Towards generalization of low x evolution equations

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Based on papers:

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

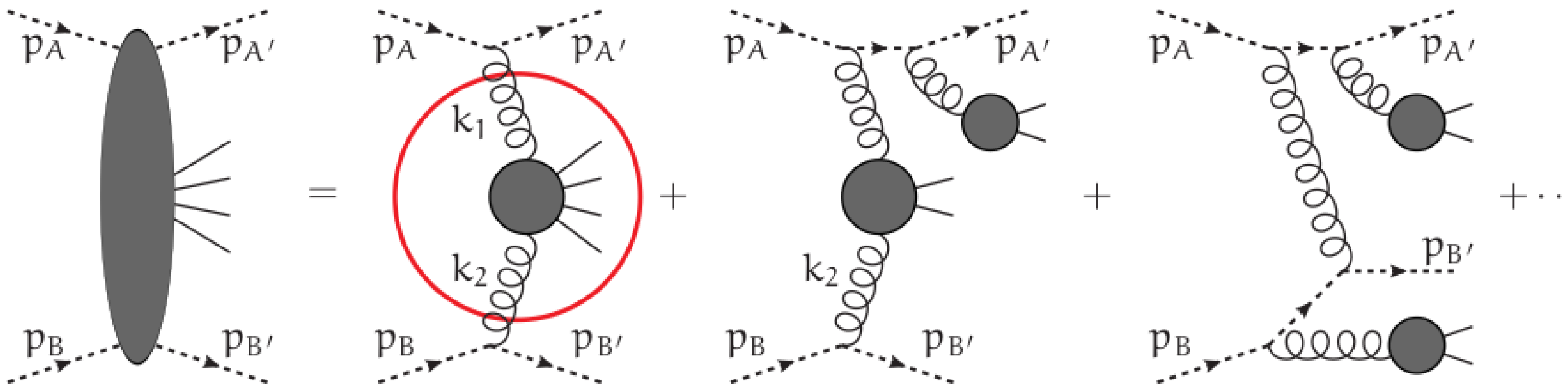
M. Hentschinski, A. Kusina, K.K; Phys. Rev. D 94, 114013 (2016)

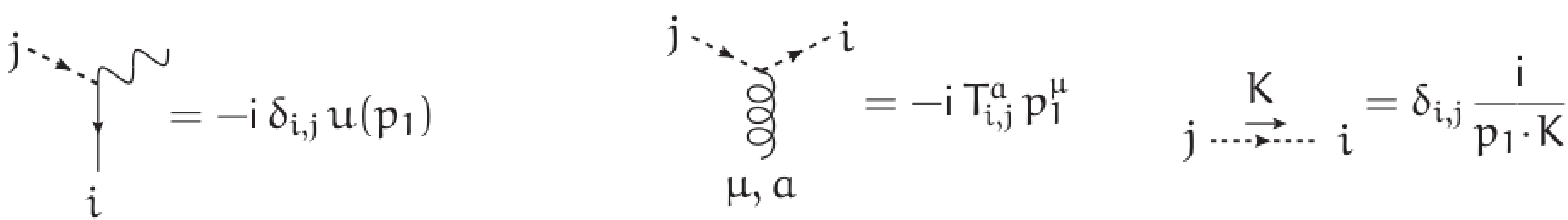
O. Gituliar, M. Hentschinski, K.K; JHEP 1601 (2016) 181

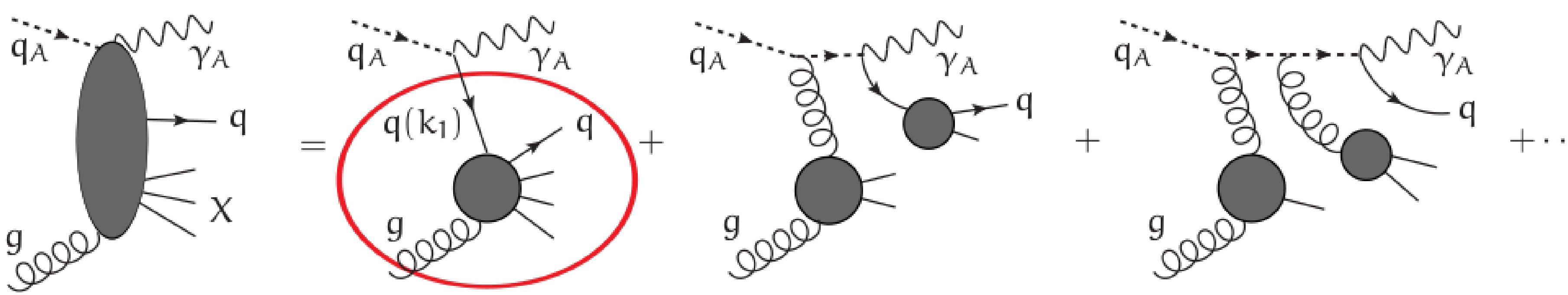
Hard coefficient functions in kt factorization:

One consider embedding off-shell amplitude in on-shell and introduces eikonal lines

Kotko, KK, van Hameren 2013,
KK, Salwa, van Hameren 2013

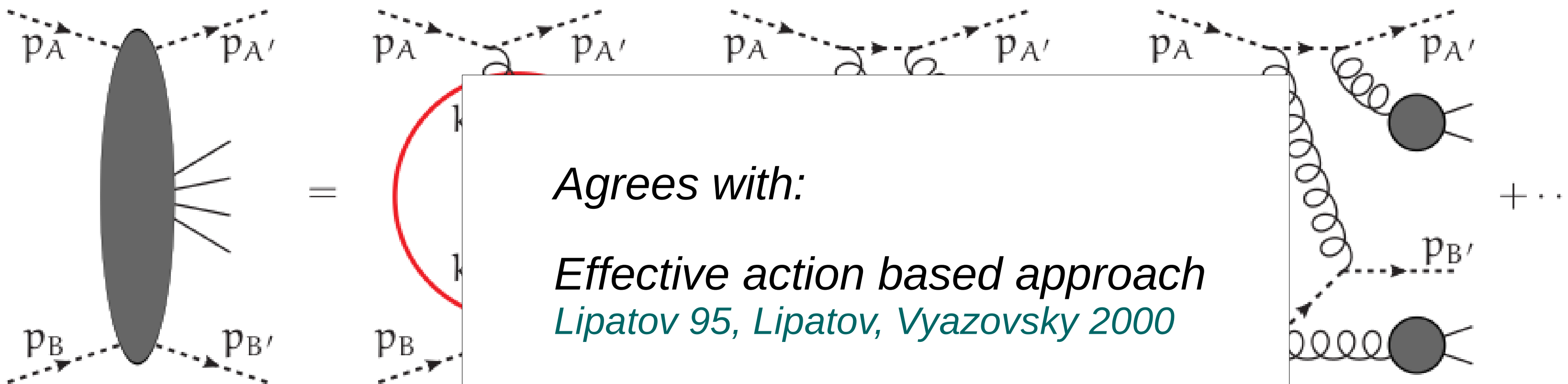






Hard coefficient functions in HEF:

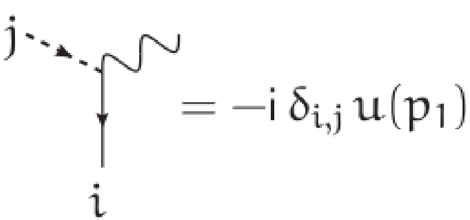
Kotko, KK, van Hameren 2013,
KK, Salwa, van Hameren 2013

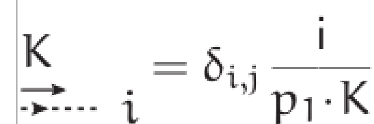


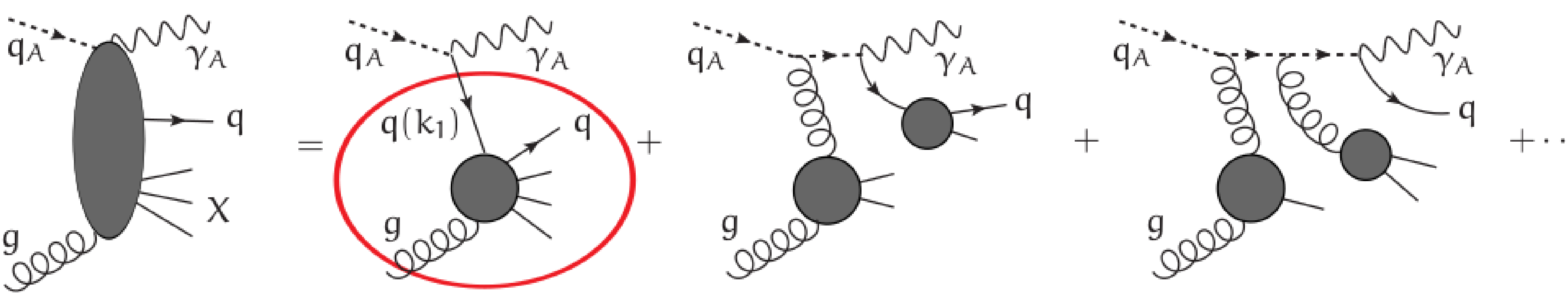
Agrees with:

Effective action based approach
Lipatov 95, Lipatov, Vyazovsky 2000

Gauge link based derivation
Kotko'14







So far we can

USE higher order corrections to BFKL/BK/JIMWLK
but:
What about evolution of quarks? Can one get in some limit complete DGLAP at least an LO?

Use CCFM includes “1/z” and “1/(1-z)” terms of splitting function, depends on hard scale
but:
does not allow to account for finite terms like “z(1-z)”. Jumps from low z to large z. Framework limited only to gluons. Limited description of data.

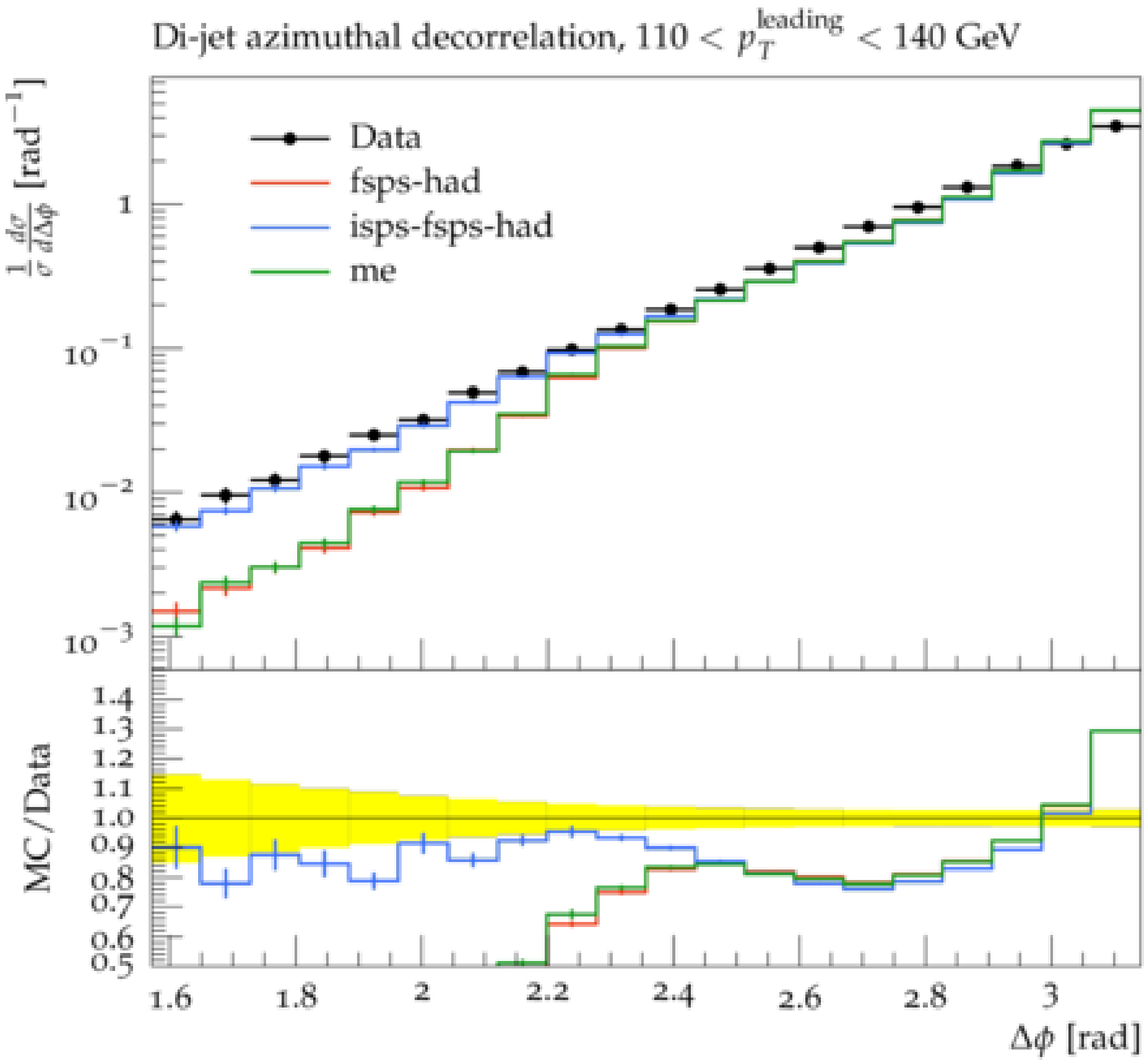
Framework by Balitsky and Tarasov: large “z”, small “z”, moderate “z”, Sudakov, nonlinearity, spin dependence. The same kinematics in the kernel as in our approach.
but:
limited so far to gluons only. Not clear how to deal with it numerically

Kimber,Martin, Ryskin, Watt or “Parton Branching” Jung at. al 1804.11152 provides full set of TMD pdfs.
but:
DGLAP based only integral version fully consistent. They should be at least refitted.

Ciafaloni,Colferai,Staśto,Salam JHEP 0708:046,2007 → anzatz for system of equations unifying DGLAP and BFKL.
but
quark splitting functions are k_t independent.

Example- dijets- azimuthal angle correlations – central region

M. Bury, A. van Hameren, H. Jung, KK, S. Sapeta, M. Serino ‘17



Simulation in KaTie+ CASCADE

KMR pdfs used unintegrated DGLAP based

Together with off-shell ME

as obtained from KaTie ME generator

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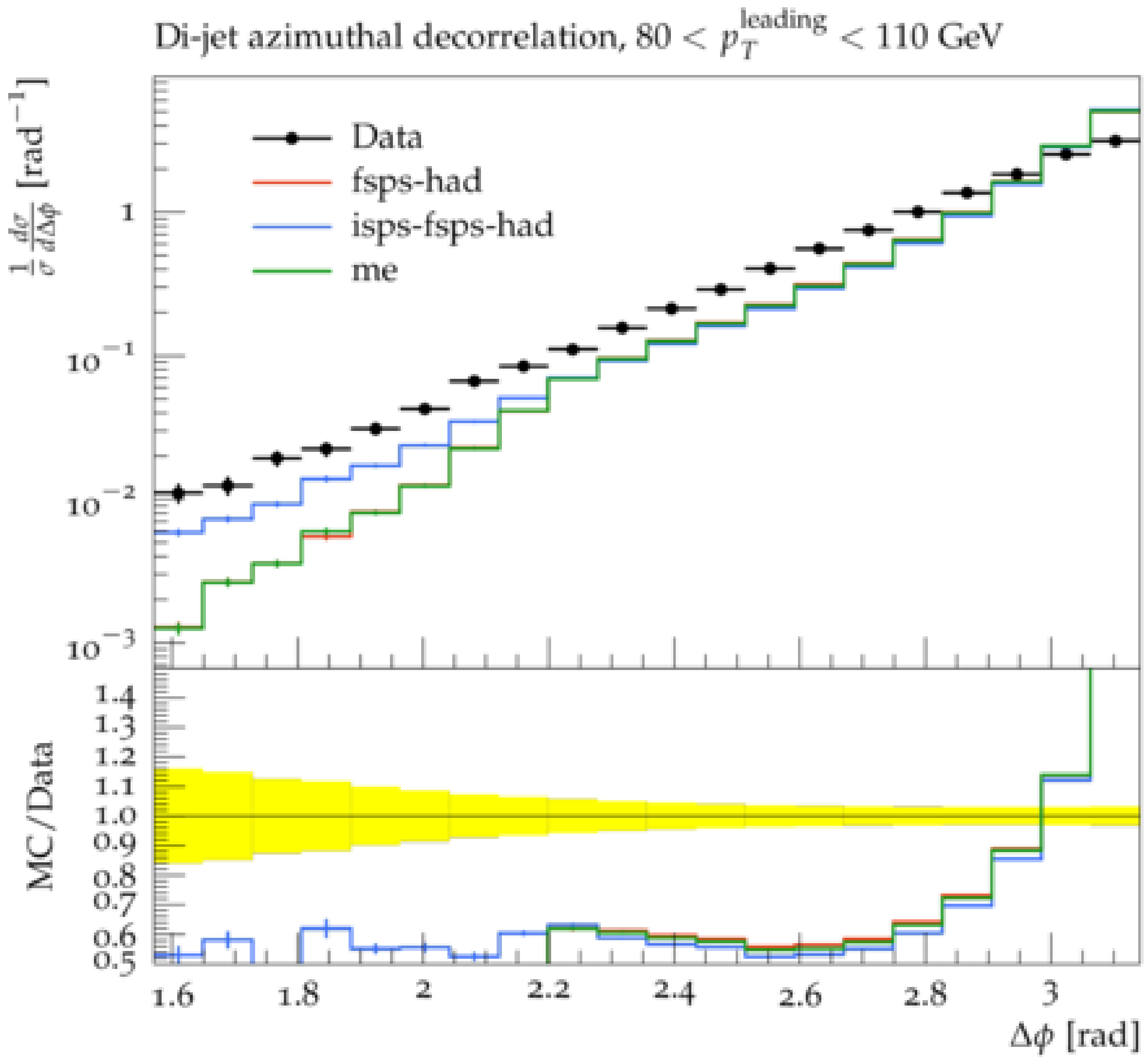

Simulation in KaTie+ CASCADE

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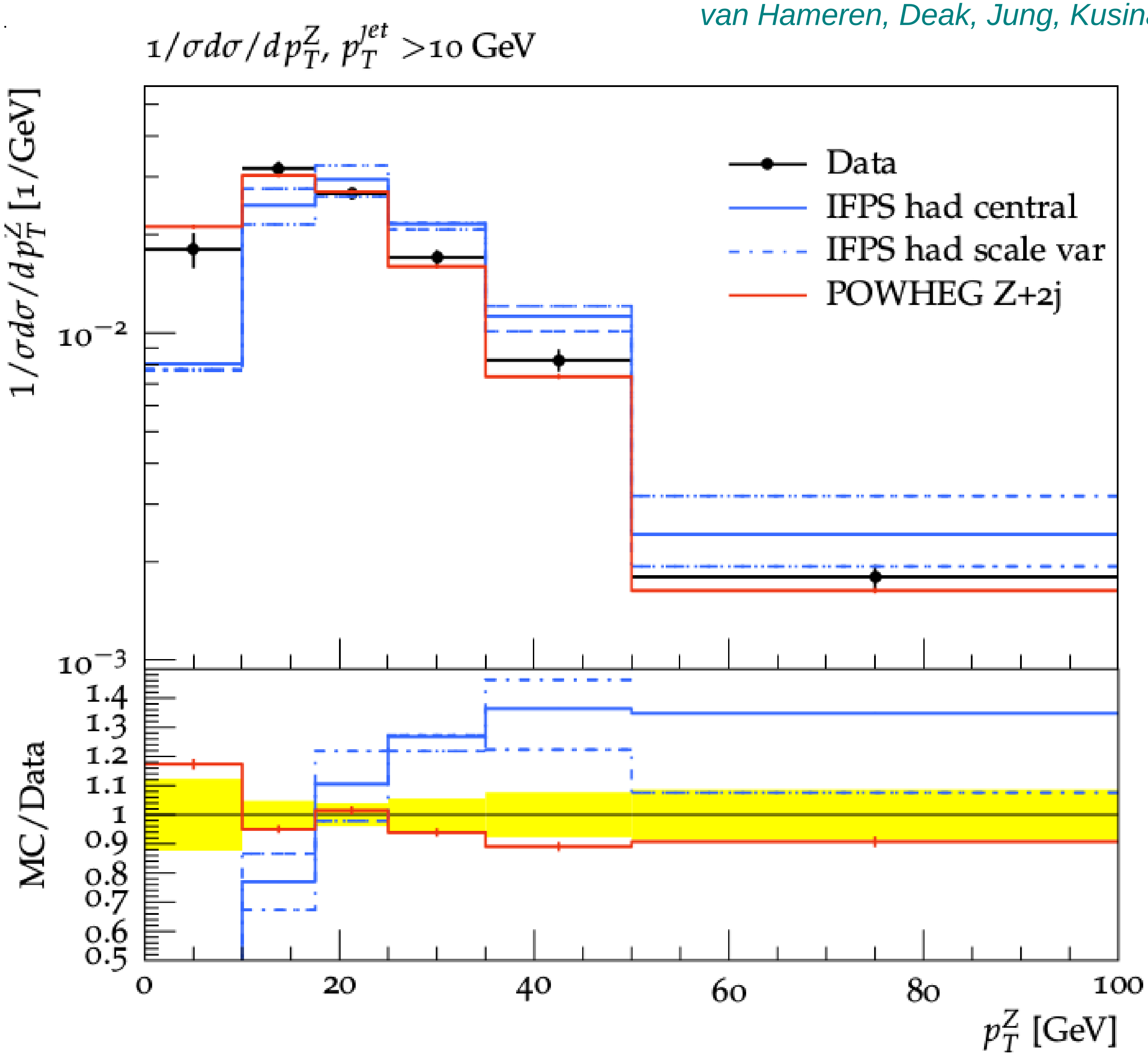
as obtained from KaTie ME generator

Example- Z+jet- p_T of Z

van Hameren, Deak, Jung, Kusina, Kutak, M. Serino in preparation



Simulation in KaTie+ CASCADE

Parton Branching pdfs used unintegrated DGLAP based

Together with off-shell ME as obtained from

KaTie ME generator

The goal

- go beyond DGLAP and BFKL by generalized splitting kernel
- coverage of all "z" regions
- extend evolution towards large x
- reproduce collinear limit (DGLAP)
- reproduce BFKL in low "z" limit
- k_T-dependent splitting functions
- in longer term goal: to describe large class of exclusive processes

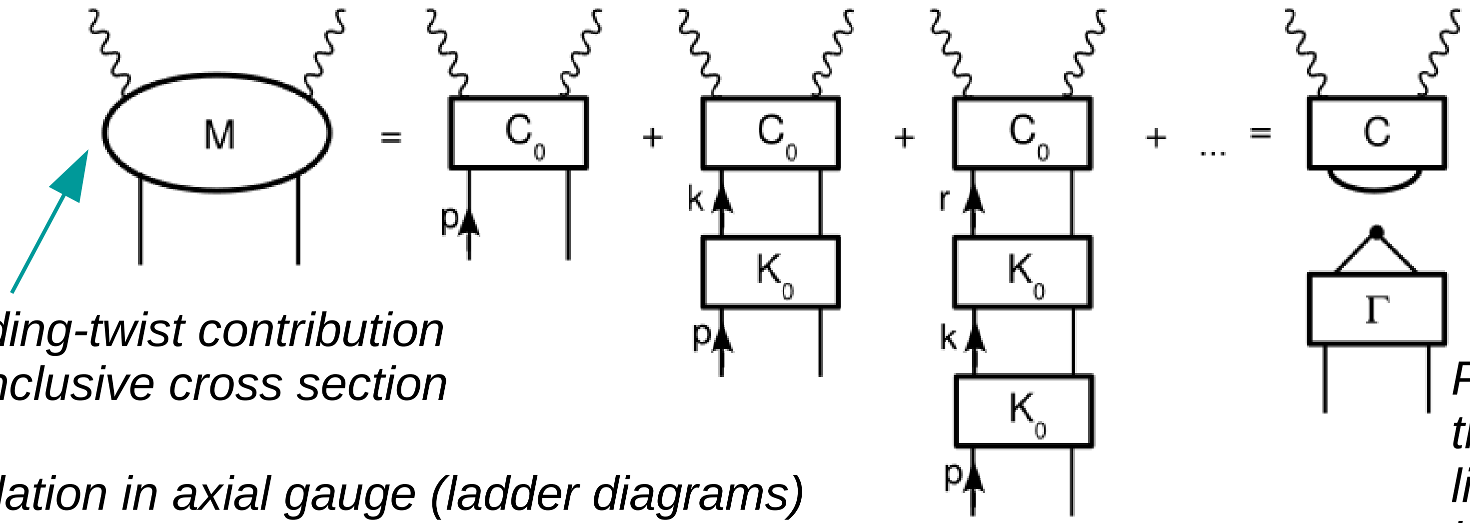
We aim at will achieving this goals by using Curci-Furmanski-Petronzio (CFP) and Catani-Hautmann (CH) formalisms.

Curci, Furmanski, Petronzio Nucl. Phys. B175 (1980) 27

Catani, Hautmann NPB427 (1994) 475524

Curci-Furmanski-Petronzio method

Factorization based on generalized ladder expansion (in terms of Two Particle Irreducible (2PI) kernels)



Renormalized and finite hard matrix element

the leading-twist contribution to the inclusive cross section

Process independent transition function linked to splitting kernel

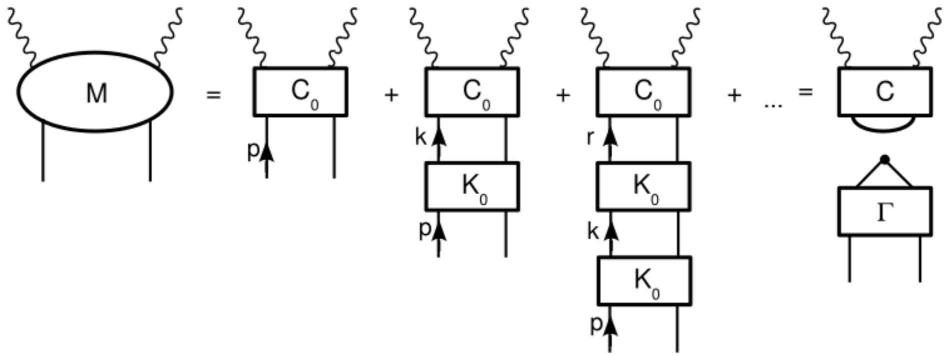
formulation in axial gauge (ladder diagrams)

C_0 hard scattering coefficient function

K_0 2PI kernels connected only by convolution in x this is achieved by introducing appropriate projector operators

How does it work….

Curci-Furmanski-Petronzio - factorization



notation from CFP paper
they studied Pqq

$$k_\mu = x_\mu + \alpha n_\mu + k_{\perp\,\mu}$$

$$M = \frac{1}{2} C_0 K_0 \not{p} = \frac{1}{2} C_0 \mathbb{P} K_0 \not{p} + \boxed{\frac{1}{2} C_0 (1 - \mathbb{P}) K_0 \not{p}}$$

finite (M – pole part is finite)

$$\frac{1}{2} C_0 \mathbb{P} K_0 \not{p} = \frac{1}{2} C_0 \mathbb{P}_\epsilon \mathbb{P}_s K_0 \not{p}$$

the projector performs integral over phase space of “k” and extracts poles

$$= \frac{1}{2} C_0 \mathbb{P}_\epsilon \mathbb{P}_{out} K_0 \mathbb{P}_{in}$$

C_0 hard scattering coefficient function

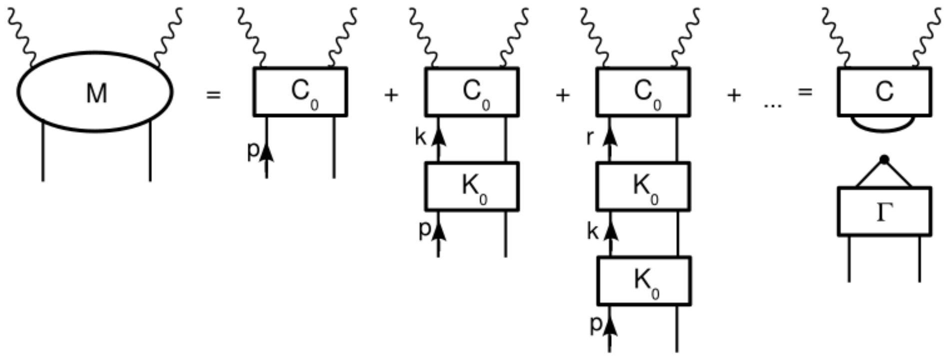
K_0 2PI kernels

$$= \int \frac{dx}{x} \frac{1}{2} C_0 \mathbb{P}_{out}\big|_{k^2=0} \, \Gamma\left(\frac{Q^2}{\mu_F^2}, x, \frac{1}{\epsilon}\right)$$

factorization convolution only in “x”

pole part
spin part

Curci-Furmanski-Petronzio - factorization



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factorization convolution only in “x”

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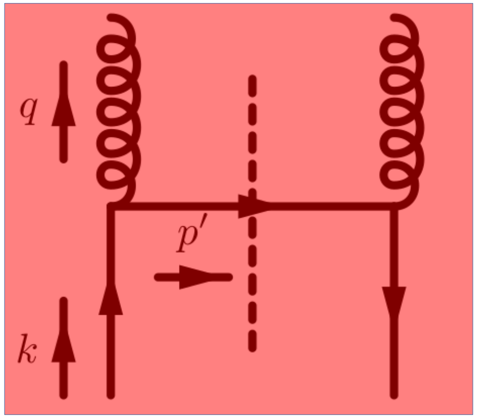
spin part

Curci-Furmanski-Petronzio - splitting function



- *incoming propagators amputated*
- *contains propagator of outgoing parton + incoming on-shell*

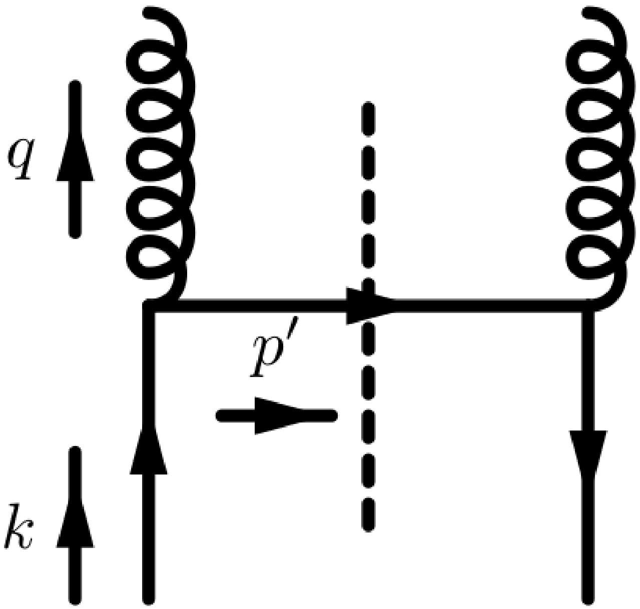
The CFP method applied to construct splitting functions

$$\Gamma \sim \hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = z \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 + q^2)\mathbb{P}_{j,\,\mathrm{in}} \otimes \hat{K}_{ij}^{(0)}(q,k) \otimes \mathbb{P}_{i,\,\mathrm{out}}$$

$$= \frac{\alpha_S}{2\pi\Gamma(1+\epsilon)} z \int_0^{\mu_F^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\left(\frac{e^{-\gamma_E}\boldsymbol{q}^2}{\mu^2}\right)^{\epsilon} P_{ij}^{(0)}(z;\epsilon)$$

The method can be used to prove factorization and to derive evolution equations

Generalization to HEF kinematics



$$q^\mu = x p^\mu + q_\perp^\mu + \frac{q^2 + \boldsymbol{q}^2}{2x\, p \cdot n} n^\mu$$

$$k^\mu = y p^\mu + k_\perp^\mu$$

ordering in "-" components

Catani, Hautmann NPB427 (1994) 475524

F. Hautmann, M. Hentschinski, H. Jung
Nucl.Phys. B865 (2012) 54-66

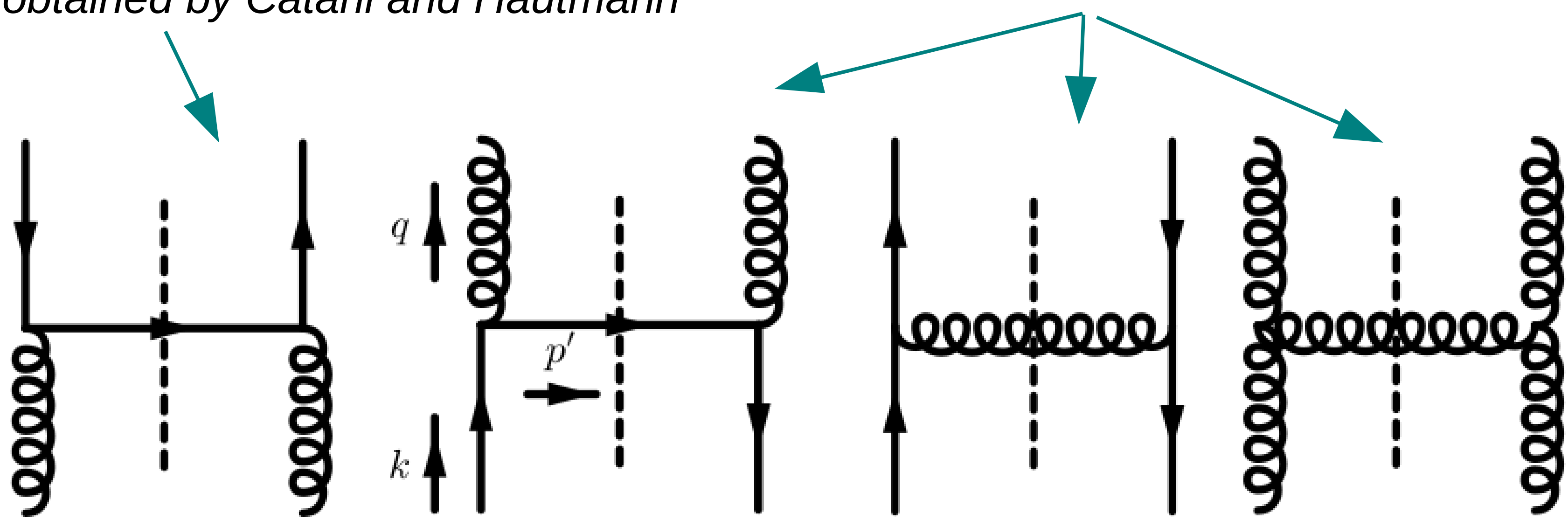
We will define and constrain splitting functions by requiring:

- *gauge invariance/current conservation of vertices*
- *collinear limit (LO)*
- *HEF limit (LO)*

Generalization to HEF kinematics

O. Gituliar, M. Hentschinski, K.K; JHEP 1601 (2016) 181
Hentschinski, Kusina, KK, Serino '17

Kernel obtained by Catani and Hautmann



P_{qg} P_{gq} P_{qq} P_{gg}

One needs:

- *appropriate projector operators*
- *generalize QCD vertices (can be obtained form Lipatov effective action or equivalently by spin helicity method)*

CH kernel

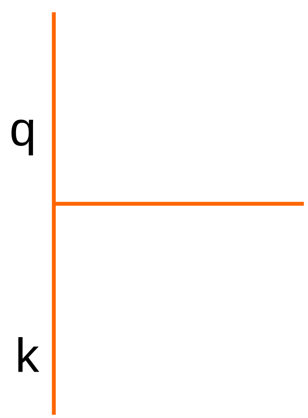
Application of the method to P_{qg}

Usage of axial gauge. The outgoing projector is the same for quark as in the original CFP

The projector for incoming gluons obtained from

$$\mathcal{M}^{g^* g^* \to q\bar{q}}(k_1, k_2; p_3, p_4) = \frac{2\, y_1\, y_2\, p_1^{\mu_1}\, p_2^{\mu_2}}{\sqrt{k_{1\perp}^2\, k_{2\perp}^2}}\, d_{\mu_1\nu_1}(k_1)\, d_{\mu_2\nu_2}(k_2)\, \hat{\mathcal{M}}^{g^* g^* \to q\bar{q}}_{\mu_1,\mu_2}(k_1, k_2; p_3, p_4)$$

$$y_1 p_1^{\mu_1} d_{\mu_1\nu_1}(k_1) = k_{1\perp\,\nu_1} \qquad y_2 p_2^{\mu_2} d_{\mu_2\nu_2}(k_2) = k_{2\perp\,\nu_2}$$



$$\mathbb{P}^{s\,\mu\nu}_{g,in} = -\frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \qquad \mathbb{P}^s_{q,out} = \frac{\not{n}}{2q\cdot n}$$

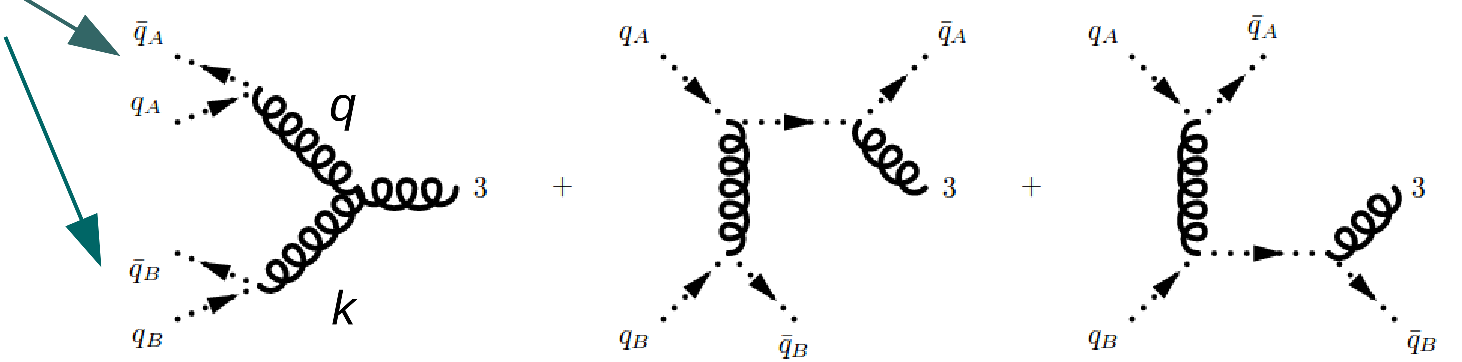
$$\tilde{\boldsymbol{q}} = \boldsymbol{q} - z\boldsymbol{k}$$

$$\bar{P}^{(0)}_{qg} \;=\; T_R \left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2}\right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right]$$

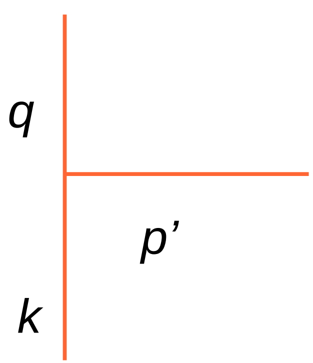
Vertices – example derivation

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

auxiliary quarks



$$\mathcal{A}(q,k,p') = (\sqrt{2})\,\frac{p_{\mu_1}\,n_{\mu_2}\,\epsilon_{\mu_3}(p')}{q^2\,k^2}\left\{\mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\,d^{\mu_1}{}_{\lambda}(q)\,d^{\mu_2}{}_{\kappa}(k)\right.$$



$$\left. + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}\right\}$$

$$\equiv (\sqrt{2})\,\frac{p_{\mu_1}\,n_{\mu_2}\,\epsilon_{\mu_3}(p')}{q^2\,k^2}\,\Gamma^{\mu_1\mu_2\mu_3}(q,k,p')$$

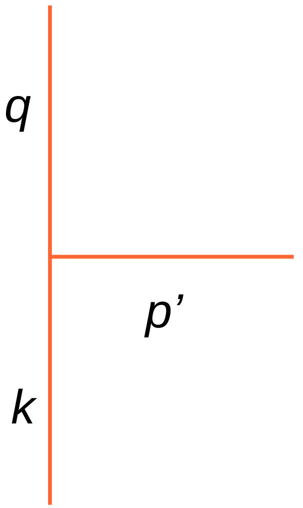
current conservation w.r.t outgoing gluon
q → general kinematics
k → HEF

Full set of projectors

Constrained by Ward identities and appropriate limits the splitting functions should have correct DGLAP and BFKL limits we have the following projectors

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{in}} = -y^2\,\frac{p^\mu p^\nu}{k_\perp^2}$$

$$\mathbb{P}^{s\,\mu\nu}_{g,\,\mathrm{out}} = -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} - k^2\,\frac{n_\mu n_\nu}{(k\cdot n)^2}$$



$$\mathbb{P}^{s}_{q,\,\mathrm{in}} = \frac{y\,\not{p}}{2}$$

$$q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \boldsymbol{q}^2}{2x\,p\cdot n}n^\mu$$

$$\mathbb{P}^{s}_{q,\,\mathrm{out}} = \frac{\not{n}}{2\,n\cdot l}$$

$$k^\mu = yp^\mu + k_\perp^\mu$$

Full set of vertices

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

$$\Gamma^{\mu}_{q^* g^* q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(k)\,\left(\gamma^{\nu} - \frac{n^{\nu}}{k\cdot n}\not{q}\right)$$

$$\Gamma^{\mu}_{g^* q^* q}(q,k,p') = igt^a\, d^{\mu}{}_{\nu}(q)\,\left(\gamma^{\nu} - \frac{p^{\nu}}{p\cdot q}\not{k}\right)$$

$$\Gamma^{\mu}_{q^* q^* g}(q,k,p') = igt^a\left(\gamma^{\mu} - \frac{p^{\mu}}{p\cdot p'}\not{k} + \frac{n^{\mu}}{n\cdot p'}\not{q}\right)$$

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = i\,g\,f^{abc}\left\{\mathcal{V}^{\lambda\kappa\mu_3}(q,k,p')\,d^{\mu_1}{}_{\lambda}(q)\,d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}\right\}$$

Obtained using spinor helicity methods

Van Hameren, Kotko, Kutak, JHEP 1301 (2013) 078

remark: can be obtained
from Lipatov effective action

Example calculation of splitting function: P_{gg} case

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

$$\mathbb{P}_{g,\,\mathrm{in}} \otimes \hat{K}_{gg}^{(0)}(q,k) \otimes \mathbb{P}_{g,\,\mathrm{out}} =$$

$$\mathbb{P}_{g,\,\mathrm{in}}^{\beta\beta'}(k)\,\mathbb{P}_{g,\,\mathrm{out}}^{\mu'\nu'}(q)(\Gamma_{g^*g^*g}^{\beta\mu\alpha})^\dagger\Gamma_{g^*g^*g}^{\nu\beta'\alpha'}\,\frac{-id^{\mu\mu'}(q)}{q^2-i\epsilon}\,\frac{id^{\nu\nu'}(q)}{q^2+i\epsilon}\,d^{\alpha\alpha'}(k-q)$$

$$\tilde{P}_{gg}^{(0)}(z,\tilde{\boldsymbol{q}},\boldsymbol{k}) = 2C_A\left\{\frac{\tilde{\boldsymbol{q}}^4}{(\tilde{\boldsymbol{q}}-(1-z)\boldsymbol{k})^2\,[\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2]}\left[\frac{z}{1-z}+\frac{1-z}{z}+\right.\right.$$
$$\left.\left.+(3-4z)\frac{\tilde{\boldsymbol{q}}\cdot\boldsymbol{k}}{\tilde{\boldsymbol{q}}^2}+z(3-2z)\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right]+\frac{(1+\epsilon)\tilde{\boldsymbol{q}}^2z(1-z)[2\tilde{\boldsymbol{q}}\cdot\boldsymbol{k}+(2z-1)\boldsymbol{k}^2]^2}{2\boldsymbol{k}^2[\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2]^2}\right\}$$

$$\tilde{\boldsymbol{q}} = \boldsymbol{q} - z\boldsymbol{k}$$

Results

M. Hentschinski, A. Kusina, K.K, M. Serino; Eur.Phys.J. C78 (2018) no.3, 174

For sake of presentation: only angular averaged kernels

$$\bar{P}_{qg}^{(0)} = T_R \left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2} \right]$$

$$\bar{P}_{gq}^{(0)} = C_F \left[\frac{2\tilde{\boldsymbol{q}}^2}{z|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{(2-z)\tilde{\boldsymbol{q}}^4 + z(1-z^2)\boldsymbol{k}^2\tilde{\boldsymbol{q}}^2}{\left(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2\right)^2} \right]$$

$$\bar{P}_{qq}^{(0)} = C_F \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[\frac{\tilde{\boldsymbol{q}}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\tilde{\boldsymbol{q}}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]$$

$$\bar{P}_{gg}^{(0)} = C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\,|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]$$

Kinematic limits P_{gg} – DGLAP BFKL

with this variable one can disentangle singularities

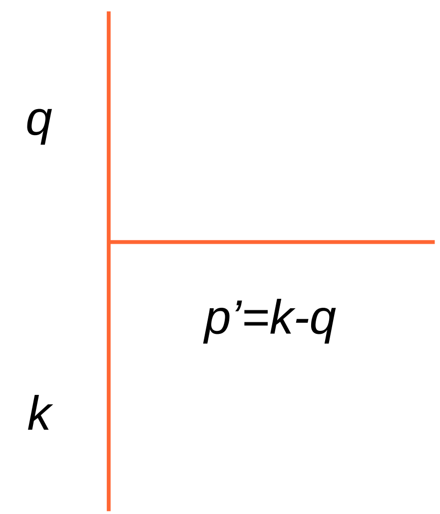
$$\tilde{\boldsymbol{p}} = \frac{\boldsymbol{k}-\boldsymbol{q}}{1-z}$$

DGLAP limit:

$$\lim_{\boldsymbol{k}^2\to 0}\int_0^{2\pi} d\phi P(z,\boldsymbol{k}^2,\tilde{\boldsymbol{p}}^2) = 2C_A\left[\frac{z}{1-z}+\frac{1-z}{z}+z\,(1-z)\right]$$

BFKL limit:

$$\begin{aligned}\lim_{z\to 0}\hat{K}_{gg}\left(z,\frac{\boldsymbol{k}^2}{\mu^2},\epsilon,\alpha_s\right) &= \frac{\alpha_s C_A}{\pi(e^{\gamma_E}\mu^2)^\epsilon}\int\frac{d^{2+2\epsilon}\tilde{\boldsymbol{p}}}{\pi^{1+\epsilon}}\Theta\left(\mu_F^2-(\boldsymbol{k}-\tilde{\boldsymbol{p}})^2\right)\frac{1}{\tilde{\boldsymbol{p}}^2}\\ &= \int\frac{d^{2+2\epsilon}\boldsymbol{q}}{\pi^{1+\epsilon}}\Theta\left(\mu_F^2-\boldsymbol{q}^2\right)\frac{\alpha_s C_A}{\pi(e^{\gamma_E}\mu^2)^\epsilon}\frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2},\end{aligned}$$



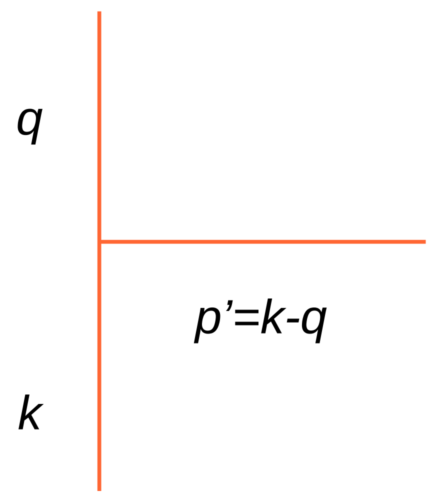
Kinematic limits P_{gg} - CCFM

$$p' \equiv k - q = y\,p\,(1-z) + \boldsymbol{k} - \boldsymbol{q} + \frac{q^2 + \boldsymbol{q}^2}{2\,x\,p \cdot n}\,n$$

$$\frac{p'}{1-z} = y\,p + \boxed{\frac{\boldsymbol{k} - \boldsymbol{q}}{1-z}} + \frac{q^2 + \boldsymbol{q}^2}{2\,(1-z)\,x\,p \cdot n}\,n$$

related to angle

$$\tilde{\boldsymbol{p}} = \frac{\boldsymbol{k} - \boldsymbol{q}}{1-z}$$



CCFM limit:

$$\lim_{\tilde{\boldsymbol{p}} \to 0} \hat{K}_{gg}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, 0, \alpha_s\right) = z \int_{\tilde{\boldsymbol{p}}^2_{min}}^{\tilde{\boldsymbol{p}}^2_{max}} \frac{d\tilde{\boldsymbol{p}}^2}{\tilde{\boldsymbol{p}}^2} \frac{\alpha_s C_a}{\pi} \left[\frac{1}{z} + \frac{1}{1-z} + \mathcal{O}\left(\frac{\tilde{\boldsymbol{p}}^2}{\boldsymbol{k}^2}\right)\right]$$

Towards evolution equation

- *For now we have real part emissions of the splitting functions.*
- *The non diagonal splitting functions do not have virtual contribution at the LO.*
- *They are divergent when p’ → 0. The diagonal once have virtual contributions.*
- *However, the distribution of gluons gets contribution from quarks….*
- *We can consider the following model*

Towards evolution equation

Real part of Pqq to be complemented by virtual corrections → can expect cancellations of singularities but Pgq is divergent

For gluonic part we use low z limit part of Pgg i.e. LO BFKL equation

$$\mathcal{F}(x,\boldsymbol{q}^2) = \mathcal{F}^0(x,\boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} \left[\mathcal{F}\left(\frac{x}{z}, |\boldsymbol{q}+\boldsymbol{p}|^2\right) - \theta(\boldsymbol{q}^2 - \boldsymbol{p}^2)\mathcal{F}\left(\frac{x}{z}, \boldsymbol{q}^2\right)\right]$$

add quark induced contribution

$$+\frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z,\boldsymbol{p},\boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

Towards evolution equation- BFKL with Regge form factor

Use simplified P_{gg} kernel i.e. BFKL limit. Introduce phase space slicing parameter to separate resolved and unresolved emissions

$$\mathcal{F}(x,\boldsymbol{q}^2) = \mathcal{F}^0(x,\boldsymbol{q}^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z}\left[\int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right) - \ln\frac{\boldsymbol{q}^2}{\mu^2}\mathcal{F}\left(\frac{x}{z},\boldsymbol{q}^2\right)\right]$$

Using Mellin transforms and some algebra we get

$$\mathcal{F}(x,\boldsymbol{q}^2) = \underbrace{\tilde{\mathcal{F}}^0(x,\boldsymbol{q}^2)}_{\text{modified}} + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \underbrace{\Delta_R(z,\boldsymbol{q}^2,\mu^2)}_{\exp(-\overline{\alpha}_s \ln 1/z \ln \boldsymbol{q}^2/\mu^2)} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)$$

Stable in $\mu \to 0$

Towards evolution equation

M. Hentschinski, A. Kusina, K.K; Phys. Rev. D 94, 114013 (2016)

For quark part the crucial difference: no virtual corrections

$$\int \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2} \to \int_{\mu^2} \frac{d\boldsymbol{p}^2}{\boldsymbol{p}^2}$$

$$\text{‘BFKL’} + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \int_{\mu^2} \frac{d^2\boldsymbol{p}}{\pi \boldsymbol{p}^2} P_{gq}(z, \boldsymbol{p}, \boldsymbol{q}) \mathcal{Q}\left(\frac{x}{z}, |\boldsymbol{p}+\boldsymbol{q}|^2\right)$$

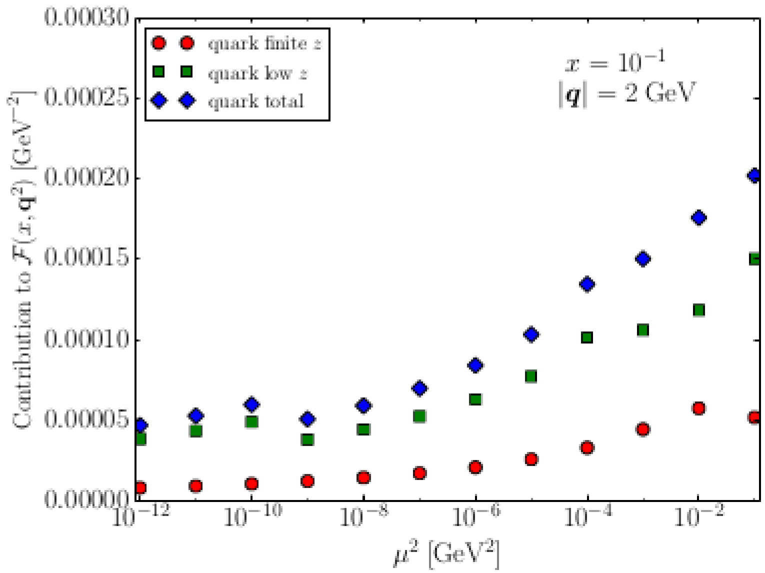
The equation for gluon reads:

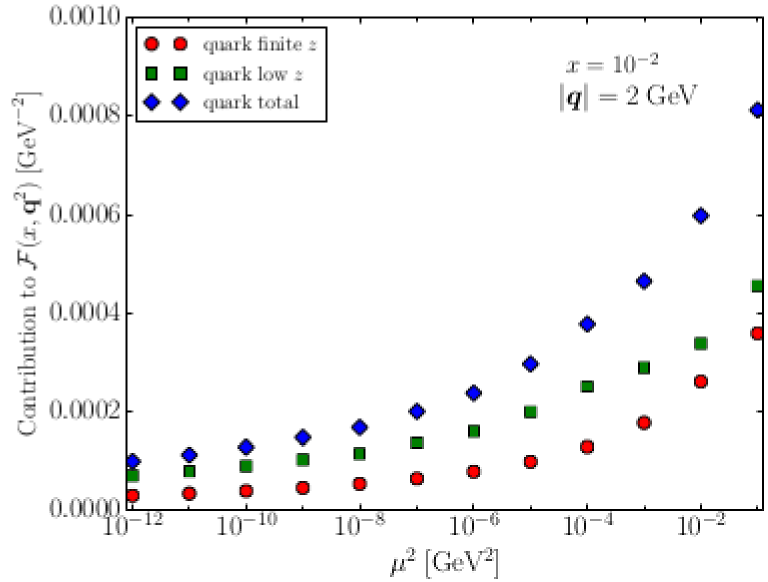
$$\mathcal{F}(x,\boldsymbol{q}^2) = \tilde{\mathcal{F}}^0(x,\boldsymbol{q}^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\int \frac{d^2\boldsymbol{p}}{\pi\boldsymbol{p}^2}\theta(\boldsymbol{p}^2-\mu^2)\Bigg[\Delta_R(z,\boldsymbol{q}^2,\mu^2)$$
$$\left(2C_A\mathcal{F}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right) + C_F\mathcal{Q}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)\right)$$
$$-\int_z^1 \frac{dz_1}{z_1}\Delta_R(z_1,\boldsymbol{q}^2,\mu^2)\left[\tilde{P}'_{gq}\left(\frac{z}{z_1},\boldsymbol{p},\boldsymbol{q}\right)\frac{z}{z_1}\right]\mathcal{Q}\left(\frac{x}{z},|\boldsymbol{q}+\boldsymbol{p}|^2\right)\Bigg]$$

where

$$P_{gq} = \tilde{P}_{gq}/z$$

Towards evolution equation - stability





Resummation of $\ln \boldsymbol{q}^2/\mu^2$ in $\Delta_R = \left(\frac{\mu^2}{\boldsymbol{q}^2}\right)^{\overline{\alpha}_s \ln 1/z}$ *cuts of μ → region*

Conclusions and outlook

- *We have applied CFP and CH technique to calculate real emissions splitting functions*
- *We used the splitting functions to construct model equation for gluon density receiving contributions from quarks*
- *We found that found that resummation of virtual contributions to P_{gg} at low x helps with treatment of singularity of P_{gq} splitting function*
- *Virtual contributions to P_{gg} and P_{qq} should be computed using the same formalism*
- *Evolution variable: will come after getting full kernels*
- *The full set of evolution equations*
- *Relation to operator definition of TMD, address nonlinearities*
- *Solution*
- *Monte Carlo implementation*