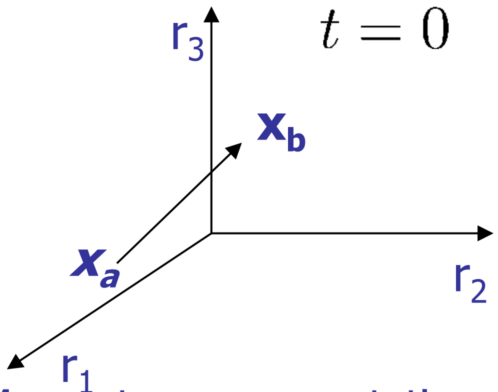
Femtoscopy signatures of collective flows in p-p collisions.

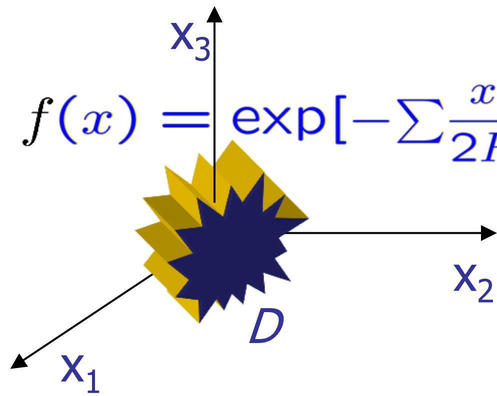
Yu.M. Sinyukov (BITP, Kiev)

The Benasque COST workshop on collectivity in small systems
August 13-15 , 2018

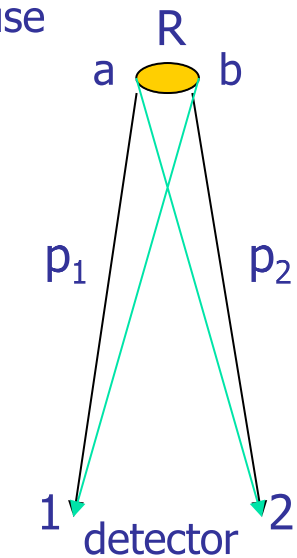
Interferometry microscope (Kopylov, Podgoretcky: 1971-1973)

The idea of the correlation femtoscopy is based on an impossibility to distinguish between registered particles emitted from different points because of particle identity.





$$f(x) = \exp[-\sum \frac{x_i^2}{2R_i^2}]$$

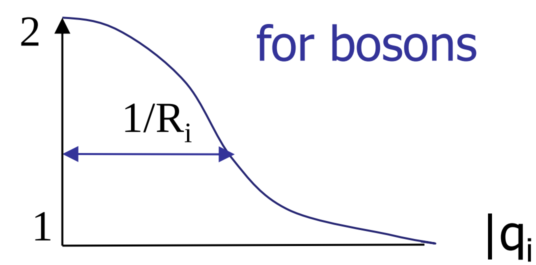


Momentum representation

$$\Psi_{x_a,x_b}(p_1,p_2) = \frac{1}{\sqrt{2}}\left[e^{-i\mathbf{p}_1\cdot\mathbf{x}_a}e^{-i\mathbf{p}_2\cdot\mathbf{x}_b} \pm e^{-i\mathbf{p}_2\cdot\mathbf{x}_a}e^{-i\mathbf{p}_1\cdot\mathbf{x}_b}\right]$$

Probabilities:

$$W_{x_a,x_b}(p_1,p_2) = |\Psi_{x_a,x_b}(p_1,p_2)|^2 = 1 \pm \cos\left[\overbrace{(\mathbf{p}_1 - \mathbf{p}_2)}^{\mathbf{q}} \cdot (\mathbf{x}_a - \mathbf{x}_b)\right]$$



for bosons

The model of independent particle emission

$$W_D(p_1,p_2) = \int_D d^3x_1 d^3x_2 f(x_1) f(x_2) W_{x_1,x_2}(p_1,p_2) = 1 \pm |\int d^3x f(x) e^{i\mathbf{q}\mathbf{x}}|^2 = 1 \pm \exp\left[-\sum q_i^2 R_i^2\right]$$

Further development of the correlation femtoscopy collisions

The theory, method and interpretation of the correlation femtoscopy measurements that are utilized by all the collaborations dealing with such kind of analysis in A+A and p+p collisions at the SPS, RHIC та LHC, are developed. It allows to study the homogeneity lengths in extremely inhomogeneous fast expanding hadron and quark-gluon systems, with accuracy 10^{-15} m and 10^{-23} s.

«Sinyukov-Makhlin formula" that allow to measure the life-time of the hot matter at "Little bang"

1987

$$R_L \approx \tau\sqrt{\frac{T_{f.o.}}{m_T}}$$

$$\underbrace{R_l^2(k_T) = \tau^2\lambda^2\left(1+\frac{3}{2}\lambda^2\right)}_{\textbf{2015}}$$

$$\lambda^2 = \frac{T}{m_T}\left(1-\bar{v}_T^2\right)^{1/2}$$

$\bar{v}_T^2$ - mean transv. velocity

Femto "**homogeneity lengths**".
The general interpretation of the femtoscopy scales as the spatio-temporal homogeneity length has been formulated

$$R_i^2 = \lambda_i^{\,2} = \frac{f(x_0, p)}{\left|f''_{x_i}(x_0, p)\right|}$$

"Bowler–Sinyukov treatment"
The method that allow to separate the quantum-statistical (QS) correlations from Coulomb ones and long-lived (l-l) resonance contributions is proposed

$$C_{tot}(q) = (1-\alpha) + \alpha K(q_{inv})(1 + C_{QS}(\mathbf{q}))$$

α - fraction of l-l resonances
K- Coulomb wave function.
C_{QS} - QS-кореляційна ф.

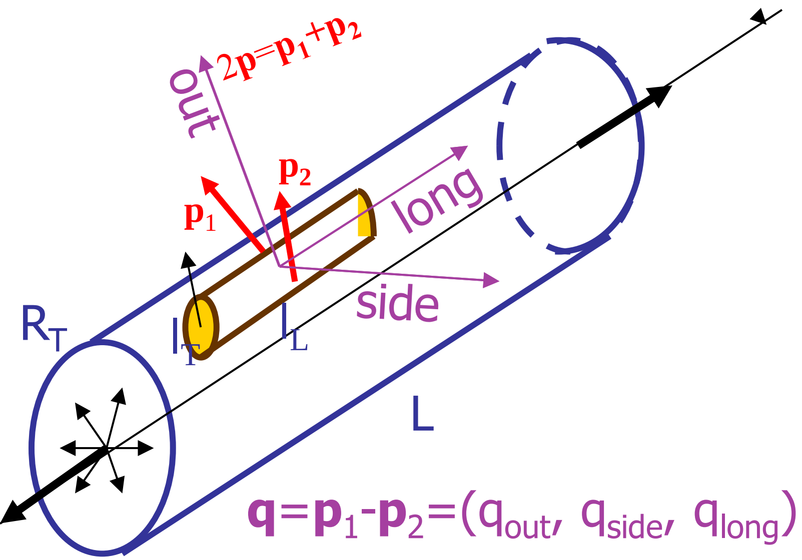
THE DEVELOPMENT OF THE FEMTOSCOPY (Yu.S.1986 – 1995)

To provide calculations analytically one should use the saddle point method and Boltzmann approximation to Bose-Einstein distribution function. Then the single particle spectra are proportional to homogeneity volume:

$$p^0 \frac{d^3N}{d^3p} \propto \prod_i \lambda_i(p)$$

and just these homogeneity lengths forms exponent in Bose-Einstein correlation function

$$C = 1 + \exp\left[-\sum q_i^2 R_i(p)^2\right]$$



Interferomerty radii:

$$R_L(p_T) \approx \lambda_L = \tau\sqrt{\frac{T_{f.o.}}{m_T}}/cosh(y), m_T = \sqrt{m^2 + p_T^2}$$

$$R_S \approx \lambda_T = R_T/\sqrt{1 + Im_T/T_{f.o.}},\ I \propto < v_T^2 >$$

$$R_o^2 \approx \lambda_T^2 + v^2\langle\triangle t^2\rangle_p - 2v\langle\triangle x_o \triangle t\rangle_p, v = \frac{p_{out}}{p_0}$$

$$C(p,q) = \frac{d^6N/d^3p_1 d^3p_2}{d^3N/d^3p_1 d^3N/d^3p_2} \approx 1 + e^{R_L^2(p)q_L^2 + R_s^2(p)q_s^2 + R_O^2(p)q_O^2}$$

QGP $\Rightarrow$ $R_{out}/R_{side} >> 1$ $Exp: R_{out}/R_{side} \approx 1$ RHIC HBT PUZZLE

First attempts to apply hydro for both types of high energy collisions: A+A and p+p

Physics Letters B 265 (1991) 159–166
North-Holland

PHYSICS LETTERS B

Pion interferometry testing the validity of hydrodynamical models

B. Lörstad [1]
Department of Particle Physics, Lund University, S-22362 Lund, Sweden

and

Yu.M. Sinyukov
Institute of Theoretical Physics, SU-252 130 Kiev, USSR

Slope of pion p_T spectra → $T_{f.o.}$, μ

$T_{f.o}$ + Interferometry radii → R_T, τ

$$\frac{dN_\pi}{dy} \approx \pi R_T^2 \tau n(T_{f.o}, \mu)$$

Experiment. data for $\frac{dN_\pi}{dy}$

Received 6 April 1991; revised manuscript received 31 May 1991

We show how a joint analysis of the transversal momentum spectrum, rapidity distributions and the interferometrical correlator can test and distinguish between different models of hydrodynamical expansion of matter. Our study shows that the hydrodynamical picture is inadequate for minimum bias events at CERN ISR energies. On the other hand, current data on ultra-relativistic nucleus–nucleus collisions are shown to be consistent with the existence of hydrodynamical flows in the matter formed in these processes. More complete and accurate interferometrical data is demanded for clarifying the matter evolution in time and space.

The simplest estimate for p+p collisions at $\frac{dN_{ch}}{d\eta} = 20, \ \sqrt{s} = 7 \text{ TeV}$

Freeze-out at τ: transversally $n(x_T) \sim e^{-x_T^2/2R_T^2}$ **, longitudinally: boost-invar.**

$$\lambda_{m.f.p.} \approx \frac{\langle r_T \rangle}{2} \approx \frac{1}{\sqrt{2}\sigma n} \approx \frac{\pi \langle r_T \rangle^2 \tau}{\sqrt{2}\sigma dN/dy} \qquad (\langle r_T \rangle = R_T \sqrt{\frac{2}{\pi}})$$

Experiment (ALICE): $R_L = 1.6 \text{ fm for } p_T = 0.25 \text{ GeV/c}$

$$\tau = \sqrt{\frac{m_T}{T} \frac{K_1(\frac{m_T}{T})}{K_2(\frac{m_T}{T})}} R_L \div \sqrt{\frac{m_T}{T}} R_L$$

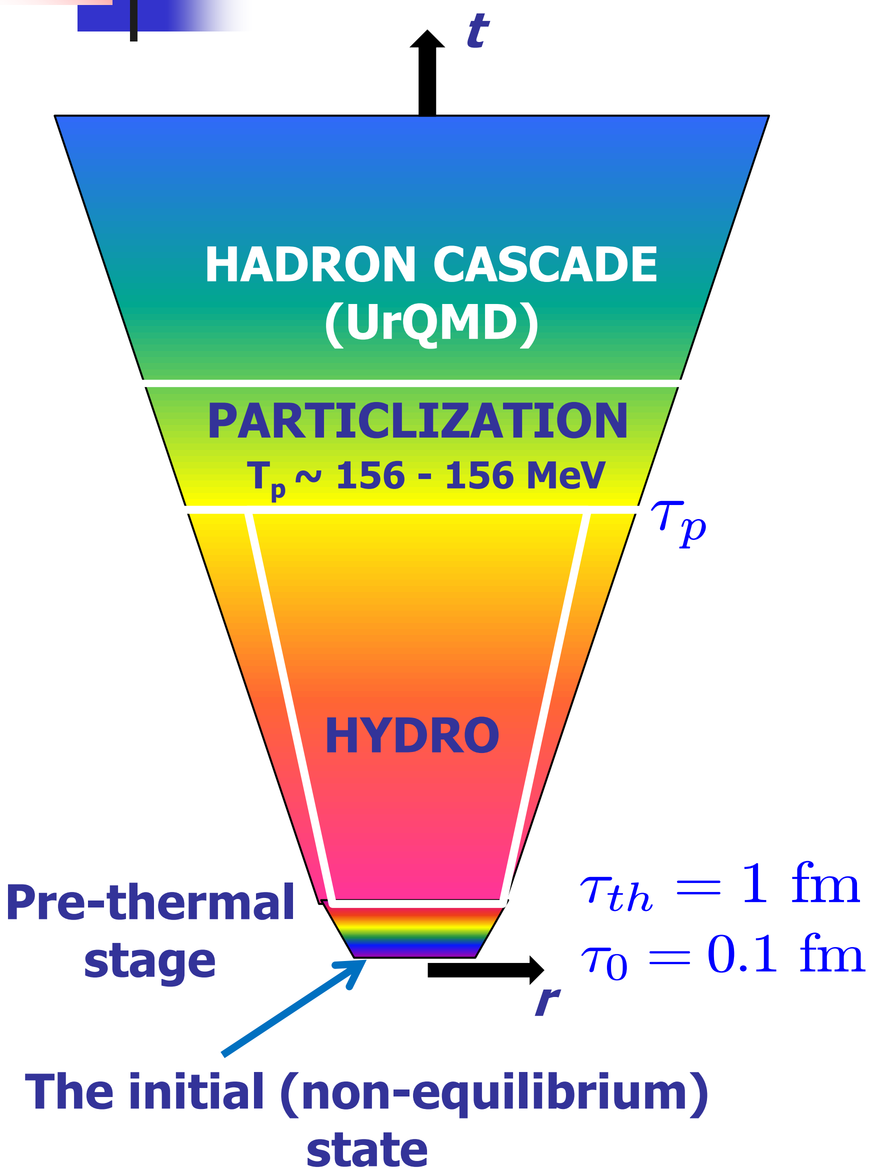
T=0.170 GeV , τ=1.5 – 2.1 fm/c

$$\sigma = 10 \div 17 \text{ mb}$$

- **Disagreement with the data:** $R_T \approx 2.8 \div 6.5 \text{ fm while } R_{s-exp} = 1.4 \text{ fm}, \ R_{o-exp} = 1.3 \text{ fm}$

Transverse flow ⟹ smaller homogeneity length (?)

Integrated HydroKinetic model: HKM $\longrightarrow$ iHKM



Complete algorithm incorporates the stages:

- **generation of the initial states:** MC Sitenko-Glauber
- **thermalization of initially non-thermal matter**;
- **viscous chemically equilibrated hydrodynamic expansion**;
- **sudden (with option: continuous) particlization of expanding medium**;
- **a switch to UrQMD cascade with near equilibrium hadron gas as input;**
- **simulation of observables.**

Yu.S., Akkelin, Hama: PRL 89 (2002) 052301;
... + Karpenko: PRC 78 (2008) 034906;
Karpenko, Yu.S. : PRC 81 (2010) 054903;
... PLB 688 (2010) 50;
Akkelin, Yu.S. : PRC 81 (2010) 064901;
Karpenko, Yu.S., Werner: PRC 87 (2013) 024914;
Naboka, Akkelin, Karpenko, Yu.S. : PRC 91 (2015) 014906;
Naboka, Karpenko, Yu.S. PRC 93 (2016) 024902.

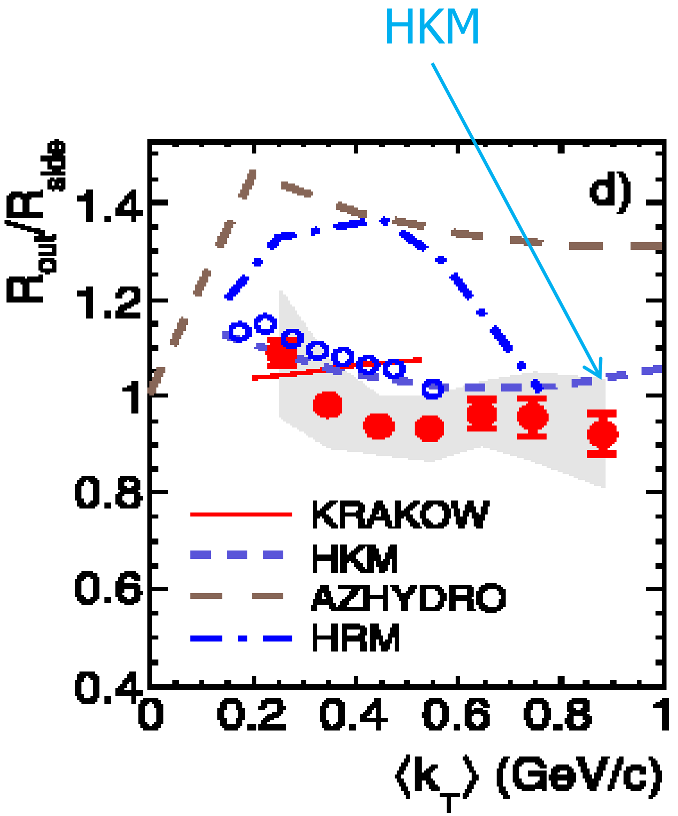
HKM prediction: solution of the HBT Puzzle

Two-pion Bose–Einstein correlations in central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV☆ ALICE Collaboration Physics Letters B 696 (2011) 328-

ALICE

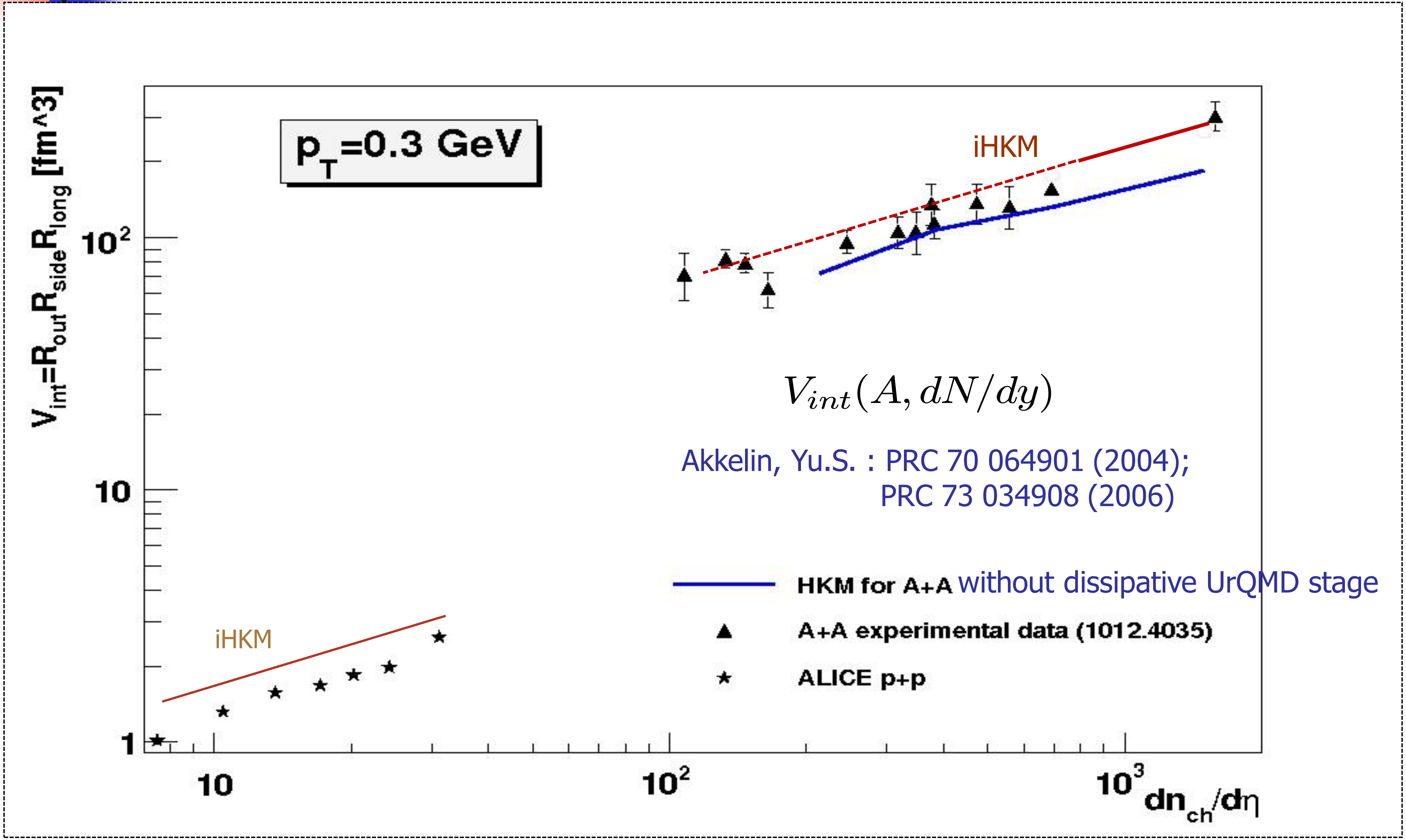
Quotations:

Available model predictions are compared to the experimental data in Figs. 2-d and 3. Calculations from three models incorporating a hydrodynamic approach, AZHYDRO [45], KRAKOW [46,47], and HKM [48,49], and from the hadronic-kinematics-based model HRM [50,51] are shown. An in-depth discussion is beyond the scope of this Letter but we notice that, while the increase of the radii between RHIC and the LHC is roughly reproduced by all four calculations, only two of them (KRAKOW and HKM) are able to describe the experimental R_{out}/R_{side} ratio.



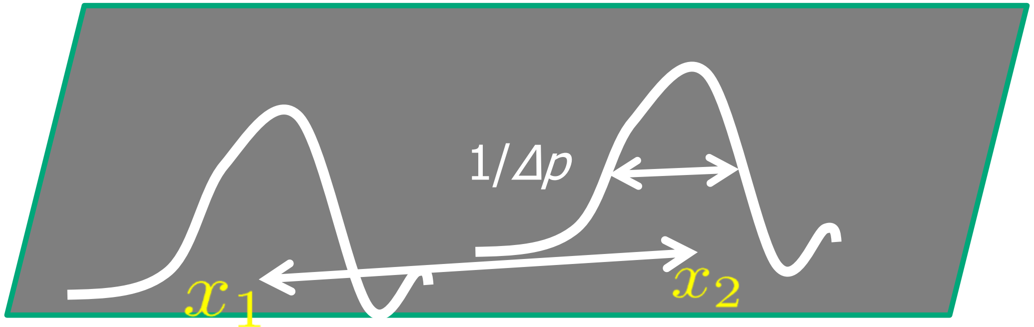
[48] I.A. Karpenko, Y.M. Sinyukov, Phys. Lett. B 688 (2010) 50.
[49] N. Armesto, et al. (Eds.), J. Phys. G 35 (2008) 054001.

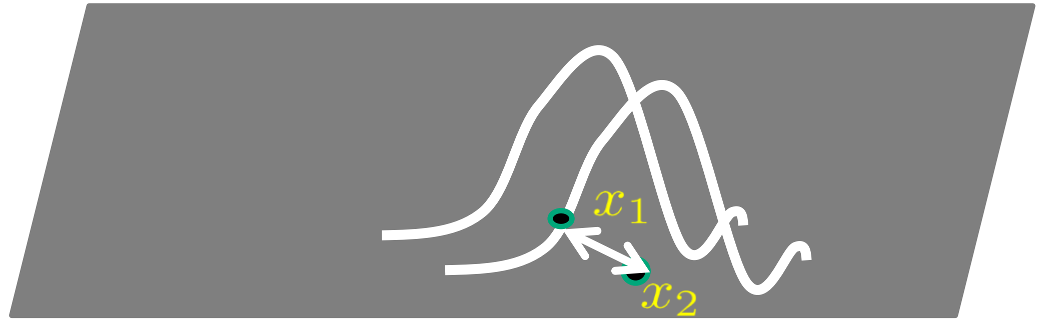
Interferometry volume Vint in LHC p-p and **central** Au-Au, Pb-Pb collisions



Uncertainty principle and distinguishability of emitters

The distance between the centers of emitters $\Delta x = x_1 - x_2$ is larger than their sizes related to the widths of the emitted wave packets $1/\Delta p$.





Distinguishable emitters $\Delta x \gg 1/\Delta p$

The states are orthogonal

Wave function for emitters

$$\psi_{x_i}(\vec{p}) = e^{i\vec{p}\vec{x}_i} e^{i\varphi(\vec{x}_i)} \tilde{f}(\vec{p})$$

Spectrum:

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = const$$

$$f(\vec{p}) = \tilde{f}^2(\vec{p}) = \frac{1}{(2\pi p_0^2)^{3/2}} e^{-\frac{\vec{p}^2}{2p_0^2}}$$

Indistinguishable emitters $\Delta x \ll 1/\Delta p$

The states are not orthogonal

Criterion : overlapping of the wave packages:

$$I_{ij} = \left| \int d^3\mathbf{x} \psi_{x_i}(t,\mathbf{x}) \psi^*_{x_j}(t,\mathbf{x}) \right|$$

Phase correlations $\left\langle e^{i\phi(x_1)} e^{-i\phi(x_2)} \right\rangle =$

$$I_{12} = \delta^3(\mathbf{x}_1 - \mathbf{x}_2) \qquad t_1 = t_2$$

$$I_{12} = e^{-\frac{p_0^2(\vec{x}_1 - \vec{x}_2)^2}{2}}$$

Yu.M. Sinyukov, V.M. Shapoval. Correlation femtoscopy of small systems. Phys. Rev. D **87** 094024 (2013)

The results for the interferometry radii

The reduction of the interferometry radii:

$$
\begin{aligned}
\frac{R_S^2}{R_{S,st}^2} &= \frac{4k_0^2 R_T^2}{1+4k_0^2 R_T^2} \\
\frac{R_O^2}{R_{O,st}^2} &= \left(R_T^2 \frac{4k_0^2 R_T^2}{1+4k_0^2 R_T^2} + T^2 v_{out}^2 \frac{4k^2 T^2}{1+4k^2 T^2} \right) / \left(R_T^2 + T^2 v_{out}^2 \right) \\
\frac{R_L^2}{R_{L,st}^2} &= \frac{4k_0^2 R_L^2}{1+4k_0^2 R_L^2}
\end{aligned}
$$

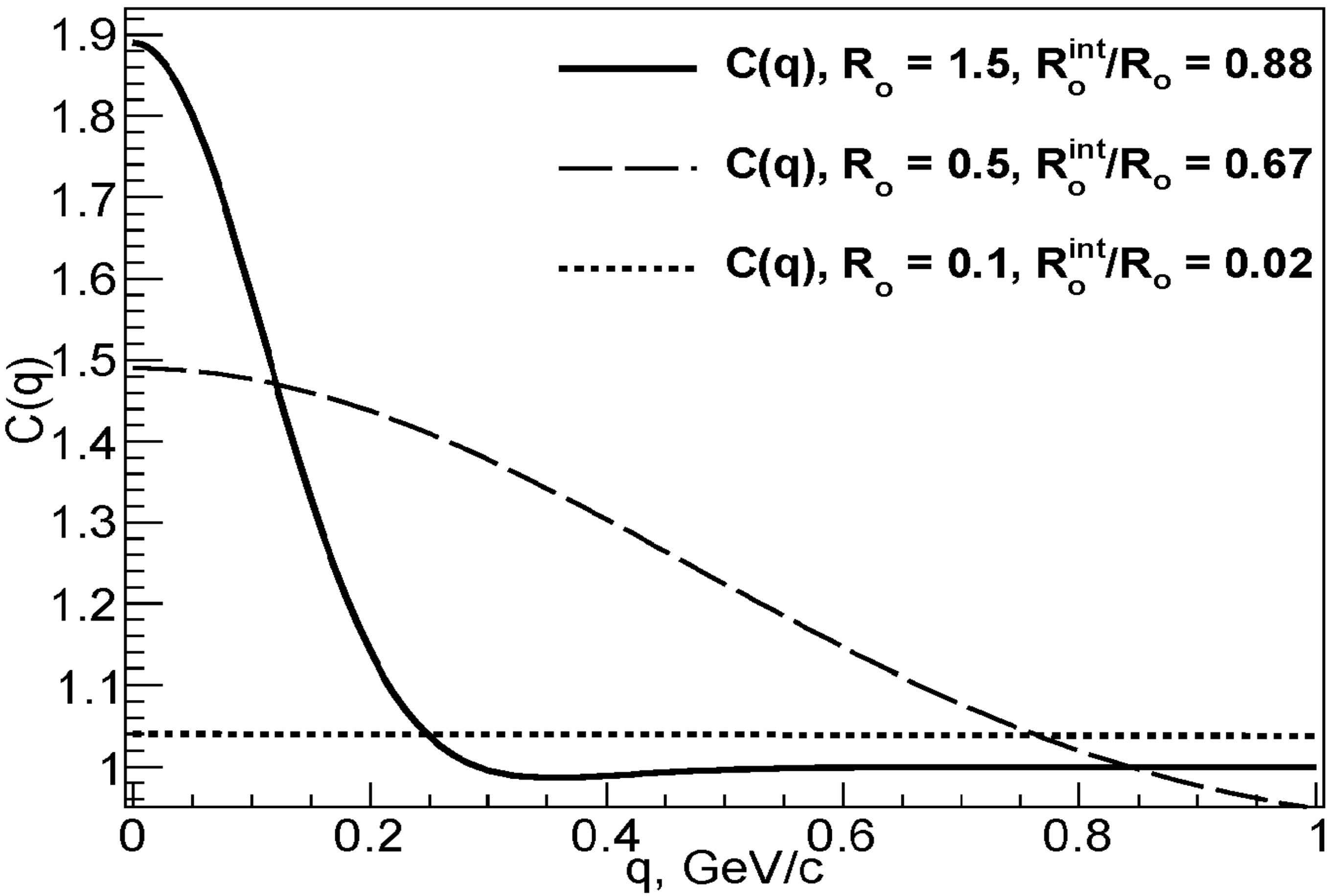
where $v_{out} = p_{out}/m,$

$$k_0^2 = p_0^2/(1+\alpha p_0^4 T^2/m^2)$$

R_st means interferometry radii in the (standard) model of independent distinguishable emitters.

In the region of the source sizes 0.5 - 2 *fm* α = 1.5 - 0.4

The Bose-Einstein correlation function for small systems



The behavior of the two-particle Bose-Einstein correlation function (*side*-projection) where the uncertainty principle and correction for double counting are utilized. The momentum dispersion k=m=0.14 GeV, p_T=0, T=R.

ELSEVIER

Physics Letters B

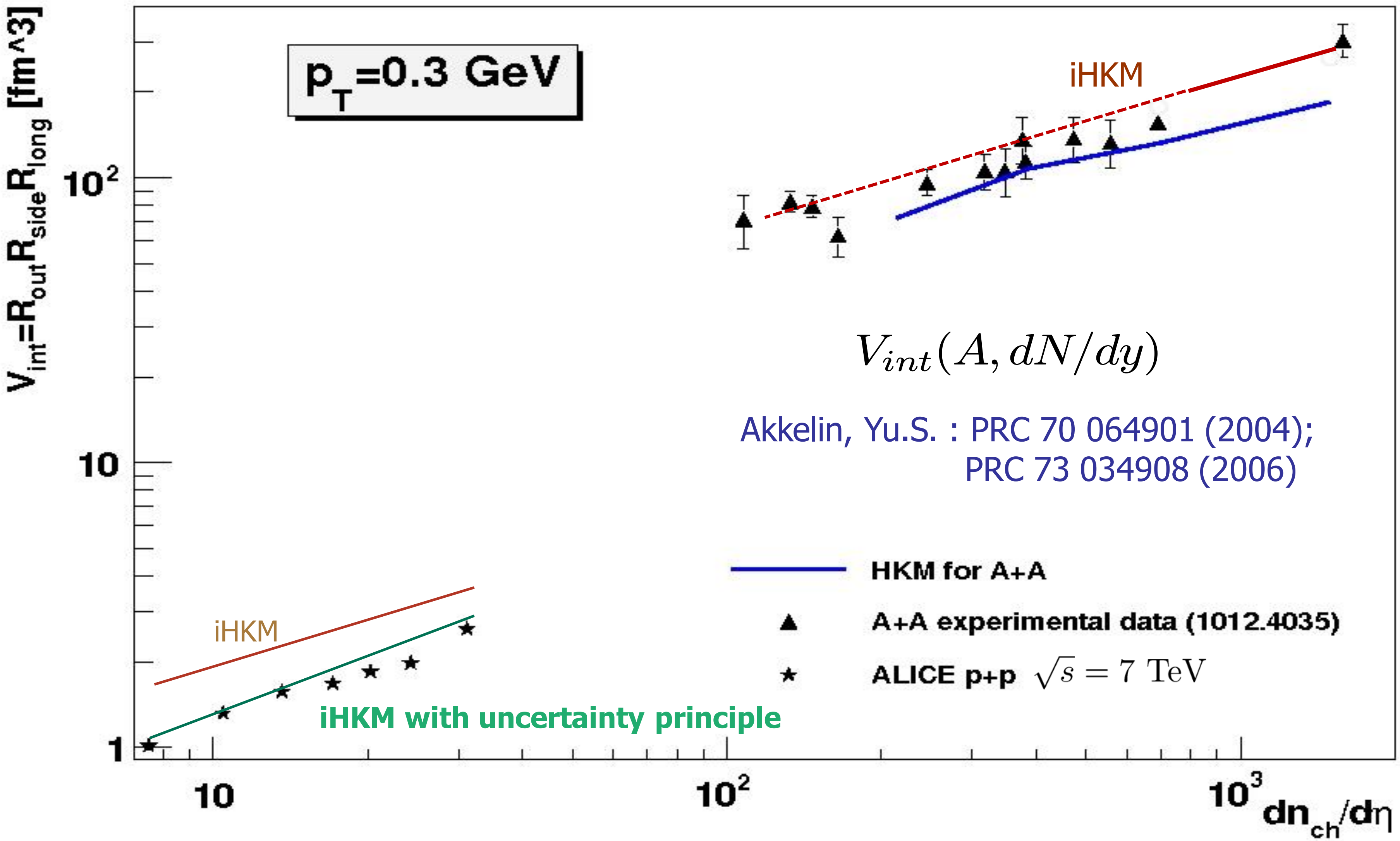
www.elsevier.com/locate/physletb

Femtoscopic scales in $p+p$ and $p+\text{Pb}$ collisions in view of the uncertainty principle

V.M. Shapoval [a], P. Braun-Munzinger [b,c], Iu.A. Karpenko [a,c], Yu.M. Sinyukov [a,*]

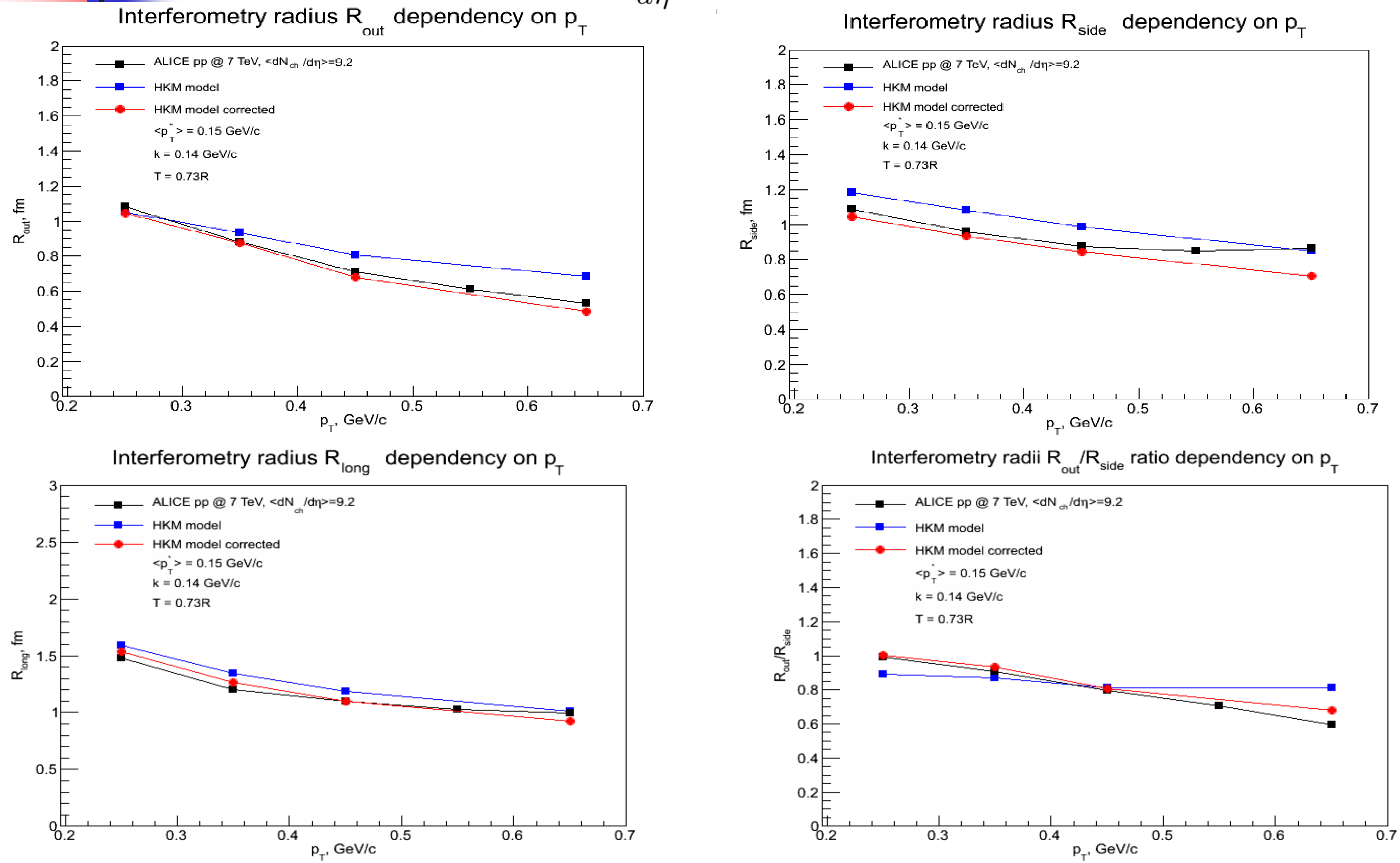
A method for quantum corrections of Hanbury-Brown/Twiss (HBT) interferometric radii produced by semi-classical event generators is proposed. These corrections account for the basic indistinguishability and mutual coherence of closely located emitters caused by the uncertainty principle. A detailed analysis is presented for pion interferometry in $p+p$ collisions at LHC energy ($\sqrt{s}=7$ TeV). A prediction is also presented of pion interferometric radii for $p+\text{Pb}$ collisions at $\sqrt{s}=5.02$ TeV. The hydrodynamic/hydrokinetic model with UrQMD cascade as 'afterburner' is utilized for this aim. It is found that quantum corrections to the interferometry radii improve significantly the event generator results which typically overestimate the experimental radii of small systems. A successful description of the interferometry structure of $p+p$ collisions within the corrected hydrodynamic model requires the study of the problem of thermalization mechanism, still a fundamental issue for ultrarelativistic $A+A$ collisions, also for high multiplicity $p+p$ and $p+\text{Pb}$ events.

Interferometry volume Vint in LHC **p-p** and **central** Au-Au, Pb-Pb collisions



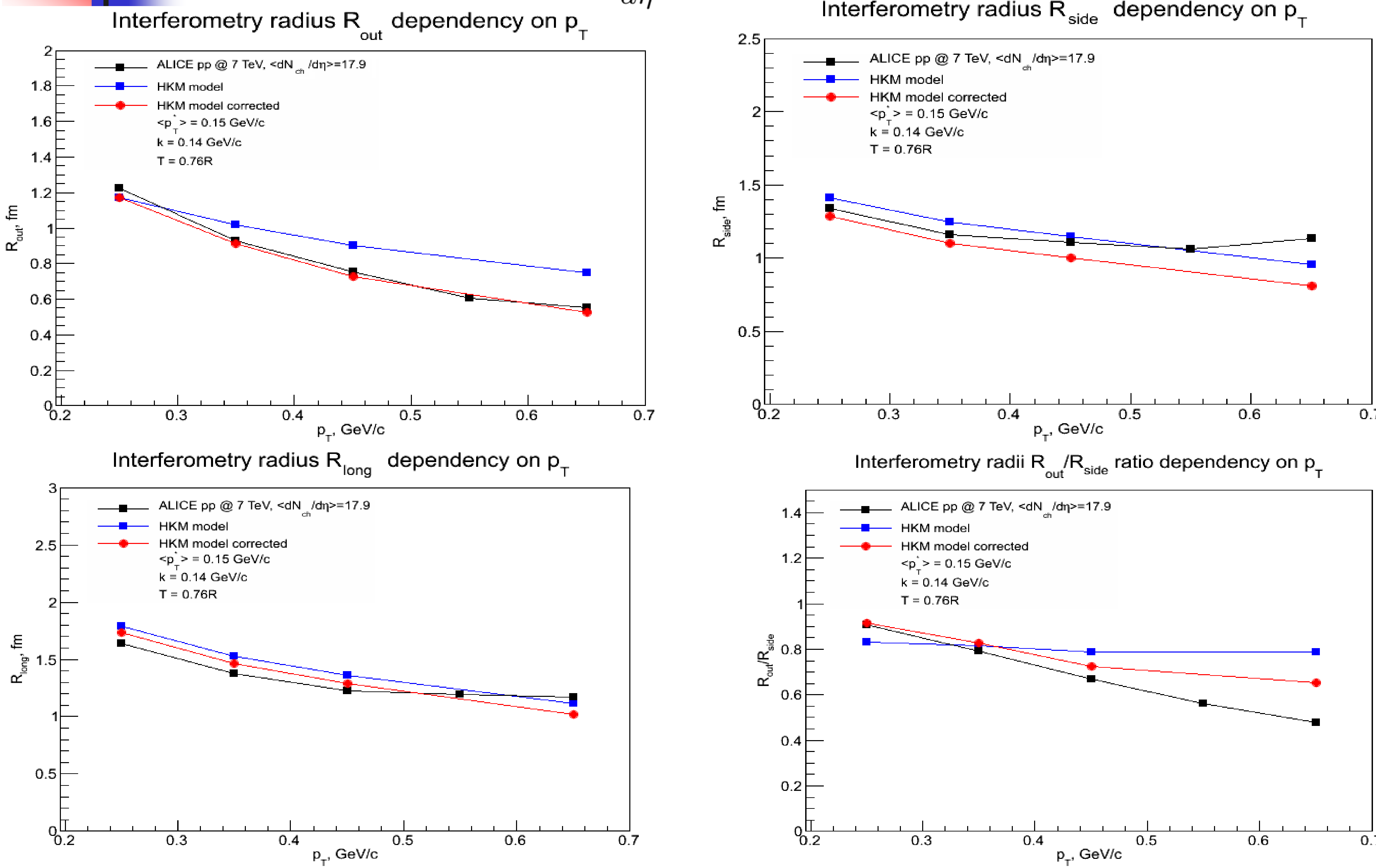
Femtoscopy scales vs p_T in the HKM after corrections $\sqrt{s} = 7$ TeV

$$\frac{dN_{ch}}{d\eta} = 9.2$$



Femtoscopy scales vs p_T in the HKM after corrections $\sqrt{s} = 7$ TeV

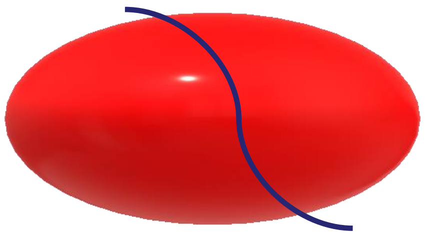
$$\frac{dN_{ch}}{d\eta} = 17.9$$



Quantum problem of boson radiation from small systems

Statistical operator of decaying thermal 3D- ellipsoidal ball with Gaussian radii R_i **,**

defined by constant temperature $T = 1/\beta$ **and chemical potential** $\mu(x) = -\mu_0 \sum_i \frac{x_i^2}{2R_i^2}$



$$\rho = \frac{1}{Z} exp[-\beta(\int d^3k \omega_k a^+(k)a(k) - \int d^3x \mu(x)\psi^+(x)\psi(x))]$$

M.Adzhymambetov, Yu.S. (2018). in preparation:

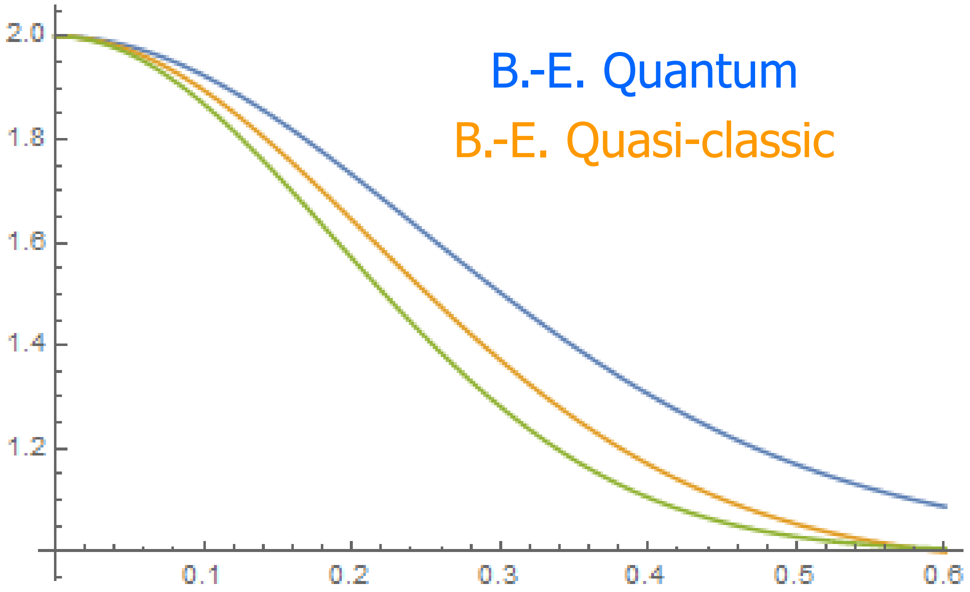
$$< a^+(\vec{k}_1)a(\vec{k}_2) >= \sum_{n=1}^{\infty} \prod_{i=1,2,3} \left(\frac{b_i^2}{2\pi sh(n\beta\omega_i)} \right)^{1/2} e^{-b_i^2 p_i^2 th(\frac{n\beta\omega_i}{2}) - \frac{b_i^2 q_i^2}{4} cth(\frac{n\beta\omega_i}{2})}$$

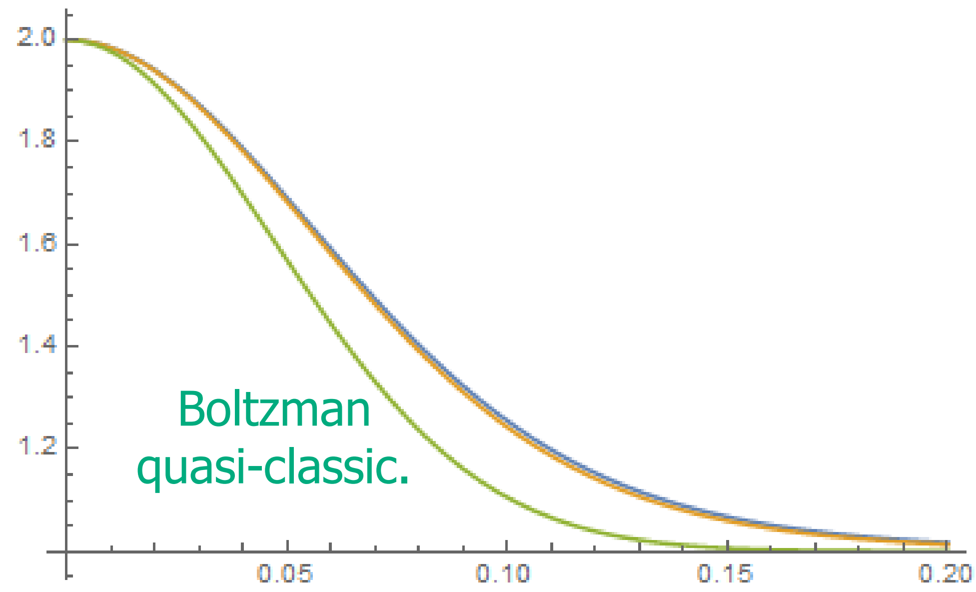
Where $\omega_i^2 = \frac{\mu_0}{mR_i^2}$ **,** $b_i^2 = \frac{R_i}{\sqrt{m\mu_0}}$**,** $p_i = \frac{k_{1i}+k_{2i}}{2}$ **,** $q_i = k_{1i} - k_{2i}$

$$< a^+(k_1)a(k_2) >_{q.c.} = \frac{1}{(2\pi)^3} \int d^3x \frac{1}{e^{\frac{p^2}{2mT} + \sum_{n=1}^{3} \frac{\mu_0 x_i^2}{2R_i^2 T}} - 1} e^{i\vec{q}\vec{x}}$$

Quasi-classic (q.c.) approach

$$C(\vec{p},\vec{q}) = 1 + \left|< a^+(\vec{k}_1)a(\vec{k}_2) >\right|^2 /(< a^+(\vec{k}_1)a(\vec{k}_1) >< a^+(\vec{k}_2)a(\vec{k}_2) >)$$

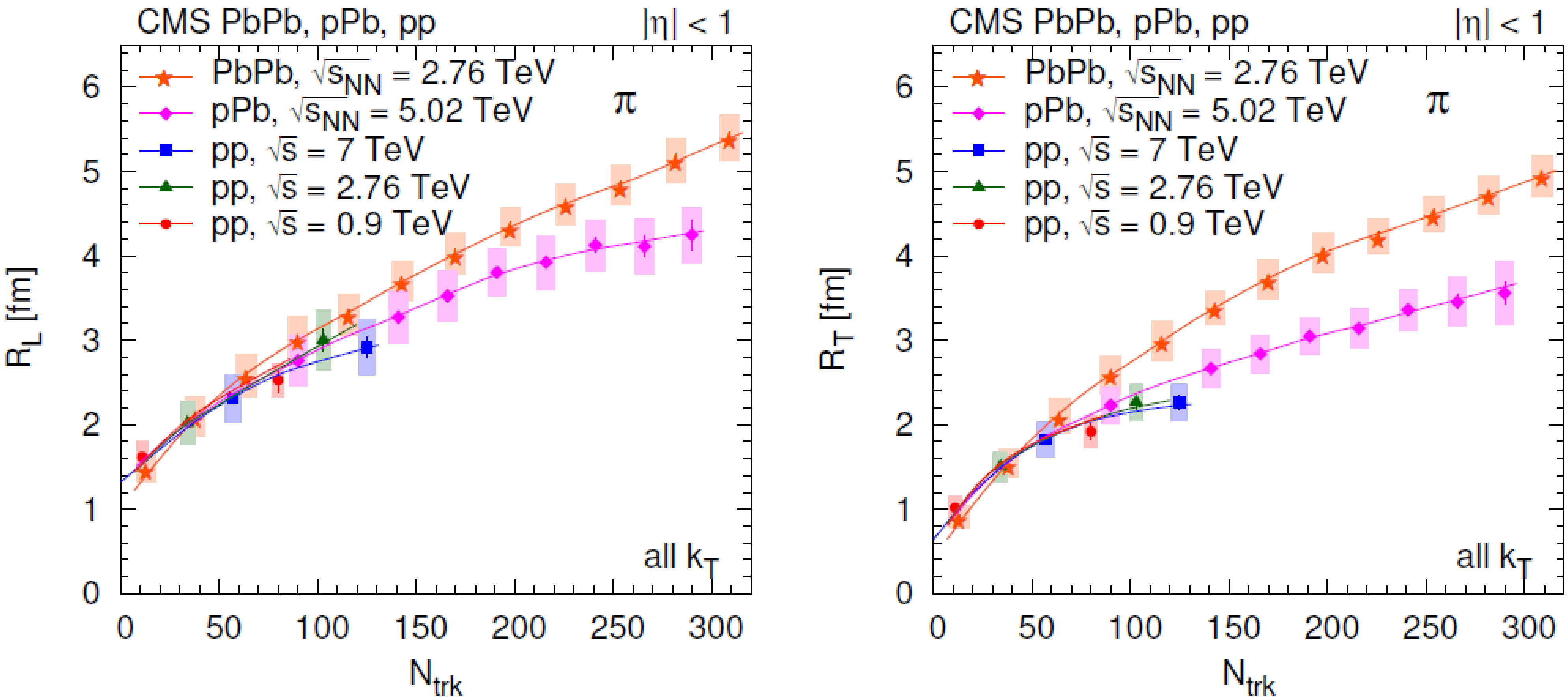




k=0.3 GeV, R=0.75 fm ~ 1/k

k=0.3 GeV, R= 3 fm >> 1/k

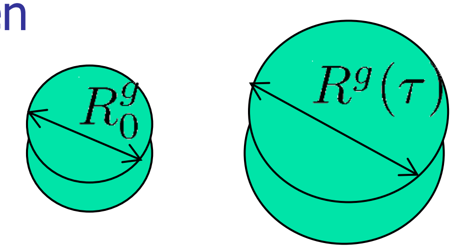
Saturation of the HBT radii in p-p



The similar observation was found by the ATLAS Collaboration

Possible explanation of the saturation

If an expansion of the matter with transv. initial size R_0^g is mostly of hydrodynamic type, then



S.V. Akkelin, Yu.M. Sinyukov, Phys. Lett. B 356 (1995) 525;
S.V. Akkelin, Yu.M. Sinyukov, Z. Phys. C 72 (1996) 501.
S.V. Akkelin, P. Braun-Munzinger, Yu.M. Sinyukov, Nucl. Phys. A 710 (2002) 439.

acceleration $a = \nabla_{x_T} p/\epsilon \propto p(\mathbf{x}_T = 0)/(R_0^g \epsilon) = c_0^2/R_0^g$

V.M. Shapoval, P. Braun-Munzinger, Yu.M. Sinyukov Phys. Lett. B 725 (2013)

$$R_T = \frac{R^g(\tau)}{\sqrt{1 + \frac{2}{\pi}\langle|v_T|\rangle^2 \beta m_T}} \approx R_0^g \left(1 + \frac{\tau^2 c_0^2}{2(R_0^g)^2} - \beta m_T \frac{\tau^2 c_0^2}{\pi^2 (R_0^g)^2}\right)$$

Saturation of R_0^g

$\Downarrow$

Saturation of R_T

On the other hand, if no long post-hydrodynamic stage,

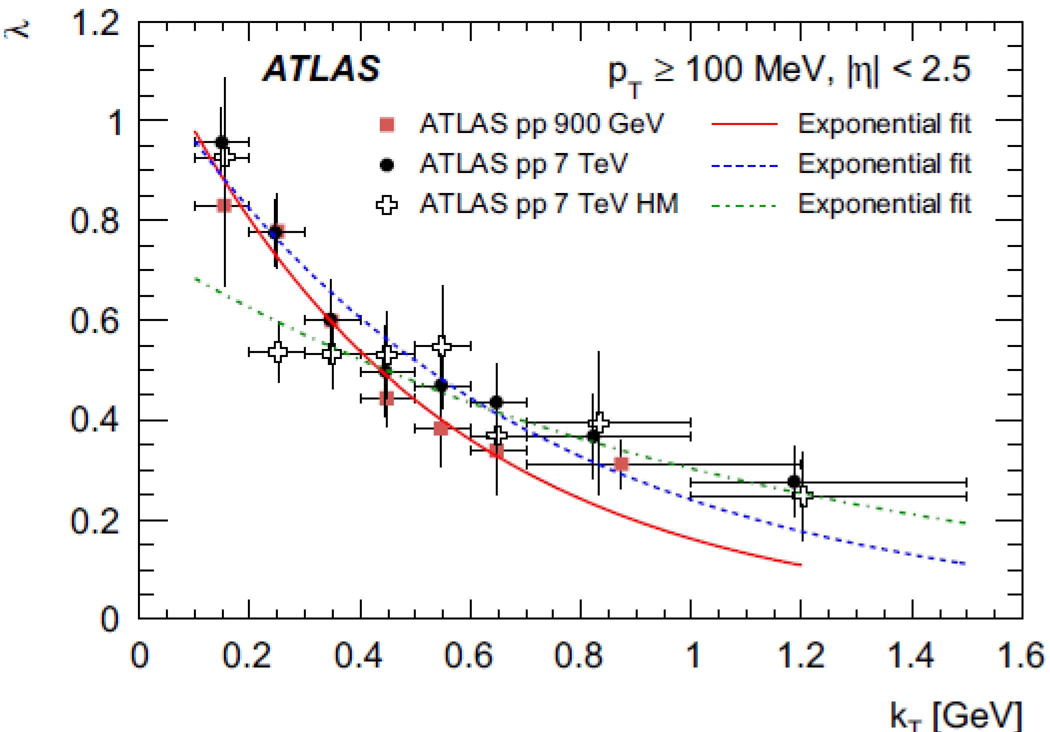
S.V. Akkelin, Yu.M. Sinyukov, Phys. Rev. C 70 (2004) 064901;
S.V. Akkelin, Yu.M. Sinyukov, Phys. Rev. C 73 (2006) 034908.

$$V_{\text{int}} \simeq C \frac{dN/dy \uparrow}{\langle f \rangle T_{\text{eff}}^3 \uparrow}$$

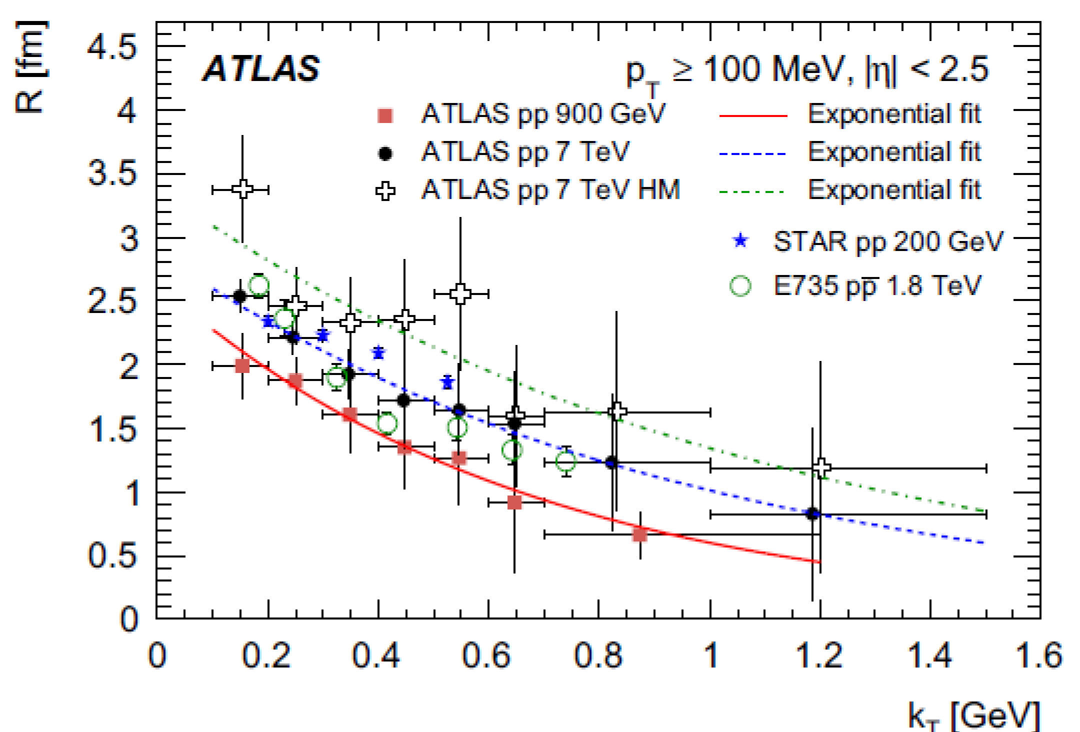
where $\langle f \rangle$ is pion average phase-space density

T_{eff} is the inverse of slope of pion spectra

Decrease of the correlation strength λ with k_T

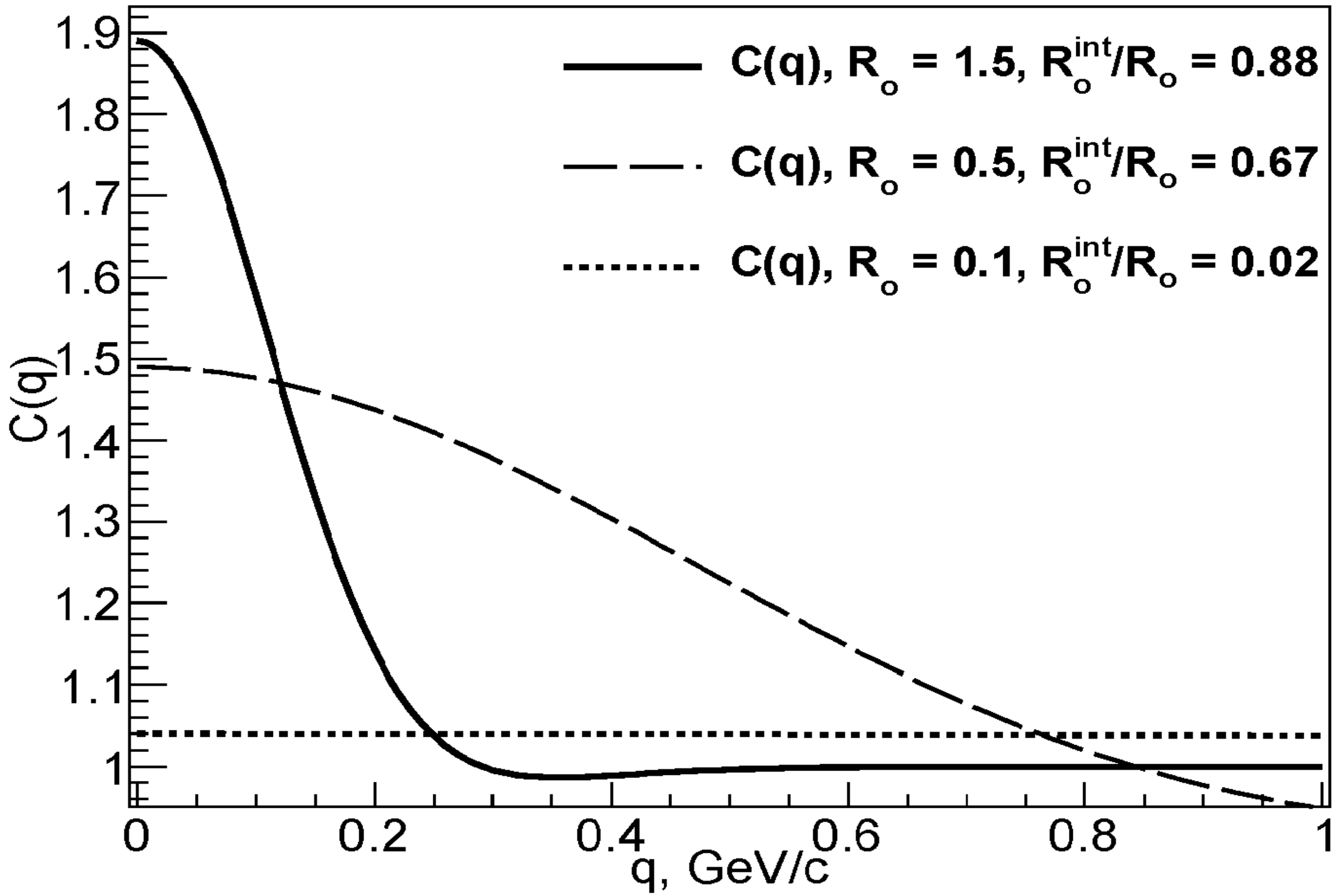


(a)



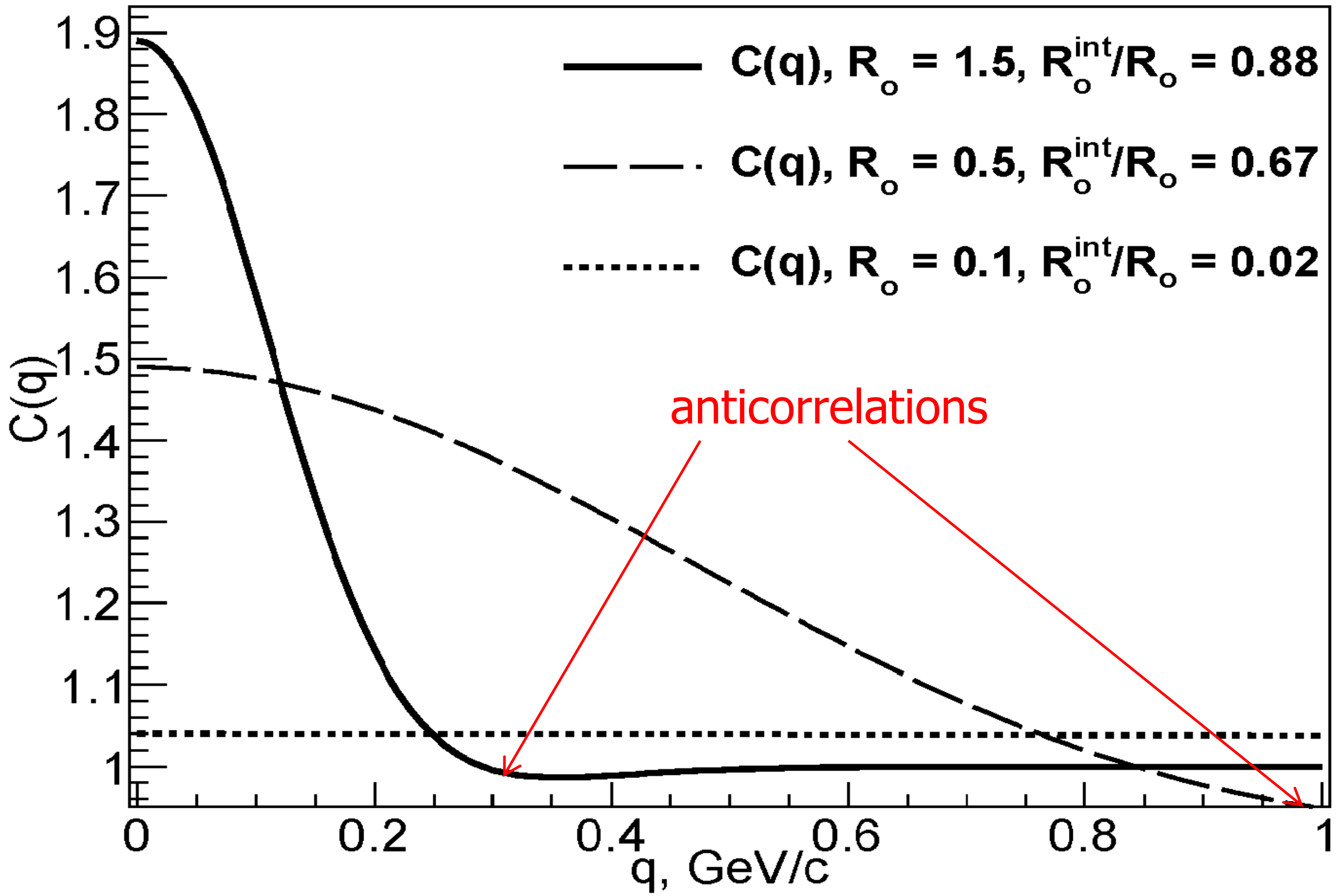
(b)

Possible explanation of λ behavior



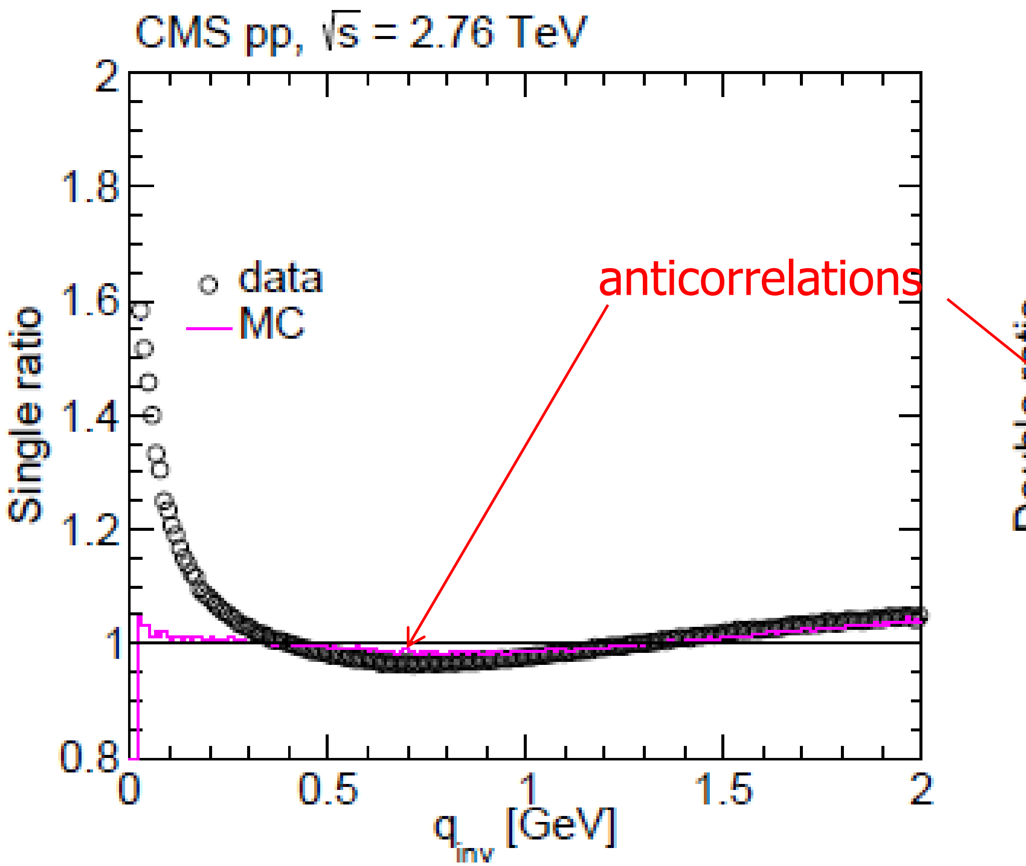
The behavior of the two-particle Bose-Einstein correlation function (*side*-projection) where the uncertainty principle and correction for double counting are utilized. The momentum dispersion k=m=0.14 GeV, p_T=0, T=R.

Possible explanation



The behavior of the two-particle Bose-Einstein correlation function (*side*-projection) where the uncertainty principle and correction for double counting are utilized. The momentum dispersion k=m=0.14 GeV, p_T=0, T=R.

Possible explanation



SUMMARY

- Our results on p+p collisions at the LHC energy demonstrates that the uncertainty principle may play an important role for such small systems and allows one to explain the observed overall femtoscopy scale (interverometry volume) and its dependence on multiplicity.
- An analysis of the p_T – dependence of the femtoscopic scales corrected for uncertainty principle does not exclude the possibility of the hydrodynamic interpretation of p+p collisions at the LHC energies.
- The comparison of V_{int} vs $dN/d\eta$ for pp and AA collisions conforms probably the result of Akkelin, Yu.S. : PRC 70 064901 (2004); PRC 73 034908 (2006) that the interferometry volume depends not only on multiplicity but also on the initial size of colliding systems.
- The saturation of the interfereometry radii, decreasing of correlation strength and anti-correlations observed recently in p-p collisions by CMS and ATLAS collaborations, can be explained by the hydrodynamic regime at large multiplicities without essential post-hydrodynamic hadron cascade stage. One should add, however, to pure hydrodynamic description the quantum-mechanics uncertainty principle and associated with it coherent effects.
- At the moment we develop detail model with such a symbiosis (S. Akkelin, Yu.S.,2018)

Acknowledgement

Thank you for your attention