An efficient non-linear Feshbach engine

New Journal of Physics **20**, 015005 (2018)

(*and a little of…*)

New Journal of Physics **18**, 103035 (2016)

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Thermodynamics of Quantum Systems

INFORMATION THEORY

The Quantum Thermodynamics Revolution

As physicists extend the 19th-century laws of thermodynamics to the quantum realm, they're rewriting the relationships among energy, entropy and information.

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NATURE | NEWS FEATURE

The new thermodynamics: how quantum physics is bending the rules

Experiments are starting to probe the limits of the classical laws of thermodynamics.

Zeeya Merali

01 November 2017

Feature: Quantum thermodynamics

The rise of the quantum machines

Technological devices are getting ever smaller, but as they approach the scale at which quantum physics matters, our understanding of how they interact with their environment evaporates. James Millen and André Xuereb explain how a better understanding of quantum thermodynamics could kick-start a new industrial revolution on the tiniest scale

"*I have been very critical of the field because there is far too much theory and not enough experiment,*" says quantum physicist Peter Hänggi

Thermodynamics of Quantum Systems

Familiar notions of work (and heat) need re-examined when dealing with quantum systems.

Work is not an observable! For quantum systems it has a probability distribution

$$P(W) = \sum_{n,M} p(n,M)\delta\left[W - (E'_M - E_n)\right]$$

How can we access/study the work for quantum systems?

The characteristic function allows us precisely this

$$\chi(u,\tau) = \int dW e^{iuW} P(W) = \mathrm{Tr}[U_\tau^\dagger e^{iu\mathcal{H}(\tau)} U_\tau e^{-iu\mathcal{H}(0)} \rho_{\mathrm{eq}}^0]$$

Cold atoms offer a uniquely versatile theoretical & experimental play ground

Fluctuation theorems: Work is not an observable , P. Talkner, E. Lutz, P. Hänggi, Phys. Rev. E **75**, 050102(R) (2007).
Colloquium: Quantum fluctuation relations: Foundations and applications , M. Campisi, P. Hänggi, P. Talkner, Rev. Mod. Phys. **83**, 771 (2011).
Measuring the Characteristic Function of the Work Distribution , L. Mazzola, G. De Chiara, M. Paternostro, Phys. Rev. Lett. **110**, 230602 (2013).
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Experimental Reconstruction of Work Distribution and Study of Fluctuation Relations in a Closed Quantum System , T. B. Batalhão et al, Phys. Rev. Lett. **113**, 140601 (2014).

Thermodynamics of Cold Atom Systems

….but thermodynamics…..with no temperature??

We examine the energetics of quantum systems within the framework of finite time thermodynamics

For a sudden change in the Hamiltonian parameters

$$\chi(t) = \langle \psi_0^I | e^{i\mathcal{H}_F t} e^{-i\mathcal{H}_I t} | \psi_0^I \rangle = \sum_j e^{i(E_j^F - E_0^I)t} \left| \langle \psi_0^I | \psi_j^F \rangle \right|^2$$

From which we can determine the associated 'quantum' work

$$\langle W \rangle = -i\partial_t \chi(t)\big|_{t=0} = \sum_j (E_j^F - E_0^I) \left| \langle \psi_0^I | \psi_j^F \rangle \right|^2$$

and define the irreversible work (equivalent to the irreversible entropy production for thermal systems)

$$\langle W_{\text{irr}} \rangle = \langle W \rangle - \Delta F$$

Non-Equilibrium Thermodynamics of Harmonically Trapped Bosons , M. Á. García-March, T. Fogarty, SC, Th. Busch, M. Paternostro, New J. Phys. **18**, 103035 (2016).

Statistics of the work distribution for a quenched Fermi gas , A. Sindona, J. Goold, N. Lo Gullo, F. Plastina, New J. Phys. **16**, 045013 (2014).

Thermodynamics of Cold Atom Systems

Notice the characteristic function is directly related to another well studied quantity: the Loschmidt echo

$$\chi(t) = \langle\psi_0^I|e^{i\mathcal{H}_F t}e^{-i\mathcal{H}_I t}|\psi_0^I\rangle = \sum_j e^{i(E_j^F - E_0^I)t}\left|\langle\psi_0^I|\psi_j^F\rangle\right|^2$$

$$\mathcal{L}(t) = |\langle\Psi_0|e^{i\mathcal{H}_F t}e^{-i\mathcal{H}_I t}|\Psi_0\rangle|^2 = \left|\sum_n e^{i(E_n^F - E_0^I)t}\langle\psi_0^I|\psi_n^F\rangle^2\right|^2$$

These figures of merit can be readily studied for cold atomic systems

For (harmonically) trapped system we can imagine varying the Hamiltonian parameters via Feshbach resonances

Non-Equilibrium Thermodynamics of Harmonically Trapped Bosons , M. Á. García-March, T. Fogarty, SC, Th. Busch, M. Paternostro, New J. Phys. **18**, 103035 (2016).

Statistics of the work distribution for a quenched Fermi gas , A. Sindona, J. Goold, N. Lo Gullo, F. Plastina, New J. Phys. **16**, 045013 (2014).

(Quantum) Otto Cycle

The Otto cycle is an ideal setting to examine the thermodynamics of quantum systems. It comprises of 4 strokes:

1. A → B: *Adiabatic (isentropic) compression*. The working medium is compressed. This stroke involves both volume and temperature changes, while the entropy remains constant.

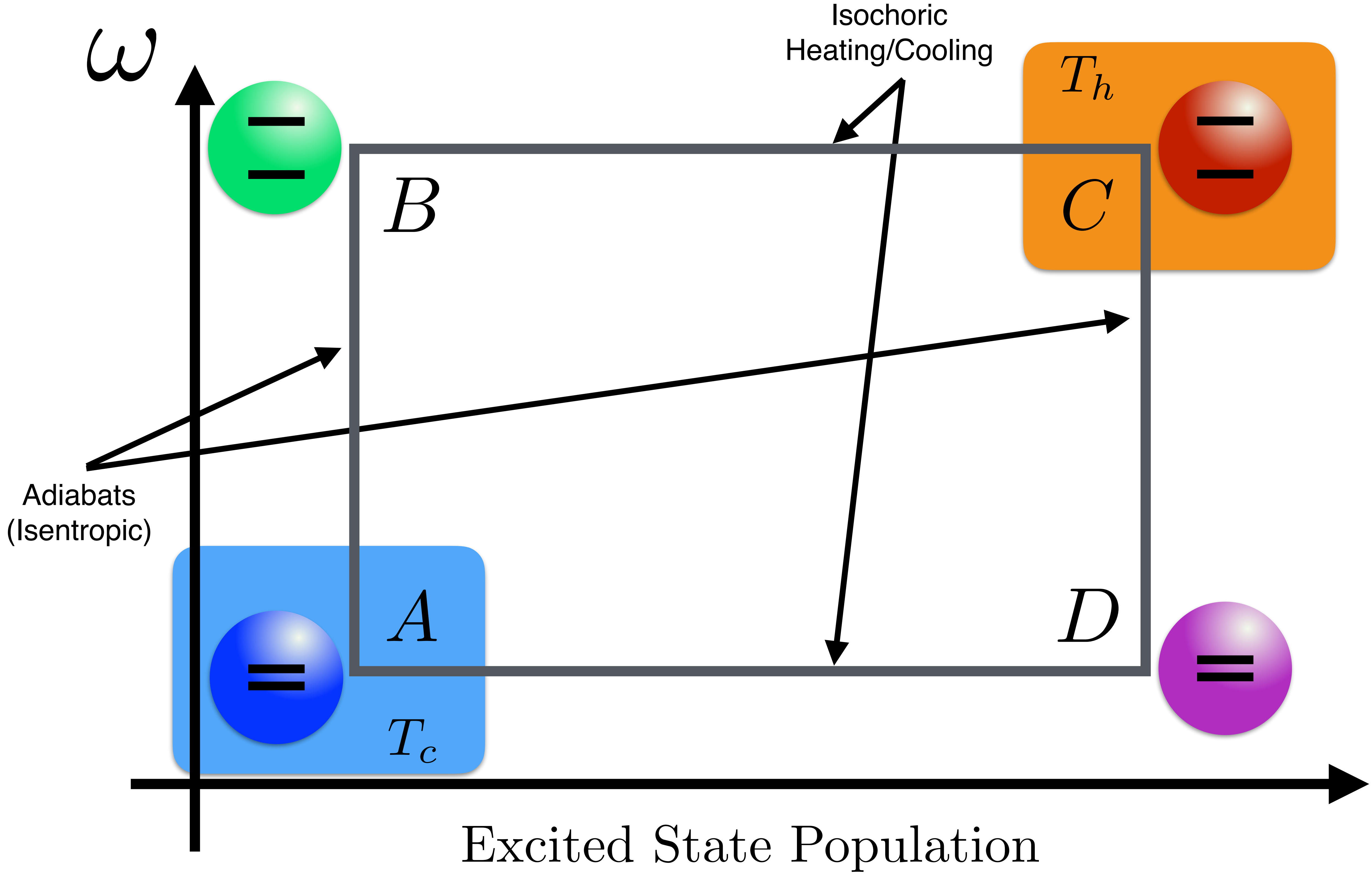
2. B → C: *Isochoric heating*. The volume of the working medium is fixed, while the temperature is increased.

3. C → D: *Adiabatic (isentropic) expansion*. The power stroke, when useful work is extracted from the engine. Again this stroke involves both volume and temperature changes, at fixed entropy.

4. D → A: *Isochoric cooling*. The working medium is cooled at a fixed volume and returned to its initial state, ready to begin the cycle again.

Quantum thermodynamic cycles and quantum heat engines, H.T. Quan, Y.-x. Liu, C. P. Sun, F. Nori, Phys. Rev. E **76**. 031105 (2007).

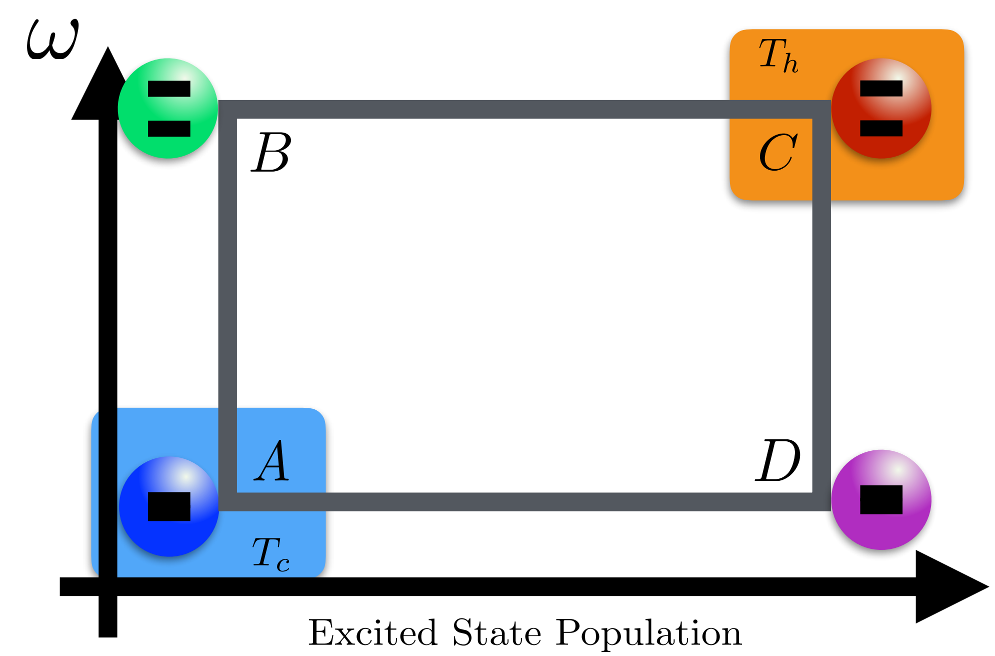
(Quantum) Otto Cycle



Quantum thermodynamic cycles and quantum heat engines, H.T. Quan, Y.-x. Liu, C. P. Sun, F. Nori, Phys. Rev. E **76**. 031105 (2007).

(Quantum) Otto Cycle

The working substance is almost never in equilibrium



Allows for a clear distinction between when work is done/to or heat is added/removed from the working substance

A significant difference between classical and quantum Otto cycles arises from the notion of adiabaticity

The fragility of quantum systems necessitates we examine how to coherently control them

For cold atoms we also have the issue of how to realise the thermalisation strokes if we insist on the working substance remaining in its ground state?

Quantum thermodynamic cycles and quantum heat engines, H.T. Quan, Y.-x. Liu, C. P. Sun, F. Nori, Phys. Rev. E **76**. 031105 (2007).
More bang for your buck: Super-adiabatic quantum engines, A. Del Campo, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014).

A Non-Linear Feshbach Engine - Basics

The basic set up has been examined for the linear case. Our interest follows 2 basic questions:

1. Can we design a quantum Otto cycle for cold atoms, i.e. without the need for an explicit temperature dependence?
2. Are there any advantages to using non-linear systems?

We consider the GPE and its associated free-space solution

$$\left[-\frac{1}{2}\psi_{xx} + g(t)|\psi(x,t)|^2 + \frac{1}{2}x^2\right]\psi(x,t) = \mu(t)\psi(x,t)$$

$$\psi(x,t) = A(t)\mathrm{sech}\left(\frac{x}{a(t)}\right)$$

$A(t)$ & $a(t)$ both functions of $g(t)$

The associated energy is then

$$\epsilon(t) = \int dx\left[\frac{1}{2}|\nabla\psi(x,t)|^2 + \frac{1}{2}x^2|\psi(x,t)|^2 - \frac{g(t)}{2}|\psi(x,t)|^4\right]$$

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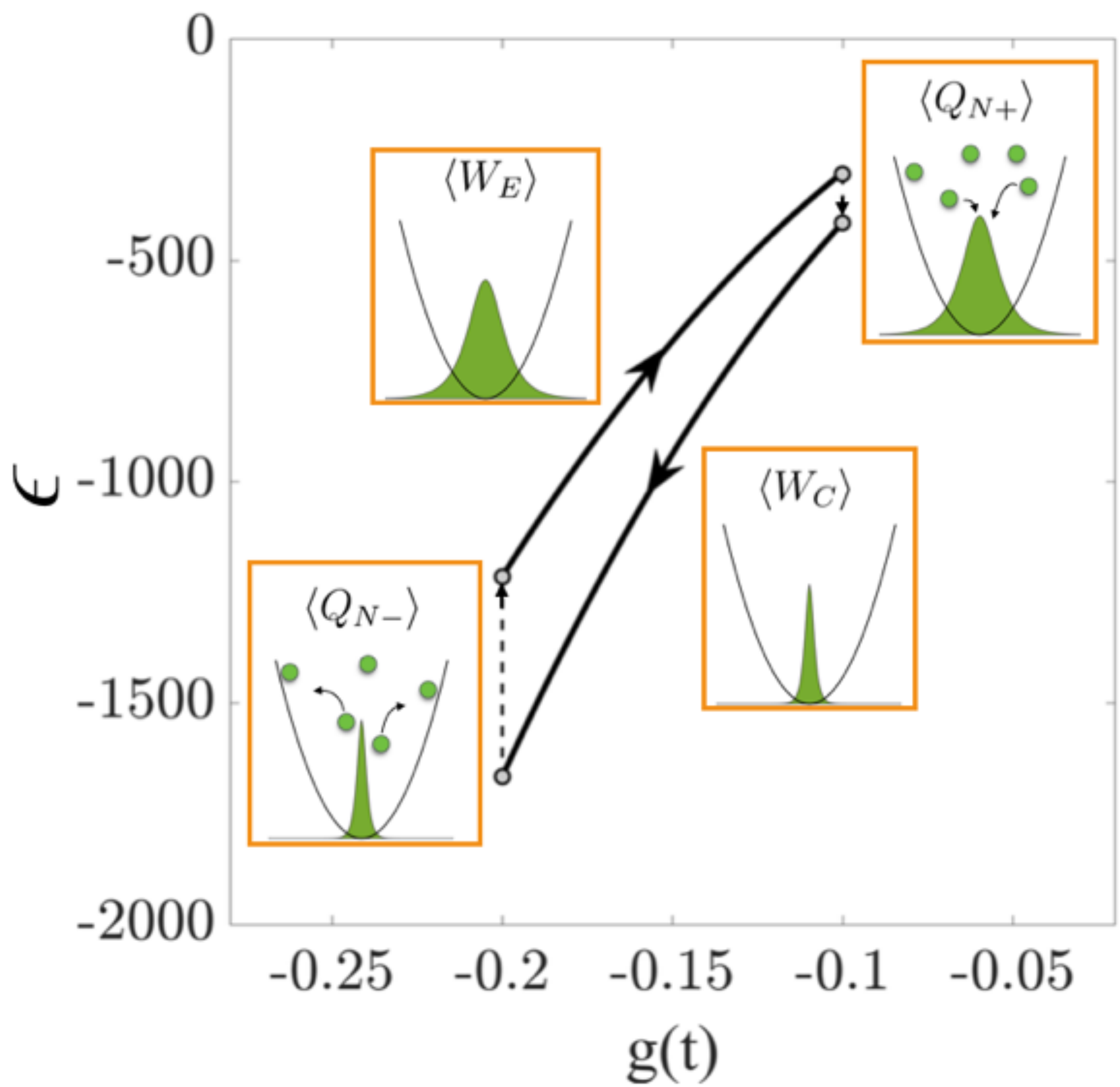
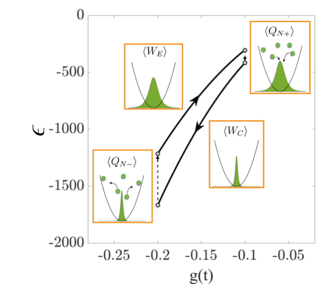
An efficient nonlinear Feshbach engine, J. Li, T. Fogarty, SC, X. Chen, Th. Busch, New J. Phys. **20**, 015005 (2018).

A Feshbach Engine - Basics

The adiabatic strokes are realised by varying the non-linear interaction strength

We model thermalisation by adding/removing atoms from the soliton





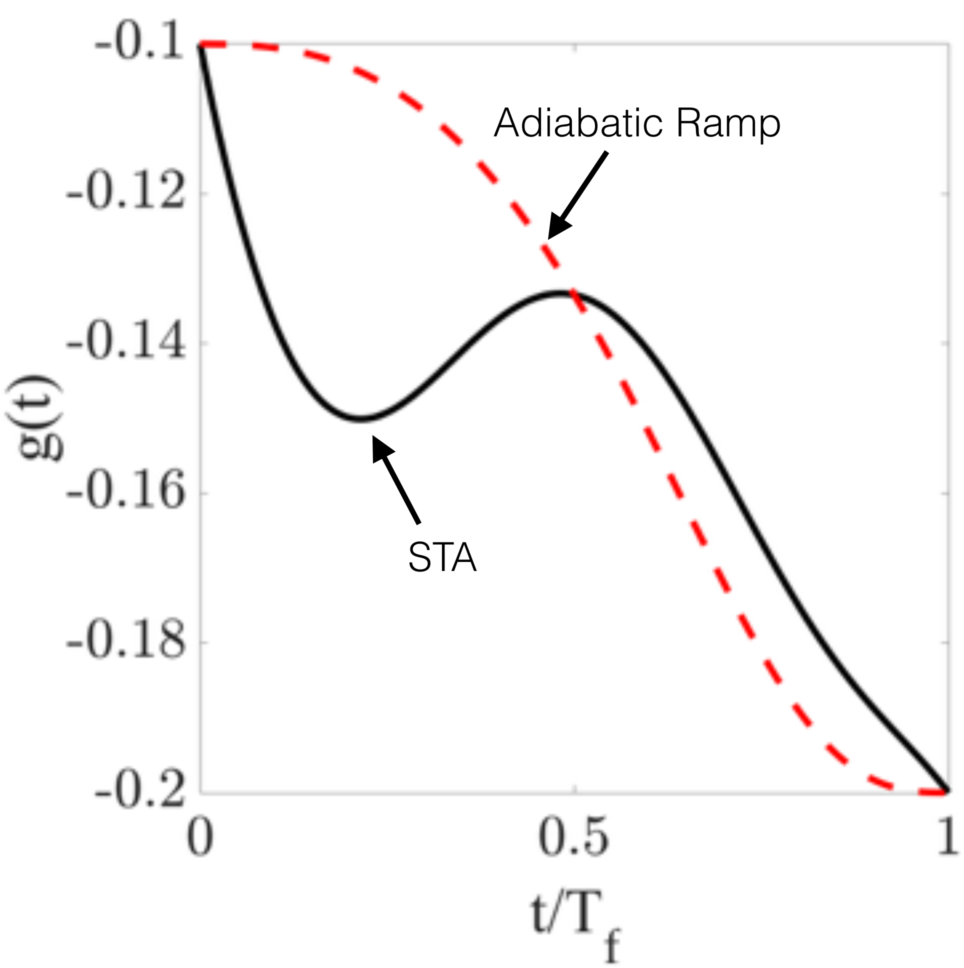
A Feshbach Engine

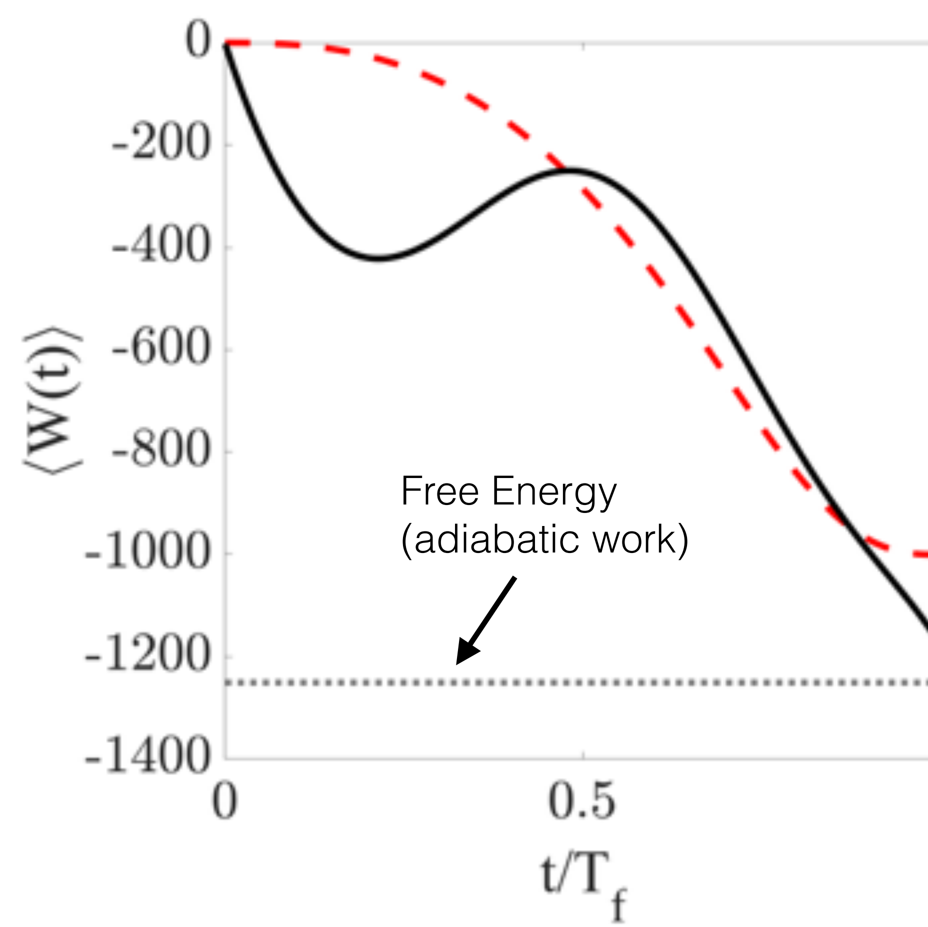
Boosting Power by STAs

We use shortcuts to adiabaticity (STA) to design a control ramp that achieves high final state fidelities with the target state for a given ramp duration

For comparison we will also use the “adiabatic” ramp

Nevertheless our scheme leads to some irreversible work being generated

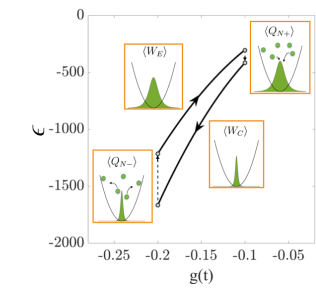




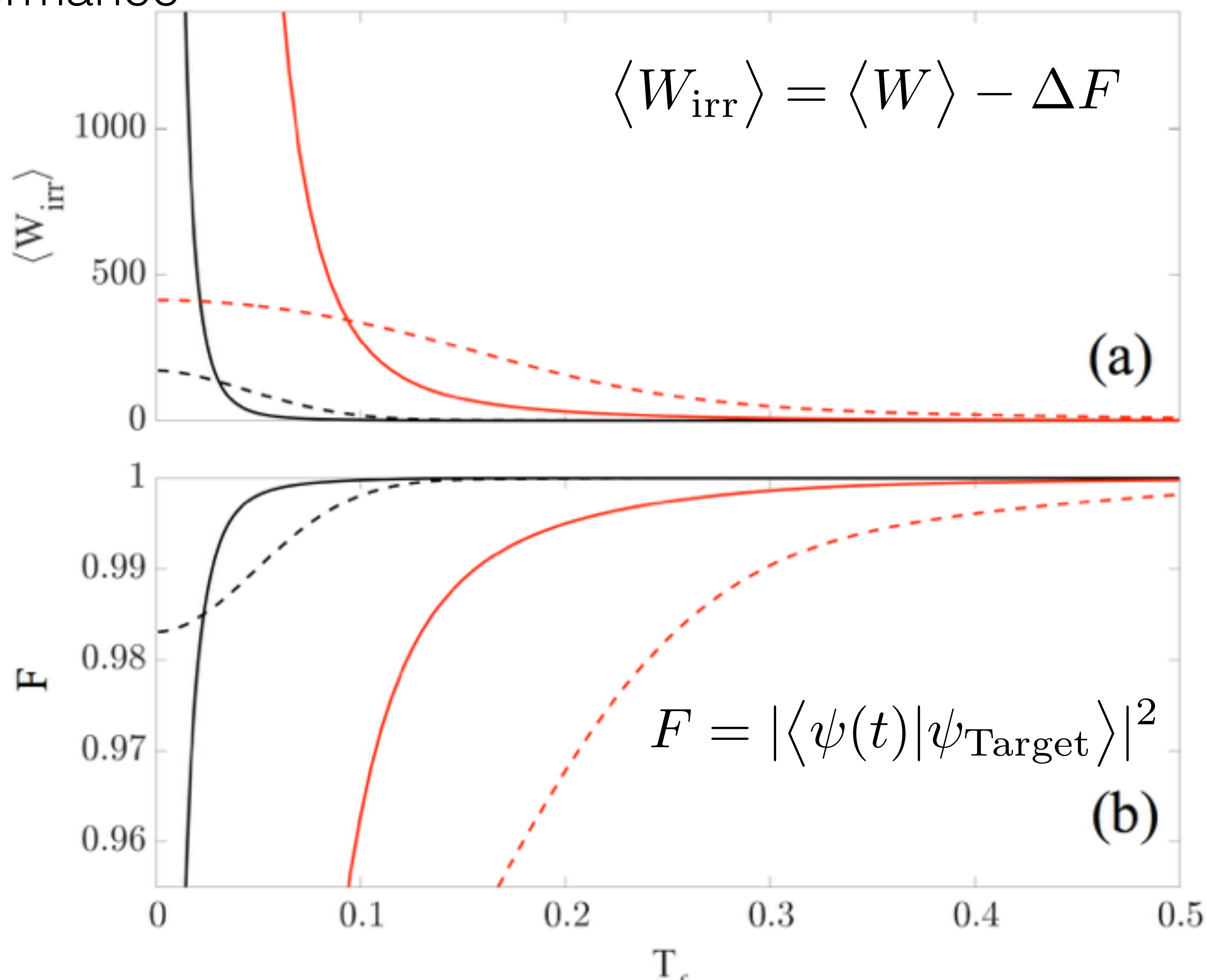
More bang for your buck: Super-adiabatic quantum engines, A. Del Campo, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014).
Shortcut to adiabatic control of soliton matter waves by tunable interaction, J. Li, K. Sun, X. Chen, Sci. Rep. **6**, 38258 (2016).
An efficient nonlinear Feshbach engine, J. Li, T. Fogarty, SC, X. Chen, Th. Busch, New J. Phys. **20**, 015005 (2018).

A Feshbach Engine

Fidelity and Irreversibility



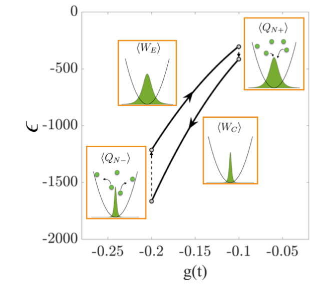
Focusing on a single adiabatic stroke we see that the STA is always effective and larger non-linearity (but same overall change in energy!) allows for better overall performance



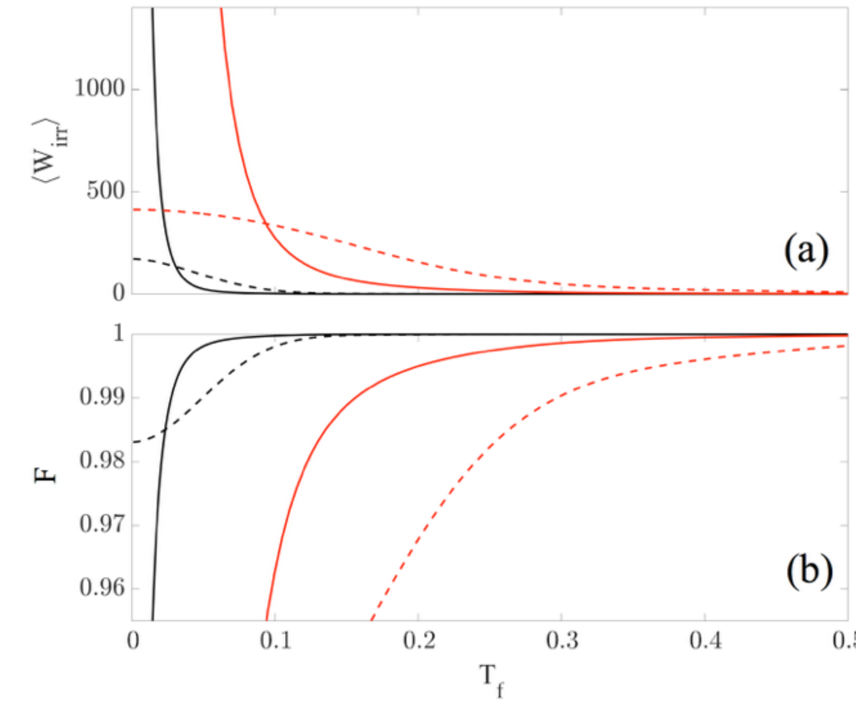
RED: Rescaled Adiabatic Ramp; **BLACK**: Using STA; SOLID: Strong nonlinear strength; DASHED: Weak nonlinear strength

A Feshbach Engine

Fidelity and Irreversibility



Combining the shortcut and larger non-linear interaction strength greatly reduces the time required to perform the stroke and still achieve (close to) the target state



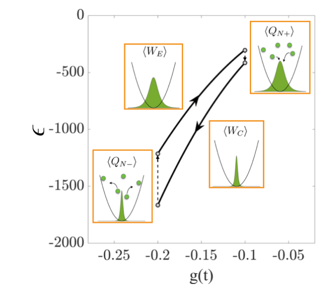
The utility of the larger non-linear interaction strength is due to its affect on the energy spectrum of the matter wave

The same basic features emerge for both ‘compression’ and ‘expansion’ stroke

Combining the two STA assisted strokes with the particle addition/subtraction strokes we can examine the overall performance of our STA assisted Feshbach-Atom Engine

A Feshbach Engine

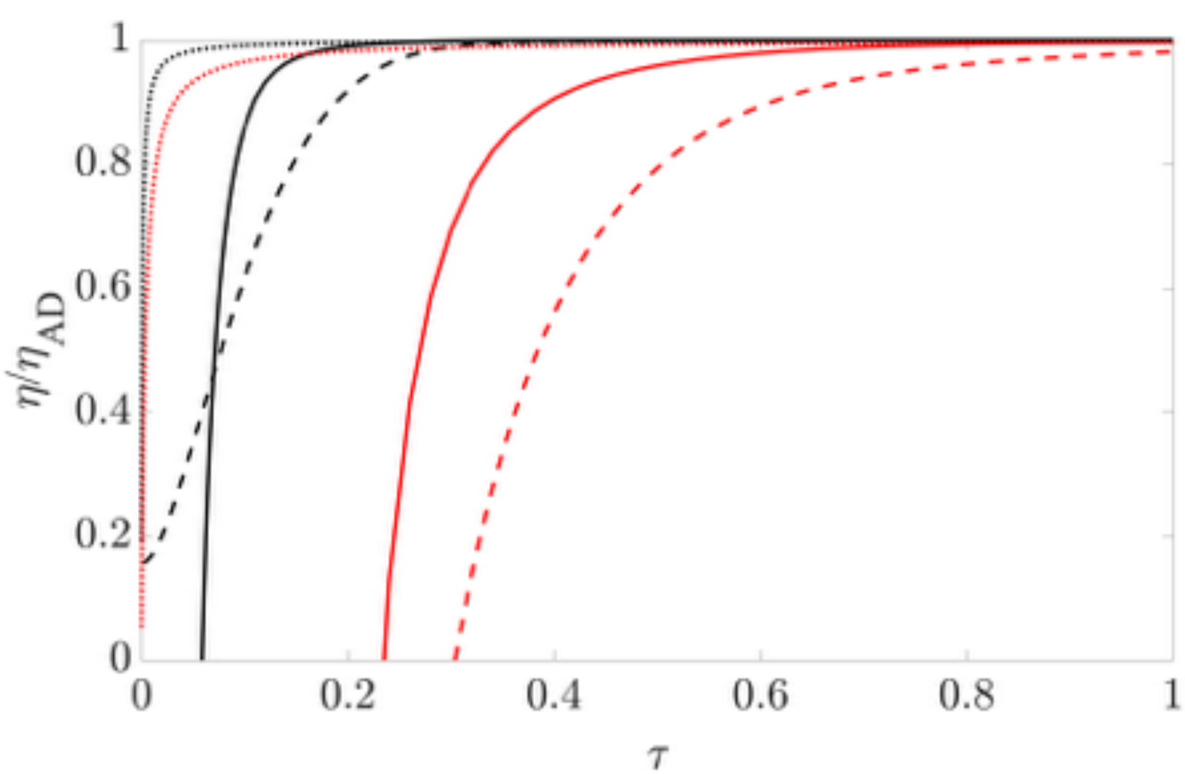
Power and Efficiency

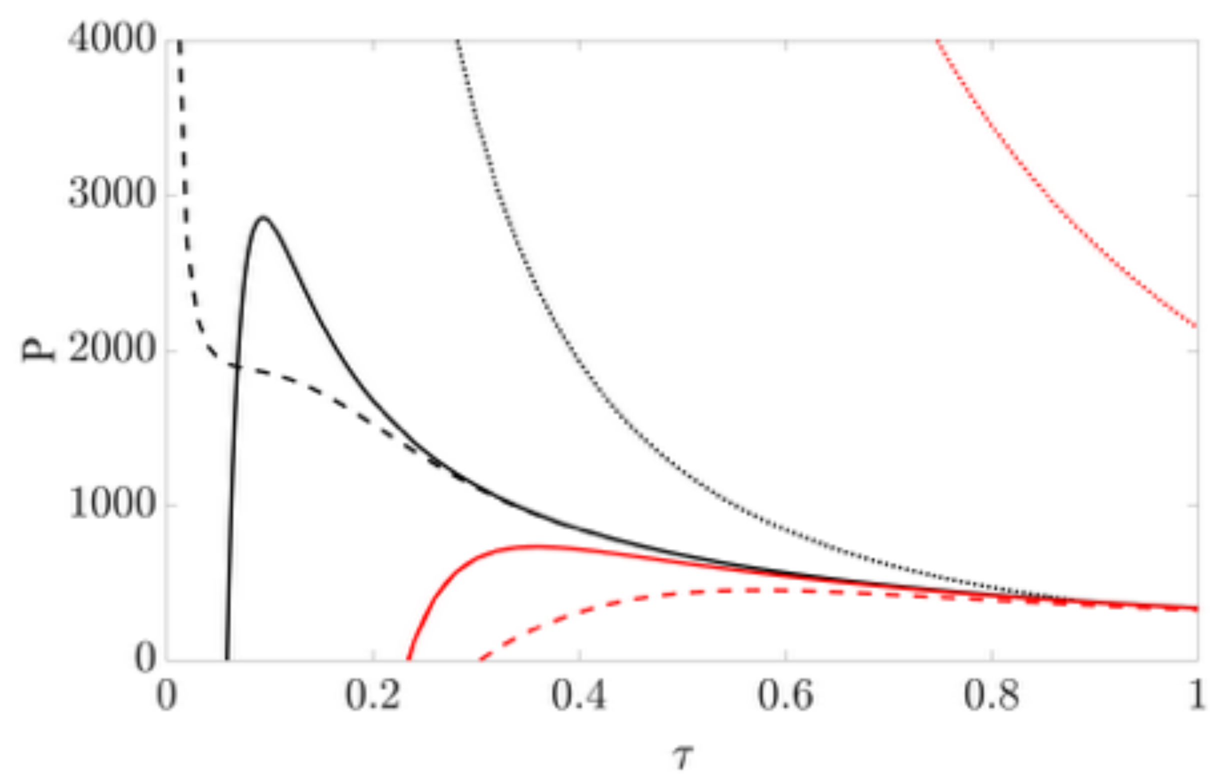


We assume the atom addition/subtraction strokes are much quicker than the adiabats (a common simplification)

From the textbook definitions of efficiency and power we again achieve better overall performance by exploiting both the STA and nonlinearities

$$\eta = -\frac{\langle W_C \rangle + \langle W_E \rangle}{\langle Q_{N_-} \rangle} \qquad P = -\frac{\langle W_C \rangle + \langle W_E \rangle}{\tau}$$





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The thermodynamic cost of quantum control

But quantum control isn't for free!

A currently active research area is in defining the (thermodynamic/energetic) cost of coherent control of quantum systems

All proposed measures nevertheless share some common traits, in particular most are related to the average/variance of the energy using the STA

We will define the cost of via

$$\langle \mathcal{E}_{STA} \rangle = \frac{1}{T_f} \int_0^{T_f} [\epsilon^I_{STA}(t) - \epsilon^I_{TRA}(t)] dt$$

(it could be viewed as a 'catalyst' as it is not dissipated - nevertheless it is still a cost that must be paid at some point)

More bang for your buck: Super-adiabatic quantum engines, A. Del Campo, J. Goold, M. Paternostro, Sci. Rep. **4**, 6208 (2014).

Cost of counterdiabatic driving and work output, Y Zheng, SC, G De Chiara, D Polettio, Physical Review A **94**, 042132 (2016).

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Universal Work Fluctuations During Shortcuts to Adiabaticity by Counterdiabatic Driving, K. Funo, J.Zhang, C.Chatou, K.Kim, M.Ueda, A. del Campo, Phys. Rev. Lett. **118**, 100602 (2017).

Energy consumption for shortcuts to adiabaticity, E. Torrontegui, I. Lizuain, S. González-Resines, A. Tobalina, A. Ruschhaupt, R. Kosloff, J. G. Muga, Phys. Rev. A **96**, 022133 (2017).

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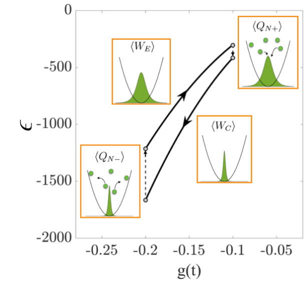
The quantum harmonic Otto cycle, R. Kosloff, Y. Rezek, Entropy **19**, 136 (2017).



A Feshbach Engine

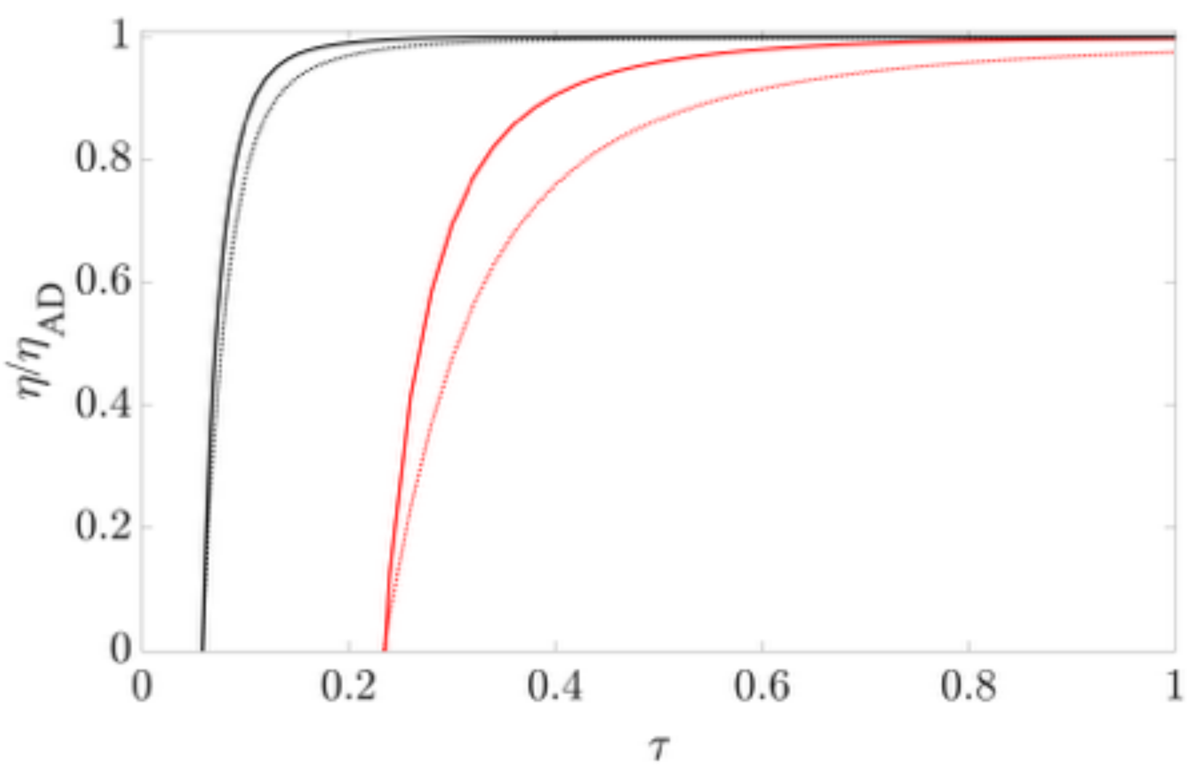
Power and Efficiency

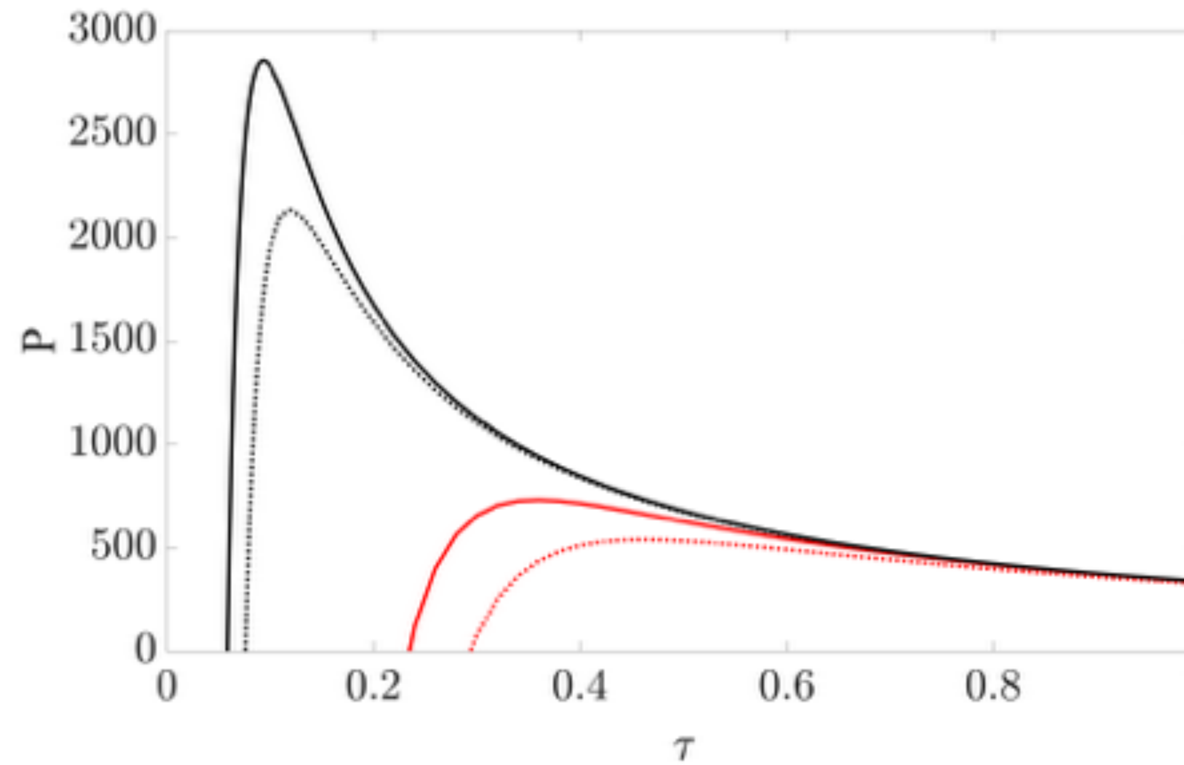
We can modify the definitions of efficiency and power to take into account the additional energetic resources necessary to realise our cycle

Even in this case we still find that the STA assisted cycle performs better

$$\eta_{cost} = -\frac{\langle W_C \rangle + \langle W_E \rangle}{\langle Q_{N_-} \rangle + \langle \mathcal{E}_{STA} \rangle_C + \langle \mathcal{E}_{STA} \rangle_E}$$

$$P_{cost} = -\frac{\langle W_C \rangle + \langle W_E \rangle - \langle \mathcal{E}_{STA} \rangle_C - \langle \mathcal{E}_{STA} \rangle_E}{\tau}$$





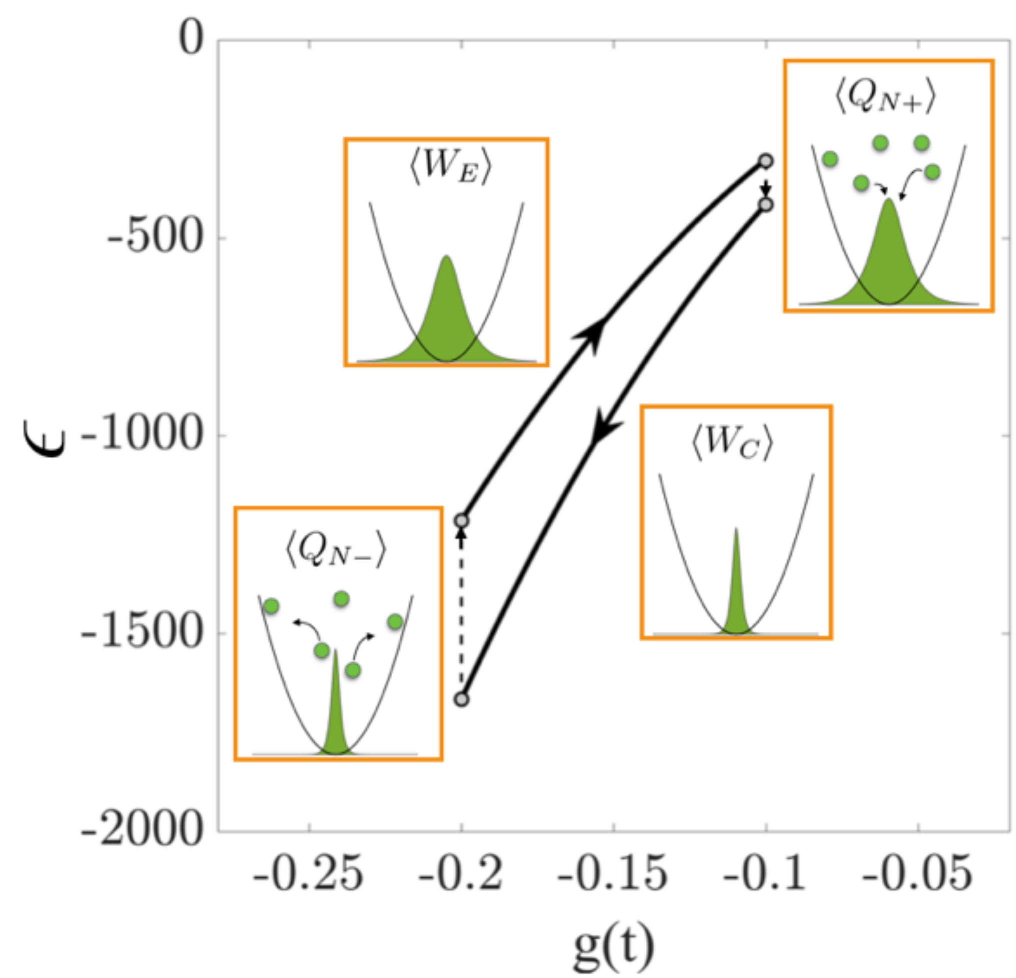
RED: Rescaled Adiabatic Ramp; **BLACK**: Using STA; THICK: without cost; THIN: with cost

The take home message(s)

Cold atomic systems are ideal test-beds for non-equilibrium quantum thermodynamics

Prototypical heat engines can be realised using cold atomic systems as working substances

The performance of cyclic thermodynamic processes can be enhanced using state-of-the-art control techniques



Nonlinearities are shown to be a potentially rich resource in further boosting the performance of these processes

Even when the additional resources required to achieve high level control are taken into account, realised cycle is still efficient

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Thanks to these guys and you all for listening

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