Josephson oscillations of strongly-correlated one-dimensional Bose gases

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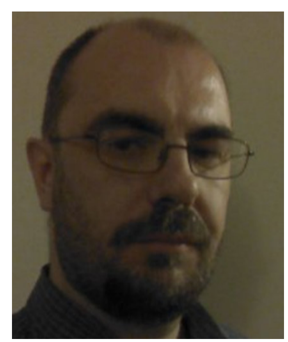
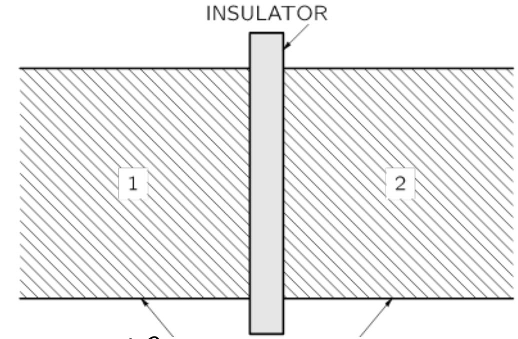


The Josephson effect

– Two phase-coherent systems, tunnel-coupled



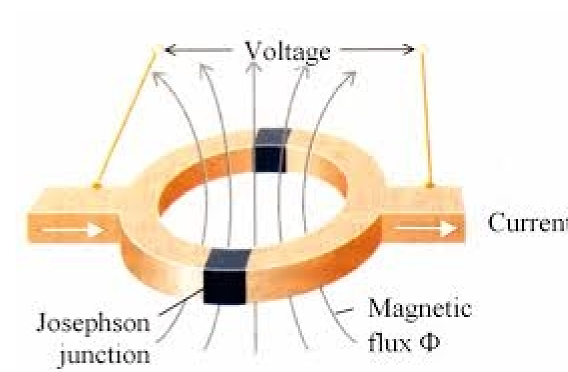
$$i\partial_t\psi_1 = E_1\psi_1 - K\psi_2$$
$$i\partial_t\psi_2 = E_2\psi_2 - K\psi_1$$

$$\psi_1 = \sqrt{n_1}e^{i\theta_1} \qquad \psi_2 = \sqrt{n_2}e^{i\theta_2}$$

$$\partial_t n = -2K\sqrt{n_1 n_2}\sin(\varphi)$$
$$\partial_t\varphi = E_2 - E_1$$

$$n = (n_1 - n_2)/2$$
$$\varphi = \theta_2 - \theta_1$$

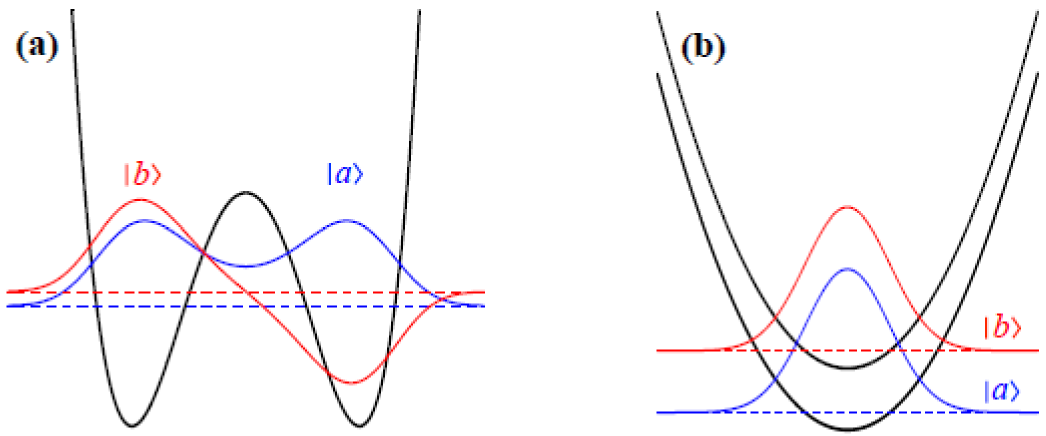
→ Heavily exploited in superconductors (Josephson junction arrays, important technological applications eg SQUIDS for magnetometers)



Bose-Josephson junctions

– Macroscopic coherence among two matter waves :

external or internal Josephson effect :



$$\Phi_1 = (\Phi_a - \Phi_b)/\sqrt{2}$$

$$\Phi_2 = (\Phi_a + \Phi_b)/\sqrt{2}$$

– The basic model : two-mode approximation of the Gross-Pitaevskii mean field equation

$$\Psi(x,t) = \psi_1(t)\Phi_1(x) + \psi_2(t)\Phi_2(x)$$

$$i\partial_t\psi_1 = E_1^0\psi_1 + U_1 n_1\psi_1 - K\psi_2$$

$$i\partial_t\psi_2 = E_2^0\psi_2 + U_2 n_2\psi_2 - K\psi_1$$

$$E_{1,2}^0 = \int [\frac{\hbar^2}{2m}|\nabla\Phi_{1,2}|^2 + \Phi_{1,2}^2 V_{ext}]d\vec{r}$$

$$U_{1,2} = g_0 \int \Phi_{1,2}^4 d\vec{r}$$

$$K = -\int [\frac{\hbar^2}{2m}(\nabla\Phi_1\nabla\Phi_2) + \Phi_1\Phi_2 V_{ext}]d\vec{r}$$

Bose-Josephson junctions

$$z = \frac{n_1 - n_2}{2(n_1 + n_2)}$$

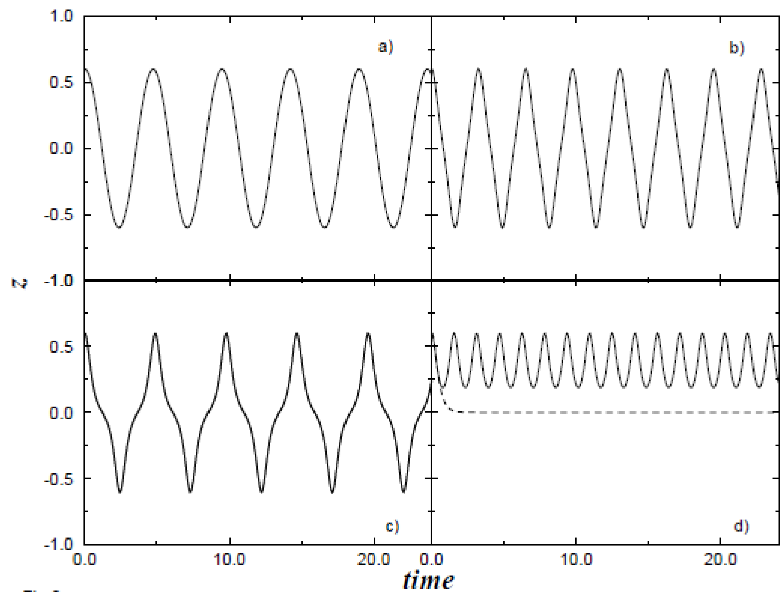
$$\phi = \theta_1 - \theta_2$$

– Main predictions : Josephson-plasma oscillations or self trapping

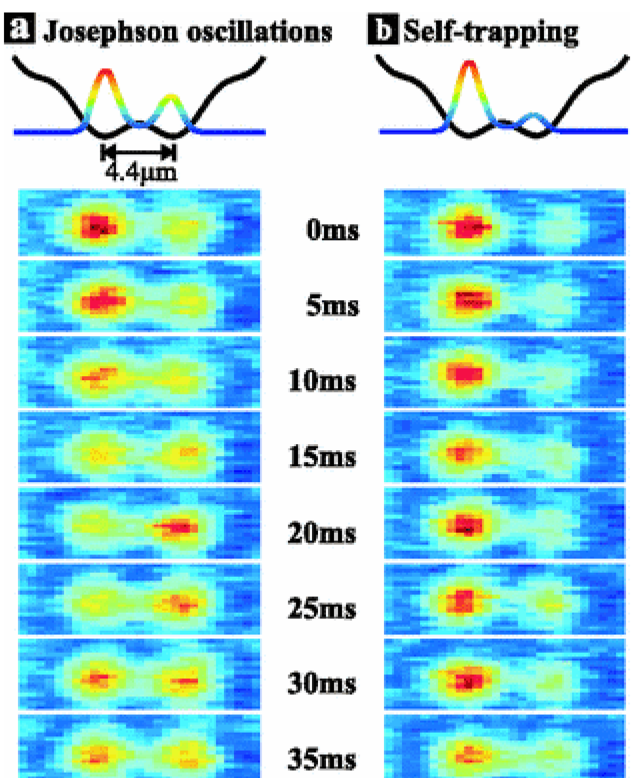
$$\partial_t z = -\sqrt{1 - z^2}\sin\phi$$
$$\partial_t \phi = \Delta E + \frac{z}{\sqrt{1 - z^2}}\cos\phi + \Lambda z$$

$$\mathcal{H} = \frac{\Lambda}{2}z^2 + \Delta E z - \sqrt{1 - z^2}\cos\phi$$

$$\omega_P = 2K\sqrt{\Lambda + 1} \qquad \Lambda = \frac{UN}{K}$$



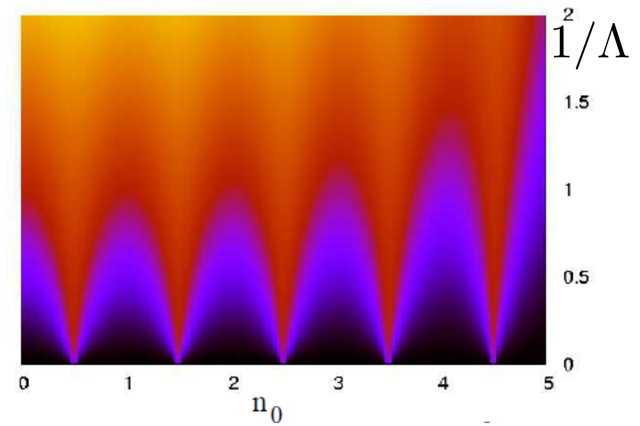
Smerzi et al, PRL 79, 4957 (1997)



– Observed in experiments
[Albiez et al, PRL 95 010402 (2005)]

Beyond the classical two-mode model...

relative-number fluctuations

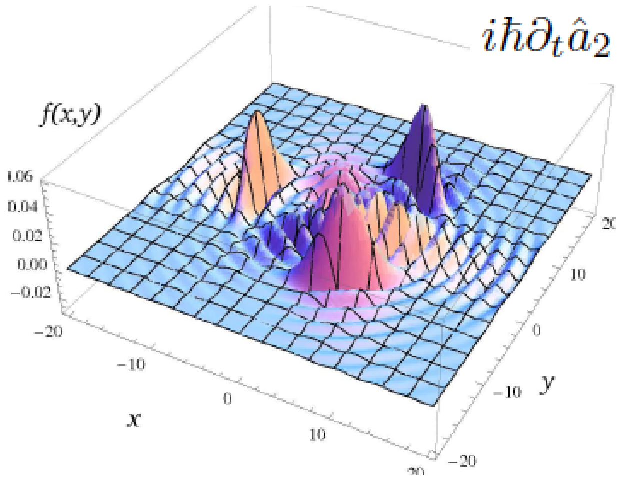


[Ferrini, Minguzzi, Hekking, PRA 2008]

– Quantum regime : two-mode Bose-Hubbard model

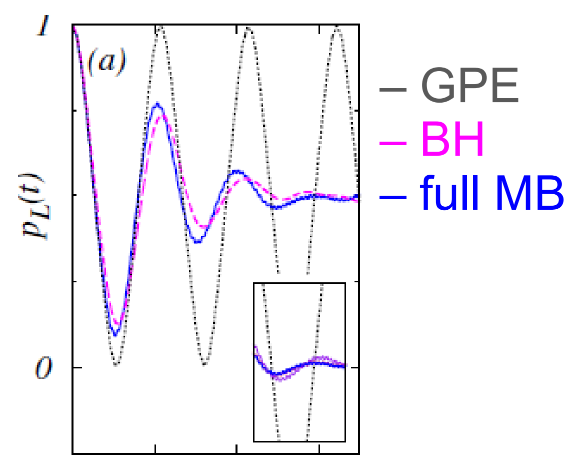
Squeezing, macroscopic superpositions (Schroedinger cats)...

$$i\hbar\partial_t \hat{a}_1 = E_1^0 \hat{a}_1 + U_1 \hat{n}_1 \hat{a}_1 - K\hat{a}_2$$
$$i\hbar\partial_t \hat{a}_2 = E_2^0 \hat{a}_2 + U_2 \hat{n}_2 \hat{a}_1 - K\hat{a}_1,$$



Tomographic reconstruction of a 3 component cat state
[Ferrini, Minguzzi, Hekking, PRA 2009]

– Quantum, beyond two-mode :
  – collapse and revivals of Josephson oscillations
  – loss of coherence



[Sakmann et al, PRL 2009]

Josephson effect in 1D elongated wires

Quantum fluctuations of the phase affect the coherent dynamics

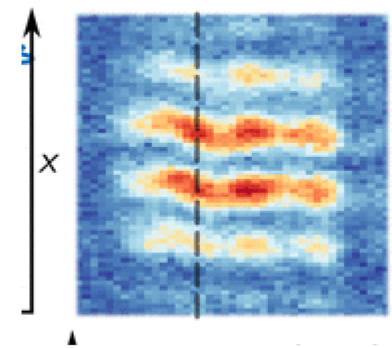
– Parallel wires *[J. Schmiedmayer group]*

Prethermalization, long-time dynamics

Realization of the Sine-Gordon Hamiltonian

Full counting statistics and higher order correlations

Relaxation of Josephson oscillations



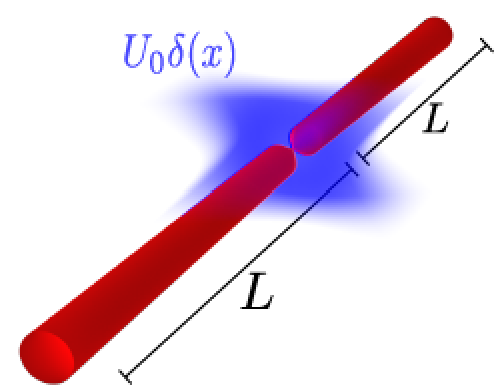
[Rauer et al, Science 2018]

– our work : head-to-tail configuration

Boundary Sine-Gordon problem

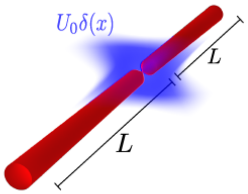
dynamics of a 1D Luttinger liquid with impurity (barrier)

Small and large-amplitude oscillations



Plan

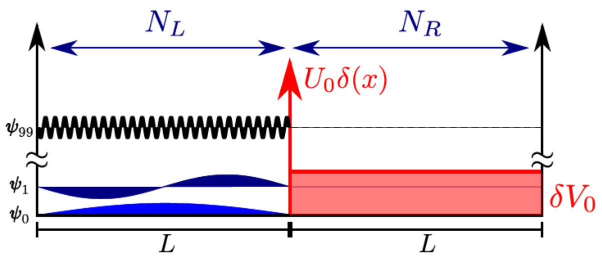
– Luttinger-liquid model for finite coupled wires



mapping on a quantum particle in a bath

Josephson and Rabi dynamical regimes

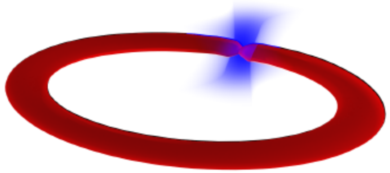
– Tonks-Girardeau exact solution for coupled wires



zero and finite temperature

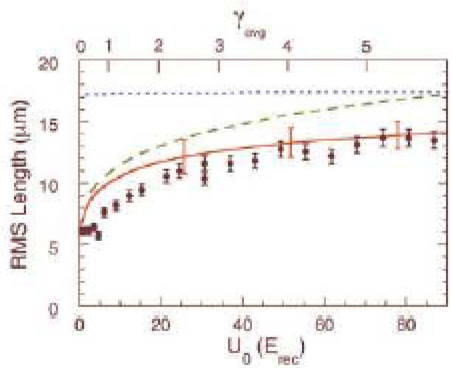
test of validity of Luttinger liquid theory

– A dual model : in a ring with weak barrier, Josephson oscillations of the current

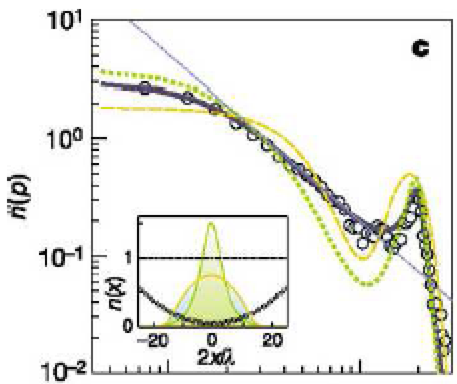


Strongly interacting 1D bosons

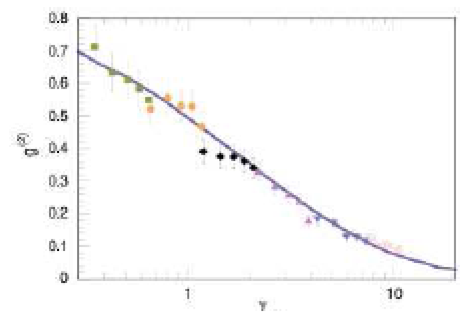
Many experimental results : density profiles, momentum distribution, collective modes, transport, number fluctuations...



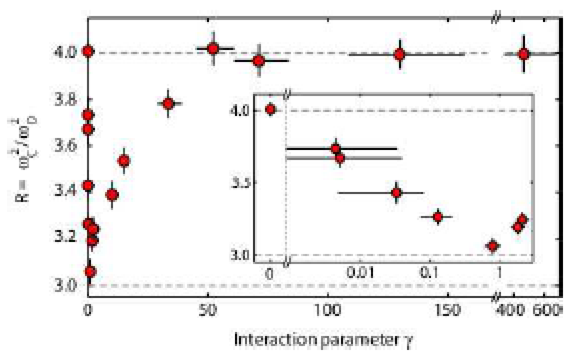
[T Kinoshita et al (2004)]



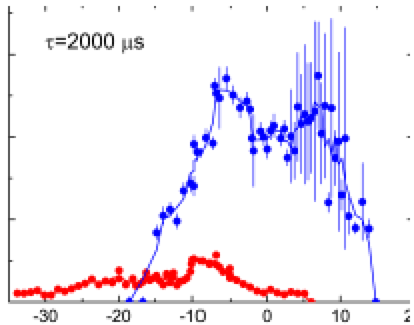
[B. Paredes et al, (2004)]



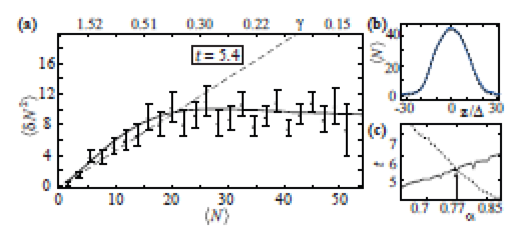
[T Kinoshita et al, (2005)]



[E Haller et al, (2009)]



[S. Palzer et al, (2009)]



[T. Jacqmin et al, (2011)]

The microscopic model

Ultracold dilute bosonic gases in 3D : binary interactions through s-wave scattering length

For atoms in a tight waveguide *[Olshanii, 1998]*

$$v(x) = g\delta(x) \qquad\qquad g = 2a_s\hbar\omega_\perp(1 - 0.4602\, a_s/a_\perp)^{-1}$$

Model Hamiltonian *[Lieb and Liniger, 1963]*

$$\mathcal{H} = \sum_i -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + g\sum_{i<j}\delta(x_i - x_j)$$

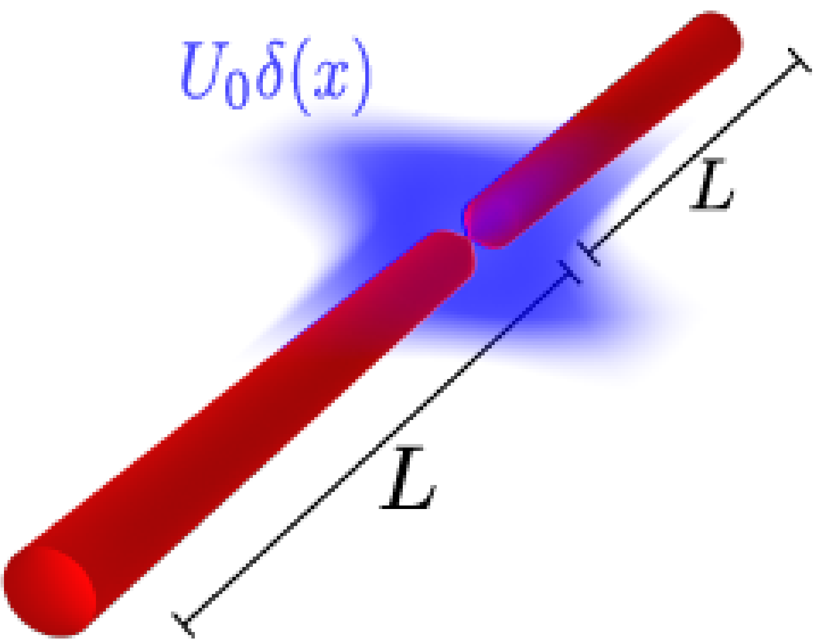
Lieb-Liniger model with external potential

Coupling strength

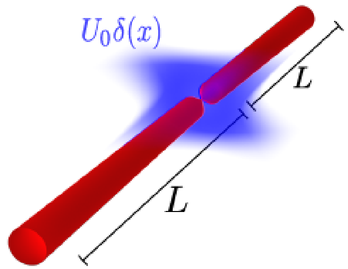
$$\gamma = gn/(\hbar^2 n^2/m)$$

Note : *strong* coupling at *weak* densities

1 – Luttinger-liquid theory for coupled wires



The Luttinger-liquid approach



Quantum hydrodynamic theory, for density and phase fluctuation fields

$$\partial_x \hat{\theta}_\pm(x)/\pi \qquad \hat{\varphi}_\pm(x)$$

Effective field theory at low-energy/large distance :

$$\hat{H}_{LL\pm} = \frac{\hbar v_\pm K_\pm}{2\pi}\int_0^L dx \left[\left(\partial_x \hat{\varphi}_\pm(x)\right)^2 + \frac{1}{K_\pm^2}\left(\partial_x \hat{\theta}_\pm(x)\right)^2 \right]$$

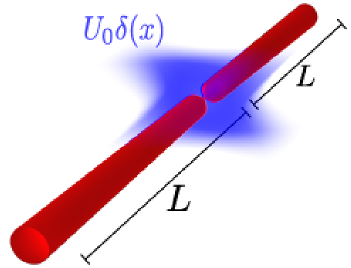
$$\hat{H}_t = -E_J \cos[\hat{\varphi}_+(0^+) - \hat{\varphi}_-(0^-)]$$

boundary sine-Gordon

Quandratic Hamiltonian in density fluctuations and superfluid velocity :
phonon excitation spectrum

Large barrier limit :
– the tunnel Hamiltonian is a perturbation coupling the Luttinger liquids
– the excitation modes of the Luttinger liquids correspond to the infinite barrier case

The Luttinger-liquid approach



In a finite wire,

– discrete phonon spectrum $\Omega_\mu = v k_\mu \quad k_\mu = \pi\mu/L; \;\; \Phi_\mu(x) = \sqrt{2/L}\cos(k_\mu x)$

– zero modes $[\hat{N}_\pm, \hat{\phi}_{0\pm}] = -i$

mode expansion of each Luttinger liquid :

$$\hat{\phi}_\pm(x) = \hat{\phi}_{0\pm} + \frac{1}{\sqrt{L}} \sum_{\mu\geq 1} \Phi_\mu(x)\hat{Q}_{\mu\pm},$$

$$\hat{n}_\pm(x) = \frac{\hat{N}_\pm}{L} + \frac{\sqrt{L}}{\hbar} \sum_{\mu\geq 1} \Phi_\mu(x)\hat{P}_{\mu\pm},$$

A *quantum particle* in a bath $\hat{N} = (\hat{N}_+ - \hat{N}_-)/2, \hat{\phi}_0 = \hat{\phi}_{0,+} - \hat{\phi}_{0,-}$

$$\hat{H}_T^{rel} = \frac{E_Q}{2}(\hat{N} - N_{ex})^2 - E_J \cos\left(\hat{\phi}_0\right)$$

$$+ \sum_{\mu\geq 1} \left[\frac{1}{2M}\left(\hat{P}_\mu + \frac{\sqrt{2}\hbar}{L}(\hat{N} - N_{ex})\right)^2 + \frac{1}{2} M \Omega_\mu^2 \hat{Q}_\mu^2 \right]$$

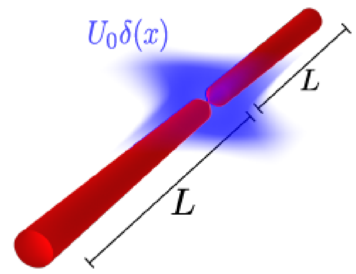
with left-right imbalance N_{exc}

$$E_Q = \frac{\hbar^2}{ML^2}$$

$$M = \hbar K / 2\pi v L$$

Caldeira-Leggett model,but intrinsec bath provided by the phonon excitations in the wire

A Josephson junction with a bath



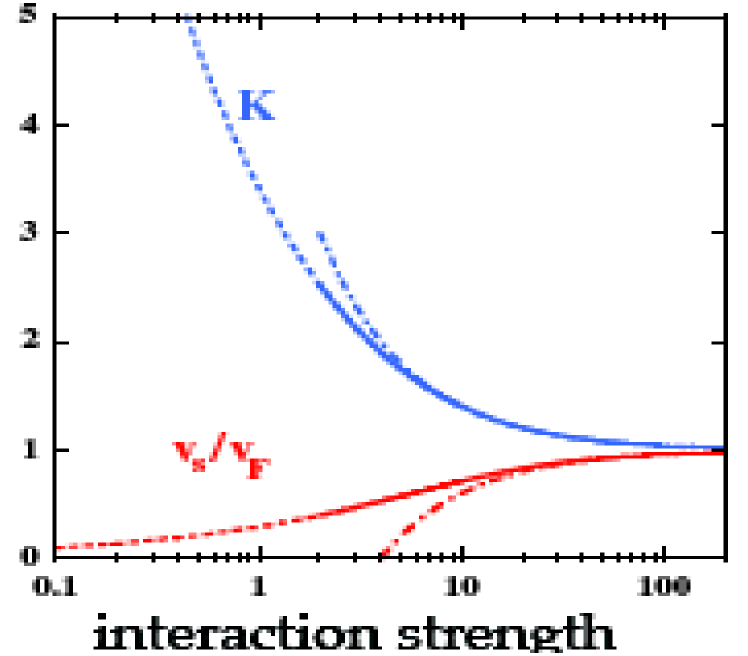
$$\hat{H}_T^{rel} = \frac{E_Q}{2}(\hat{N} - N_{ex})^2 - E_J \cos\left(\hat{\phi}_0\right) + \hat{H}_{bath}$$

The parameters of this junction depend on interactions :

$$E_Q = \frac{\hbar^2}{ML^2} \qquad M = \hbar K / 2\pi v L$$

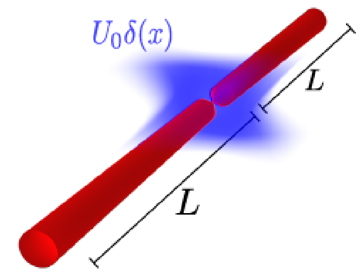
Luttinger parameter and sound velocity from Lieb-Liniger Bethe Ansatz solution

At increasing interactions, E_Q increases



[M. Cazalilla, JPhysB 2004]

A Josephson junction with a bath



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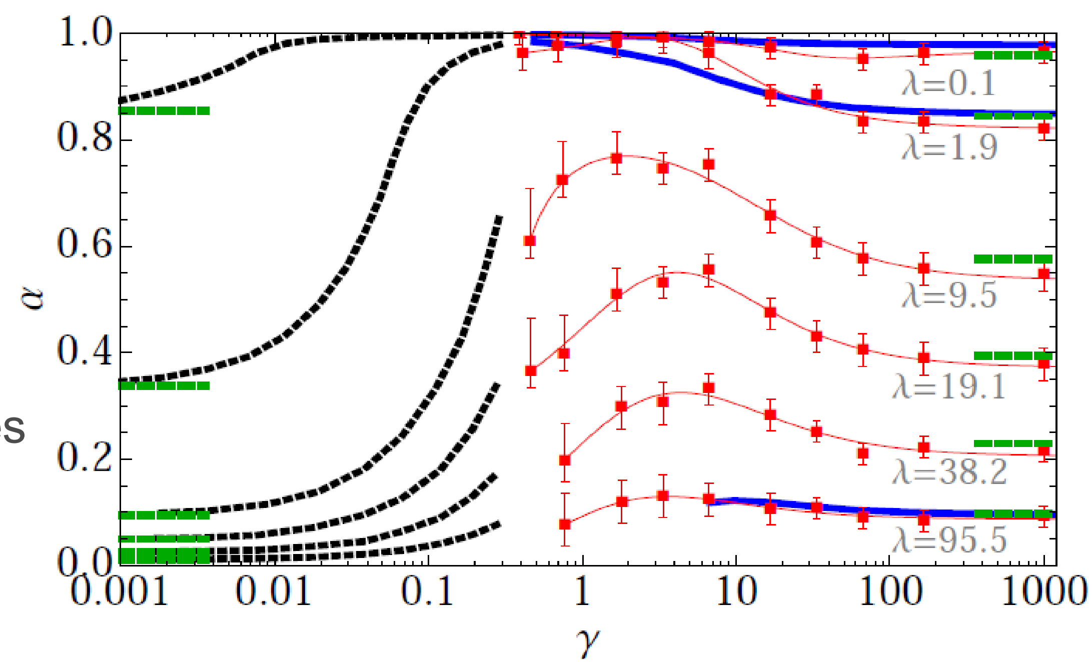
$$E_Q = \frac{\hbar^2}{ML^2} \qquad M = \hbar K / 2\pi v L$$

Luttinger parameter and sound velocity from Lieb-Liniger Bethe Ansatz solution

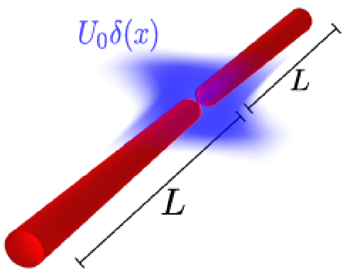
The tunnel energy is renormalized by quantum fluctuations

At increasing interactions, E_J decreases



[*M. Cominotti et al, PRL 113, 025301, (2014)*]

The Josephson regime : $E_Q \ll E_J$



$$\hat{H}_T^{rel} = \frac{E_Q}{2}(\hat{N} - N_{ex})^2 - E_J \cos\left(\hat{\phi}_0\right) + \hat{H}_{bath}$$

$E_Q \ll E_J$ Equation of motion for number imbalance

Josephson oscillations among the two wires

$$\ddot{\hat{N}} + \omega_0^2 \cos(\hat{\phi}_0)\hat{N} + \int_0^t dt' \,\gamma_N(t,t')\dot{\hat{N}}(t') = \xi_N(t)$$

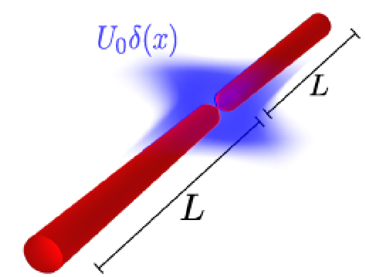
Josephson frequency $\omega_0 = \sqrt{E_J E_Q}/\hbar$

damping $\gamma_N(t,t') \simeq (E_J/\hbar\sqrt{K})\delta(t-t')$

noise (thermal) $\langle \xi_N(t)\xi_N(t')\rangle = \eta\delta(t-t')$ $\eta = 2E_J^2 k_B T/\hbar^2 M L v$

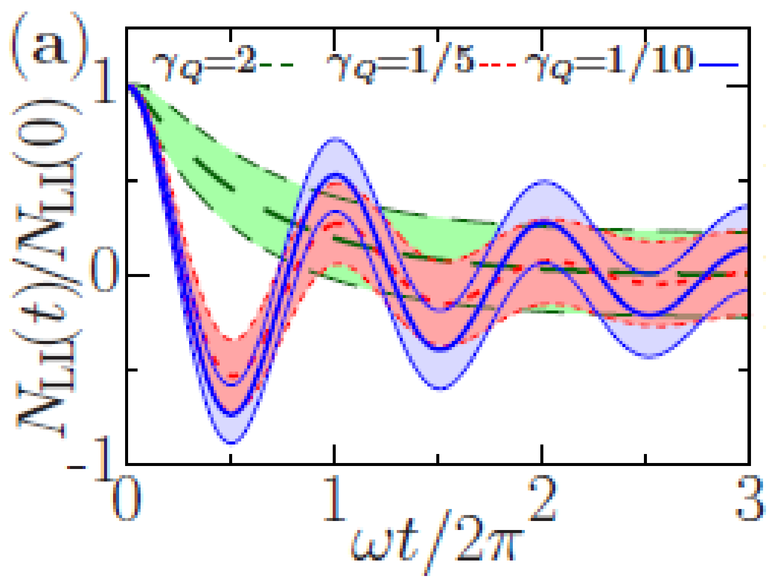
A generalization of the two-mode model, with a damping due to phonons propogating in the extended wires

Small-amplitude Josephson oscillations



$E_Q \ll E_J$

Dynamics of the quantum particle :
Josephson oscillations with damping and frequency shift due to the bath



$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega_0 = \sqrt{E_J E_Q}/\hbar$$
$$\gamma = E_J/K\hbar$$

$$E_Q \simeq 100\mathrm{Hz} \quad E_J \simeq 80 - 200\mathrm{Hz}$$

$$\Delta N_{\mathrm{LL}} = \langle (\hat{N} - \langle \hat{N} \rangle)^2 \rangle^{1/2} = \sqrt{\frac{ML^2}{\hbar^2} k_B T}$$

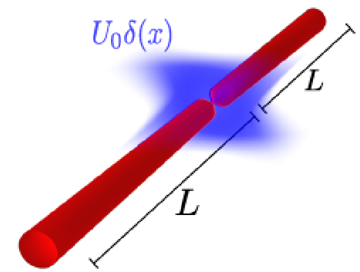
Thermalization at long times (due to the bath modes)

from underdamped to overdamped oscillations by tuning interactions or barrier strength

at weak interactions : damping -->0, Josephson oscillations as in the two-mode model

at increasing interactions, E_J decreases and E_Q increases...

The Rabi regime : $E_Q \gg E_J$



$$\hat{H}_T^{rel} = \frac{E_Q}{2}(\hat{N} - N_{ex})^2 - E_J \cos\left(\hat{\phi}_0\right) + \hat{H}_{bath}$$

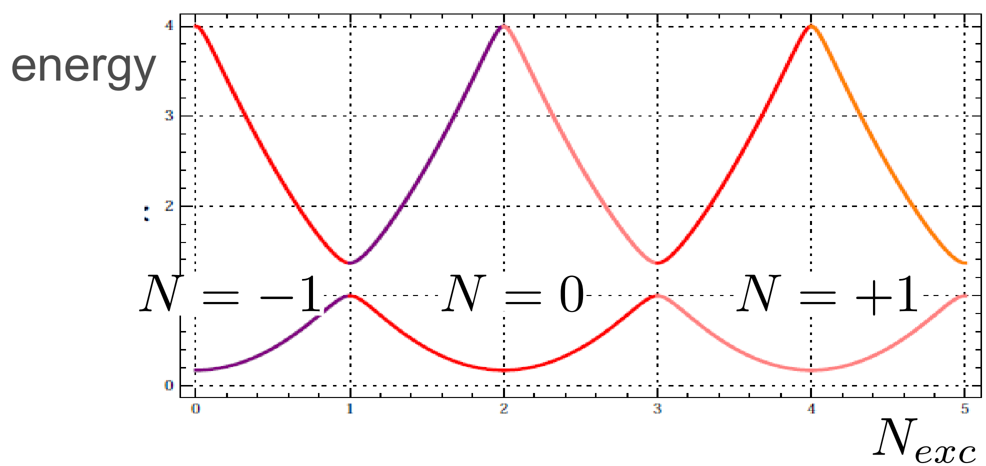
$E_Q \gg E_J$ The ‘qubit’ regime

oscillations among states with
well defined, different N

Rabi frequency $\omega_0 = E_J/\hbar$

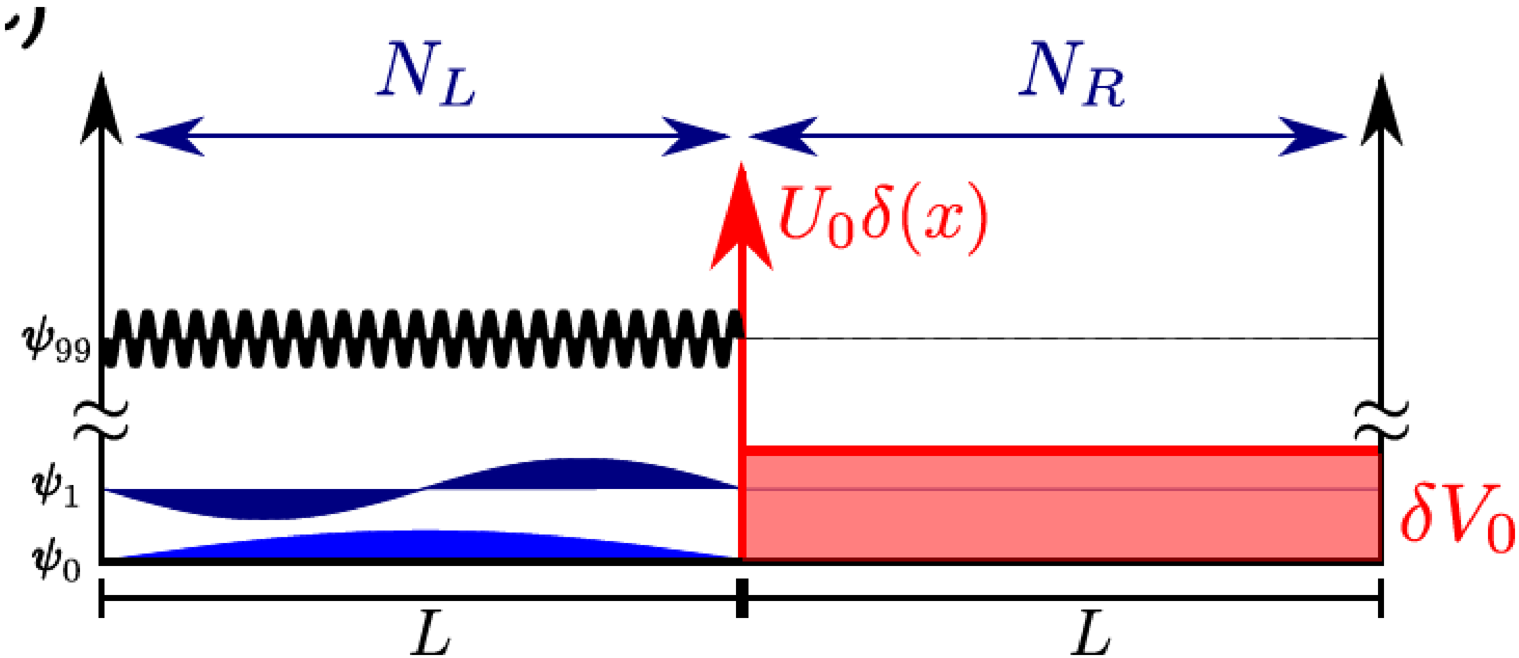
No damping : too large energy scale
associated to phonon modes :

$$\Delta E = \hbar\pi v/L = E_Q/K$$



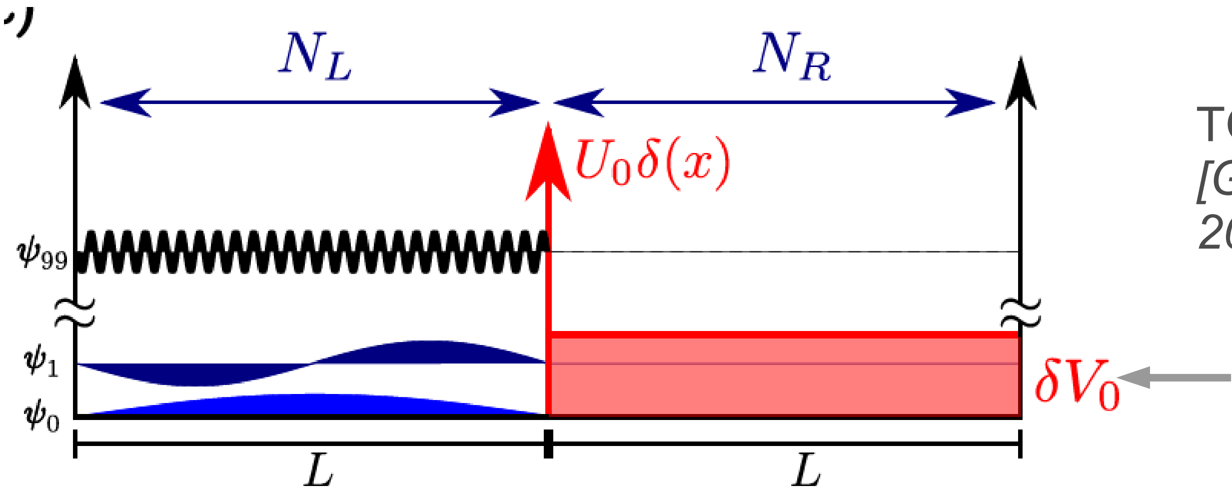
Undamped Rabi oscillations of particle imbalance

2 – Exact Tonks-Girardeau solution for coupled wires



An exact microscopic solution for a Tonks-Girardeau gas

Infinite interactions – Luttinger parameter K=1
exact solution for the quantum dynamics both at zero and finite temperature



TG gas in a split trap – see *[Goold and Busch, PRA 2008])*

oscillations induced by a sudden quench of δV_0

Exact solution for the full quantum evolution
– many-body wavefunction from Bose-Fermi mapping

$$\Psi_{TG}(x_1, ..., x_N, t) = \Pi_{1\leq j \leq l \leq N} \mathrm{sgn}(x_j - x_l) \det[\psi_k(x_i, t)]$$

$$i\hbar\partial_t\psi_j = (-\hbar^2\partial_x^2/2m + V(x,t))\psi_j$$

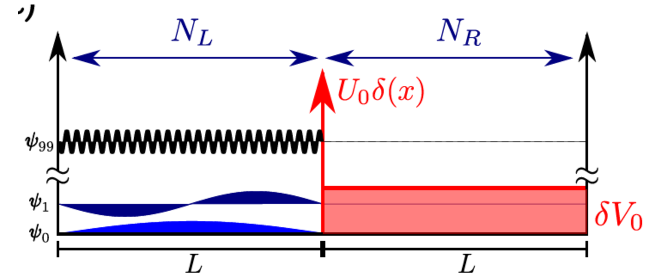
$\psi_j(x,0)$ j-th eigenstate with imbalace trap on

$$n(x,t) = \sum_j f(\epsilon_j)|\psi_j(x,t)|^2$$

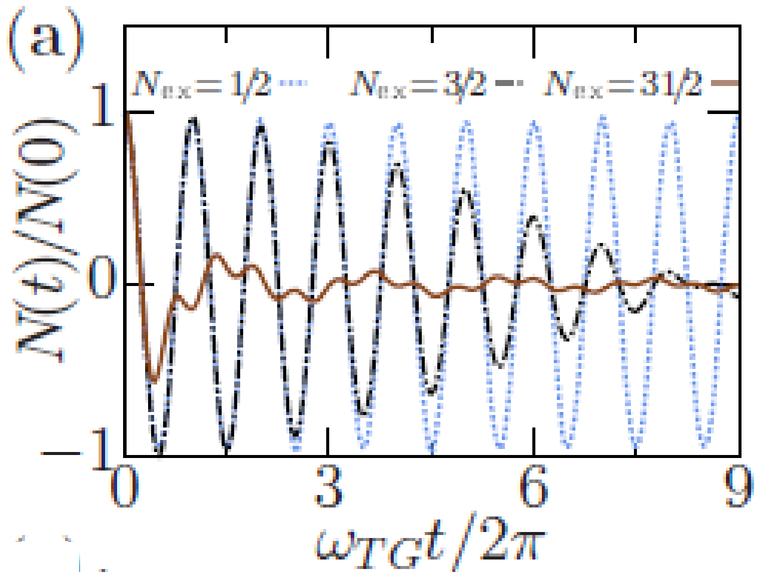
$$N(t) = N_L - N_R \qquad N_L = \int_{-L}^{0} dx\, n(x,t) \qquad N_R = \int_0^L dx\, n(x,t)$$

TG exact solution for the Josephson oscillations

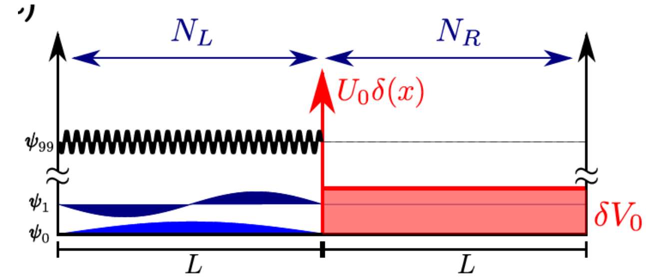
Particle-number oscillations $N(t) = N_L - N_R$



At zero temperature, increasing imbalance δV_0



TG exact solution for the Josephson oscillations



Particle-number oscillations $N(t) = N_L - N_R$

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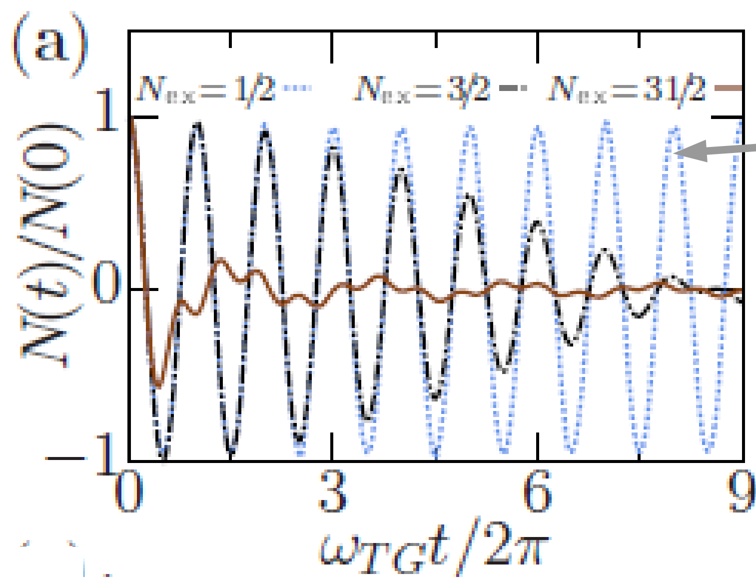


Small imbalance : undamped oscillations

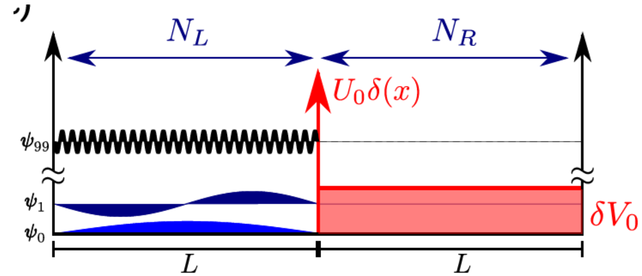
$$E_J = \epsilon_{N_T+1} - \epsilon_{N_T} \quad E_Q = \hbar^2\pi^2 N_T/2mL^2$$

$$E_J/E_Q = 4 \times 10^{-3}$$

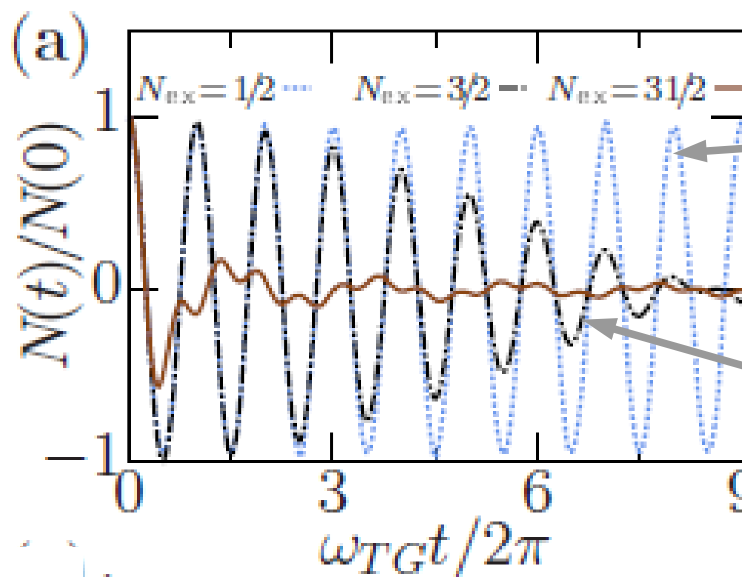
Rabi regime : *OK with Luttinger liquid predictions!*

TG exact solution for the Josephson oscillations

Particle-number oscillations $N(t) = N_L - N_R$



At zero temperature, increasing imbalance δV_0



Small imbalance : undamped oscillations with frequency

$$\omega_0 = E_J/\hbar$$

Damped oscillations at larger imbalance

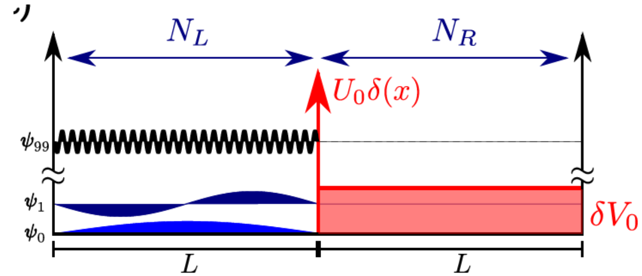
Large-amplitude oscillations : beyond LL

$$E_J = \epsilon_{N_T+1} - \epsilon_{N_T} \quad E_Q = \hbar^2\pi^2 N_T/2mL^2$$

$$E_J/E_Q = 4 \times 10^{-3}$$

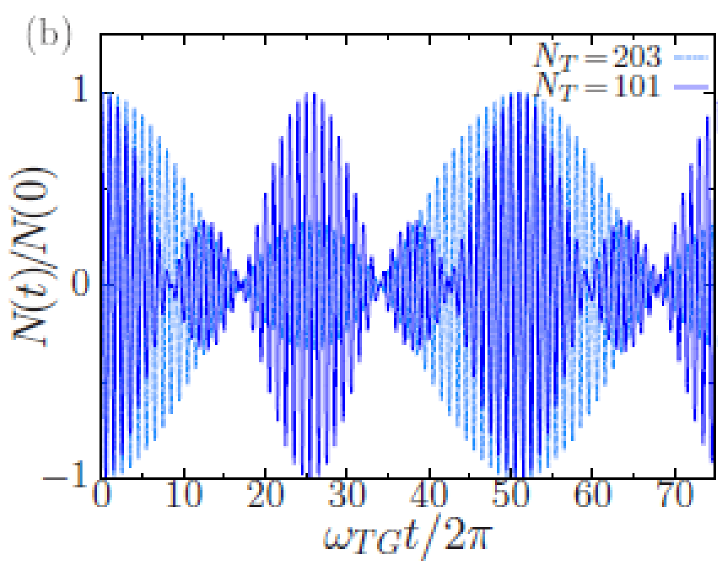
Rabi regime : *OK with Luttinger liquid predictions!*

TG exact solution for the Josephson oscillations



Particle-number oscillations $N(t) = N_L - N_R$

At zero temperature, long-time dynamics



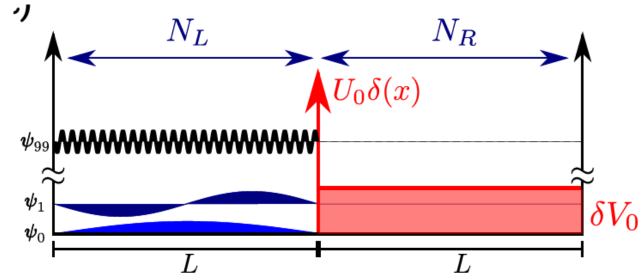
Intermediate imbalance, long times : collapses and revivals due to finite size

– *beyond our LL approach where a continuous spectrum is assumed*

$$E_J = \epsilon_{N_T+1} - \epsilon_{N_T} \quad E_Q = \hbar^2\pi^2 N_T/2mL^2$$

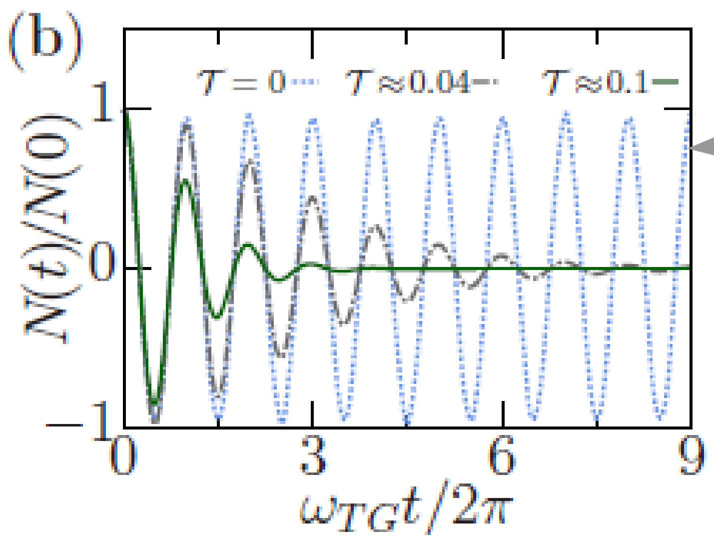
$$E_J/E_Q = 4 \times 10^{-3}$$

TG exact solution for the Josephson oscillations



Particle-number oscillations $N(t) = N_L - N_R$

At fixed small δV_0, increasing temperature



Zero temperature : undamped oscillations in the Rabi regime

$$E_J = \epsilon_{N_T+1} - \epsilon_{N_T} \quad E_Q = \hbar^2\pi^2 N_T/2mL^2$$

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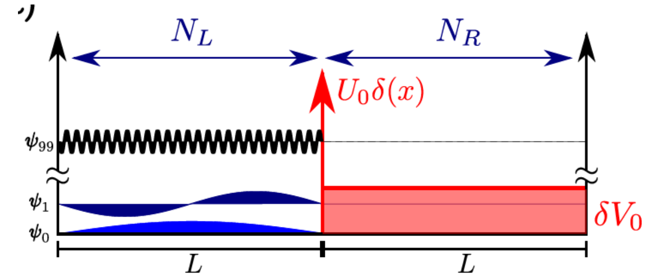
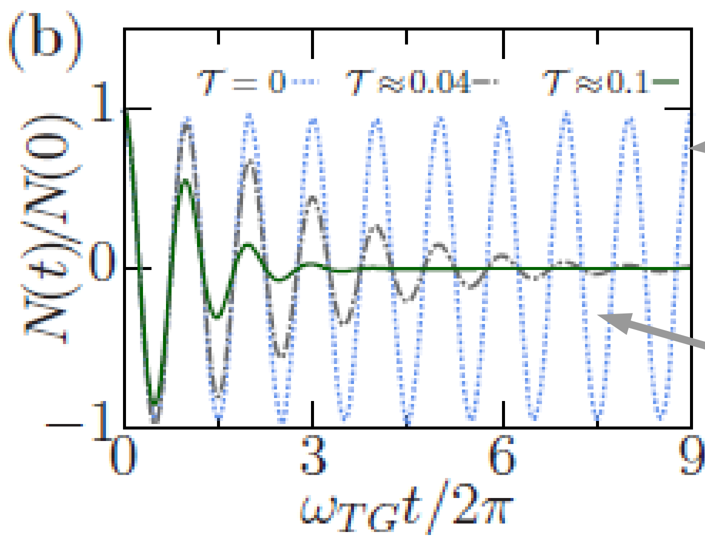


Particle-number oscillations $N(t) = N_L - N_R$

At fixed small δV_0, increasing temperature



Zero temperature : undamped oscillations in the Rabi regime

Damped oscillations at finite temperature

Not predicted by the LL theory

$$E_J = \epsilon_{N_T+1} - \epsilon_{N_T} \quad E_Q = \hbar^2\pi^2 N_T/2mL^2$$

$$E_J/E_Q = 4 \times 10^{-3}$$

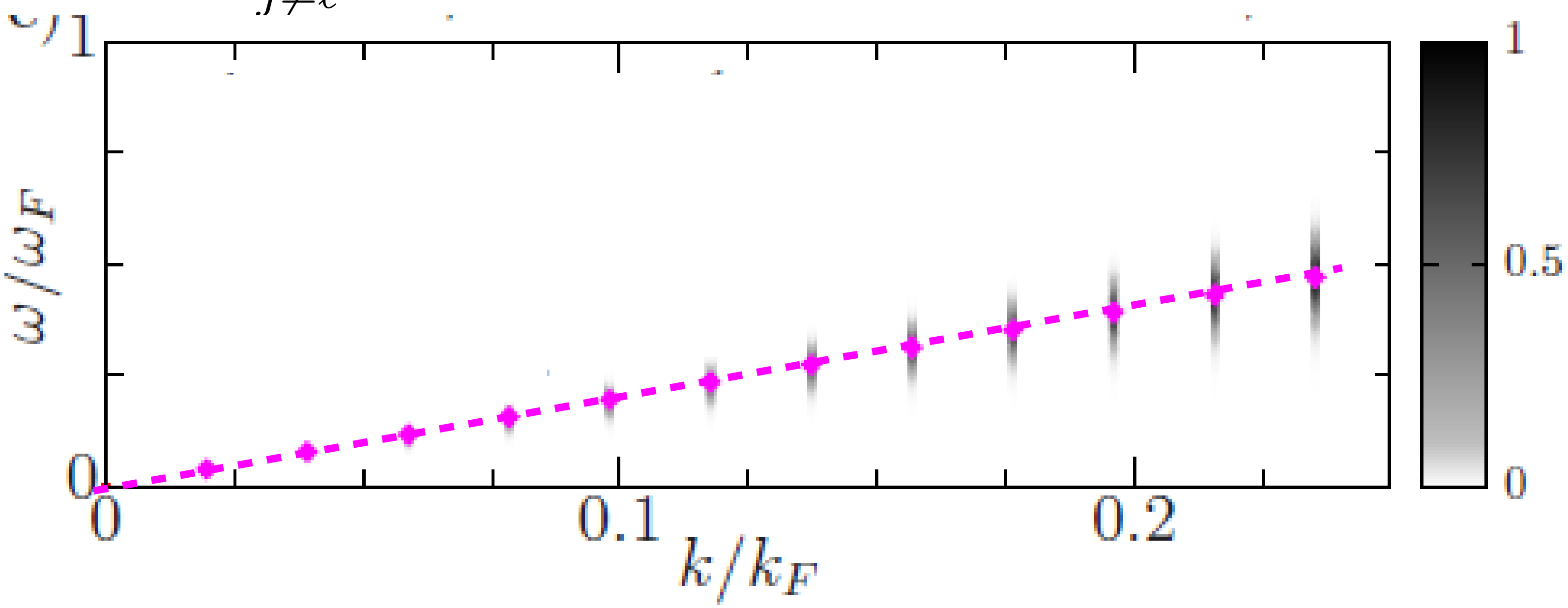
???

TG exact solution vs Luttinger-liquid approach

Small-oscillation regime, where both theories hold

Excitation spectrum (from linear-response)

$$S(k,\omega) = \sum_{j\neq\ell} 2\pi\delta(\omega-\omega_{j,\ell}) f(\epsilon_j)[1-f(\epsilon_\ell)]\delta(k-(k_\ell-k_j)))$$



Luttinger liquid linear spectrum

In general a good approximation
at low energy

Exact TG spectrum : broad
particle-hole excitations

TG exact solution vs Luttinger-liquid approach

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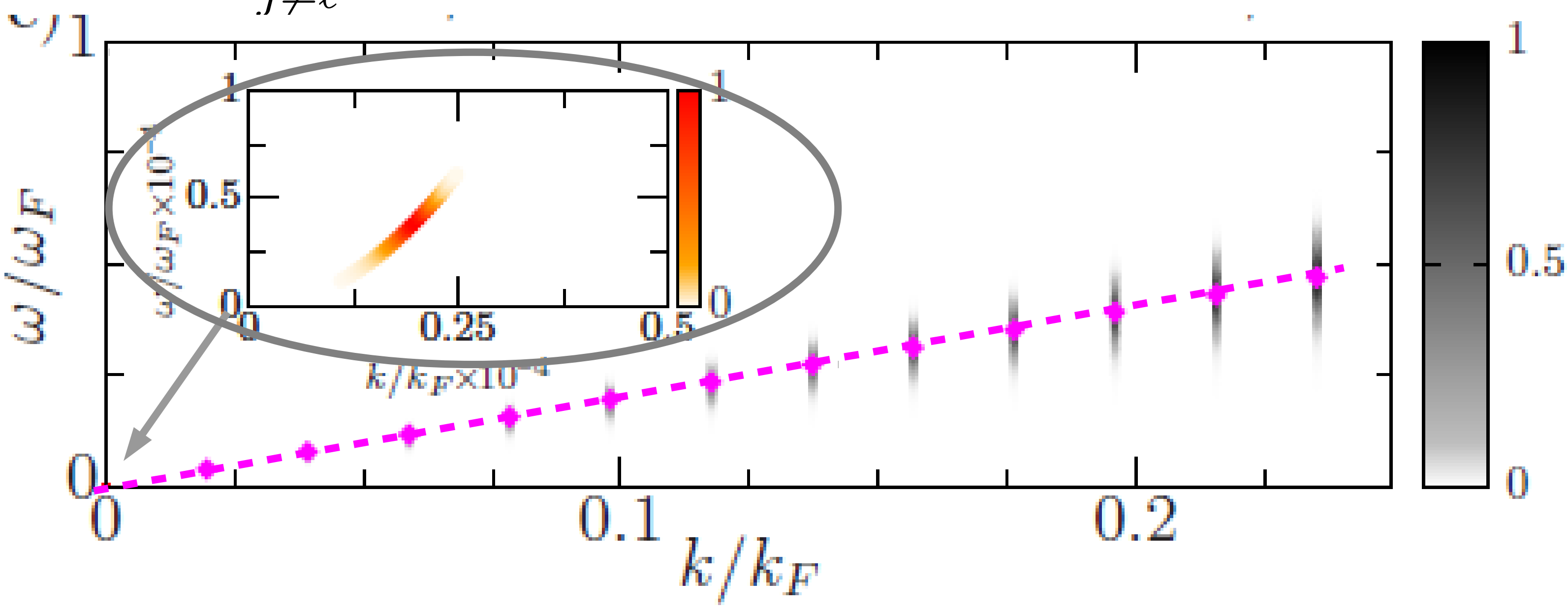


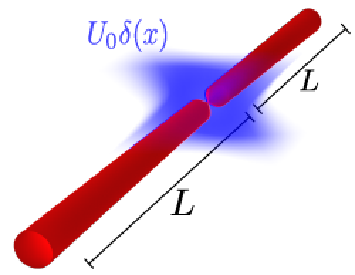
Luttinger liquid linear spectrum

In general a good approximation at low energy

Exact TG spectrum : broad particle-hole excitations

Has some low-energy modes not included the LL theory !!

Josephson oscillations : Luttinger-liquid vs TG exact solution



$$\hat{H}_T^{rel} = \frac{E_Q}{2}(\hat{N} - N_{ex})^2 - E_J \cos\left(\hat{\phi}_0\right) + \hat{H}_{bath}$$

A quantum particle (the Josephson junction) in the presence of a phonon bath

$E_Q \ll E_J$ Josephson oscillations, from underdamped or overdamped at increasing interactions

– undamped oscillations in the two-mode model

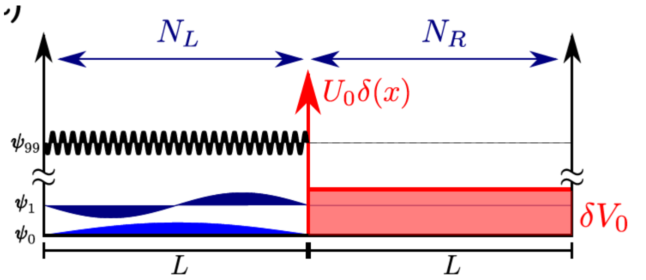
– no more oscillations in the strongly interacting regime

$E_Q \gg E_J$ Rabi oscillations undamped

OK Luttinger with TG exact solution at small oscillations

TG has extra modes which damp the oscillations at finite temperature

Josephson oscillations : the TG exact solution has also....



Large-amplitude oscillations

Long-time dynamics

Thermal and quantum noise

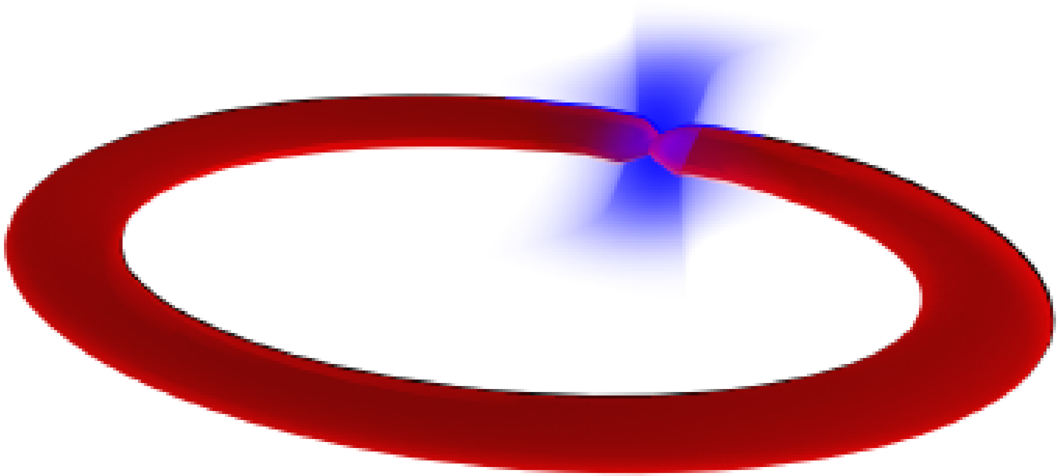
Exact description of excitation modes with barrier

In both TG and LL, the damping is due to the presence of excited modes

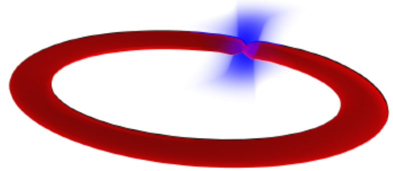
→ A microscopic explanation for the damping of Josephson oscillations

[Polo, Ahufinger, Hekking, Minguzzi, arXiv:1712.06949]

3 – Luttinger-liquid theory for a ring with barrier



A dual Luttinger-liquid problem



a ring with a weak barrier and applied gauge flux

$$\hat{H}_{LL} = \frac{\hbar v}{2\pi}\int_0^L dx K(\partial_x \hat{\varphi}(x) - \frac{2\pi}{L}\Omega)^2 + \frac{1}{K}(\partial_x \hat{\theta}(x))^2$$

phase field

density fluctuation field

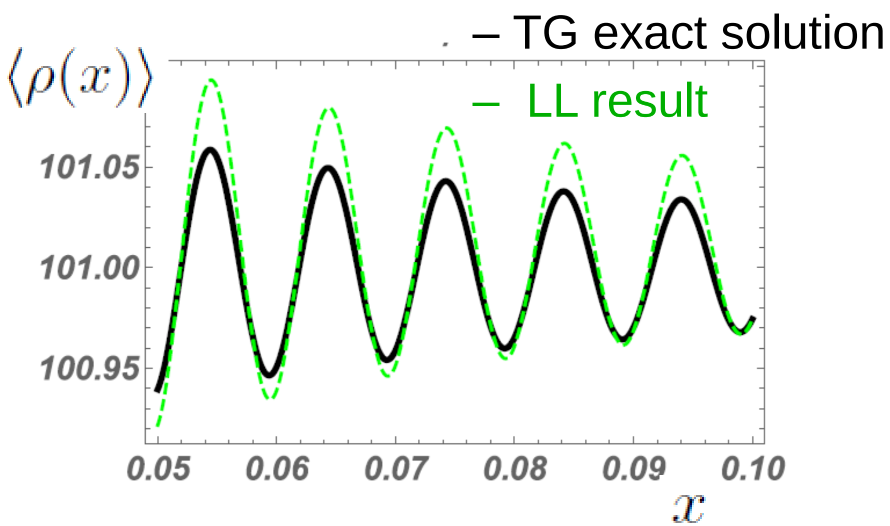
$$H_b = 2n_0 U_{\text{eff}} \cos(2\hat{\theta}(x=0))$$

Weak barrier : density modulation $\rightarrow$ 'backscattering' term in LL

Barrier : Friedel oscillations and dephasing

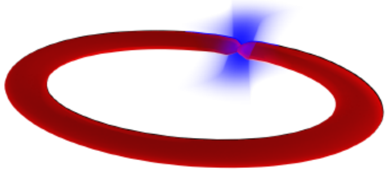
$$\frac{\langle \rho(x) \rangle}{\rho_0} \simeq 1 + \frac{\alpha \cos(2\pi \rho_0 x + 2\theta_B)}{x}$$

$$\boxed{\theta_B = \pi/2 \quad \text{for weak barrier}}$$



[Didier, Minguzzi, Hekking, PRA (2009)]

A dual Luttinger-liquid problem



As in the two-wire case, again a Josephson-junction problem :

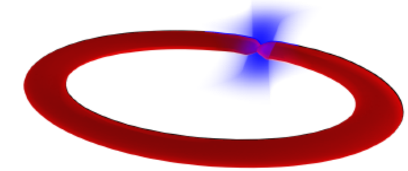
$$\hat{H}_T = E_Q^{\mathrm{r}} \left(\hat{J} - \Omega\right)^2 - E_J^{\mathrm{r}} \cos(2\hat{\theta}_0) + \sum_{\mu \geq 1} \left[\frac{1}{2M_{\mathrm{r}}} \left(\hat{P}_\mu + \frac{4\pi\sqrt{2}\hbar}{L} \left(\hat{J} - \Omega\right) \right)^2 + \frac{1}{2} M_{\mathrm{r}} \Omega_\mu^2 \hat{Q}_\mu^2 \right]$$

$$E_Q^{\mathrm{r}} = \hbar^2 (2\pi)^2 / M_{\mathrm{r}} L^2 \qquad E_J^{\mathrm{r}} = 2 n_0 U_{\mathrm{eff}} \qquad M_{\mathrm{r}} = \frac{\hbar\pi}{vLK}$$

Average current (zero mode) : quantum particle / Josephson-junction Hamiltonian

Phonon excitations : intirinsic bath, leading to damping of oscillations

The Josephson regime : $E_Q^{\rm r} \ll E_J^{\rm r}$



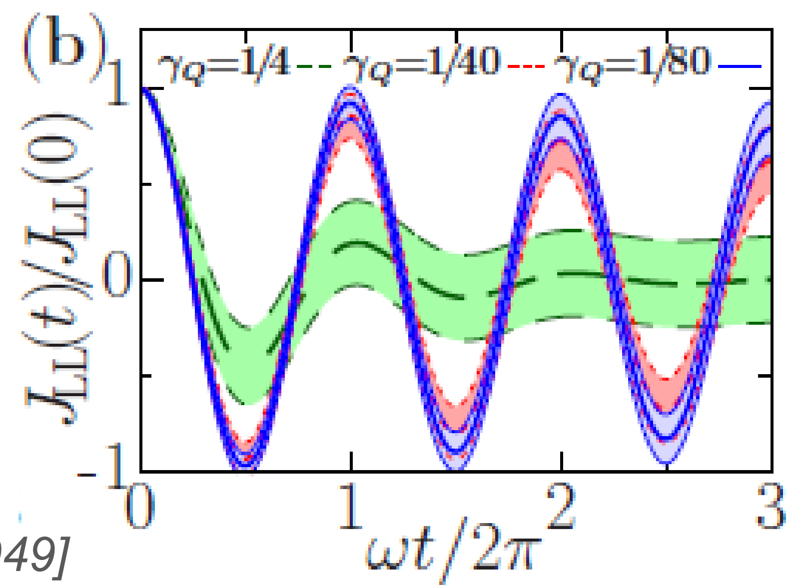
$$\hat{H}_T = E_Q^{\rm r}\left(\hat{J} - \Omega\right)^2 - E_J^{\rm r}\cos(2\hat{\theta}_0) + H_{bath}$$

$$E_Q^{\rm r} = \hbar^2(2\pi)^2/M_{\rm r}L^2 \qquad E_J^{\rm r} = 2n_0 U_{\rm eff} \qquad M_{\rm r} = \frac{\hbar\pi}{vLK}$$

upon a quench of the applied gauge field : oscillations of average current

Josephson frequency $\omega_0 = \sqrt{E_Q^{\rm r} E_J^{\rm r}}/\hbar$

damping rate $\gamma = 4n_0 U_{eff} K$

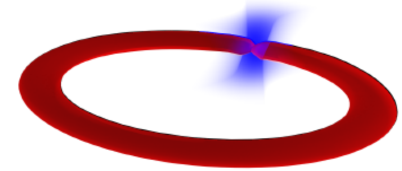


Damping *decreases* at increasing interactions

– different from two-wire case

[Polo, Ahufinger, Hekking, Minguzzi, arXiv:1712.06949]

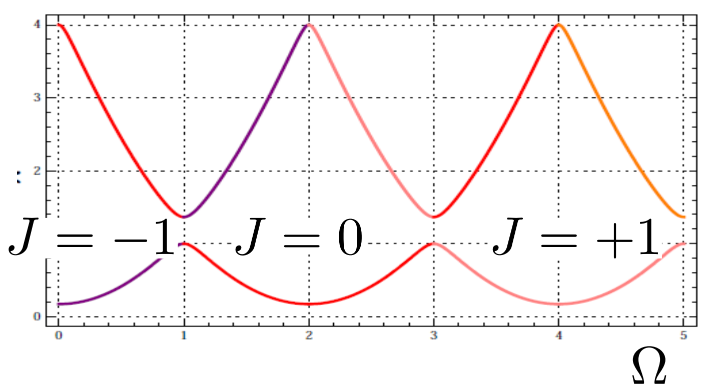
The Rabi regime : $E_Q^{\mathrm{r}} \gg E_J^{\mathrm{r}}$

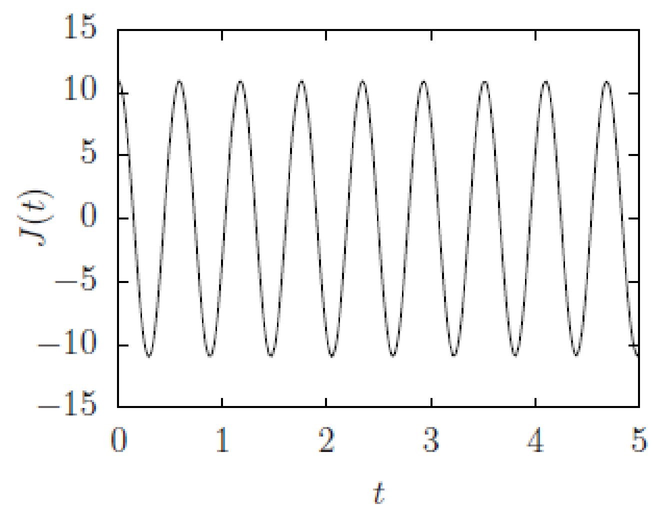


$$\hat{H}_T = E_Q^{\mathrm{r}} \left(\hat{J} - \Omega \right)^2 - E_J^{\mathrm{r}} \cos(2\hat{\theta}_0) + H_{bath}$$

upon a quench of the applied gauge field : oscillations of average current

energy



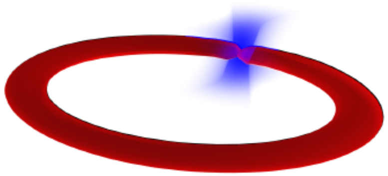


Undamped Rabi oscillations among angular momentum states : the flux qubit

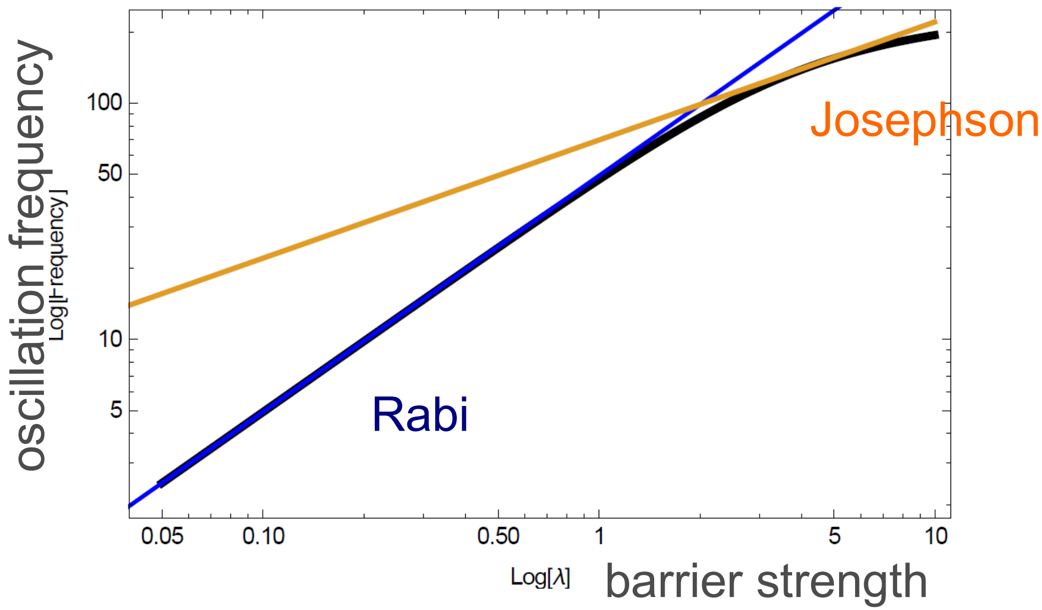
Rabi frequency $\omega_0 = E_J^{\mathrm{r}} / \hbar$

No effect of the bath : too large phonon level spacing

From Rabi to Josephson oscillations in the exact TG solution



upon a quench of the applied gauge field :
oscillation frequency vs barrier strength



At increasing barrier strength : clear change of regime from Rabi to Josephson oscillations

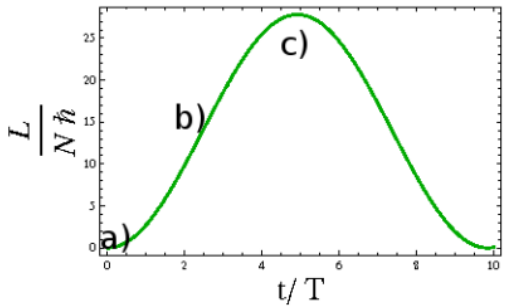
At stronger barriers : breakdown of the ‘weak barier limit’ used in the Luttinger Liquid theory

[Dubessy, Polo, et al, in preparation]

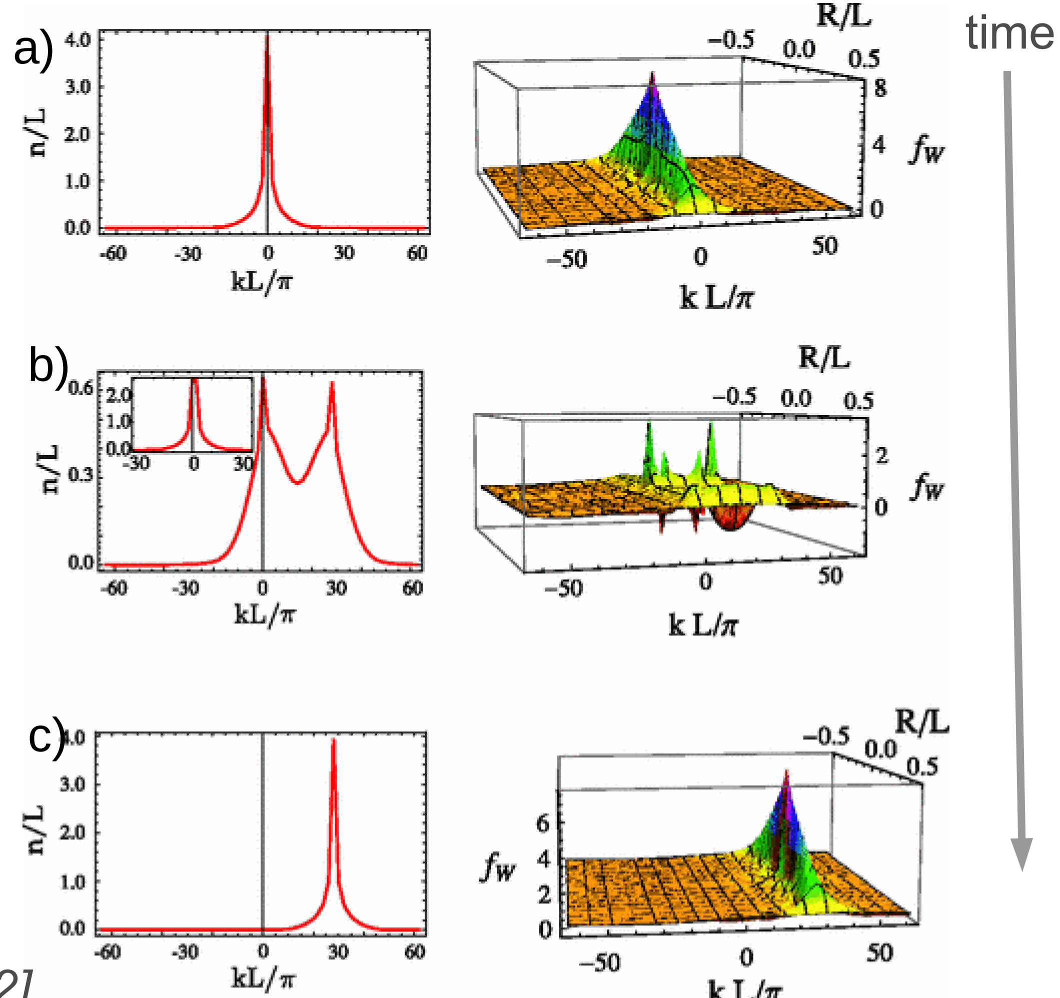
Nonclassical states

Exact Tonks-Girardeau solution following a sudden quench of the gauge field

Rabi oscillation of the current :



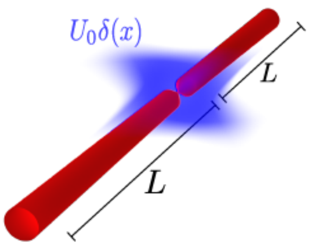
Macroscopic superposition
of two 'Fermi spheres' :
negative Wigner function



[Schenke, Minguzzi, Hekking, PRA 2012]

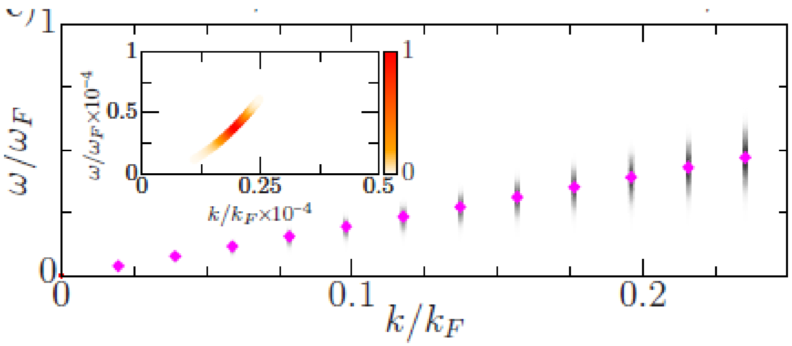
Conclusions

– Effective LL theory and exact TG solution for Josephson oscillations of 1D Bose coupled wires : Josephson and Rabi regimes

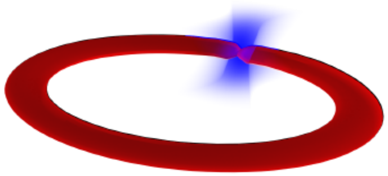


– Low-energy modes responsible for damping

→ beyond LL



– Dual Josephson oscillations in a ring with weak barrier



[J. Polo, V. Ahufinger, F. Hekking, A. Minguzzi, arXiv:1712.06949]

Outlook

– Quantum noise of LL

– At arbitrary interactions LL : beyond small-amplitude oscillations, beyond small tunnel energy/weak barrier

– Other approaches to quantum dynamics at arbitrary interactions

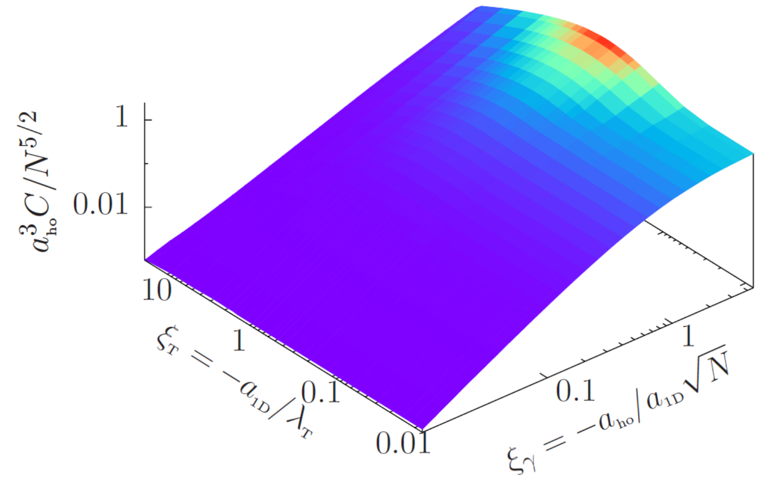
– Damping of Josephson oscillations in the parallel-wire configuration

– Josephson effect for multicomponent BEC → See poster by Enrico Compagno

Other recent results : Tan’s contact

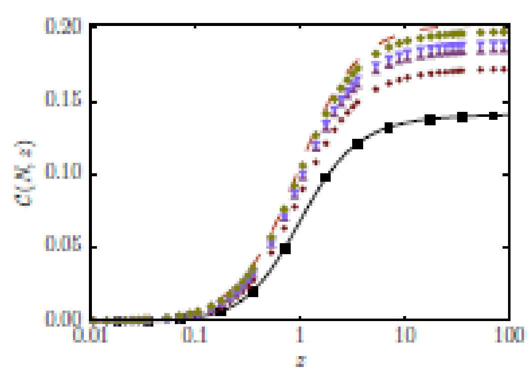
– Tan’s Contact for Trapped Lieb-Liniger Bosons at Finite Temperature,

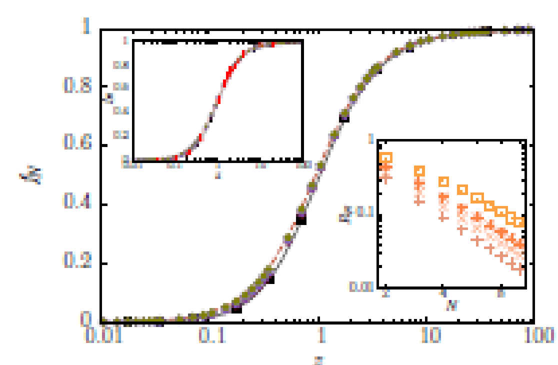
[Hepeng Yao, David Clément, Anna Minguzzi, Patrizia Vignolo, Laurent Sanchez-Palencia, arXiv:1804.04902]



– Contact and ground-state energy for harmonically-trapped one-dimensional interacting bosons: from two to many

[Matteo Rizzi, Christian Miniatura, Anna Minguzzi, Patrizia Vignolo , arXiv:1805.02463]



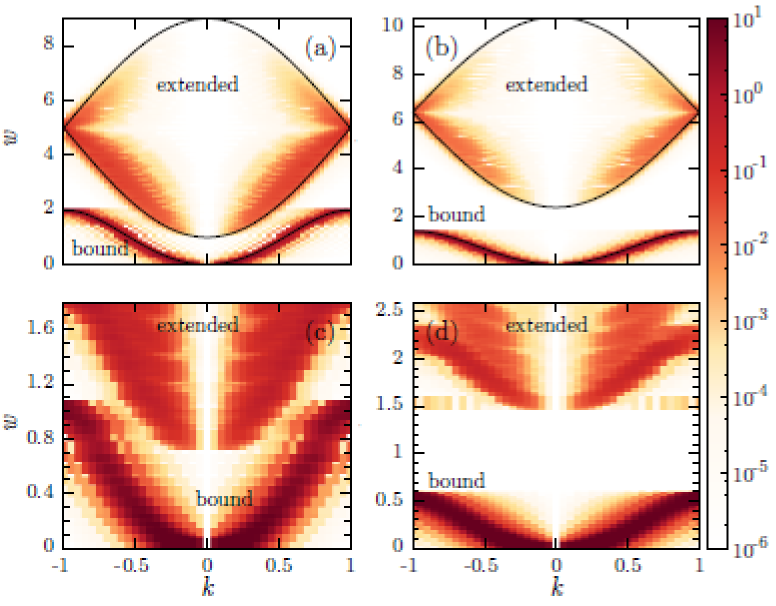


Other recent results : dimers and solitons

– Raise and fall of a bright soliton in an optical lattice

[Piero Naldesi, Juan Polo Gomez, Anna Minguzzi, Boris Malomed, Maxim Olshanii, Luigi Amico, arXiv:1804.10133]

→ See talk by Juan Polo



– Traces of integrability in scattering of one-dimensional dimers on a barrier

[Juan Polo Gomez, Anna Minguzzi, Maxim Olshanii , arXiv:1806.01820]