CGC approach: odd harmonics in angular correlations

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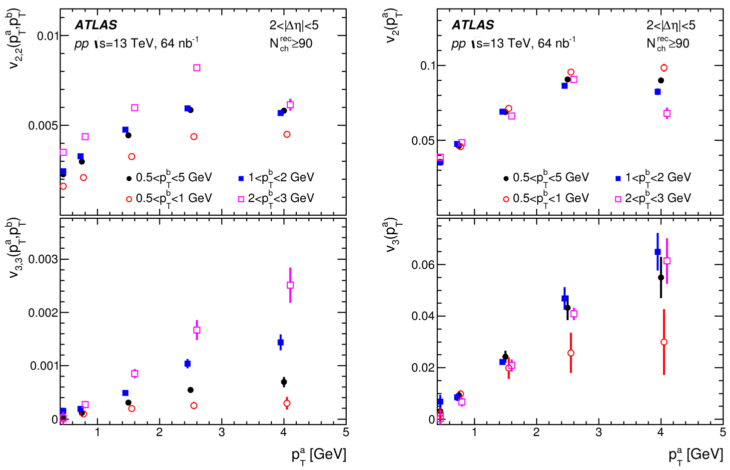
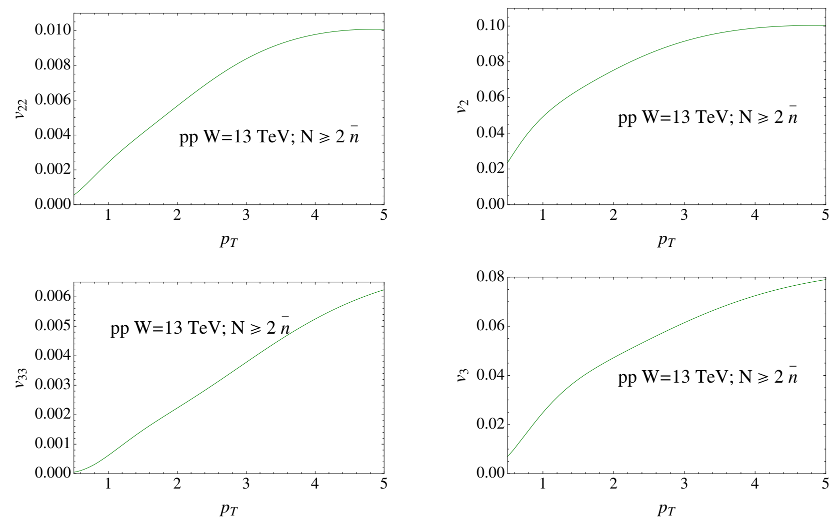
E. Gotsman and E.L.: *"CGC/saturation approach: re-visiting the problem of odd harmonics in angular correlations,"* arXiv:1807.02783

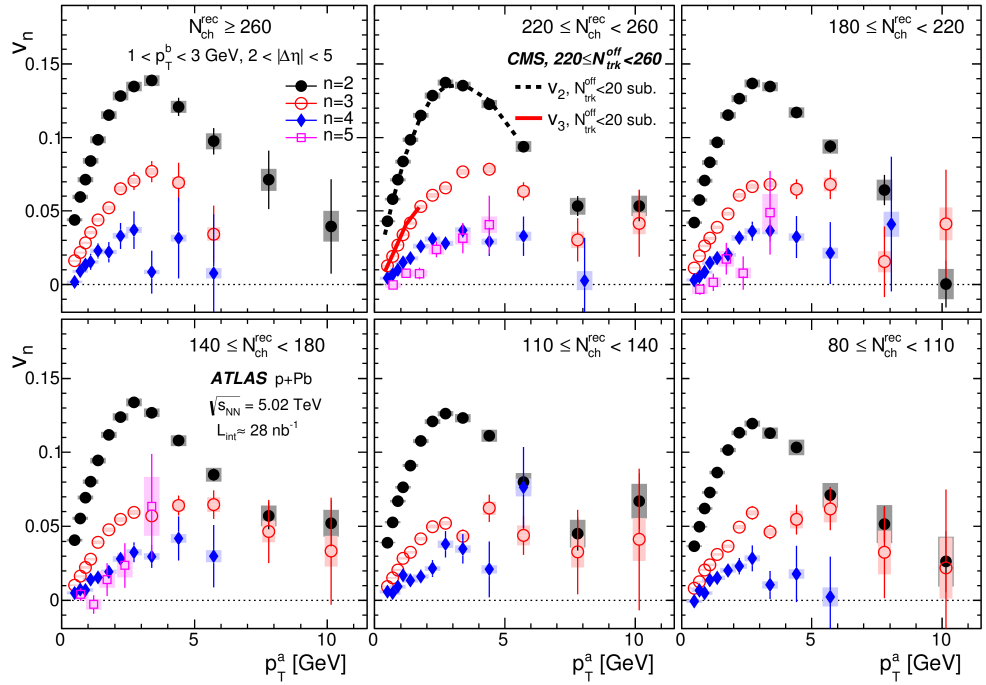
Motivations:

$v_{2n+1} = 0$ **is the only prediction of CGC/saturation approach in a qualitative disagreement with the experiment**

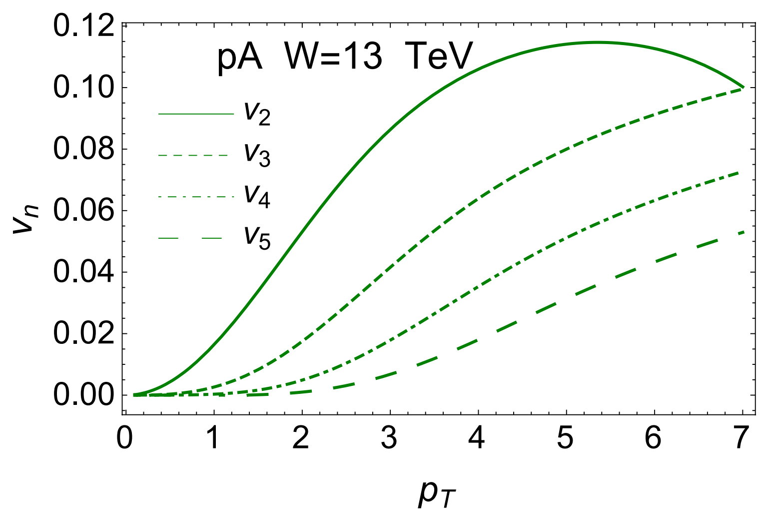




- **ATLAS data**
- **Our estimates**



$N_{ch}^{rec} \geq 260$
$1 < p_T^b < 3$ GeV, $2 < |\Delta\eta| < 5$
n=2
n=3
n=4
n=5
$220 \leq N_{ch}^{rec} < 260$
CMS, $220 \leq N_{trk}^{off} < 260$
v_2, N_{trk}^{off}<20 sub.
v_3, N_{trk}^{off}<20 sub.
$180 \leq N_{ch}^{rec} < 220$
$140 \leq N_{ch}^{rec} < 180$
ATLAS p+Pb
$\sqrt{s_{NN}}$ = 5.02 TeV
$L_{int} \approx$ 28 nb^{-1}
$110 \leq N_{ch}^{rec} < 140$
$80 \leq N_{ch}^{rec} < 110$
v_n
p_T^a [GeV]



pA W=13 TeV
v_2
v_3
v_4
v_5
v_n
p_T

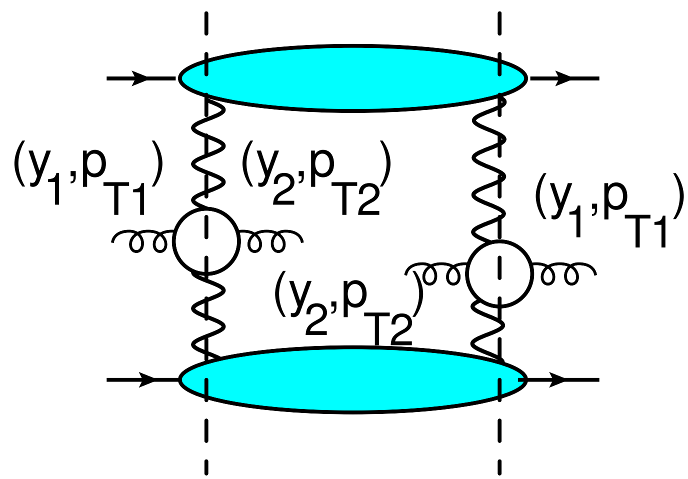
$\varphi \longrightarrow \pi - \varphi$ symmetry:

- **Is this symmetry based on the general features of QCD ?**
- **Is this symmetry is an artifact of leading order approximation ?**
- **Does this symmetry hold only for the totally inclusive measurements ?**

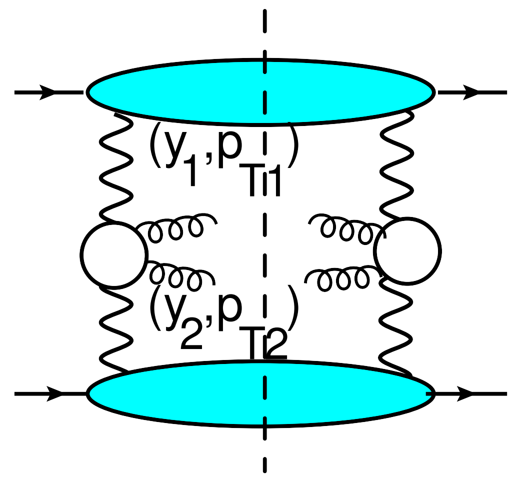
NO • YES • YES

- **How v_n depend on the multiplicity of the event ?**

Dilute-dilute system scattering: the origin of the symmetry



BE enhancement



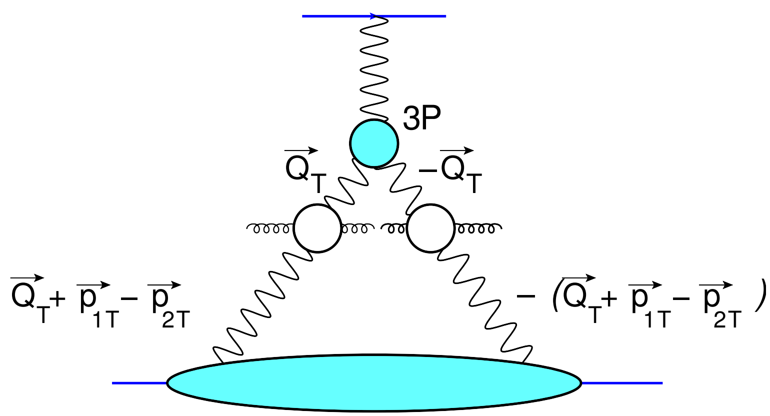
Central diffraction

	BE enhancement	Central diffraction
Function of	$(\vec{p}_{1,T} - \vec{p}_{2,T})^2$	$(\vec{p}_{1,T} + \vec{p}_{2,T})^2$
Multiplicities:	$N \geq 2\,\bar{N}$	$N \ll \bar{N}$

$\bar{N}$ **is the average multiplicity in the inclusive event.**

$\bar{\alpha}_S |y_1 - y_2| \ll 1$

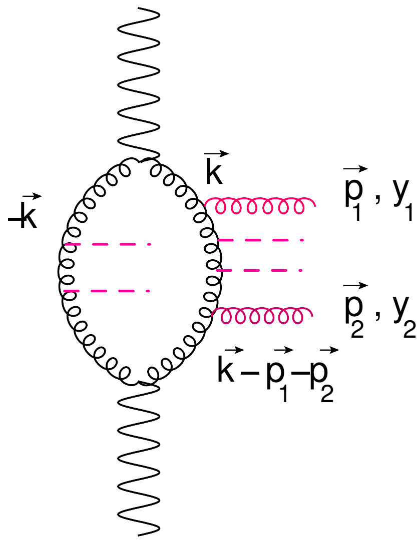
Can we calculate angular correlations in CGC?

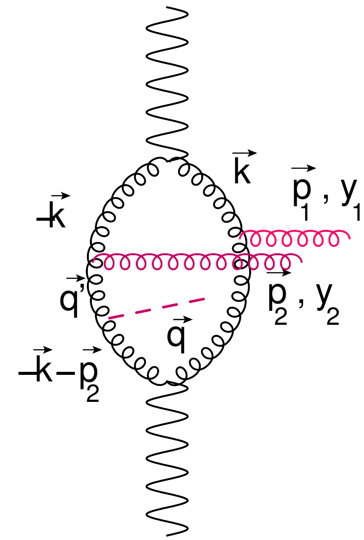


- $\propto \exp\left(-\frac{B_{3\mathbb{P}} B_p}{B_{3\mathbb{P}} + B_p} \Delta \vec{p}^{\,2}_{12}\right) \quad \rightarrow \quad \exp\left(-B_{3\mathbb{P}}\, \Delta \vec{p}^{\,2}_{12}\right)$

A: YES

Sudakov suppression of CD production





$$e^{-\frac{\bar{\alpha}_S}{4}\ln^2\left(\frac{p_T^2(1+\cosh(y_1-y_2))}{2\,Q_s^2(y_1\approx y_2)}\right)}$$

E. Gotsman and E.L.: PRD 96 (2017) 074011

Dilute-dense system scattering: parton model

Parton model: BFKL Pomeron with fixed dipole sizes

- $$d\,N\,(Y)\,/d Y \;=\; \Delta\left(N\,(Y)\;-\;N^2\,(Y)\right)$$
- $$N\,(Y)\;=\;\frac{\gamma\,e^{\Delta\,Y}}{1+\gamma\left(e^{\Delta Y}-1\right)}\;=\;\frac{\gamma\,z}{1+\gamma(z-1)}$$

Generating function:

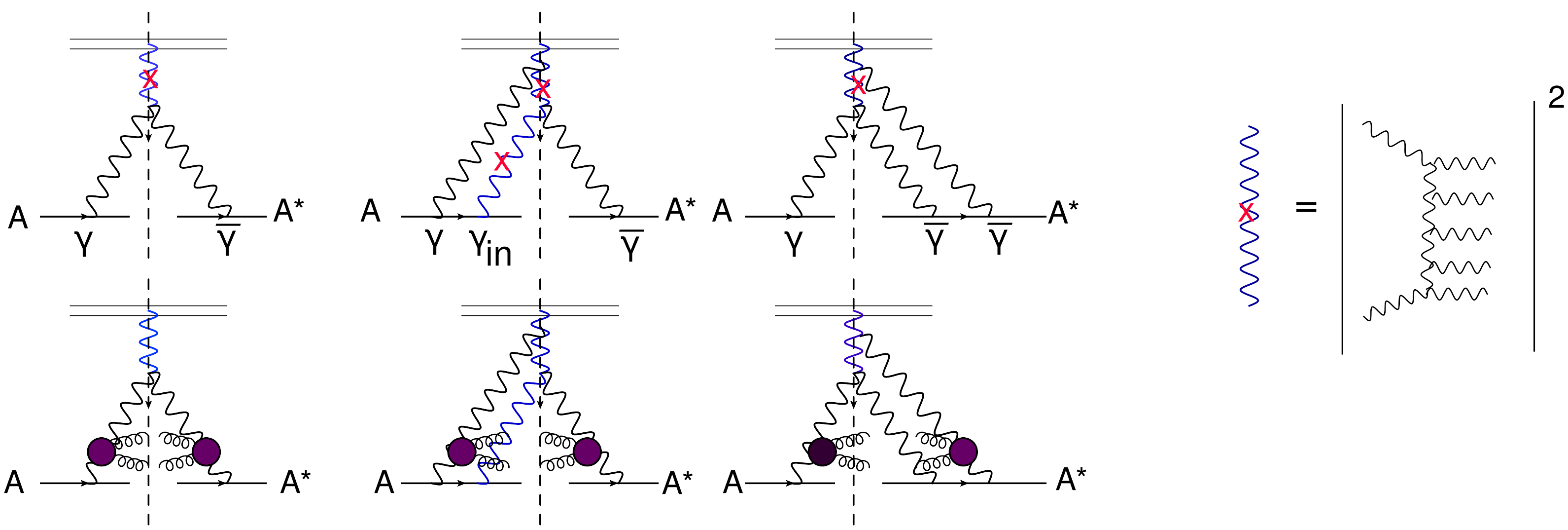
- $$Z\,(w,\bar{w},v;Y)\;=\;\sum_{k=0,l=0,m=0}^{\infty}P\,(k,l,m;Y)\,w^k\,\bar{w}^l\,v^m$$

It has the form: E.L & Prygarin (2008)

- $$Z\,(w,\bar{w},v;Y)\;=\;\frac{w}{(1-w)(z-1)\;+\;1}+\frac{\bar{w}}{(1-\bar{w})(z-1)\;+\;1}-\frac{w+\bar{w}-v}{(1-w-\bar{w}+v)(z-1)+1}$$

where $z = e^{\Delta Y}$.

- $$N\,(\gamma,\bar{\gamma},\gamma_{\rm in};Y)\;=\;1-Z\,(1-\gamma,1-\bar{\gamma},1-\gamma_{\rm in};Y)$$
- $$N\,(\gamma,\bar{\gamma},\gamma_{\rm in};Y)\;=\frac{\gamma z}{\gamma(z-1)+1}+\frac{\bar{\gamma}z}{\bar{\gamma}(z-1)+1}-\frac{(\gamma+\bar{\gamma}-\gamma_{\rm in})z}{(\gamma+\bar{\gamma}-\gamma_{\rm in})(z-1)+1}$$



For inclusive measurements the CD production = BE enhancement

- $\sum_{n=0}^{\infty} \sigma_n^{\rm CD} = \sum_{n=2\bar{N}}^{\infty} \sigma_n^{\rm BE} \Longrightarrow v_{2n-1} = 0$

For measurements with $n \geq 2\bar{N}$ CD contribution =

$\sum_{n=0}^{\infty} \sigma_n^{\rm CD} - \sum_{n=0}^{n<2\bar{N}} \sigma_n^{\rm CD}$; $v_{2n-1} \propto \int \cos\left((2n-1)\varphi\right) d\varphi \sum_{n=0}^{n<2\bar{N}} \sigma_n^{\rm CD}$

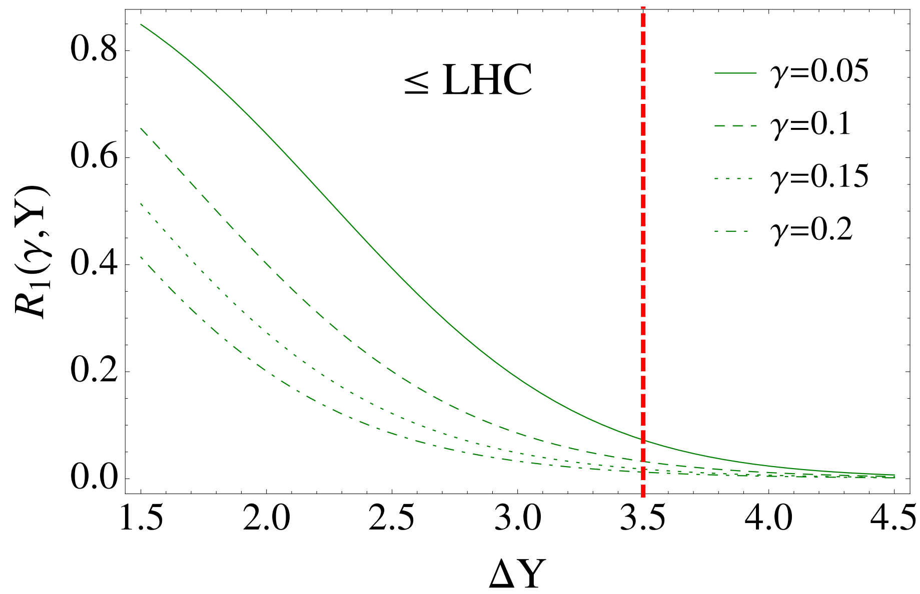
- $\sum_{n=0}^{n<2\bar{N}} \sigma_n^{\rm CD} \longrightarrow 0$ **for** $Y \gg 1$
- $R = \left(\sum_{n=0}^{n<2\bar{N}} \sigma_n^{\rm CD}\right) \Big/ \left(\sum_{n=0}^{\infty} \sigma_n^{\rm CD}\right)$

- $$\sigma_{\rm CD} = \Gamma^2\left(2I\!P \to 2G\right)\;\gamma \frac{\partial}{\partial \gamma}\,\bar{\gamma}\frac{\partial}{\partial \bar{\gamma}} N\left(Y,\gamma,\bar{\gamma},\gamma_{in}\right)\Big|_{\gamma=\bar{\gamma},\gamma_{in}=0} = \Gamma^2\left(2I\!P \to 2G\right)\;\frac{2\gamma^2\,z\,\left(z-1\right)}{\left(1+2\gamma\left(z-1\right)\right)^3}$$

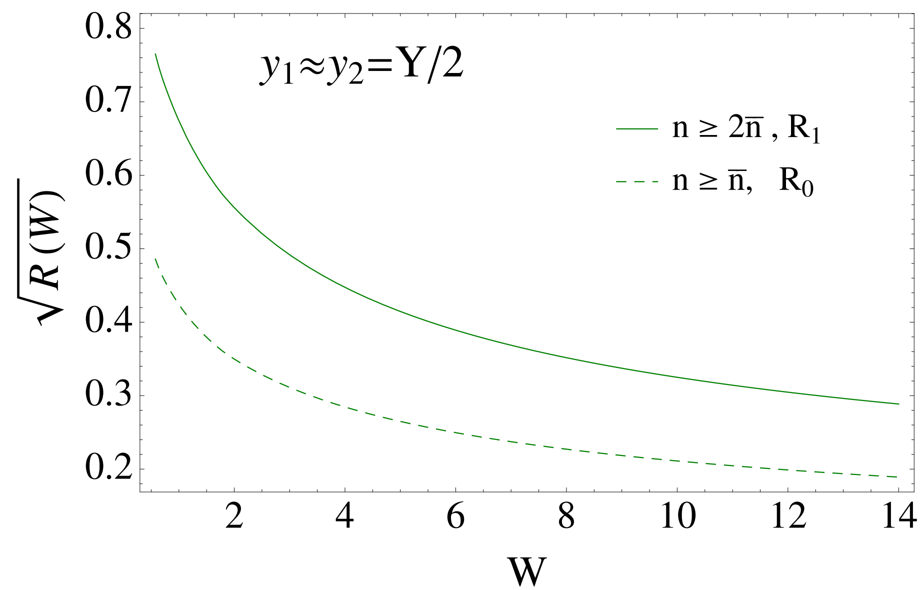
- $$\sigma^{BE}_n = \frac{1}{N^2_c-1}\Gamma^2_G\,\frac{\gamma^k_{\rm in}}{k!}\frac{\partial}{\partial \gamma^k_{\rm in}}\,N\left(\gamma,\bar{\gamma},\gamma_{\rm in};Y\right)\Big|_{\gamma_{\rm in}=0,\gamma=\bar{\gamma}} = \frac{1}{N^2_c-1}\Gamma^2_G\gamma^k_{\rm in}\frac{z\left(z-1\right)^{k-1}}{\left(1+2\,\gamma\,\left(z-1\right)\right)^{k+1}}$$
$$= \frac{1}{N^2_c-1}\Gamma^2_G\left(2\gamma\right)^k\,\frac{z\left(z-1\right)^{k-1}}{\left(1+2\,\gamma\,\left(z-1\right)\right)^{k+1}}$$

- $$\sigma^{CD}_n = \Gamma^2\left(2I\!P \to 2G\right)\;\gamma\frac{\partial}{\partial \gamma}\,\bar{\gamma}\frac{\partial}{\partial \bar{\gamma}}\,\frac{\gamma^k_{\rm in}}{k!}\frac{\partial}{\partial \gamma^k_{\rm in}}\,N\left(\gamma,\bar{\gamma},\gamma_{\rm in};Y\right)\Big|_{\gamma_{\rm in}=0,\gamma=\bar{\gamma}}$$
$$= \left(k+2\right)\left(k+1\right)\Gamma^2\left(2I\!P \to 2G\right)\gamma^2\,\gamma^k_{\rm in}\frac{z\left(z-1\right)^{k+1}}{\left(1+2\,\gamma\left(z-1\right)\right)^{k+3}}$$
$$= \left(k+2\right)\left(k+1\right)\Gamma^2\left(2I\!P \to 2G\right)\;\gamma^2\;\left(2\,\gamma\right)^k\;\frac{z\left(z-1\right)^{k+1}}{\left(1+2\,\gamma\left(z-1\right)\right)^{k+3}}$$

Schwimmer model: $\gamma = g_{I\!P} G_{3I\!P} S_A\left(b\right)$



≤ LHC
γ=0.05
γ=0.1
γ=0.15
γ=0.2
$R_1(\gamma, Y)$
ΔY



$y_1 \approx y_2 = Y/2$
$n \geq 2\bar{n}$, R_1
$n \geq \bar{n}$, R_0
$\sqrt{R(W)}$
W

DIS in QCD

Simplified non-linear equation: leading twist approach

For $\tau = x_{01}^2 \, Q_s^2 > 1$

- $\int K\left(\vec{x}_{01}; \vec{x}_{02}, \vec{x}_{12}\right) d^2 x_{02} \;\rightarrow\; \pi \int_{1/Q_s^2(Y,b)}^{x_{01}^2} \frac{d x_{02}^2}{x_{02}^2} \;+\; \pi \int_{1/Q_s^2(Y,b)}^{x_{01}^2} \frac{d|\vec{x}_{01} - \vec{x}_{02}|^2}{|\vec{x}_{01} - \vec{x}_{02}|^2}$

- $\frac{\partial^2 \widetilde{N}\left(Y;\vec{x}_{01},\vec{b}\right)}{\partial Y \; \partial \ln r^2} \;=\; \bar{\alpha}_S \left\{ \left(1 \;-\; \frac{\partial \widetilde{N}\left(Y;\vec{x}_{01},\vec{b}\right)}{\partial \ln x_{01}^2} \right) \widetilde{N}\left(Y; \vec{x}_{01}, \vec{b} \right) \right\}$

- $\widetilde{N}\left(Y; \vec{x}_{01}, \vec{b} \right) \;=\; \int^{x_{01}^2} d x_{02}^2 \, N\left(Y; \vec{x}_{02}, \vec{b} \right) / x_{01}^2$

The leading twist:

- $$\chi(\gamma) = \begin{cases} \frac{1}{\gamma} & \text{for } \tau \geq 1; \\ \frac{1}{1-\gamma} & \text{for } \tau \leq 1; \end{cases}$$

From solution $\tau \leq 1$ the boundary conditions

- $N\left(Y; \zeta = 0_-(\xi = -\xi_s), b\right) \;=\; N_0\left(b\right);$ $\quad \frac{\partial \ln N\left(Y;\zeta=0_-(\xi=-\xi_s),b\right)}{\partial \zeta} \;=\; \frac{1}{2};$

Solution:

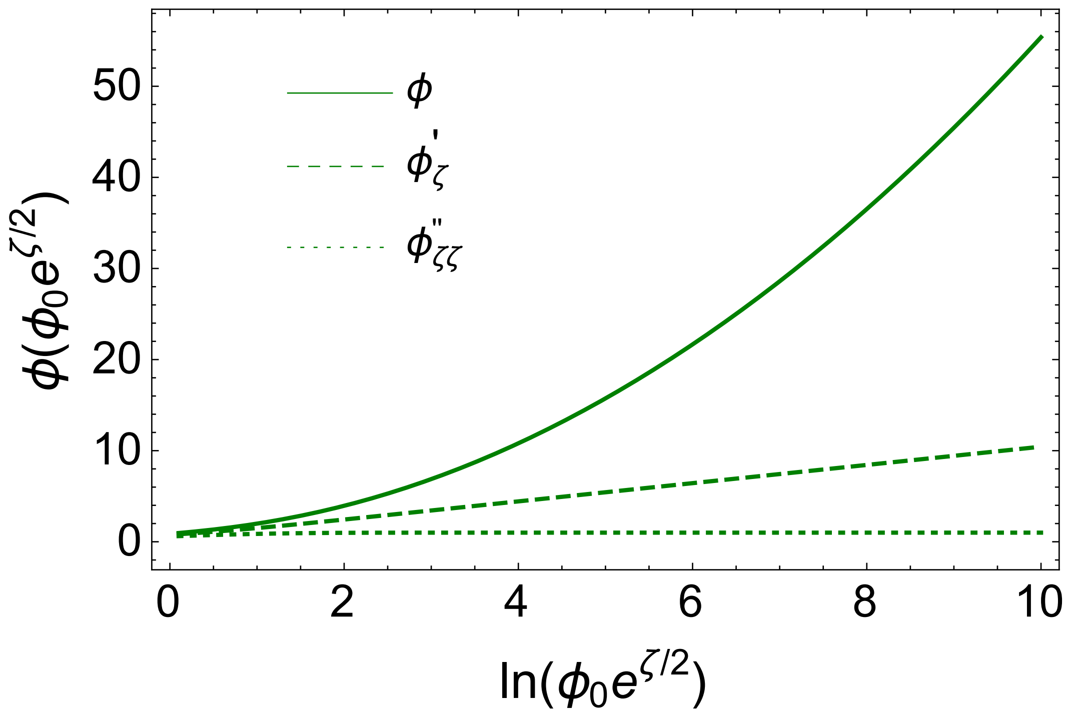
- $$\widetilde{N} \;=\; \int_{\xi_s}^{\xi} d\xi' \left(1 \;-\; e^{-\phi(\xi',Y)}\right)$$
- $$\frac{\partial^2\phi}{\partial Y\,\partial\xi} \;=\; \bar{\alpha}_S \left(1 \;-\; e^{-\phi(Y;\xi)}\right)$$
- $$\frac{\partial^2\phi}{\partial\zeta^2} \;-\; \frac{\partial^2\phi}{\partial x^2} \;=\; \frac{1}{4}\left(1 \;-\; e^{-\phi(Y;\xi)}\right)$$

with $\zeta \;=\; \ln(\tau) \;=\; \xi_s + \xi$ and $x = \xi_s - \xi$.

Traveling wave solution :

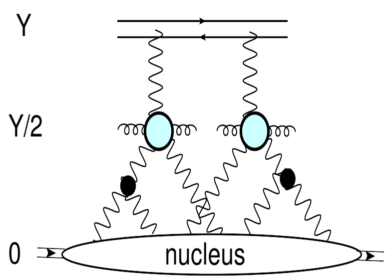
- $$\int_{\phi_0}^{\phi} \frac{d\phi'}{\sqrt{c \;+\; \frac{1}{2(\lambda^2-\kappa^2)}\left(\phi' - 1 + e^{-\phi'}\right)}} \;=\; \kappa\,x + \lambda\,\zeta$$
- $$\sqrt{2}\,\int_{\phi_0}^{\phi} \frac{d\phi'}{\sqrt{\phi' \;-\; 1 + \;e^{-\phi'}}} \;=\; \zeta$$

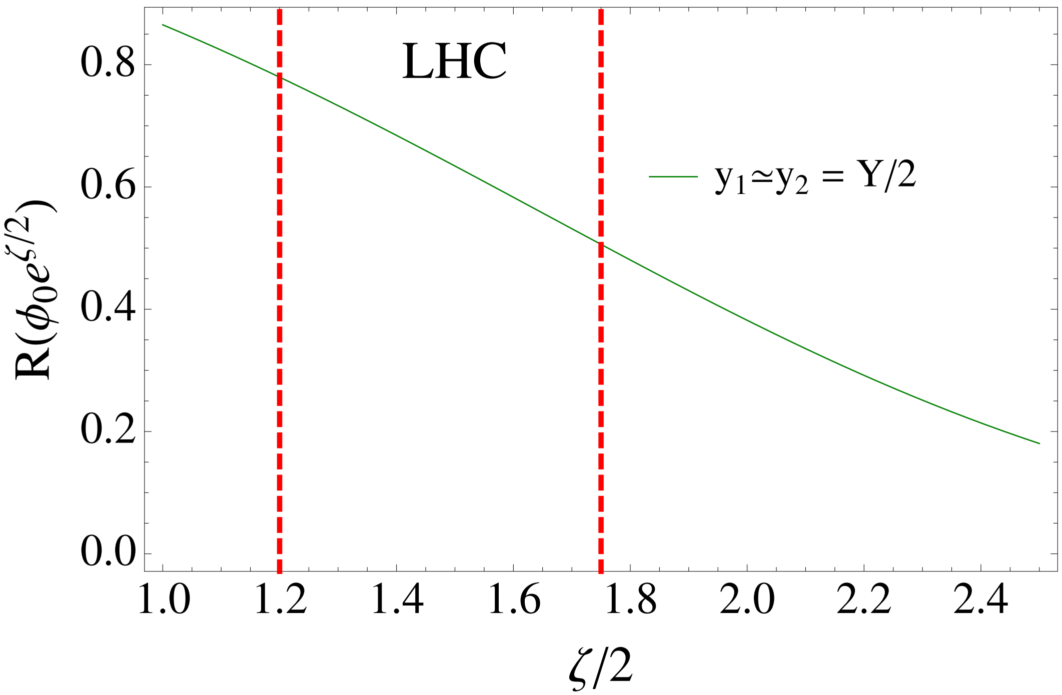
• $$\int_0^{\phi} d\phi' \left\{ \frac{1}{\sqrt{2\left(\phi' - 1 + e^{-\phi'}\right)}} - \frac{1}{\phi'} \right\} = \ln\left(\phi_0 e^{\frac{1}{2}\zeta}\right)$$



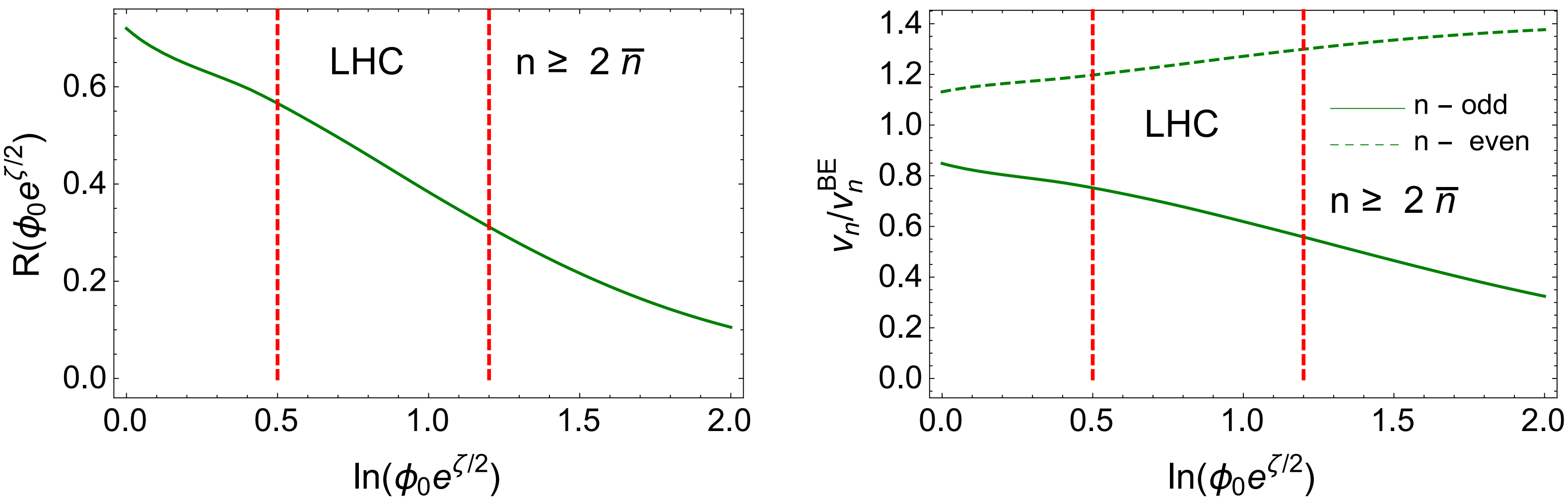
$$N\left(\gamma, \bar{\gamma}, \gamma_{\rm in}; \zeta\right) = 1 - e^{-\phi\left(\gamma e^{\frac{1}{2}\zeta}\right)} - e^{-\phi\left(\bar{\gamma} e^{\frac{1}{2}\zeta}\right)} + e^{-\phi\left(\left(\gamma + \bar{\gamma} - \gamma_{\rm in}\right) e^{\frac{1}{2}\zeta}\right)}$$

$$R\left(\phi_0 e^{\frac{1}{2}\zeta}\right) = \frac{\sigma_0^{\rm CD} + \sigma_1^{\rm CD}}{\sigma^{\rm BE}} = \frac{\left\{\phi'^{2}_{\zeta} + \phi'^{3}_{\zeta}\right\} e^{-\phi\left(2\,\phi_0\, e^{\frac{1}{2}\zeta}\right)}}{\left(\phi_0 e^{\frac{1}{2}(\zeta - \frac{1}{2}\zeta)}\right)^2 \left(1 - e^{-2\,\phi\left(\phi_0 e^{\frac{1}{4}\zeta}\right)}\right)^2}$$





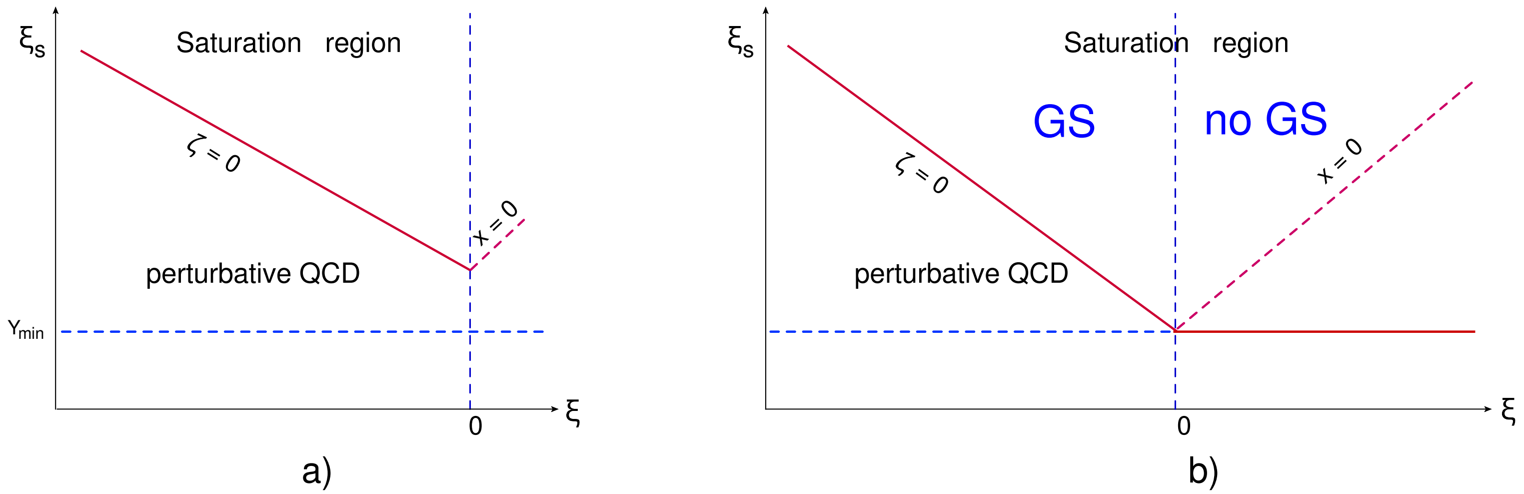
Estimates for DIS (proton) with nucleus scattering :



- **For DIS with proton $\phi_0 \approx 0.1 - 0.2$.**
- **For DIS with nucleus $\phi_0 \approx 0.2 - 0.4$.**
- **Schwimmer model for proton-A collision gives small ϕ_0.**

$$\tau_m = x_{01}^2\, Q_s^2\,(Y = Y_{min})?$$

$\tau_m \leq 1 \Longleftarrow$ **the same as Fig.a.** $\tau_m \geq 1 \Longleftarrow$ **?.**



- **Initial condition at** $Y = Y_{min}$: $\phi = \phi_0\, e^{\xi}$ **with** $\phi_0 = 1.$

- $$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = \frac{1}{4}\left(1 - e^{-\phi(Y;\xi)}\right) \xrightarrow{\xi \gg 1} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = \frac{1}{4}$$

$$t = \zeta.$$

$t \le x$:

- $\phi_1(z) = \frac{1}{8}\zeta^2 + \frac{1}{2}\left(e^{\phi_0} - 1\right)\zeta + \phi_0$

$t \ge x$:

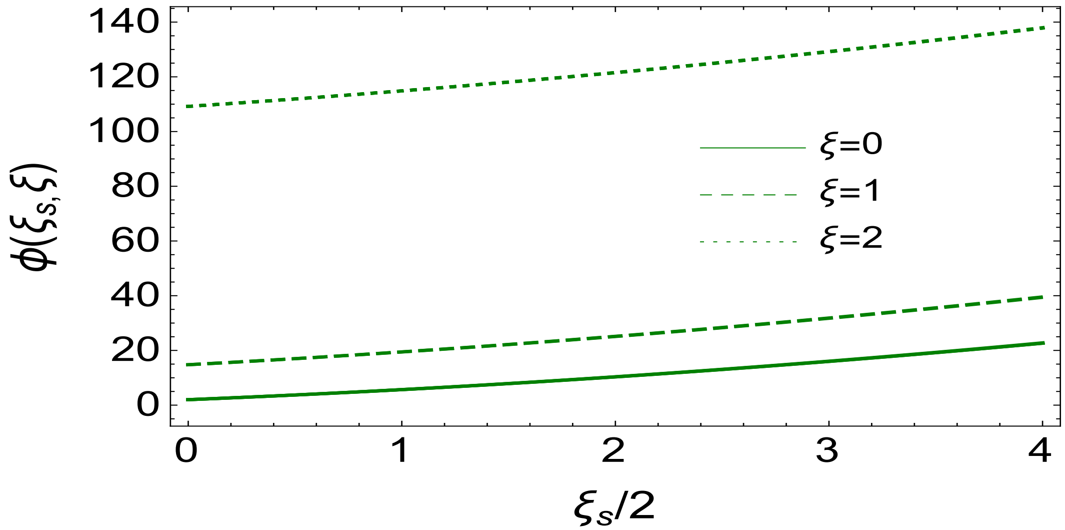
- $\phi_2(\xi_s, \xi) = \frac{1}{4}\xi_s\,\xi + F_1(\xi_s) + F_2(\xi)$

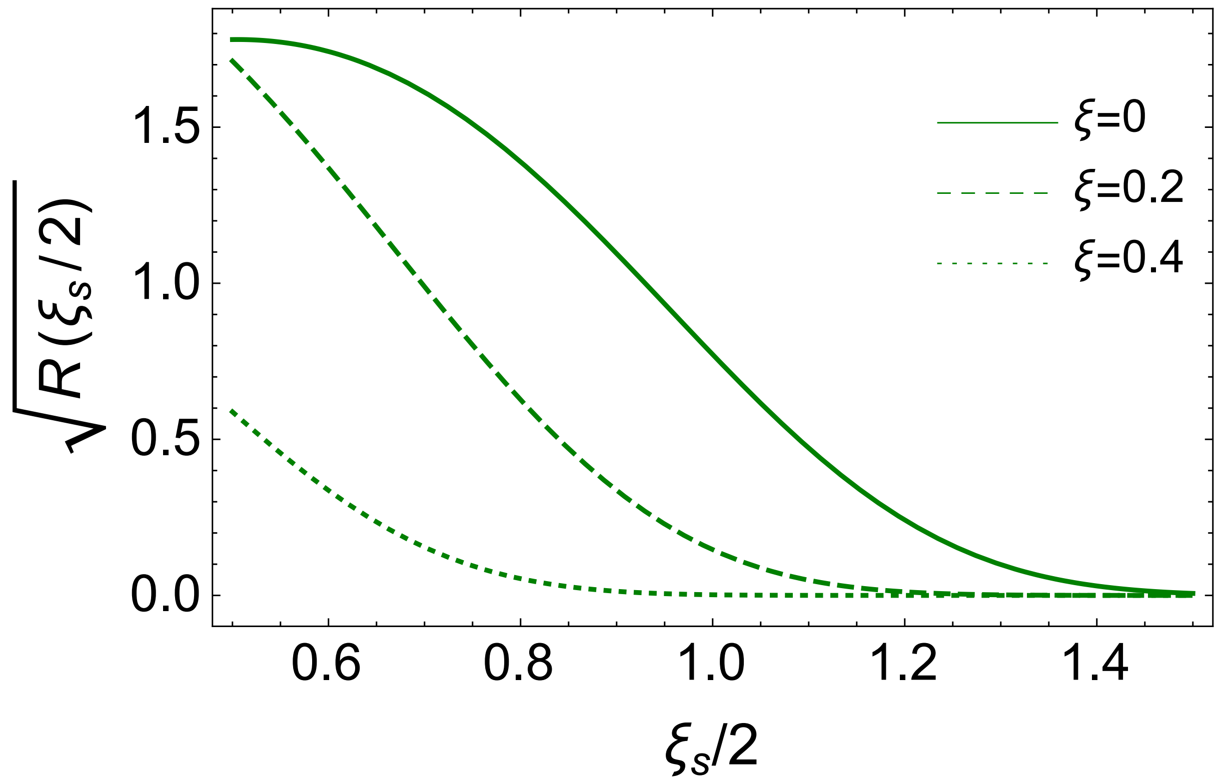
$t = x\ (\xi = 0)$:

- $\phi_1(\xi = 0) = \phi_2(\xi = 0)$ **and** $\phi_2(\xi_s = 0) = \phi_0 e^{\xi}$

Solution:

- $\phi_2(z, \xi) = \zeta^2/8 - \xi^2/8 + \phi_0\,e^{\xi} + \frac{1}{2}\left(e^{\phi_0} - 1\right)\xi_s$





$\sqrt{R(\xi_s/2)}$
1.5
1.0
0.5
0.0
0.6
0.8
1.0
1.2
1.4
$\xi_s/2$
ξ=0
ξ=0.2
ξ=0.4

Conclusions

- Selection of the events with different multiplicities of produced particles lead to the violation of $\phi \to \pi - \phi$ symmetry.
- For $Q^2 > Q_s^2\,(A; Y_{\min}; b)$ this violation is so large that we can neglect the symmetry: $v_{2n} \approx v_{2n-1}$;
- For $Q^2 < Q_s^2\,(A; Y_{\min}; b)$ this violation is negligibly small: $v_{2n-1} = 0$;