Cooling a quantum gas by losses

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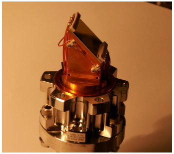
Small and Medium Sized Cold Atom Systems, Benasque, 2018, Jul 29 - Aug 04



Different $j-$body loss processes

Density decrease : $\boxed{\frac{dn}{dt} = -\kappa_j n^j}$

- **1-body process.** background gas, spin-flip
- **2-body process.** e.g. dipolar collisions for atoms in the low-field seeking state
- **3-body process.** Formation of a deeply bound dimer in 3-body collision
- **Higher order ?**

Usually consider as detrimental

Grisin et al., Rauer et al. 2016 : cooling via 1-body losses in 1D homogeneous Bose gases in the quasi-condensate regime

Effect of j-body losses on BEC or quasi-BEC ? Role of confining potential ? State produced by losses ?

I.B. et al. arXiv :1806.08759 (2018), M. Schemmer et al. arXiv :1806.09940 (2018),

A. Jonhson et al. Phys. Rev. A 96, 013623 (2017), M. Schemmer et al. Phys. Rev. A 95, 043641 (2017)

Outline

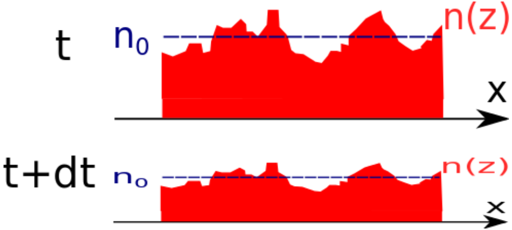
Outline

Effect of losses : qualitative picture

- Quasi-BEC or BEC : collective modes.

Long wave length : phonons govern by repulsive interactions

- Losses $\Rightarrow$ decrease of density fluctuations



$\Rightarrow$ decrease of energy in each collective mode

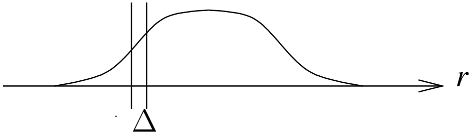
$\boxed{\Rightarrow \text{cooling}}$

- Stocastic nature of losses : increase of density fluctuations

$\boxed{\Rightarrow \text{heating}}$

$\boxed{\Rightarrow \textbf{Stationnary value of } y = k_B T/(mc^2)}$

Discretisation of the problem

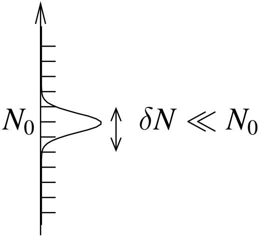


- Effect of losses in each cell

BEC or quasi-condensate gases : small density fluctuations

Small volume Δ.

$N = N_0 + \delta N$



Conjugate operator :

phase operator

$[\theta, \delta N] = i$

Effect of losses on δN and θ ?

Effect on atom-number distribution

Stocastic process :

Number of lost events during dt : $dN_e = \langle dN_e \rangle + d\xi_e$, $\langle d\xi_e^2 \rangle = \langle dN_e \rangle$

Modification of atom number :

$$dN = -\frac{\kappa_j}{\Delta^{j-1}} N^j dt + d\xi, \quad \boxed{\langle d\xi^2 \rangle = j \frac{\kappa_j}{\Delta^{j-1}} N^j dt}$$

Effect on δN : $N = N_0 + \delta N, \quad dN_0 = -\kappa_j (N_0^j / \Delta^{j-1}) dt$

$$d\delta N = -j\kappa_j n_0^{j-1} dt \delta N + d\xi$$

Dissipative term
reduction of density fluctuations
reduction of interaction energy

Stocastic term
Increase of density fluctuations
Increase of interaction energy

Effect on phase distribution

Phase diffusion

If number of lost atoms (N_l) recorded :
$\Rightarrow$ increase of knowledge on δN
$\Rightarrow \langle \delta N^2 \rangle$ decreases $\Rightarrow \langle \theta^2 \rangle$ increases

Bayes formula $P(\delta N|N_l) \propto P(N_l|\delta N)$

$$P(N_l|\delta N) \propto e^{-(N_l-\kappa_j N^j dt/\Delta^{j-1})^2/(2\sigma_l^2)}$$

To lowest order in δN : $N^j = N_0^j + jN_0^{j-1}\delta N$

$$\Rightarrow P(N_l|\delta N) \propto e^{-(\delta N-\overline{\delta N})^2/(2\sigma_{\delta N}^2)}$$

$$d\langle \theta^2 \rangle = \frac{1}{4\sigma_{\delta N}^2} = \frac{j\kappa_j n_0^{j-1}}{4n_0\Delta} dt$$

Continuous limit and reduced dimensions

Continuous limit

$$\begin{cases} d\delta n = -j\kappa_j n_0^{j-1}\delta n\, dt + d\eta \\ \langle d\eta(\mathbf{r})d\eta(\mathbf{r}')\rangle = j\kappa_j n_0^j \delta(\mathbf{r}-\mathbf{r}')dt \end{cases}$$

$$\langle d\theta(\mathbf{r})d\theta(\mathbf{r}')\rangle = \frac{j}{4}\kappa_j n_0^{j-2}\delta(\mathbf{r}-\mathbf{r}')dt$$

Classical fiel limit : $n_0 \to \infty$ at fixed $\delta n/n_0$

$\Rightarrow$ Noise terms negligible

Reduced dimension : effective κ

Loss rate $\ll \omega_\perp$: atoms stay in transverse ground state

Confinement on a transverse width $\gg$ volume for $j-$body process

$\kappa_j = \kappa_j^{3D} \int d^2x_\perp |\psi(x_\perp)|^{2j}$ (2D)

$\kappa_j = \kappa_j^{3D} \int dx_\perp |\psi(x_\perp)|^{2j}$ (1D)

Case Homogeneous gas : intrinsic dynamic

Bogoliubov Hamiltonian

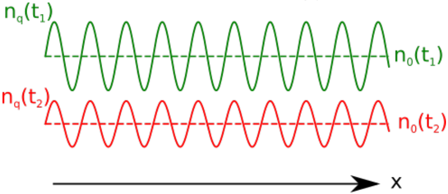
Linearisation in $\delta n(\mathbf{r})$ and $\theta(\mathbf{r})$

Collective modes : Fourier modes. $H = \sum_k H_k$

$$H_k = A_k \delta n_k^2 + B_k \theta_k^2, \qquad B_k = \hbar^2 k^2 n_0/(2m)$$

Long wave-length modes (phonons) : $A_k = g/2$

Effect of decrease of n : reduction of energy in each mode $\Rightarrow$ cooling



Homogeneous gas : Evolution of energy in phonon modes

$H_k = \frac{g}{2}\delta n_k^2 + \frac{\hbar^2 k^2 n_0}{2m}\theta_k^2$

- **Small loss rate** $\Rightarrow$ Equipartition : $\frac{g}{2}\langle\delta n_k^2\rangle = \frac{\hbar^2 k^2 n_0}{2m}\langle\theta_k^2\rangle = \langle H_k\rangle/2$
- **Effect of modification of δn :**

$d\langle\delta n_k^2\rangle = -2j\kappa_j n_0^{j-1}\langle\delta n_k^2\rangle dt + j\kappa_j n_0^j dt$

- **Effect of modification of θ :** $d\langle\theta_k^2\rangle = \frac{1}{4}j\kappa_j n_0^{j-2}dt$

Change of mode energy and of $y = \langle H_k\rangle/(gn_0) \simeq T_k/(gn_0)$

$$d\langle H_k\rangle/dt = \kappa_j n_0^{j-1}\left(-\langle H_k\rangle(j+\frac{1}{2}) + j\frac{g}{2}n_0 + j\frac{\hbar^2 k^2}{8m}\right)$$

$$dy/dt \simeq \kappa_j n_0^{j-1}\left(-y(j-\frac{1}{2}) + \frac{j}{2}\right)$$

$\boxed{\text{Stationnary value : } y_\infty = 1/(2-1/j)}$

Homogeneous gas : Evolution of energy in phonon modes

$$H_k = \frac{g}{2}\delta n_k^2 + \frac{\hbar^2 k^2 n_0}{2m}\theta_k^2$$

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$$d\langle\delta n_k^2\rangle = -2j\kappa_j n_0^{j-1}\langle\delta n_k^2\rangle dt + j\kappa_j n_0^j dt$$

- **Effect of modification of θ :** negligible for phonons

Change of mode energy and of $y = \langle H_k\rangle/(gn_0) \simeq T_k/(gn_0)$

$$d\langle H_k\rangle/dt = \kappa_j n_0^{j-1}\left(-\langle H_k\rangle(j+\frac{1}{2}) + j\frac{g}{2}n_0 + \cancel{j\frac{\hbar^2 k^2}{8m}}\right)$$

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$$\boxed{\text{Stationnary value : } y_\infty = 1/(2-1/j)}$$

General case : gas intrinsic dynamcis

Evolution of mean profile under losses

$n_0(\mathbf{r},t)$ evolves in time and mean velocity field $\nabla\theta_0$

- Small loss rate : adiabatic following and $\nabla\theta_0$ negligible
- Local Density Approximation

$$\mu(n_0(\mathbf{r},t)) = \mu_p(t) - V(\mathbf{r})$$

Evolution of fluctuations

Bogoliubov : Linearisation in $\delta n(\mathbf{r})$ and $\varphi(\mathbf{r}) = \theta - \theta_0$
Hydrodynamic modes : long wavelengths

$$H_{\mathrm{hdyn}} = \frac{\hbar^2}{2m}\int d^d\mathbf{r}\, n_0(\nabla\varphi)^2 + \frac{m}{2}\int d^d\mathbf{r}\frac{c^2}{n_0}\delta n^2$$

$$mc^2(\mathbf{r}) = n_0\partial_n\mu|_{\mathbf{r}}$$

Collective hydrodynamic modes

Diagonalisation of H_{hdyn}

At any time $H_{\mathrm{hdyn}} = \sum_\nu H_\nu$,

$$H_\nu = \frac{\hbar\omega_\nu}{2}(x_\nu^2 + p_\nu^2)$$

$$\begin{cases} x_\nu = \dfrac{m}{\hbar\omega_\nu} \displaystyle\int d^d\mathbf{r} \frac{c^2 \delta n}{n_0} g_\nu(\mathbf{r}) \\ p_\nu = \int d^d\mathbf{r} \varphi(\mathbf{r}) g_\nu(\mathbf{r}) \end{cases}$$

Time-depend mode function g_ν : $\nabla \cdot \left(n_0 \nabla(\frac{c^2}{n_0} g_\nu)\right) = -\omega_\nu^2 g_\nu$

Effect of losses

Evolution of $\langle H_\nu \rangle \simeq T_\nu$?

Evolution of T_ν

Small loss rate

- Modification of $n_0(\mathbf{r})$ and $g_\nu(\mathbf{r})$: keep invariant $A_\nu = \langle H_\nu \rangle / (\hbar\omega_\nu)$
- Coupling between modes introduced by losses neglected
- Equipartition at all time

Differential equation for $y_\nu = \langle H_\nu \rangle / (mc_p^2)$

$$\frac{d}{dt} y_\nu = \kappa_j n_p^{j-1} \left[-(j\mathcal{A} - \mathcal{C}) y_\nu + j\mathcal{B} \right]$$

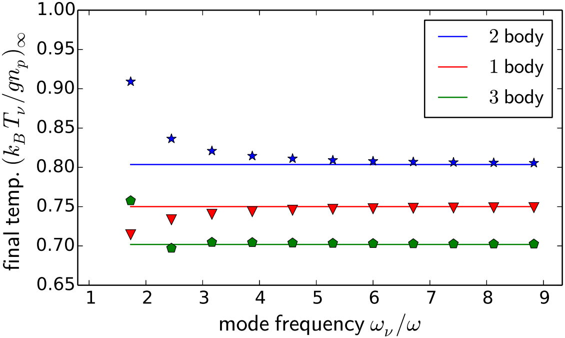
$\mathcal{A}, \mathcal{B}, \mathcal{C}$: integrals involving $n_0(\mathbf{r})$ and $g_\nu(\mathbf{r})$.

- $\mathcal{A}$: reduction of density fluctuations due to loss process
- $\mathcal{C}$: time evolution of $mc_p^2/(\hbar\omega_\nu)$
- $\mathcal{B}$: density fluctuations due to stocastic nature of losses

See I. Bouchoule et al., arXiv :1806.08759

Application of formalism : Asymptotic temperature for 1D gas in a harmonic potential

Functions g_ν : Legendre polynomials

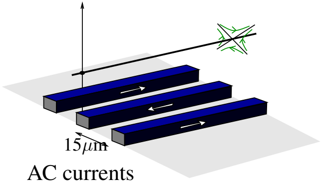


Large$-\nu$ limit : $y_\infty \simeq \dfrac{\frac{j}{\pi}\int_0^{\pi/2} d\alpha\ \sin^{2j}\alpha}{\frac{2j}{\pi}\int_0^{\pi/2} d\alpha\ \sin^{2j-2}\alpha - \int_0^{\pi/2} d\alpha\ \sin^{2j+1}\alpha}$

Outline

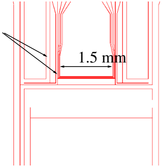
Experimental setup : trapping atoms with an atom-chip

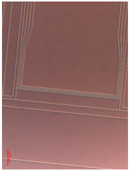
- Magnetic confinement of ^{87}Rb atoms : $V = \mu_B |\mathbf{B}|$
- Cu micro-wires deposited on an AlN substrate



AC currents
$\omega_\perp = 0.1 - 100\text{kHz}$

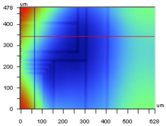
Chip design (wire edges shown)

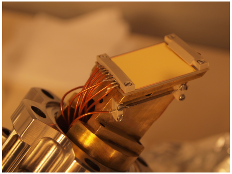




- Planarisation and insulation with resist, covered with Au miror

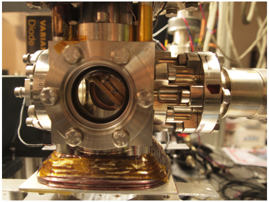
Interferometric image

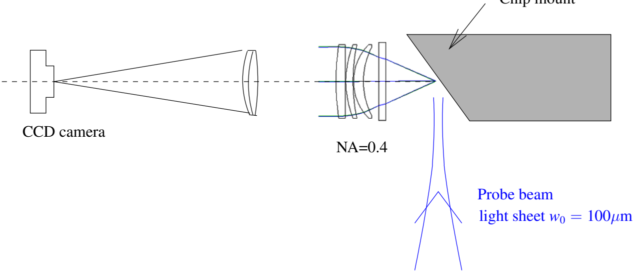




Fabrication : LPN, CNRS, help of S. Bouchoule

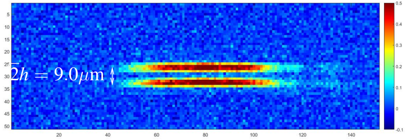
Experimental setup : realising and imaging 1D gases





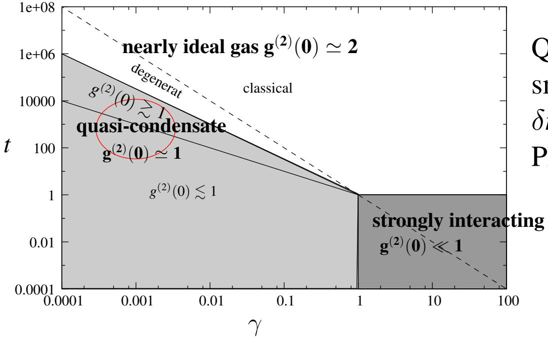
- $N_{\mathrm{at}} = 3 - 10 \times 10^3$
- $\omega_z = 8 - 15$ Hz
- $\omega_\perp = 1.5 - 3$ kHz
- $\mu \simeq T = 50 - 100$ nK
- $l_c/\xi \simeq 10$: deep into quasi-BEC

A typical in-situ absoprtion image



Regimes of 1D Bose gas with repulsive contact interaction

Contact repulsive interaction : $g\delta(z_i - z_j)$
Thermodynamic : Yang-Yang (60')
Dimensionless parameters : $t = \hbar k_B T/(mg^2)$, $\gamma = mg/\hbar^2 n$

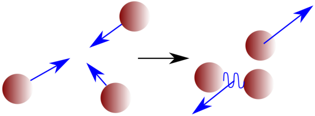


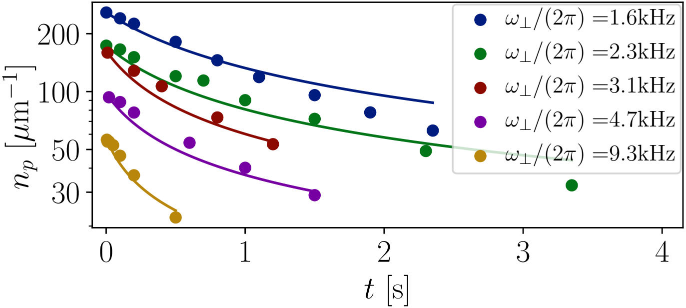
Quasi-bec regime :
small density fluctuations
$\delta n \ll n_0$
Phase fluctuations remain

Decay of atom number under 3-body loss process

- ω_{RF} of radio-frequency field increased $\Rightarrow$ no 1-body losses
- Losses dominated by 3-body process

$$\frac{dn}{dt} = -K_3 n^3$$





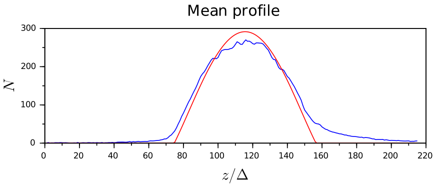
Thermometry in qBEC regime via density ripples analysis

- Trapping potential suddenly turned off

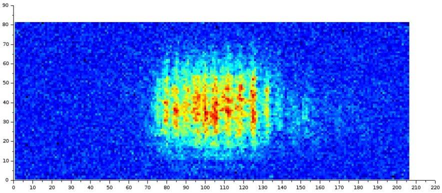
transverse expansion $\rightarrow$ instantaneous switching off of interactions

- 8 ms time of flight $\rightarrow$

phase fluctuation transform into density fluctuations $\Rightarrow$ density ripples



Single shot image

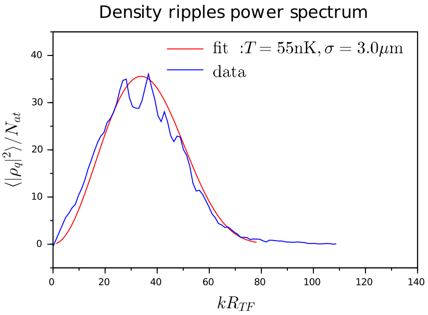


Statistical analysis on $\simeq$ 50 images

$\Rightarrow$ extract power spectrum $\langle |\rho_q|^2 \rangle$

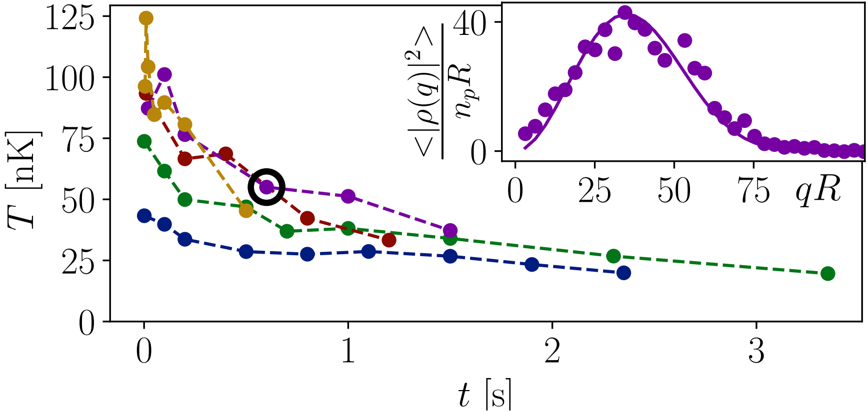
Power spectrum of density ripples and thermometry

We fit the power spectrum to obtain the temperature.



Sensitive to phononic modes

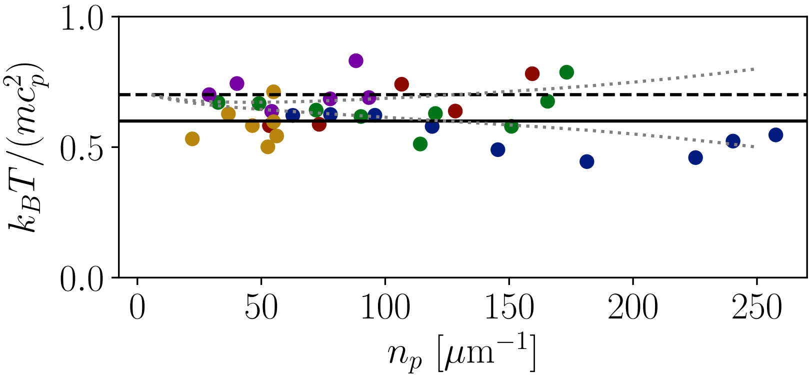
Evolution of temperature during the loss process



Decrease of the temperature up to a factor 4.

⇒ Losses associated to cooling

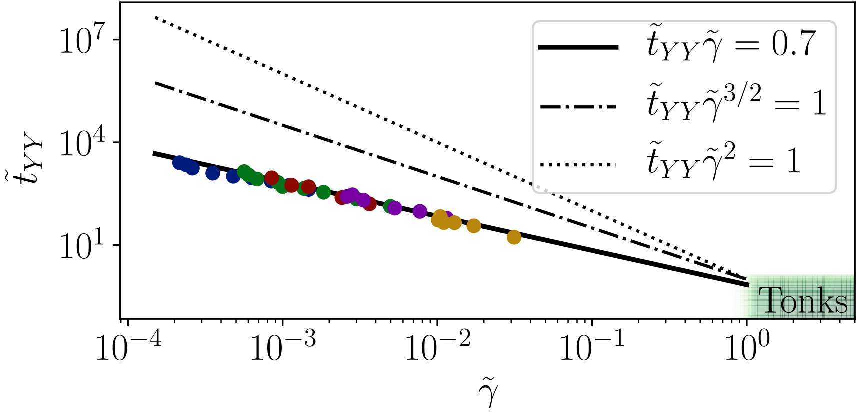
Ratio $k_B T/(mc^2)$



- Quasi-1D : transverse swelling non negligible. mc^2 non linear in n
- Stationnary ratio attained as soon as the gas is deep into the quasi-bec regime and reach the 1D regime.

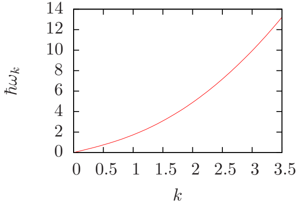
Zone explored in the phase diagram

We generalise the 1D paramaters to quasi-1D : $\tilde{t} = \hbar^2 k_B T n^2/(m^3 c^4)$ and $\tilde{\gamma} = m^2 c^2/(\hbar^2 n^2)$



Outline

Beyond phonons. 1-body losses, homogeneous gases



Bogoliubov Hamiltonian : $H_k = A_k \delta n_k^2 + B_k \theta_k^2$
$B_k = \hbar^2 k^2/(2mn_0)$, $A_k = g/2 + \hbar^2 k^2/(8mn_0)$
Phonons : $k \ll \sqrt{mgn_0}/\hbar$, $\omega_k \simeq k\sqrt{gn_0/m}$
Particles : $k \gg \sqrt{mgn_0}/\hbar$, $\omega_k \simeq \hbar k^2/(2m)$

- Small loss rate : adiabatic invariant $\tilde{E}_k = \langle H_k \rangle / (\hbar\omega_k)$

$$\Rightarrow \frac{d}{dt}\tilde{E}_k = \Gamma\left(-\tilde{E}_k + \left(\sqrt{\frac{\hbar^2 k^2/2m + 2gn_0}{\hbar^2 k^2/2m}} + \sqrt{\frac{\hbar^2 k^2/2m}{\hbar^2 k^2/2m + 2gn_0}}\right)/4\right)$$

Different modes acquiere different temperature

Phonons : $k_B T_{\text{phonon}} \underset{t\to\infty}{\simeq} \rho_0(t) g$

Particles : $k_B T_{\text{part}} \underset{t\to\infty}{\simeq} \frac{\hbar^2 k^2}{2m}\frac{1}{\Gamma t}$

Large t : $T_{\text{part}} \gg T_{\text{phonon}}$

$\Rightarrow$ **Generalised Gibbs ensemble**

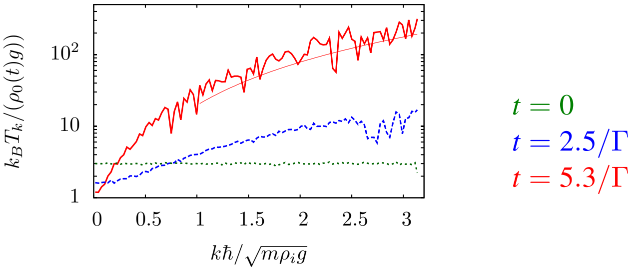
Robustness versus non-linear couplings for 1D gases

Beyond Bogoliubov : truncated Wigner approximation

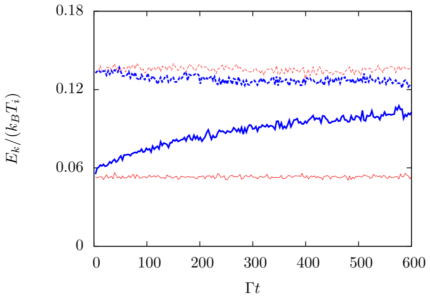
Wigner function evolves according to trajectories :

$$i\hbar d\psi = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\psi + g|\psi|^2\psi - i\frac{\Gamma}{2}\psi \right) dt + d\xi$$

$$\langle d\xi^*(z) d\xi(z') \rangle = \Gamma dt \delta(z - z')/2$$



Long-lived non-thermal states and link with integrability



top curves :
$k = 6.0\sqrt{mg\rho_i}/\hbar$
bottom curbes :
$k = 6.0\sqrt{mg\rho_i}/\hbar$
$\tilde{g} = 0.4g$ ($m'/m = 3$)
$\tilde{g} = 0$

Breaking integrability

Two-1D Bose gases :

$$i\hbar\partial\psi/\partial t = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial z} + (\tilde{g}|\varphi|^2 + g|\psi|^2)\psi,$$
$$i\hbar\partial\varphi/\partial t = -\frac{\hbar^2}{2m'}\frac{\partial^2\varphi}{\partial z} + (g|\varphi|^2 + \tilde{g}|\psi|^2)\varphi.$$

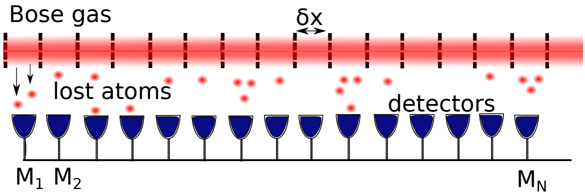
$\tilde{g} = 0 \Rightarrow$ 2-independant 1D Bose gases
$\tilde{g} \neq 0 \Rightarrow$ non-integrable system

Outline

Learning from losses and quantum feedback cooling

- Time and position-resolved detector : information on density fluctuations present in the gas
- Backaction condition on the recorded losses

$\Rightarrow$Cooling collective modes of the gas



Learning from losses and quantum feedback cooling

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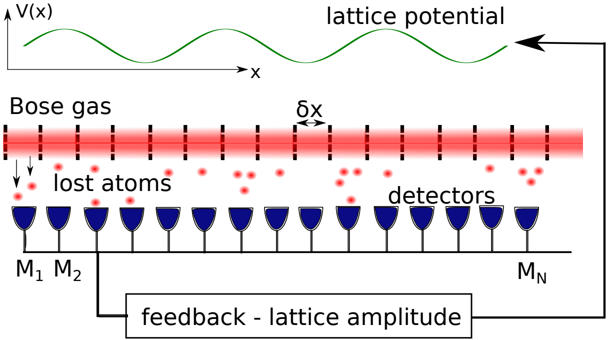


Quantum Monte Carlo wave-function analysis

Wave-function evolution during Δt for a single "cell" of the gas

Fock state expansion : $|\psi\rangle = \sum_n c_n |n\rangle$

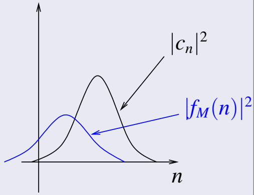
Monte-Carlo evolution :

M recorded lost atoms :

$c_n \to f_M(n) c_n$

$\Rightarrow$ (i) shift of center (depends on M)

$\Rightarrow$ (ii) narrowing of the distribution



For a given Bogoliubov mode of a homogeneous gas (wave-vector k)

Expansion in n_k basis :

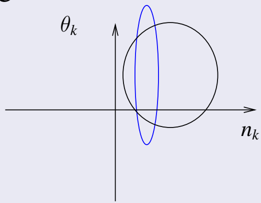
$|\psi_k\rangle = \int dn_k c(n_k) |n_k\rangle$

Monte-Carlo evolution :

M_k Fourier transform of the M's :

$c(n_k) \to f_{M_k}(n_k) c(n_k)$

Wigner function evolution



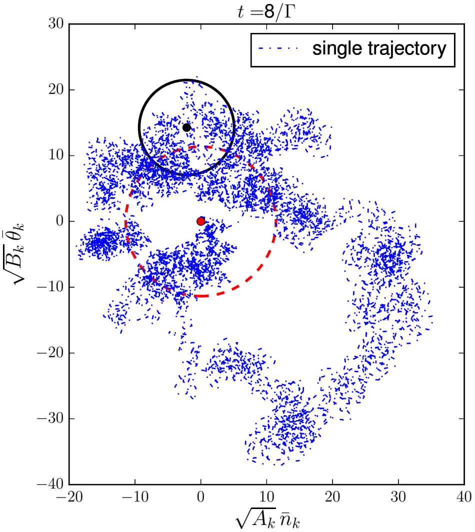
Evolution of Wigner function for a single Bogoliubov mode and for a given quantum trajectory

$H_k = A_k n_k^2 + B_k \theta_k^2$

Assume $g(t) = g_0 e^{\Gamma t}$
$\Rightarrow gn_0 = \text{cste}$
Loss rate : $\Gamma = \omega_k/400$

Evolution of center :
trajectory dependent

Width of the distribution :
goes to ground state width

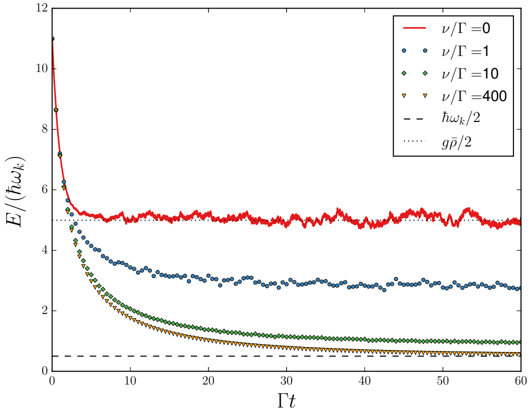


Quantum feedback : cooling to groud state

Feedback : periodic potential of amplitude $V \Rightarrow \hat{H}_{\rm fb} = V(t)\hat{n}_k$

$V(t) = -\hbar\nu\langle\theta_k\rangle$: computed using the acquired losses information and integrating equation of motion

Average over trajectories



Outline

Conclusion and prospects

Conclusion

- First observation of 3-body losses cooling
- Extension of previous theoretical work
- Non-thermal nature of the state resulting from losses
- Proposal for quantum feedback to ground state

Prospects

- Elucidating the effect of 1-body losses (stationnary ratio $k_B T/(mc^2)$ not observed experimentally)
- Extend work on non-thermal states to the case of trapped systems
- Extend this work to strongly interacting regime of 1D Bose gases
- Take into account an eventual position-dependent loss term : link with evaporative cooling