Staircase in Magnetization and Entanglement Entropy of Spinor Condensates

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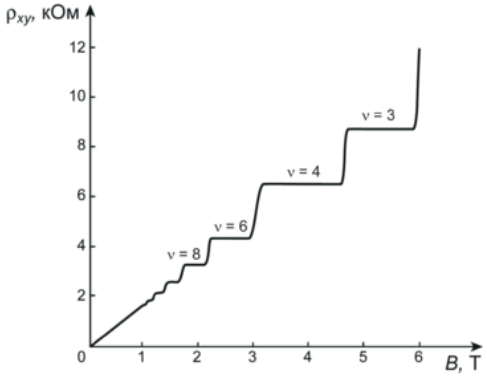
Staircase Response

- When an observable parameter of a system responds discretely to the continuous tuning of a control parameter, the corresponding response function takes the form of a staircase.
- A classic example is the integer quantum Hall effect, where the Hall conductivity responds discretely to continuous tuning of the applied magnetic field.

As a firm signature of quantization in many-body systems, staircase response functions can also be used to develop precise measurement devices.



New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing, G. Dorda, and M. Pepper
Phys. Rev. Lett. **45**, 494 – Published 11 August 1980

An article within the collection: Letters from the Past - A PRL Retrospective

Staircase Response Functions in Bosons

Rotating Scalar BECs

- A spinless, non-inteacting, rotating BEC in a harmonic trap is characterized by Landau levels, similar to a 2D electron gas in a magnetic field and has been predicted to display a staircase response in the presence of weak interactions.
- The effective Hamiltonian is $H = U_I + \Omega L_z$, where U_I is the interaction term and L_z is the orbital angular momentum in the z direction. The ground state is an eigenstate of L_z, whose eigenvalue depends on the strength of U_I relative to Ω.
- The response function is in general calculated using numerical methods — hard to scale up the number of atoms.

Outline of Our Results

Our Result

We show that the magnetization vector of spinor BEC, under commonly used interacting Hamiltonians responds discretely to continuous tuning of the applied magnetic field or the condensate density.

One Axis Twisting Hamiltonian

- A staircase response in the magnetization can be observed in a system of N interacting (pseudo) spin -1/2 atoms, under the one-axis twisting Hamiltonian,

$$H = \chi S_z^2 - pS_z.$$

  χ is the strength of interaction and p is the effective magnetic field.
- This Hamiltonian was proposed by M. Ueda in 1991 to produce spin-squeezed states and subsequently, has been implemented in many cold atomic systems.
- The eigenstates $|m\rangle$ of S_z are also eigenstates of H and the eigenenergies are

$$H|m\rangle = E_m|m\rangle \text{ where } E_m = \chi m^2 - pm$$

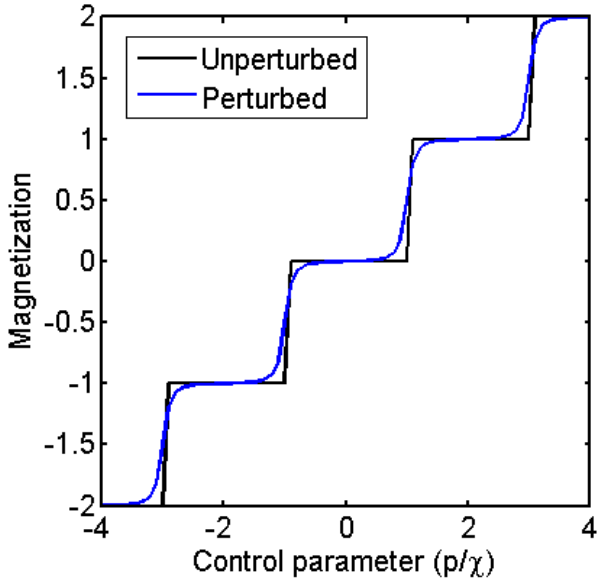
Staircase in One Axis Twisting Hamiltonian

The ground state magnetization is obtained by minimizing the energy over m.

$$m_{gs} = \left[\frac{p}{2\chi}\right].$$

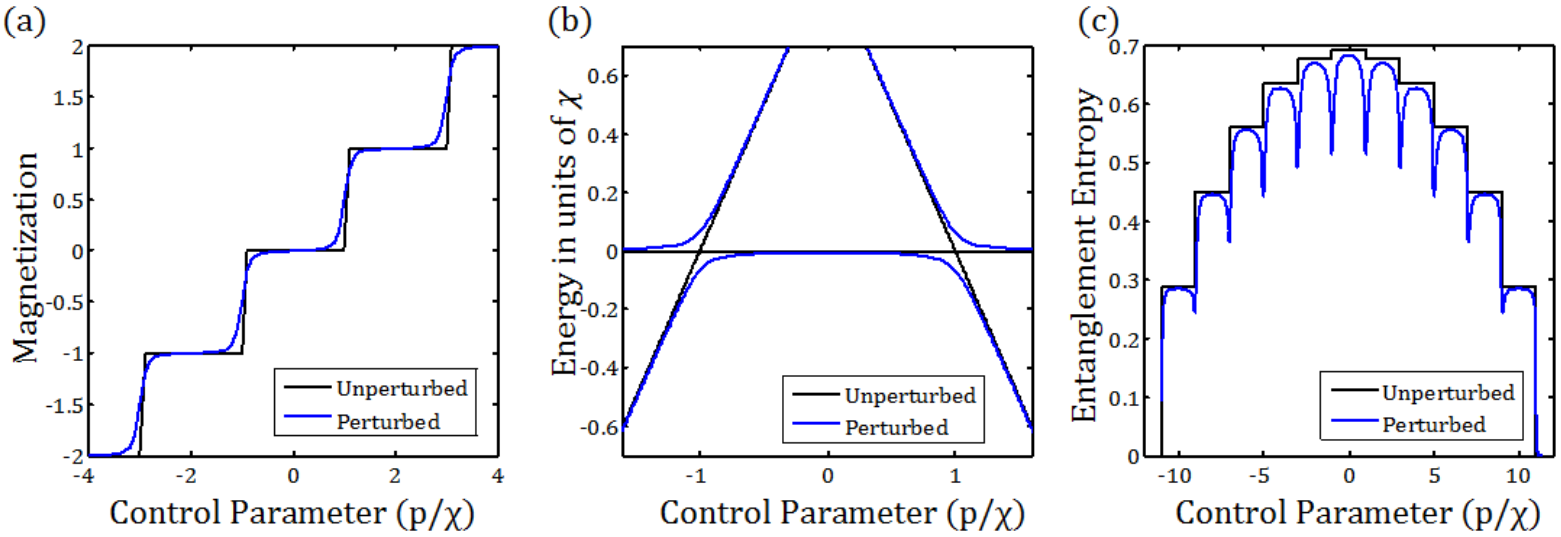
It responds discretely, to a continuous adiabatic change in p, the applied magnetic field.

Therefore, the response of the system's magnetization to adiabatic changes in the control parameter takes the form of a staircase.



How do we observe it?

- Every jump in the staircase corresponds to a level crossing between two magnetization eigenstates and therefore, cannot be crossed adiabatically.
- Adding a small perturbative field in the x direction, ϵS_x, to the Hamiltonian opens up a non-zero gap at the level crossing making it possible to maintain adiabaticity.

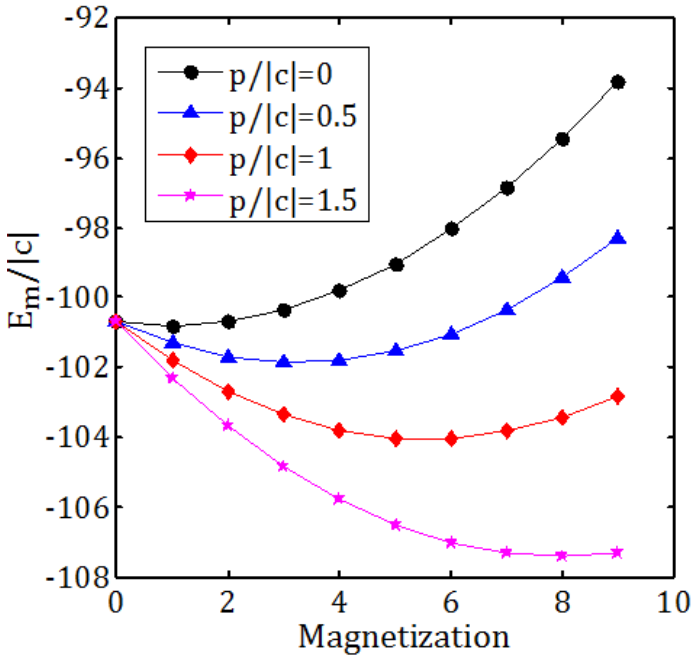


A General Statement

General Statement

If $\{\psi_1, \psi_2, \cdots\}$ are the eigenstates of a Hamiltonian H' with eigenvalues E_m convex in m, the ground state of the Hamiltonian $H = H' - p\sum_m m|\psi_m\rangle\langle\psi_m|$ has a staircase structure with respect to the control parameter p.

- The eigenenergies of H are $E_m = E'_m - pm$. The minima of this energy can be shifted in integer steps by tuning p.
- In the figure, c is energy scale of E'_m and it shows E_m as a function of m for various value of p.



Anti-ferromagnetic spin-1 condensates

- A staircase response in the direction of magnetization can be observed in a system of N interacting spin-1 anti-ferromagnetic atoms, under the Hamiltonian,

$$H = cS^2 - pS_z.$$

  $c > 0$ is the strength of interaction and p is the applied magnetic field.
- This is the Hamiltonian of a system of ^{23}Na BEC in an optical dipole trap.
- The common eigenstates $|s, m\rangle$ of S^2 and S_z, where $S^2|s, m\rangle = s(s+1)|s, m\rangle$ and $S_z|s, m\rangle = m|s, m\rangle$ are also eigenstates of H and the eigenenergies are

$$H|s, m\rangle = E_{s,m}|m\rangle \text{ where } E_{s,m} = cs(s+1) - pm$$

Staircase in magnetization of an antiferromagnetic BEC

- The ground state is obtained by minimizing the energy over s, m and takes the form $|s, s\rangle$, with

$$s = 2\left[\frac{p-c}{4c}\right].$$

  The magnetization responds discretely, to a continuous adiabatic change in c, the interaction strength.
- We show now that by adding a suitable perturbation to the Hamiltonian, this staircase structure can be transferred to the direction of the magnetization.

Staircase in the direction of the magnetization.

- Let us perturb the Hamiltonian by adding a $Q_{xz} = \sum_{i=1}^{N} \{L_{xi}, L_{zi}\}$, where L_{xi} is the spin-x operator for the $i-$th atom. The new Hamiltonian is $H = cS^2 - pS_z + \alpha Q_{xz}$ with $\alpha << c$.
- The staircase structure remains, with a set of perturbed ground states:

$$|s, s\rangle + \frac{\alpha}{p} q_s |s, s-1\rangle$$

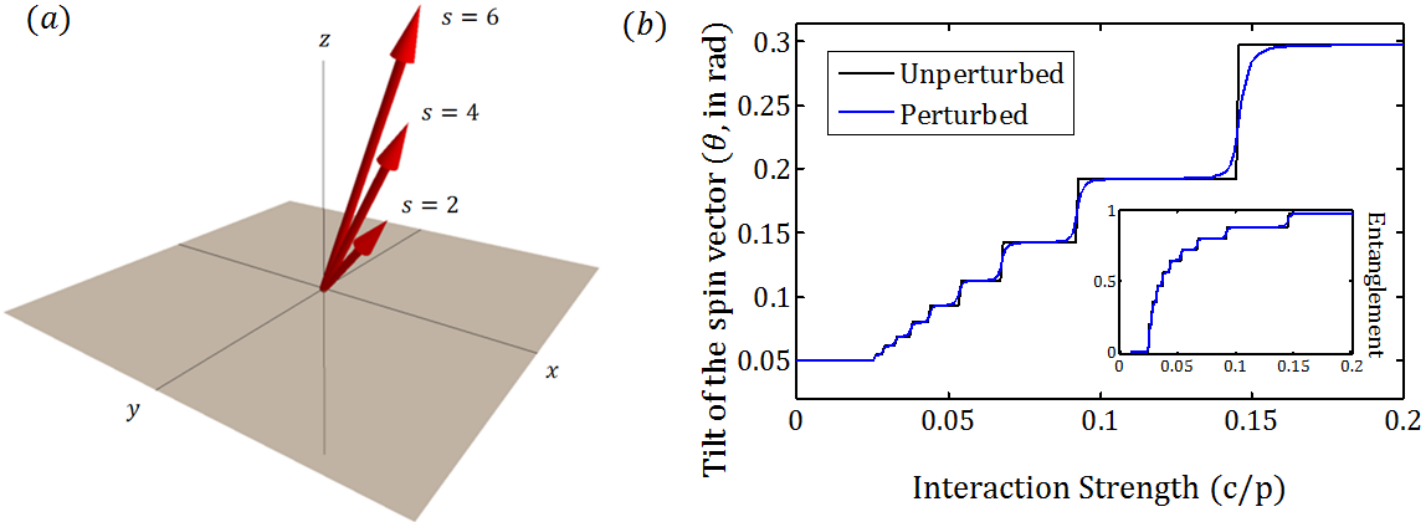
with $s = 2\left[\frac{p-c}{4c}\right]$ and $q_s = \langle s, s|Q_{xz}|s, s-1\rangle$.

- The magnetization vector is now $\vec{m} = (\frac{\alpha}{p}\sqrt{2s}, 0, s)$ and takes discrete steps in the tilt angle with the $z-$axis.

Staircase in the direction of the magnetization.

The tilt angle of the magnetization vector is

$$\theta_s = \arctan\left(\frac{2N+3}{2s+3}\frac{\alpha}{2p}\right)$$



Experimental Considerations
Detection Noise

There are three effects that limit our experimental ability to observe this staircase response function.

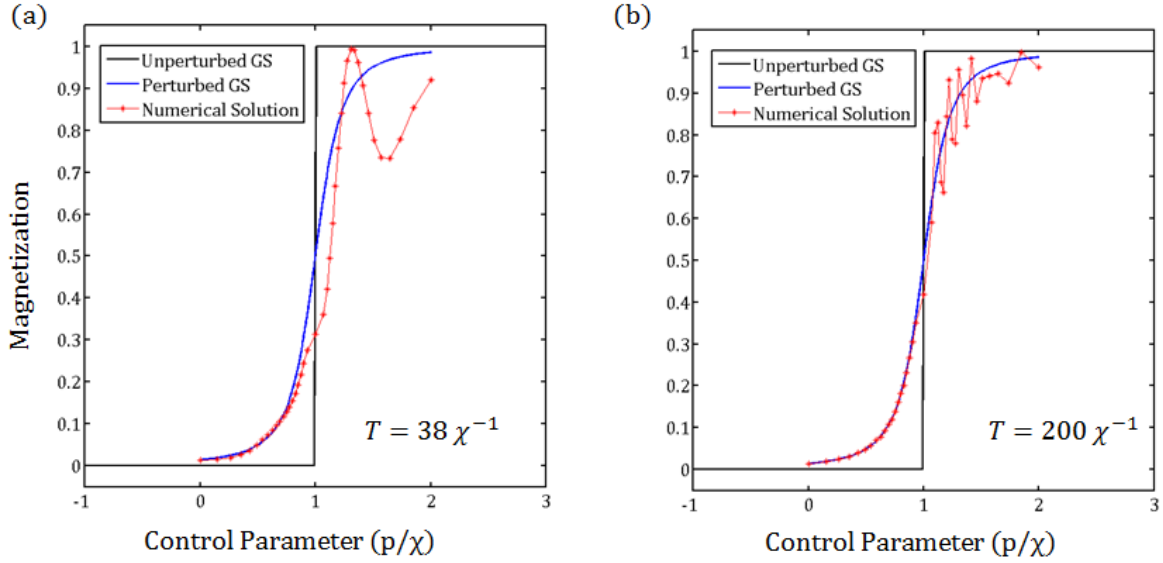
1. Atom detection noise.
2. Loss of atoms from the trap.
3. Phase noise.

The current limits on the detection of atoms is ± 3 atoms for a BEC in an atom chip (M. Fadel et. al., Science 2018). A single atom detection limit has been reported in ion trap and neutral atom quantum simulators (H. Bernien et. al., Nature 2018 and J. Zhang et. al., Nature 2017).

Experimental Considerations

How fast can an adiabatic ramp be?

- In order to avoid the smearing out of the staircase due to particle loss, one has to design an adiabatic ramp that takes a shorter time (T) than the atom loss time scale.
- An efficient adiabatic ramp is generated by maintaining a constant landau-Zener parameter throughout the ramp. The Landau Zener parameter is defined, in terms of the energy gap Δ as $\Gamma = \frac{\Delta^2}{d\Delta/dt}$



Experimental Considerations

Atom loss and phase noise

- For BECs in a chip trap, typically, $\chi \sim 3$Hz (M. F. Riedel et. al., Nature 2010) and therefore, using the parameters of fig (a) in the previous slide, $T \approx 13$s, which is lesser than the typical atom loss timescale (~ 20s).
- In ion trap and neutral atom quantum simulators, $\chi \sim 0.5$KHz and therefore, such an experiment is more easily feasible in these systems.
- In order to estimate the phase noise, we consider the total number of phase cycles during the adiabatic ramp, given by $n = \int \Delta(t)dt$. This is about 24 for the parameters in fig (a) and 120 for fig (b) above and adiabatic ramps with close to 100 phase cycles have been implemented recently, in ^{87}Rb BECs (T. M. Hoang et. al., PNAS 2016).

Conclusions

- We have described a staircase response function that appears in the magnetization of spinor condensates,, in response to continuous tuning of the applied magnetic field or condensate density.
- The considered Hamiltonians can be implemented in trapped atoms and simulated in trapped ions, double well systems and optical tweezers.
- Reference: arXiv:1804.03745

The Group

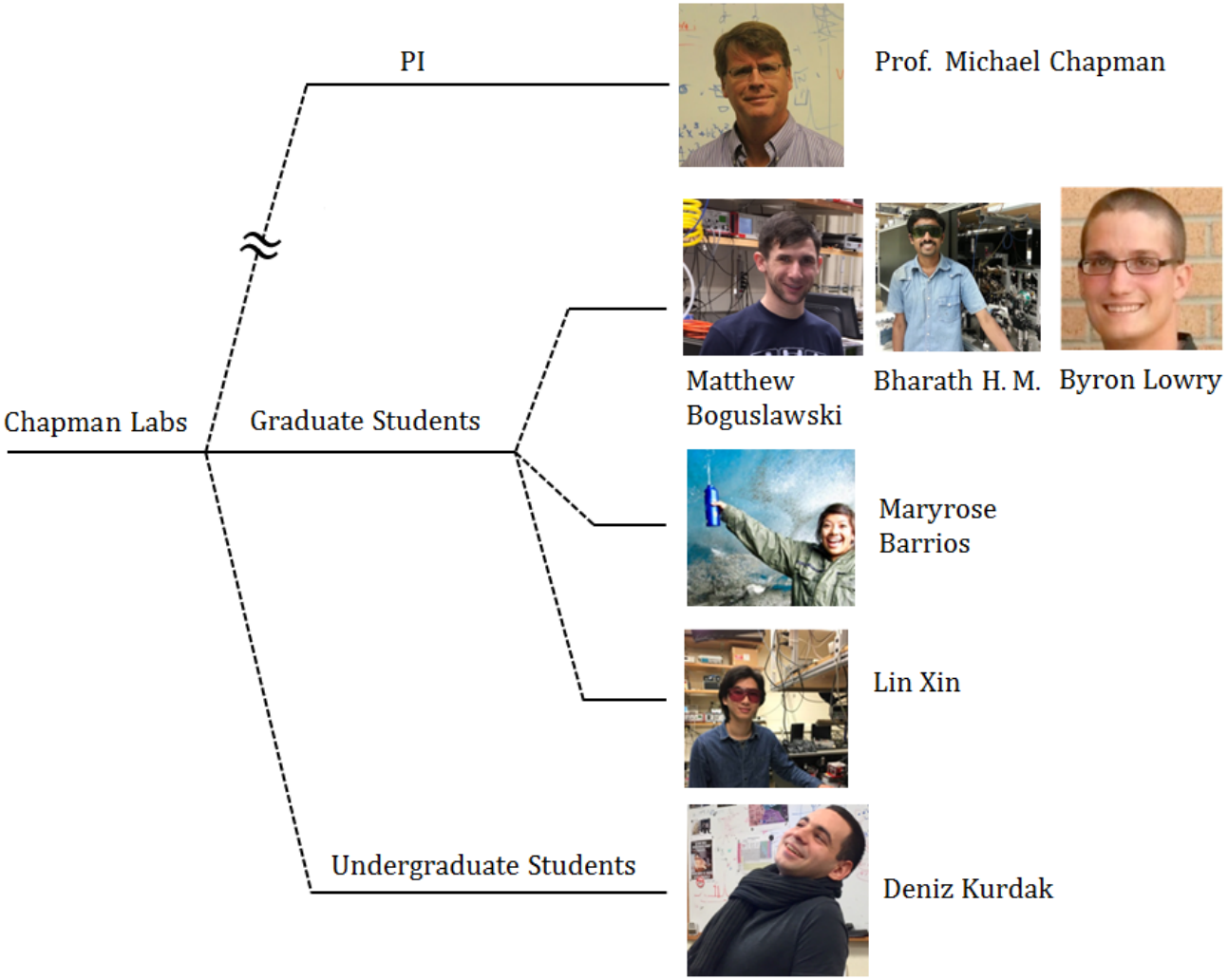


Figure : Hyperfine structure of Chapmanlabs