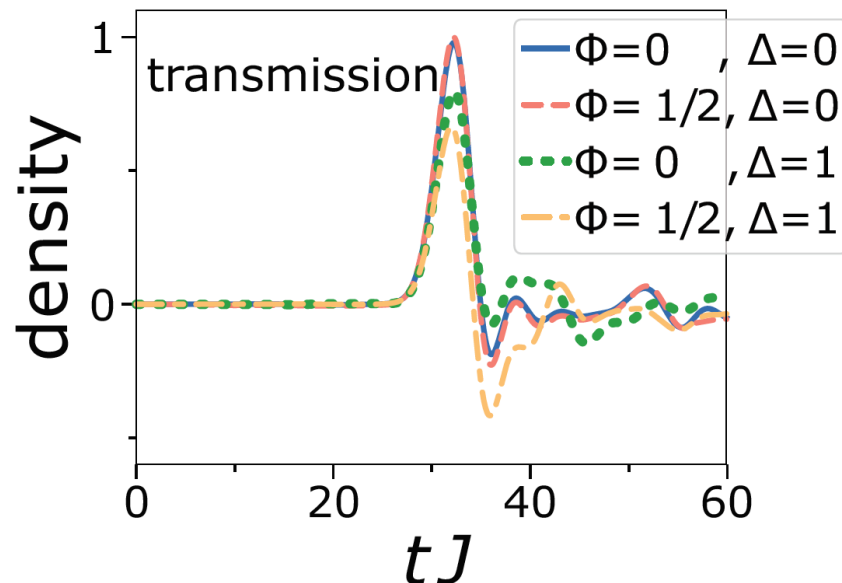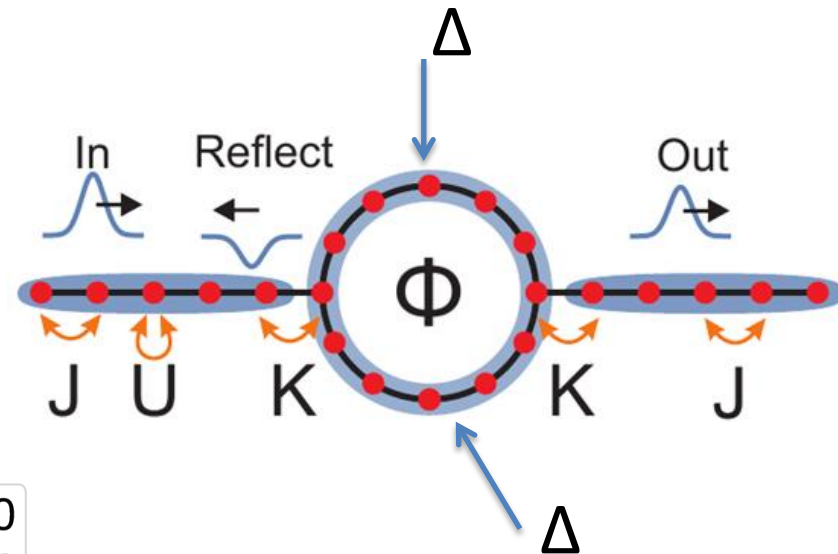
$$c_\pm = \sqrt{gn_0} \pm (A - v_0)$$

- **Ring with flux A**: Flux causes a (direction dependent) shift in speed of sound, but no change in interference pattern

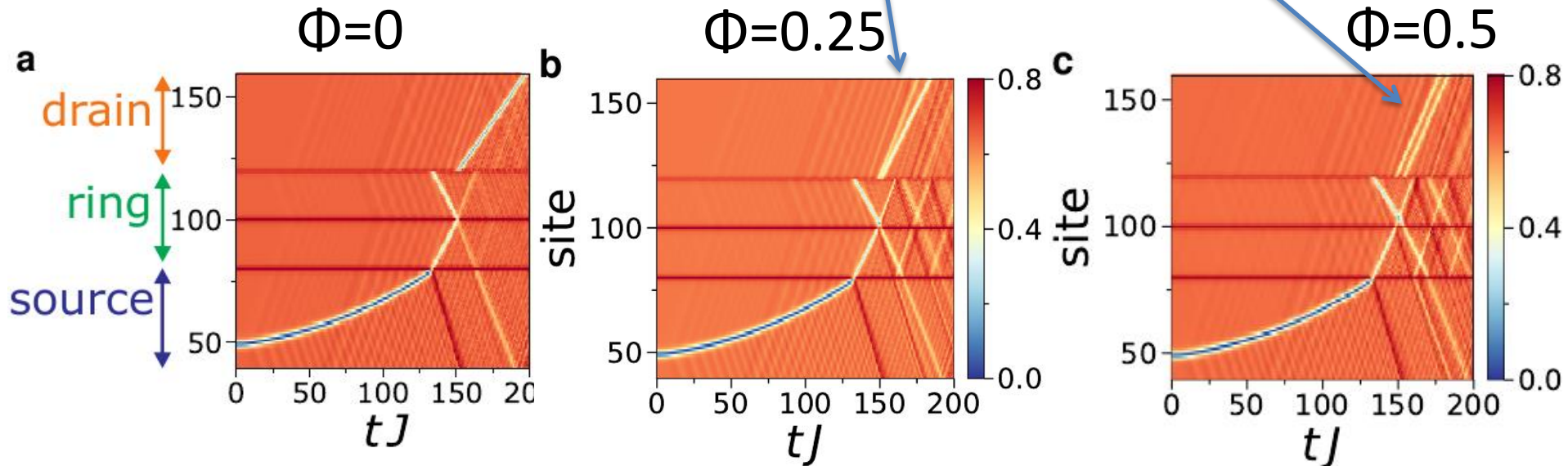→**No Aharonov-Bohm** effect for excitations on top of a bosonic condensate

Breaking translational invariance

- Add two potential barriers Δ into the ring for Bose-Hubbard
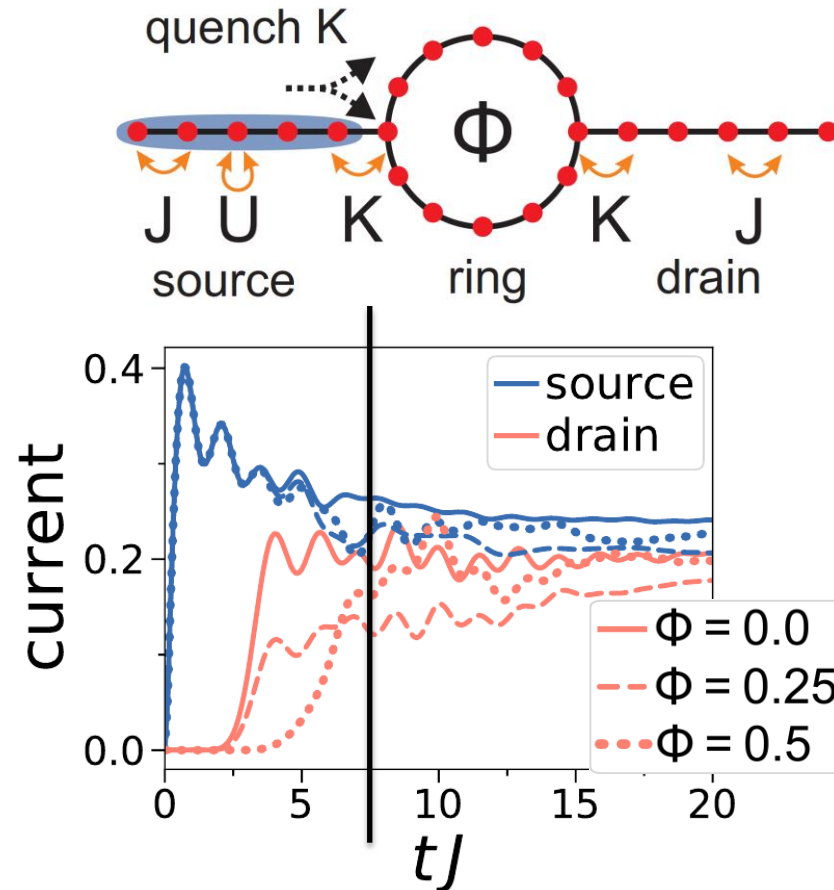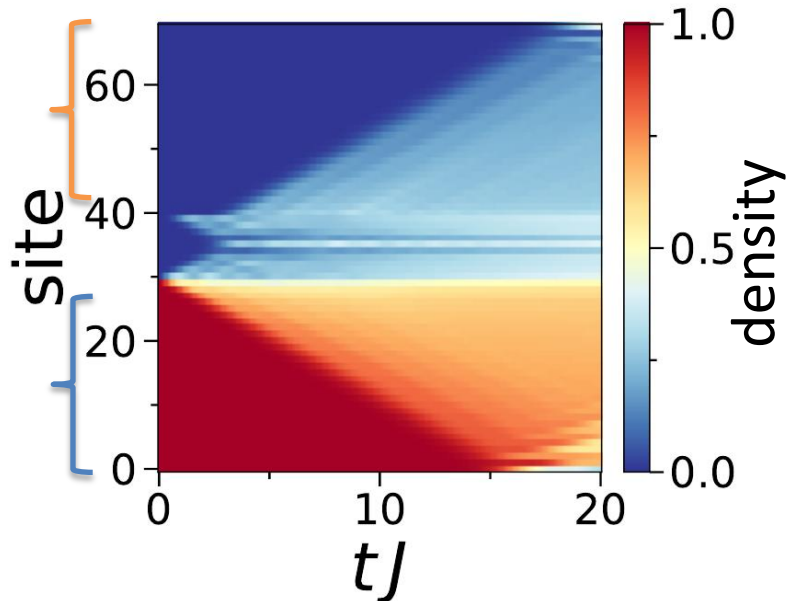
- Restores flux dependence

Grey soliton in ring

- Gray soliton passing through ring
- Soliton splits into two in transmission with flux
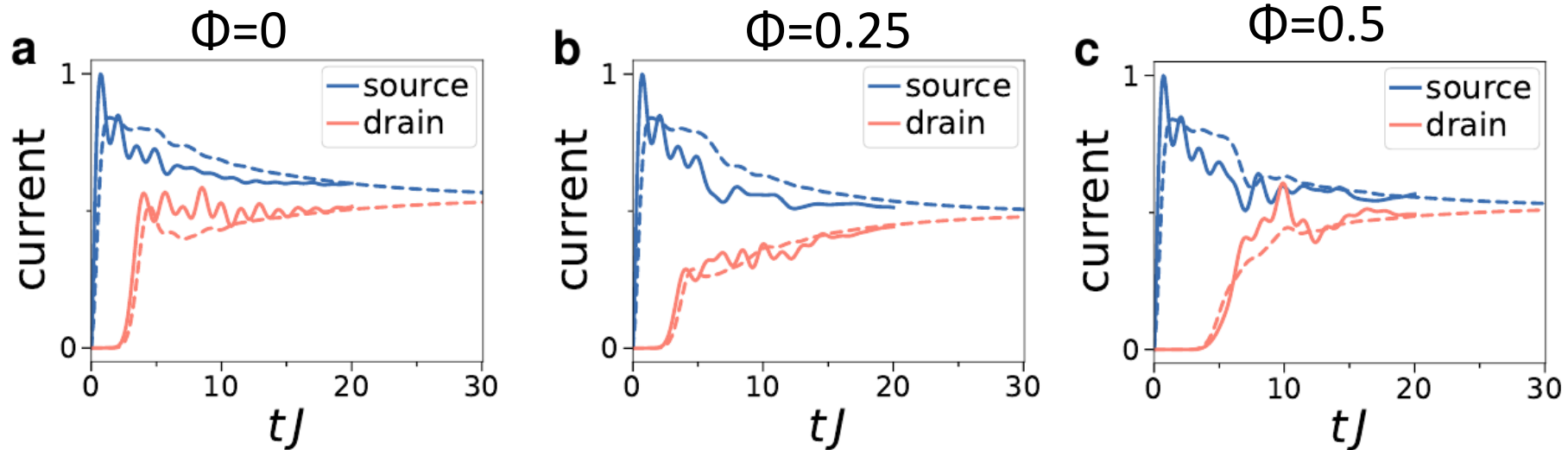
→Use to measure rotation

Φ=0

Φ=0.25

Φ=0.5

Quench of ring

- Simulate full closed system ring + long leads (hard-core bosons)
- Initially fill only source lead with atoms
- Switch on coupling K to ring (K=1)

Towards the steady-state

- <u>Solid</u>: Full simulation of ring+lead with **tDMRG**

- <u>Dashed</u>: Approximate leads as **Markovian baths:** Open system with Lindblad + ring

- For Φ=0.5, current into drain starts later

- For Φ=0.25, current increases slower

- **Dynamics depends on flux**, however **steady-state** nearly **independent** of **flux**

Conclusion

Y-junction:

- Different types of reflection (Andreev-like, positive-negative, positive) controlled by the coupling strength
- GPE shows different behavior than Bose-Hubbard (no weak coupling regime)

Ring:

- Aharonov-Bohm effect **suppressed for bosons**
- Can be restored by breaking translational invariance
- Solitons could measure flux in ring device
- Quench dynamics is **flux dependent**, **steady-state not**

arXiv:1807.03616
arXiv:1706.05180