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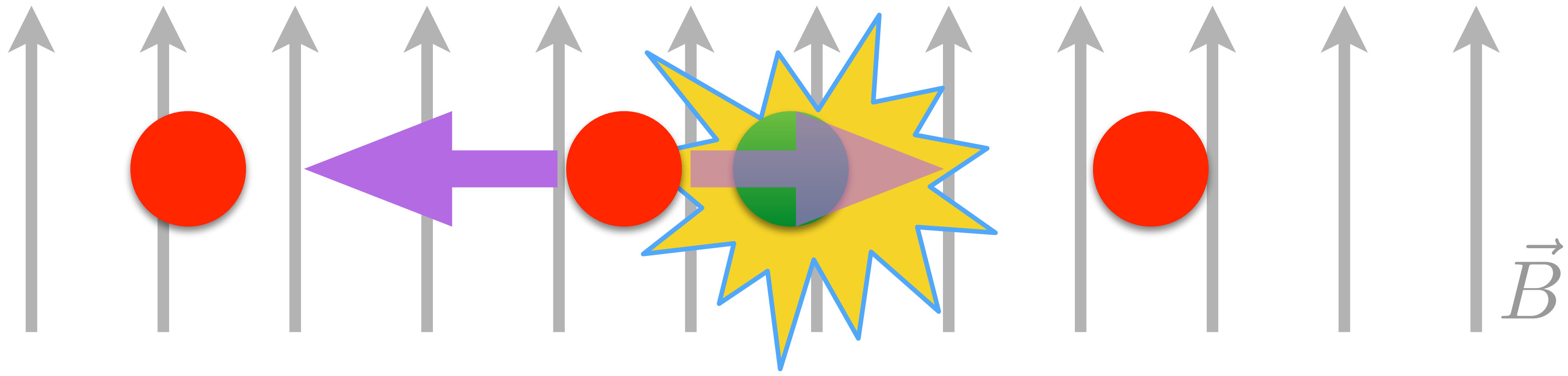
CUI THE HAMBURG CENTRE FOR ULTRAFAST IMAGING

ULTRACOLD ATOMIC COLLISIONS FOR PRECISE MAGNETOMETRY



Antonio Negretti

Benasque, August 2, 2018

Why magnetic field sensing?

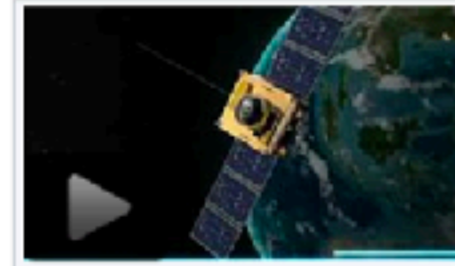
Magnetometer - Wikipedia

https://en.wikipedia.org/wiki/Magnetometer 110% magnetormeter

La Repubblica Corriere IlFatto FAZ ZEIT quant-ph cond-mat Physics APS Nature NatPhys Science MailUHH Gmail ILP WordRef LEO Printers - CUPS 1.4.4

Uses [edit]

Magnetometers have a very diverse range of applications, including locating objects such as submarines, sunken ships, hazards for tunnel boring machines, hazards in coal mines, unexploded ordnance, toxic waste drums, as well as a wide range of mineral deposits and geological structures. They also have applications in heart beat monitors, weapon systems positioning, sensors in anti-locking brakes, weather prediction (via solar cycles), steel pylons, drill guidance systems, archaeology, plate tectonics and radio wave propagation and planetary exploration.



Magnetometers can measure the magnetic fields of planets.

Depending on the application, magnetometers can be deployed in spacecraft, aeroplanes (*fixed wing* magnetometers), helicopters (*stinger* and *bird*), on the ground (*backpack*), towed at a distance behind quad bikes (*sled* or *trailer*), lowered into boreholes (*tool*, *probe* or *sonde*) and towed behind boats (*tow fish*).

Archaeology [edit]

Main article: Magnetic survey (archaeology)

Magnetometers are also used to detect archaeological sites, shipwrecks and other buried or submerged objects. Fluxgate gradiometers are popular due to their compact configuration and relatively low cost. Gradiometers enhance shallow features and negate the need for a base station. Caesium and Overhauser magnetometers are also very effective when used as gradiometers or as single-sensor systems with base stations.

The TV program *Time Team* popularised 'geophys', including magnetic techniques used in archaeological work to detect fire hearths, walls of baked bricks and magnetic stones such as basalt and granite. Walking tracks and roadways can sometimes be mapped with differential compaction in magnetic soils or with disturbances in clays, such as on the Great Hungarian Plain. Ploughed fields behave as sources of magnetic noise in such surveys.

Auroras [edit]

Magnetometers can give an indication of auroral activity before the light from the aurora becomes visible. A grid of magnetometers around the world constantly measures the effect of the solar wind on the Earth's magnetic field, which is then published on the K-index.[29]

Coal exploration [edit]

Whilst magnetometers can be used to help map basin shape at a regional scale, they are more commonly used to map hazards to coal mining, such as basaltic intrusions (dykes, sills and volcanic plugs) that destroy resources and are dangerous to longwall mining equipment. Magnetometers can also locate zones ignited by lightning and map siderite (an impurity in coal).

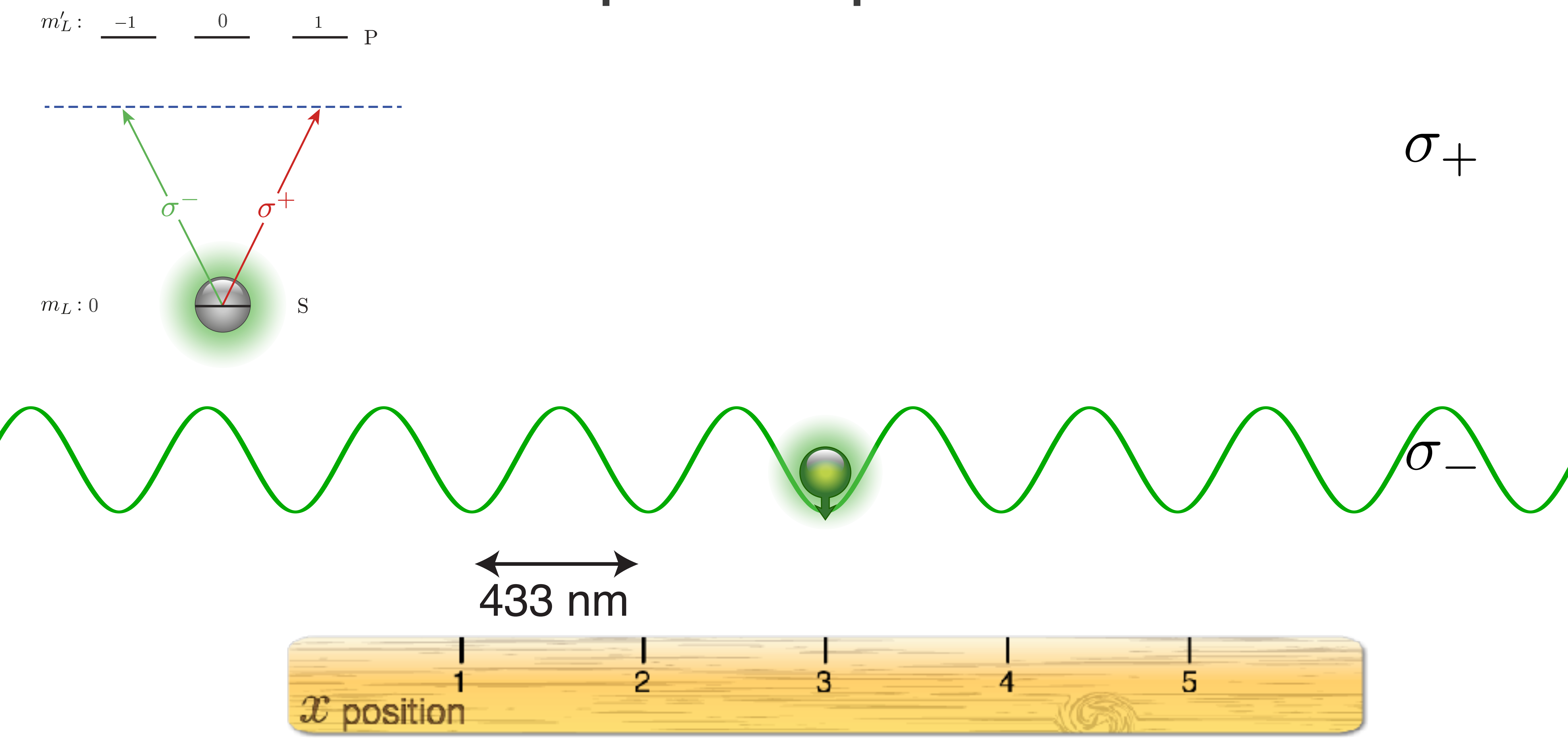
The best survey results are achieved on the ground in high-resolution surveys (with approximately 10 m line spacing and 0.5 m station spacing). Bore-hole magnetometers using a Ferret can also assist when coal seams are deep, by using multiple sills or looking beneath surface basalt flows.[*citation needed*]

Modern surveys generally use magnetometers with GPS technology to automatically record the magnetic field and their location. The data set is then corrected with data

- https://en.wikipedia.org/wiki/Magnetometer

Quantum simulation with atoms in optical lattices

State-dependent optical lattices

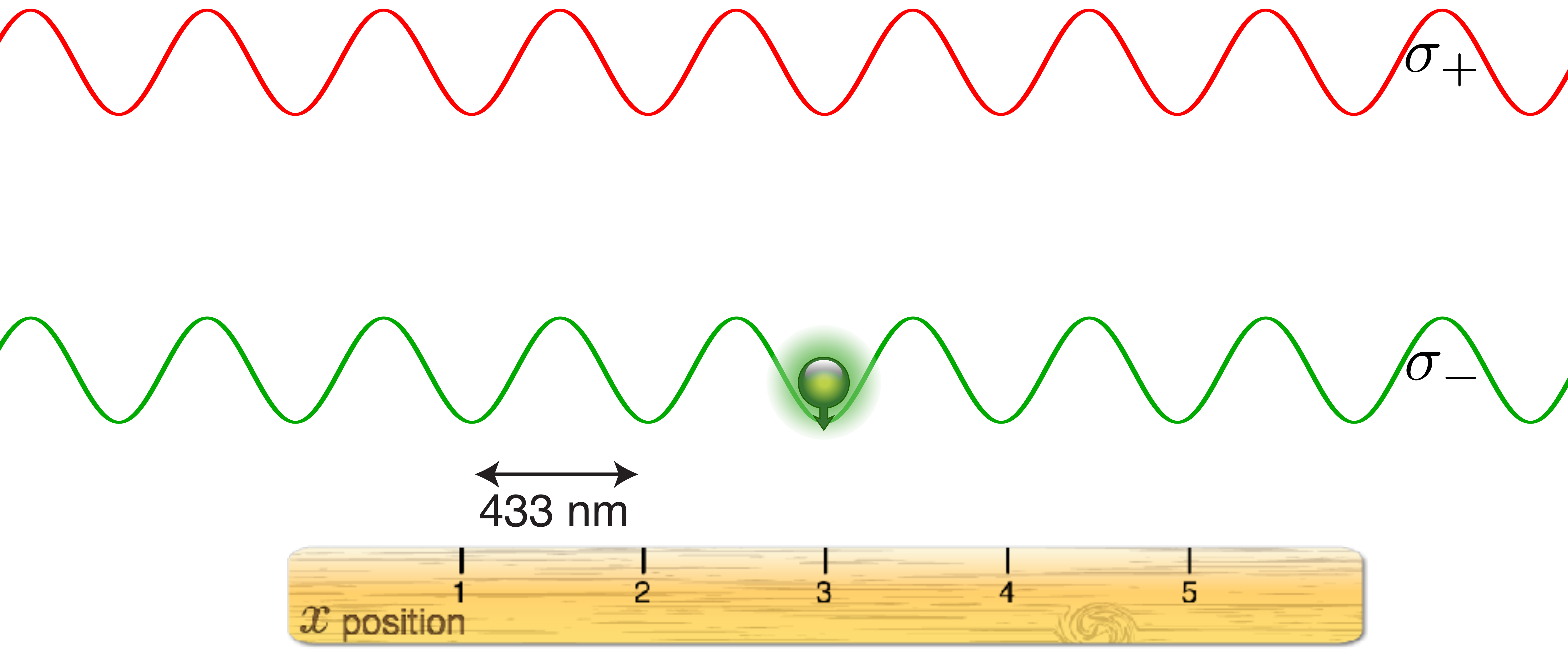


Animation courteously from Andrea Alberti

- **Theory:** D. Jaksch et al., Phys. Rev. Lett. **82**, 1975 (1999)
- **Experiments:** Mandel et al., Phys. Rev. Lett. **91**, 010407 (2003); A. Steffen, et al., PNAS **109**, 9770 (2012).

Quantum simulation with atoms in optical lattices

State-dependent optical lattices



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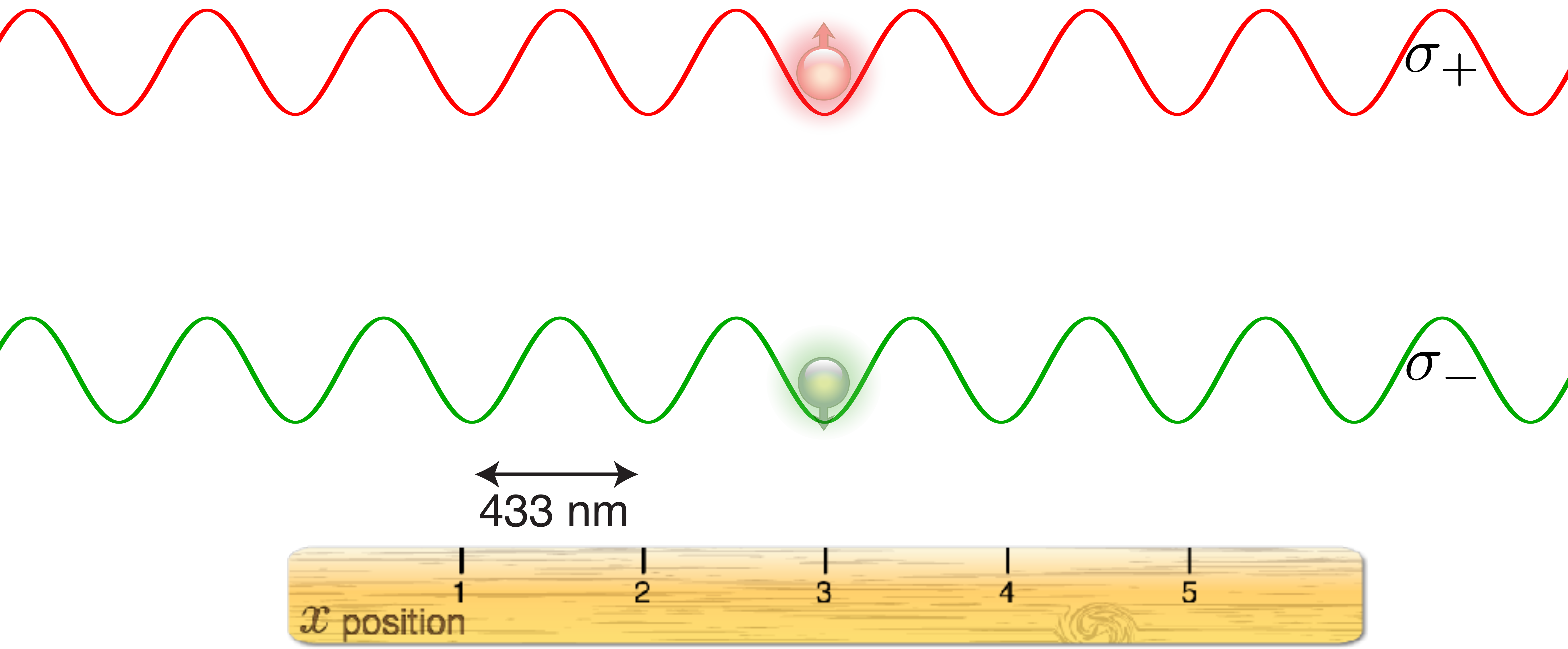
Quantum simulation with atoms in optical lattices

State-dependent optical lattices



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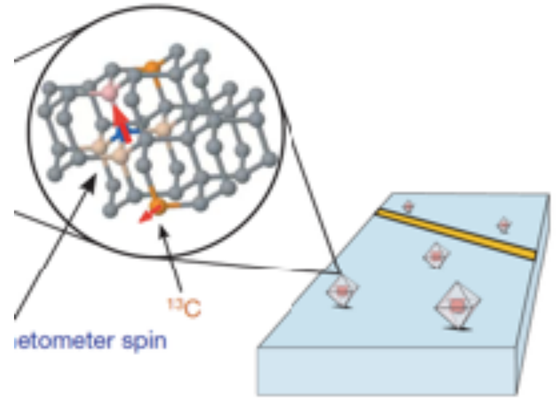
Magnetic field sensors

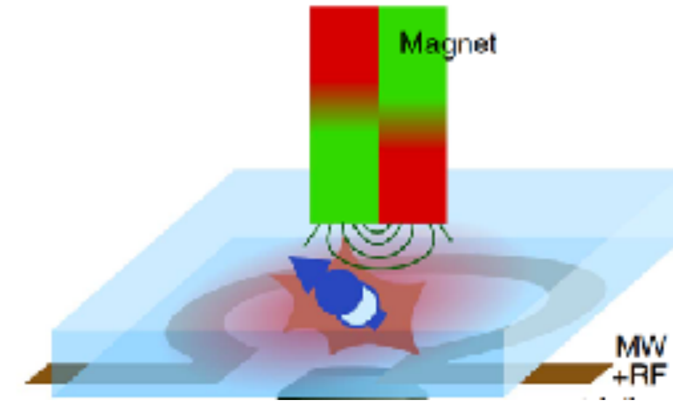
Nitrogen vacancy centres in diamond

J. Maze *et al.*, Nature **455**, 644 (2008)

T. Wolf *et al.*, PRX **5**, 041001 (2015)

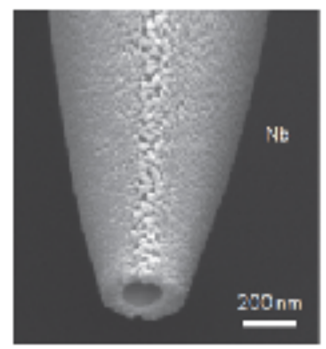
S. Zaiser *et al.*, Nat. Comm. **7** (2016)





Superconducting circuits (SQUID)

D. Vasyukov, *et al.*, Nature Nanotechnology **8**, 639 (2013)



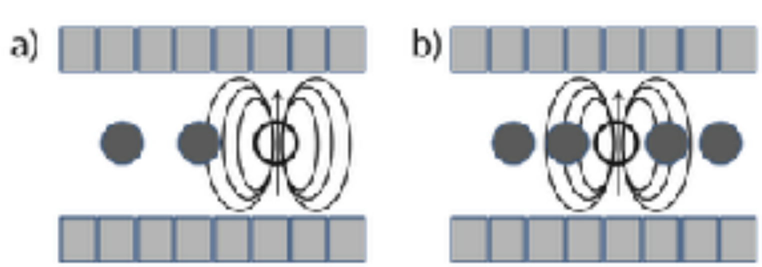
Thermal atomic vapors (Faraday rotation)

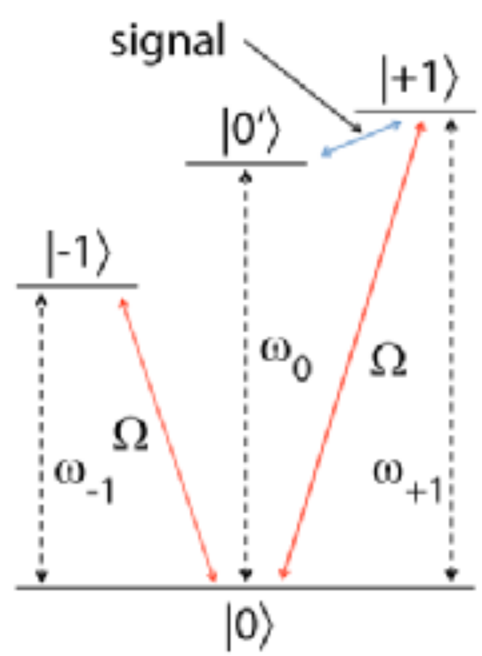
R. J. Sewell *et al.*, Phys. Rev. Lett. **109**, 253605 (2012)
K. Jensen *et al.*, Sci. Rep. **6**, 29638 (2016)

Trapped ions

I. Baumgart *et al.*, PRL **116**, 240801 (2016)

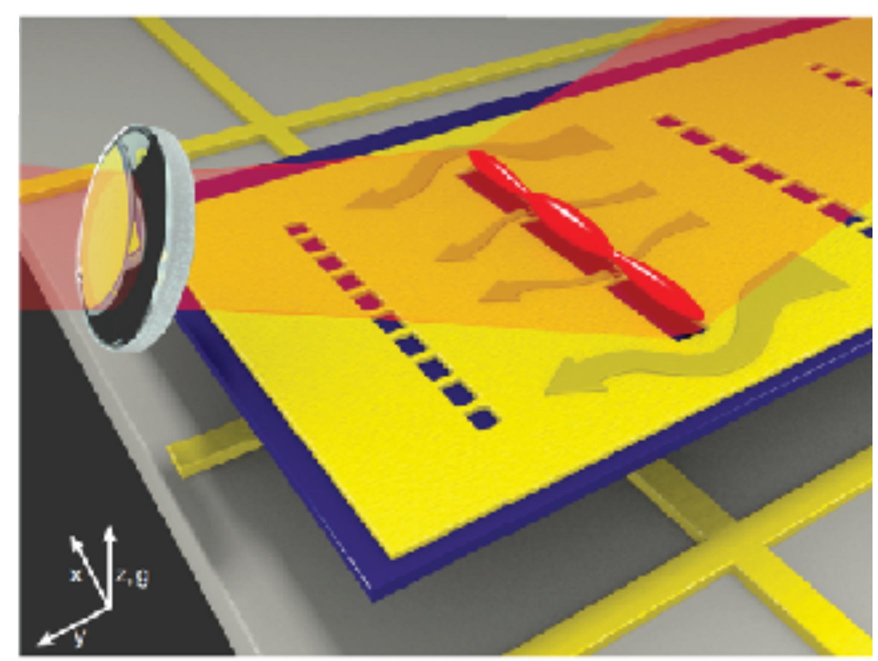
F. Schmidt-Kaler and R. Gerritsma, EPL **99**, 53001 (2012)





Cold atomic ensembles and BECs

S. Wildermuth *et al.*, Nature **435**, 440 (2005)
M. Vengalattore *et al.*, PRL **98**, 200801 (2007)
M. Koschorreck *et al.*, Applied Physics Letters **98**, 074101 (2011)
N. Behbood *et al.*, Applied Physics Letters **102**, 173504 (2013)
F. Yang *et al.*, Phys. Rev. Applied **7**, 034026 (2017)



- C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. **89**, 035002 (2017)

Quantum sensing

Possible definitions for (quantum) sensing:

(I) Quantum object with quantised energy levels for measuring a classical or quantum physical quantity (e.g. atoms, ions, superconducting devises, etc.)

(II) Use quantum coherence (i.e., wavelike or temporal superpositions) to measure a physical quantity

(III) Use of entanglement to improve the precision of a measurement beyond what is possible classically

Desiderata for a quantum sensor:

(1) The quantum system has discrete and resolvable energy levels

(2) It must be possible to initialise the quantum system into a well-known state and read out its state

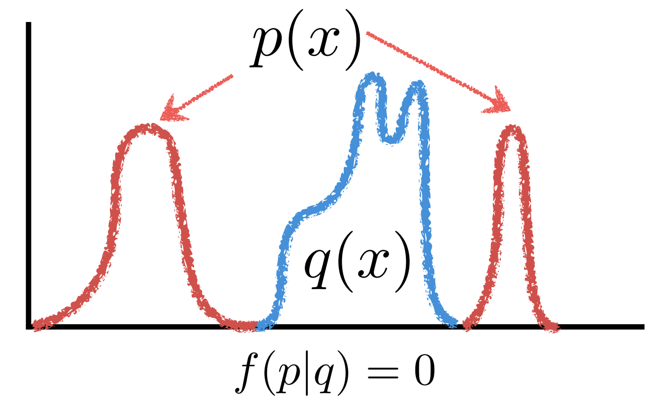
(3) The quantum system can be coherently manipulated with, e.g., time-dependent fields

(4) The quantum system interacts with a relevant physical quantity (e.g., magnetic field) and it is quantified by a coupling parameter which relates changes in the transition energy to changes in the physical quantity

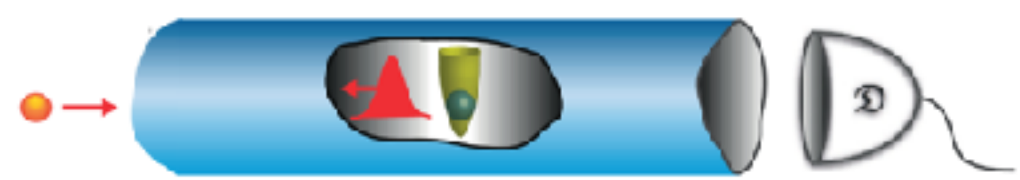
- C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. **89**, 035002 (2017)

Outline

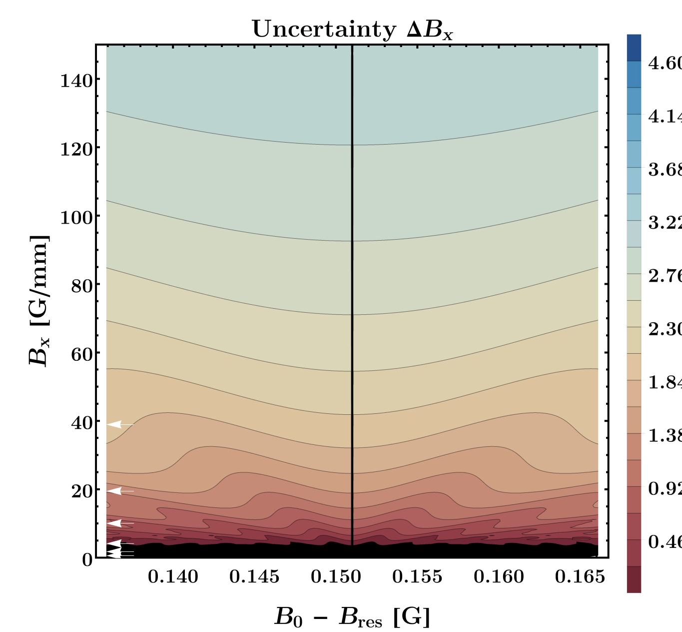
1. Introduction
   - Quasi-1D quantum scattering
   - Sensitivity and Fisher information



2. Magnetic field detection via controlled collisions
   - The sensor's idea
   - Sensitivity and robustness



3. Magnetic gradiometry in multiple waveguides
   - General principle
   - Results



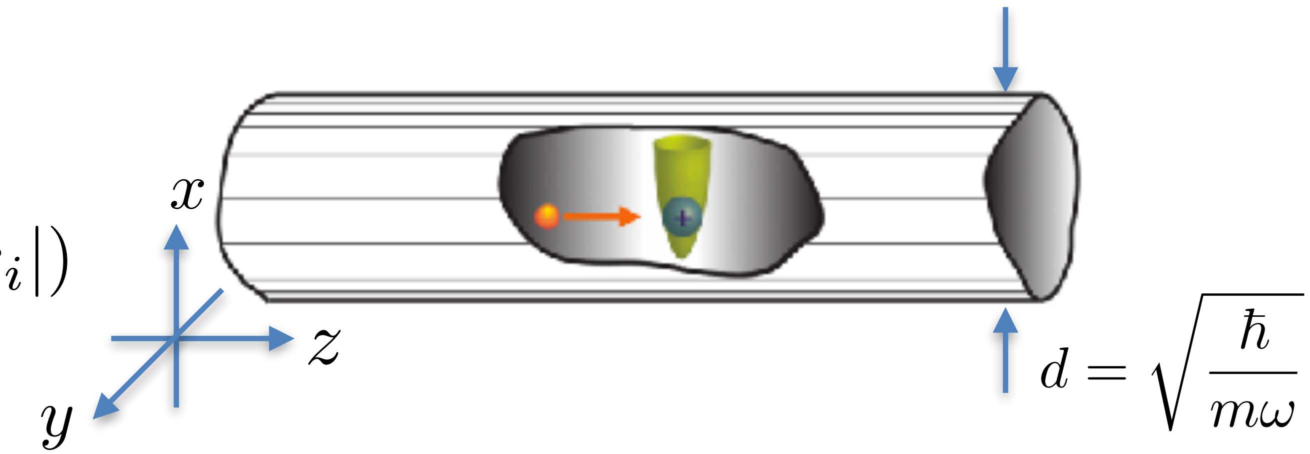
4. Summary

Quasi-1D scattering and CIR

Hamiltonian of the system:

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2(x^2+y^2) + V(|\mathbf{r}-\mathbf{r}_i|)$$

Short-range interaction potential



Scattering wavefunction:

Transverse ground state

$$\psi(\mathbf{r}) \xrightarrow[|z|\to\infty]{} e^{ikz}\phi_{00}(x,y) + f e^{ik|z|}\phi_{00}(x,y)$$

Energy: $E = \hbar\omega + \frac{\hbar^2 k^2}{2m}$

s-wave ultracold scattering → Scattering amplitude $f = -\frac{1}{1+ika_{1D}}$

$$ka_{1D} = -\frac{kd}{2}\left(\frac{d}{a} - C\right)$$

$$C = -\zeta_H\left(\frac{1}{2}, 1 - \frac{\hbar^2k^2/2m}{2\hbar\omega}\right)$$

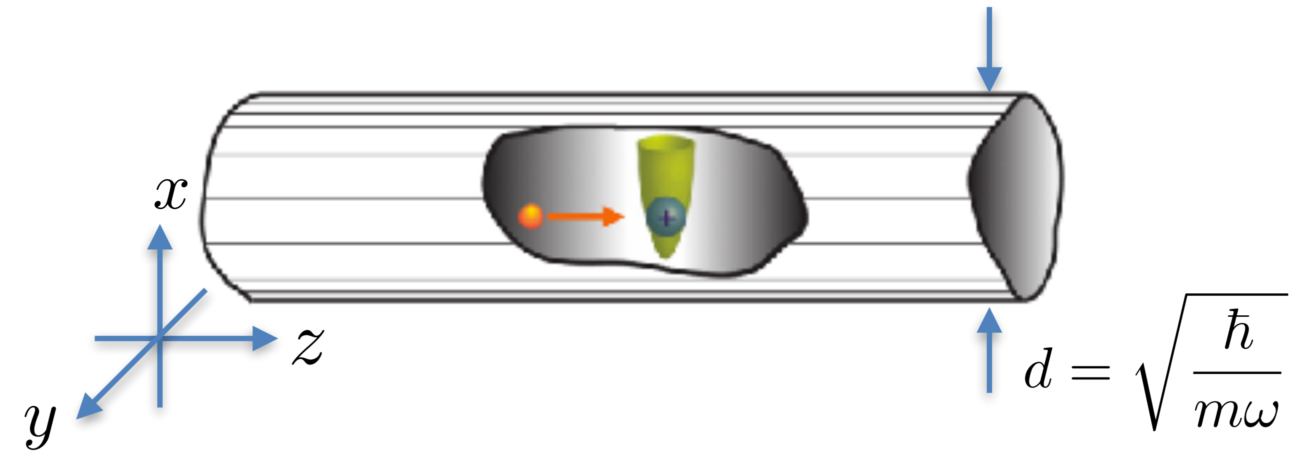
- M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998)
- T. Bergeman, M. G. Moore, M. Olshanii, Phys. Rev. Lett. **91**, 163201 (2003)

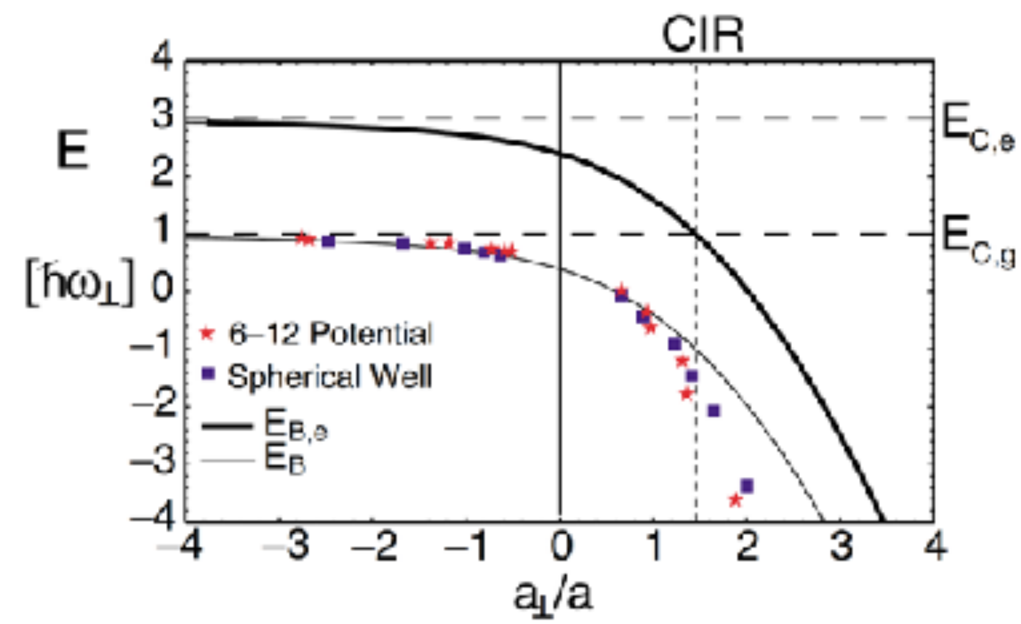
Quasi-1D scattering and CIR

Same pseudo-potential approximation as in 3D:

$$V(\mathbf{r}) \to V(z) = g_{1D}\delta(z)$$

Coupling strength $\quad g_{1D} = \frac{2\hbar^2 a}{md^2}\frac{1}{1 - C\frac{a}{d}}$





Control of interactions in low dimensions

Confinement-Induced Resonance: $|g_{1D}| \to \infty$

Transmission drops to zero, i.e. perfect reflection!

Experimental realisation of the Tonks-Girardeau limit:

- B. Paredes *et al.*, Nature **429**, 277 (2004)
- T. Kinoshita *et al.*, Science **305**, 1125 (2005)

Experimental CIR studies:

- H. Moritz *et al.*, PRL **94**, 210401 (2005)
- E. Haller *et al.*, PRL **104**, 153203 (2010)
- S. Sala *et al.*, PRL **110**, 203202 (2013)

- M. Olshanii, Phys. Rev. Lett. **81**, 938 (1998)
- T. Bergeman, M. G. Moore, M. Olshanii, Phys. Rev. Lett. **91**, 163201 (2003)

Quasi-1D scattering and CIR

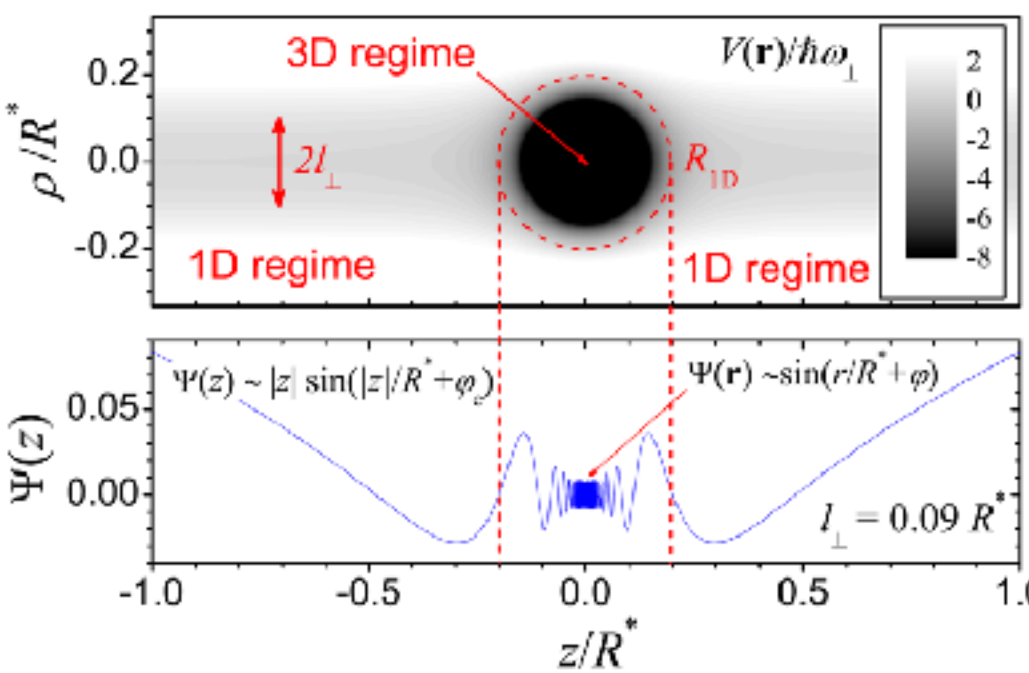
Atom-ion interaction

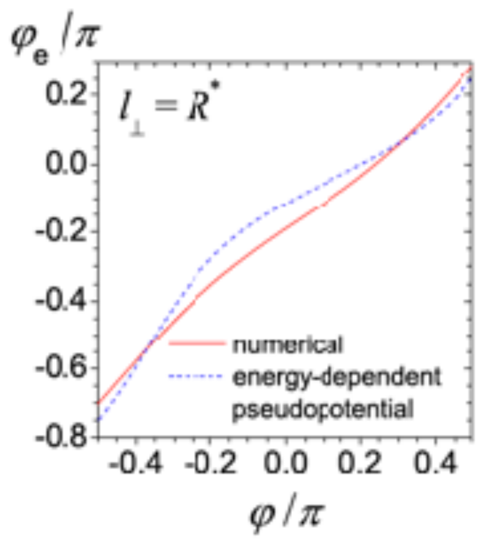
$$V(r) = -\frac{1}{2}\mathbf{p}\cdot\mathbf{E} = -\frac{C_4}{r^4}$$

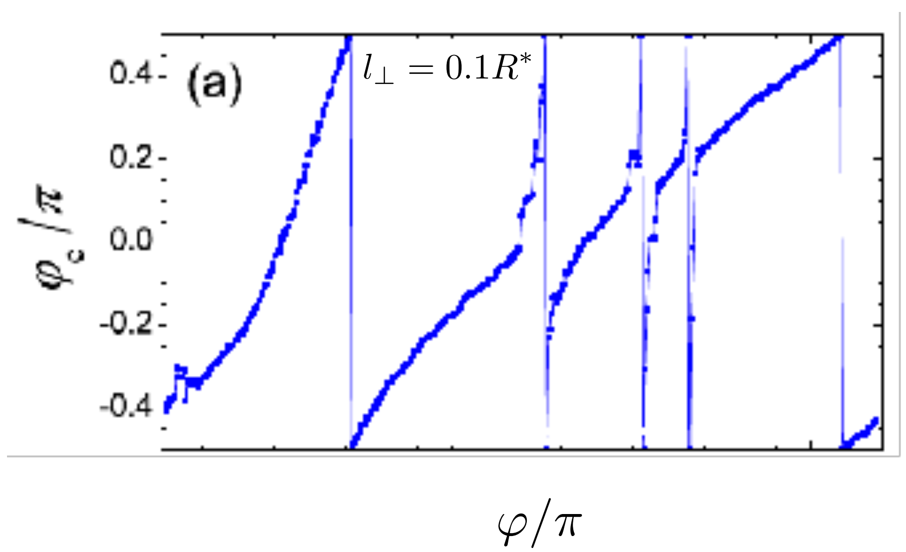
$$R^* = \sqrt{2\mu C_4/\hbar^2}$$

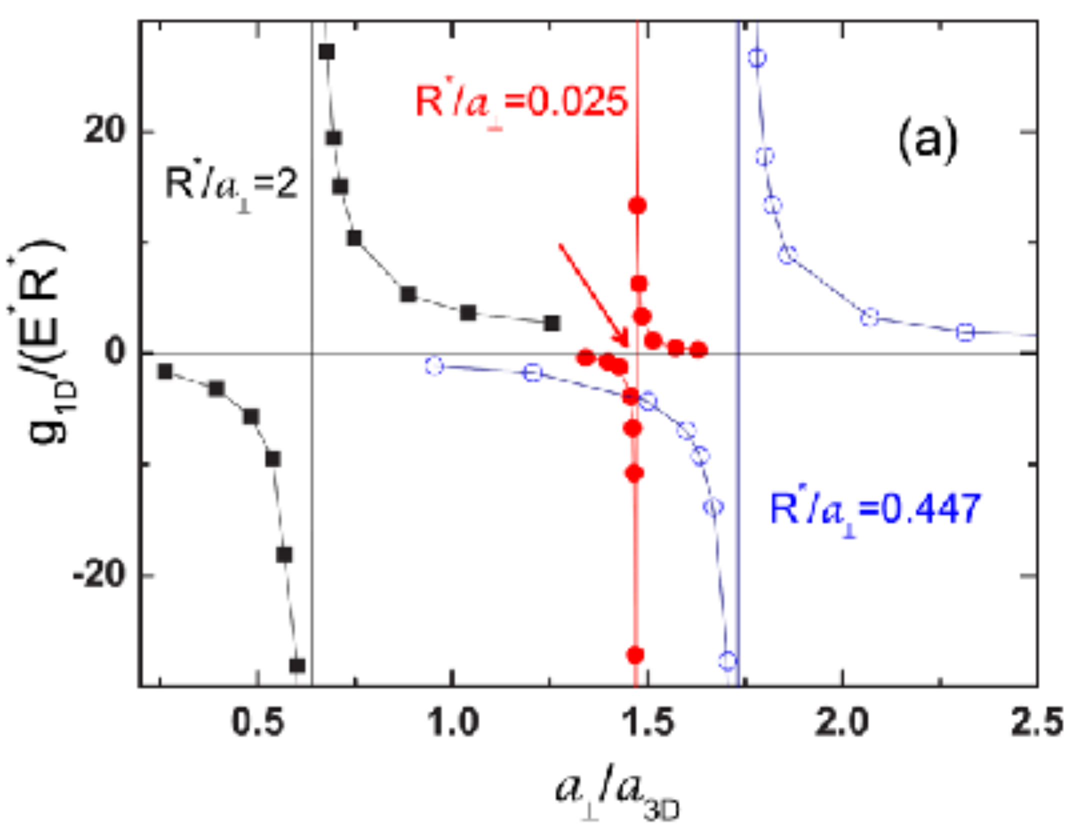
$$E^* = \hbar^2/[2\mu(R^*)^2]$$

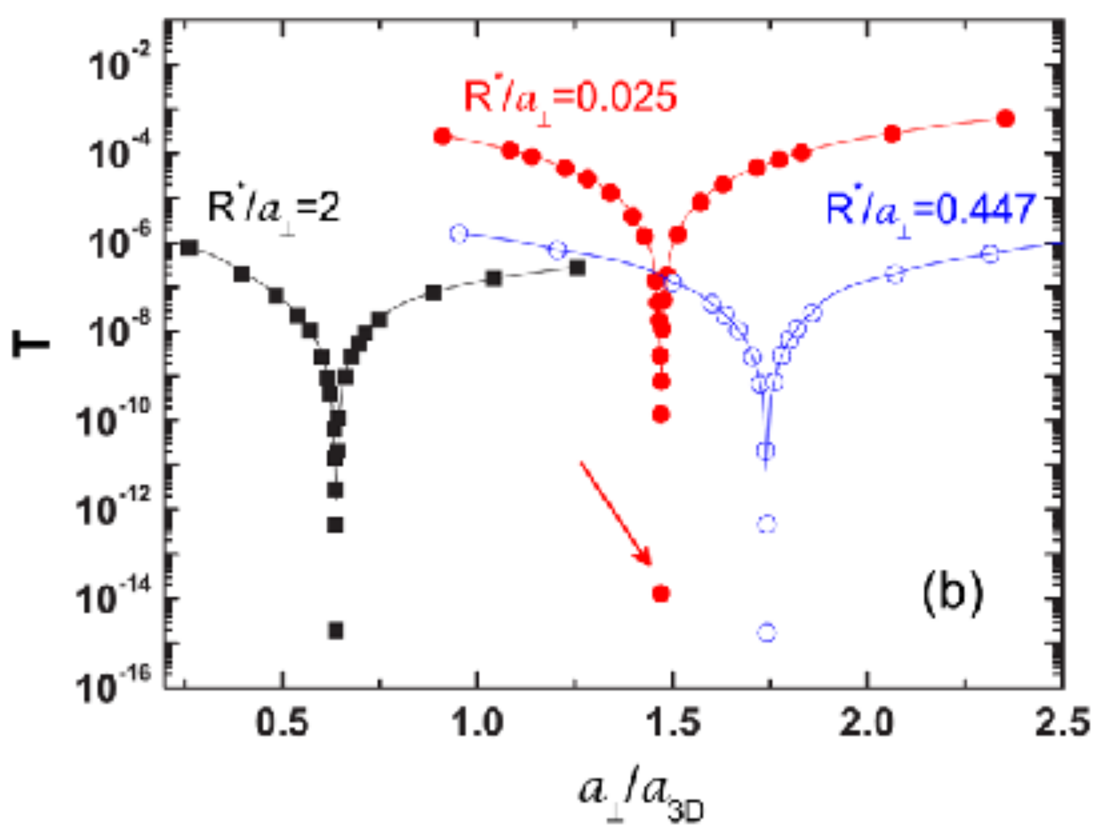
$$\mu \rightarrow \text{Reduced mass}$$











- Z. Idziaszek, T. Calarco, P. Zoller, Phys. Rev. A **76**, 033409 (2007)
- V. Melezhik, **AN**, Phys. Rev. A **94**, 022704 (2016)

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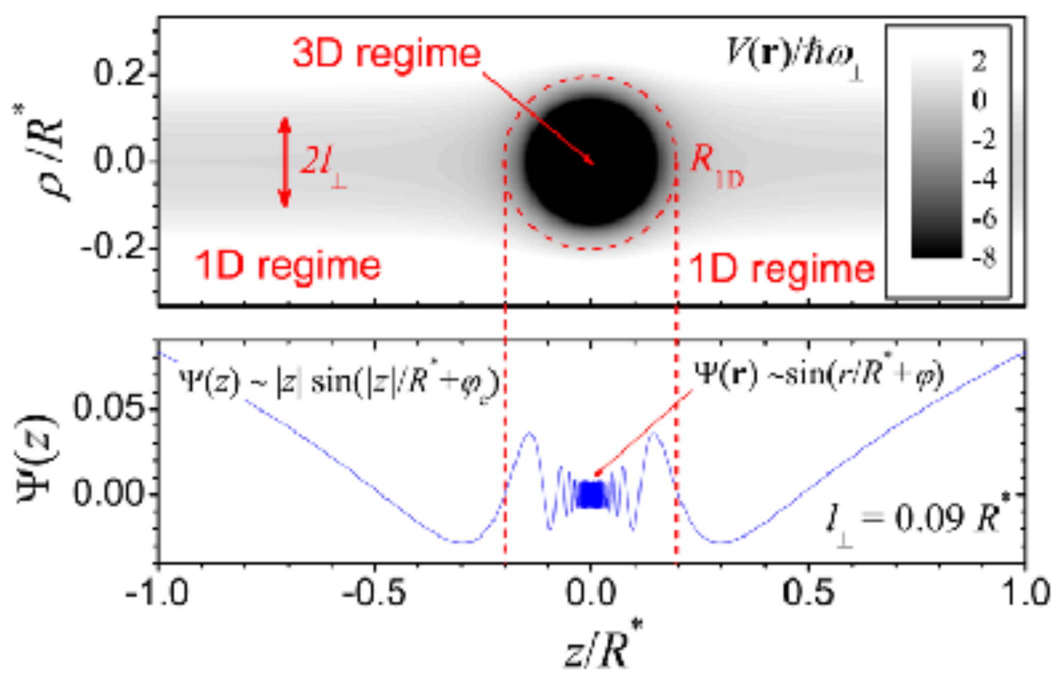
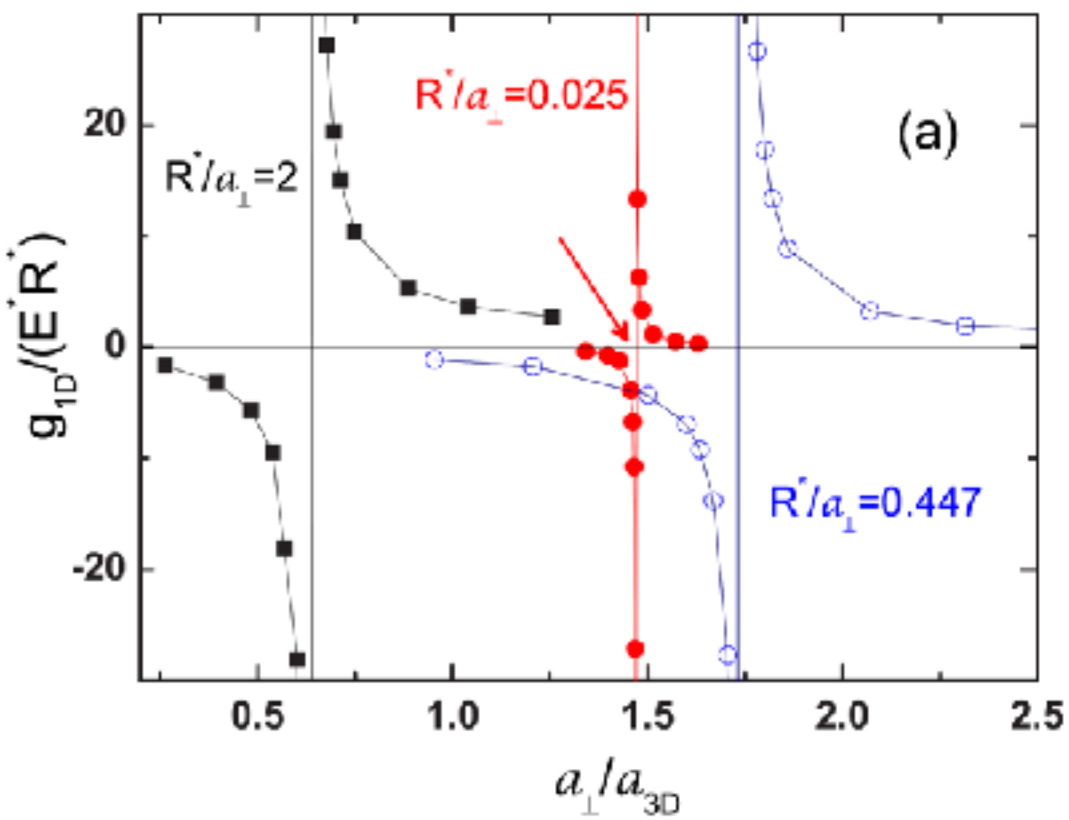
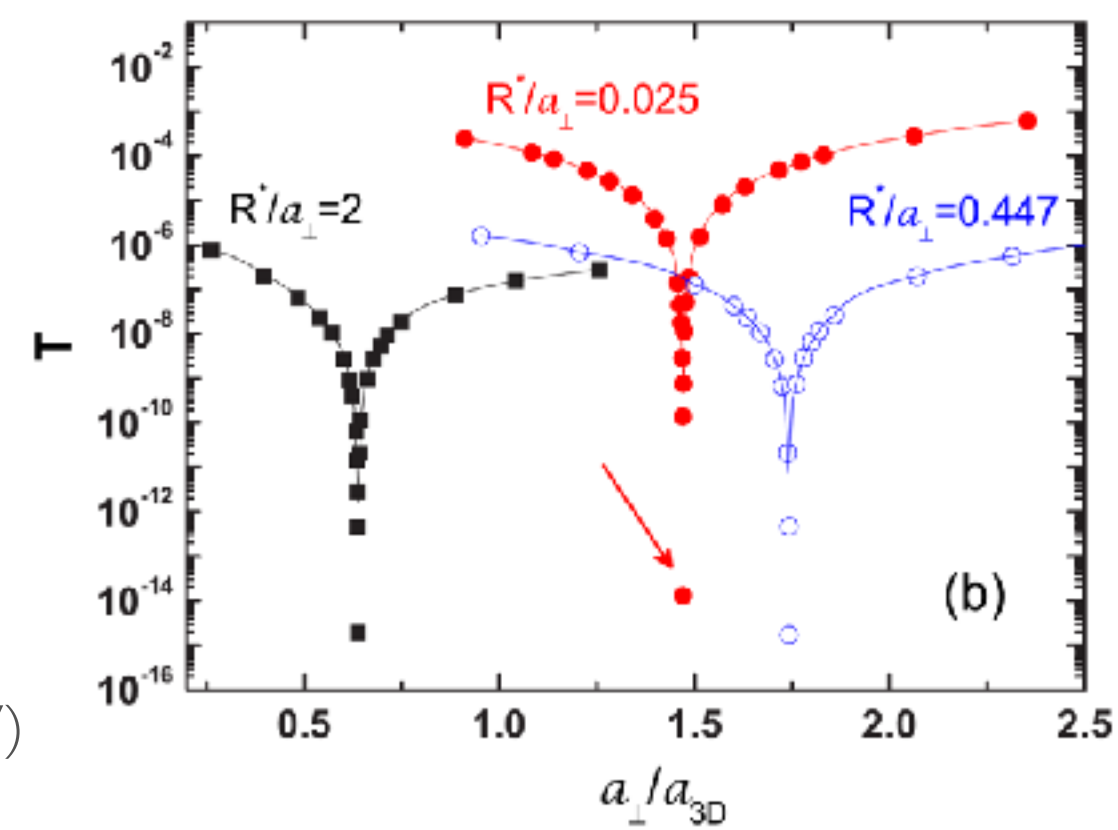


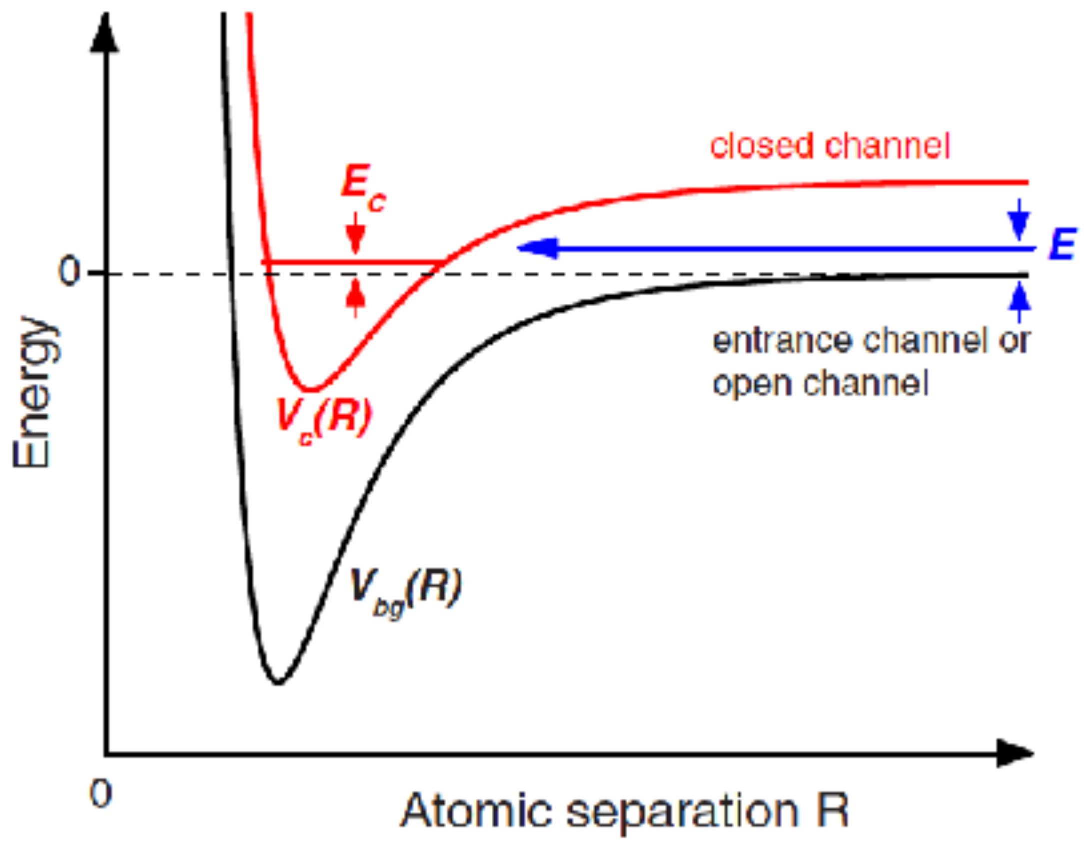


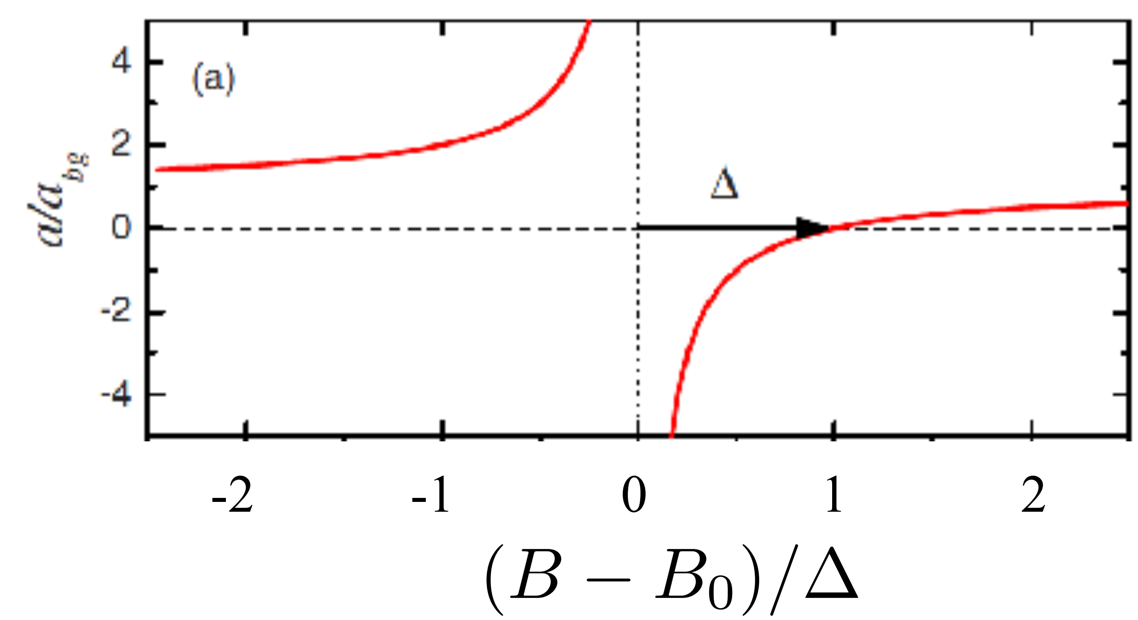


- Resonance shift
- Finite energy corrections
- Broader resonances

- Z. Idziaszek, T. Calarco, P. Zoller, Phys. Rev. A **76**, 033409 (2007)
- V. Melezhik, **AN**, Phys. Rev. A **94**, 022704 (2016)

Feshbach resonances

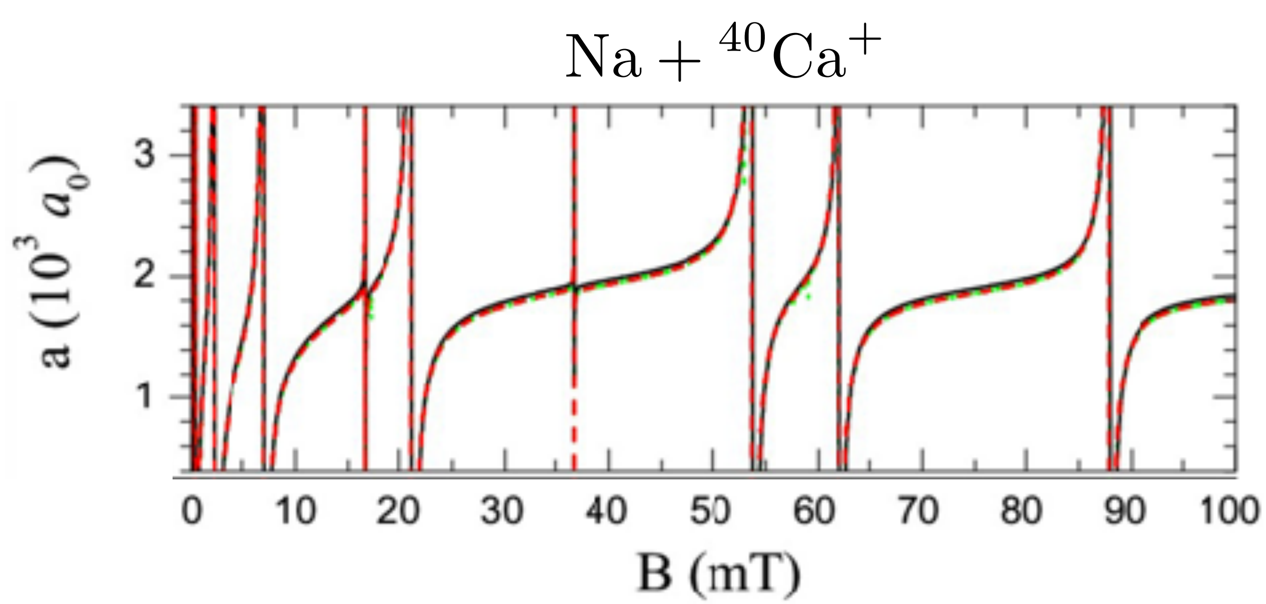




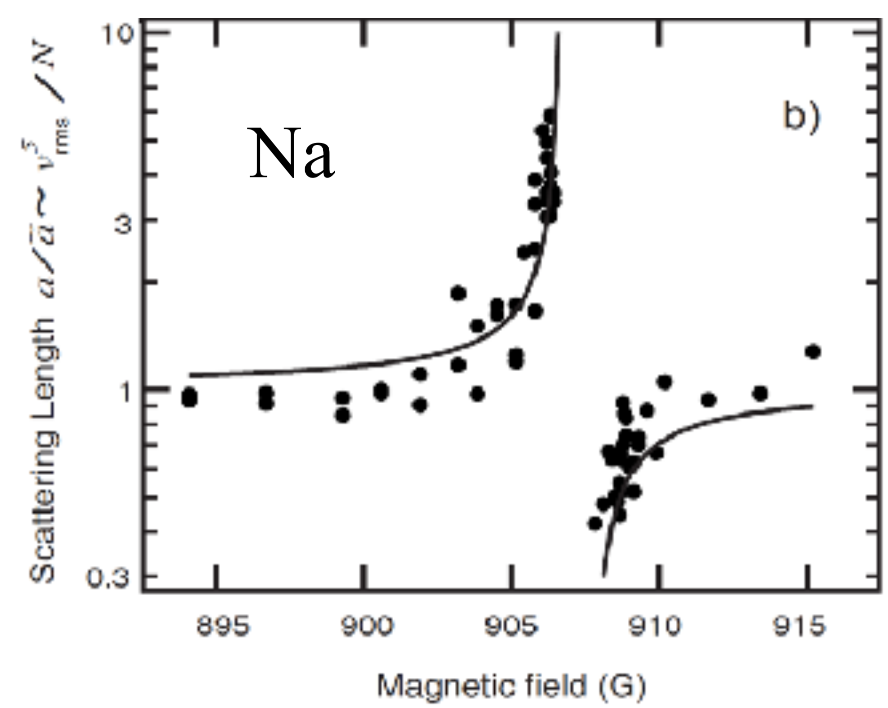
Scattering length depends on the magnetic field!

$$a(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta}{B - B_{\mathrm{res}}}\right)$$

zero-energy limit



Many broad and narrow resonances

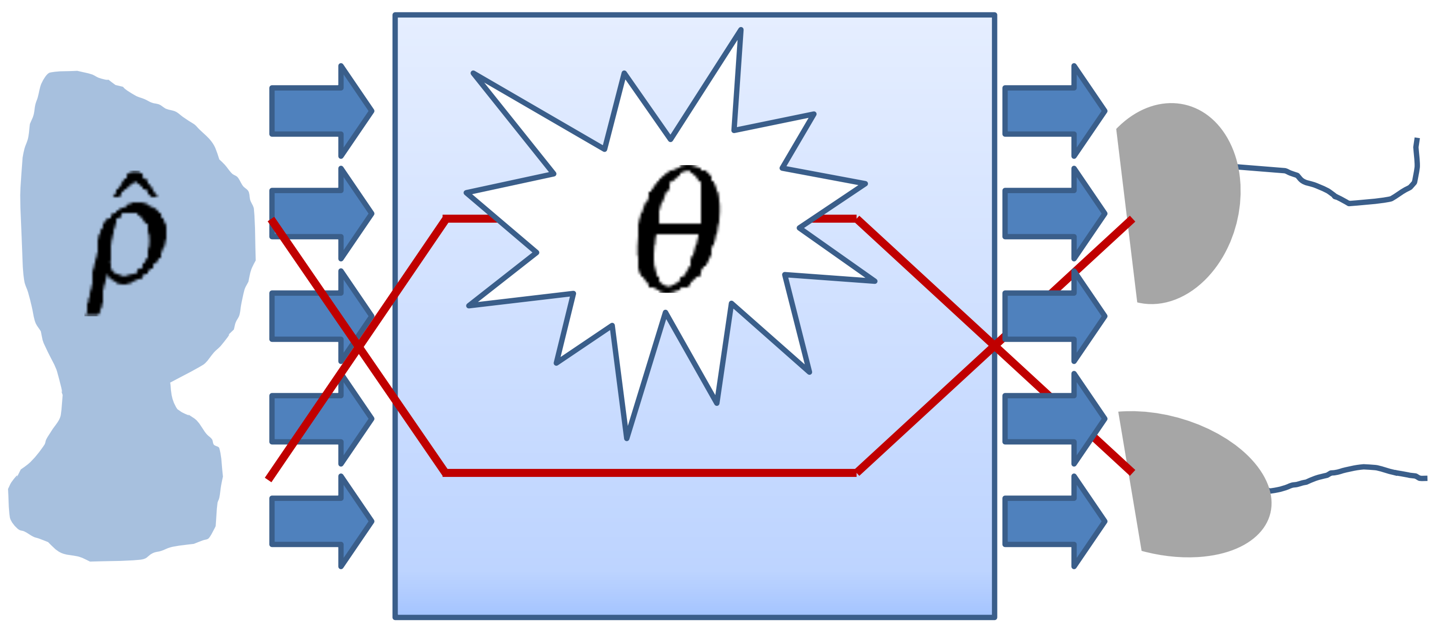


- C. Chin, R. Grimm, P. Julienne, E. Tiesinga, RMP **82**, 1225 (2010)
- Z. Idziaszek, T. Calarco, P. S. Julienne, A. Simoni, PRA **79**, 010702 (2009)

Cramer-Rao lower bound and Fisher information

Problem: How to infer the value of an unknown parameter?

General metrological scheme:



Estimated parameter: $\theta = ?$

Maximum Likelihood Estimator

Goal: To minimize $\Delta\theta$

From what estimate? $\longrightarrow$ Measurement outcomes: random var. x

Maximum estimation sensitivity? $\longrightarrow$ Cramer-Rao lower bound:

$$(\Delta\theta)^2 \geqslant \frac{1}{m}\frac{1}{F}$$

Fisher information

$$F = \sum_x \frac{1}{p(x|\theta)}\left(\frac{\partial p(x|\theta)}{\partial\theta}\right)^2$$

Quantum mechanics $\longleftarrow$ Distribution of outcomes: $p(x|\theta)$

- C. W. Helstrom, Quantum detection and estimation theory (Acad. Press, 1976)

Cramer-Rao lower bound and Fisher information

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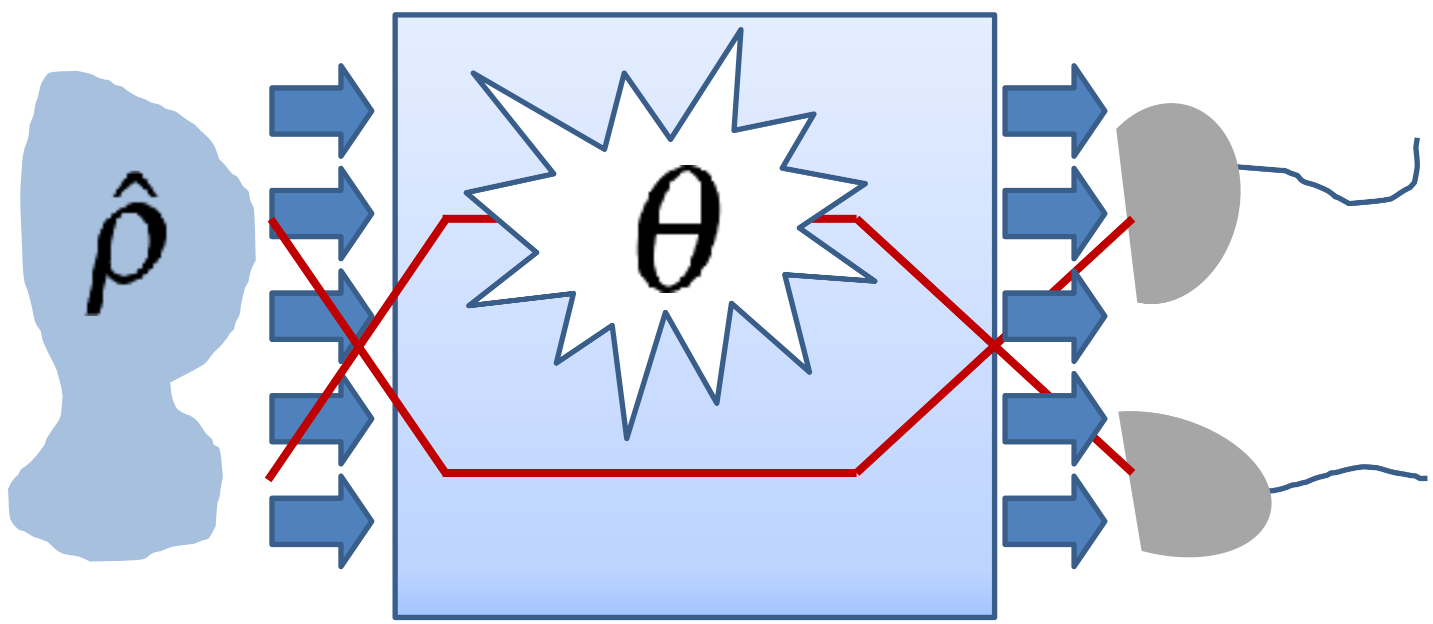

Estimated parameter: $\theta = ?$

Precision of the estimation: $\Delta\theta$

Goal: To minimize $\Delta\theta$

From what estimate? → Measurement outcomes: random var. x

Maximum estimation sensitivity? → Cramer-Rao lower bound:

$$(\Delta\theta)^2 \geqslant \frac{1}{m}\frac{1}{F}$$

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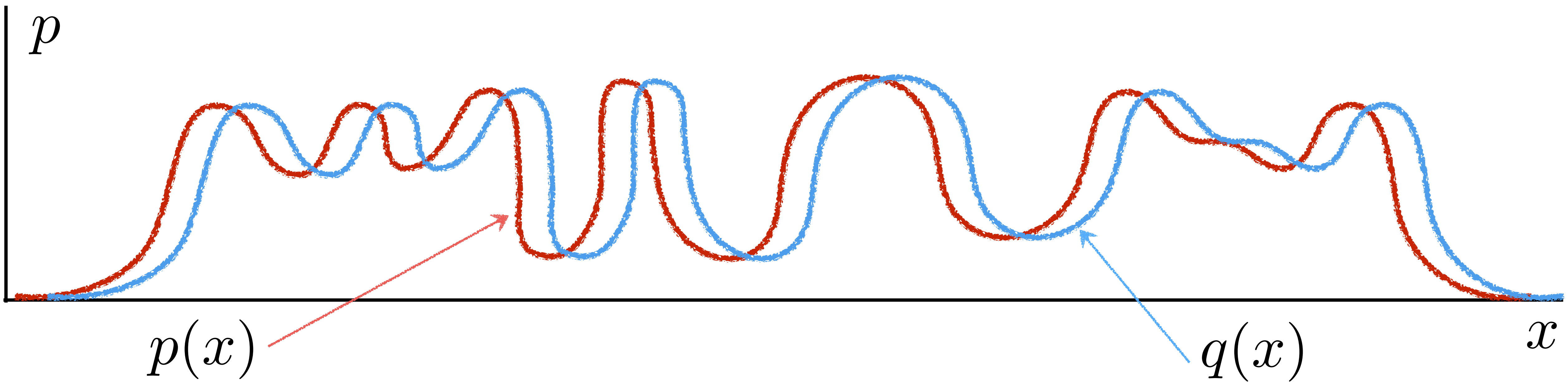
Cramer-Rao lower bound and Fisher information

What is this Fisher information *F*?

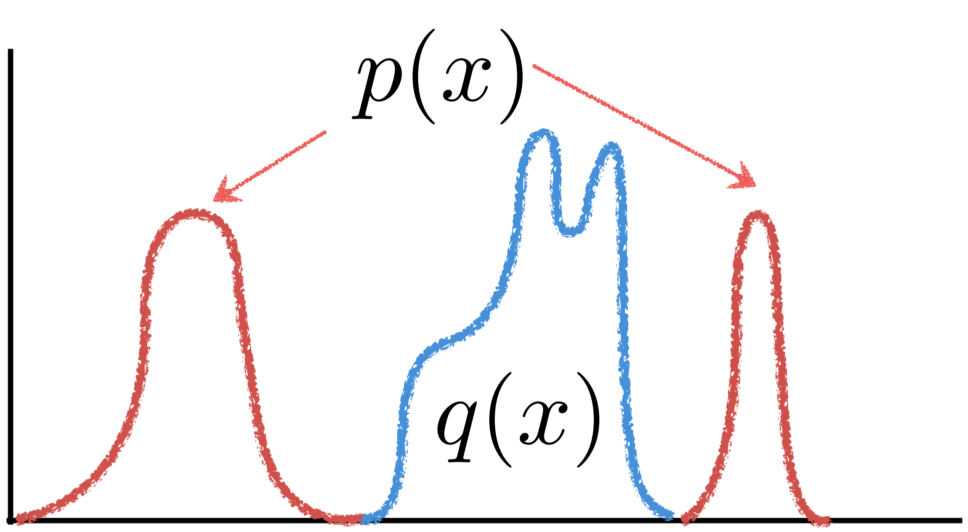


Fidelity: $f(p|q) = \sum_n \sqrt{p_n q_n}$

The same $f(p|p) = 1$

In general $0 \leqslant f(p|q) \leqslant 1$

Different distributions



$f(p|q) = 0$

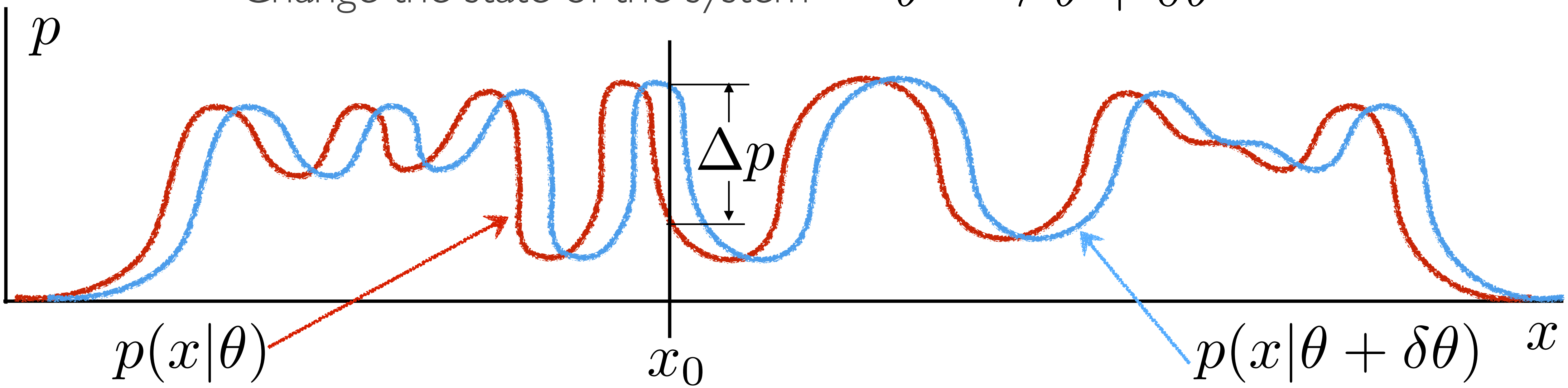
$$p(x) = \mathrm{Tr}[\hat{\rho}\, |x\rangle\langle x|]$$

System state

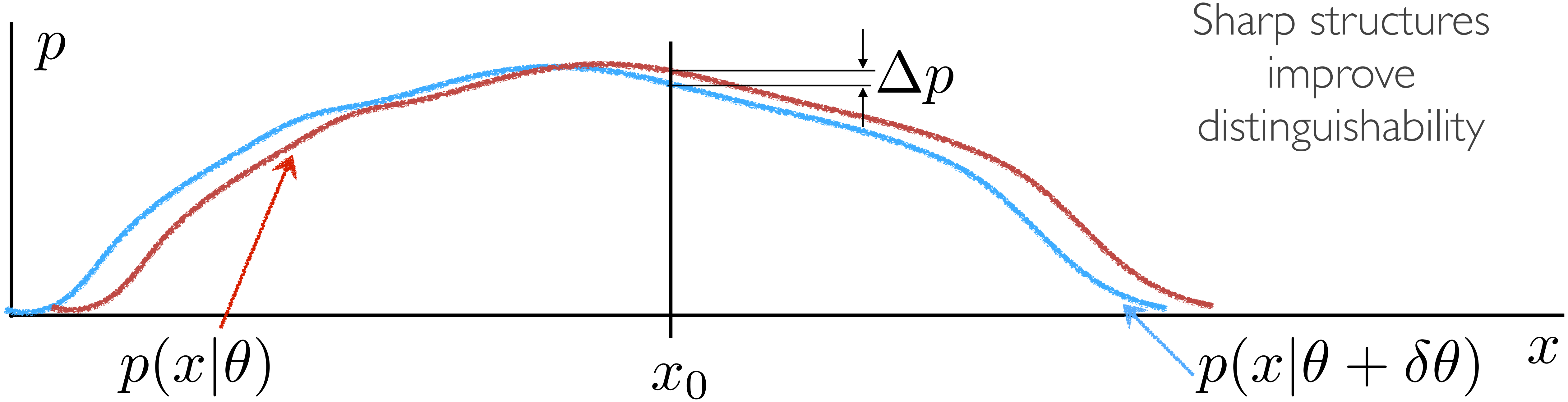
Measurement

Cramer-Rao lower bound and Fisher information

Change the state of the system $\theta \longrightarrow \theta + \delta\theta$



Sharp structures improve distinguishability



For small changes

$$f\left(p(\theta)\middle|p(\theta+\delta\theta)\right) = 1 - \frac{1}{8}F\delta\theta^2$$

Estimation uncertainty

$$\Delta\theta \geqslant \frac{1}{\sqrt{F}}$$

$$F = \sum_x \frac{1}{p(x|\theta)}\left(\frac{\partial p(x|\theta)}{\partial\theta}\right)^2$$

Outline

1. Introduction
   - Quasi-1D quantum scattering
   - Sensitivity and Fisher information

2. Magnetic field detection via controlled collisions
   - The sensor's idea
   - Sensitivity and robustness

3. Magnetic gradiometry in multiple waveguides
   - General principle
   - Results

4. Summary

General idea of the sensor

Setup: Atomic waveguide

Measurement:
transmitted vs reflected particle

Scattering wave function: $\psi(z) \xrightarrow[|z|\to+\infty]{} e^{ikz} + f e^{ik|z|}$

probability amplitude of reflection

Reflected particle: $\psi(z) \xrightarrow[z\to-\infty]{} e^{ikz} + \boxed{f} e^{-ikz}$

Transmitted particle: $\psi(z) \xrightarrow[z\to+\infty]{} e^{ikz} + f e^{ikz} = \boxed{(1+f)} e^{ikz}$

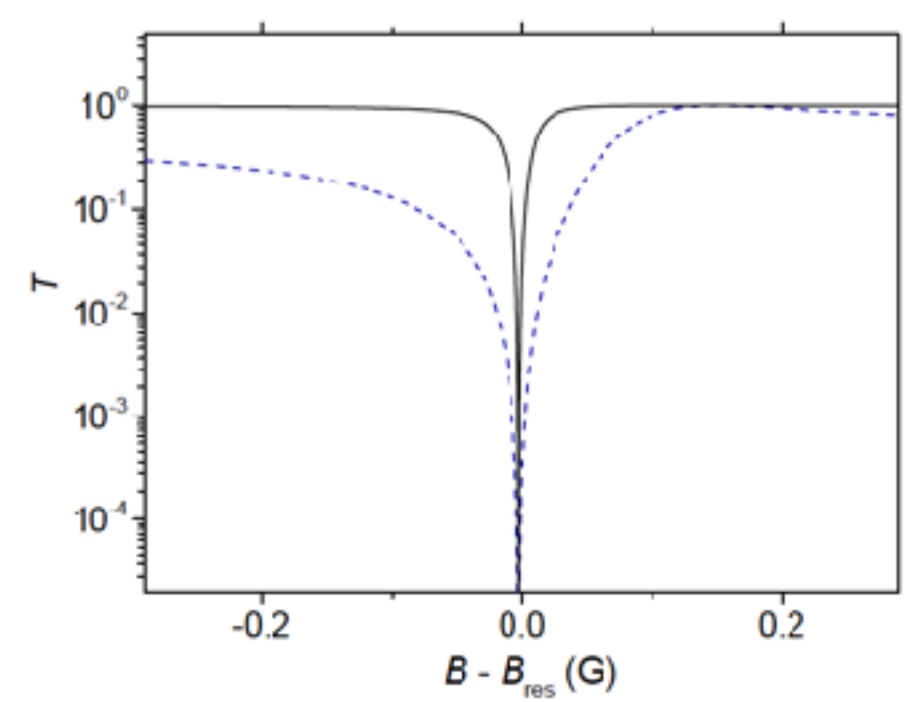
probability amplitude of transmission

$$f = -\frac{1}{1 + ika_{1D}}$$

$$ka_{1D} = -\frac{kd}{2}\left(\frac{d}{a} - C\right)$$

Transmittance:

$$T(B) = |1+f|^2$$



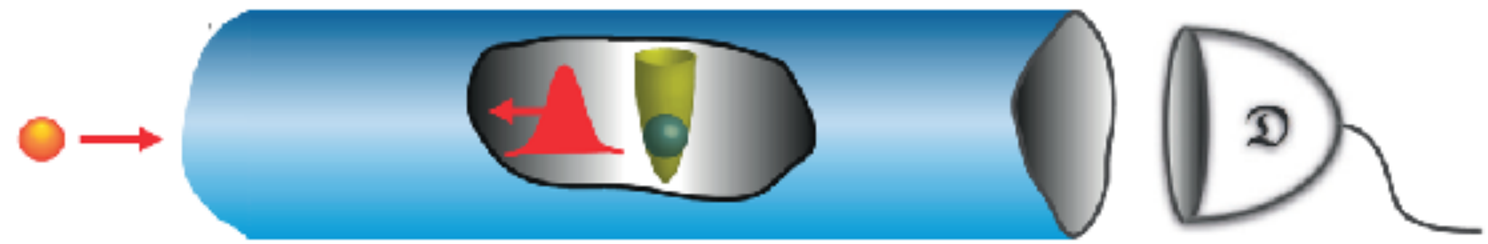
Magnetic field sensitivity

Probability of transmitting the atom:

$$p(+|B) = T(B)$$

Probability of reflecting the atom:

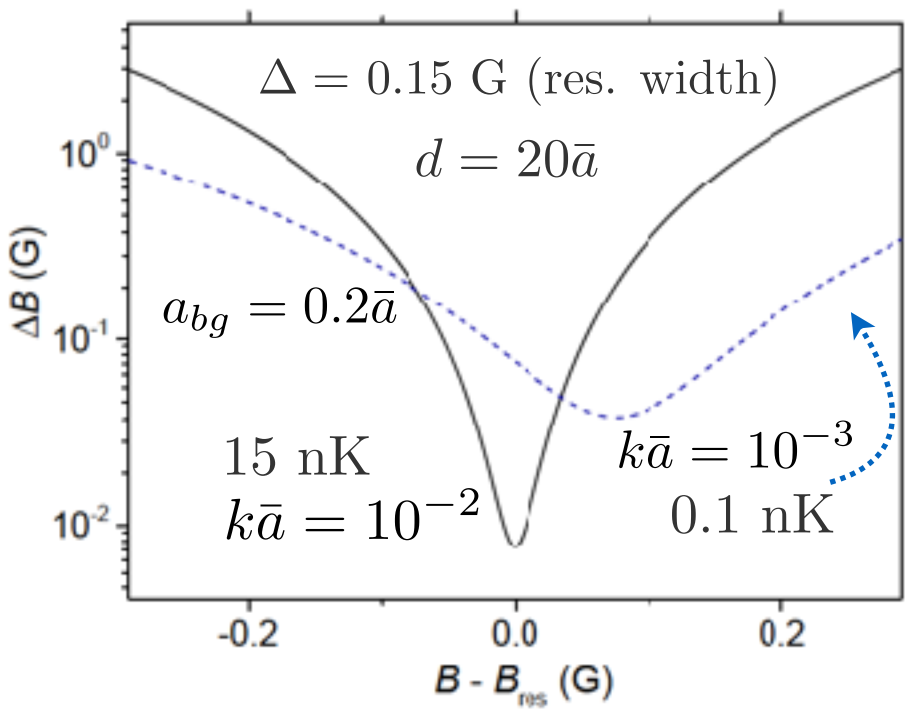
$$p(-|B) = 1 - T(B)$$

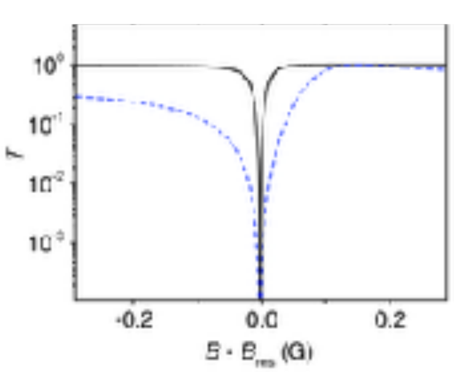


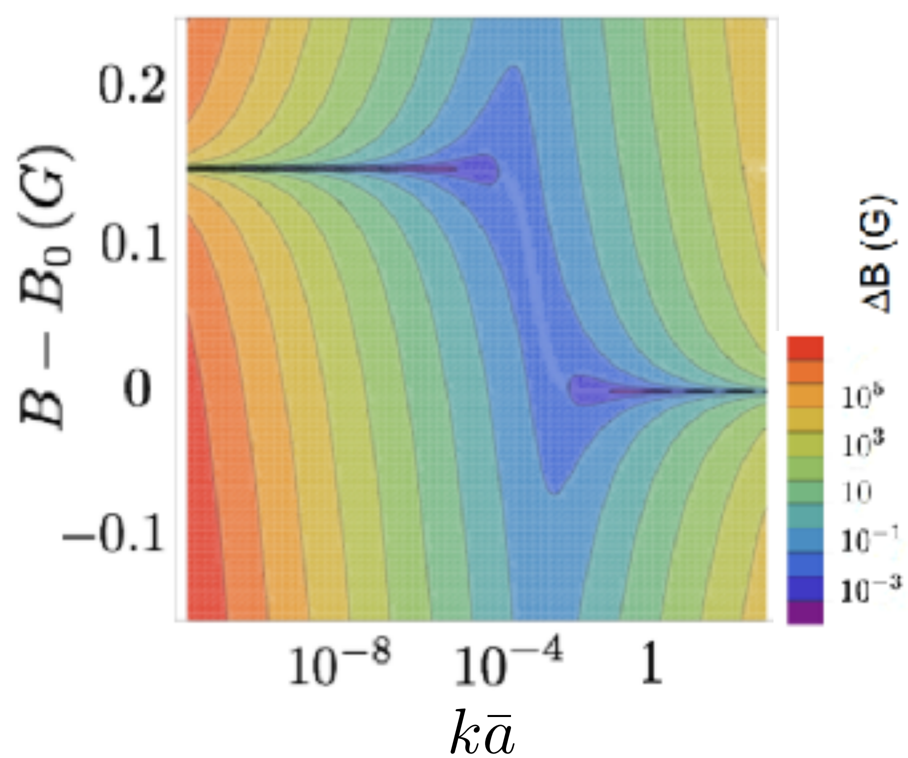
Fisher information:

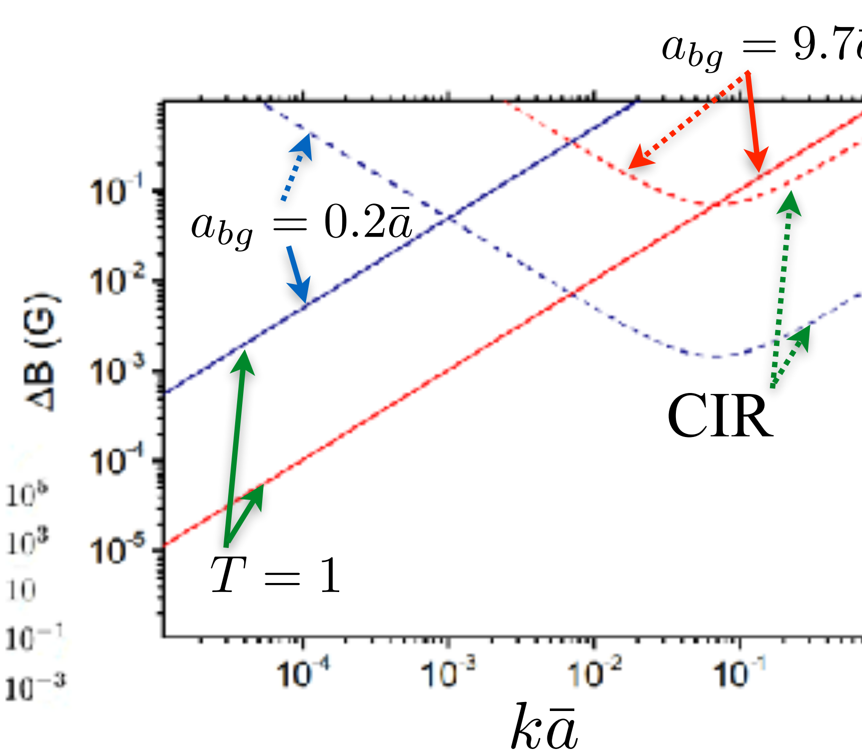
$$F = \sum_{s=\pm} \frac{1}{p(s|B)} \left(\frac{\partial p(s|B)}{\partial B} \right)^2$$

Sensitivity for *T*=0, e.g.,
for lanthanide atoms:









$$\bar{a} = \frac{2\pi}{\Gamma(\frac{1}{4})^2} \left(\frac{2\mu C_6}{\hbar^2} \right)^{1/4}$$

For Cs-Cs $\bar{a} = 95.5\ a_0$

- K. Jachymski, T. Wasak, *et al.*, PRL **120**, 013401 (2018)

Imperfections

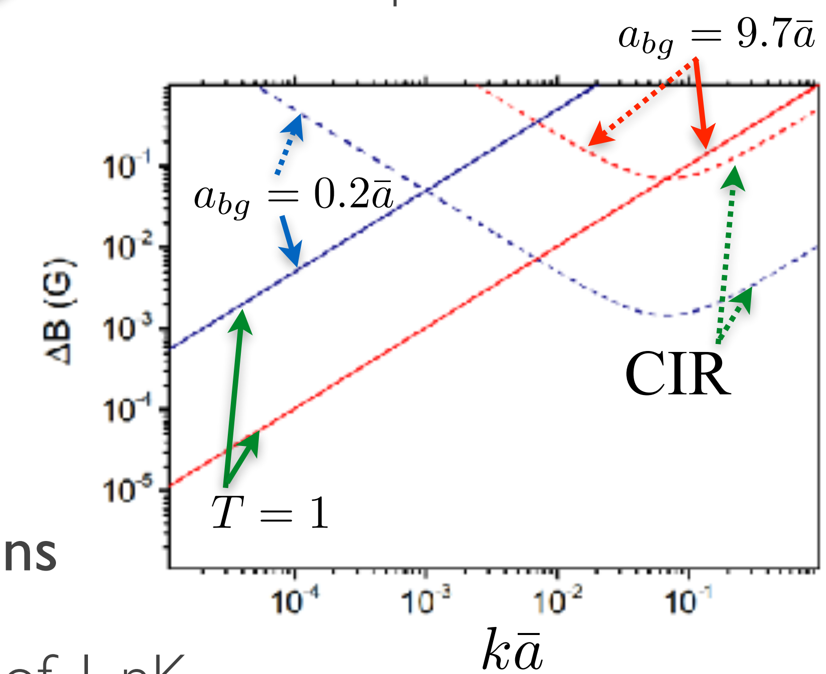
Finite longitudinal momentum

$$\Delta B \simeq a_{\mathrm{bg}} \Delta \left(\frac{1}{pd^2} + \frac{C^2 p}{4} \right) \qquad C \approx 1.4603$$

→ at the CIR position

$$\Delta B \simeq \frac{\Delta p d^2}{4 a_{\mathrm{bg}}}$$

→ when $T = 1$



Fluctuations of the resonance position due to finite-energy corrections

These result in uncertainties on the order of 1 nT for an energy width of 1 nK

Imperfect detectors

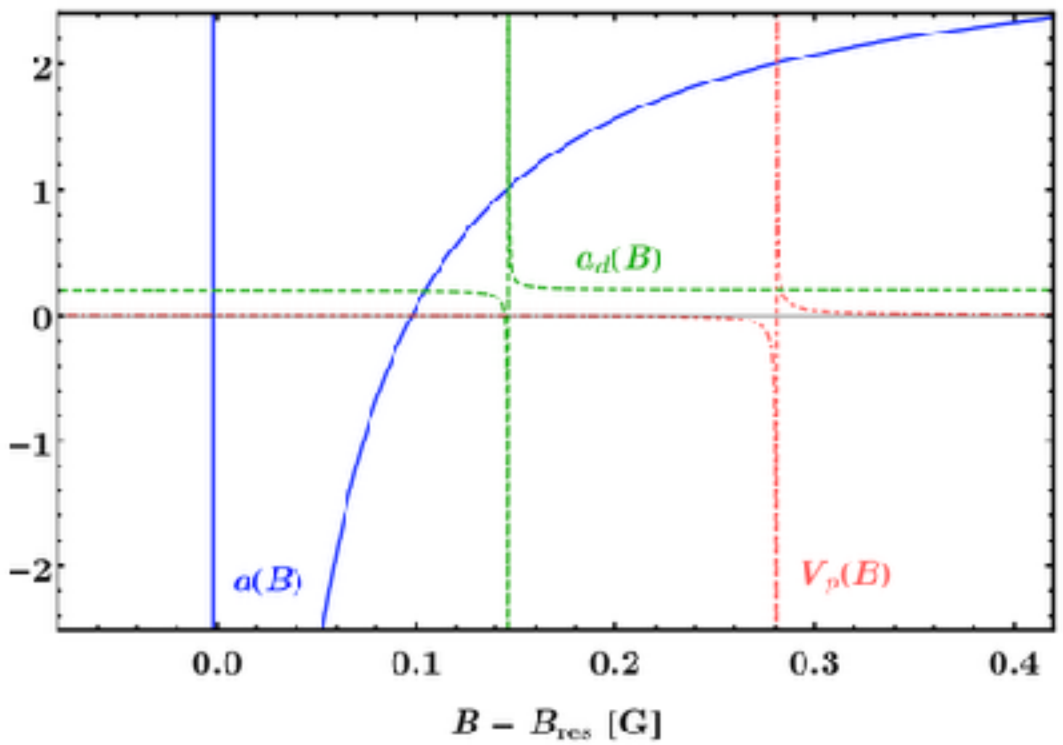
We measure whether the injected atom was transmitted or reflected with efficiency η

$$P(\pm|B) \mapsto \eta P(\pm|B) \;\Rightarrow\; F \mapsto \eta F \;\Rightarrow\; \Delta B \mapsto \Delta B / \sqrt{\eta}$$

◉ K. Jachymski *et al.*, PRL **120**, 013401 (2018)

Long-range interactions: Do they help?

Long-range interactions imply contribution from higher partial-waves



- B. E. Granger and D. Blume, PRL **92**, 133202 (2004)
- P. Giannakeas, F. K. Diakonos, P. Schmelcher, PRA 86, 042703 (2012)

- T. Wasak, K. Jachymski, T. Calarco, AN, PRA **97**, 052701 (2018)

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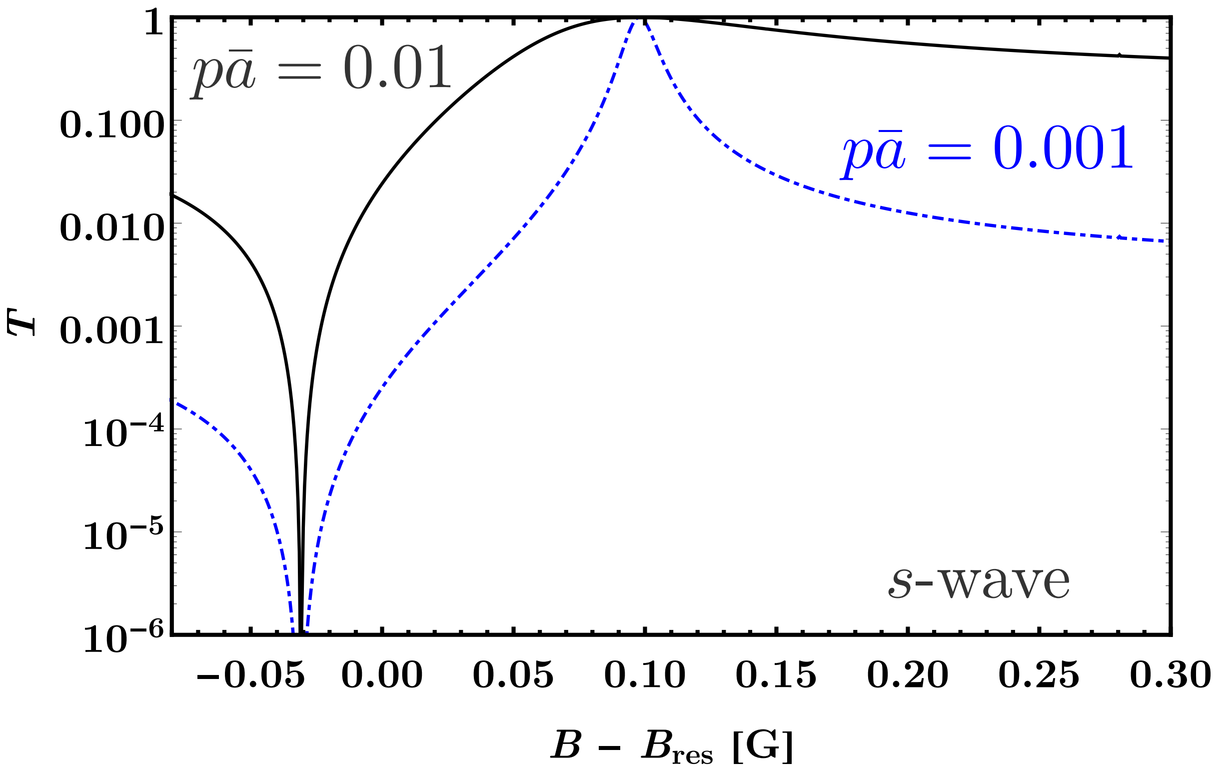

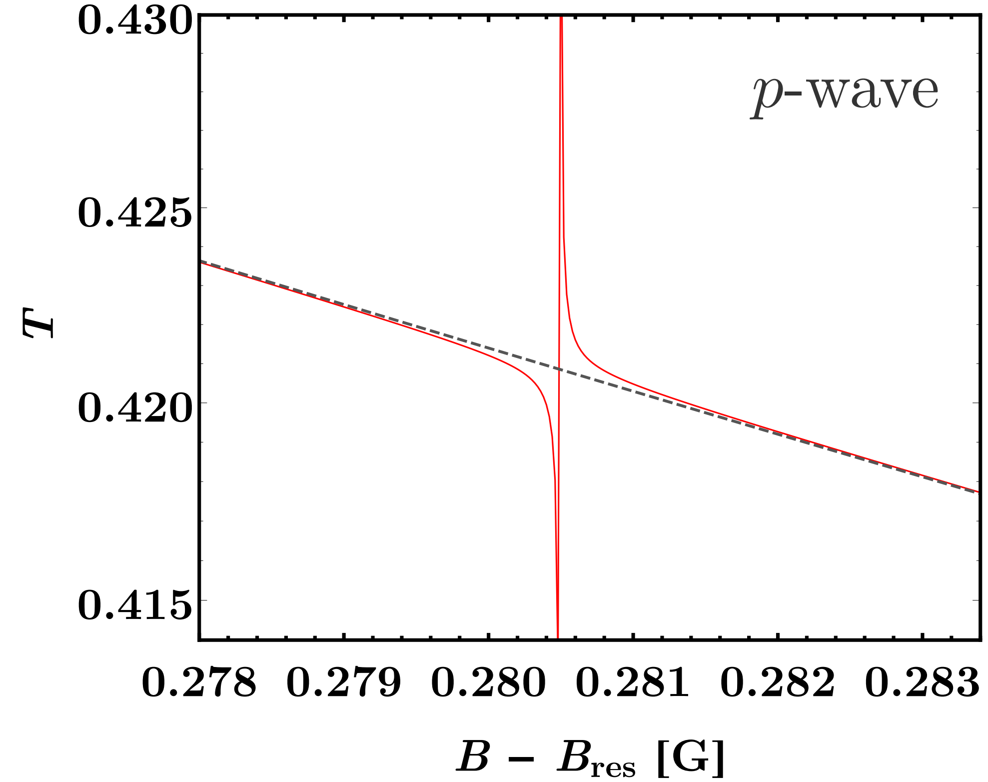

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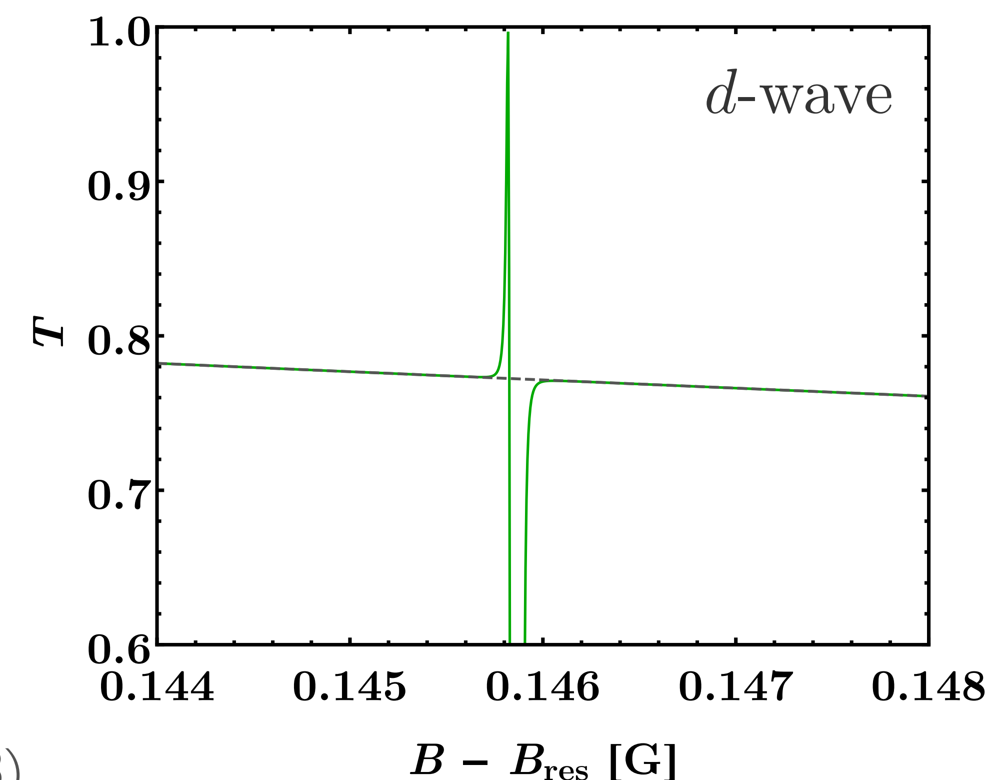
For Cs-Cs $\bar{a} = 95.5\ a_0$

$\Delta = 0.15$ G (res. width)

$d = 20\bar{a}$



- T. Wasak, K. Jachymski, T. Calarco, AN, PRA **97**, 052701 (2018)

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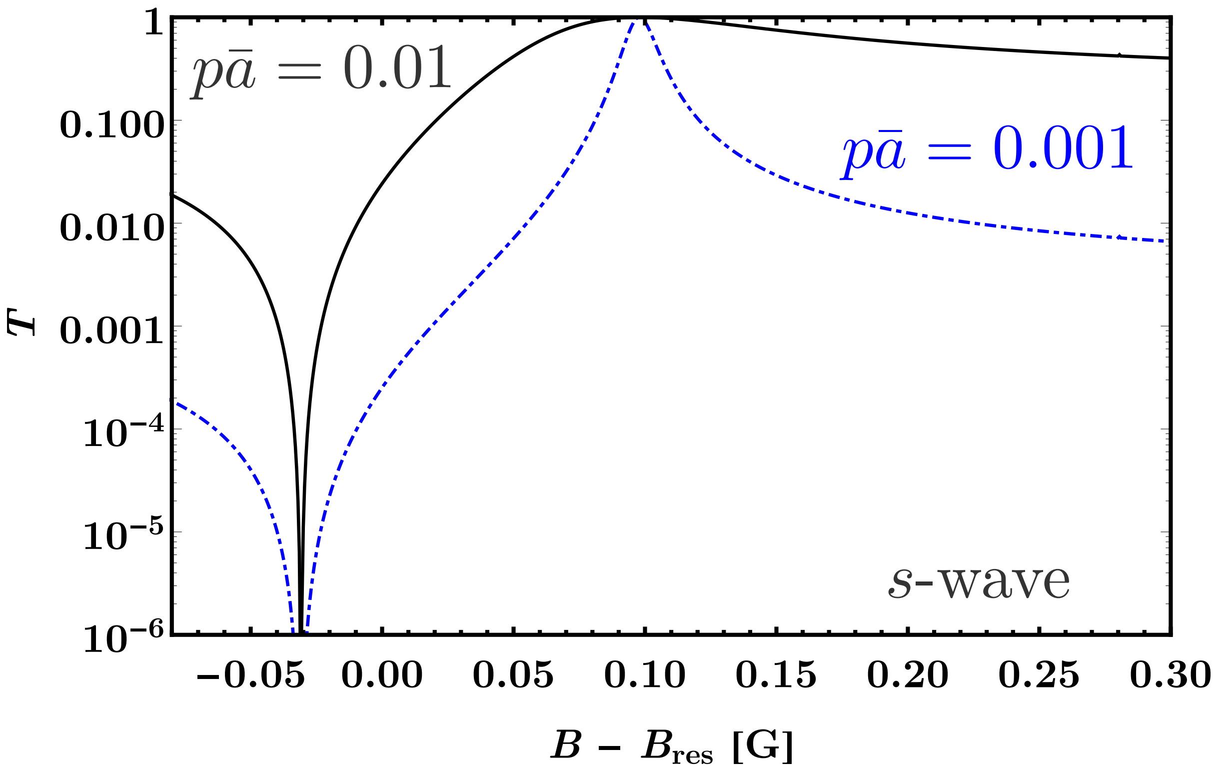

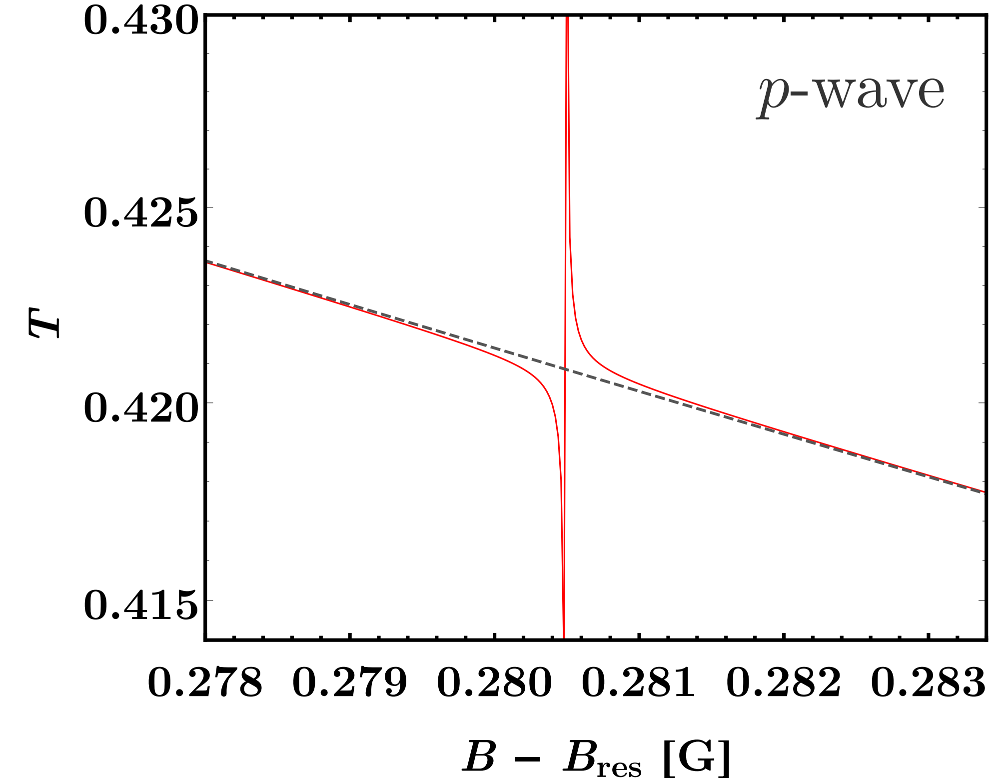

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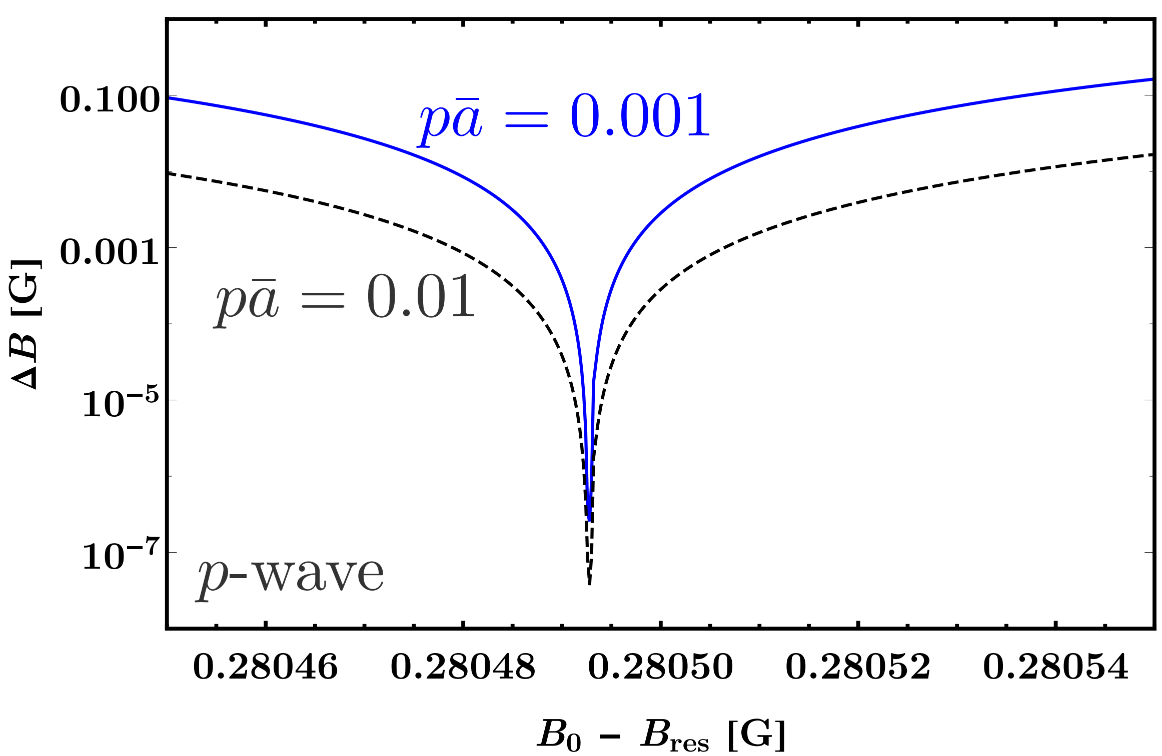

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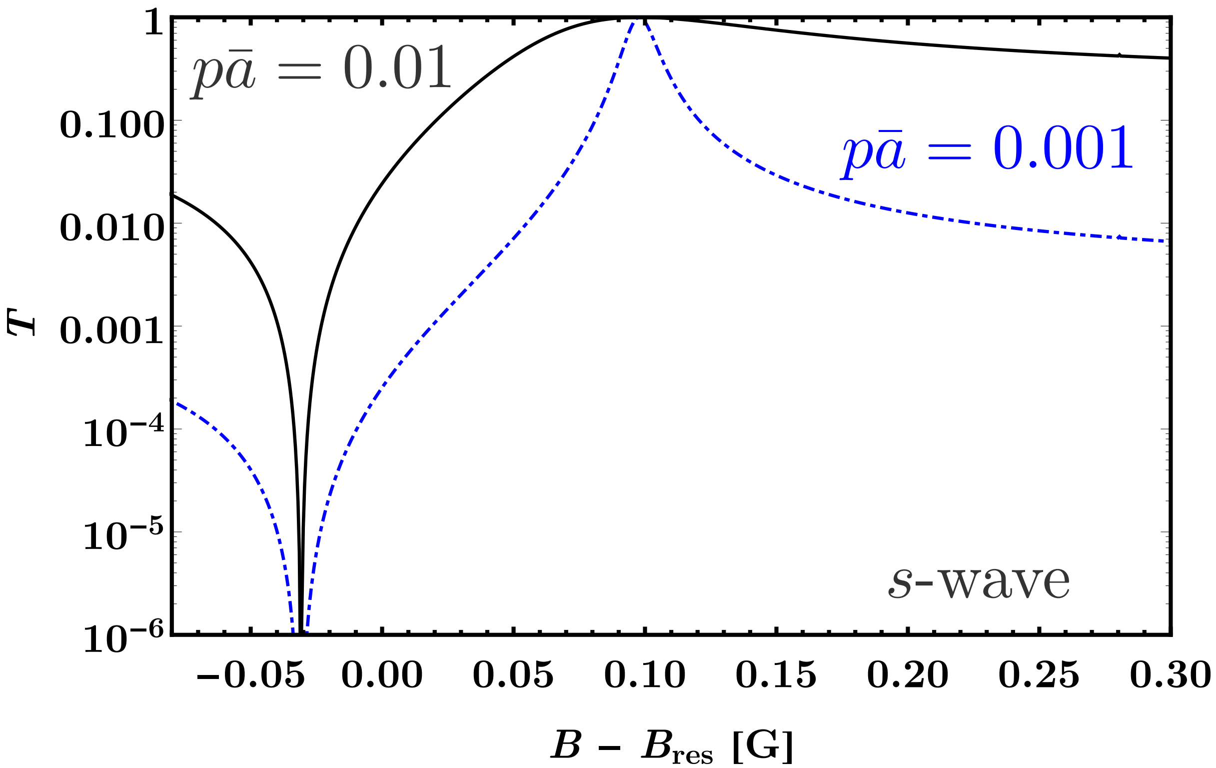

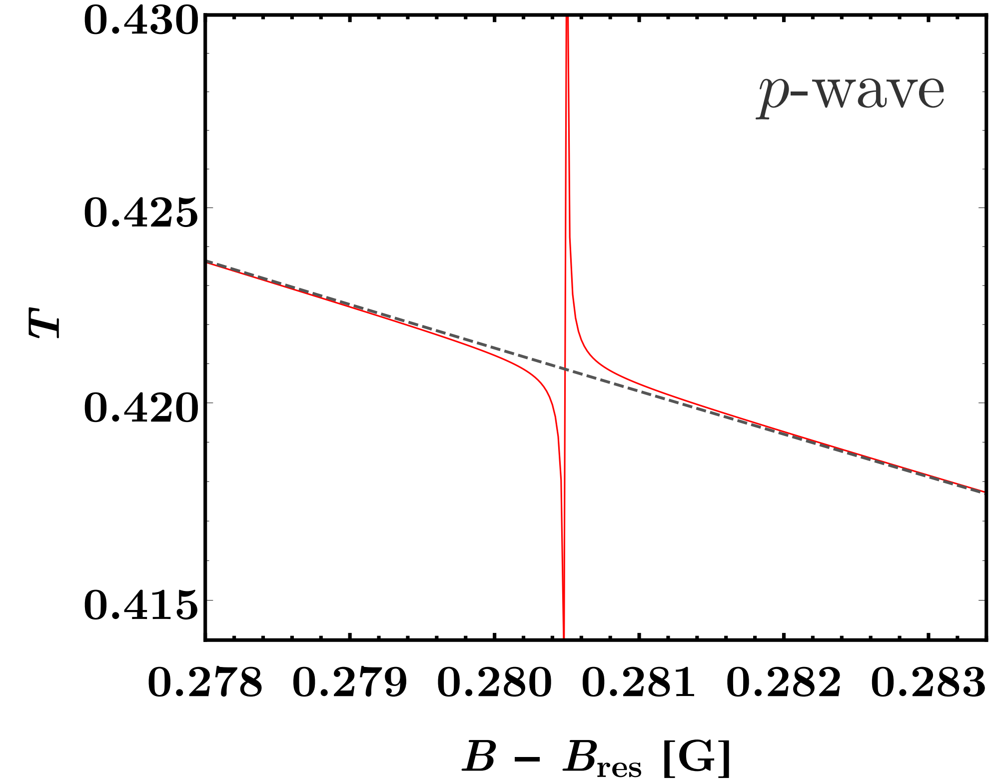

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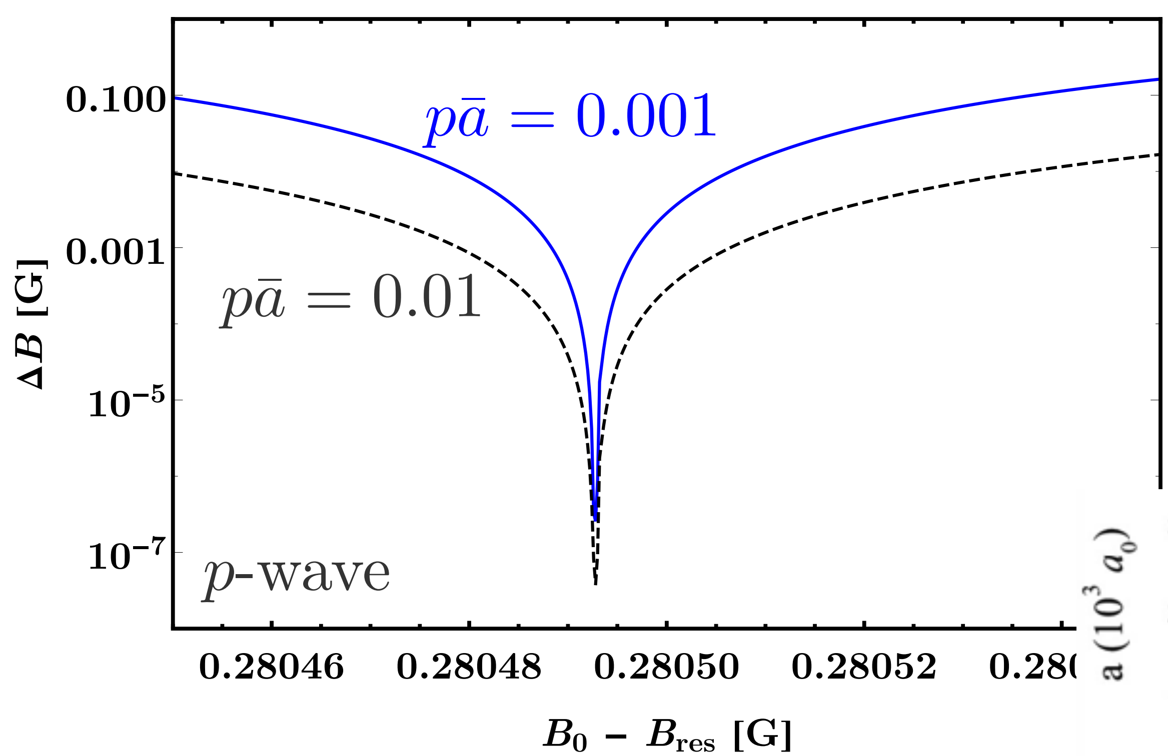
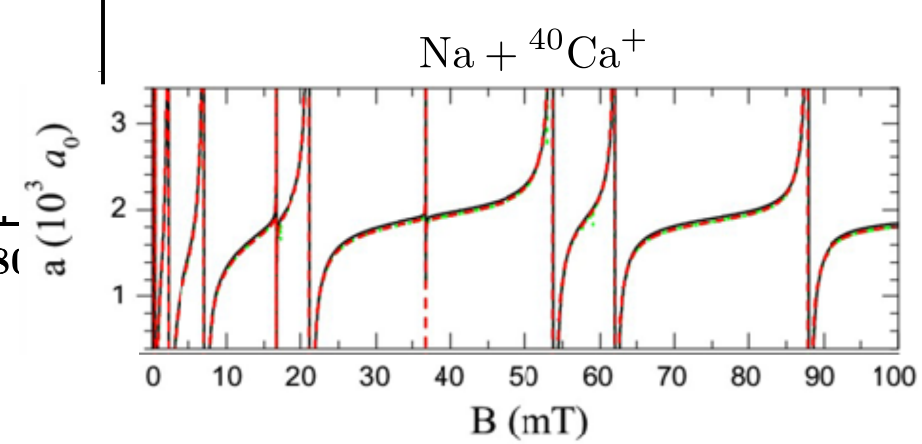


Conclusion:
Long-range interactions provide narrower resonances yielding higher sensitivity, BUT harder to resolve

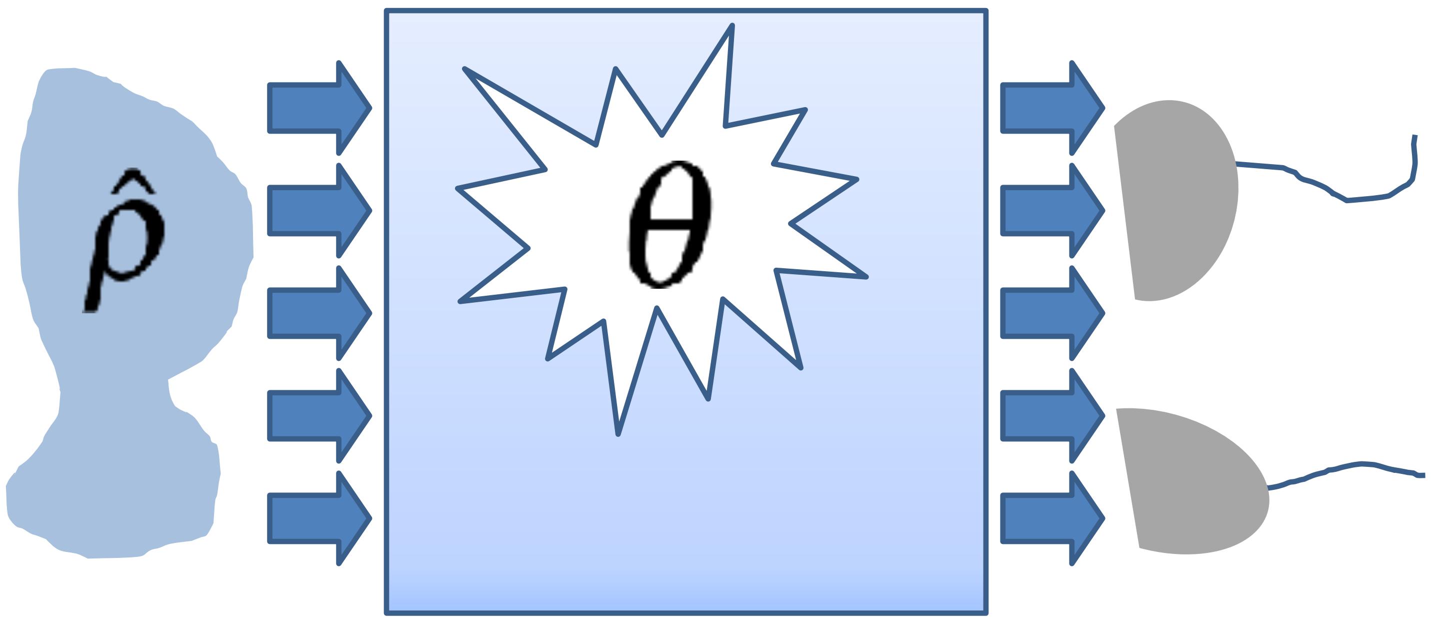


- T. Wasak, K. Jachymski, T. Calarco, AN, PRA **97**, 052701 (2018)

Quantum Fisher information

What is quantum in the Fisher Information?

General metrological scheme:



State of the system after transformation $\hat{\rho}(\theta)$

QM provides pdfs:

$$p(x|\theta) = \mathrm{Tr}[\hat{\rho}(\theta)\hat{\Pi}(x)]$$

Measurement operator:

$$\hat{\Pi}(x) = |x\rangle\langle x|$$

CRLB: maximum information for given measurement → Fisher information *F*

F depends on the specific choice of the measurement

Maximum precision allowed by QM → **Quantum Fisher Information**

$$\max_{\text{measurements}} F \equiv F_Q \longrightarrow F_Q = 2\sum_{i,j} \frac{(p_i - p_j)^2}{p_i + p_j} |\langle i|\hat{h}|j\rangle|^2$$

Eigenvalues and eigenvectors of $\hat{\rho}(\theta)$

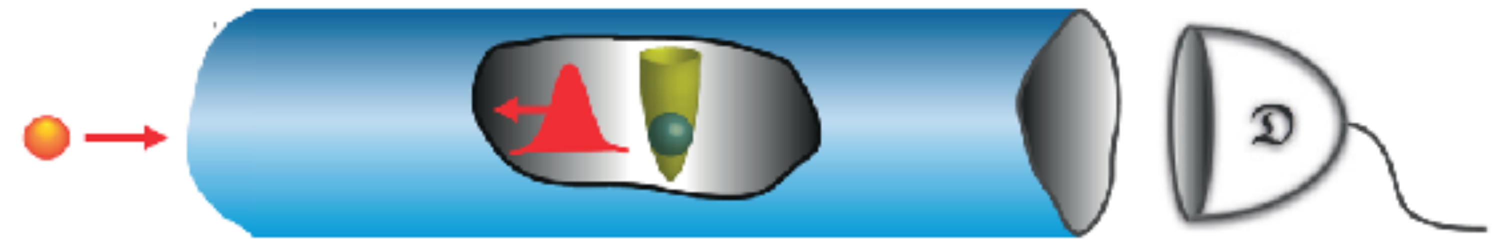
Generator of transformation

$$\hat{\rho}(\theta + \delta\theta) = e^{i\delta\theta\hat{h}}\hat{\rho}(\theta)e^{-i\delta\theta\hat{h}}$$

◉ S. L. Braunstein, C. M. Caves, PRL **72**, 3439 (1994)

Magnetic field sensitivity

- Can we do better?
- Is the scheme optimal?



before $|\psi_+\rangle$ $\xrightarrow{\text{collision}}$ after $r|\psi_-\rangle + t|\psi_+\rangle$

Rewrite amplitudes by using

$$\cot \delta_{1D} \equiv k a_{1D}$$

$$r = i \sin \delta_{1D} e^{i\delta_{1D}}$$

$$t = \cos \delta_{1D} e^{i\delta_{1D}}$$

Generator of the transformation

$$\boxed{\hat{h} = (\hat{1} + \hat{\sigma}_x) \frac{d\delta_{1D}(B)}{dB}}$$

Quantum Fisher Information (QFI)

$$F_Q = 4\left(\frac{d\delta_{1D}(B)}{dB}\right)^2 = F$$

$$T(B) = \cos^2 \delta_{1D}(B)$$

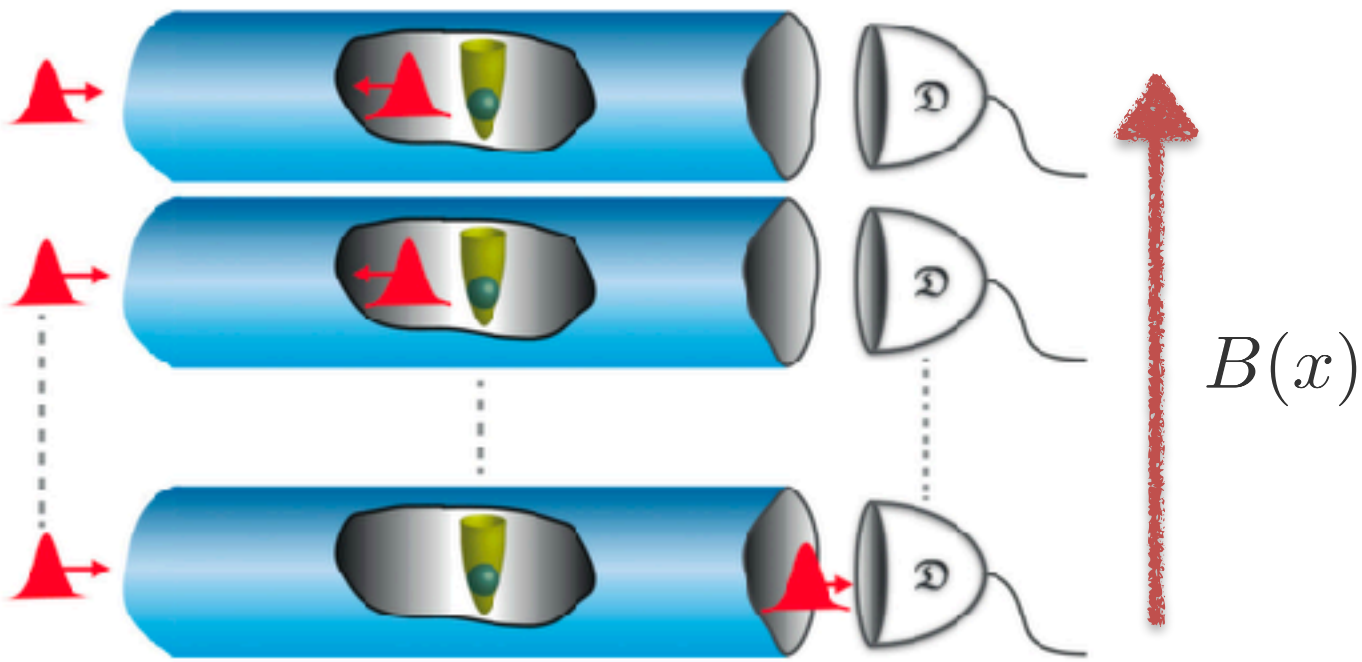
1. We know the **state**
2. We know the **generator**
3. We can calculate **QFI**

$\Rightarrow$ $\boxed{\textbf{QFI=CFI}}$

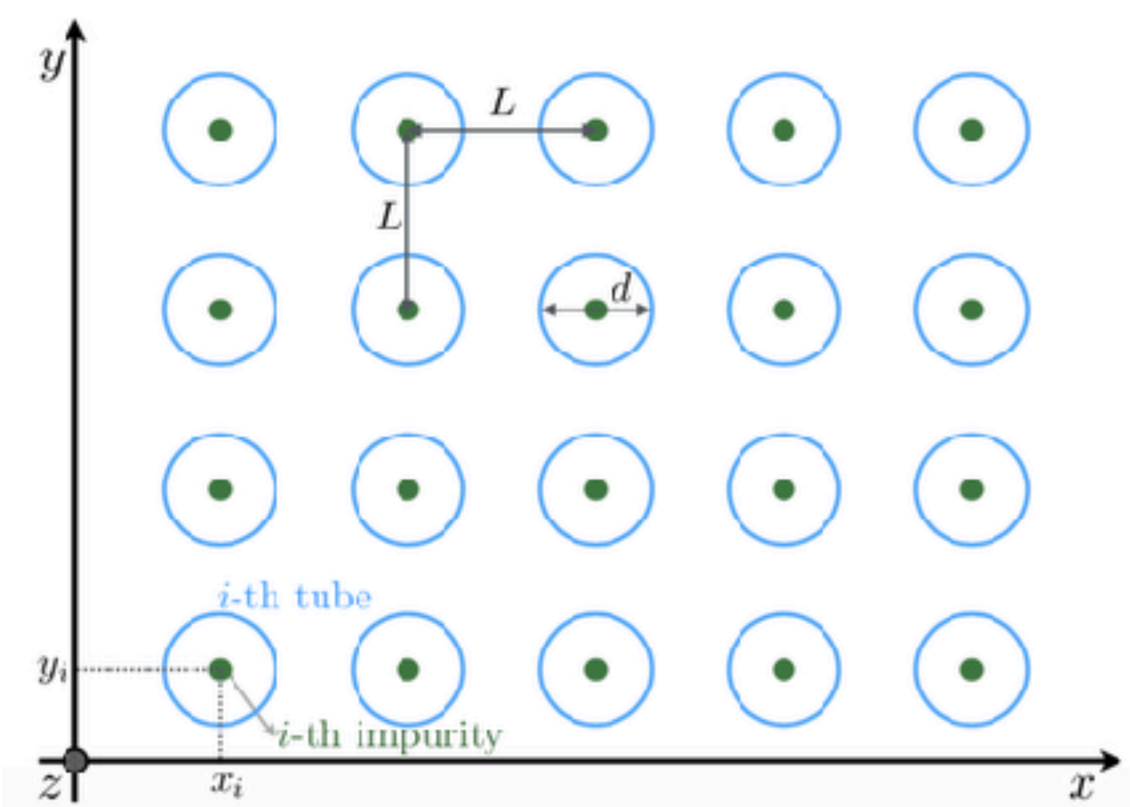
K. Jachymski *et al.*, PRL **120**, 013401 (2018)

Magnetic field gradient estimation

- T. Wasak, K. Jachymski, T. Calarco, AN, PRA **97**, 052701 (2018)



Estimation problem with 1+2 parameters:
- Field strength
- Field gradient components



<u>Assumption:</u> The field varies smoothly

$$B(\mathbf{r}_i) = B_0 + B_{x_i} x_i + B_{y_i} y_i \qquad B_x \equiv \partial_x B, \;\; B_y \equiv \partial_y B$$

Fisher information becomes a matrix

$$F_{i,j} = \sum_x \frac{1}{p(x|\boldsymbol{\theta})} \frac{p(x|\boldsymbol{\theta})}{\partial \theta_i} \frac{p(x|\boldsymbol{\theta})}{\partial \theta_j} \qquad \boldsymbol{\theta} = (B_0, B_x, B_y)$$

$$\Delta\theta_i \geq \frac{1}{\sqrt{m}} [F^{-1}]_{i,i}^{1/2}$$

For each of the tubes

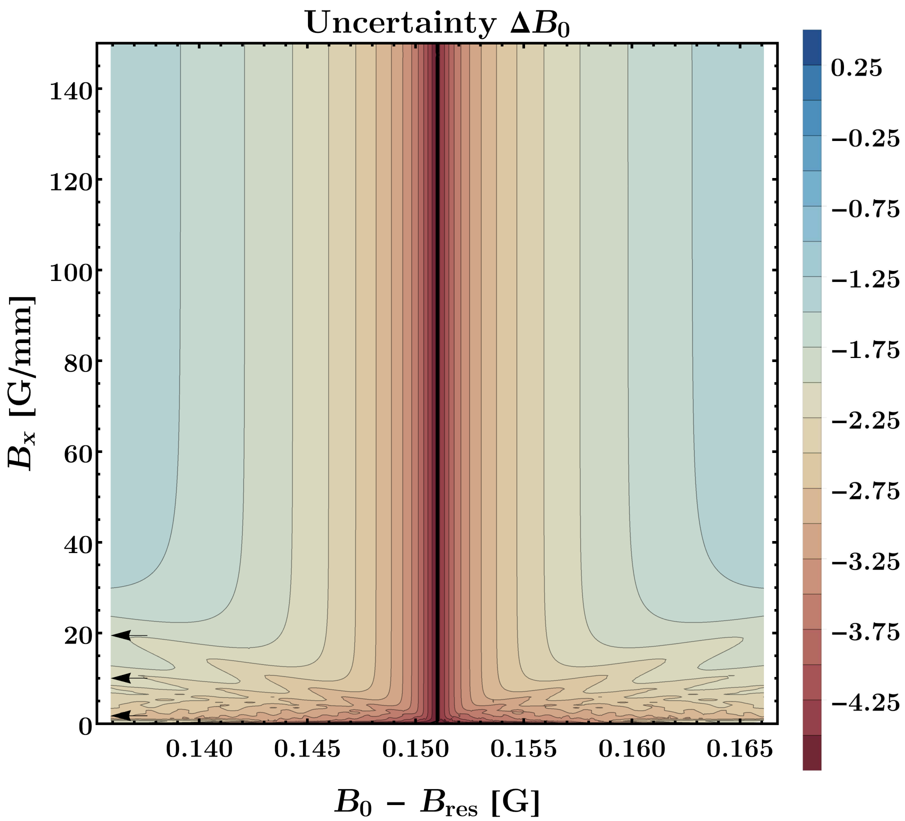
Results: Scenario for which T=1 (e.g. Cs atoms)

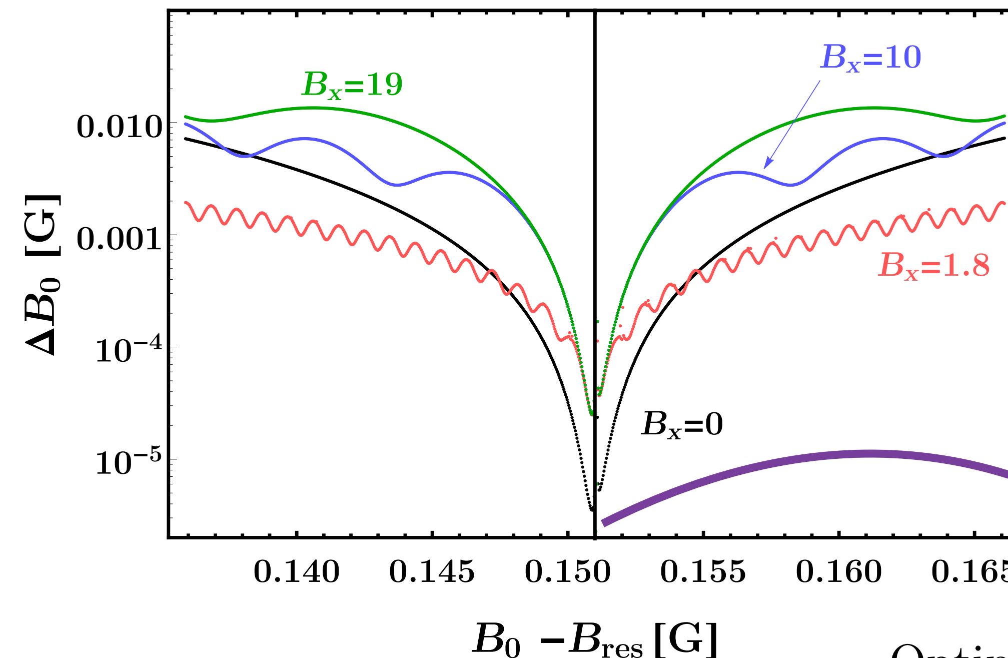
M = 51 × 51 tubes; L = 523 nm; B_y=0 $\quad \Delta = 0.15$ G (res. width)

$d = 20\bar{a}$

$a_{bg} = 9.7\bar{a}$

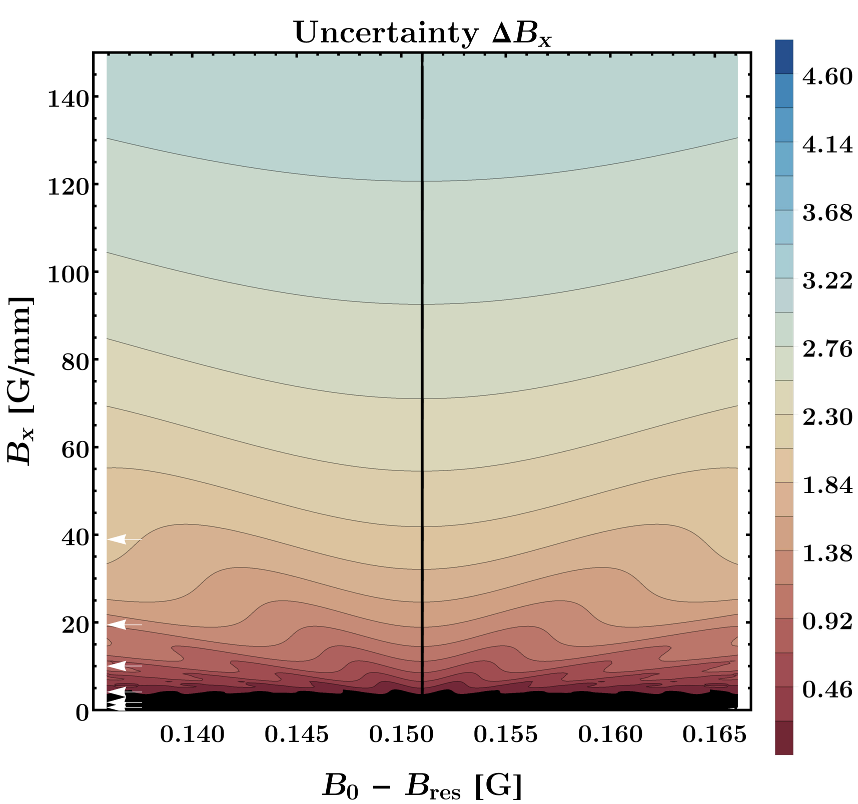
$k\bar{a} = 10^{-4}$

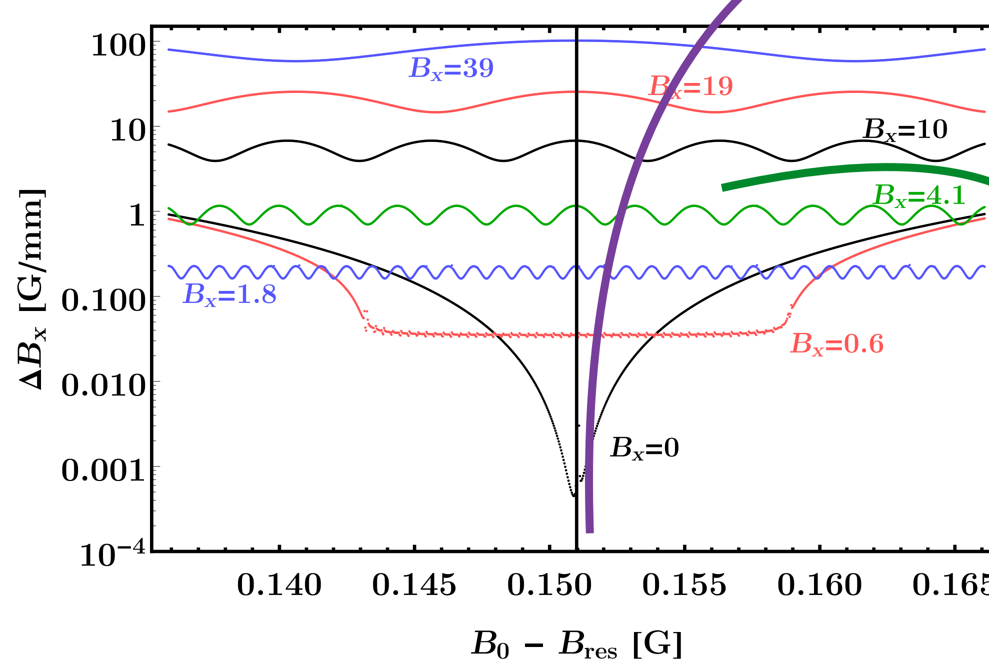


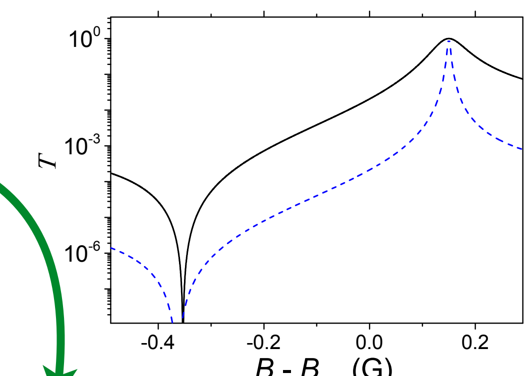


Optimal operating point:

$B_0 - B_{res} \simeq \Delta$







Highest performance for small gradients!

◉ T. Wasak, K. Jachymski, T. Calarco, **AN**, PRA **97**, 052701 (2018)

Conclusions and outlook

- Cold collisions in waveguides can be indeed useful for magnetometry with sensitivity in the range of nT and 100 nT/mm, but also for precise Feshbach characterisation
- Long-ranged interactions provide narrower resonances, thus enhancing the sensitivity, but harder to resolve (atom-ion systems afford richness of Feshbach resonances)
- With our simple scheme, i.e. measurement of the transmitted vs. reflected atoms, no quantum enhancement is possible
- Possible strategies to improve the sensitivity:

➡Repeated measurements: $\Delta B = \frac{1}{\sqrt{m}}\frac{1}{\sqrt{F}}$

➡Coherent input state of N atoms: $\Delta B = \frac{1}{\sqrt{N}}\frac{1}{\sqrt{m}}\frac{1}{\sqrt{F}}$

➡Entangled input state of N atoms (e.g. GHZ-state): $\Delta B = \frac{1}{N}\frac{1}{\sqrt{m}}\frac{1}{\sqrt{F}}$

$$\frac{1}{\sqrt{2}}\left(|\psi_+\rangle^{\otimes N} + |\psi_-\rangle^{\otimes N}\right)$$

- Multiple impurities in the waveguides (unfortunately, no significant improvement)

Thank you for your attention!

- T. Wasak, K. Jachymski, T. Calarco, **AN**, Phys. Rev. A **97**, 052701 (2018)
- K. Jachymski, T. Wasak, Z. Idziaszek, P. S. Juliene, **AN**, T. Calarco, Phys. Rev. Lett. **120**, 013401 (2018)